

The logo for LAVA (Lattice Virtual Academy) features the word "LAVA" in large, white, sans-serif capital letters. The letters are overlaid on a background of a volcano erupting at night, with bright orange and yellow lava fountains and a dark blue sky. A white dashed grid is superimposed over the letters, and a pattern of white dots is visible on the right side of the image.

LAVA

lattice virtual academy

## Algorithms (Part 1)

Presented by Mike Peardon

## Essentials

- **Review of probability and statistics:** Laws of large numbers, CLT, sampling and properties of sample averages. Variance and covariance.

## Monte Carlo

- **Introducing Monte Carlo:** Why use Monte Carlo? Variance reduction and importance sampling. Examples.
- **Drawing from simple distributions:** Transformation and rejection methods. Box-Muller,  $SU(2)$  heatbath.

## Markov Chain Monte Carlo

- **Markov chains:** Properties and convergence. Detailed Balance. Metropolis.
- **Local (bosonic) theories:** Gibbs samplers (Cabbibo-Marinari). Over-relaxation. Example:  $SU(2)$  Yang-Mills.
- **Autocorrelations:** Definitions and examples. Connection to physics - correlation lengths and scaling, topology freezing.

## Fermions

- **The fermion determinant:** Path integral and Grassmann integral. Gaussian representation and pseudofermions. Non-locality problem.

## Hybrid Monte Carlo

- **Hamiltonian dynamics:** Equations of motion. Dynamics on Lie manifolds. Symplectic integrators (leap-frog) and step-size errors. Metropolis, reversibility and area preservation.
- **Fermions and pseudofermions:** Equations of motion. The resulting linear systems.
- **Advanced methods:** Higher-order and Omelyan. Multiple time-scales. Force-gradient, Hasenbusch.

## Correlation functions

- **Hadronic two-point functions:** Wick contractions. Point and smeared sources.
- **Disconnected diagrams:** All-to-all techniques and stochastic representations.

## Multi-level methods

- **Introduction and examples:** Factorising path integrals. Fermion methods.

## Linear systems

- **Krylov spaces:** conjugate gradient and BiCG. Preconditioning (even-odd).
- **Eigensolvers:** Lanczos and Arnoldi. Tridiagonal forms. re-orthogonalisation and round-off.
- **Deflation and Multigrid:**