Hadron Physics

- Prerequisites: Lattice Essentials, some Renormalization and Improvement, some Data Analysis for the exercises.
- Topic too broad. Must be split into two: Hadron structure and Hadron spectroscopy.
- Some things in common: hadronic interpolators, group theory, two point functions, use of translational invariance, . . .
- Modern hadron spectroscopy includes multiple hadron scattering states. It is a huge topic on its own. We have not devised a syllabus for this but can discuss.
- Hadron structure including unstable particles is at its infancy. For the moment restrict ourselves in hadron structure to hadrons that do not undergo strong decay.
  We only cover the pion and the nucleon as examples.
- Form factors for semileptonic (and leptonic) decays or even $K \to \pi\pi$ are also hadron structure and, in the latter case, related to hadron spectroscopy.
  These will be covered elsewhere: "Precision Frontier", including flavour $\to$ Gregorio.

Course Strategy

- Should be self-contained, including the quantum field theory (QFT) necessary.
- Suggestion also for other lectures: illustrative examples desirable.
  But review of lattice results or just “selling” own work undesirable.
- One will have to brush over many technicalities to stay focused.
  Therefore, there must be auxiliary (text) material and exercise problems.
- Three aspects: phenomenological application, QFT, lattice QCD methods.
  The guiding idea is to highlight specific techniques.
  1. Start each topic with phenomenological motivation.
  2. Develop connection between physics and specific QCD matrix element(s).
  3. Discuss lattice implementation and specifics of lattice approach (renormalization etc.).
- Example: Dark Matter.
  1. Phenomenology: Dark Matter and detection of WIMPs.
  2. QFT: σ terms, decoupling at heavy flavour thresholds, expectation.
  3. Lattice: methods to compute quark line connected and disconnected diagrams.

(incomplete) Course Strategy II

- Example: neutrino oscillations.
  1. Phenomenology: neutrino sector of the (new) SM, long baseline neutrino mixing experiments.
  2. QFT: Connection of cross section with axial and vector matrix elements, Lorentz-decomposition of the axial matrix element into form factors, axial Ward identity, parametrizations of form factors.
  3. Lattice: computation method, excited states (and connection to ChPT).
- Example: DIS.
  1. Phenomenology: Deep inelastic scattering.
  2. QFT: structure functions, PDFs, collinear factorization, conformal spin/twist expansion, OPE, Mellin transform and convolution, DGLAP evolution, sum rules.
  3. Lattice: computation and renormalization of matrix elements, mixing with lower dimensional operators.
- Example: bi-local operators for PDFs etc. and position space methods.
- Example: exclusive processes, deeply virtual Compton scattering (DVCS) and generalized parton distributions/form factors (GPDs/GFFs).

Syllabus (not yet appropriately sorted)

- The hadron spectrum: stable particles and resonances. What will be covered and what not. (Hadron spectroscopy is a separate topic.)
- Mention: isoQCD vs QCD. Why photons make trouble. What corrections to expect.
- Recap: 2-point functions, masses/energies, amplitude.
- Overview what matrix elements will be of interest.
  Example: $\pi \to \ell\nu_\ell$: $\langle\Omega|A_\mu|\pi(p)\rangle = i\sqrt{2}F_\pi p_\mu$. Matrix element from a 2-pt function ($1 \to 0$ process). Local operator, dimension, Lorentz-structure, normalization.
- Differential cross sections at low and high momentum transfer and connection to form factors and structure functions. Inclusive vs. exclusive.
- Charges: interaction with Dark Matter at small recoil. Decoupling of heavy flavours.
- Recap: interpolators, parity, spin/helicity/chirality, quantum numbers, normalization of states, baryon spinors.
- Charges from 3-pt functions in the forward limit ($1 \to 1$ process).

Syllabus II

- Lattice techniques for isovector matrix elements.
- Techniques for quark line disconnected contributions.
- Electron and neutrino scattering (electromagnetic and axial form factors, proton radius, neutrino oscillations).
- The continuum, infinite volume limit at the physical quark mass point.
  ChPT to understand the volume- and mass-dependence.
- Excited state pollution with connection to ChPT.
- Neutron EM dipole moment, other charges and form factors, connection to BSM searches.
- DIS, structure functions, twist as conformal spin, collinear factorization and OPE, Mellin transform and DGLAP evolution.
- Moments of PDFs as local matrix elements/generalized charges, hard coefficient functions.
- The energy-momentum tensor and its renormalization. Higher moments and operator mixing.

Syllabus III

- Light front quantization in the continuum: why and how. Relation to the OPE.
- Deeply virtual Compton scattering (DVCS) and GPDs/GFFs.
- Adding k_T: a zoo of different functions characterizing baryon structure. (Translation table as handout/auxilliary material?)
- Lattice structure functions: quasi-/pseudo-/etc-PDFs: basics.
- Collinear factorization of non-local lattice matrix elements.
- Results on quasi-PDFs etc. and challenges.
- Moments of LCDAs ($1 \to 0$ processes).

Existing material

- INT Summer School on Problem Solving in Lattice QCD (2021)
  https://sites.google.com/uw.edu/lqcdschool2021/home
  Material not available online but one can ask.
  (Some videos covering other topics are on youtube.)
- INT Summer School on Lattice QCD for Nuclear Physics 2012
  https://archive.int.washington.edu/PROGRAMS/12-2c/Lectures.html
  Many resources including James Zanotti on Hadron Structure.
- Yong Zhao at Methods of Effective Field Theory and Lattice Field Theory 2021
  https://indico.ph.tum.de/event/6882/timetable/
  Parton Distributions Functions, TMDs etc.
- Meinulf Göckeler at Dubna Summer School 2011
  http://theor.jinr.ru/~diastp/summer11/lectures/Goeckeler.pdf
  Hadron Structure from Lattice QCD.

Existing material II

- Constantia Alexandrou introductory article 2011
  https://arxiv.org/pdf/1111.5960.pdf
  Hadron Structure in Lattice QCD.
- Arwed Schiller at Summer School on Lattice QCD for Beginners ECT* 1998
  https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=2be1588d9493856ea8d2cdefa1cd17e3e726eb49
  Introductory Lectures on the structure of the hadrons from the Lattice.
- ???

Exercises

- Illustrate the concepts introduced in the lectures, include aspects which are skipped/skimmed over. Very important for the learning success and self assessment.
- Some challenges: different levels of technical knowledge, unsupervised.
    - Analytical exercises: provide solutions.
    - Numerical exercises:
        - Availability of compilers/libraries (versions) — keep any code/scripts provided simple and well commented.
        - Providing data — need a simple (text) format.
        - Problems of debugging, provide solution code or complete code that the student just has to modify.
        - Transparent and precise instructions.
    - Indicate background knowledge that is assumed.
- Keep in mind when developing the lectures. Develop after the content/structure is clear (avoid replication across lectures).

Example: INT Summer School on Problem Solving in Lattice QCD

Hadron structure exercises

- Sigma terms and direct dark matter detection
  - Wick contractions for the three-point functions
  - Spectral decomposition of the three- and two-point functions
  - Extracting the scalar matrix elements via the Feynman-Hellmann theorem
  - Mass decomposition of the nucleon: derivation of heavy quark sigma terms
  - Calculating the WIMP-nucleon scattering cross-section
- Position space methods and parton-distribution functions
  - Comparing different unpolarised PDF sets
  - Moments of unpolarised PDF data sets
  - Transforming the unpolarised PDFs to Ioffe-time

Other topics covered: High-Performance Computing, Hadron Spectroscopy and Resonances, Flavor Physics, Nuclear Physics on the Lattice, Nonzero Temperature and Density QCD, Machine Learning, and Quantum Computing and Simulation. All exercises used python.

Lattice practices, . . . but material on the exercises generally not available.

General thoughts

- Longer exercises possible: Monte-Carlo simulation of a quantum mechanical system (Metropolis, heatbath, over-relaxation algorithms, correlation functions, . . .), scalar field theory, . . .
- Tutorials for main software packages: MILC, Chroma, openQCD, GRID/GPT, . . .
- Exercises/tutorials need to be well tested before being made available.
- Designing and testing exercises is some work. Involve additional people.

Conventions

- These considerations are for "experts" only, not part of the syllabus.
- Suggestion: use Euclidean spacetime for time evolution and $\gamma_5(\not{D}+m)\gamma_5(\not{D}+m)^\dagger$ but Minkowski conventions regarding Γ-matrices and $q^2 = q_0^2 - \vec{q}^2$ etc.
- Minkowski: $\gamma_0\gamma_\mu\gamma_0 = \gamma_\mu^\dagger$. Therefore, $(i\overrightarrow{\not\partial} - m)u = 0 \Rightarrow \bar{u}(i\overleftarrow{\not\partial} - m) = 0$ with $\bar{u} = u^\dagger\gamma_0$.
- Euclidean: $\gamma_4\gamma_4\gamma_4 \neq -\gamma_4^\dagger$. If $(\overrightarrow{\not\partial} + m)u = 0 \Rightarrow \bar{u}(\overleftarrow{\not\partial} + m) = 0$ then $\bar{u} \neq u^\dagger\gamma_4$.
- Just replacing $\gamma_j^E = i\gamma_j$, $\gamma_4 = \gamma_0$, $\gamma_5^E = \gamma_5$ so that $\{\gamma_\mu^E, \gamma_\nu^E\} = 2\delta_{\mu\nu}\mathbb{1}$ is not sufficient.
- As a consequence, 2-point functions $\langle[\bar{q}_1\Gamma q_2](t)[\bar{q}_1\Gamma q_2]^\dagger(0)\rangle$ are sometimes negative, due to $\gamma_4\Gamma\gamma_4 = \pm\Gamma^\dagger$, depending on Γ. False assumptions of lattice codes must then be corrected by hand, e.g., by turning around the sign consistently at the sink.
- We prefer to keep the Γ-algebra in Minkowski space and provide a translation table for the results. (Something similar is done in perturbation theory, where only the d-dimensional integrals are computed in Euclidean spacetime.)

Conventions II

- Note that the concrete γ-representation [Dirac ($\gamma_5 \neq \text{diag}(\cdots)$), chiral ($\gamma_5 = \text{diag}(-1,-1,1,1)$) or DeGrand-Rossi ($\gamma_2 \mapsto -\gamma_2$, $\gamma_5 = \text{diag}(1,1,-1,-1)$)] is not needed, as long as $\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}\mathbb{1}$. However, switching between chiral and DeGrand-Rossi means right-$\longleftrightarrow$left-handed.
- Ensembles can be generated with other conventions than those used in the measurement since the quarks are integrated out.
- Within measurements it is important that the same γs are used in the operators/interpolators as in the propagator, in particular γ_μ must correspond to the hopping in the μ direction. This can be an issue when computing connected and disconnected diagrams separately.
- Baryons are Fermions! This means the interpolators have a spin index α and the spinors of the created states a helicity λ: $\sum_\lambda \bar{u}_N^\lambda(p)\frac{1}{2}(\mathbb{1}+\gamma_0)u_N^\lambda(p) = 2(E(\vec{p}) + m_N)$. This can be notationally cumbersome and may distract from the physics $\to$ hand-out.
- $\Rightarrow$ additional written notes and problems!