Proton in Basis Light Front Quantization and Prospects for Light Nuclei

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ECT* Trento
Tomography of Light Nuclei at an EIC
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Dirac's forms of relativistic dynamics [Dirac, Rev. Mod. Phys. **21**, 392 1949]
Instant form is the well-known form of dynamics starting with $x^0 = t = 0$

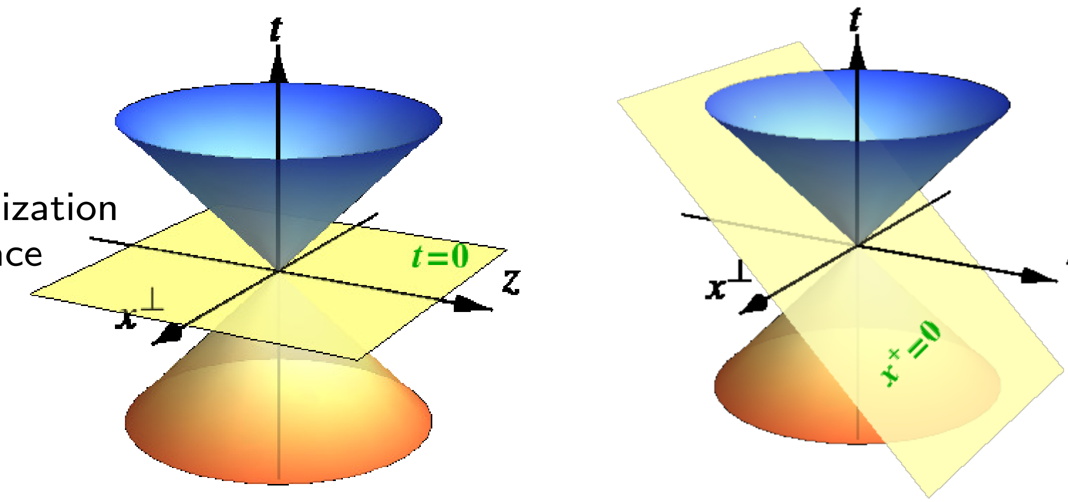
$K^i = M^{0i}$, $J^i = \frac{1}{2}\varepsilon^{ijk} M^{jk}$, $\varepsilon^{ijk} = (+1,-1,0)$ for (cyclic, anti-cyclic, repeated) indeces

Front form defines relativistic dynamics on the light front (LF): $x^+ = x^0 + x^3 = t + z = 0$

$$P^{\pm} \triangleq P^0 \pm P^3,\ \vec{P}^{\perp} \triangleq (P^1, P^2),\ x^{\pm} \triangleq x^0 \pm x^3,\ \vec{x}^{\perp} \triangleq (x^1, x^2),\ E^i = M^{+i},$$
$$E^+ = M^{+-},\ F^i = M^{-i}$$

	instant form	front form	point form
time variable	$t = x^0$	$x^+ \triangleq x^0 + x^3$	$\tau \triangleq \sqrt{t^2 - \vec{x}^2 - a^2}$

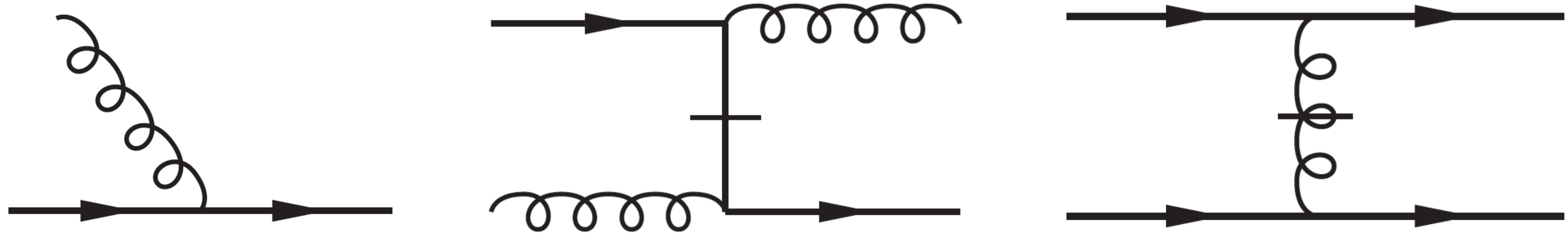
quantization surface



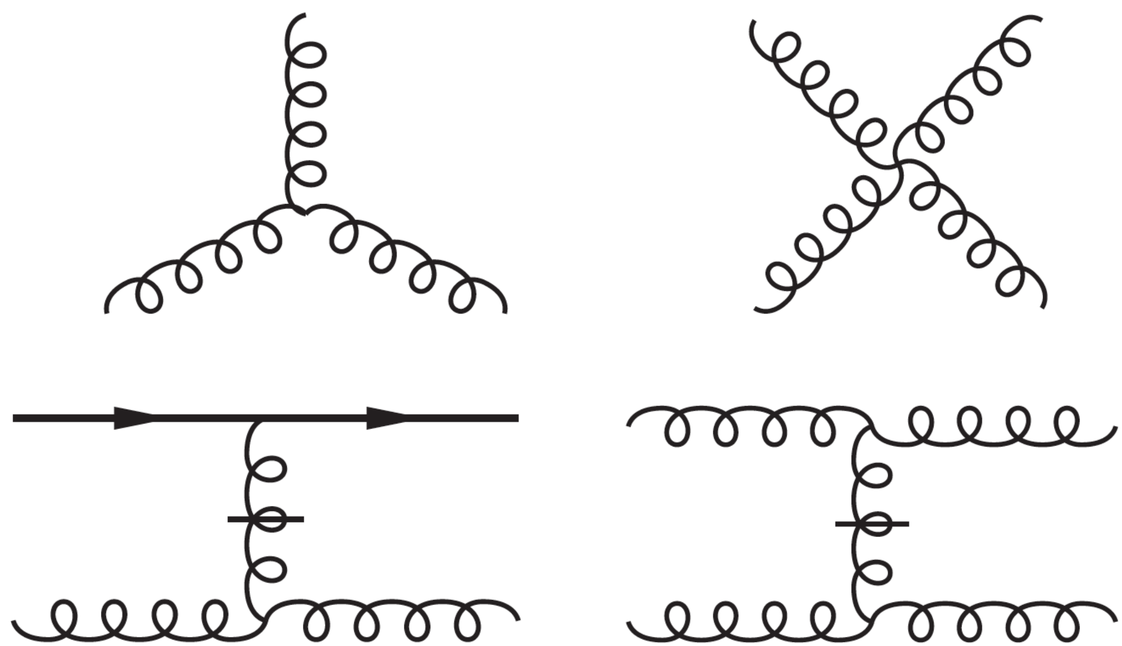
	instant form	front form	point form
Hamiltonian	$H = P^0$	$P^- \triangleq P^0 - P^3$	P^μ
kinematical	$\vec{P}, \vec{J}$	$\vec{P}^{\perp}, P^+, \vec{E}^{\perp}, E^+, J^-$	$\vec{J}, \vec{K}$
dynamical	$\vec{K}, P^0$	$\vec{F}^{\perp}, P^-$	$\vec{P}, P^0$
dispersion relation	$p^0 = \sqrt{\vec{p}^2 + m^2}$	$p^- = (\vec{p}_{\perp}^2 + m^2)/p^+$	$p^\mu = m v^\mu \ (v^2 = 1)$

Adapted from talk by Yang Li

Light Front (LF) Hamiltonian Defined by its Elementary Vertices in LF Gauge



QED & QCD



QCD

Light-Front Wavefunctions (LFWFs)

$$|\psi_h(P,j,\lambda)\rangle = \sum_n \int [\mathrm{d}\mu_n]\, \psi_{n/h}(\{\vec{k}_{i\perp}, x_i, \lambda_i\}_n) |\{\vec{p}_{i\perp}, p_i^+, \lambda_i\}_n\rangle$$

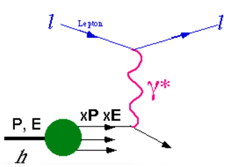
LFWFs are *frame-independent* (boost invariant) and depend only on the relative variables: $x_i \equiv p_i^+/P^+, \vec{k}_{i\perp} \equiv \vec{p}_{i\perp} - x_i \vec{P}_\perp$

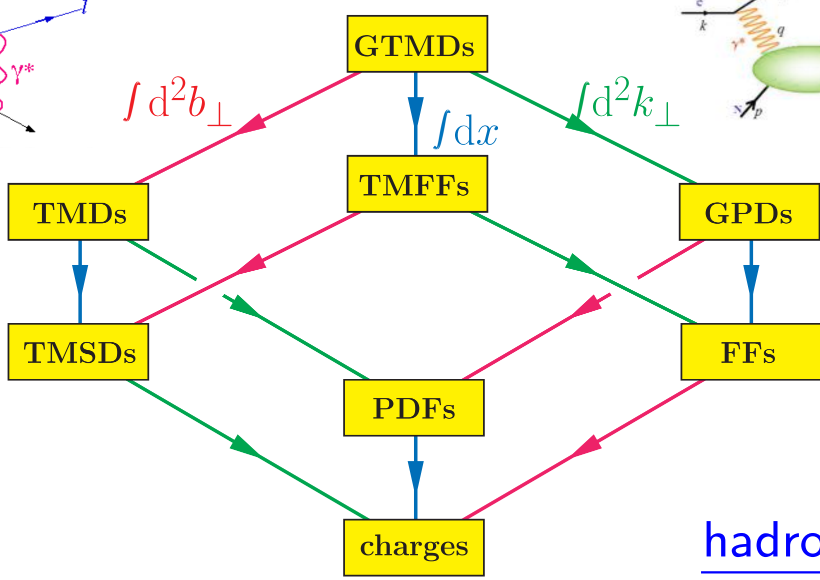
LFWFs provide intrinsic information of the structure of hadrons, and are indispensable for exclusive processes in DIS [Lepage '80]

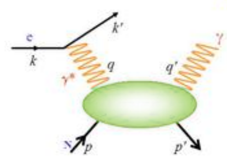
- Overlap of LFWFs: structure functions (e.g. PDFs), form factors, ...
- Integrating out LFWFs: light-cone distributions (e.g. DAs)

"Hadron Physics without LFWFs is like Biology without DNA!"

— Stanley J. Brodsky

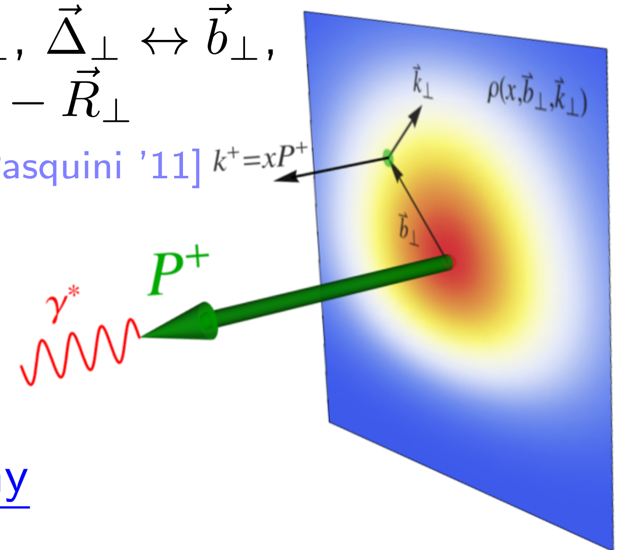






$\vec{k}_\perp \leftrightarrow \vec{r}_\perp, \vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp,$
$\vec{b}_\perp = \vec{r}_\perp - \vec{R}_\perp$

[Lorcé & Pasquini '11]



hadron tomography

Discretized Light Cone Quantization

[H.C. Pauli & S.J. Brodsky, PRD32 (1985)]

$\Downarrow$

Basis Light Front Quantization

[J.P. Vary, et al., PRC81 (2010)]

$$\phi\left(\vec{k}_\perp, x\right) = \sum_\alpha \left[f_\alpha\left(\vec{k}_\perp, x\right) a_\alpha + f_\alpha^*\left(\vec{k}_\perp, x\right) a_\alpha^\dagger \right]$$

where $\{a_\alpha\}$ satisfy usual (anti-) commutation rules.

Furthermore, $f_\alpha(\vec{x})$ are arbitrary except for conditions:

Orthonormal: $$\int f_\alpha\left(\vec{k}_\perp, x\right) f_{\alpha'}^*\left(\vec{k}_\perp, x\right) \frac{d^2 k_\perp dx}{(2\pi)^3 2x(1-x)} = \delta_{\alpha\alpha'}$$

Complete: $$\sum_\alpha f_\alpha\left(\vec{k}_\perp, x\right) f_\alpha^*\left(\vec{k}'_\perp, x'\right) = 16\pi^3 \sqrt{x(1-x)}\,\delta^2\left(\vec{k}_\perp - \vec{k}'_\perp\right)\delta\left(x - x'\right)$$

For mesons we adopt (later extended to baryons): [Y. Li, et al., PLB758 (2016)]

$$f_{\alpha=\{nml\}}\left(\vec{k}_\perp, x\right) = \phi_{nm}\left(\vec{k}_\perp / \sqrt{x(1-x)}\right)\chi_l(x)$$

ϕ_{nm} 2D-HO functions as in AdS/QCD

χ_l Jacobi polynomials times $x^a(1-x)^b$

BLFQ
Symmetries & Constraints

Baryon number $\sum_i b_i = B$

Charge $\sum_i q_i = Q$

Angular momentum projection (M-scheme) $\sum_i (m_i + s_i) = J_z$

All $J \geq J_z$ states in one calculation

Longitudinal momentum (Bjorken sum rule) $\sum_i x_i = \sum_i \frac{k_i}{K} = 1$

Longitudinal mode regulator (Jacobi) $\sum_i l_i \leq L$

Transverse mode regulator (2D HO) $\sum_i (2n_i + |m_i| + 1) \leq N_{max}$

Finite basis regulators

"Internal coordinates" $\vec{k}_{i\perp} = \vec{p}_{i\perp} - x_i \vec{P}_\perp \quad \Rightarrow \quad \sum_i \vec{k}_{i\perp} = 0$

$H \rightarrow H + \lambda H_{CM}$

Preserve transverse boost invariance

Global Color Singlets (QCD)

Light Front Gauge

Optional Fock-Space Truncation

Overview of BLFQ/tBLFQ applications to mesons and baryons

Common features

Transverse confinement from 2D HO (in common with LF Holography)
Longitudinal confinement (Y. Li, et al, PLB 2016, PRD 2017)
Basis states from exact solutions of this reference Hamiltonian
Compare results with experiment, lattice, DSE/BSE, . . .

Distinct features

For Veff
1) perturbative one-gluon exchange (Y. Li, et al, PLB 2016, PRD 2017)
2) NJL model for light meson applications (S. Jia, et al, PRC 2019)

For Fock space truncation
1) Valence sector
2) Valence sector plus dynamical gluon

For observables
1) Single state properties and decays
2) Transitions between states
3) Non-perturbative probes (tBLFQ)

Complementary Methods

BLFQ on Quantum Computers

Heavy Quarkonia [Y.Li,PLB758,2016; PRD96,2017]

- Effective Hamiltonian in the $q\bar{q}$ sector

$$H_{\text{eff}} = \overbrace{\underbrace{\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF kinetic energy}} + \underbrace{\kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q+m_{\bar{q}})^2}\frac{\partial}{\partial x}\left(x(1-x)\frac{\partial}{\partial x}\right)}_{\text{confinement}}}^{H_0} + \underbrace{V_g}_{\text{one-gluon exchange}}$$

where $x = p_q^+ / P^+$, $\vec{k}_\perp = \vec{k}_{q\perp} = \vec{p}_{q\perp} - x\vec{P}_\perp = -\vec{k}_{\bar{q}\perp} = -\left(\vec{p}_{\bar{q}\perp} - (1-x)\vec{P}_\perp\right)$, $\vec{r}_\perp = \vec{r}_{q\perp} - \vec{r}_{\bar{q}\perp}$.

  - Confinement
    transverse holographic confinement [S.J.Brodsky,PR584,2015]
    longitudinal confinement [Y.Li,PLB758,2016]
  - One-gluon exchange with running coupling
    $V_g = -\frac{4}{3}\frac{4\pi\alpha_s(Q^2)}{Q^2}\bar{u}_{\sigma'}\gamma^\mu u_\sigma \bar{v}_s \gamma_\mu v_{s'}$

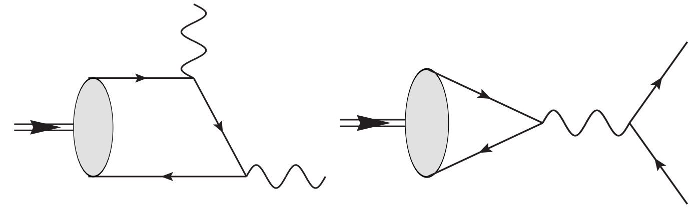
- Basis representation
  - valence Fock sector: $|q\bar{q}\rangle$
  - basis functions: eigenfunctions of H_0 (LF kinetic energy+ confinement)

Yang Li, Pieter Maris and James P. Vary, Phys. Rev. D 96, 016022 (2017); arXiv: 1704.06968

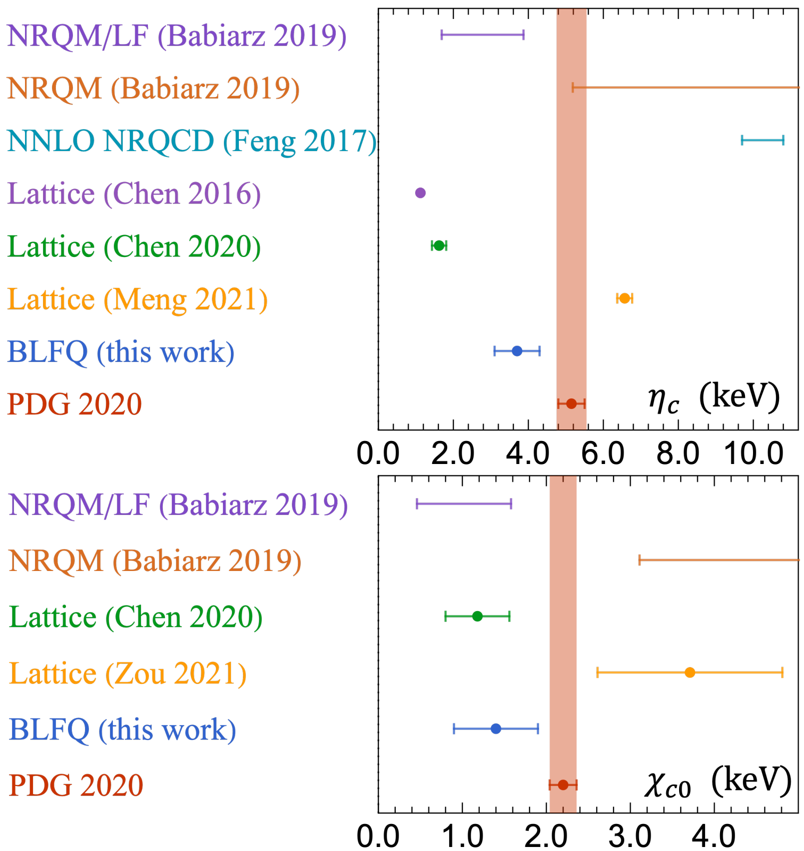
Yang Li, Meijian Li and James P. Vary, Phys. Rev. D 105, L071901 (2022); arXiv: 2111.14178

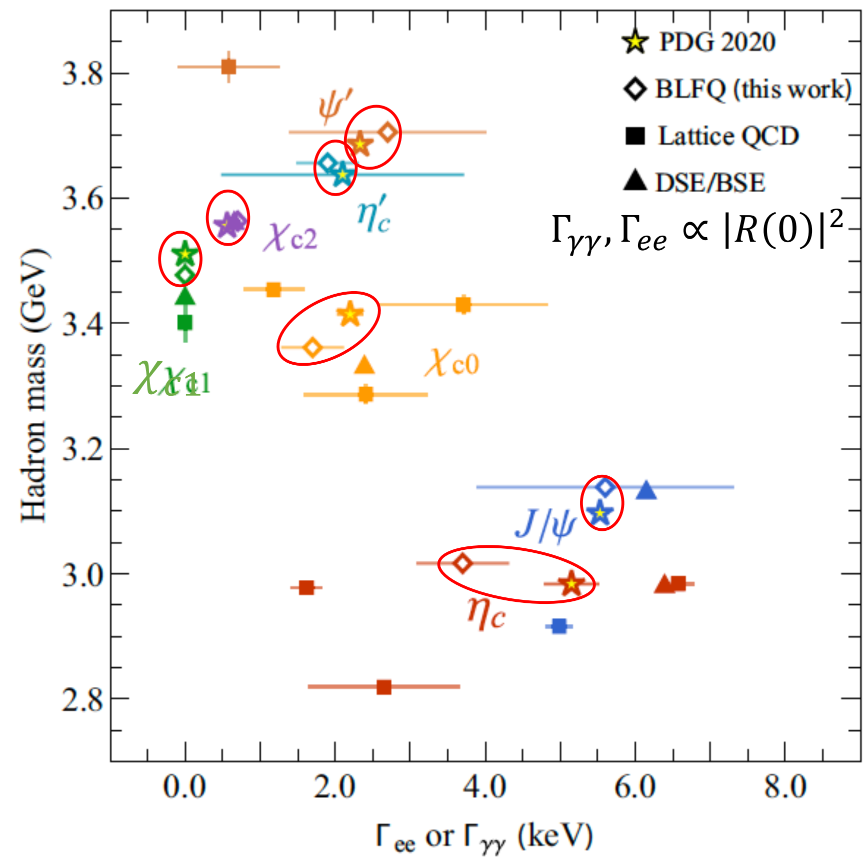
Diphoton width $\Gamma_{\gamma\gamma}$ of charmonia in BLFQ

- ✓ Notoriously challenging
- ✓ BLFQ predictions are very competetive!
  - ✓ No parameters were adjusted!



$\Gamma_{P,S,T\to\gamma\gamma}$ $\Gamma_{V\to ee}$

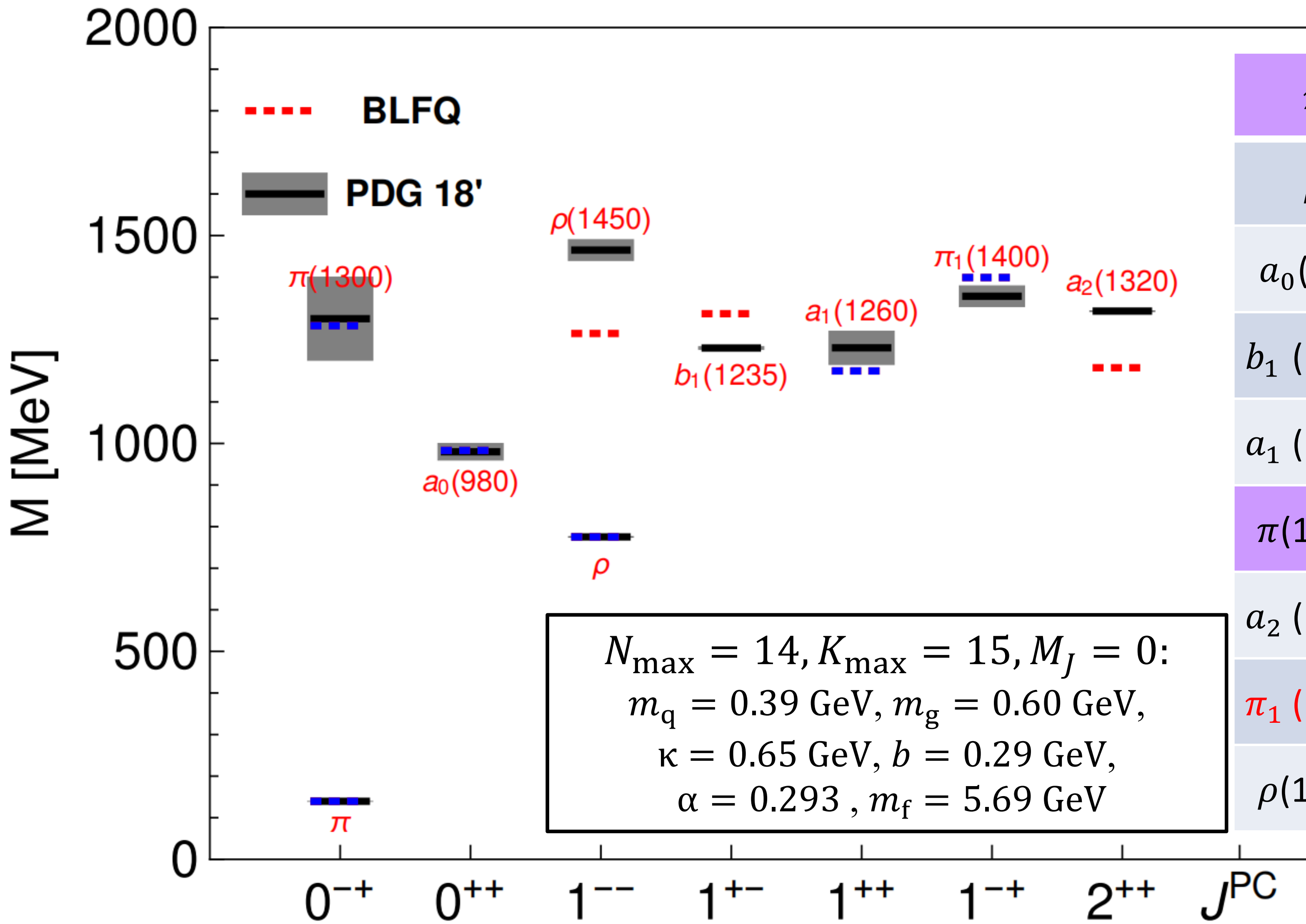




Lattice: Dudek '06, Chen '16, Chen '20, Meng '21, Zou '21;
DSE: Chen '17
NRQCD: Feng '15 & '17
NRQM: Babiarz '19 & '20

Comparison of theoretical prediction of masses and dilepton/diphoton widths combined

Light Meson Mass Spectrum Including One Dynamical Gluon



	Norm$q\bar{q}$	DC[MeV]
π	0.492	138
ρ	0.486	129
$a_0(980)$	0.370	0
b_1 (1235)	0.30	0
a_1 (1260)	0.324	11
$\pi(1300)$	0.284	53
a_2 (1320)	0.320	0
π_1 (1400)	0.002	0
$\rho(1450)$	0.312	46

$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

Fix the parameters by fitting six blue states

- $\pi_1(1400)$: $|q\bar{q}g\rangle$ dominates
- $\pi(1300)$: the DC is smaller than the DC of pion

[Lan, et al., (BLFQ Collaboration) PLB 825, 136890 (2022); arXiv 2106.04954]

Light-Front Hamiltonian (Model I)

$$|P_{baryon}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, q\bar{q}\rangle + \cdots\cdots$$

$$P^{-} = H_{K.E.} + H_{trans} + H_{longi} + H_{Interact}$$

$$H_{K.E.} = \sum_{i} \frac{p_i^2 + m_q^2}{p_i^+}$$

[S. Xu et al, PRD 104 094036(2021)]

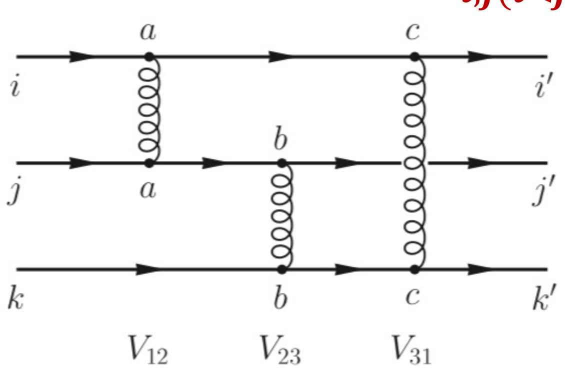
$$H_{trans} \sim \kappa_T^4 r^2$$

[S. J. Brodsky, G. de Teramond arXiv: 1203.4025]

$$H_{longi} \sim -\sum_{ij} \kappa_L^4 \partial_{x_i} \left(x_i x_j \partial_{x_j} \right)$$

[Y. Li, X. Zhao , P Maris , J. P. Vary, PLB 758(2016)]

$$H_{Interact} = -\frac{C_F 4\pi\alpha_s}{Q^2} \sum_{i,j(i<j)} \bar{u}_{s_i'}(k_i')\gamma^{\mu} u_{s_i}(k_i) \bar{u}_{s_j'}(k_j')\gamma_{\mu} u_{s_j}(k_j)$$



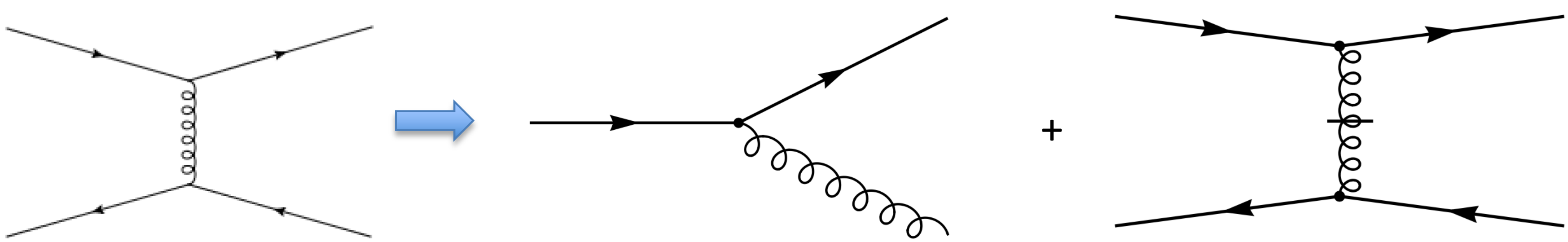
Parameters are determined through fitting nucleon mass and EMFFs.

[S. Xu, C. Mondal, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph]]

Light-Front Hamiltonian (Model II)

$$|P_{baryon}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\,q\bar{q}\rangle + \cdots\cdots$$

- $H_{Interact} \rightarrow H_{Interact} = H_{Vertex} + H_{inst}$



$N_{\max} = 9, K = 16.5$

Confining interaction (κ, m_g)

Parameters are determined through fitting nucleon mass and EMFFs.

m_u	m_d	κ	m_g	m_{int}	b_{inst}	b	g
0.32 GeV	0.25 GeV	0.54 GeV	0.50 GeV	1.80 GeV	3.00 GeV	0.70 GeV	2.40

Different Mass Asymmetry of u and d

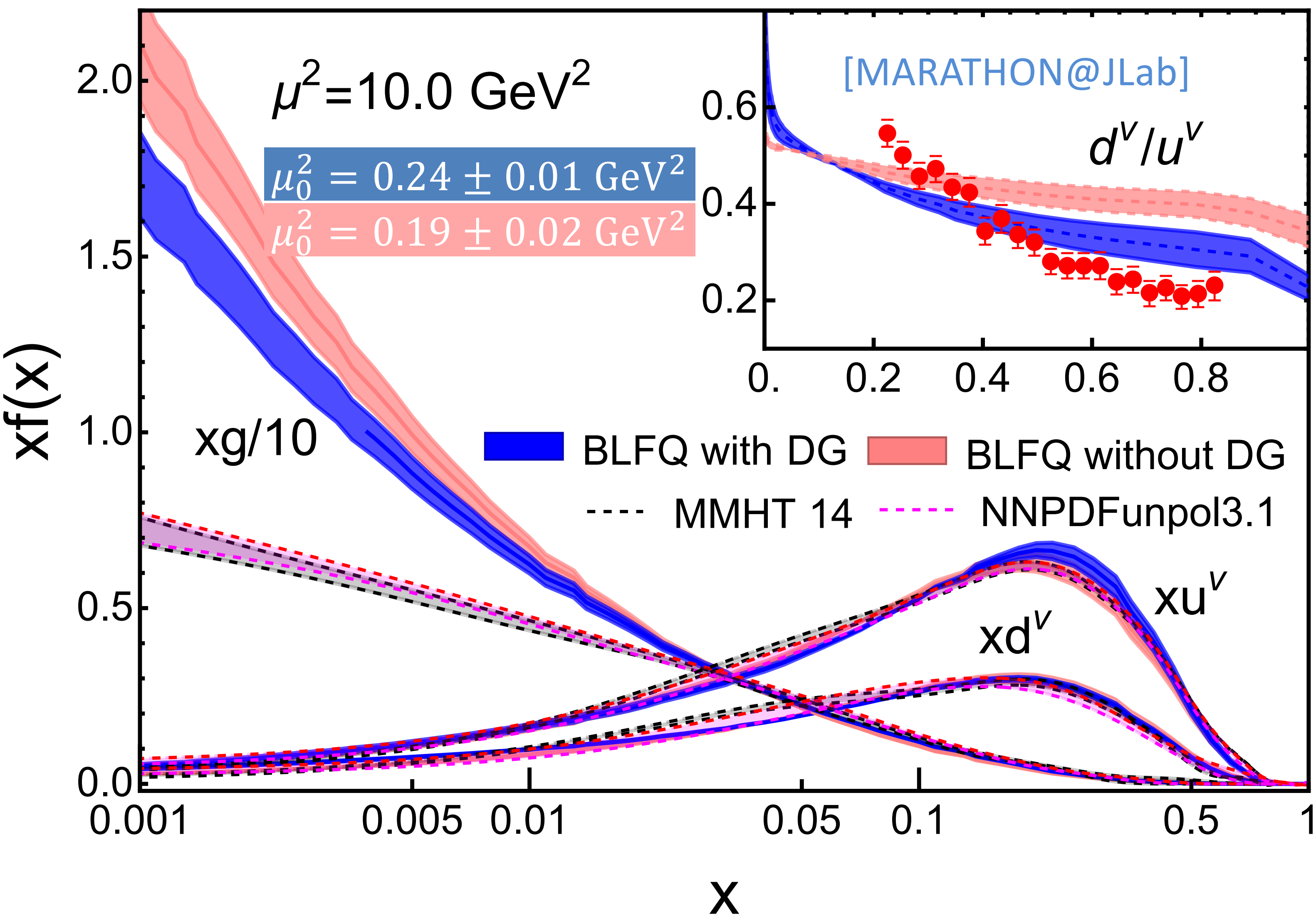
UV Cutoff In Instantaneous term

Connection with Light-front QCD Hamiltonian

$$
\begin{aligned}
P^{-}_{LFQCD} =\ & \frac{1}{2}\int d^3x \overline{\widetilde{\psi}}\gamma^+ \frac{(i\partial^\perp)^2+m^2}{i\partial^+}\widetilde{\psi} - A^i_a (i\partial^\perp)^2 A_{ia} \\
& -\frac{1}{2}g^2 \int d^3x \mathrm{Tr}\left[\widetilde{A}^\mu, \widetilde{A}^\nu\right]\left[\widetilde{A}_\mu, \widetilde{A}_\nu\right] \\
& +\frac{1}{2}g^2 \int d^3x \overline{\widetilde{\psi}}\gamma^+ T^a \widetilde{\psi}\frac{1}{(i\partial^+)^2}\overline{\widetilde{\psi}}\gamma^+ T^a\widetilde{\psi} \\
& -g^2 \int d^3x \overline{\widetilde{\psi}}\gamma^+ \left(\frac{1}{(i\partial^+)^2}\left[i\partial^+\widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\right)\widetilde{\psi} \\
& +g^2 \int d^3x \mathrm{Tr}\left(\left[i\partial^+\widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\frac{1}{(i\partial^+)^2}\left[i\partial^+\widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\right) \\
& +\frac{1}{2}g^2 \int d^3x \overline{\widetilde{\psi}}\widetilde{\not{A}}\frac{\gamma^+}{i\partial^+}\widetilde{\not{A}}\widetilde{\psi} \\
& +g \int d^3x \overline{\widetilde{\psi}}\widetilde{\not{A}}\widetilde{\psi} \\
& +2g \int d^3x \mathrm{Tr}\left(i\partial^\mu \widetilde{A}^\nu\left[\widetilde{A}_\mu, \widetilde{A}_\nu\right]\right)
\end{aligned}
$$

First-principles interactions are included

Unpolarized Parton Distribution Functions



- Initial scale increases with the inclusion of dynamical gluon
- Overall results improve with the inclusion of dynamical gluon

[S. Xu, C. Mondal, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph]]

Angular Momentum Distributions

- Proton spin decomposition

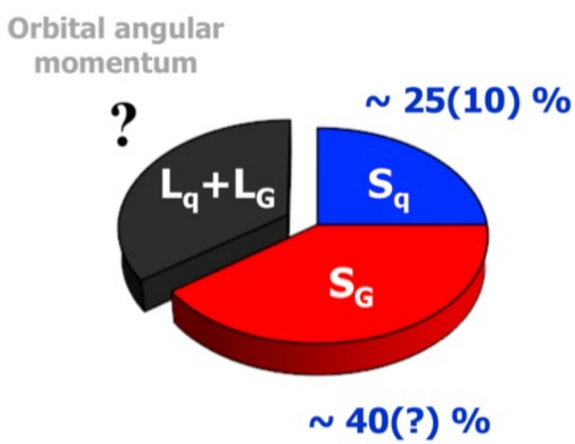
[Jaffe-Manohar 90']

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + (L_q + L_g)$$

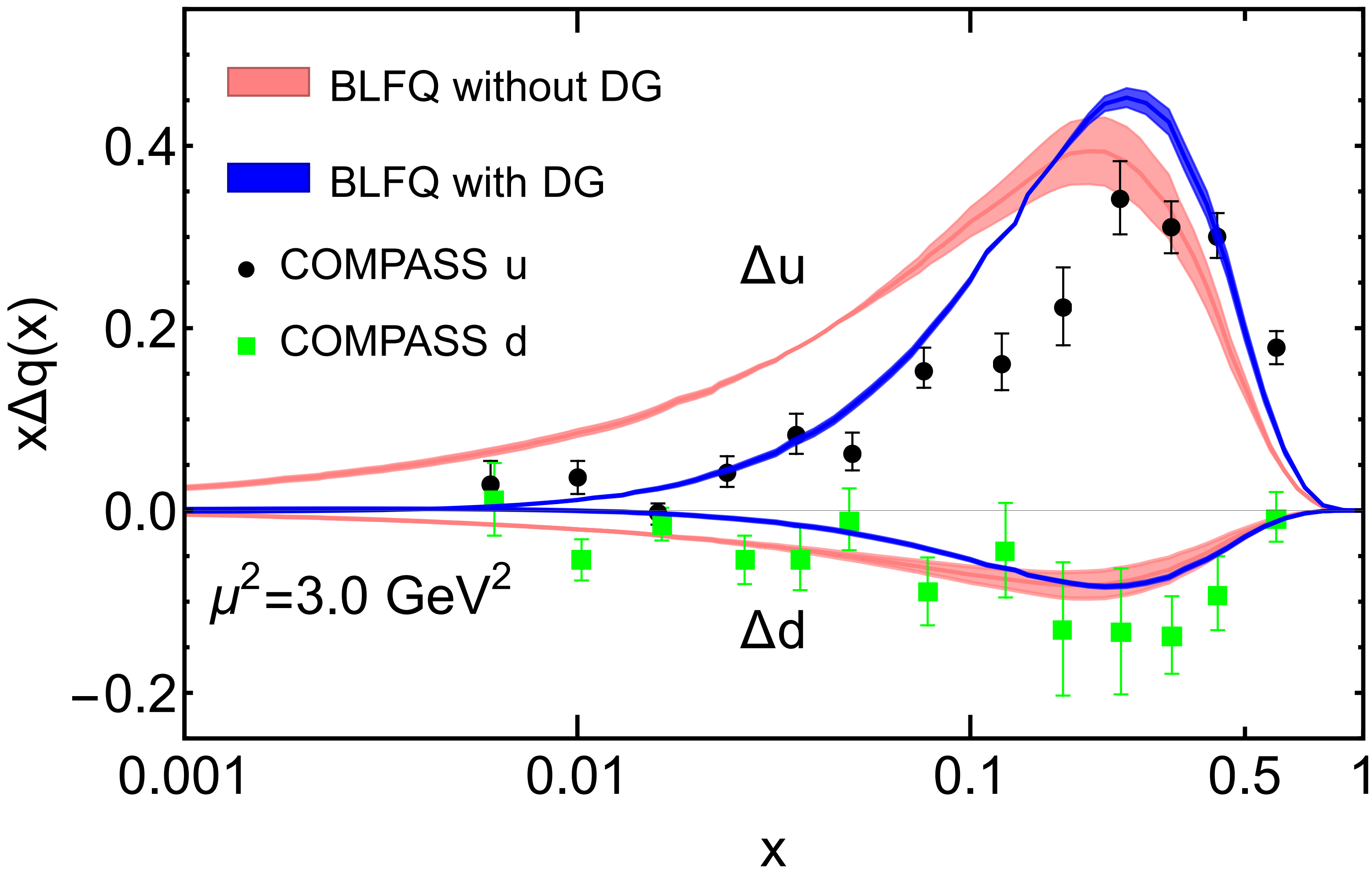
Quark spin, obtained from Δq, in quark model $\Delta\Sigma=1$

Gluon spin, obtained from Δg

Quark/gluon orbital angular momentum, obtained from GTMD $F_{1,4}$



Helicity Parton Distribution Functions

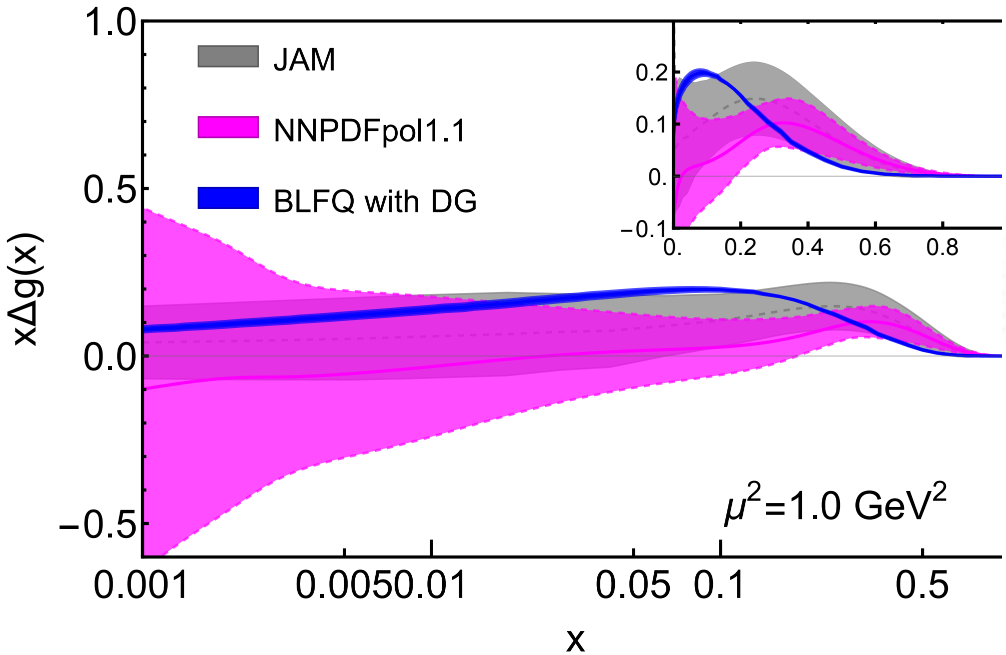


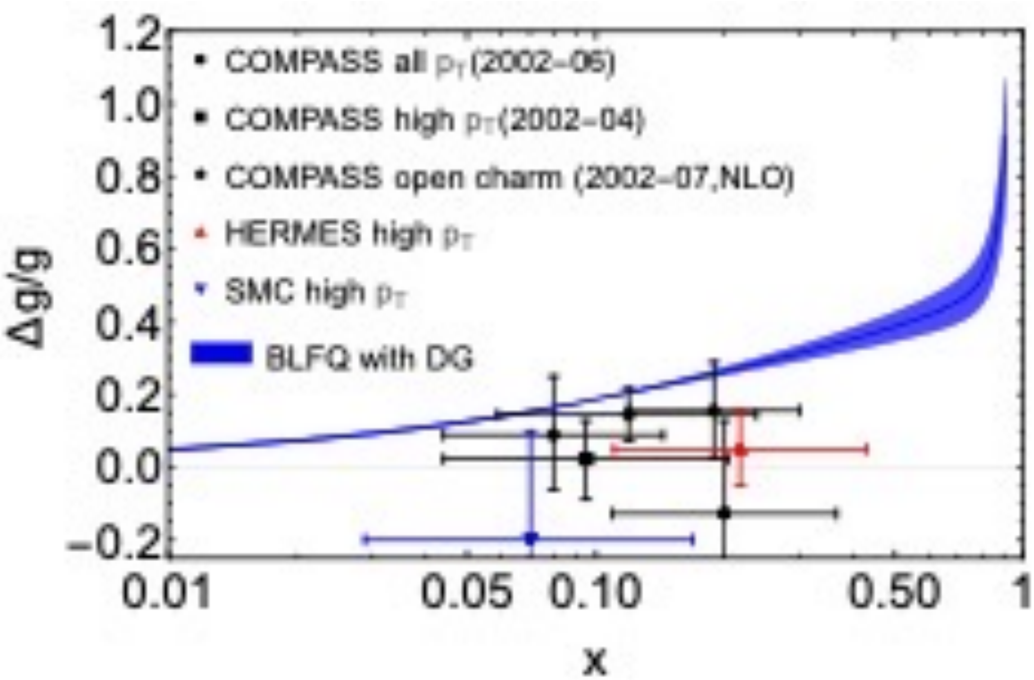
- $\Delta\Sigma_q \approx 0.7 \quad \Delta\Sigma_u \approx 0.86 \quad \Delta\Sigma_d \approx -0.16$
- Valence quark distributions at x<0.1 and x>0.5 regions show improvement with DG

[S. Xu, C. Mondal, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph]]

Helicity Parton Distribution Functions

N. Sato et al. [JAM], PRD93 (2016)
E. R. Nocera et al. [NNPDF], NPB 887 (2014)





[S. Xu, C. Mondal, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph]]

- Δg is positive over the entire x range, qualitatively agreeing with global fits
- $\Delta g/g$ increases with x, measurements of $\Delta g/g$ at EICs are promising
- $\Delta G = \int_0^1 \Delta g(x) = 0.131 \pm 0.003$, comparable with $\Delta G^{[0.002,0.3]} = 0.2 \pm 0.1$ from PHENIX collaboration [PRL 103 012003 (2009)]

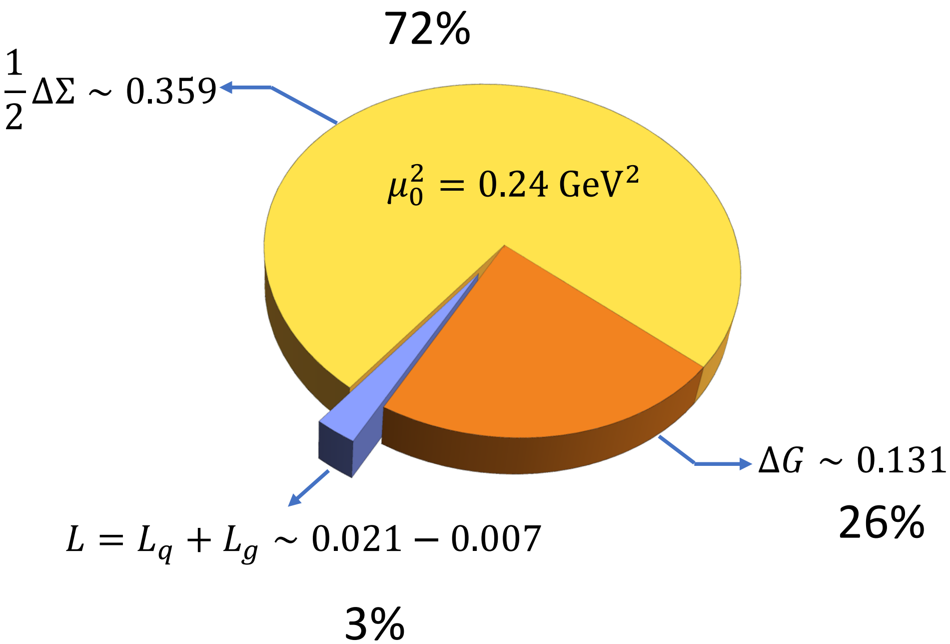
Proton Spin Decomposition

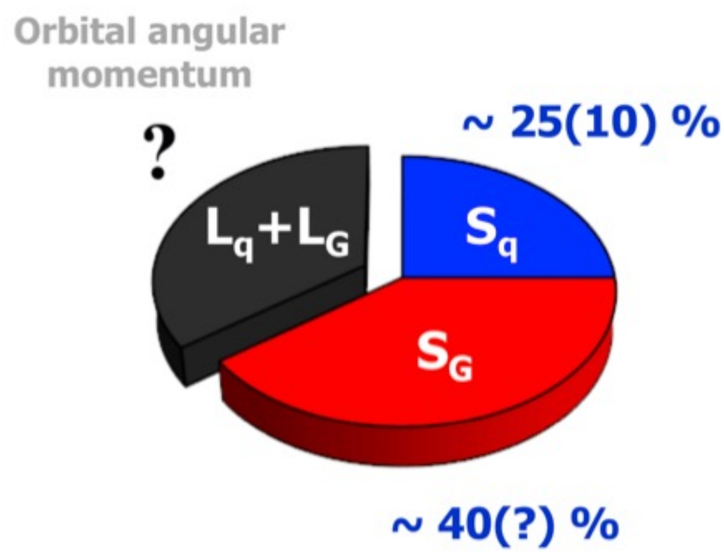
- Fock Sector Expansion

$$|\text{proton}\rangle = |qqq\rangle + |qqq\ g\rangle + |qqq\ q\bar{q}\rangle + |qqq\ gg\rangle + |qqq\ q\bar{q}\ g\rangle + \cdots$$

44% 56%

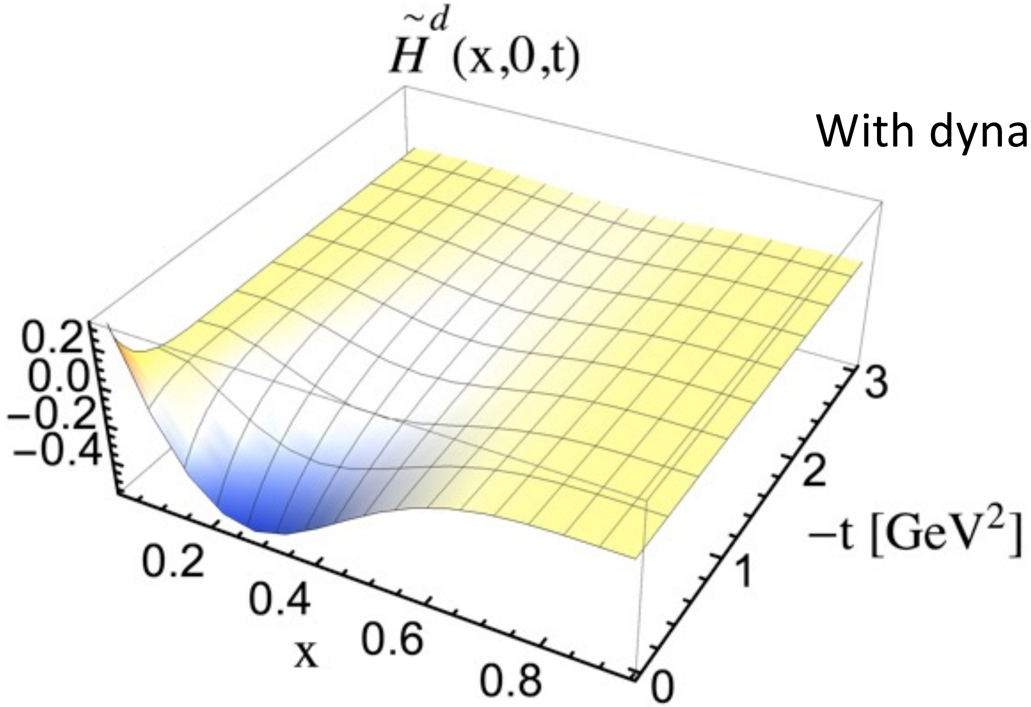
Jaffe-Manohar decomposition: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$

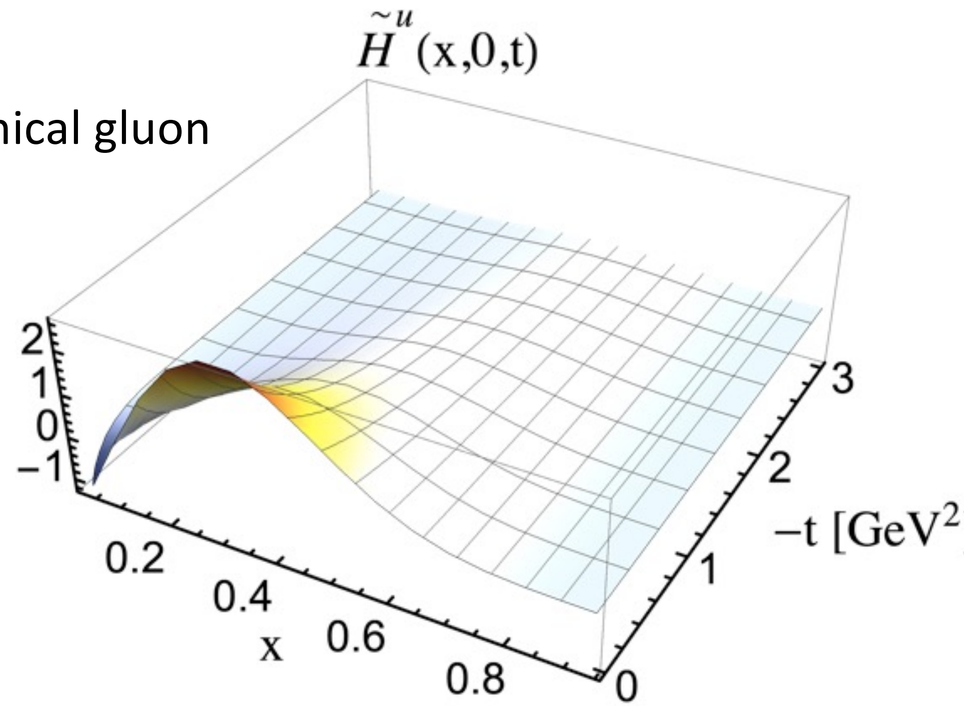




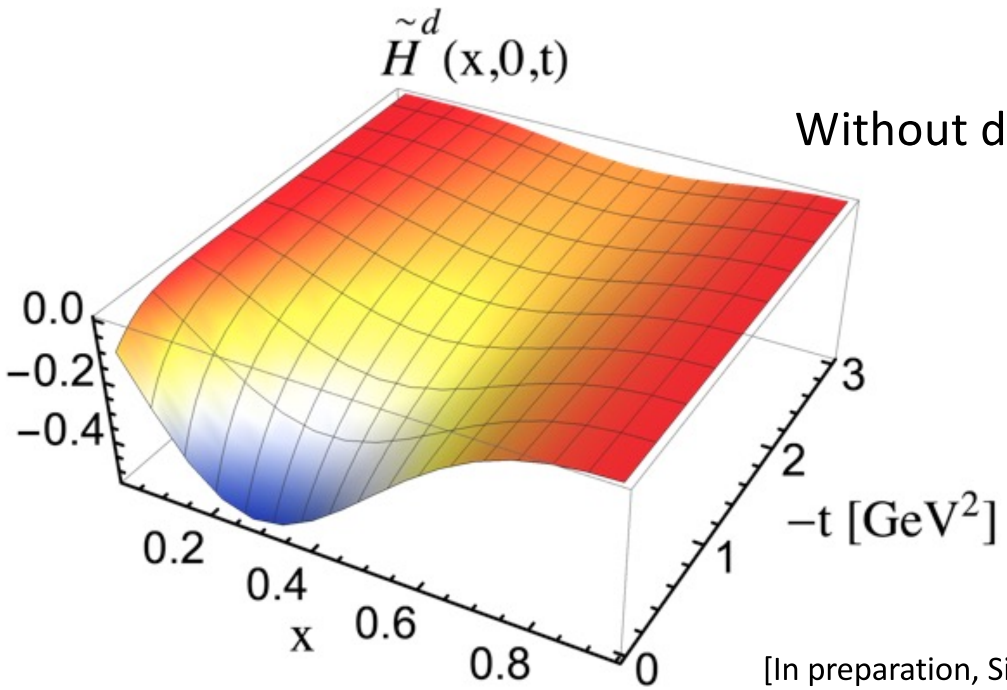
Generalized Parton Distribution Functions (GPD)

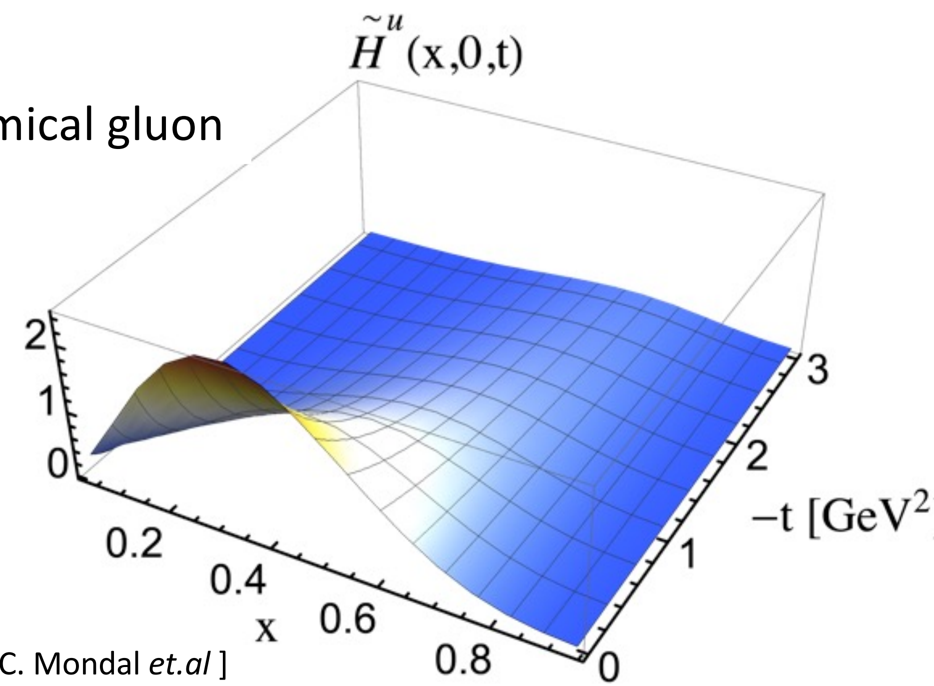
With dynamical gluon





Without dynamical gluon

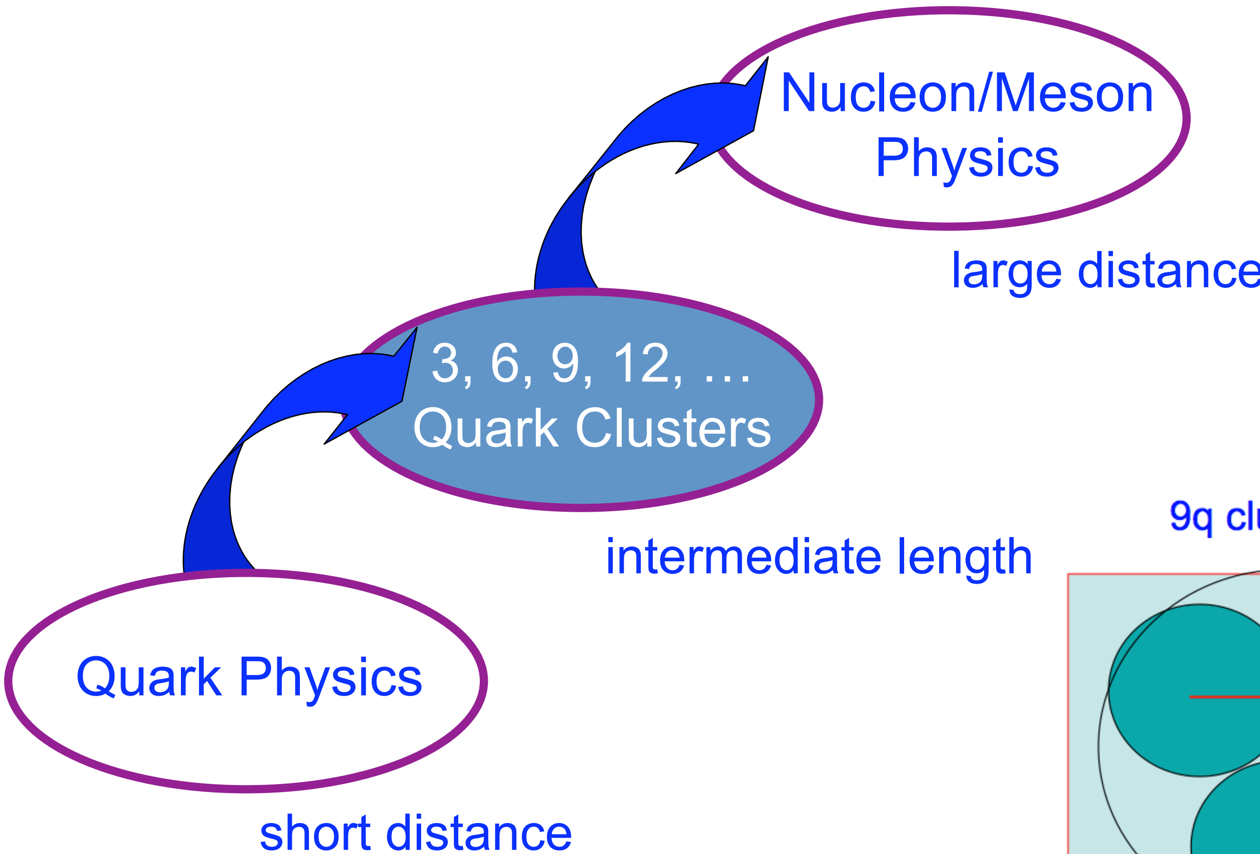




[In preparation, Siqi Xu, C. Mondal *et.al*]

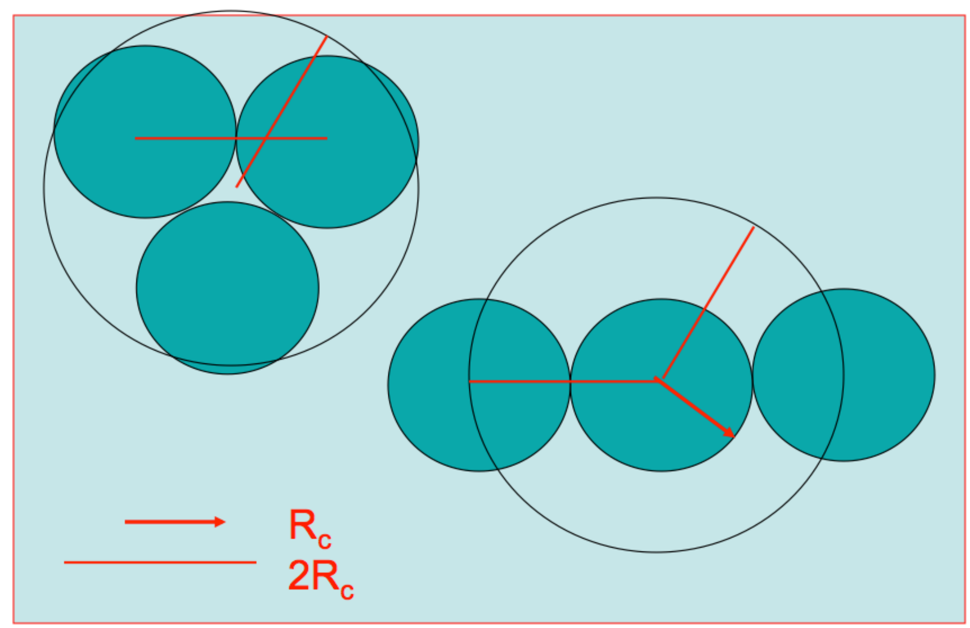
Looking ahead: under what conditions do we require a quark-based description of nuclear structure?

"Quark Percolation in Cold and Hot Nuclei"



- Probes with Q > 1 GeV/c
- Spin content of the proton
- Nuclear form factors
- DIS on nuclei – Bjorken x > 1
- DIS in Duality region
- Nuclear Equation of State

9q cluster at geometrical limits of formation



Also looking ahead: can such a sequence of EFTs be constructed in light-front field theory?

H.J. Pirner and J.P. Vary, Phys. Rev. C. **84**, 015201(2011)
H.J. Pirner and J.P. Vary, Phys. Rev. Lett. **46**, 1376 (1981)

Characteristic predictions of the Quark Cluster Model (QCM) for DIS

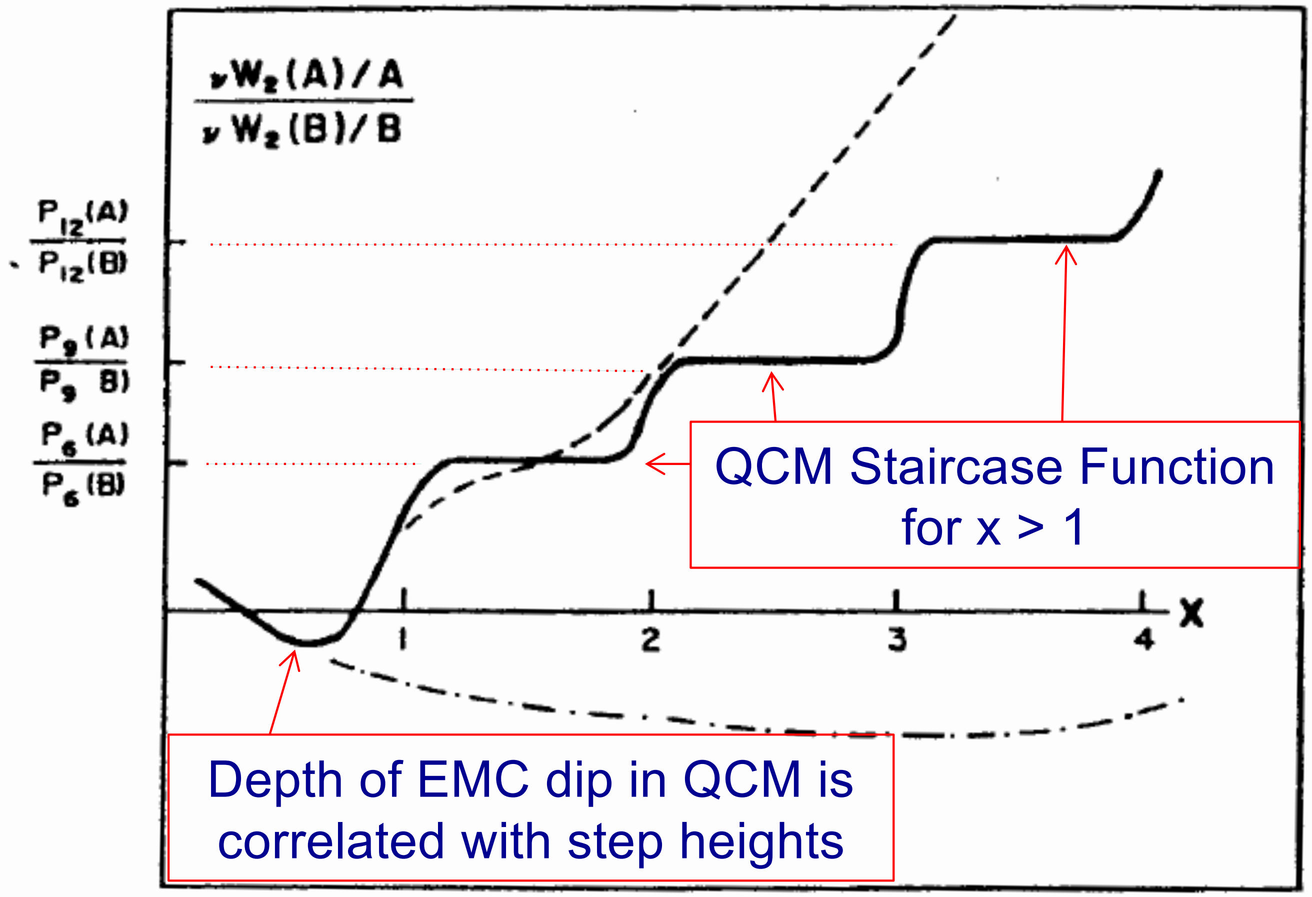


Fig. 2. Characteristic behaviour of the ratio of nuclear structure functions per nucleon for different models over a wide kinematic range of x. The QCM gives the solid curve. The dashed curve is due to the model of reference 22. The dashed-dot curve approximates the predictions of references 23 and 24.

J.P. Vary, Proc. VII Int'l Seminar on High Energy Physics Problems, "Quark Cluster Model of Nuclei and Lepton Scattering Results," Multiquark Interactions and Quantum Chromodynamics, V.V. Burov, Ed., Dubna #D-1, 2-84-599 (1984) 186 [staircase function for x > 1]

See also: numerous other conference proceedings

PHYSICAL REVIEW C VOLUME 33, NUMBER 3 MARCH 1986

Quark cluster probabilities in nuclei

A = 2, 3 & 4 in detail: p_i as function of 2Rc

M. Sato* and S. A. Coon
Department of Physics, University of Arizona, Tucson, Arizona 85721

H. J. Pirner
CERN, Geneva, Switzerland

J. P. Vary
Iowa State University, Ames, Iowa 50011

(Received 28 October 1985)

Computational challenge to use ab initio nuclear structure to evaluate QCM probabilities – consider 9-quark cluster probability in 4He & develop geometrical constraints using:

$$\theta_c(z) \equiv \theta(z-2R_c) = 1 \text{ for } z \geq 2R_c$$
$$\bar{\theta}_c(z) \equiv 1-\theta_c(z)$$

+ full A-body density matrix

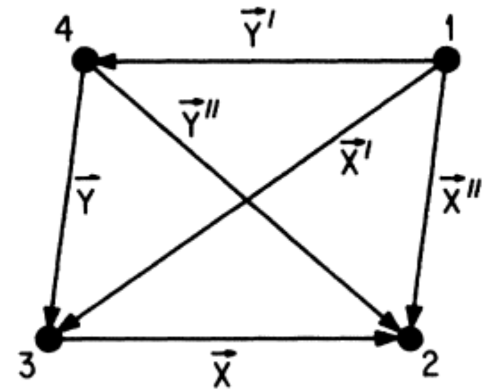
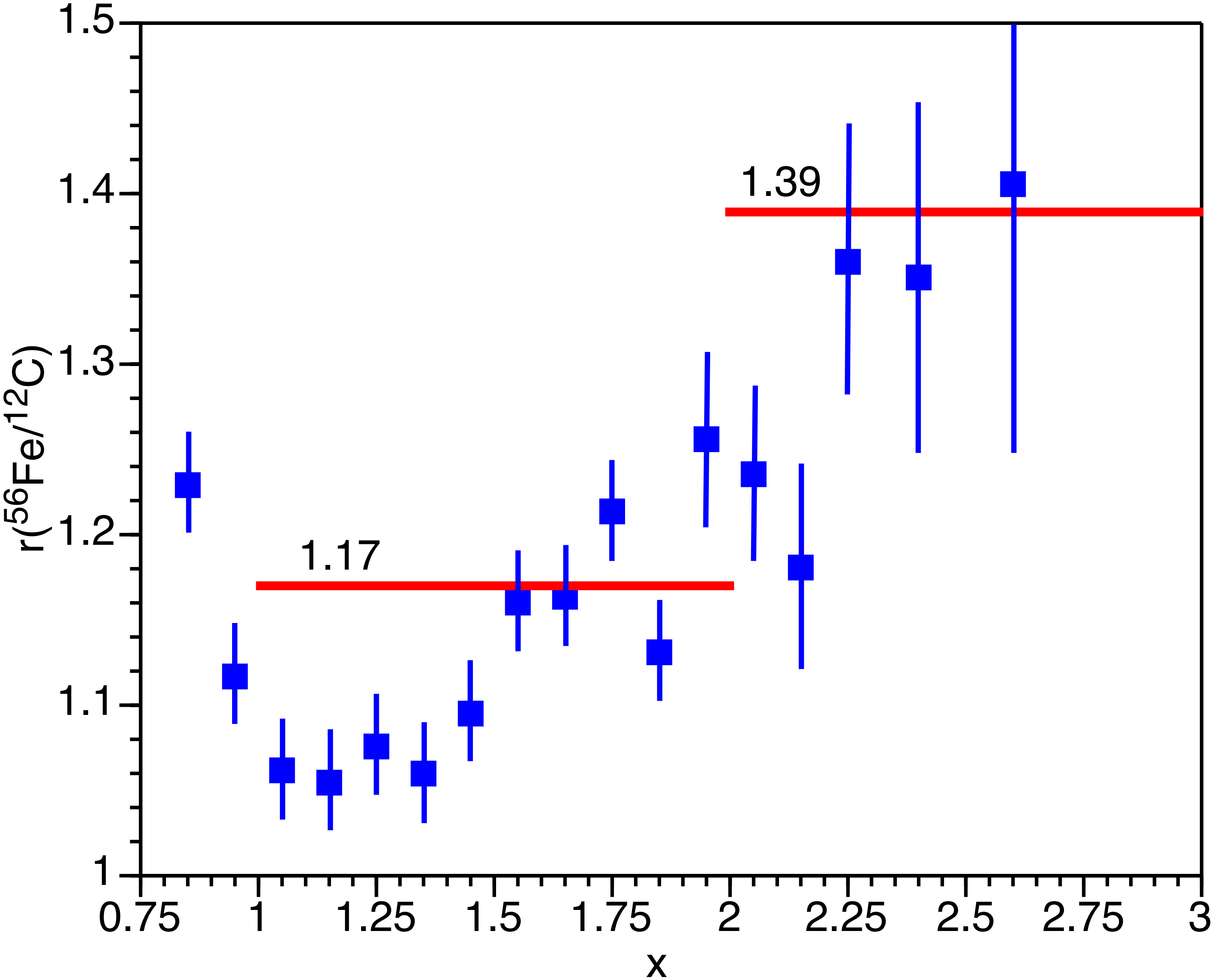


FIG. 5. Coordinate vectors of three-quark subsystems used in Eqs. (5) to define quark cluster probabilities in the $A=4$ nucleus.

$$\begin{aligned}\tilde{p}_9^{(4)} = \int d^3\mathbf{x}'d^3\mathbf{x}''d^3\mathbf{y}'\rho_4(\mathbf{x}',\mathbf{x}'',\mathbf{y}') &\times\{\theta_c(x')\theta_c(x)\theta_c(y)[\bar{\theta}_c(x'')\bar{\theta}_c(y')+\bar{\theta}_c(y')\bar{\theta}_c(y'')+\bar{\theta}_c(x'')\bar{\theta}_c(y'')-2\bar{\theta}_c(x'')\bar{\theta}_c(y')\bar{\theta}_c(y'')]\\ &+\theta_c(y')\theta_c(y'')\theta_c(y)[\bar{\theta}_c(x'')\bar{\theta}_c(x')+\bar{\theta}_c(x'')\bar{\theta}_c(x)+\bar{\theta}_c(x')\bar{\theta}_c(x)-2\bar{\theta}_c(x'')\bar{\theta}_c(x')\bar{\theta}_c(x)]\\ &+\theta_c(x'')\theta_c(x)\theta_c(y'')[\bar{\theta}_c(x')\bar{\theta}_c(y')+\bar{\theta}_c(x')\bar{\theta}_c(y)+\bar{\theta}_c(y')\bar{\theta}_c(y)-2\bar{\theta}_c(x')\bar{\theta}_c(y')\bar{\theta}_c(y)]\}\ .\end{aligned}$$

Comparison between Quark-Cluster Model and JLAB data



Data: K.S. Egiyan, et al., Phys. Rev. Lett. **96**, 082501 (2006)

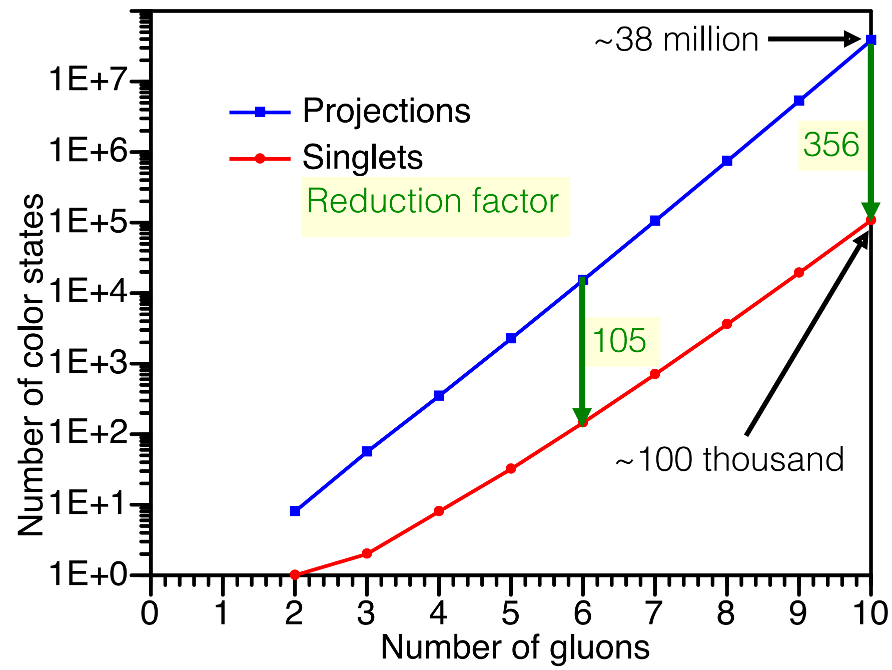
Theory: H.J. Pirner and J.P. Vary, Phys. Rev. Lett. **46**, 1376 (1981) and Phys. Rev. C **84**, 015201 (2011); nucl-th/1008.4962; M. Sato, S.A. Coon, H.J. Pirner and J.P. Vary, Phys. Rev. C **33**, 1062 (1986)

Example of "Full BLFQ" for glueballs

Color basis space dimensions of each multi-gluon space-spin configuration

Figure extended from Vary, et al., 2010 ->

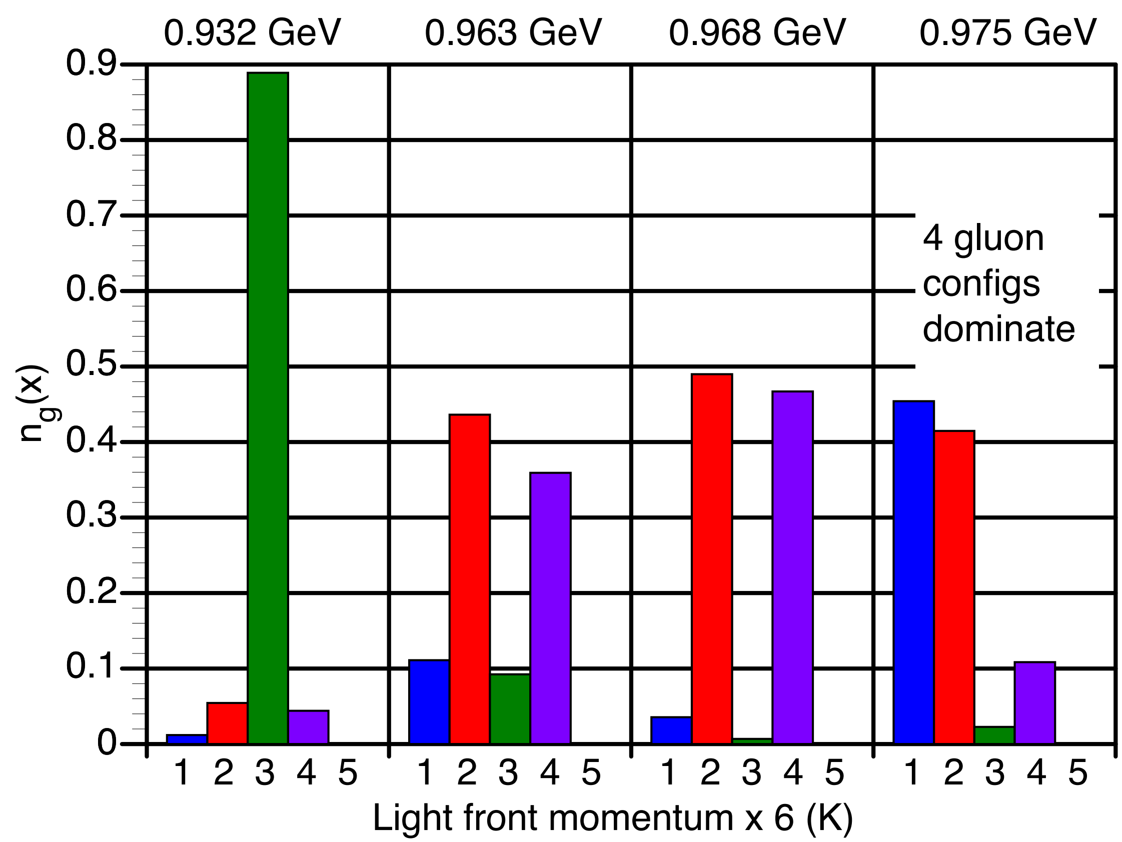
J. P. Vary, et al., Few Body Sys. 59, 56 (2018)



Hamiltonian: kinetic + triple-gluon coupling.
Distribution functions for
4 lowest mass eigenstates
$N_{max} = K = 6$
b=0.5 GeV
g_S=0.5
m_g = 0.25 GeV

Fock sectors up through 6 gluons
Hamiltonian matrix dimension ~ 2000;
Calculation runs in 3 mins on laptop.

Next step: add 2 other vertices



Deuteron wavefunction

Eigenvalue equation: $P^{-}_{QCD} P^{+} |\text{Deuteron}\rangle = M_D^2 |\text{Deuteron}\rangle$ where $M_D \sim 1.84\ GeV$

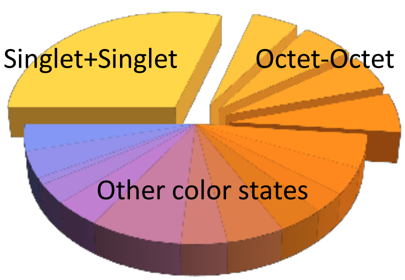
Hamiltonian in Light Front:

$$\boldsymbol{H_{K.E.}} = \sum_i \frac{\boldsymbol{p_i^2 + m_q^2}}{\boldsymbol{p_i^+}}$$

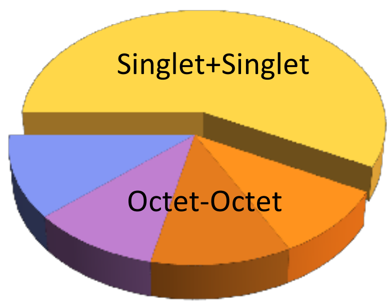
$$\boldsymbol{H_{Interact} = H_{Vertex} + H_{inst} = g\bar{\psi}\,\gamma^{\mu} T^{a}\,\psi\,A_{\mu}^{a} + \frac{g^2 C_F}{2}\,j^{+}\frac{1}{(i\partial^{+})^2}j^{+}}$$

Fock sector expansion: $|\text{Deuteron}\rangle = \psi_{6q}|qqq\ qqq\rangle + \psi_{6q1g}|qqq\ qqq\ g\rangle$

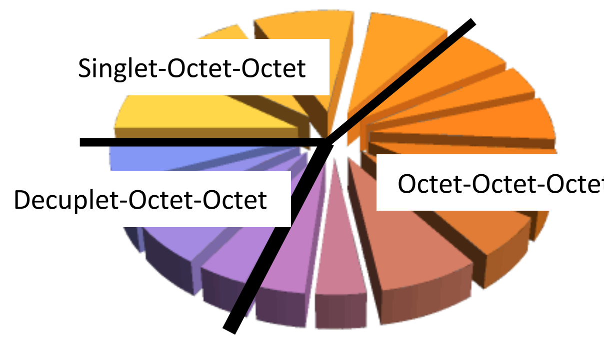
21 color state in total
(1 Singlet-Singlet + 4 Singlet-Octet-Octet+ 16 others)



5 color states
(1 Singlet-Singlet + 4 Octet-Octet)



16 color states
(4 Singlet-Octet-Octet + 8 Octet-Octet_Octet
+ 4 Decuplet-Octet-Octet)



Summary and Outlook

Basis Light Front Quantization approach to mesons and baryons yields competitive descriptions and predictions

- Bound states and transitions of hadrons are described
- Time-dependent scattering applications are showing progress
- Plan: expand the Fock spaces – g's & sea q's in Full BLFQ
- Plan: renormalization, counterterms and regulators – RGPEP
- Plan: initial investigations of the deuteron including dyn. gluon
- Efficient utilization of supercomputing resources
- Well positioned to exploit advances in quantum computing

Thank you for your attention