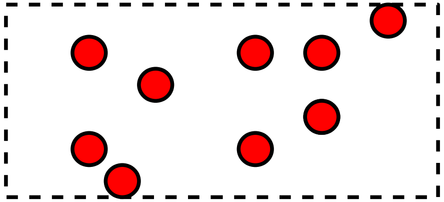
Quantum kinetics of quenched two-dimensional Bose superfluids

Clément Duval

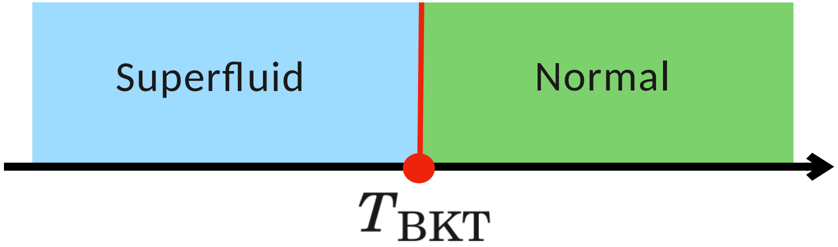
Quantum Science Generation workshop
Trento, May 2023

Uniform 2D Bose gas in equilibrium



$$\hat{H} = \int d^2\boldsymbol{r}\Big(-\frac{1}{2m}\hat{\psi}^\dagger\Delta_{\boldsymbol{r}}\hat{\psi} + \frac{g}{2}\hat{\psi}^\dagger\hat{\psi}^\dagger\hat{\psi}\hat{\psi}\Big)$$

$$g > 0,\ T \geq 0$$



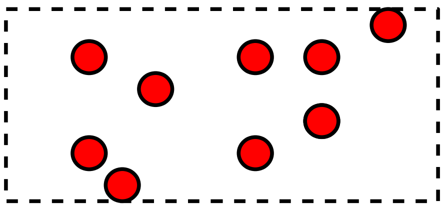
- At very low T → No condensation, but quasi-long range order → Superfluidity $\quad \langle\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r}+\Delta\mathbf{r})\rangle \sim (1/\Delta\mathbf{r})^\alpha$

  Hohenberg, Phys. Rev. 158, 383 (1967)

- At large T → Exponential decay of spatial correlations → ``Normal'' phase $\quad \langle\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r}+\Delta\mathbf{r})\rangle \sim \exp(-\Delta\mathbf{r}/l)$

  Kosterlitz, Thouless, J. Phys. C: Solid State Phys., 6 1181 (1973)

Quench protocole



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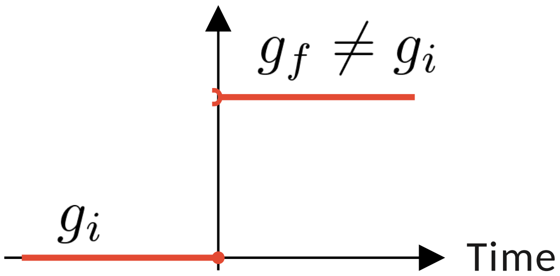
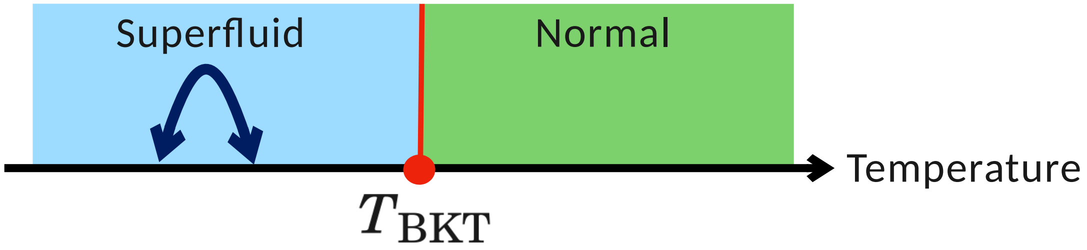
$$g > 0,\ T \geq 0$$

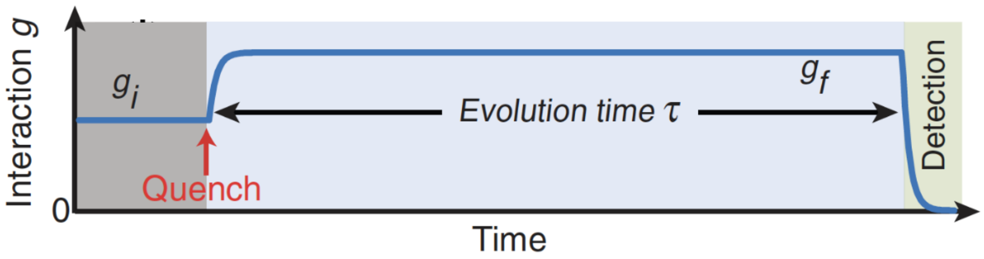
→ What happens if we push the system **out-of-equilibrium**?

Quench g inside the superfluid phase:



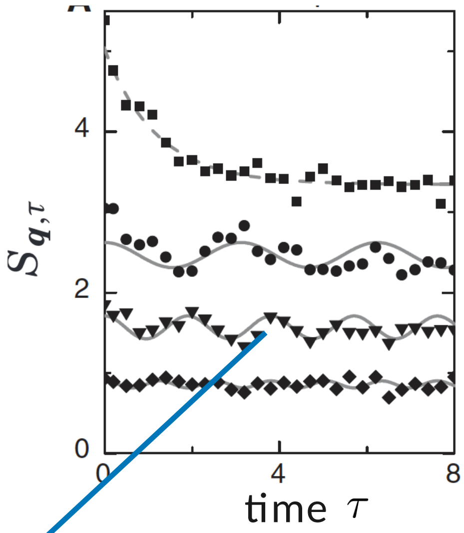


Typical experimental setup & observations



Hung, Gurarie, Chin, Science **341**, 1213 (2013)

[optical version: Steinhauer et al., Nature Com **13**, 2890 (2022)]



Structure factor
$S_{\boldsymbol{q},\tau} \equiv \langle \delta\hat{\rho}_{\boldsymbol{q},\tau} \delta\hat{\rho}_{-\boldsymbol{q},\tau} \rangle$

Undamped oscillations

$\rightarrow$ Interferences between post-quench quasiparticles

Quantum hydrodynamic formalism

2D uniform Bose gas $\hat{H} = \int d^2\boldsymbol{r}\Big(-\frac{1}{2m}\hat{\psi}^\dagger \Delta_{\boldsymbol{r}}\hat{\psi} + \frac{g}{2}\hat{\psi}^\dagger\hat{\psi}^\dagger\hat{\psi}\hat{\psi}\Big)$

- Local density and phase $\hat{\psi}(\boldsymbol{r}) = e^{i\hat{\theta}(\boldsymbol{r})}\sqrt{\hat{\rho}(\boldsymbol{r})}$ Popov 1972, 1983
- Low $T \rightarrow$ weak density fluctuations $\hat{\rho}(\boldsymbol{r}) = \rho_0 + \delta\hat{\rho}(\boldsymbol{r})$ $\delta\hat{\rho} \ll \rho_0$

3rd-order expansion

$$\rightarrow \boxed{\hat{H} \simeq \int d^2\boldsymbol{r}\Big[\underbrace{\frac{\rho_0}{2m}(\nabla_{\boldsymbol{r}}\hat{\theta})^2 + \frac{g}{2}(\delta\hat{\rho})^2 + \frac{1}{8m\rho_0}(\nabla_{\boldsymbol{r}}\delta\hat{\rho})^2}_{\text{quadratic}} + \underbrace{\frac{1}{2m}(\nabla_{\boldsymbol{r}}\hat{\theta})\delta\hat{\rho}(\nabla_{\boldsymbol{r}}\hat{\theta})}_{\text{cubic}}\Big]}$$

Quantum hydrodynamic formalism

2D uniform Bose gas $\hat{H} = \int d^2\boldsymbol{r}\Big(-\frac{1}{2m}\hat{\psi}^\dagger\Delta_{\boldsymbol{r}}\hat{\psi} + \frac{g}{2}\hat{\psi}^\dagger\hat{\psi}^\dagger\hat{\psi}\hat{\psi}\Big)$

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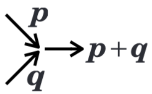
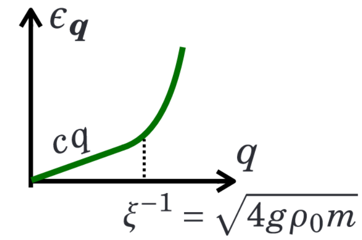
3rd-order expansion

$$\longrightarrow \boxed{\hat{H} = \int_{\boldsymbol{q}} \epsilon_{\boldsymbol{q}}\, a^\dagger_{\boldsymbol{q}} a_{\boldsymbol{q}} + \int_{\boldsymbol{p},\boldsymbol{q}} \Lambda_{\boldsymbol{p},\boldsymbol{q}} \left(\hat{a}_{\boldsymbol{p}}\hat{a}_{\boldsymbol{q}}\hat{a}^\dagger_{\boldsymbol{p}+\boldsymbol{q}} + \mathrm{h.c.}\right)}$$

Free-theory $\longrightarrow$ independent quasi-particles (low q phonons)

Cubic interaction $\longrightarrow$ three-particle processes





Coherent dynamics

- **Coherent** (prethermal) dynamics: $\hat{H} \simeq \int_{\boldsymbol{q}} \epsilon_{\boldsymbol{q}} a^{\dagger}_{\boldsymbol{q}} a_{\boldsymbol{q}}$

$$i\hbar \frac{d\hat{a}}{d\tau} = [\hat{a}, H] \longrightarrow \boxed{\hat{a}_{\boldsymbol{q},\tau} = \hat{a}_{\boldsymbol{q},0} \exp(-i\epsilon_{\boldsymbol{q}} \tau)}$$

$\longrightarrow$ **Coherent dynamics** with characteristic time scale $\tau_{\phi} \sim \epsilon^{-1}_{q=1/\xi} \sim (g\rho_0)^{-1}$

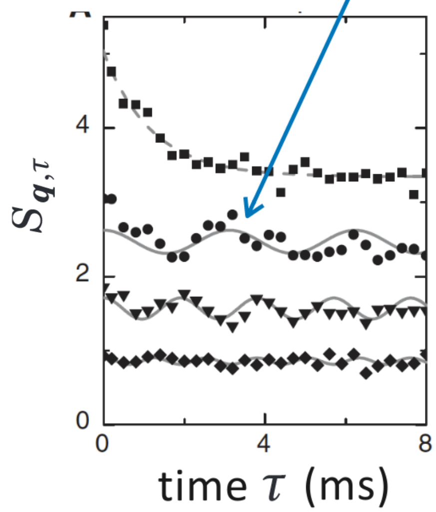
$\longrightarrow$ QP momentum distributions **time independent**

$$\langle \hat{a}^{\dagger}_{\boldsymbol{q},\tau} \hat{a}_{\boldsymbol{q},\tau} \rangle \equiv n_{\boldsymbol{q},\tau} = n_{\boldsymbol{q},0}$$

$$\langle \hat{a}_{\boldsymbol{q},\tau} \hat{a}_{-\boldsymbol{q},\tau} \rangle = \underbrace{|\langle \hat{a}_{\boldsymbol{q},\tau} \hat{a}_{-\boldsymbol{q},\tau} \rangle|}_{\equiv m_{\boldsymbol{q},t} = m_{\boldsymbol{q},0}} e^{-2i\epsilon_{\boldsymbol{q}} \tau}$$

post-quench value

Undamped oscillations



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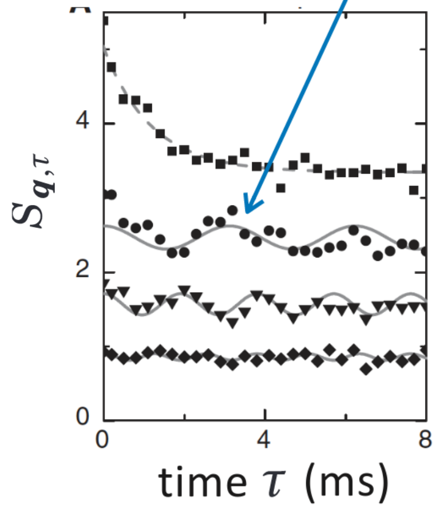
$$\longrightarrow S_{\boldsymbol{q},\tau} = \frac{E_{\boldsymbol{q}}}{\epsilon_{\boldsymbol{q}}}[2n_{\boldsymbol{q},0} + 1 + 2\cos(2\epsilon_{\boldsymbol{q}}\tau) m_{\boldsymbol{q},0}]$$

$$S_{\boldsymbol{q},\tau} = \frac{E_{\boldsymbol{q}}}{\epsilon_{\boldsymbol{q}}} \coth\left[\frac{\epsilon^i_{\boldsymbol{q}}}{2k_B T_i}\right]\left[1 + \underbrace{\frac{\epsilon^i_{\boldsymbol{q}} - \epsilon_{\boldsymbol{q}}}{\epsilon_{\boldsymbol{q}}} \sin^2(\epsilon_{\boldsymbol{q}}\tau)}\right]$$

pre-quench temperature

interference between
QP emitted at the quench

Undamped oscillations



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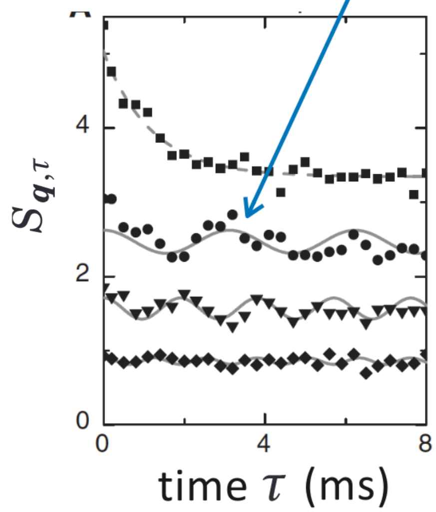
pre-quench temperature

Undamped oscillations



$\rightarrow$ What if the quasiparticles are not independent?

Relaxation of phonons

Include cubic interactions at **low energy**

$$\hat{H} = \int_{\boldsymbol{q}} \epsilon_{\boldsymbol{q}} a^{\dagger}_{\boldsymbol{q}} a_{\boldsymbol{q}} + \int_{\boldsymbol{p},\boldsymbol{q}} \Lambda_{\boldsymbol{p},\boldsymbol{q}} \left(\hat{a}_{\boldsymbol{p}} \hat{a}_{\boldsymbol{q}} \hat{a}^{\dagger}_{\boldsymbol{p}+\boldsymbol{q}} + \mathrm{h.c.} \right) \qquad \epsilon_{\boldsymbol{q}} \simeq cq \qquad \Lambda_{\boldsymbol{q},\boldsymbol{p}} \simeq \frac{3}{4m} \sqrt{\frac{g\rho_0}{2c}} \sqrt{pq|\boldsymbol{p}+\boldsymbol{q}|}$$

$\longrightarrow$ time evolution of phonon distributions?

$$n_{\boldsymbol{q},\tau} = \langle \hat{a}^{\dagger}_{\boldsymbol{q},\tau} \hat{a}_{\boldsymbol{q},\tau} \rangle \qquad m_{\boldsymbol{q},\tau} = |\langle \hat{a}_{\boldsymbol{q},\tau} \hat{a}_{-\boldsymbol{q},\tau} \rangle|$$

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Theoretical approaches

EOM hierarchy Bertini, Essler, Groha, Robinson, PRL **115**, 180601 (2015)
Regemortel, Kurkjian, Wouters, Carusotto, PRA **98**, 053612 (2018)

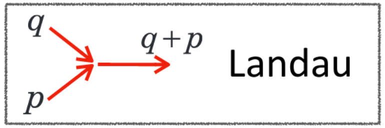
Keldysh field theory Buchhold, Diehl, EPJD **69**, 224 (2015), PRA
Buchhold, Heyl, Diehl, PRA **94**, 013601 (2016)

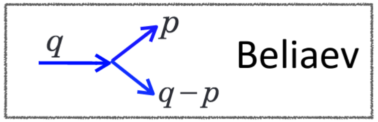
Quantum kinetic equations

- Phonon momentum distribution

$$\partial_\tau n_{\boldsymbol{q},\tau} = 2\int_0^\infty dp\, \mathcal{K}^L_{\boldsymbol{p},\boldsymbol{q}} \left[n_{\boldsymbol{p}+\boldsymbol{q}} \left(n_{\boldsymbol{p}} + n_{\boldsymbol{q}} + 1 \right) - n_{\boldsymbol{p}} n_{\boldsymbol{q}} \right]$$

$$+2\int_0^q dp\, \mathcal{K}^B_{\boldsymbol{p},\boldsymbol{q}} \left[n_{\boldsymbol{p}} n_{\boldsymbol{q}-\boldsymbol{p}} - n_{\boldsymbol{q}} \left(n_{\boldsymbol{p}} + n_{\boldsymbol{q}-\boldsymbol{p}} + 1 \right) \right]$$





with energy conservation $\partial_\tau \int_{\boldsymbol{q}} c q n_{\boldsymbol{q},\tau} = 0$

- Long-time properties

thermal equilibrium $n_{\boldsymbol{q},\infty} = [\exp(cq/T) - 1]^{-1}$

near-equilibrium dynamics $\partial_\tau \delta n_{\boldsymbol{q},\tau} \simeq -2\gamma_{\boldsymbol{q}} \delta n_{\boldsymbol{q},\tau}$ $\quad \delta n_{\boldsymbol{q},\tau} = n_{\boldsymbol{q},\tau} - n_{\boldsymbol{q},\infty}$

Landau $\gamma^L_{\boldsymbol{q}} = \dfrac{\sqrt{3}\pi}{8\rho_0 c} q T^2$

Beliaev $\gamma^B_{\boldsymbol{q}} = \dfrac{\sqrt{3}\pi}{32\rho_0 c} q^3$

CD, Cherroret, PRA **107**, 043305 (2023)

Chung, Bhattacherie, NJP **11**, 123012 (2009)

Quantum kinetic equations

- Phonon anomalous distribution

$$\partial_\tau m_{\boldsymbol{q},\tau} = 2\int_0^\infty dp\,\mathcal{K}^L_{\boldsymbol{p},\boldsymbol{q}}\left(n_{\boldsymbol{p}+\boldsymbol{q}}m_{\boldsymbol{q}} + m_{\boldsymbol{p}}m_{\boldsymbol{p}+\boldsymbol{q}} - n_{\boldsymbol{p}}m_{\boldsymbol{q}}\right) + 2\int_0^q dp\,\mathcal{K}^B_{\boldsymbol{p},\boldsymbol{q}}\left[m_{\boldsymbol{p}}m_{\boldsymbol{q}-\boldsymbol{p}} - m_{\boldsymbol{q}}\left(n_{\boldsymbol{p}}+n_{\boldsymbol{q}-\boldsymbol{p}}+1\right)\right]$$

CD, Cherroret, PRA **107**, 043305 (2023)

thermal equilibrium $m_{\boldsymbol{q},\infty} = 0$

near-equilibrium dynamics $\partial_\tau m_{\boldsymbol{q},\tau} \simeq -2\gamma_{\boldsymbol{q}} m_{\boldsymbol{q},\tau}$

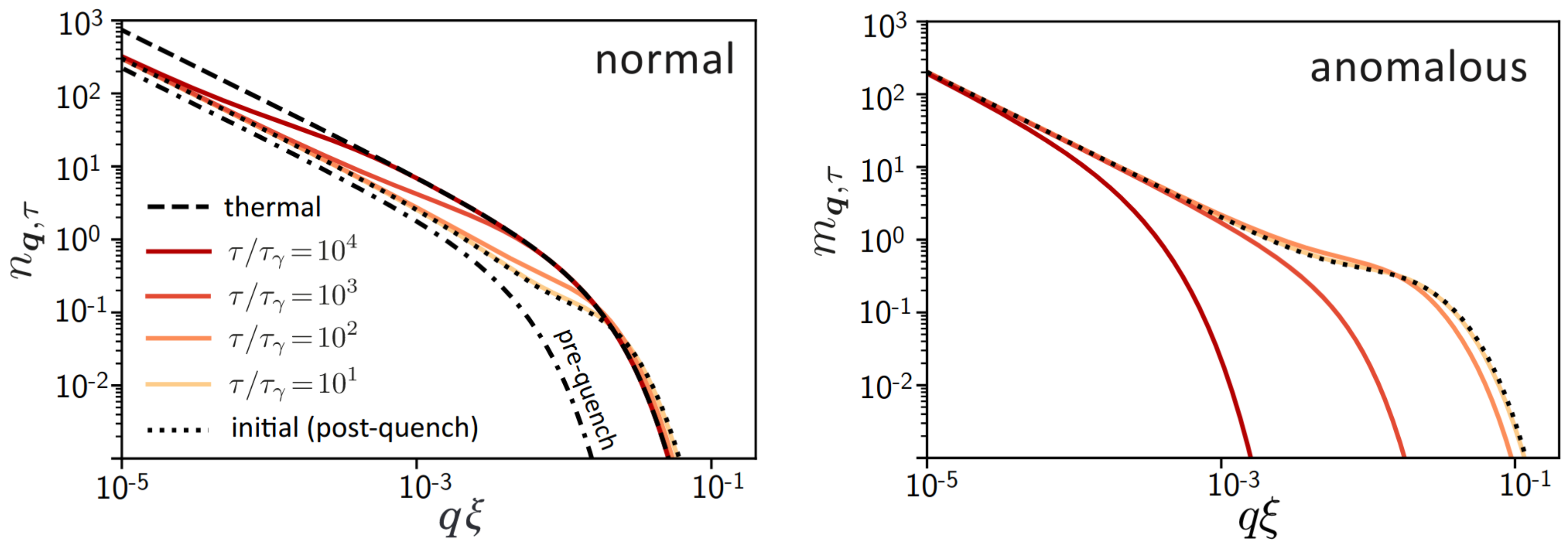
see also

1D: Buchhold, Diehl, EPJD **69**, 224 (2015)

3D: Regemortel et al., PRA **98**, 053612 (2018)

Relaxation time $\tau_\gamma \sim [2\gamma^L_{q=1/\xi}]^{-1} \sim \rho_0\xi^2\dfrac{g\rho_0}{T^2}$

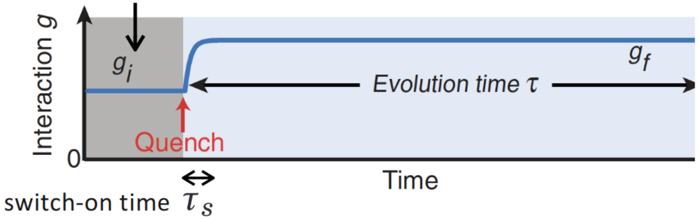
Numerical results: dynamics of phonon distributions



CD, Cherroret, PRA **107**, 043305 (2023)

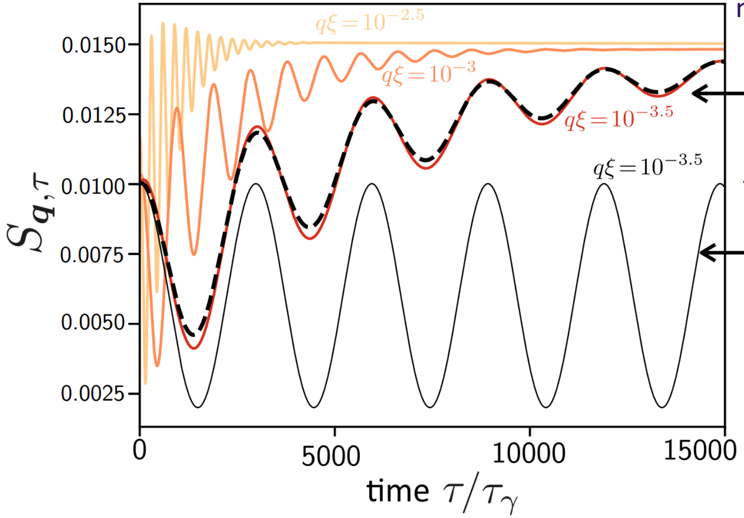
Numerical results: dynamics of phonon distributions

2D superfluid at equilibrium



Structure factor $S_{\boldsymbol{q},\tau} \equiv \langle \delta\hat{\rho}_{\boldsymbol{q},\tau}\delta\hat{\rho}_{-\boldsymbol{q},\tau}\rangle$

$$S_{\boldsymbol{q},\tau} = \frac{E_{\boldsymbol{q}}}{\epsilon_{\boldsymbol{q}}}[2n_{\boldsymbol{q},\tau}+1+\underbrace{2\cos(2\epsilon_{\boldsymbol{q}}\tau)}_{\text{fast coherent dynamics}}\underbrace{m_{\boldsymbol{q},\tau}}_{\text{slow relaxation}}]$$



near equilibrium

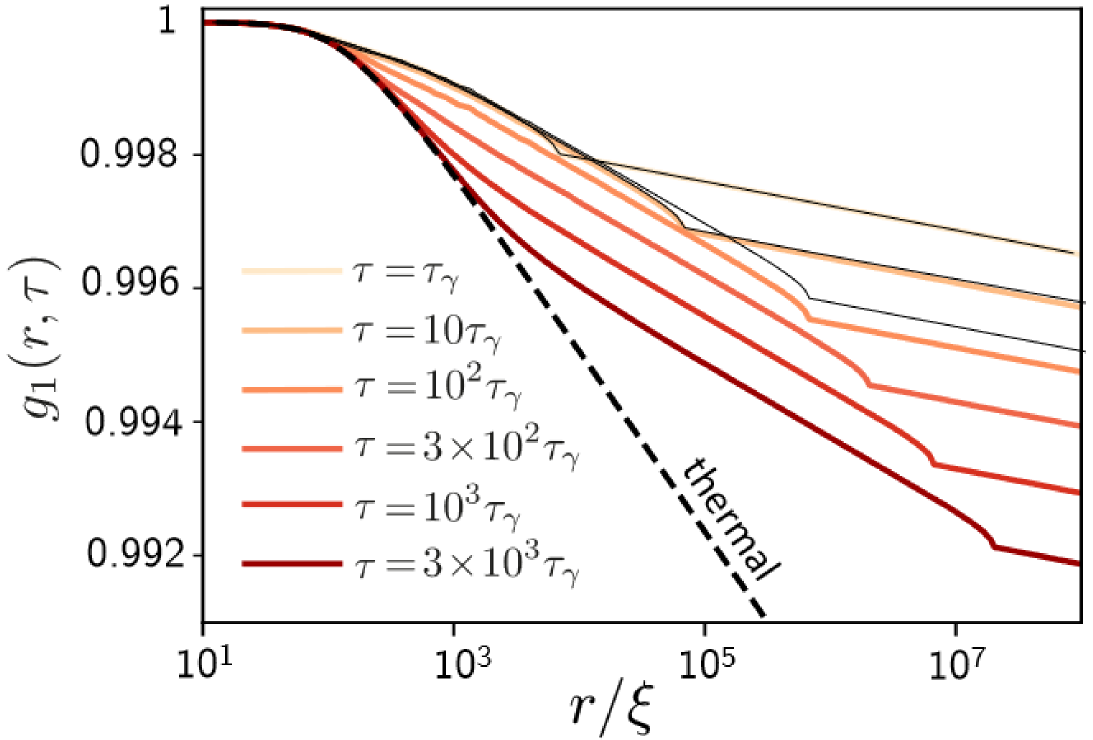
$$\delta n_{\boldsymbol{q},\tau} \simeq \delta n_{\boldsymbol{q},0}e^{-2\gamma_{\boldsymbol{q}}\tau}$$
$$m_{\boldsymbol{q},\tau} \simeq m_{\boldsymbol{q},0}e^{-2\gamma_{\boldsymbol{q}}\tau}$$

free theory

$$n_{\boldsymbol{q},\tau} \simeq n_{\boldsymbol{q},0}$$
$$m_{\boldsymbol{q},\tau} \simeq m_{\boldsymbol{q},0}$$

Numerical results: Spatial coherence

$$g_1(\Delta\mathbf{r},\tau) = \langle\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r}+\Delta\mathbf{r})\rangle$$



Conclusion

- We have derived a quantum kinetic theory for 2D superfluid after a quench

  → Small (coherent) time scales

  → Late time scales & thermalization

- Perspectives:

  → Nonperturbative regime?

  → Quench through BKT transition?

Thank you for your attention