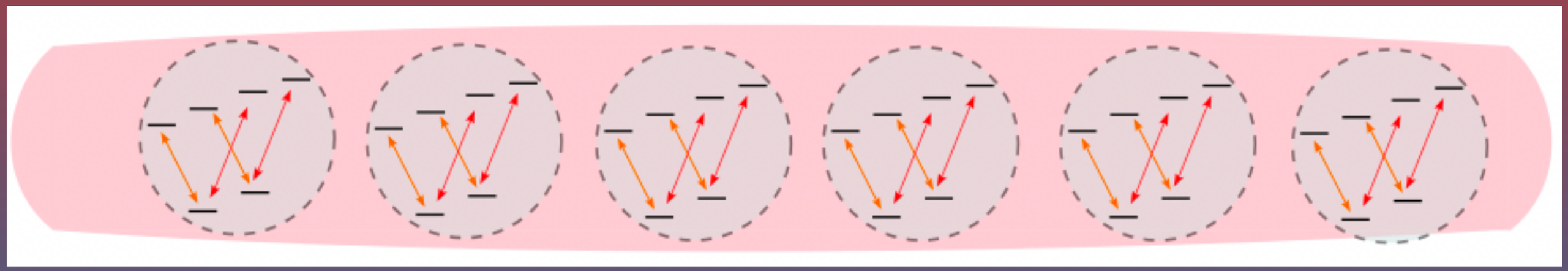
Quantum simulation of SU(2) 1D dynamics with ions qudits

Giuseppe Calajò



Collaborations:

P. Silvi, G. Magnifico and S. Montangero (Padova University)
M. Ringbauer (Innsbruck University)

T-NiSQ

QUANTERA

QSG Workshop

4/5/2023

INFN PADOVA

Gauge Theories

Main ingredients:

1) Quantum matter

2) Quantum fields

3) <u>Local</u> gauge symmetries implying <u>local</u> constrain

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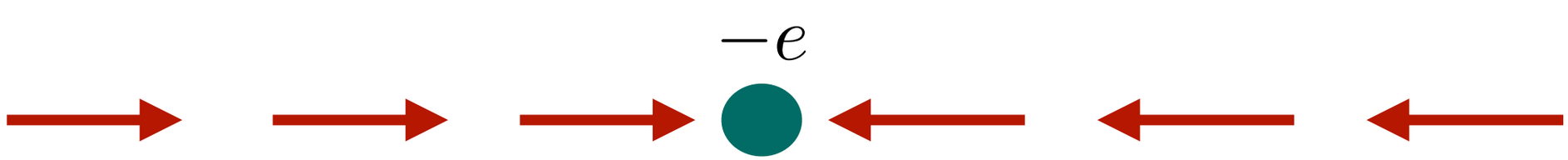
Example: Gauss's law in QED

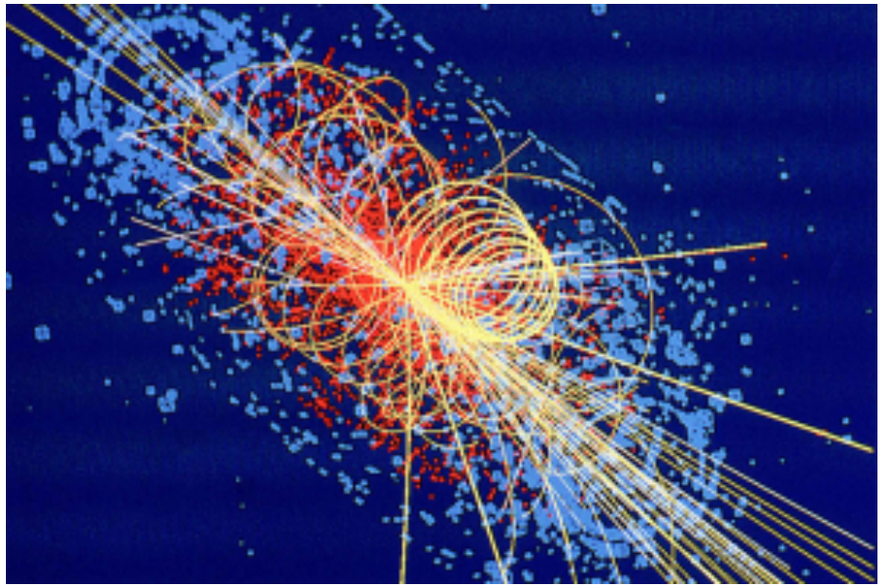
$$\nabla \cdot \mathbf{E} = \rho$$



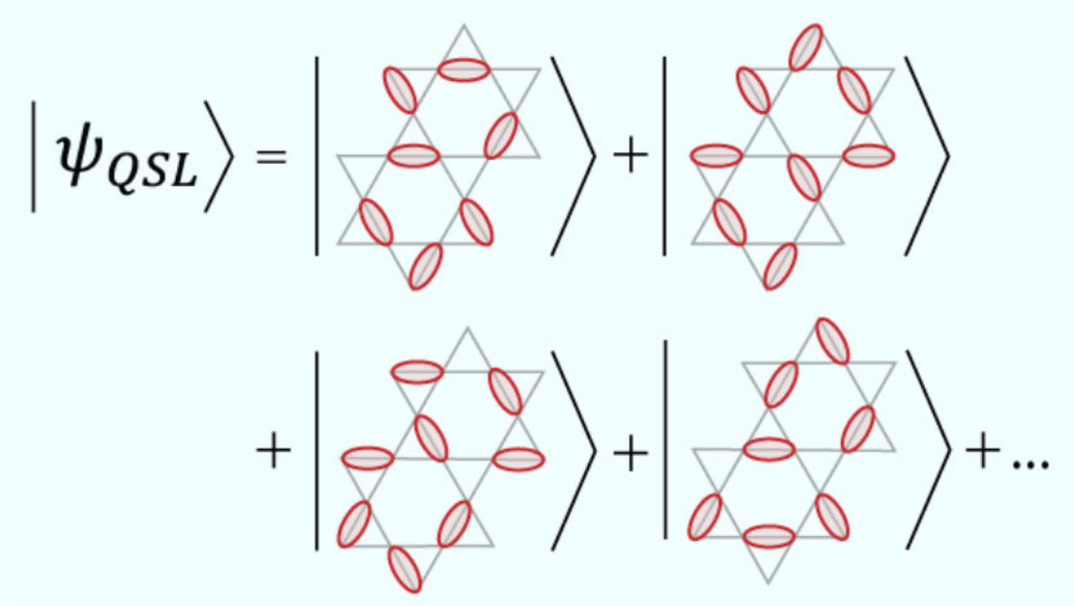
Gauge Theories

Fundamental in many area of physics

High energy physics: standard model



Emergent theories in condensed matter: e.g. spin liquids



*G. Semeghini, et al. Science, 2021, **374**(6572) (2021)*

Simulating Gauge theories

- ***Extremely tough to tackle these problems***
- ***Monte Carlo technics very successful in capturing equilibrium properties but fail at finite density and out of equilibrium***

M.C. Banalus, et al,Eur. Phys. J. D ***74****, 165 (2020); M. Dalmonte and S. Montangero Contemporary Physics*, *57*(3), 388-412 (2016);

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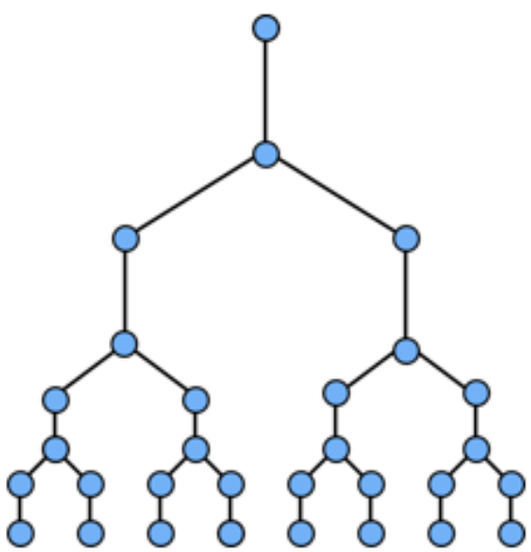
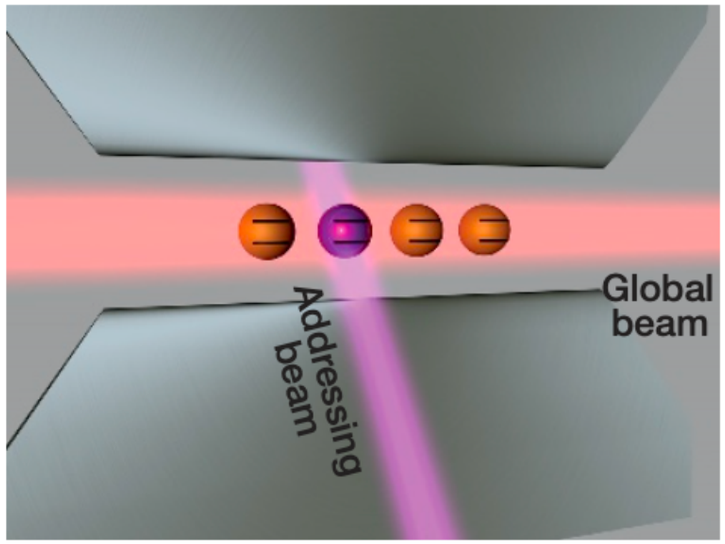
Alternative route: Lattice Gauge Theory

Tensor networks



Quantum simulation



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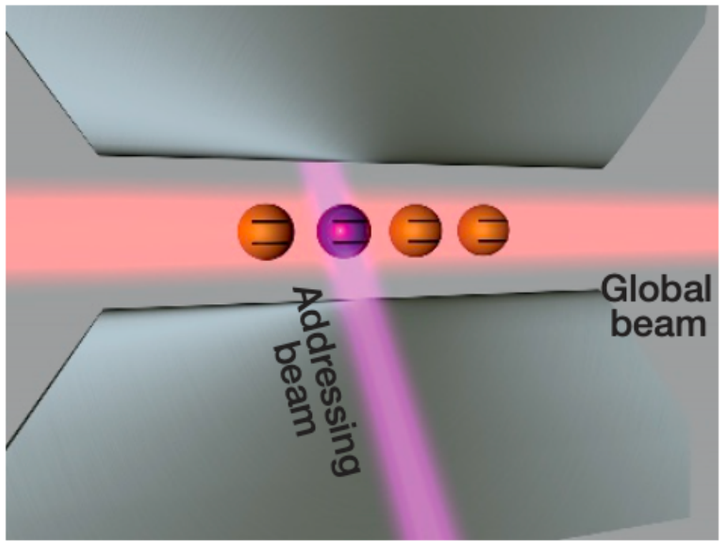
Abelian Gauge theories



Digital simulation of the Schwinger model (1D) dynamics

E. Martinez, et al, Nature ***534****, 516-519 (2016);* ***4 qubit***

N. H. Nguyen, et al, arXiv:2112.14262 (2022); ***6 qubit***

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Variational eigensolver (not dynamics) in 2D

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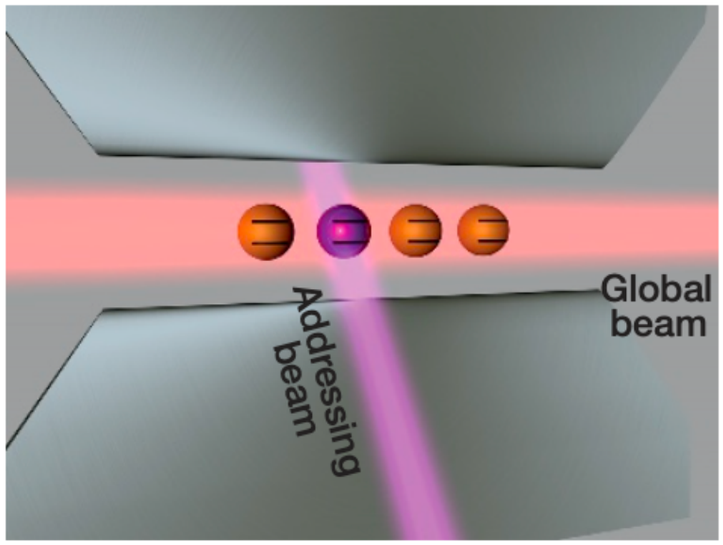

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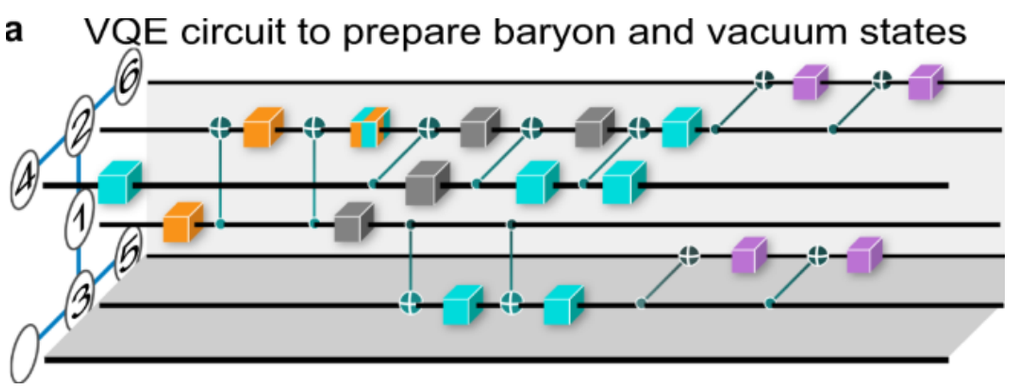

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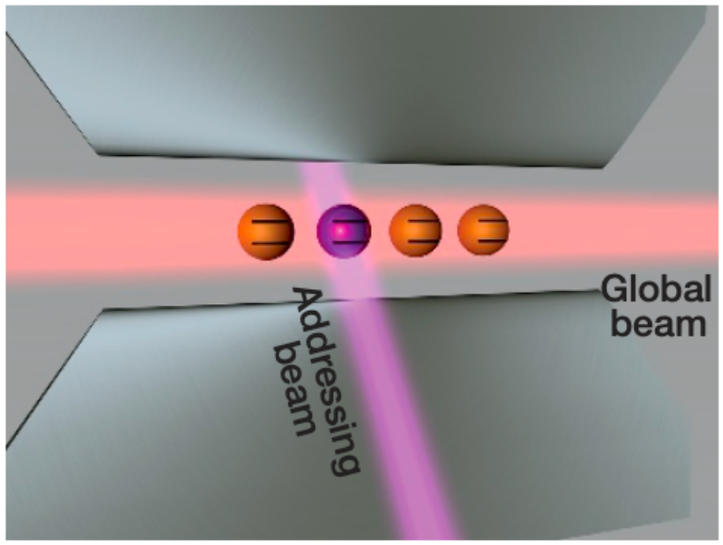

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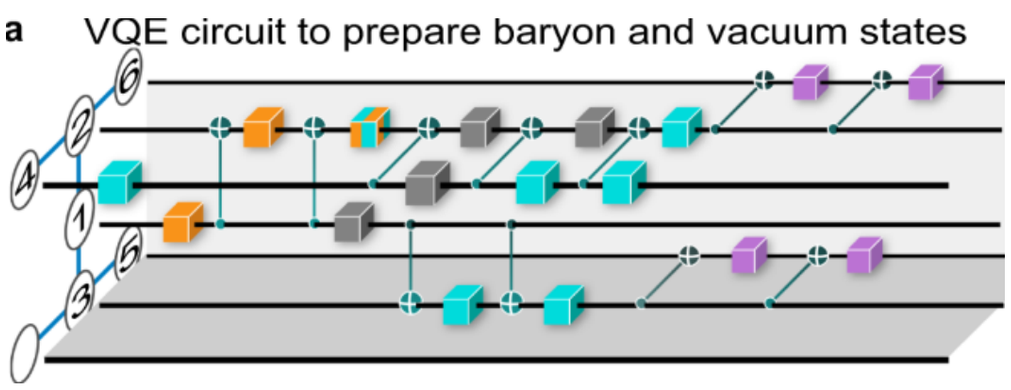

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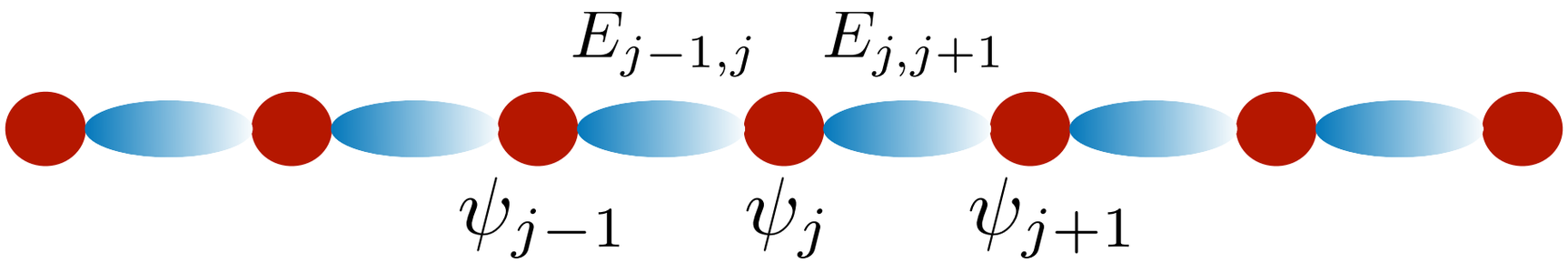
Can we simulate nonabelian dynamics?

Hamiltonian lattice gauge theory



- ***Space discretized time kept continuous***
- ***Matter lives on the sites***
- ***Field lives on the links***

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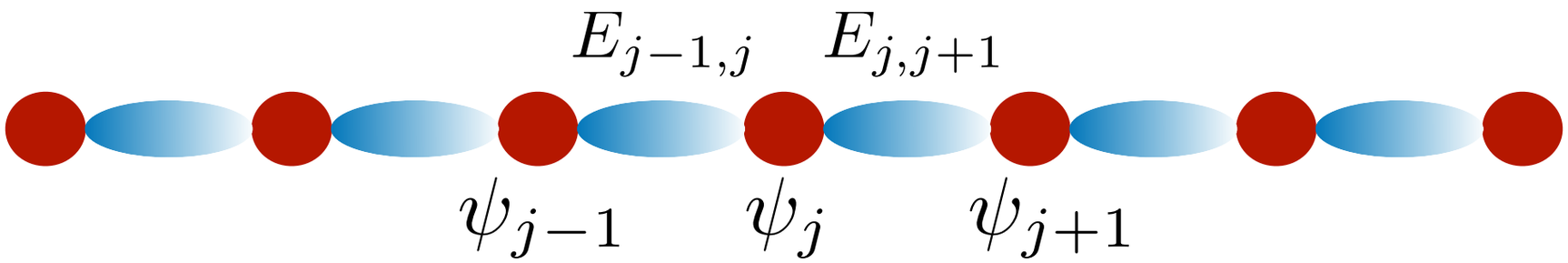

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Matter Lagrangian

$$\mathcal{L}_{\mathrm{Dirac}} = \bar{\psi}\gamma^{\mu} i\partial_{\mu}\psi - m\bar{\psi}\psi$$

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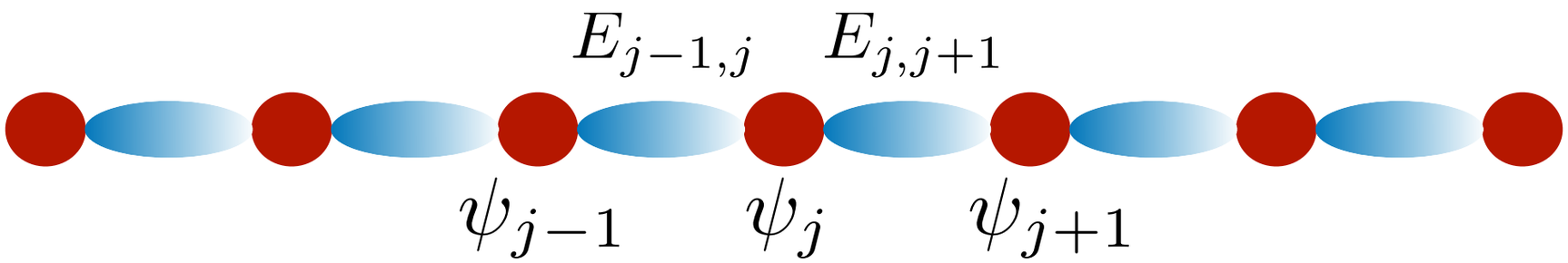

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$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}\gamma^\mu i\partial_\mu\psi - m\bar{\psi}\psi$$

$$\xrightarrow{x = ja \quad t := t}$$

Continuum limit

$$a \to 0$$

$$H_{\text{Dirac}} = \frac{1}{2a}\sum_j \left[-i\hat{\psi}_j^\dagger\hat{\psi}_{j+1} + \text{H.c.}\right] + m\sum_j (-1)^j \hat{\psi}_j^\dagger\hat{\psi}_j$$

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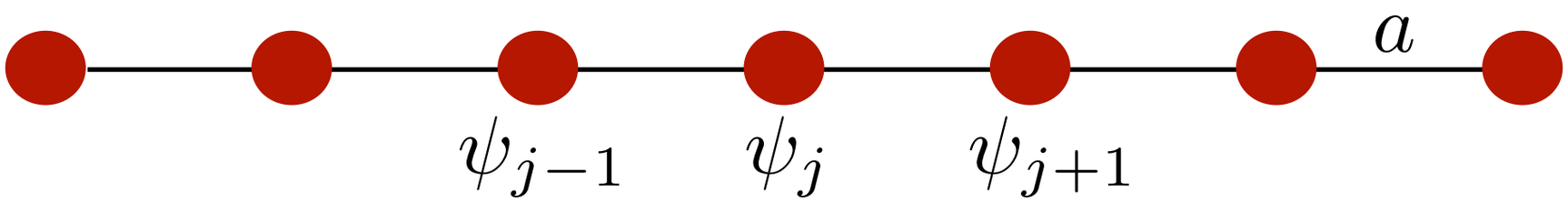

1D Matter Hamiltonian

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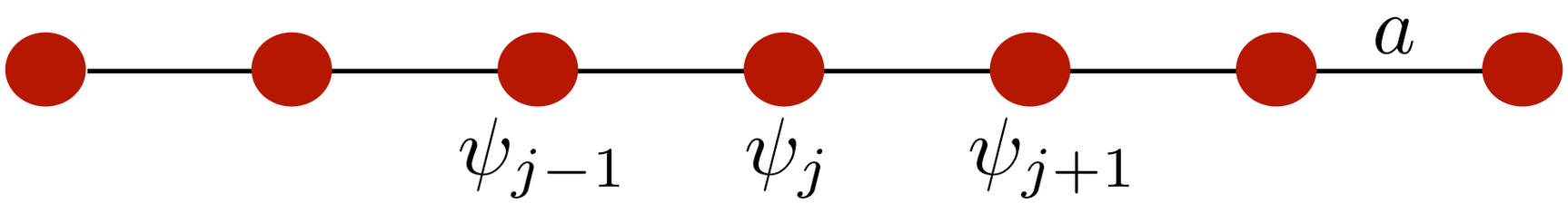

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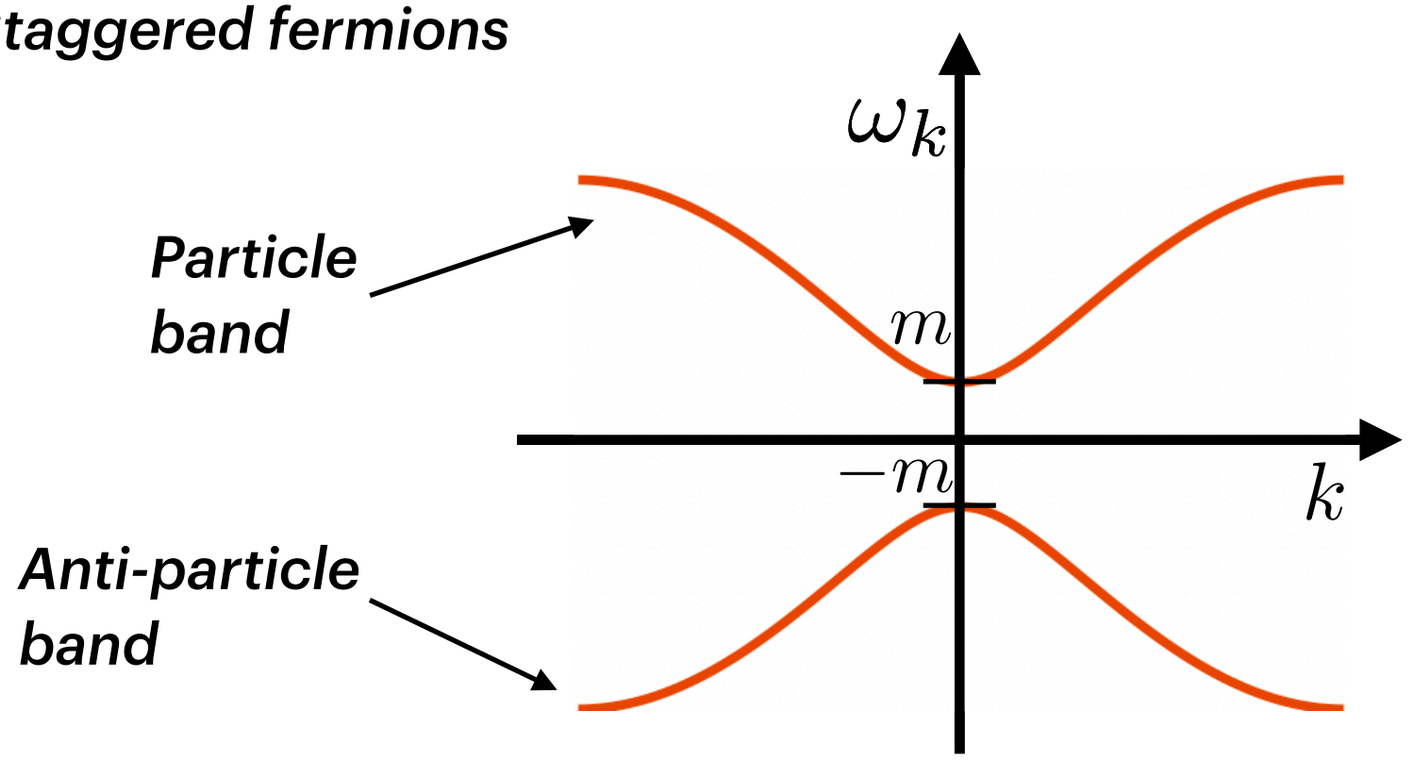
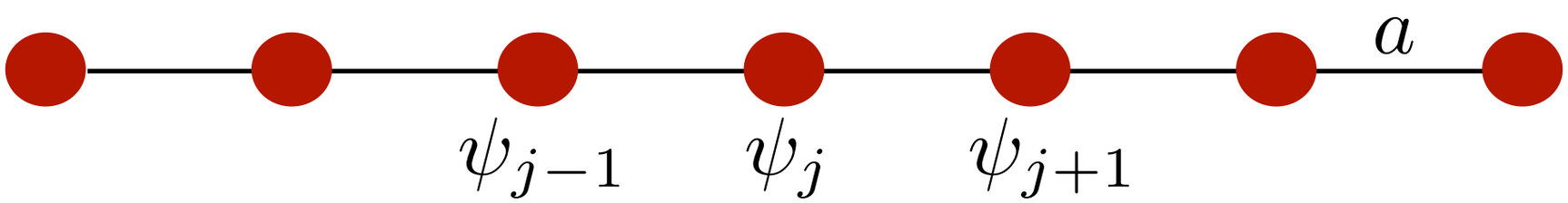
Staggered fermions



Hamiltonian lattice gauge theory



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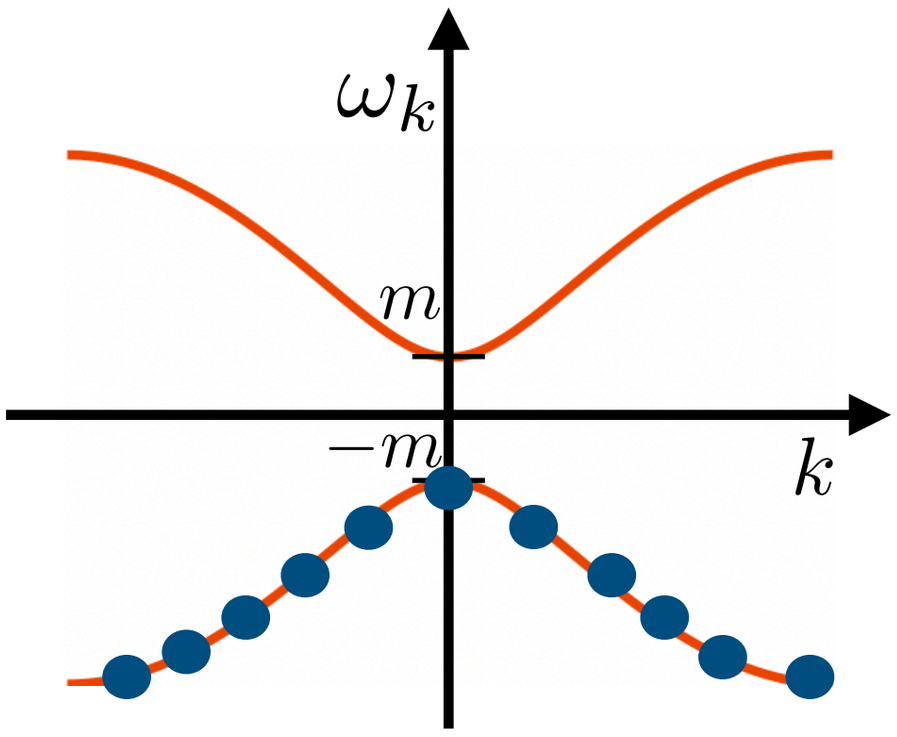
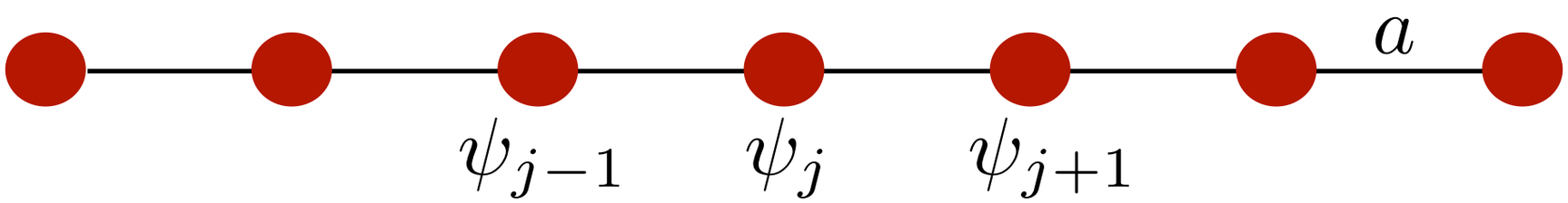
Staggered fermions

Ground state: Fermi sea



Hamiltonian lattice gauge theory



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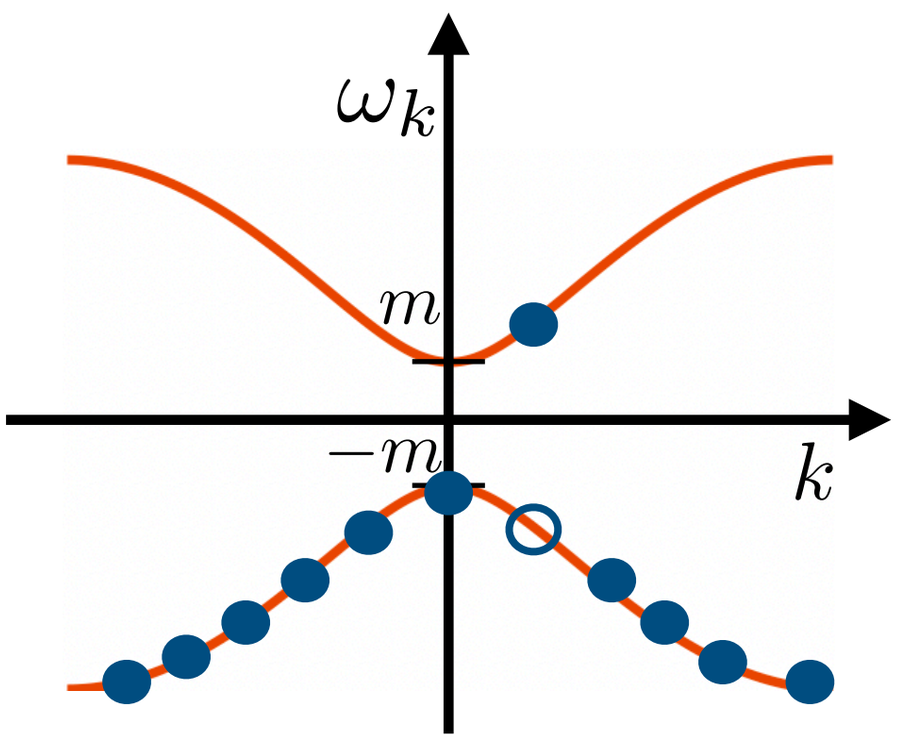
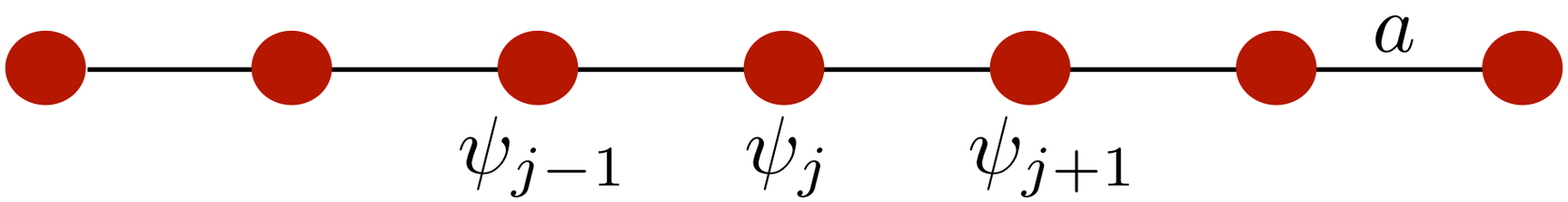
Staggered fermions

Excited state:
Fermion-antifermion pairs



Hamiltonian lattice gauge theory

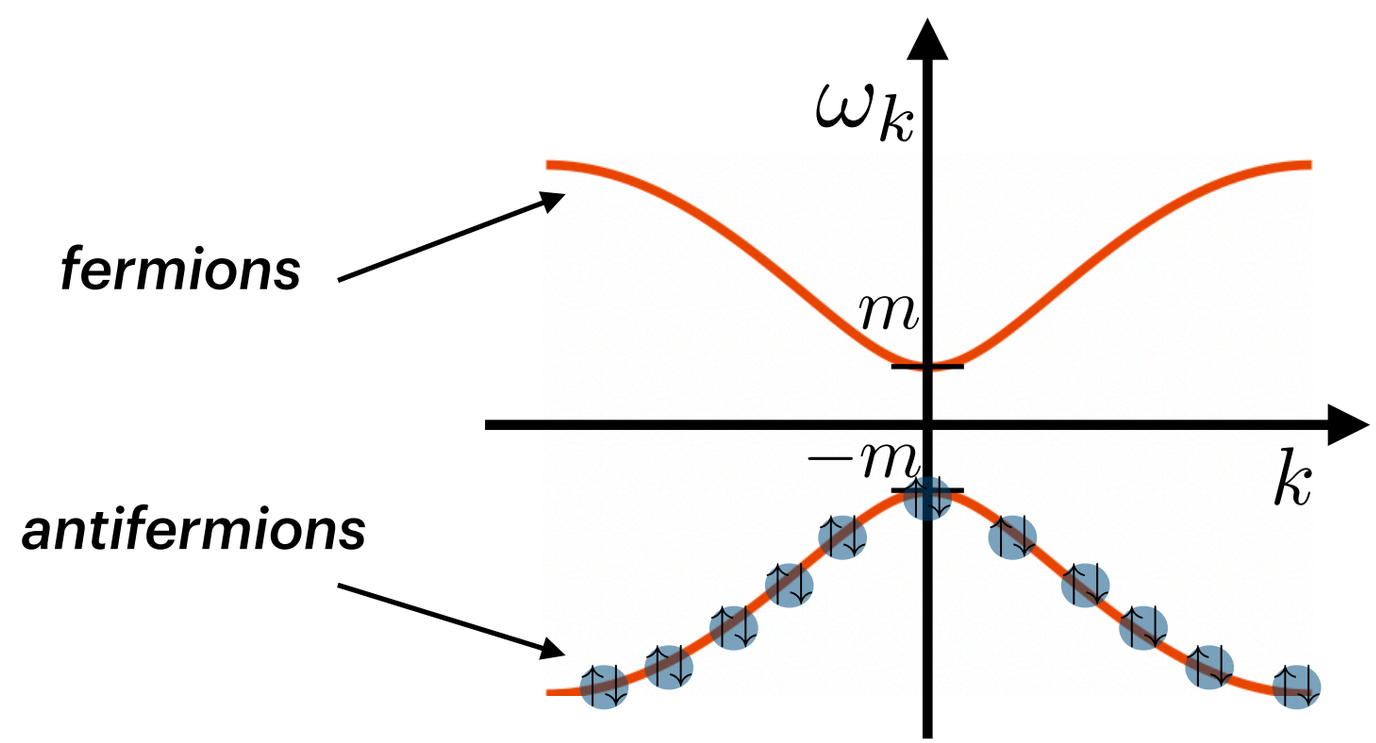


1D Matter Hamiltonian with colors

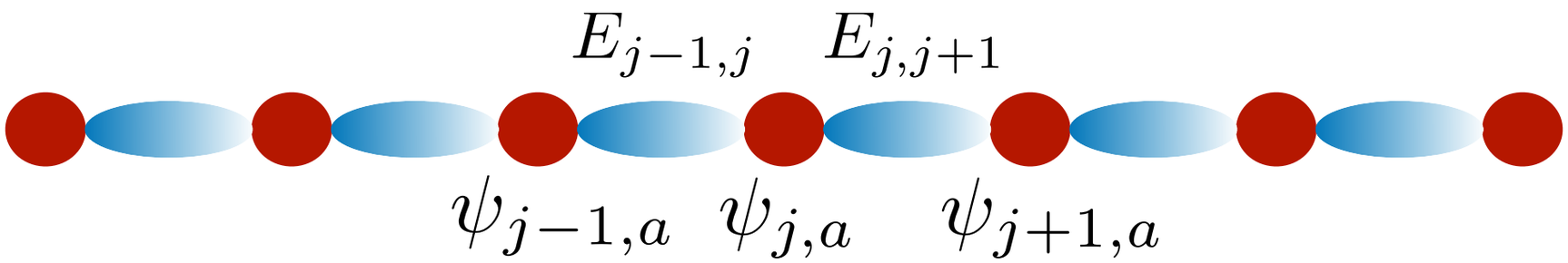
$$H_{\text{Dirac}} = \frac{1}{2a}\sum_j \sum_{a,b=\uparrow,\downarrow} \left[-i\hat{\psi}^\dagger_{ja}\hat{\psi}_{j+1b} + \text{H.c.}\right] + m\sum_j (-1)^j \hat{\psi}^\dagger_{ja}\hat{\psi}_{ja}$$

SU(2) two colors: $|\uparrow\rangle \; |\downarrow\rangle$

Ground state: Fermi sea



Hamiltonian lattice gauge theory

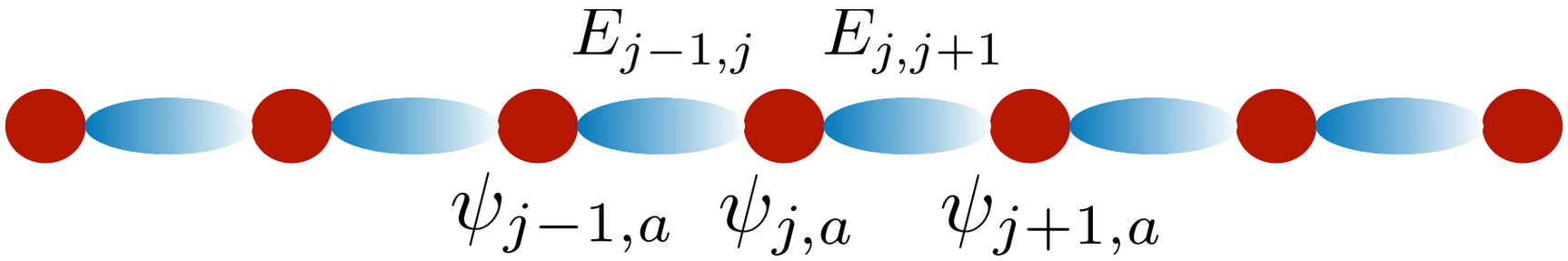


Impose invariance under SU(2) local transformation

$$\hat{\psi}_{j,a} \to \hat{\psi}_{j,a} e^{i\Lambda_j^a \hat{\sigma}_a}$$

$\hat{\sigma}_a \in SU(2)$ ***2x2 complex matrices***

Hamiltonian lattice gauge theory



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$$H = \frac{1}{2a} \sum_j \sum_{a,b=\uparrow\downarrow} \left[-i\hat{\psi}_{ja}^\dagger \hat{U}_{j,j+1,ab} \hat{\psi}_{j+1,b} + \text{H.c.} \right] + m \sum_j (-1)^j \hat{\psi}_{ja}^\dagger \hat{\psi}_{ja}$$

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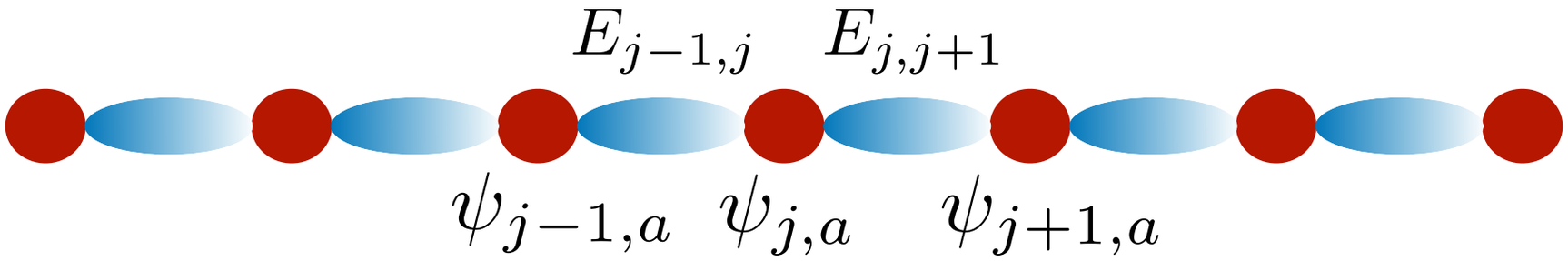

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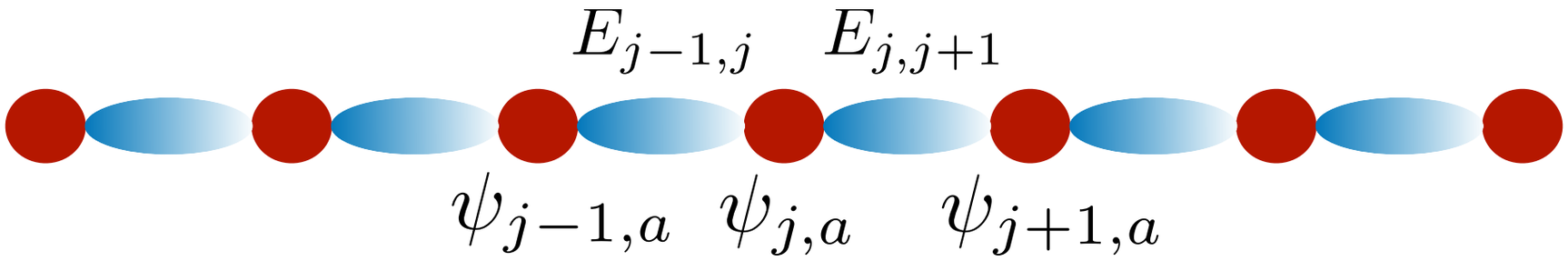
$$\hat{U}_{j,j+1,ab} = e^{ig\hat{A}_{j,j+1,ab}}$$

Parallel transport

Hamiltonian lattice gauge theory



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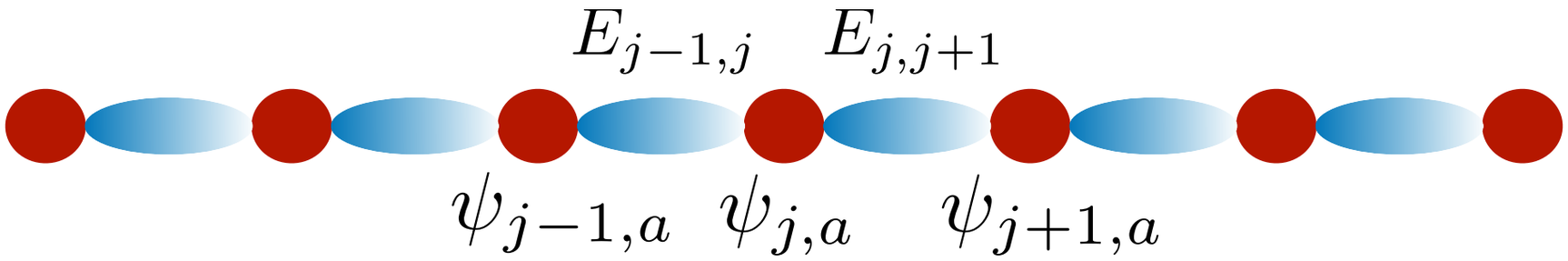

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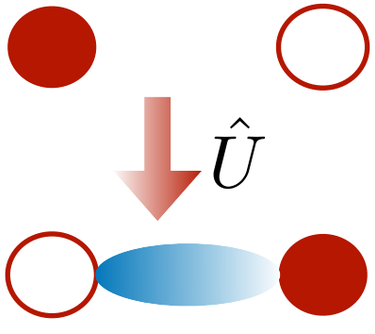

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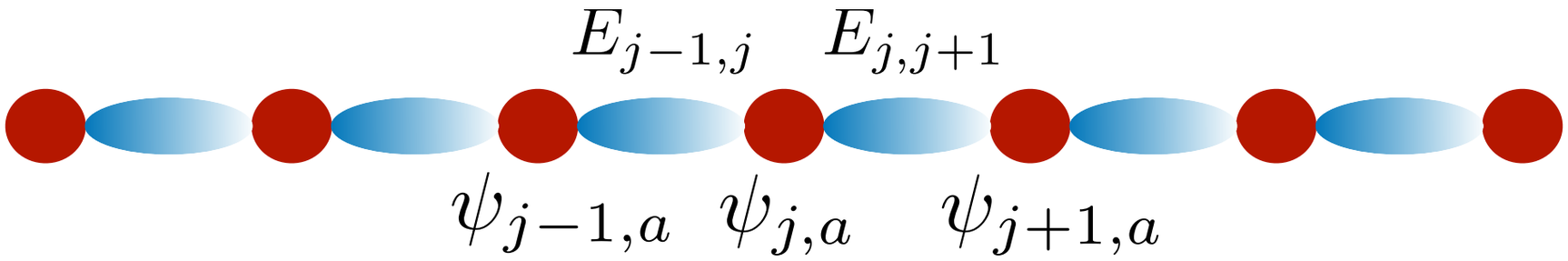

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$\hat{U}_{j,j+1,ab} = e^{ig\hat{A}_{j,j+1,ab}}$ ***Parallel transport***

$\hat{E}_{j,j+1}$ ***Electric field operator***

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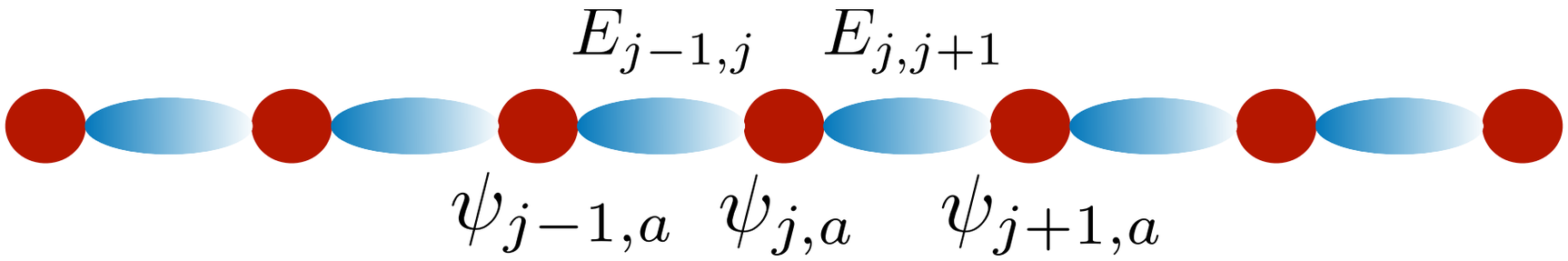

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Parallel transport

$$\hat{E}_{j,j+1}$$

Electric field operator

Angular momentum commutation rules $\hat{E} := \hat{L}_z \qquad \hat{U} := \hat{L}_+$

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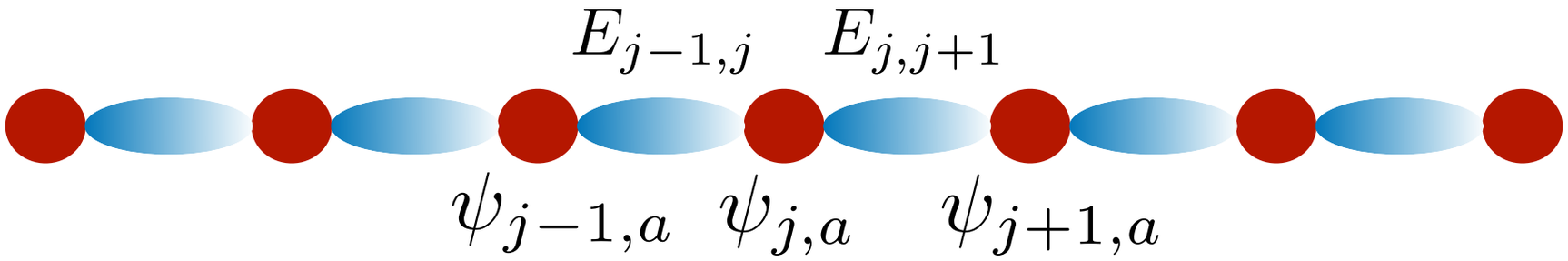

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Gauss law

$$\boxed{\hat{G}_j^a |\psi_{\text{phys}}\rangle = 0}$$

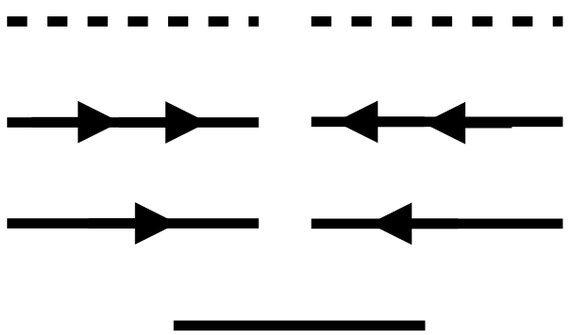
$\hat{G}_j^a$ ***generator of SU(2) symmetry transforms matter and fields***

U. J. Wiese, Ann. Phys. (Berlin) ***525****, No. 10–11, 777–796 (2013); E. Zohar, J. I. Cirac and B. Reznik Rep. Prog. Phys.* ***79*** *(2016);*

Quantum link model

Infinite dimensional field Hilbert space

$$\hat{E}|\mathcal{E}\rangle = \mathcal{E}|\mathcal{E}\rangle$$



Quantum link model

- ***Link model:*** *truncate field Hilbert space*

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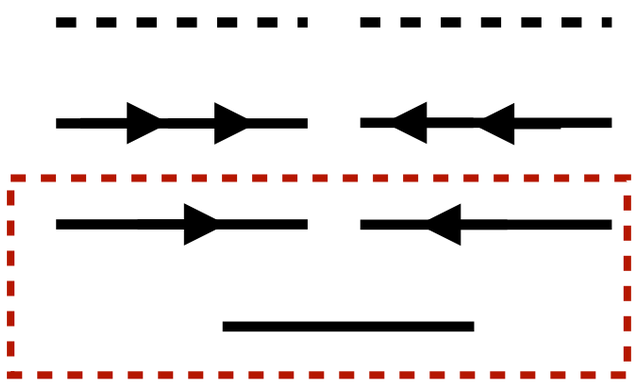

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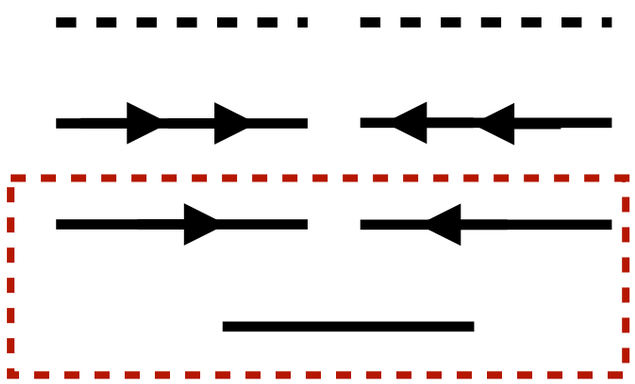

- ***Decompose link in pair of rishons***

$$\hat{U}_{j,j+1,ab} = \xi^L_{j,a}\xi^{\dagger R}_{j+1,b} \qquad \xi^{L/R}_{j,a}$$ ***fermion operators***

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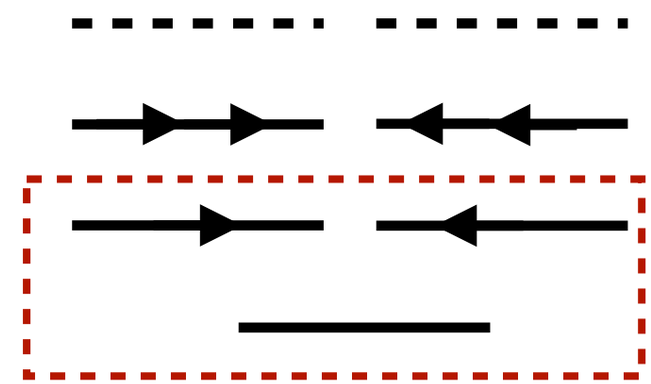

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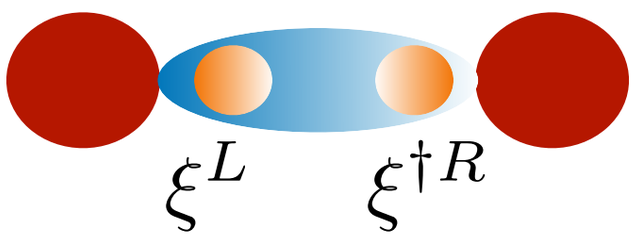

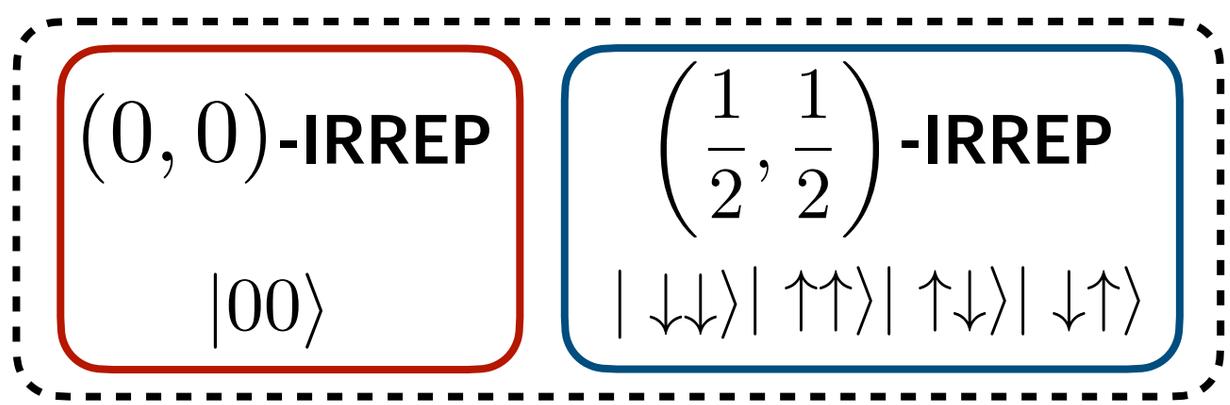

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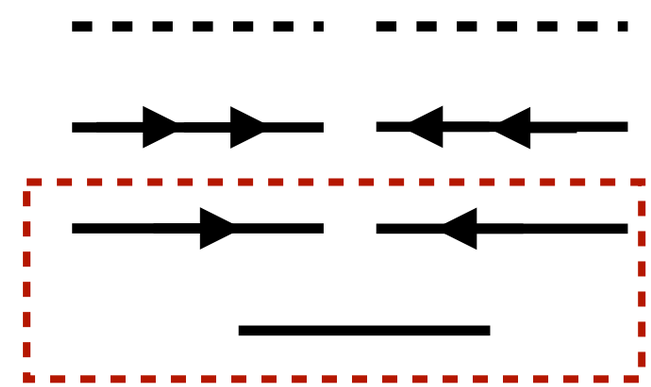

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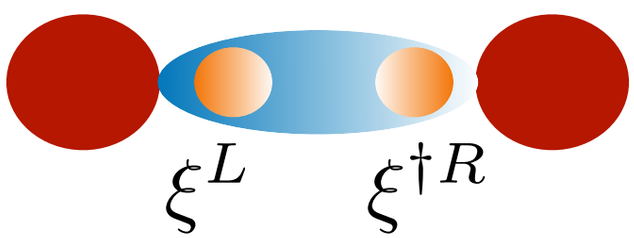

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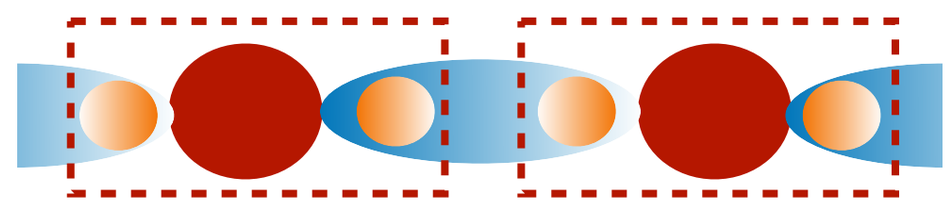

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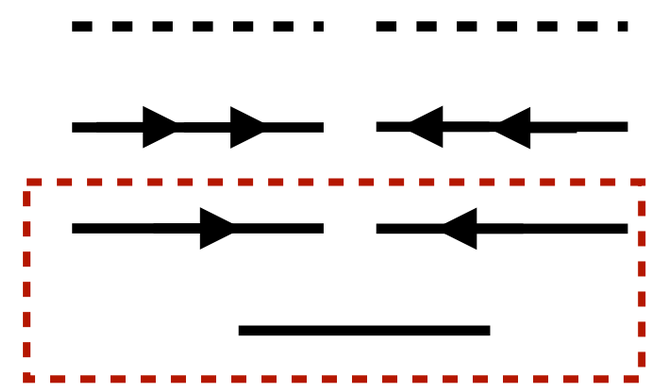

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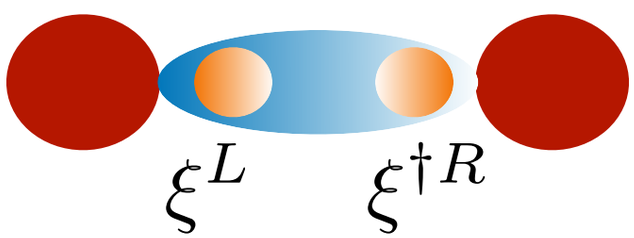

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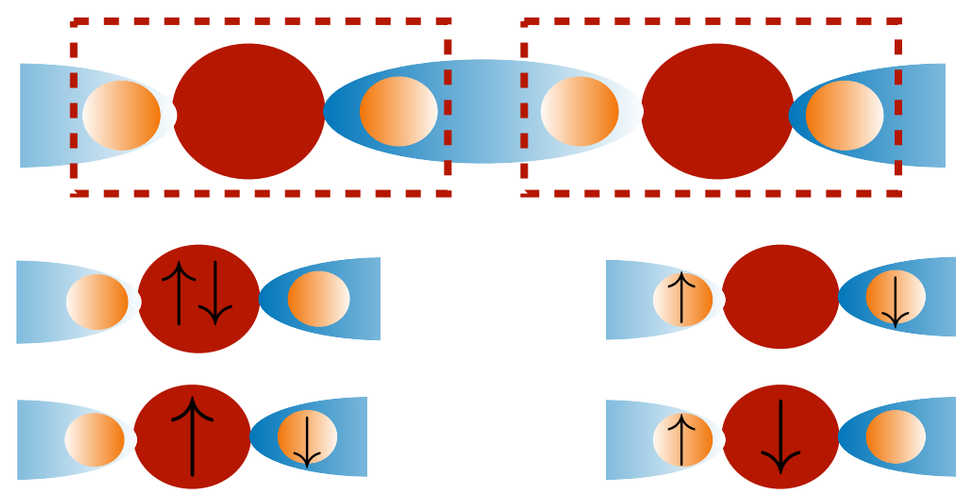

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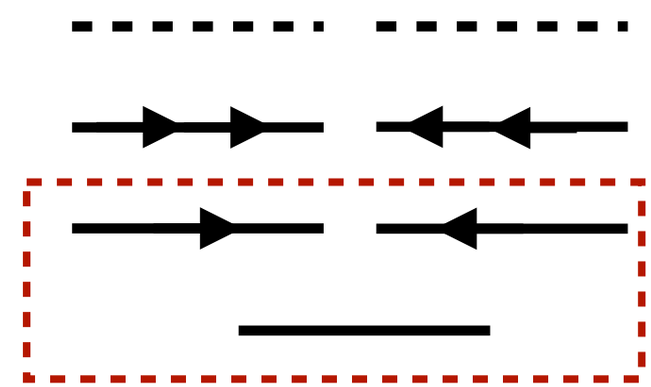

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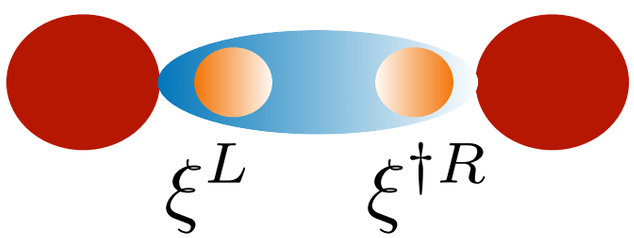

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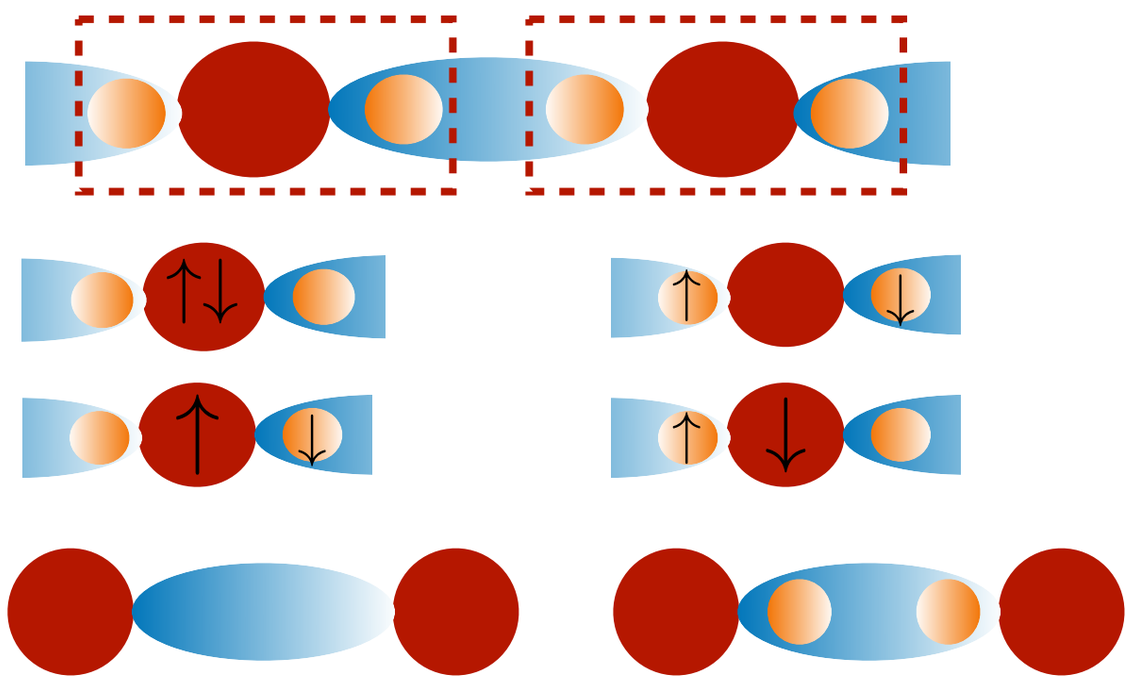
- ***Link parity constrain:*** *even number of rishon per link*



SU(2) truncated model

SU(2) invariant Hamiltonian

$$H = \frac{1}{2a}\sum_{j}\sum_{a,b=\uparrow\downarrow}\left[-i\hat{\psi}^{\dagger}_{j,a}\hat{U}_{j,j+1,ab}\hat{\psi}_{j+1,b} + \text{H.c.}\right] + m\sum_{j}(-1)^{j}\hat{\psi}^{\dagger}_{j,a}\hat{\psi}_{j,a} + \frac{ag^2}{2}\sum_{j}\hat{E}^2_{j,j+1}$$

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Local dressed basis

Model with local dimension 6

$$H = J\sum_j \left[\hat{A}^{(1)}_j \hat{B}^{(1)}_{j+1} + \hat{A}^{(2)}_j \hat{B}^{(2)}_{j+1}\right] + m\sum_j (-1)^j \hat{M}_j + g^2 \sum_j \hat{C}_j$$

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Mass term

Field term

Diagonal matrices!

$$\hat{M} = \begin{pmatrix} 0 & & & & & \\ & 0 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 2 & \\ & & & & & 2 \end{pmatrix}$$

$$\hat{C} = \begin{pmatrix} 0 & & & & & \\ & 2 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 0 & \\ & & & & & 2 \end{pmatrix}$$

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Hopping terms

$$\hat{A}^{(1)} = \begin{pmatrix} 0 & & & -\sqrt{2} & & \\ & 0 & 1 & & & \\ & 1 & 0 & & & -1 \\ -\sqrt{2} & & & 0 & -\sqrt{2} & \\ & & & -\sqrt{2} & 0 & \\ & & -1 & & & 0 \end{pmatrix} \qquad \hat{B}^{(1)} = \begin{pmatrix} 0 & & \sqrt{2}i & & & \\ & 0 & & -i & & \\ -\sqrt{2}i & & 0 & & \sqrt{2}i & \\ & i & & 0 & & i \\ & & -\sqrt{2}i & & 0 & \\ & & & -i & & 0 \end{pmatrix}$$

Sparse matrices!

Recovering SU(2) dynamics

Pairs production

$$H = m \sum_j (-1)^j \hat{M}_j + g^2 \sum_j \hat{C}_j$$

Dirac ground state

Recovering SU(2) dynamics

Pairs production

$$H = J \sum_j \left[\hat{A}_j^{(1)} \hat{B}_{j+1}^{(1)} + \hat{A}_j^{(2)} \hat{B}_{j+1}^{(2)} \right] + m \sum_j (-1)^j \hat{M}_j + g^2 \sum_j \hat{C}_j$$

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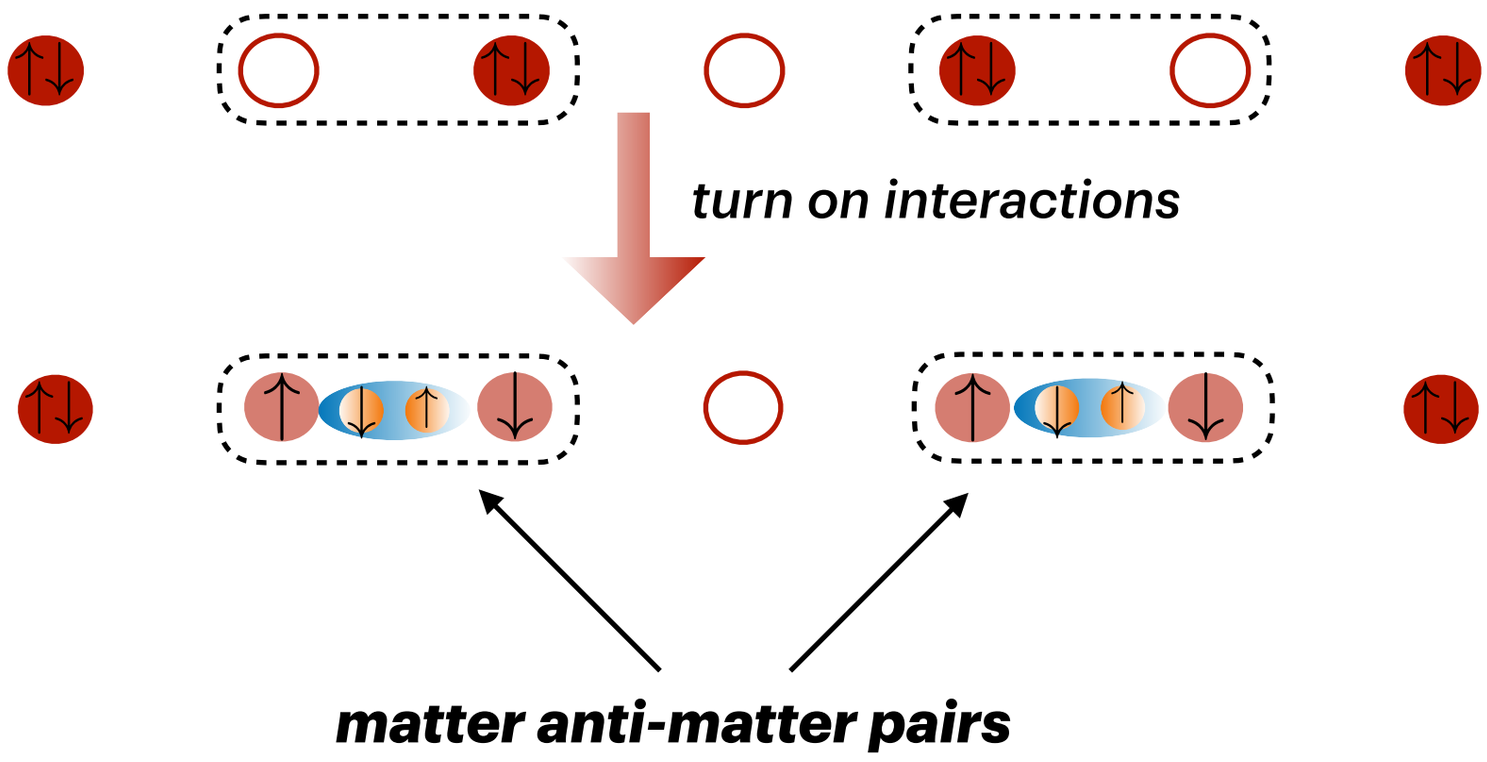

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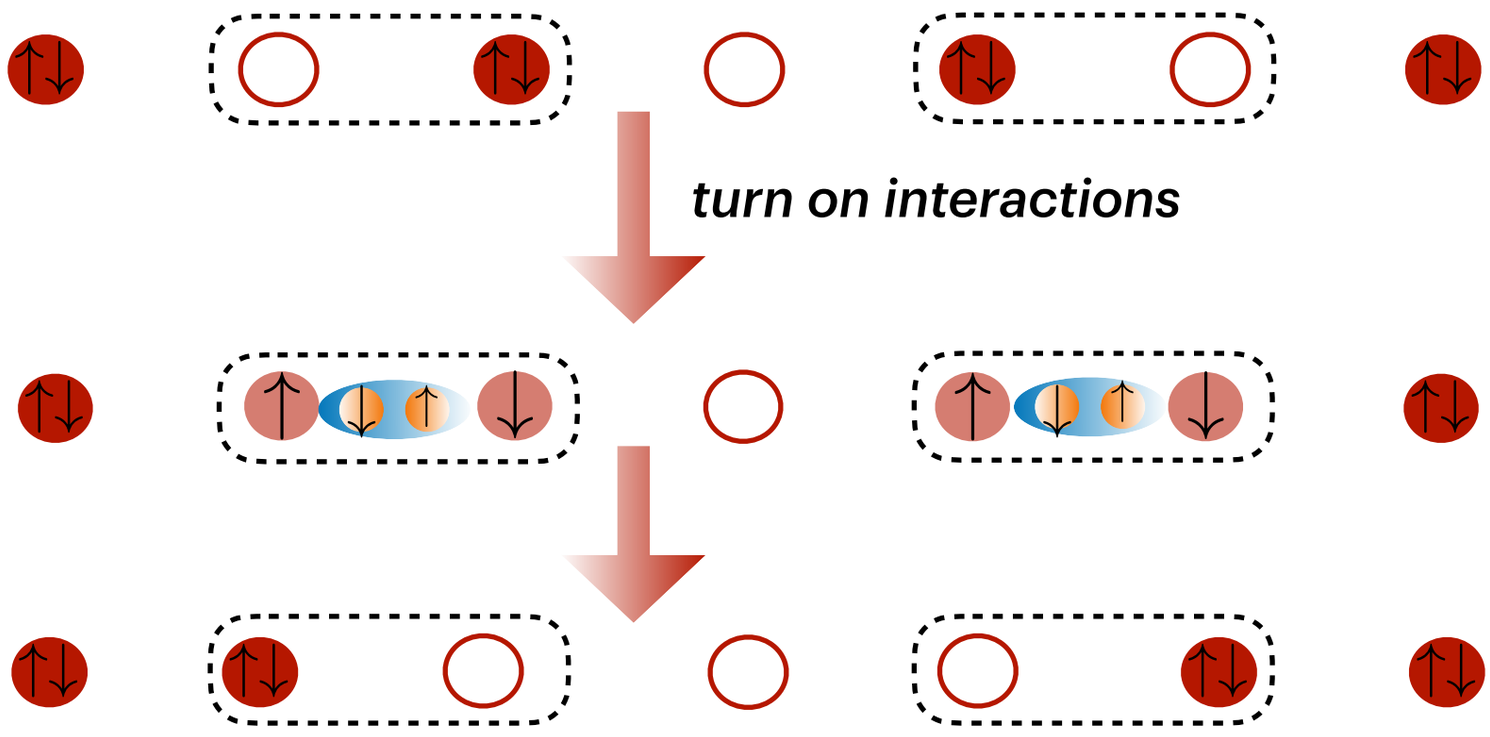

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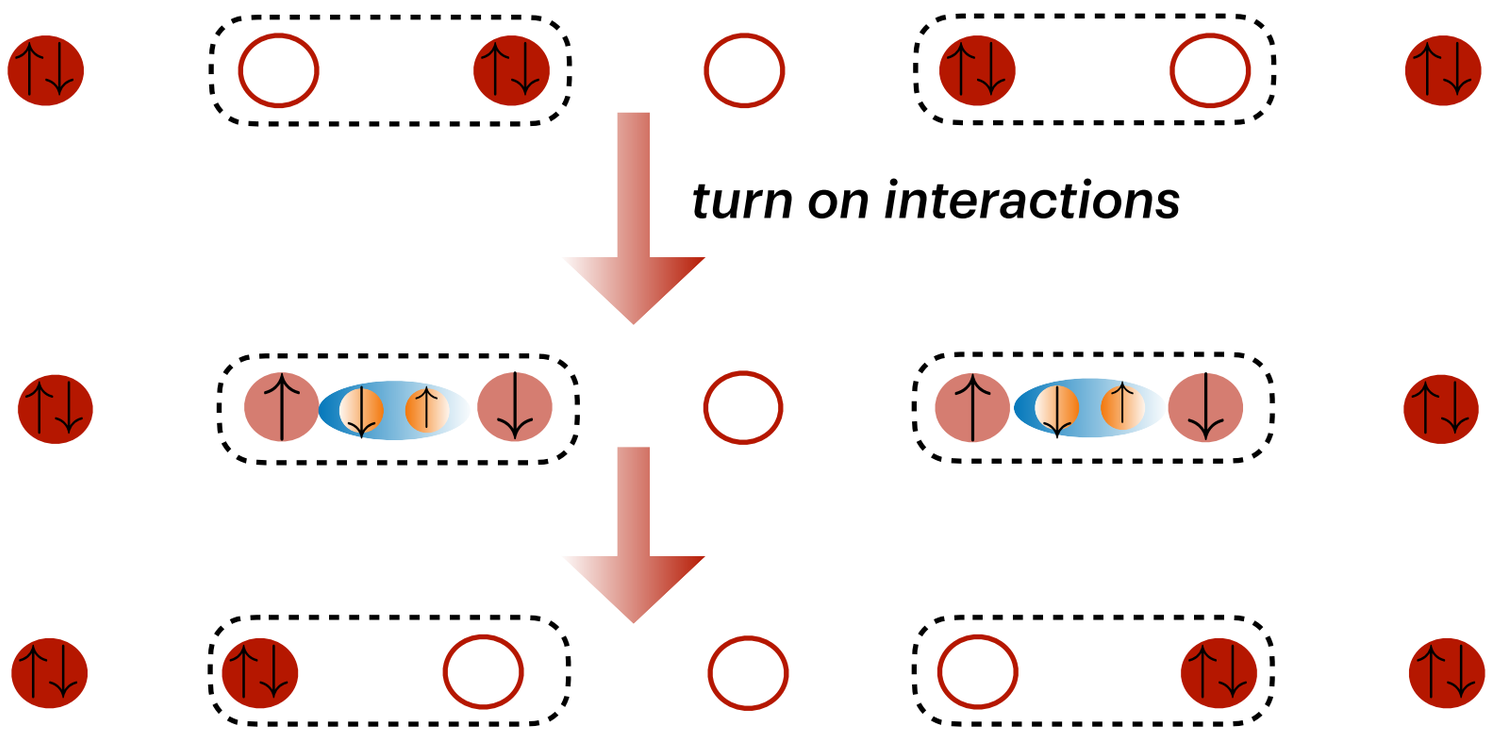
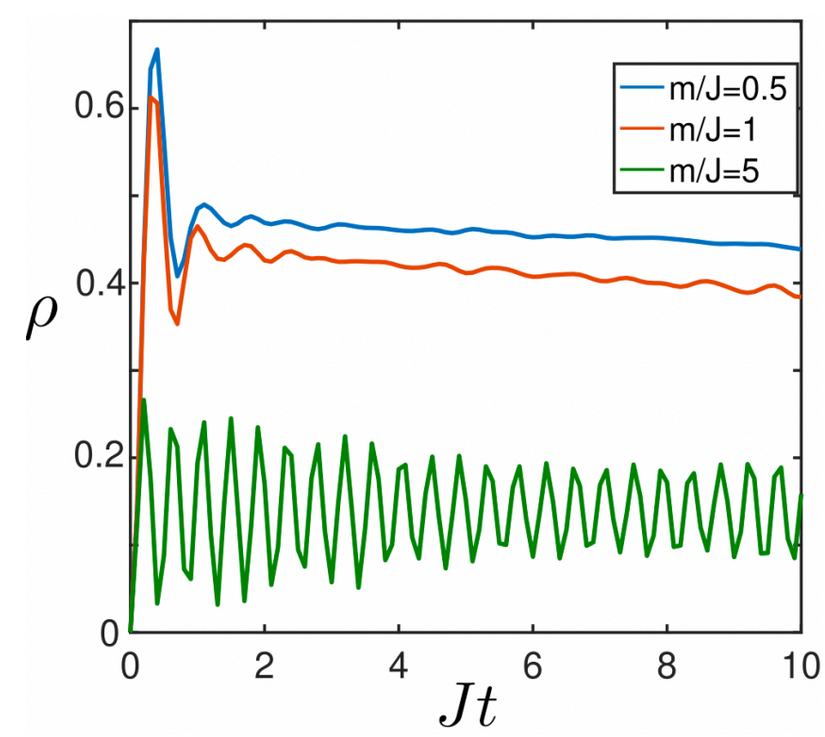


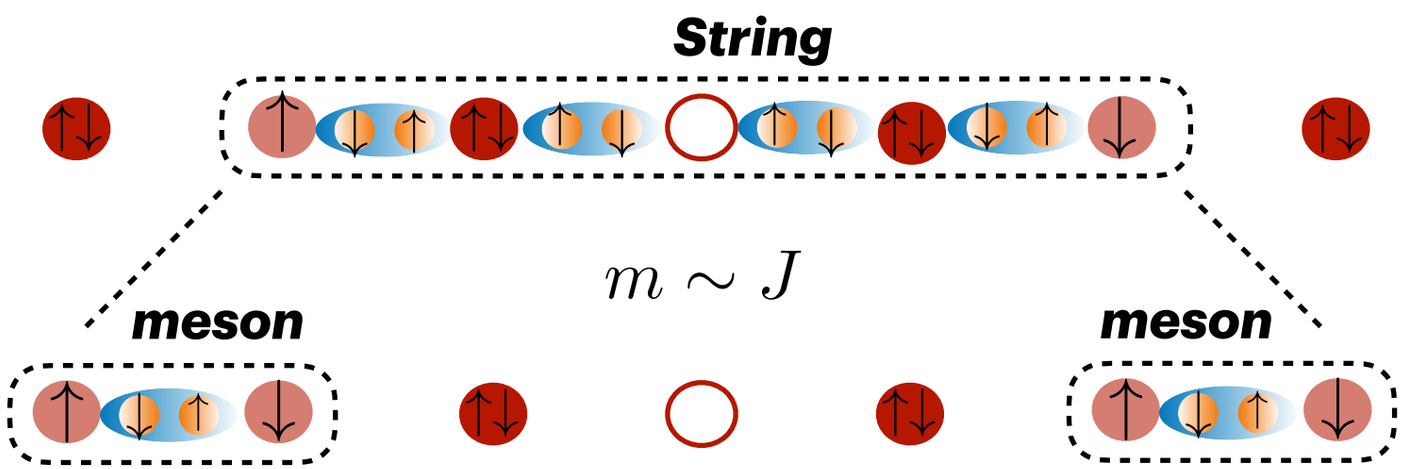
pairs density: $\rho = \dfrac{N_{\text{pairs}}}{N_{\text{sites}}}$

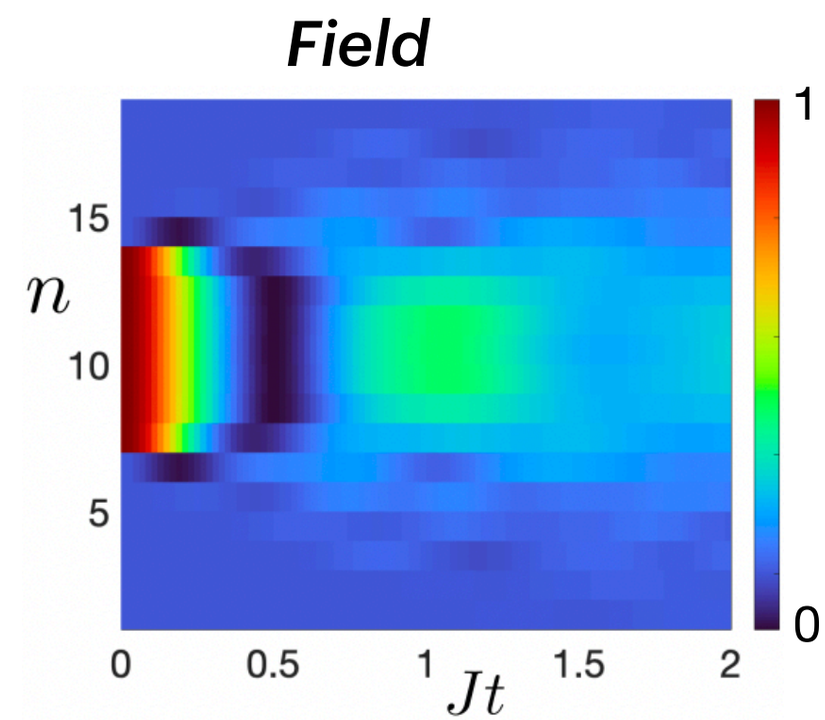
20 sites



Recovering SU(2) dynamics

String breaking





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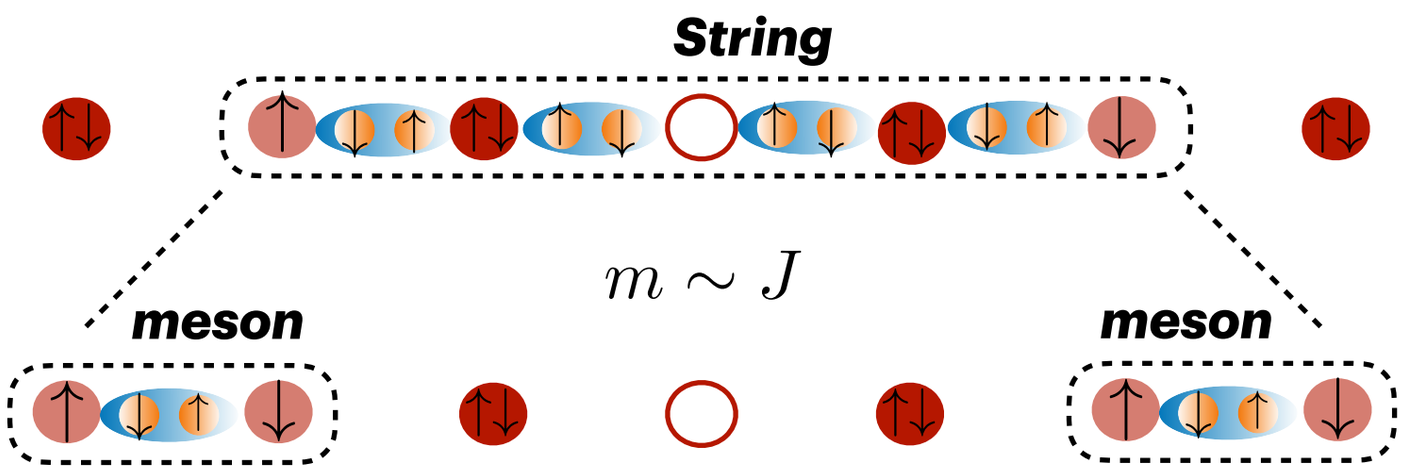
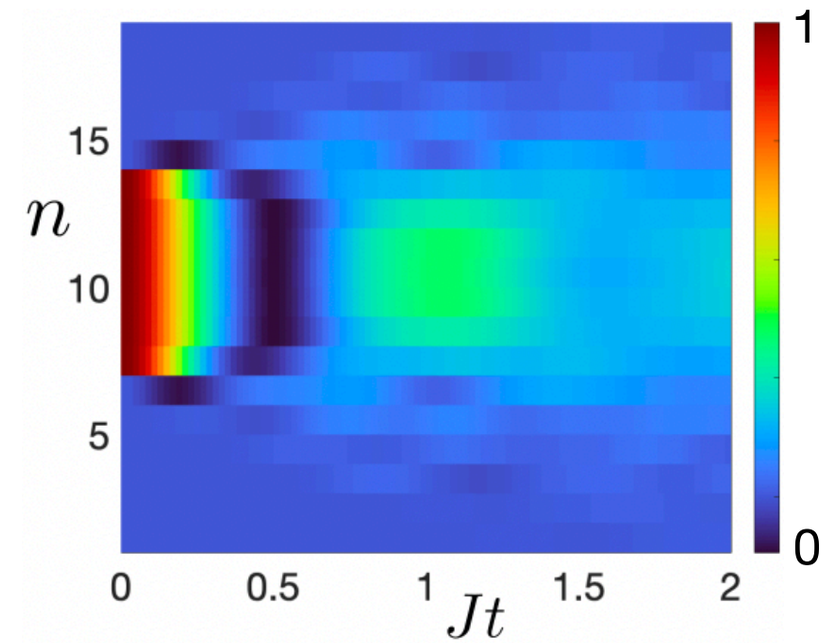
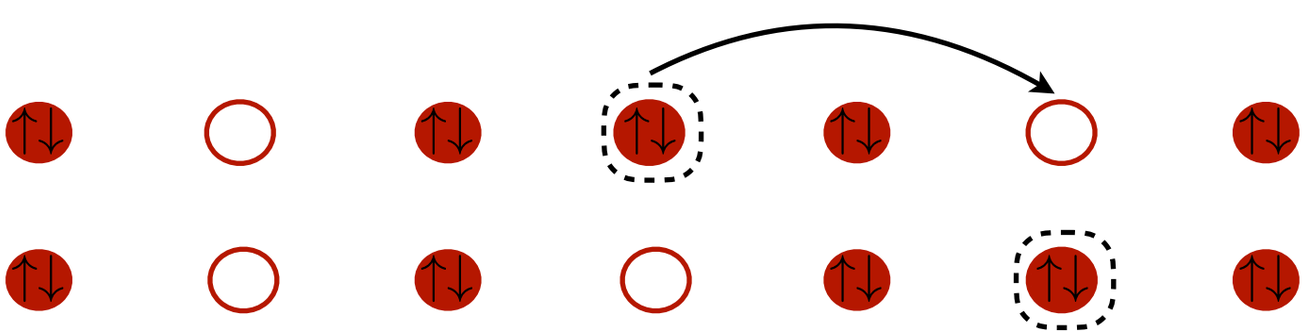


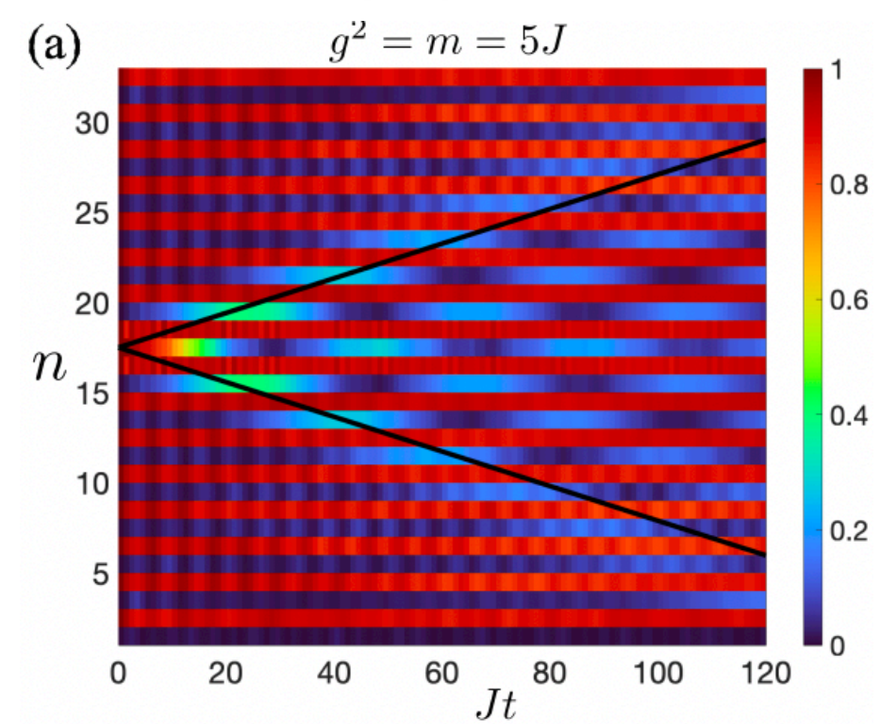
Field



Barion hopping

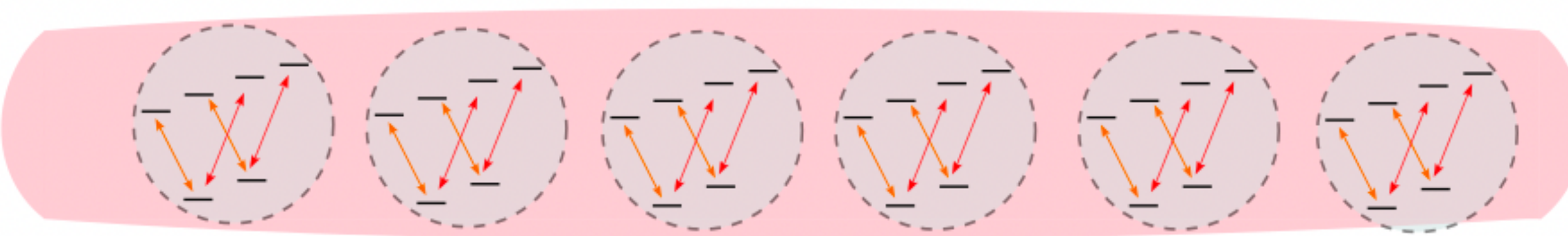


Barion population



Our proposal

Quantum simulation of a SU(2) 1D gauge theory with <u>6 levels ions qudits</u>



See also other qudit based proposals for lattice gauge theories

D. González-Cuadra, T. V. Zache, J. Carrasco, B. Kraus, and P. Zoller Phys. Rev. Lett. ***129****, 160501 (2022);*
T. V. Zache, D. González-Cuadra, and P. Zoller, arXiv:2303.08683 (2023)

nature physics

LETTERS

https://doi.org/10.1038/s41567-022-01658-0

Check for updates

A universal qudit quantum processor with trapped ions

Martin Ringbauer[1]✉, Michael Meth[1], Lukas Postler[1], Roman Stricker[1], Rainer Blatt[1,2,3], Philipp Schindler[1] and Thomas Monz[1,3]

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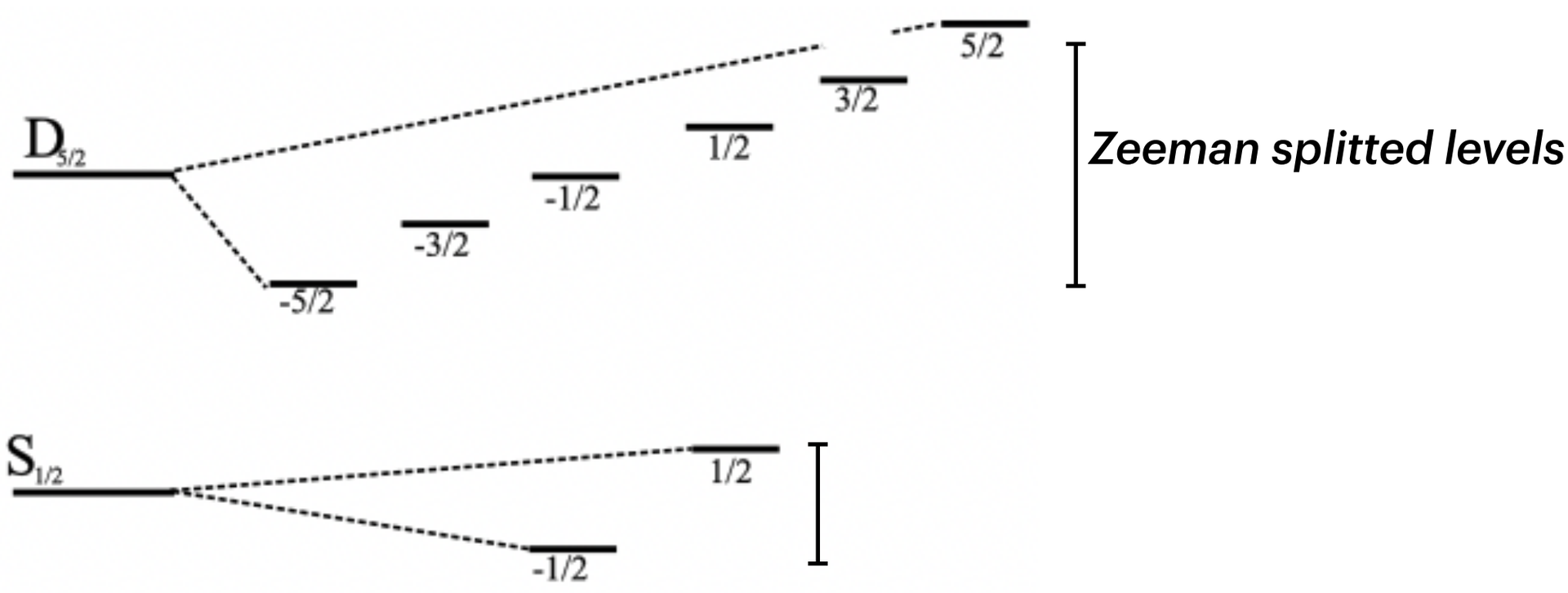

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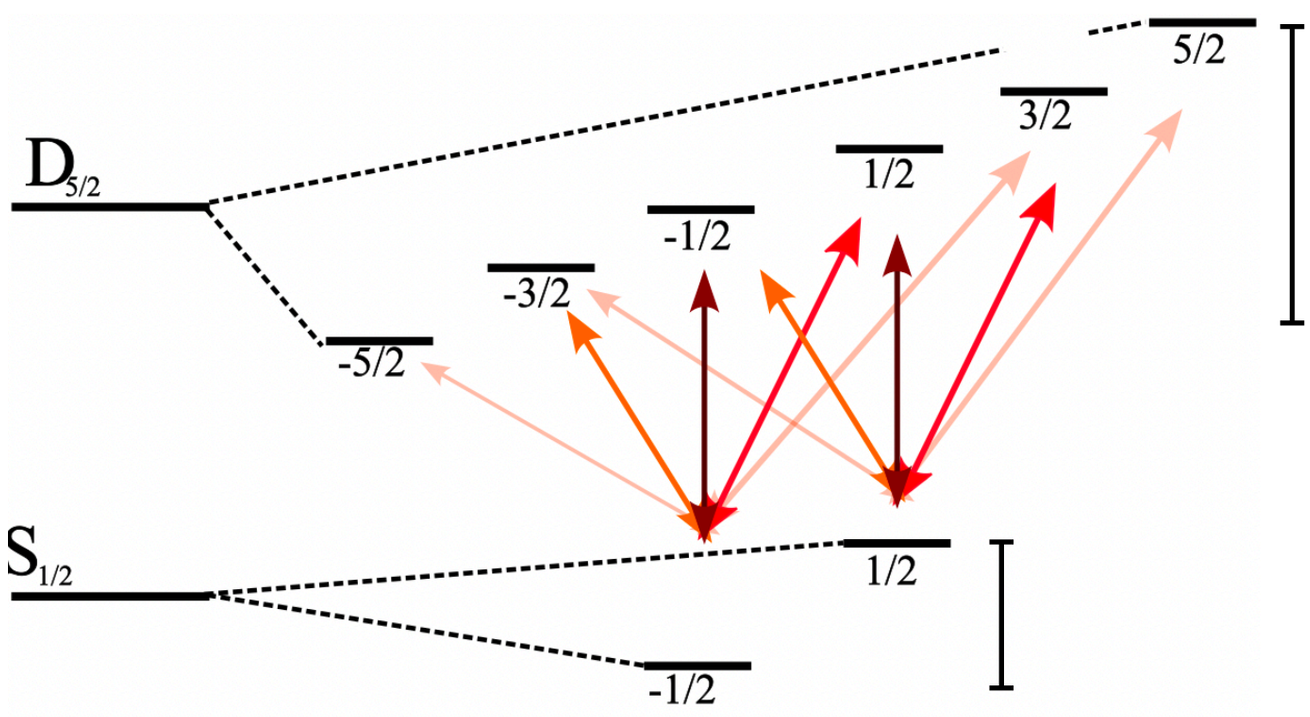

Zeeman splitted levels

$$\Delta m = 0, \pm 1, \pm 2$$

Quadrupole allowed transitions

8 levels fully connected qudit

nature physics

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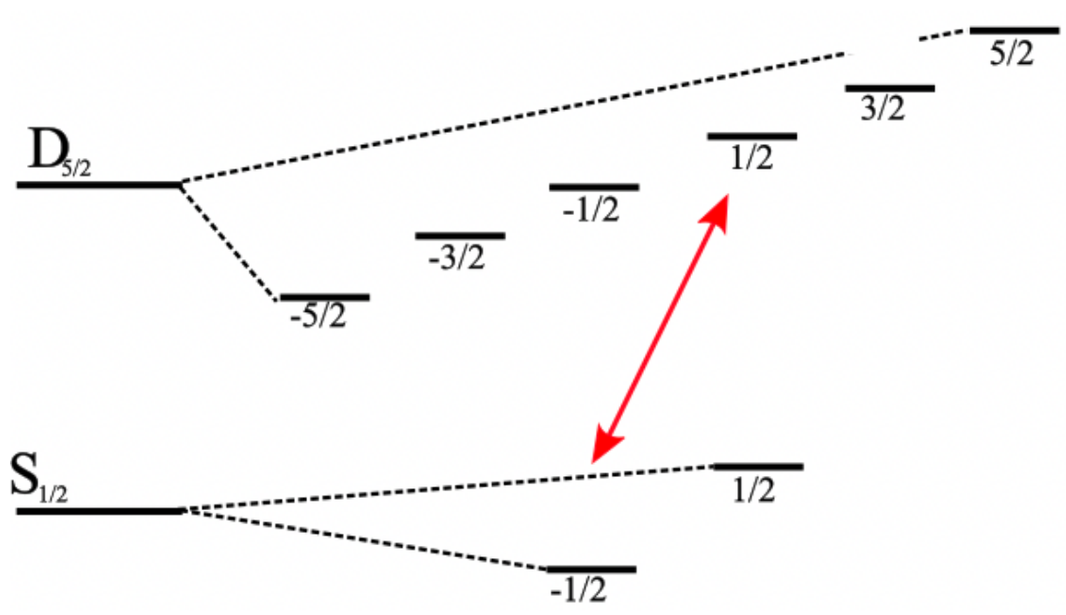

Single qudit operations:
decomposition in single qubit rotations

$$R(\theta, \phi) = e^{-i\theta\hat{\sigma}_\phi/2}$$

High fidelities

nature physics

LETTERS

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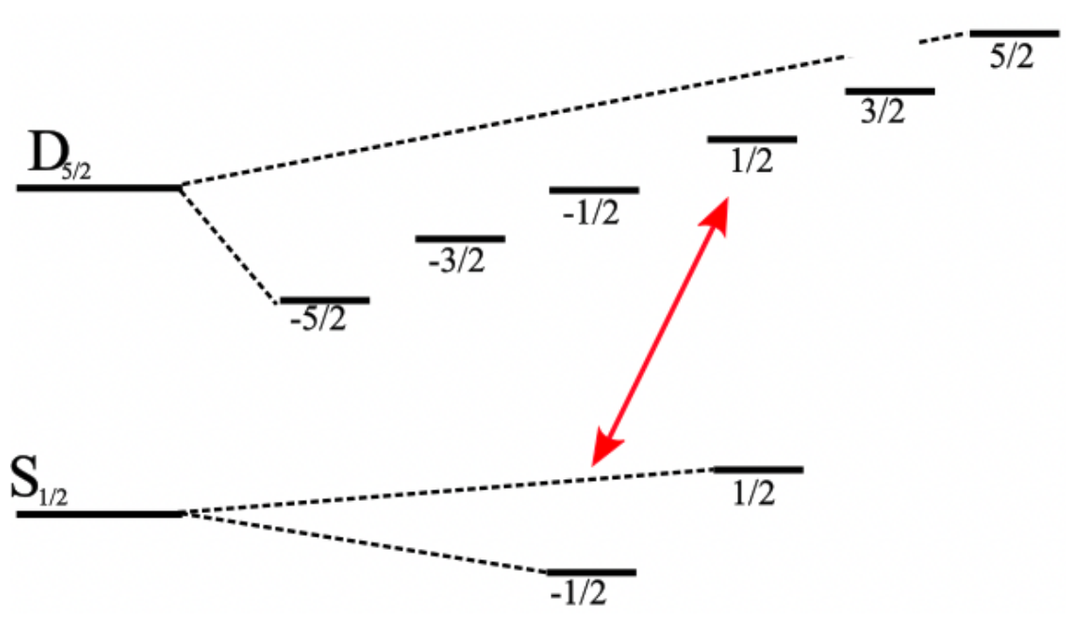

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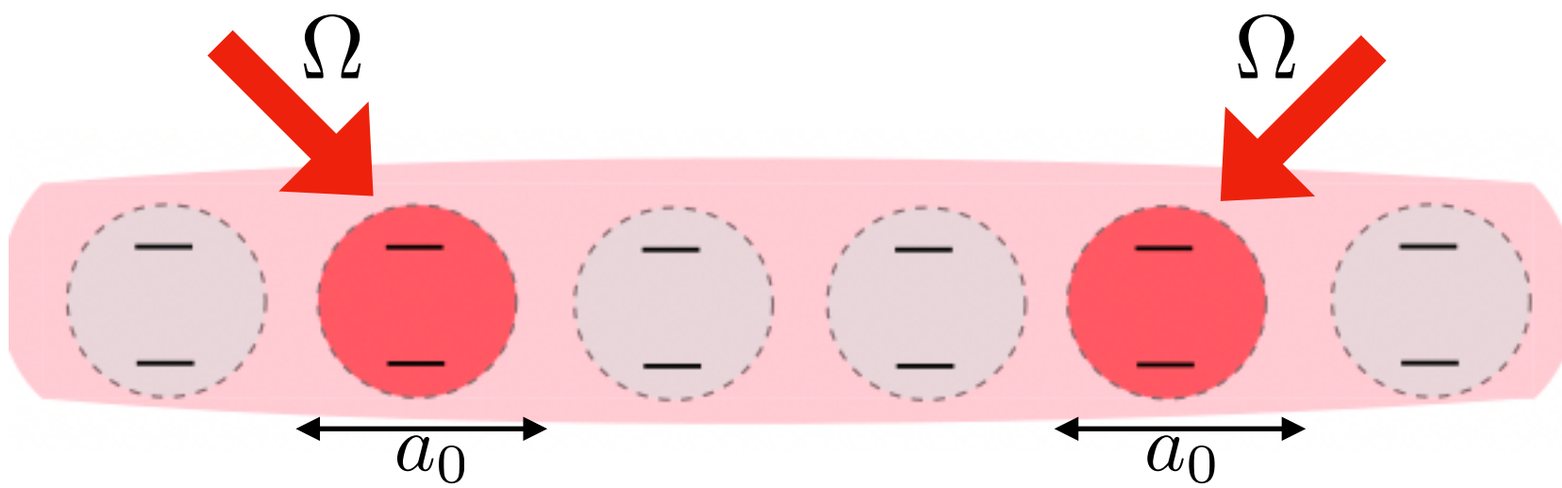
High fidelities

Two qudit operations:
decomposition in Molmer Sorensen gates

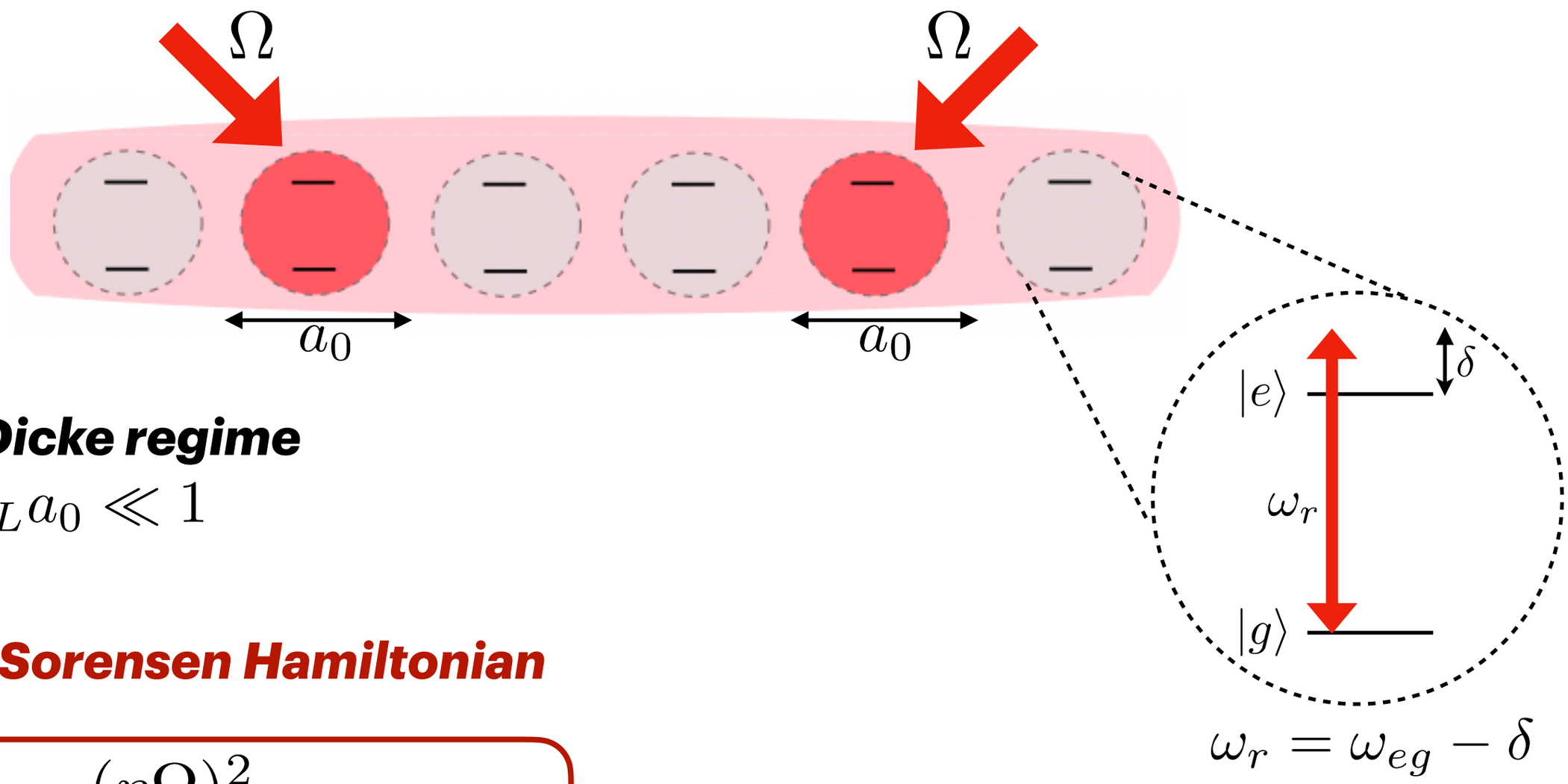
Qubit Molmer Sorensen gate



Lamb-Dicke regime

$$\eta = k_L a_0 \ll 1$$

Qubit Molmer Sorensen gate



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$$\eta = k_L a_0 \ll 1$$

Molmer-Sorensen Hamiltonian

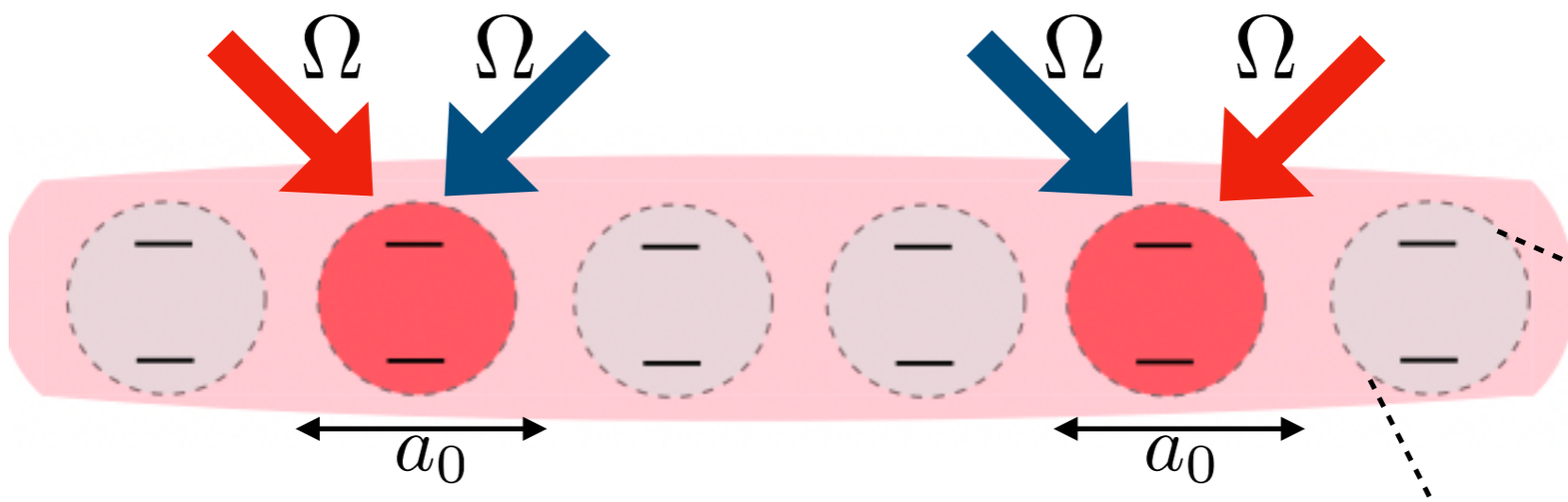
$$\boxed{H_{\mathrm{MS}} \simeq \frac{(\eta\Omega)^2}{2(\nu - \delta)} \hat{\sigma}_{\phi,j} \hat{\sigma}_{\phi',j'}}$$

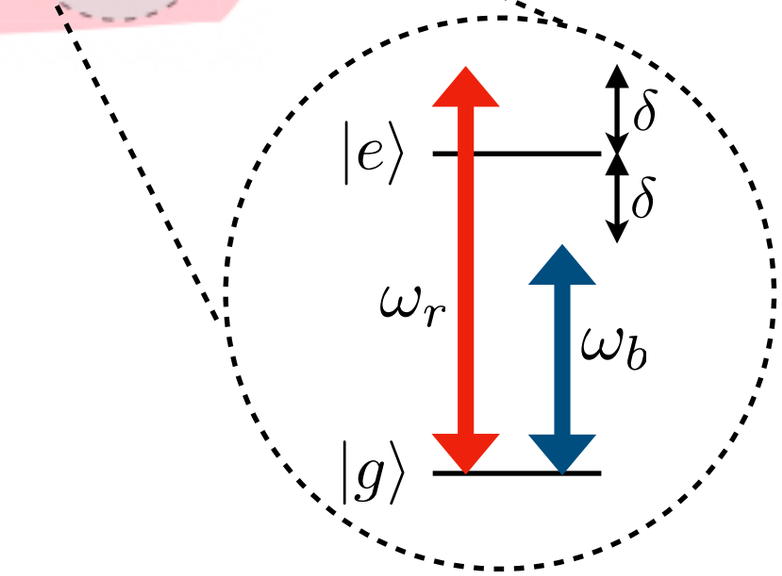
$$\eta\Omega \ll |\nu - \delta|$$

ν vibrational frequency

$\phi \in x - y$ plane

Qubit Molmer Sorensen gate





$$\omega_r = \omega_{eg} - \delta$$

$$\omega_b = \omega_{eg} + \delta$$

Lamb-Dicke regime

$$\eta = k_L a_0 \ll 1$$

Molmer-Sorensen Hamiltonian

$$H_{\mathrm{MS}} \simeq \frac{(\eta\Omega)^2}{2(\nu - \delta)} \hat{\sigma}_{\phi,j} \hat{\sigma}_{\phi',j'}$$

$$\eta\Omega \ll |\nu - \delta|$$

ν vibrational frequency

$\phi \in x - y$ plane

Pairs of lasers:
<u>insensitive to thermal motion</u>

Encoding the model into qudits

$$H = J \sum_j \left[\hat{A}_j^{(1)} \hat{B}_{j+1}^{(1)} + \hat{A}_j^{(2)} \hat{B}_{j+1}^{(2)} \right] + m \sum_j (-1)^j \hat{M}_j + g^2 \sum_j \hat{C}_j$$

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Diagonal matrices:
single qudit operations

Hopping terms

$$\hat{A}^{(1)} = \begin{pmatrix} 0 & & & -\sqrt{2} & & & \\ & 0 & 1 & & & & \\ & 1 & 0 & & & & -1 \\ -\sqrt{2} & & & 0 & -\sqrt{2} & & \\ & & & -\sqrt{2} & 0 & & \\ & -1 & & & & & 0 \end{pmatrix} \sim \hat{\sigma}_x^{n,m}$$

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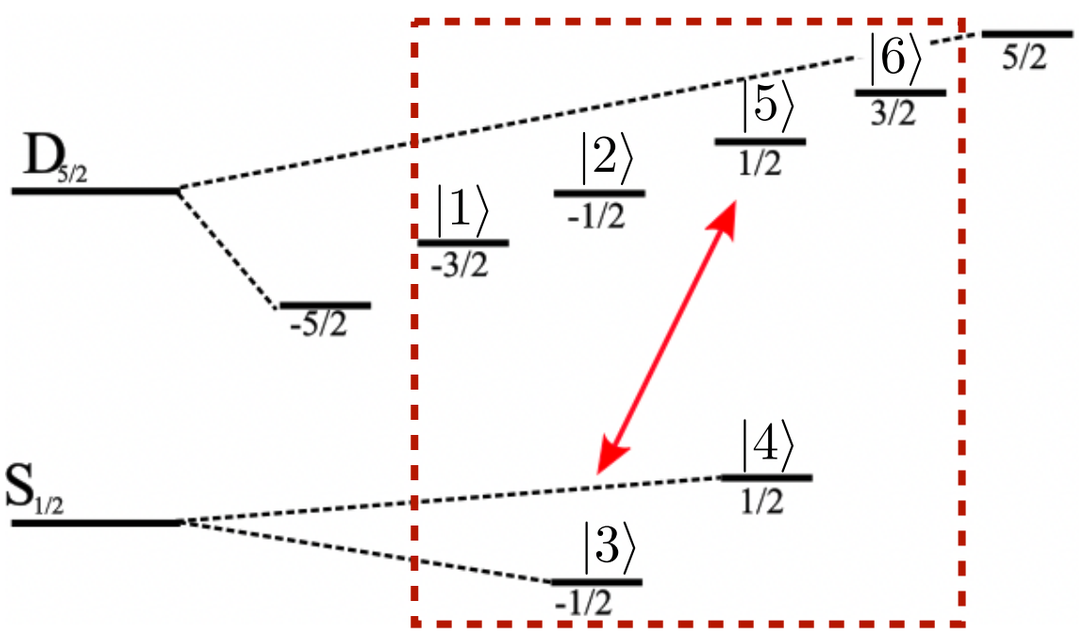

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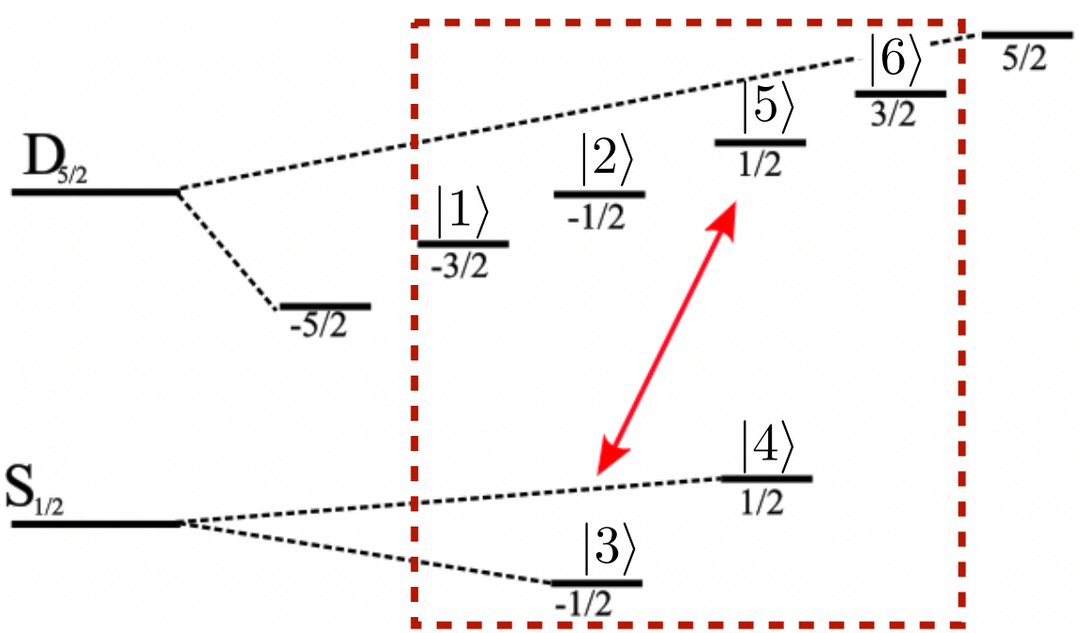

*Decomposed in MS **qubit** gates with **only direct transitions***

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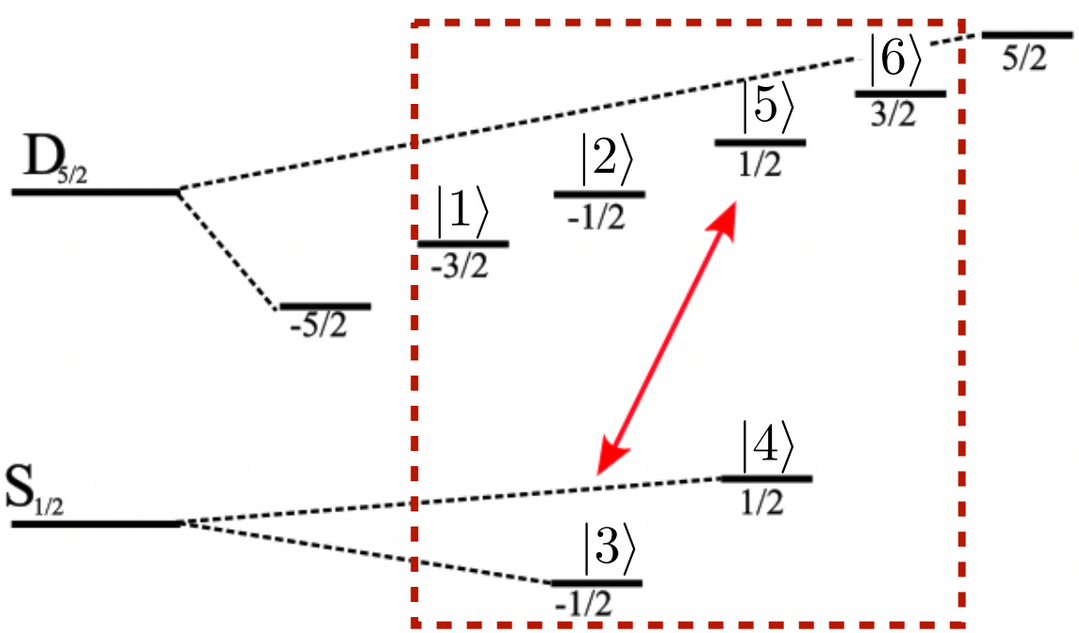
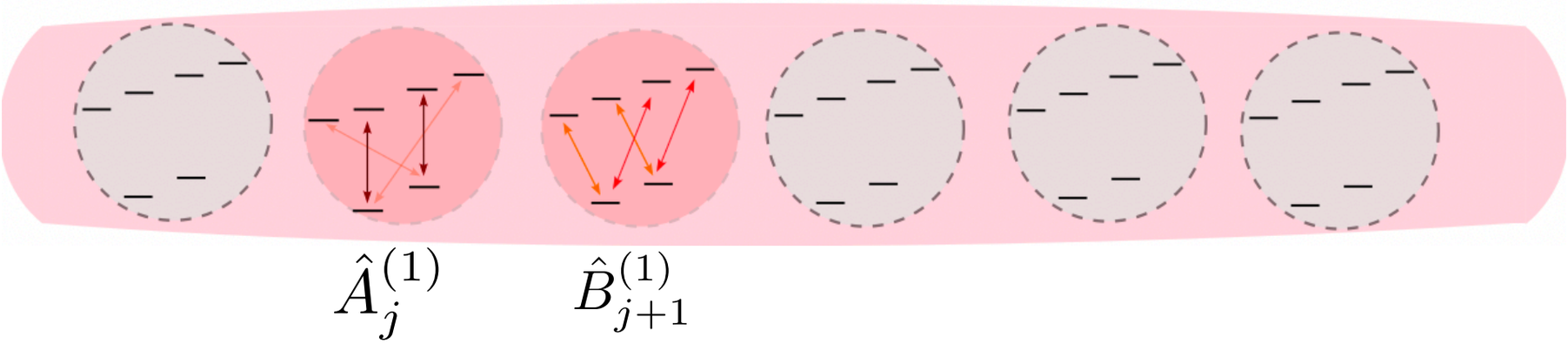


*Decomposed in MS **qubit** gates with **only direct transitions***

32 MS gates necessary...

...but fidelity bad after 10 MS

Qudit Molmer Sorensen gate



***Generalized MS gate for qudits:* simultaneously drive 4 transitions**

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For qudit gates see also: *D. P.J. Low, B. M. White, A. A. Cox, M. L. Day, and C. Senko, Phys. Rev. Research* ***2****, 033128 (2020)*

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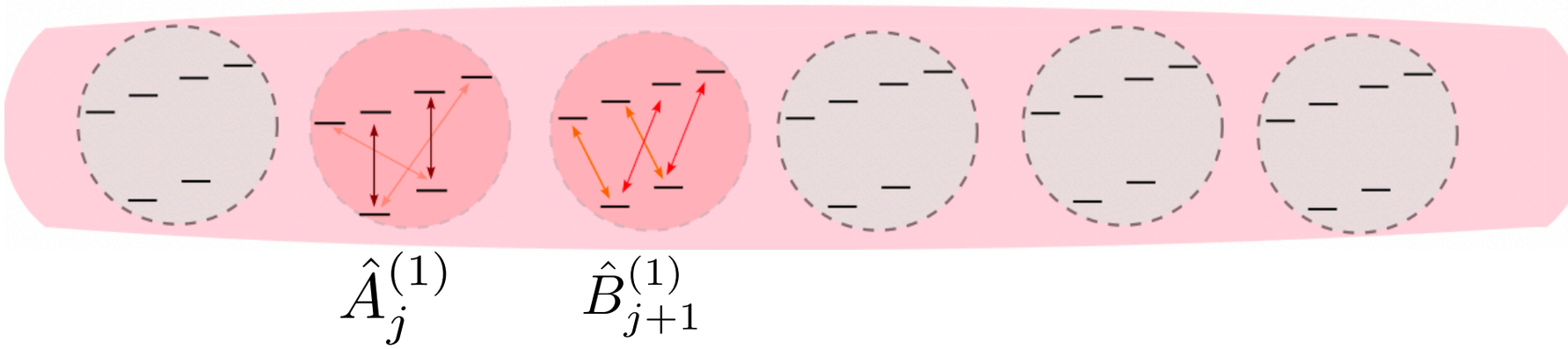

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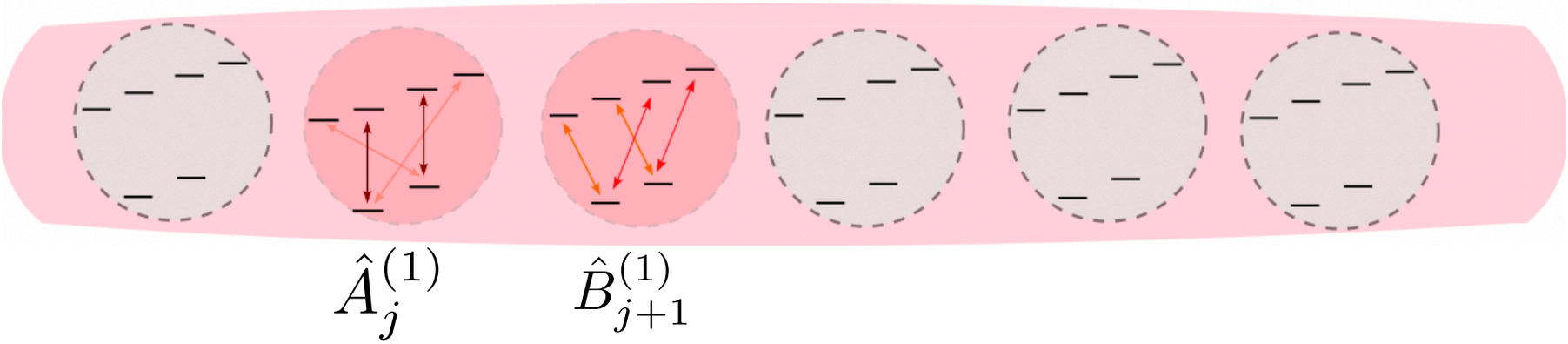

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Price to pay:
unwanted single qudit rotations $(\hat{A}_j^{(1)})^2 \quad (\hat{B}_{j+1}^{(1)})^2$

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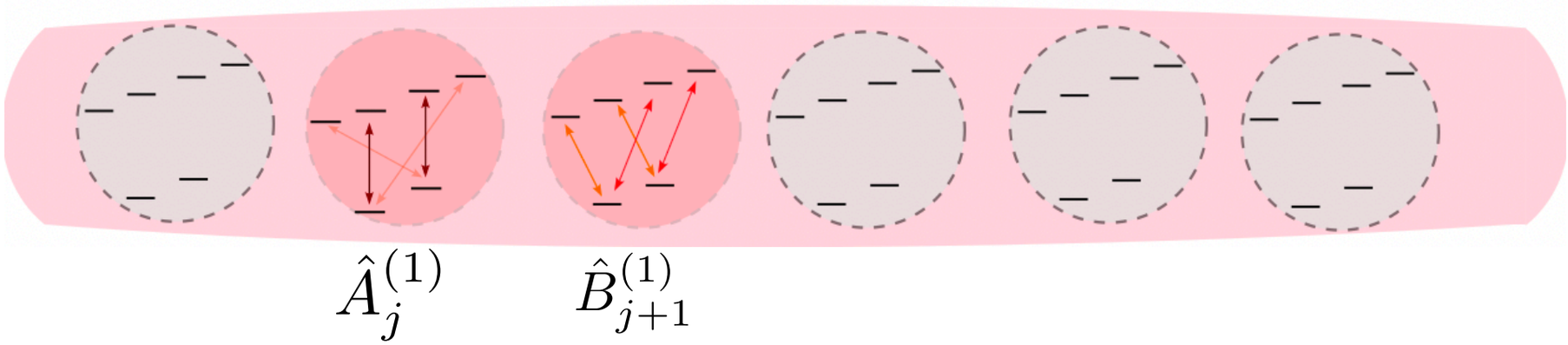

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Just diagonal matrices!

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Digital simulation of the model

- ***Suzuki-Trotter evolution***

$$U(t) \simeq \left(\Pi_j e^{iH_j t_f/n}\right)^n$$

n ***Trotter steps***

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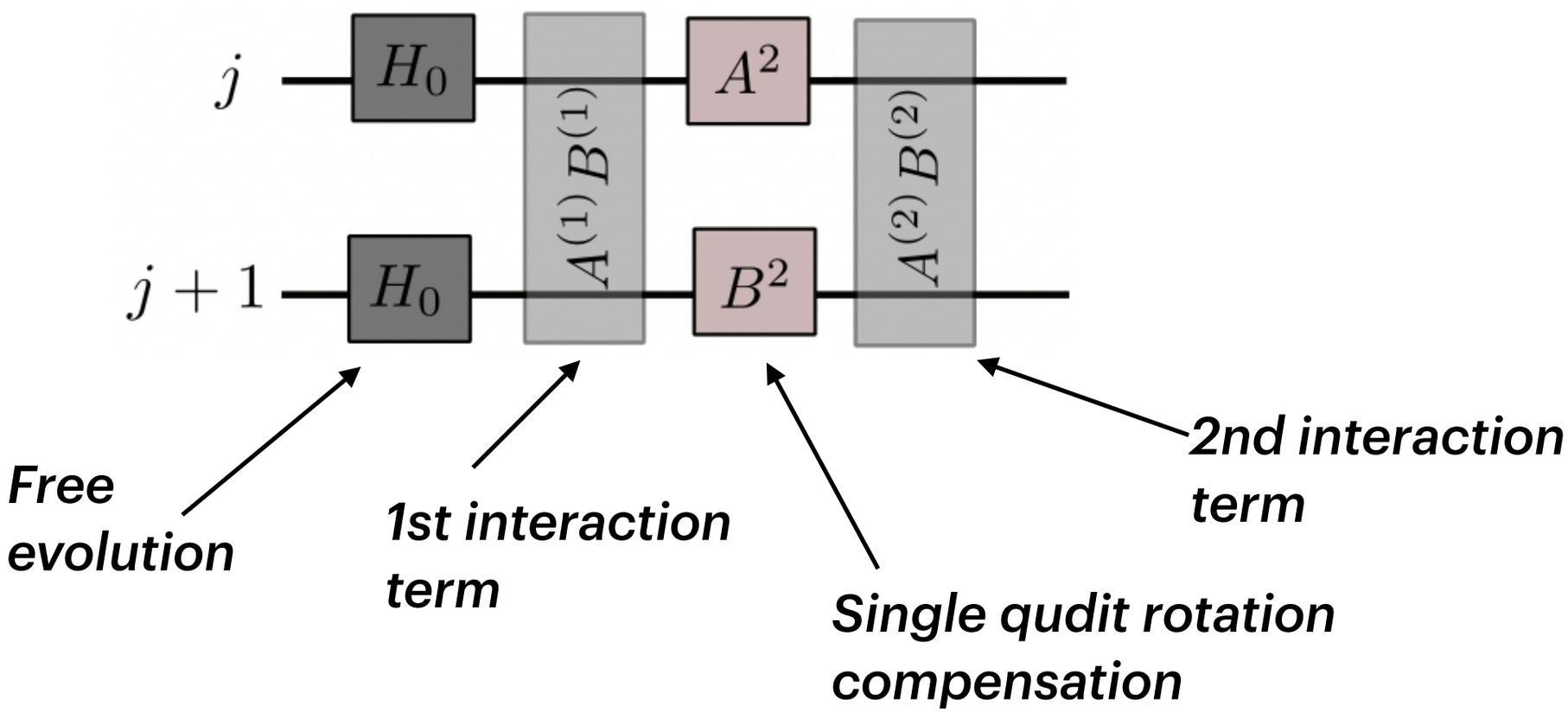
- ***Circuit decomposition***



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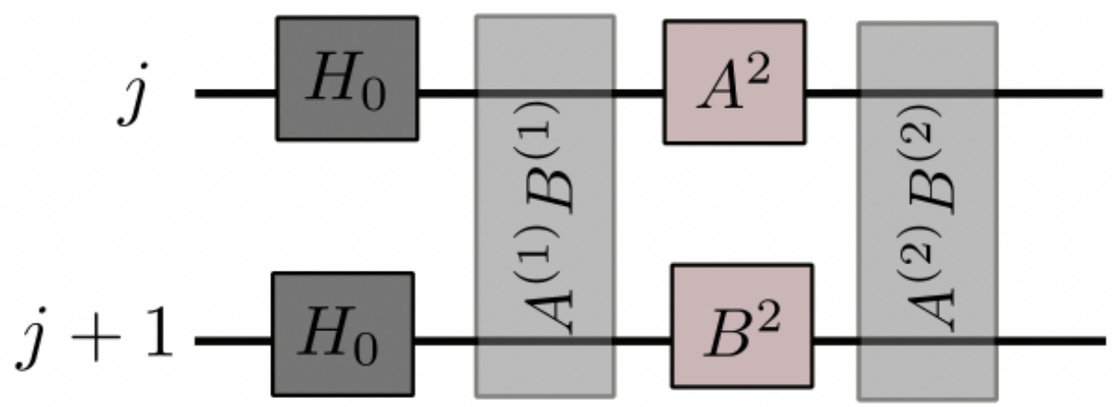
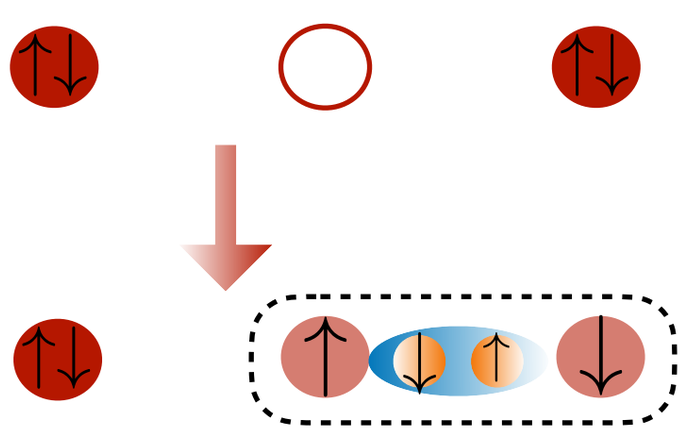
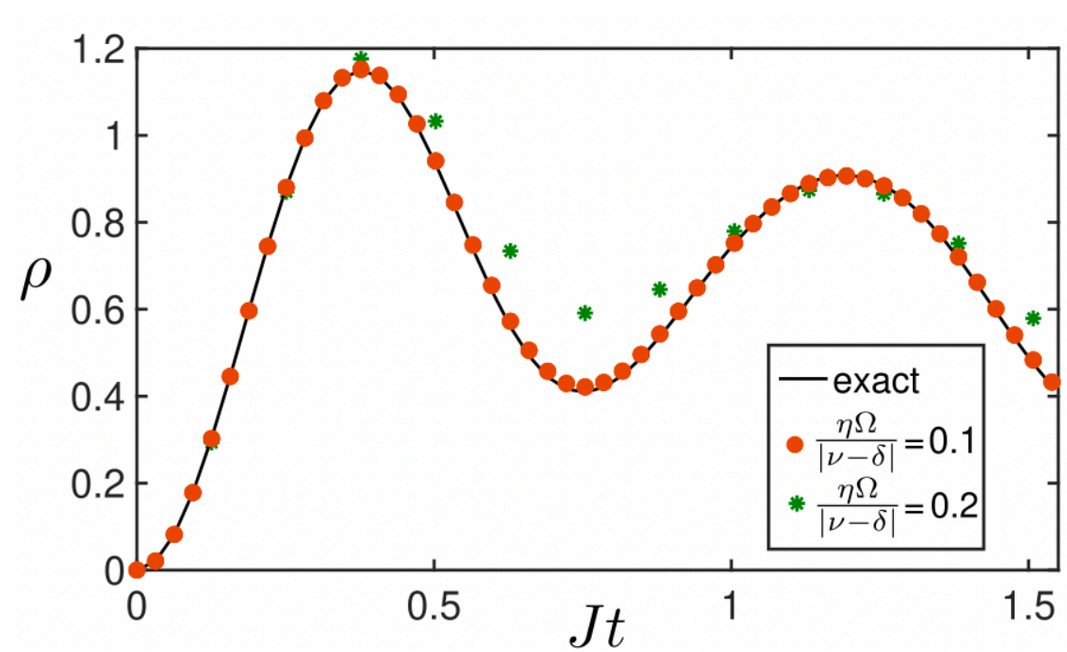


- ***Full simulation with vibrational mode***

Pairs production for 3 sites

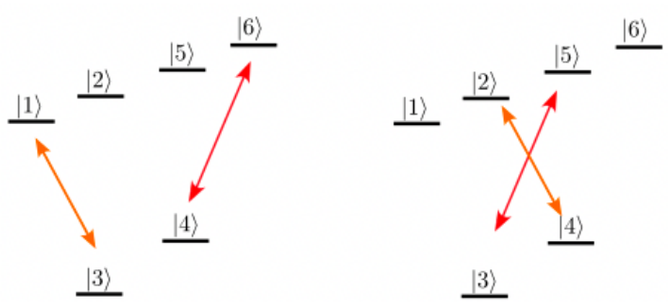


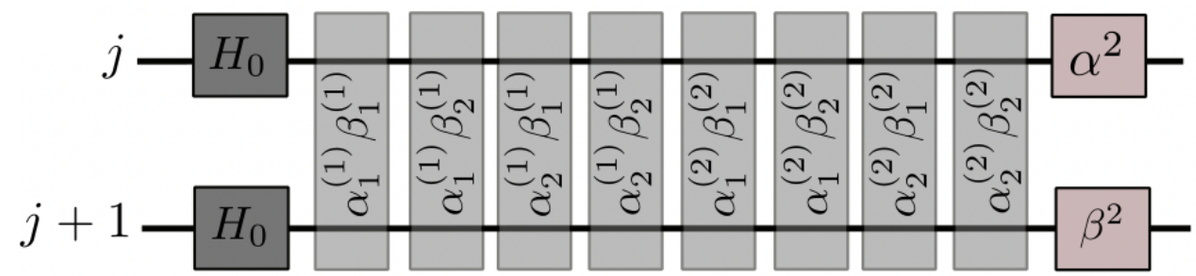


Comments and limitations

- ***Higher use of control resources and <u>calibration problems</u>***

Intermediate protocol with two simultaneously driven transitions





Larger circuit depth (8 MS)

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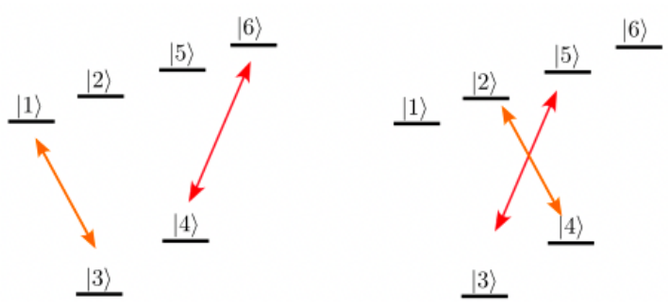

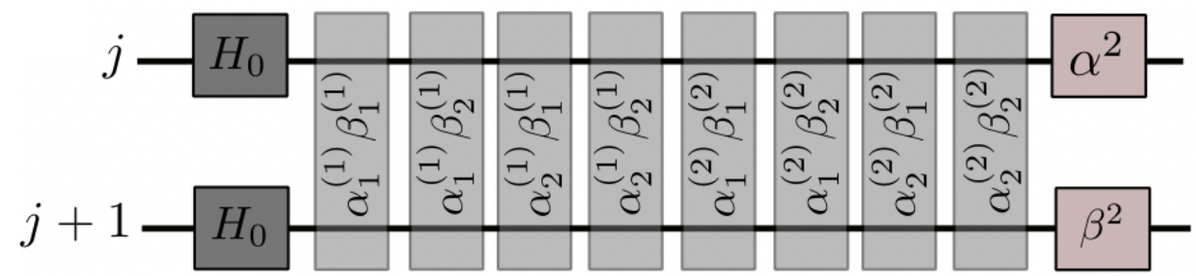

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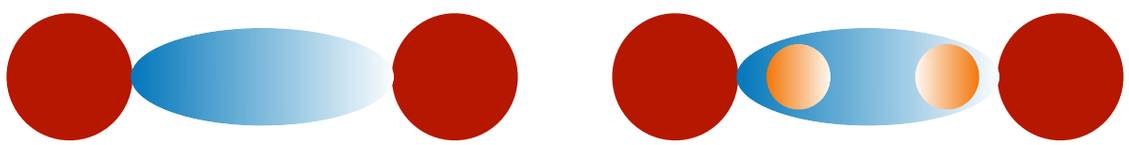
- ***Check link parity constrain with post selection***



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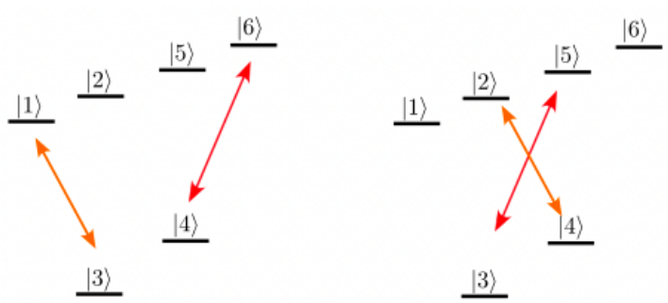

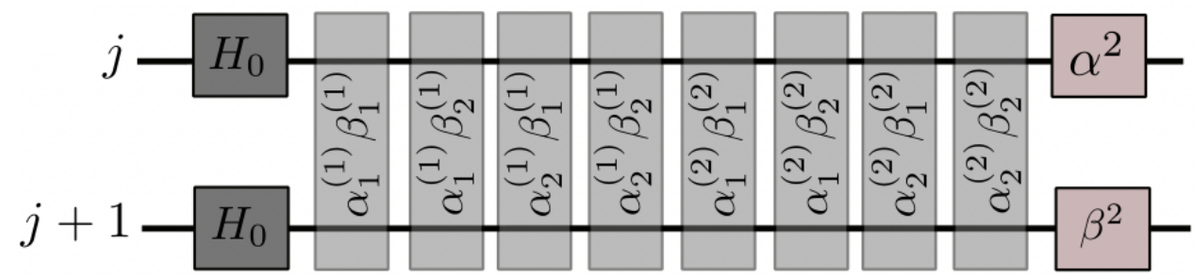

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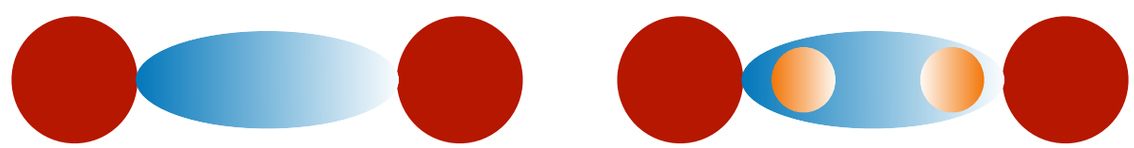
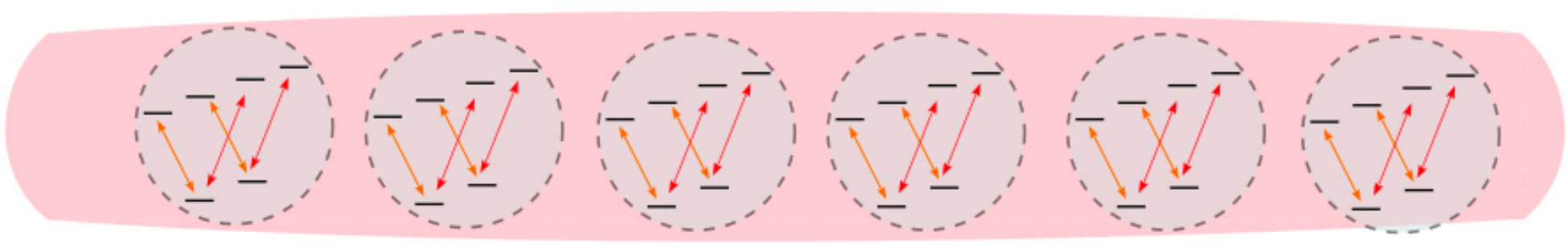


- ***Effect of magnetic field fluctuations.***

  Even if mitigated by magnetic shielding

- ***Include other source of errors:*** *off-resonant driving, imperfect cooling, photon scattering,...*

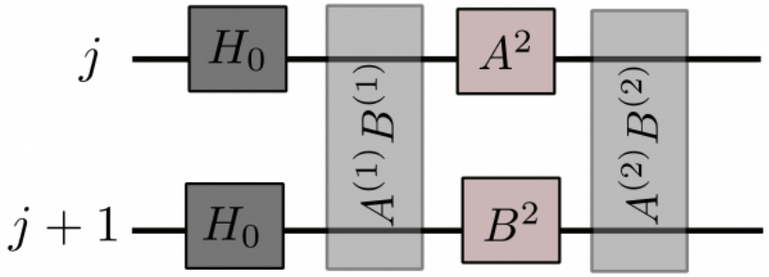
Conclusions



- ***Convenient rishon representation for 1D SU(2) model restricted to 6 dimensions***

- ***Efficient encoding with ions qudit involving only direct transitions***

- ***Shallow circuit for digital simulation using simultaneous MS gates***

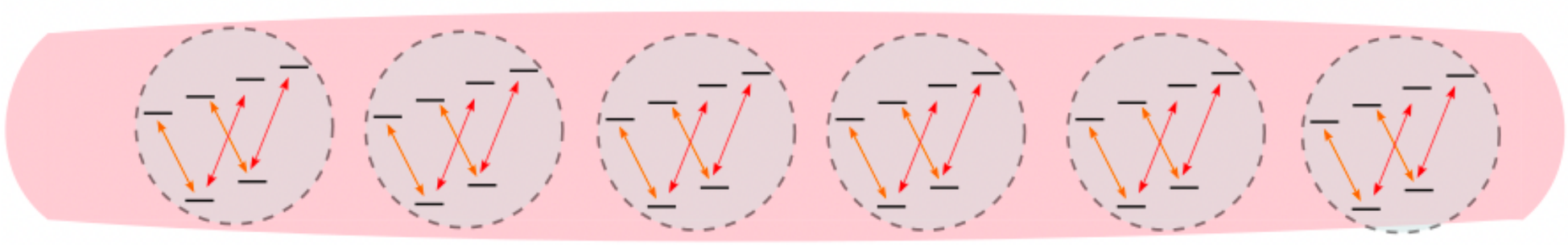


T-NiSQ

QUANTERA

INFN PADOVA

Conclusions



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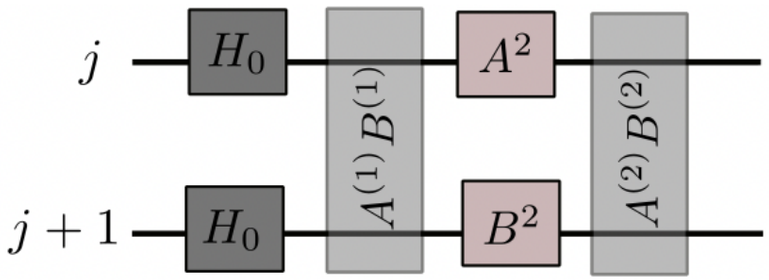

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QUANTERA

Thank you for your attention!

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