Hadron Vacuum Polarization contribution to a_μ on the lattice

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General description

Level = Beginner [at least for most lectures]

Required/Assumed knowledge for the first lecture

Calculation of two-point correlation functions.

Basics of hadron spectroscopy.

Basics of integration over Grassmann variables.

Notion of continuum and infinite volume limits.

For the rest of the lectures, it would be good to know. . .

Definition of magnetic moment in QFT. Perturbative (QED + EW) contributions to the muon anomalous magnetic moment a_μ. HVP from dispersive relations.

QED and strong isospin-breaking corrections to isospin symmetric QCD.

[probably only required for few advanced lectures].

Proposal for a first introductory lecture on the HVP

Goals:

- Provide definition of the HVP.
- Introduce the time-momentum representation of the HVP.
- Define connected and disconnected Wick contractions.
- Introduce flavour decomposition for connected current-current correlator.
- Hierarchy between flavour contributions.

The idea is to have a first 10-15 minutes long introductory video on the HVP in the style of S. Hands trial introductory lectures, e.g. http://pcwww.liv.ac.uk/~shands/LaVA/fermions_intro.mp4

[Setting the stage]

I assume you all have familiarity with the QFT definition of anomalous magnetic moment in terms of the muon-photon vertex, however repetitia iuvant.

$$\langle \mu(p')|J^\rho_{\rm em}|\mu(p)\rangle = -ie\bar{u}(p')\Gamma^\rho(p',p)u(p), \qquad J^\rho_{\rm em} = e\bar{\mu}\gamma^\rho\mu$$

Lorentz invariance, electromagnetic Ward-identity and parity, constrain general structure of $\Gamma^\rho(p',p)$

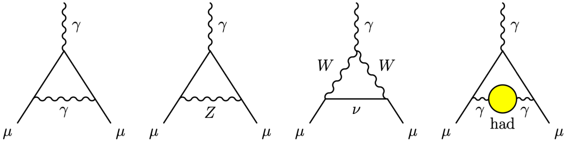
$$\Gamma^\rho(p',p) \overset{q=p'-p}{=} \gamma^\rho F_1(q^2) + \frac{i}{2m_\mu}\sigma^{\rho\nu}q_\nu F_2(q^2), \qquad \sigma^{\rho\nu} = \frac{i}{2}[\gamma^\rho,\gamma^\nu]$$

Exercise(?): derive this form-factor decomposition

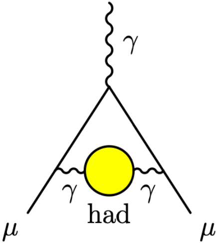
Electric charge of $\mu^\pm$ is $\pm e \implies F_1(0) = 1.$ Muon anomaly $a_\mu \equiv (g_\mu - 2)/2 = F_2(0).$

At tree level $F_2 = 0$, beyond tree level SM (and BSM) fields contribute to a_μ via loops. E.g. :

[Source: PDG]



[The (LO-)HVP contribution]



	contribution to $a_\mu \times 10^{10}$
QED	11658471.9 ± 0.0
EW	15.4 ± 0.1
LO-HVP [BMWc-20, from lattice]	707.5 ± 5.5
LO-HVP [from dispersive approach]	693.3 ± 2.5
Hlbl [from dispersive approach]	9.2 ± 1.9

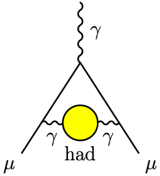
a_μ is completely dominated by QED contributions.

However, main uncertainty comes from non-perturbative QCD contributions. In particular from the LO-HVP.

The LO-HVP can be computed also using dispersion relations, which relate it to the cross-section for e^+e^- annihilation into hadrons (see appropriate section). Cross-section data are available from many experiments (KLOE, BABAR, BES-3, . . .).

However, the dispersive determination is NOT a first principle SM calculation, since e^+e^- data can be polluted as well by New Physics effects!!

[The LO-HVP on the lattice]



$$\equiv a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 \underbrace{f(Q^2, m_\mu)}_{\text{leptonic kernel}} \hat{\Pi}(Q^2), \qquad \hat{\Pi}(Q^2) = 4\pi^2 \left[\Pi(Q^2) - \Pi(0)\right]$$

$\Pi(Q^2)$ is the contribution to the vacuum polarization from quark anti-quark loops:

$$\Pi^{\mu\nu}(Q) \equiv \int d^4x\, e^{iQx} \langle j^\mu_{\mathrm{em}}(x)\, j^\nu_{\mathrm{em}}(0) \rangle = \left(Q^\mu Q^\nu - Q^2 g^{\mu\nu}\right) \Pi(Q^2)$$

E.m. quark current $j^\mu_{\mathrm{em}} = \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d - \frac{1}{3}\bar{s}\gamma^\mu s + \frac{2}{3}\bar{c}\gamma^\mu c$ (+ bottom,top, treated perturbatively)

Time momentum representation: Choose $Q_\mu = \{\omega, 0, 0, 0\}$, and perform first integral over dQ^2:

$$a_\mu^{\mathrm{HVP}} = 2\alpha^2 \int_0^\infty dt\, K(t) C(t), \quad C(t) = -\frac{1}{3} \sum_{i=1,2,3} \int d^3x\, \langle J^i_{\mathrm{em}}(\vec{x}, t) J^i_{\mathrm{em}}(0) \rangle$$

$$K(t) = 4 \int_0^\infty \frac{d\omega}{\omega} f(\omega^2, m_\mu) \left[\omega^2 t^2 - 4\sin^2\left(\frac{1}{2}\omega t\right)\right], \qquad K(t \ll m_\mu^{-1}) \propto t^4, \quad K(t \gg m_\mu^{-1}) \propto t^2$$

[Evaluating the HVP]

The main ingredient in the lattice calculation of a_μ^{HVP} is the current-current correlator $C(t)$.

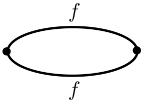
$$\langle J^i_{\mathrm{em}}(x) J^i_{\mathrm{em}}(0)\rangle = \sum_{f,f'=u,d,s,c,\ldots} q_f q_{f'} \times \left\langle \bar{f}(x)\gamma^i f(x)\bar{f}'(0)\gamma^i f'(0)\right\rangle$$

q_f is the electric charge of the quark-flavour f.

Lattice action is quadratic in $\bar{f}f \implies$ fermion fields integrated analytically (see S. Hand video on Grassmann variables).

Wick theorem for Grassmann variables gives rise to two distinct topologies of diagrams:

$$q_f^2 \times \left\langle \bar{f}(x)\gamma^i f(x)\bar{f}(0)\gamma^i f(0)\right\rangle = q_f^2 \times \frac{1}{\mathcal{Z}}\int [dU]\, e^{-S[U]}\ \mathrm{Tr}\left[\gamma^i \overbrace{S_f(0,x;U)}^{f\text{-quark propagator}} \gamma^i\ S_f(x,0;U)\right]$$

$= q_f^2 \times$ 

[flavour diagonal]

[Evaluating the HVP]

The main ingredient in the lattice calculation of $a_\mu^{\rm HVP}$ is the current-current correlator $C(t)$.

$$\langle J^i_{\rm em}(x) J^i_{\rm em}(0) \rangle = \sum_{f,f'=u,d,s,c,\dots} q_f q_{f'} \times \left\langle \bar{f}(x)\gamma^i f(x) \bar{f}'(0)\gamma^i f'(0) \right\rangle$$

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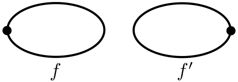
$= q_f q_{f'} \times$  [flavour diag.+off-diag. contributions]

[flavour decomposition]

All in all, current-current correlator $C(t)$ can be separated as:

$$C(t) = \underbrace{C_u(t) + C_d(t) + C_s(t) + C_c(t)}_{\text{connected contributions from single flavours}} + \underbrace{C_{disc.}(t)}_{\text{total quark-disconnected}} + \text{bottom, top (perturbative)}$$

Connected contributions admits a spectral decomposition (plug completeness relation $\mathbb{1} = |n\rangle\langle n|$ between the two e.m. currents)

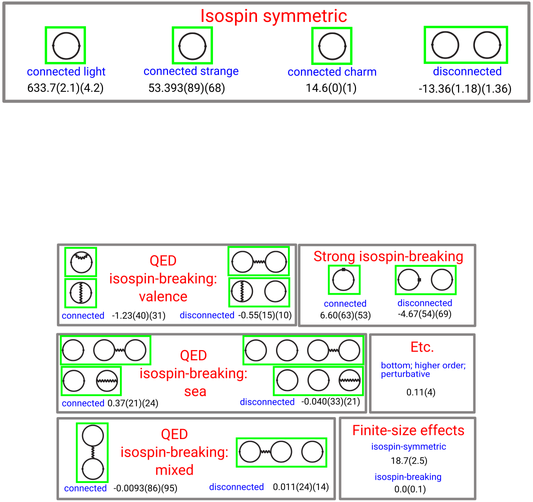
$$C_f(t) = \sum_n C_f^{(n)} e^{-M_n^f t}, \qquad C_f^{(n)} = q_f^2 \times |\langle 0|\bar{f}\gamma^i f|n\rangle|^2_{\text{conn.}}$$

Intermediate (vector) states $|n >$ excited by $\bar{u}\gamma^i u$, $\bar{d}\gamma^i d$ are lighter than those excited by $\bar{s}\gamma^i s$ and $\bar{c}\gamma^i c$.
E.g. lightest vector state in light (u-d) sector are $\pi\pi$ states, in strange-sector KK states and ϕ, in charm sector J/Ψ.
Since lighter states propagate over larger time distances and since $K(t \gg m_\mu) \propto t^2$ (enhances the tail of $C(t)$), one has a hierarchy in flavour contributions to the muon anomaly:

$$a_{\mu,\ell=u+d}^{\text{HVP}} \gg a_{\mu,s}^{\text{HVP}} \gg a_{\mu,c}^{\text{HVP}}$$

Disconnected contribution to a_μ^{HVP} is $\mathcal{O}(2\%)$ of the total.

From BMWc paper: Nature vol. 593 (2021)



Many steps to achieve this result. . .

Unprecedented statistical accuracy, achieved through:

- Large number of gauge configurations.
- Low/all mode averaging for the computation of quark propagators.

Very good control on the various sources of systematics effects:

- Physical point simulations/extrapolations.
- Finite volume effects.
- Continuum extrapolation.

Computation of QED and strong isospin-breaking corrections to isospin symmetric QCD.

[References]

Selected advanced material [to be completed]

Papers and books:

T. Blum, Lattice calculation of the lowest order hadronic contribution to the muon anomalous magnetic moment, https://arxiv.org/pdf/hep-lat/0212018.pdf.

D. Bernecker and H. Meyer, Vector Correlators in Lattice QCD: methods and applications, https://arxiv.org/pdf/1107.4388.pdf.

F. Jegerlehner, The Anomalous Magnetic Moment of the Muon, Springer Tracts in Modern Physics 274, 2017 https://bib-pubdb1.desy.de/record/393196/files/978-3-319-63577-4.pdf .

White paper 2020, https://arxiv.org/abs/2006.04822.

BMWc-20, Leading hadronic contribution to the muon magnetic moment from lattice QCD, https://arxiv.org/pdf/2002.12347.pdf.

Slides:

V. Lubicz, The muon $g-2$ and lattice QCD, https://agenda.infn.it/event/27839/contributions/142112/attachments/84439/111749/g-2%20Lattice.pdf

Videos:

Z. Fodor, HEP Seminar Jan 26, 2022 - $(g-2)_\mu$ from lattice QCD and experiments: 4.2 sigma?, https://www.youtube.com/watch?v=PxEx1cCHsuM