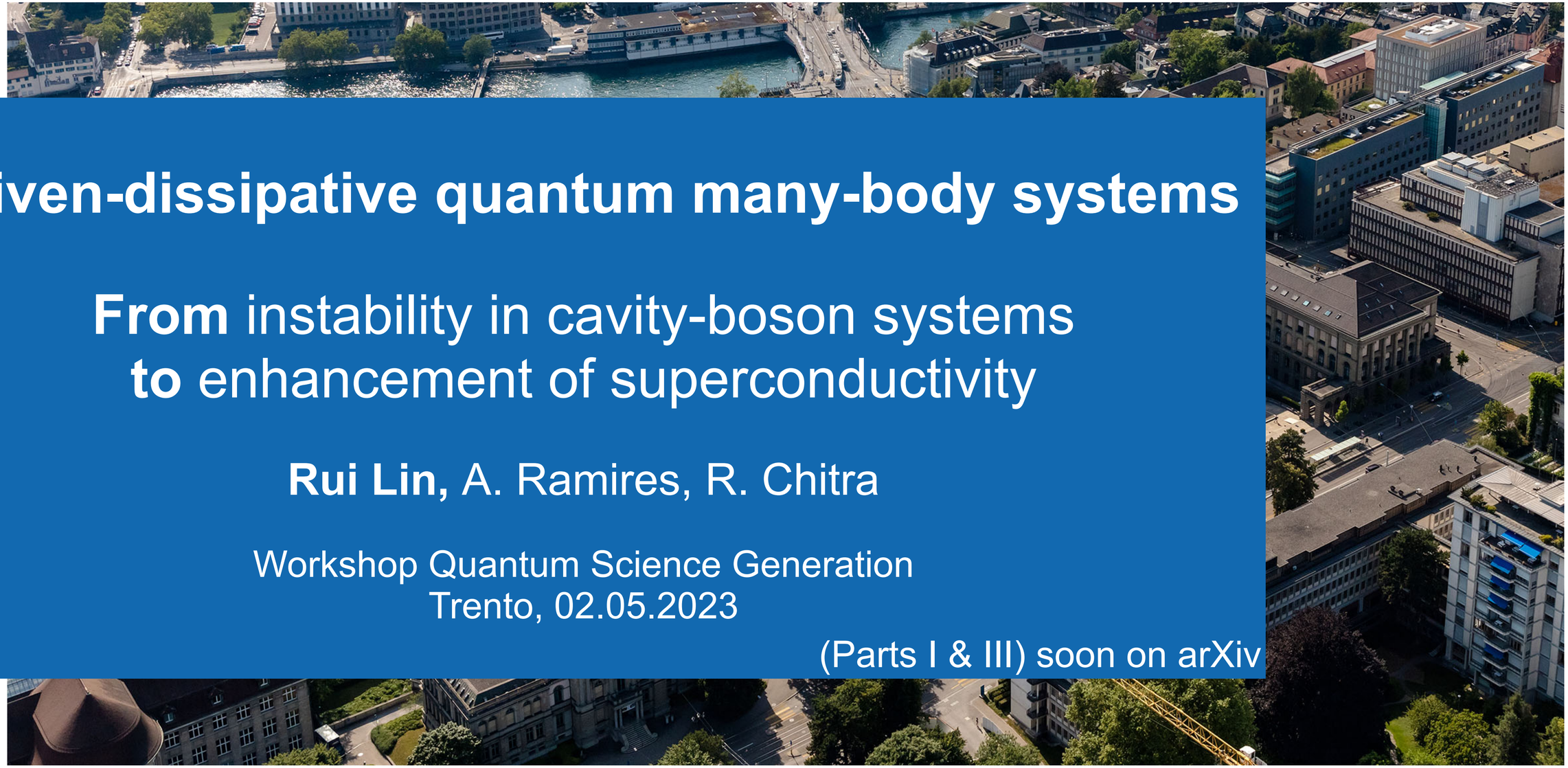
ETH zürich



Driven-dissipative quantum many-body systems

From instability in cavity-boson systems **to** enhancement of superconductivity

Rui Lin, A. Ramires, R. Chitra

Workshop Quantum Science Generation
Trento, 02.05.2023

(Parts I & III) soon on arXiv

I DRIVEN & DISSIPATIVE SYSTEMS

Introduction

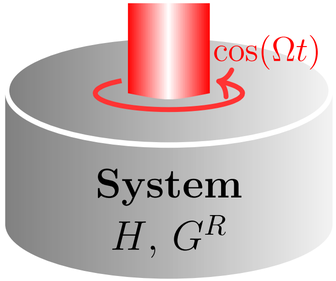
DRIVEN

Cond. matter & Quantum gas

Floquet engineering

- Effective strobo. Hamiltonian
- Synthetic dimension

...



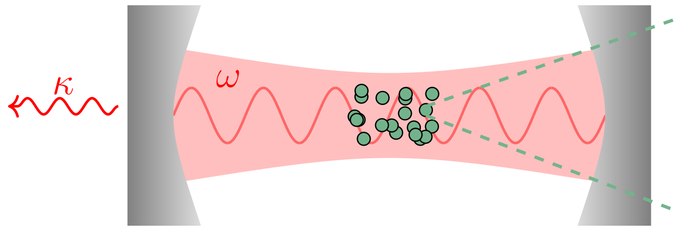
DISSIPATIVE

Quantum optical

Lindblad form

- Third quantization
- Weak symmetry

...



Introduction

DRIVEN

Cond. matter &

Floquet engineer

Effective strobo.

Synthetic dimens

DISSIPATIVE

Quan ptical

orm

uantization

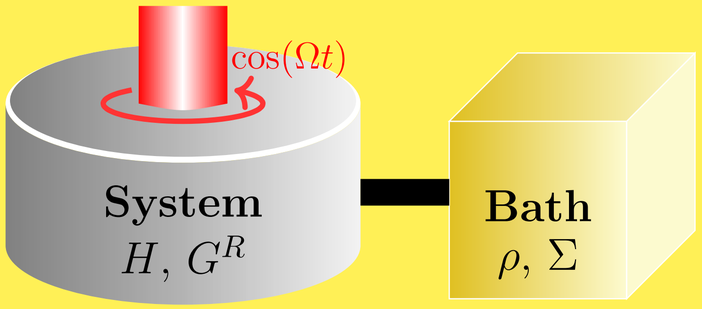
ymmetry

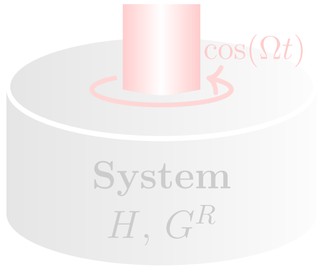
Closed

Co-rotating

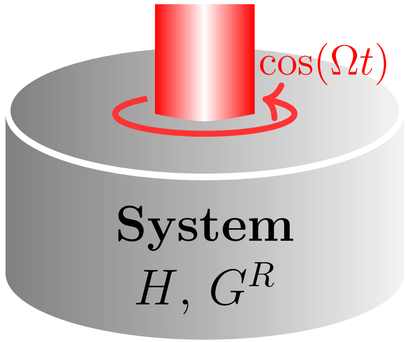
?

Unified description





Driven quantum systems — Floquet



Drive: $H(t) = \sum_{n\in\mathbb{Z}} H_n e^{in\Omega t}$

Floquet matrix

$$\mathbf{H} = \begin{pmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & H_0 + 2\Omega & H_1 & 0 & 0 & 0 & \cdots \\ \cdots & H_{-1} & H_0 + \Omega & H_1 & 0 & 0 & \cdots \\ \cdots & 0 & H_{-1} & H_0 & H_1 & 0 & \cdots \\ \cdots & 0 & 0 & H_{-1} & H_0 - \Omega & H_1 & \cdots \\ \cdots & 0 & 0 & 0 & H_{-1} & H_0 - 2\Omega & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \end{pmatrix}$$

"Lattice sites"

+

"Hoppings"

=

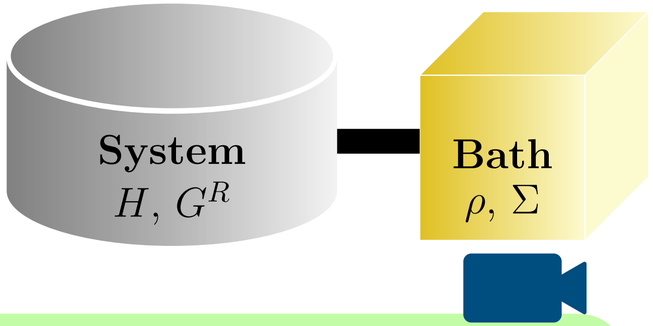
Synthetic dimension

PRX **4**, 031027 (2014), Goldman, Dalibard
PRL **106**, 236401 (2011) Tsuji, Oka, Werner, Aoki

Dissipative quantum systems — Keldysh

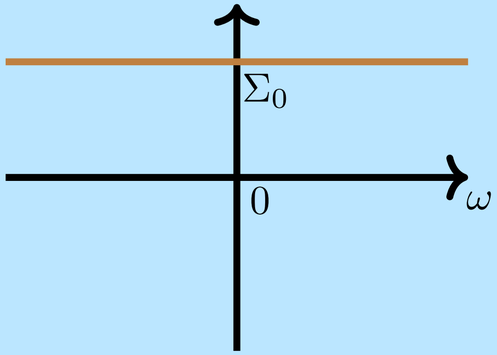
Dissipation: $$\sum_{m,n} W_{m,n}(b_m^\dagger c_n + c_n^\dagger b_m) + \sum_m \xi_m b_m^\dagger b_m$$

c_n: system
b_m: bath

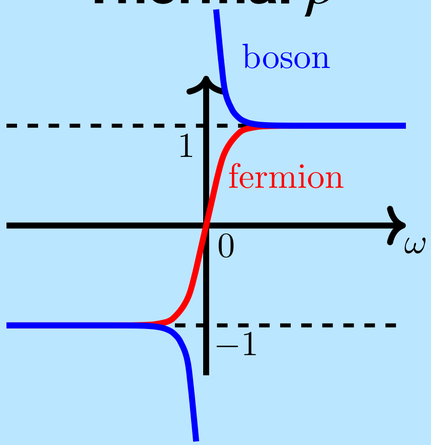


Thermal bath

Spectral Σ (Markovian)



Thermal ρ



Fluctuation-Dissipation relation

$$G^K = \rho(G^R - G^A)$$

Thermal Spectral

$$G^{R/A} = (\omega \pm i\Sigma - H)^{-1}$$

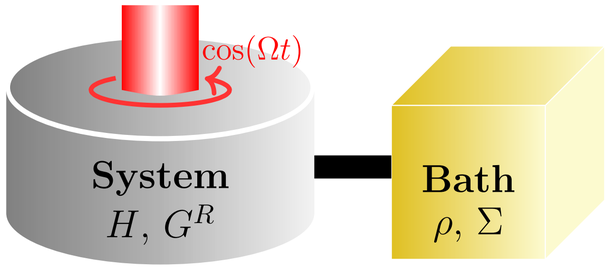
Rev. Mod. Phys. **86**, 779 (2014) Aoki, Tsuji, et al.
Rep. Prog. Phys. **79**, 096001 (2016), Sieberer, Buchhold, Diehl

Driven-Dissipative quantum systems

Fluc-Diss: $\mathbf{G}^K = \mathbf{G}^R \underline{\rho} - \underline{\rho}\mathbf{G}^A$ $\qquad \mathbf{G}^{R/A} = (\omega \pm i\Sigma - \mathbf{H})^{-1}$

Intrinsic relative rotation:

Generally $[\mathbf{H}, \underline{\rho}] \neq 0$ when $H_{\pm 1} \neq 0$



Floquet structure in bath

$$\mathbf{H} = \begin{pmatrix} & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & \boxed{H_0 + 2\Omega} & H_1 & 0 & 0 & 0 & \\ \cdots & H_{-1} & H_0 + \Omega & H_1 & 0 & 0 & \cdots \\ \cdots & 0 & H_{-1} & H_0 & H_1 & 0 & \cdots \\ \cdots & 0 & 0 & H_{-1} & H_0 - \Omega & H_1 & \cdots \\ \cdots & 0 & 0 & 0 & H_{-1} & H_0 - 2\Omega & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \end{pmatrix}$$

$$\underline{\rho}(\omega) = \begin{pmatrix} & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & \boxed{\rho(\omega + 2\Omega)} & 0 & 0 & 0 & 0 & \cdots \\ \cdots & 0 & \rho(\omega+\Omega) & 0 & 0 & 0 & \cdots \\ \cdots & 0 & 0 & \rho(\omega) & 0 & 0 & \cdots \\ \cdots & 0 & 0 & 0 & \rho(\omega-\Omega) & 0 & \cdots \\ \cdots & 0 & 0 & 0 & 0 & \rho(\omega - 2\Omega) & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \end{pmatrix}$$

Condensed Matter Field Theory (2010), Altland, Simons

Driven-Dissipative quantum systems

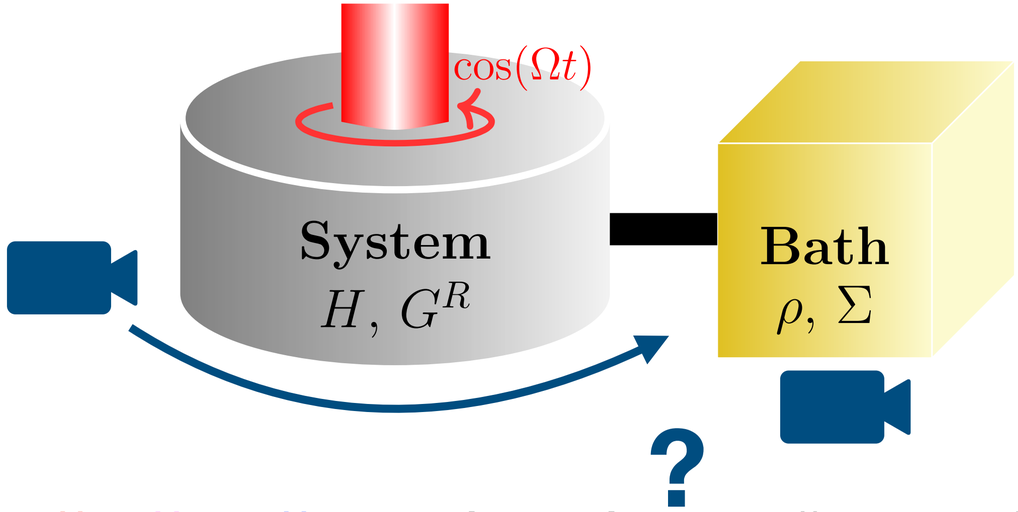
Fluc-Diss: $\mathbf{G}^K = \mathbf{G}^R \underline{\rho} - \underline{\rho} \mathbf{G}^A$ $\qquad \mathbf{G}^{R/A} = (\omega \pm i\Sigma - \mathbf{H})^{-1}$

Intrinsic relative rotation:

Generally $[\mathbf{H}, \underline{\rho}] \neq 0$ when $H_{\pm 1} \neq 0$

Floquet struct

$$\mathbf{H} = \begin{pmatrix} \vdots & \vdots & & & & & \\ \cdots & H_0 + 2\Omega & H_1 & & & & \\ \cdots & H_{-1} & H_0 + \Omega & & & & \\ \cdots & 0 & H_{-1} & & & & \\ \cdots & 0 & 0 & H_{-1} & H_0 - \Omega & H_1 & \cdots \\ \cdots & 0 & 0 & 0 & H_{-1} & H_0 - 2\Omega & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \end{pmatrix} \qquad \begin{pmatrix} \vdots & \vdots & \vdots & & & & \\ 0 & 0 & 0 & & & & \cdots \\ 0 & 0 & 0 & & & & \cdots \\ (\omega) & 0 & 0 & & & & \cdots \\ 0 & \rho(\omega - \Omega) & 0 & & & & \cdots \\ 0 & 0 & \rho(\omega - 2\Omega) & & & & \cdots \\ \vdots & \vdots & \vdots & & & & \end{pmatrix}$$



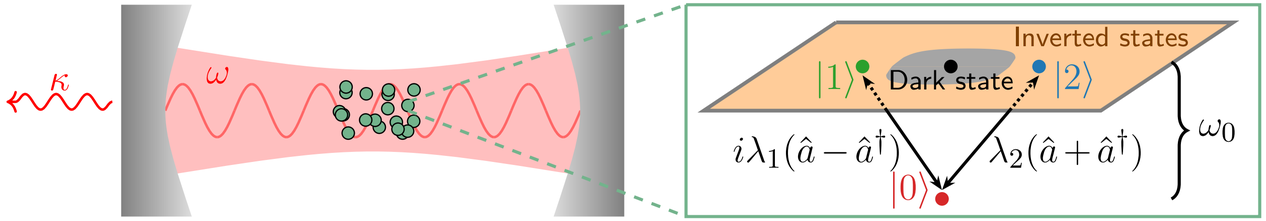
Condensed Matter Field Theory (2010), Altland, Simons

II CAVITY-BOSON SYSTEMS

within LINDBLAD FORM

A V-shaped cavity-boson systems

$$\partial_t \rho = -i[H,\rho] + \kappa\left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right)$$ κ Disspation



Ground state v.s. steady state

How?

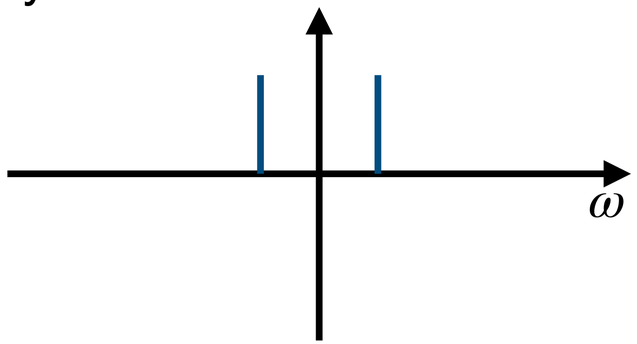
Invalid energy minimization arguments

PRL **128**, 153601 (2022) **RL** et al.
PRL **128**, 143602 (2022) Rosa-Medina et al.

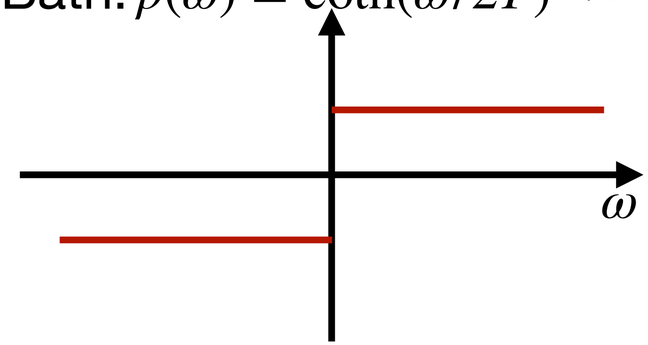
Lindblad form in Floquet-Keldysh

Static system

System: $\mathscr{A} = -\,\mathrm{Im}G^R$



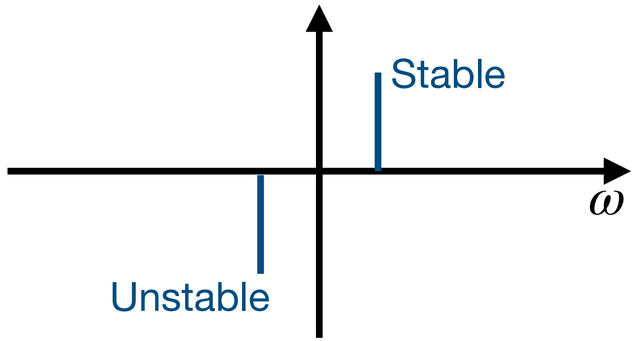
Bath: $\rho(\omega) = \coth(\omega/2T) \overset{T\to 0}{\sim} \mathrm{sgn}(\omega)$



Fluctuation-dissipation relation

$$G^K = 2\rho(\omega)\mathrm{Im}G^R$$

System-bath: $\mathscr{R} = -\,\mathrm{Im}G^K$

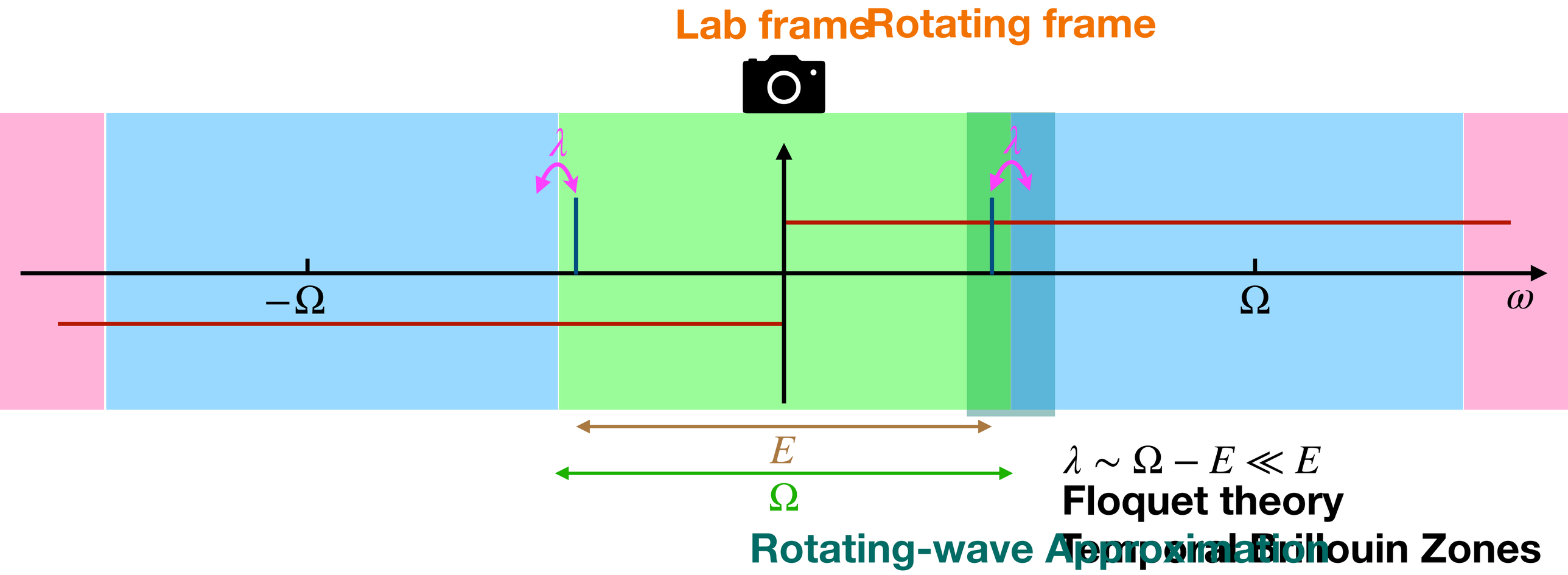


$\mathscr{A}$: Spectral function

$\mathscr{R}$: Response function

ρ: Thermal distribution

Lindblad form in Floquet-Keldysh



$\lambda \sim \Omega - E \ll E$

Floquet theory

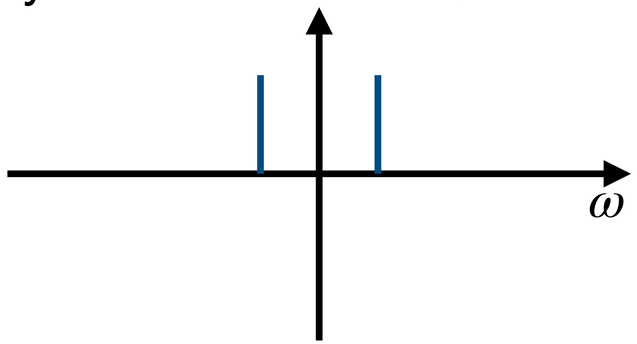
Temporal Brillouin Zones

Rotating-wave Approximation

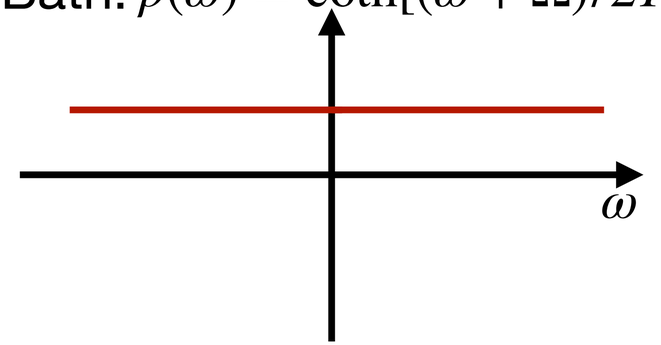
Driven-dissipative stabilization

Driven system

System: $\mathscr{A} = -\,\mathrm{Im}G^R$



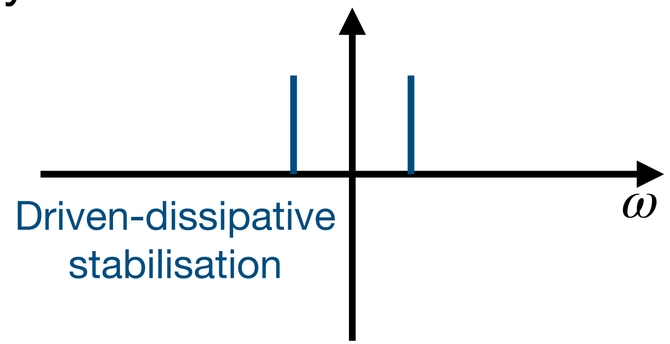
Bath: $\rho(\omega) = \coth[(\omega + \Omega)/2T] \overset{\Omega\to\infty}{\sim} 1$



Fluctuation-dissipation relation

$$\mathrm{Im}G^K = 2\rho(\omega)\mathrm{Im}G^R$$

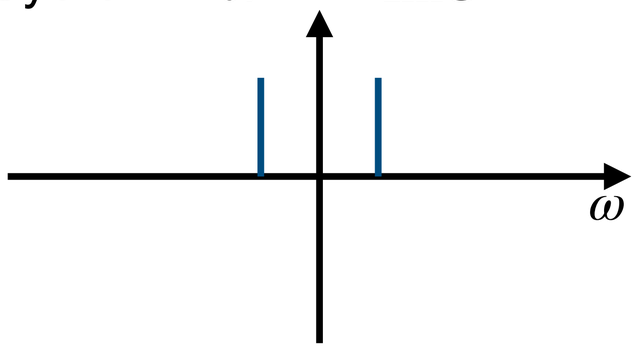
System-bath: $\mathscr{R} = -\,\mathrm{Im}G^K$



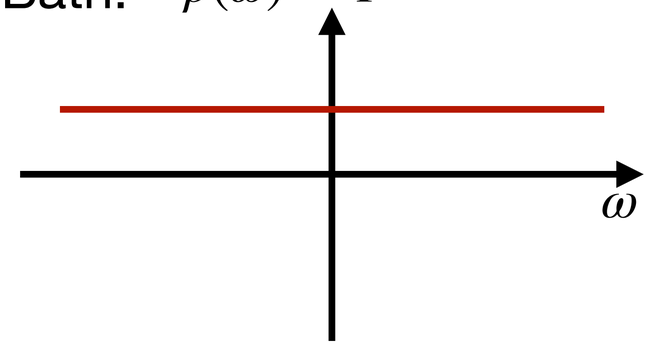
Driven-dissipative destabilization

Driven particle-like ($a^\dagger$) system

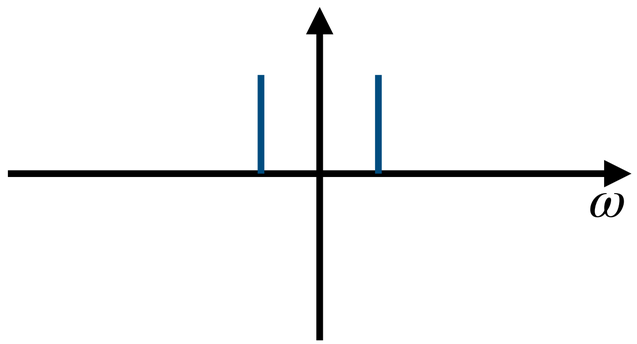
System: $\mathscr{A} = -\,\mathrm{Im}G^R$



Bath: $\rho(\omega) \sim 1$

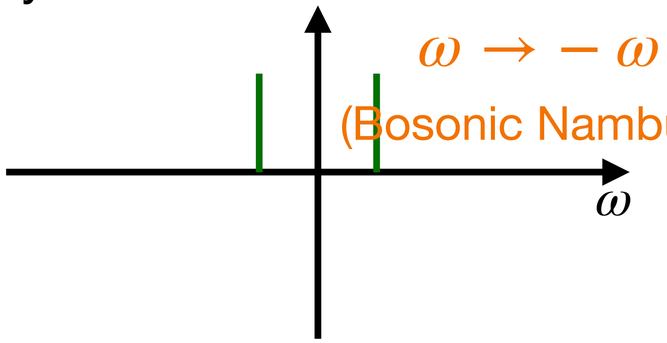


System-bath: $\mathscr{R} = -\,\mathrm{Im}G^K$



Driven hole-like (a) system

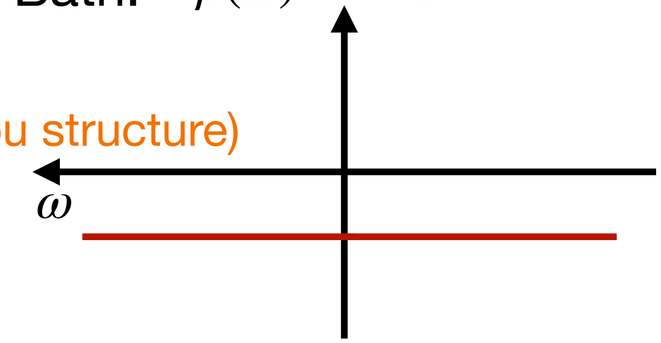
System: $\mathscr{A} = -\,\mathrm{Im}G^R$



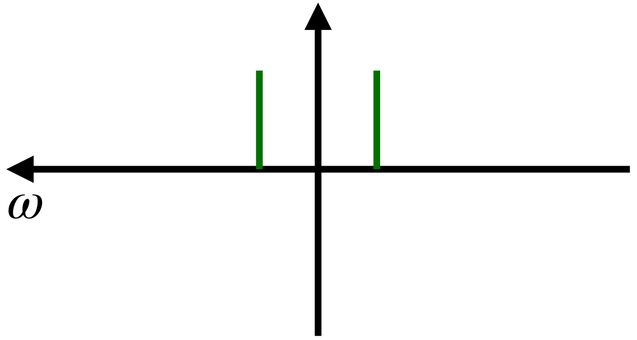
$\omega \to -\omega$

(Bosonic Nambu structure)

Bath: $\rho(\omega) \sim -1$

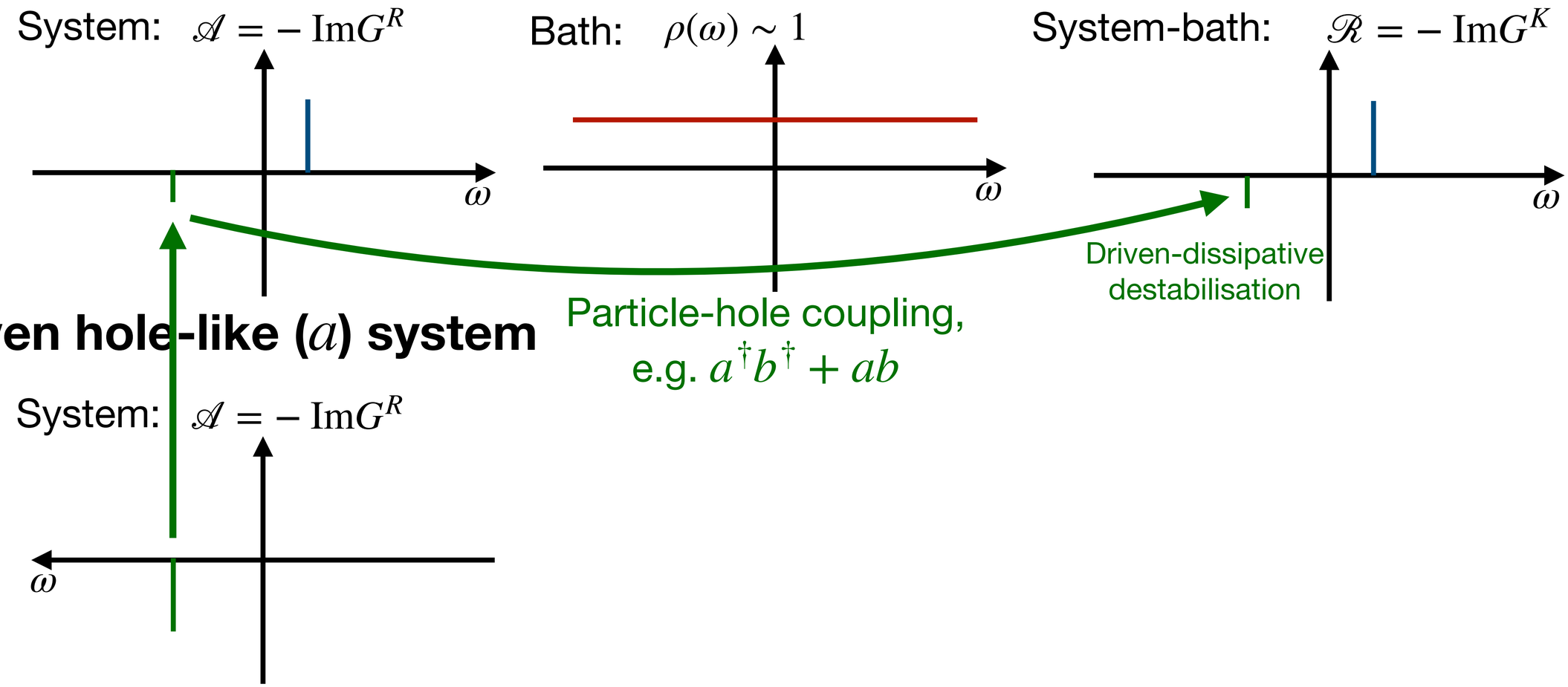


System-bath: $\mathscr{R} = -\,\mathrm{Im}G^K$



Driven-dissipative destabilization

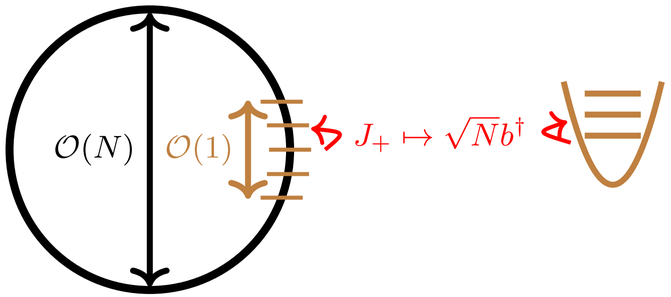
Driven particle-like ($a^\dagger$) system



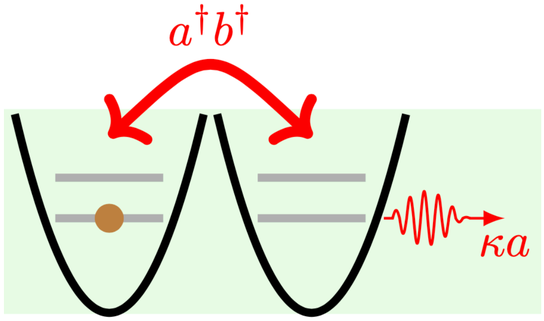
Driven hole-like (a) system

Driven-dissipative destabilization

The Holstein-Primakoff bosonic modes…



manifest **driven-dissipative instability**



with **particle-hole (counter-rotating)** couplings.

$$H = \omega_c a^\dagger a + \omega_0 b^\dagger b + \lambda(a^\dagger b^\dagger + ba)$$

III SUPERCONDUCTORS

beyond LINDBLAD FORM

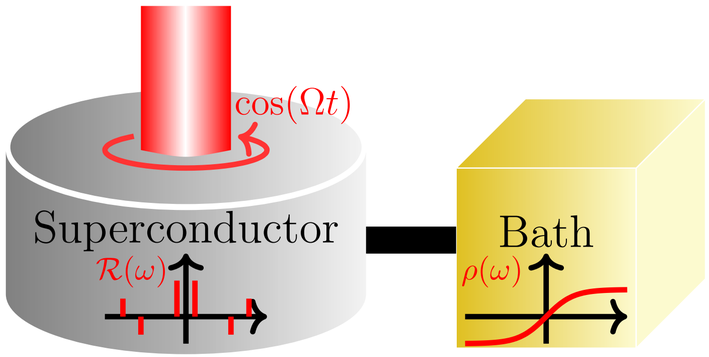
Gap equation of superconductors

Mean-field gap equation:

$$\Delta = \frac{ig}{2}\int \frac{d\omega}{2\pi}\mathscr{R}(\omega, \Delta)$$ (with Floquet structure)

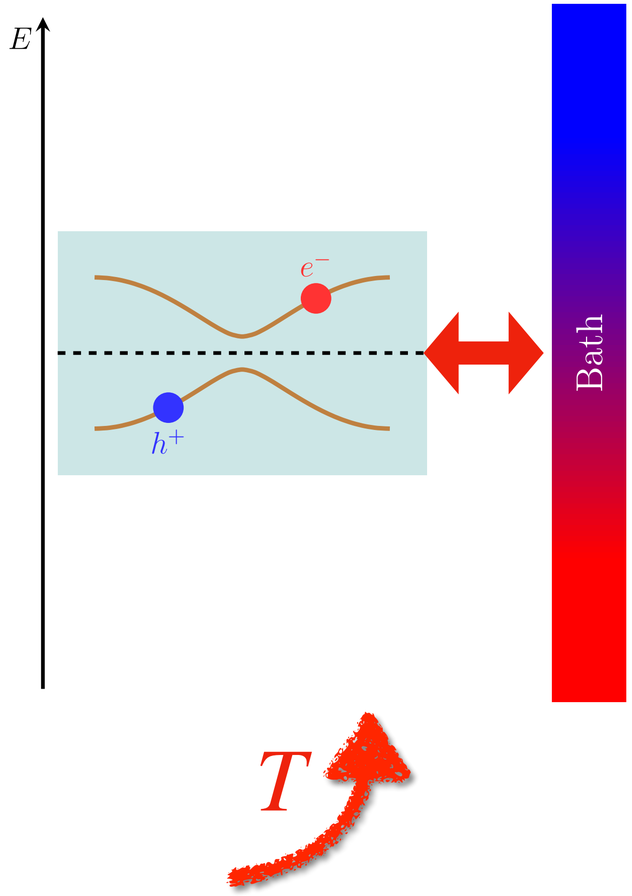
$\Downarrow$ Static system

$$\Delta = ig\sum_{\mathbf{k}} \frac{\Delta}{\sqrt{\epsilon_{\mathbf{k}}^2 + \Delta^2}} \tanh\left(\frac{\sqrt{\epsilon_{\mathbf{k}}^2 + \Delta^2}}{2T}\right)$$

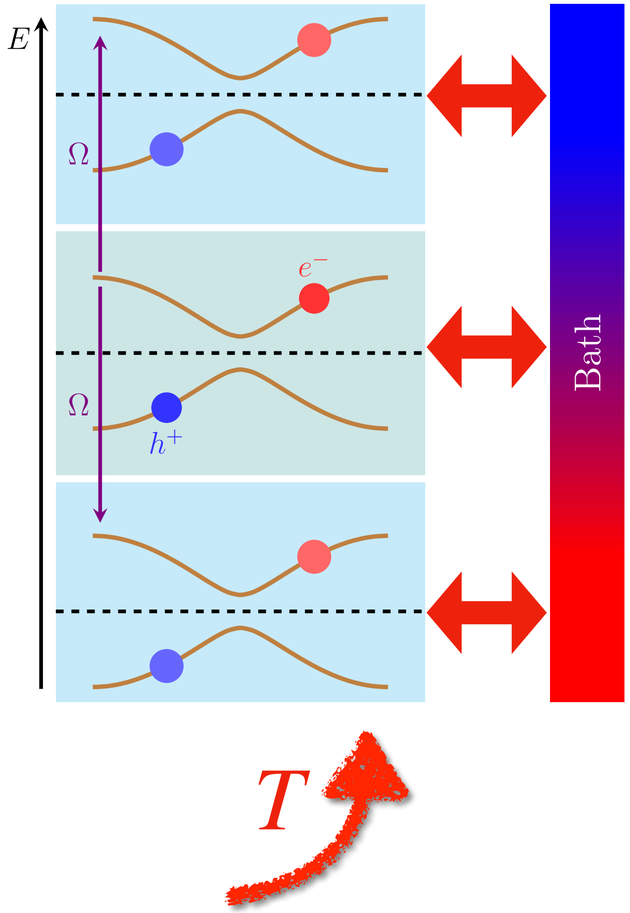


Enhancement of superconductivity

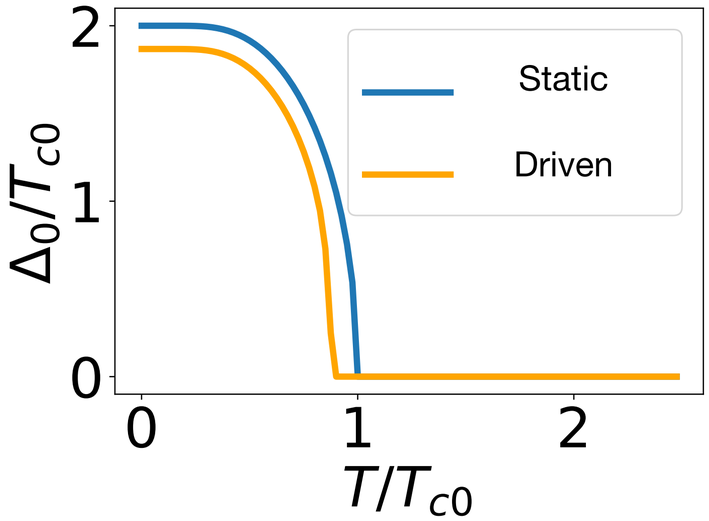
$H_D = 0$



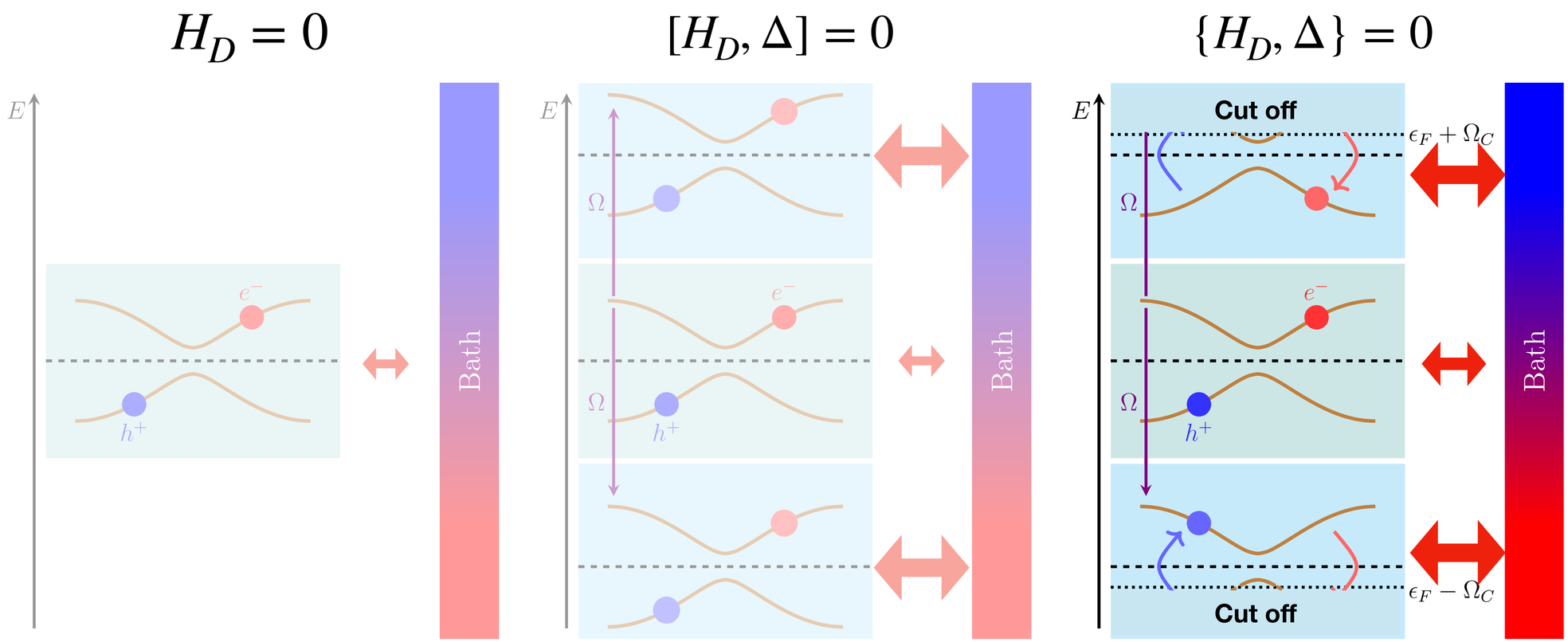
$H_D = \tau_0 \cos(\Omega t)$



Time-averaged Gap

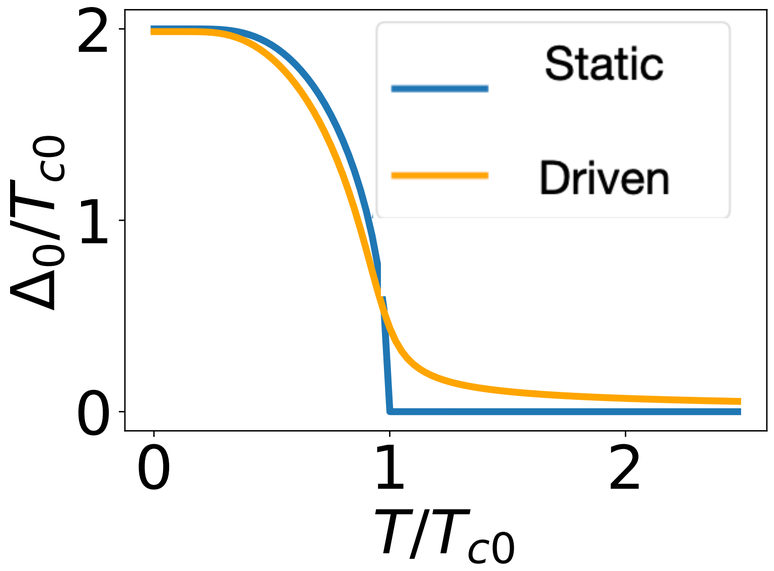


Enhancement of superconductivity

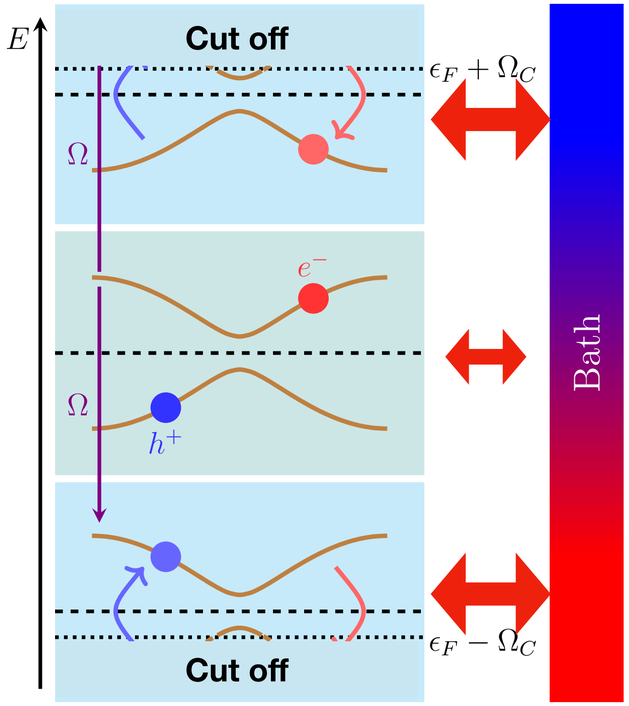


Enhancement of superconductivity

Time-averaged Gap



$\{H_D, \Delta\} = 0$



$\Delta_0(T) \sim 1/T$ **until** $T_c \approx \dfrac{a^2}{8\Omega^2}\dfrac{\Omega}{\delta\Omega}T_{c0}$

Ω: Driving freq., $\delta\Omega$: Cutoff-drive detuning

Conclusions and Outlook

Floquet-Keldysh $\mathbf{G}^K = \mathbf{G}^R \underline{\rho} - \underline{\rho} \mathbf{G}^A$ $[\mathbf{G}^R, \underline{\rho}] \neq 0$

- **Field theory** for systems described by **Lindblad form**
- **Framework** for more **general driven-dissipative** systems
- **Floquet** $\rightarrow$ **Driven-dissipative engineering**
  with a focus on **particle-hole coupling?**