Handling kinematics in far-forward processes at EIC

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Purpose

Describe simple method for connecting variables in "physics frames" and "detector frame"

Uses natural basis vectors,
avoids explicit Lorentz transforms

→ Analytic expressions
→ Uncertainty propagation
→ Validation with MC generators

References

W. Cosyn, C. Weiss PRC 102 (2020) 065204 [INSPIRE]

A. Bacchetta, W. Cosyn, C. Weiss, contribution to future update of article J. Adam et al. "Accelerator and beam conditions…" (2021) [ZENODO]

Ch. Hyde, P. Nadel-Turonski, C. Weiss et al.,
JLab LDRD 2014/15

Outline

Collinear frames

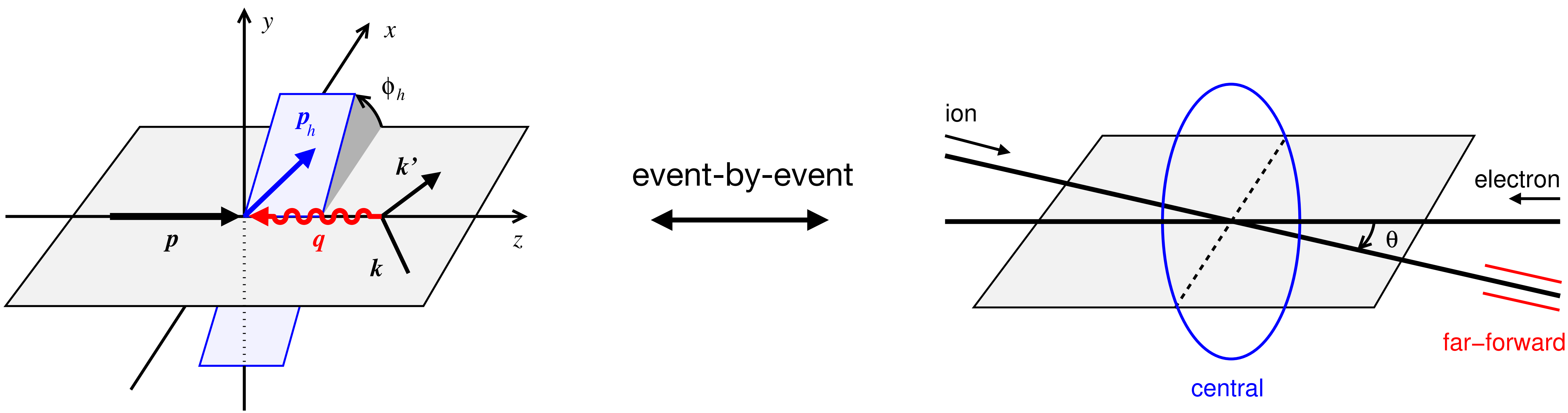
Natural basis vectors

Physics frames ↔ detector frame

Example: LC variables of final-state hadron
↔ pseudorapidity

Far-forward processes with nuclei ←

Physics ↔ detector frame



Physics frames

Usually p, q collinear

Initial state characterized
by invariant variables x, y, Q^2

Invariant variables ↔
momentum components

Detector frame

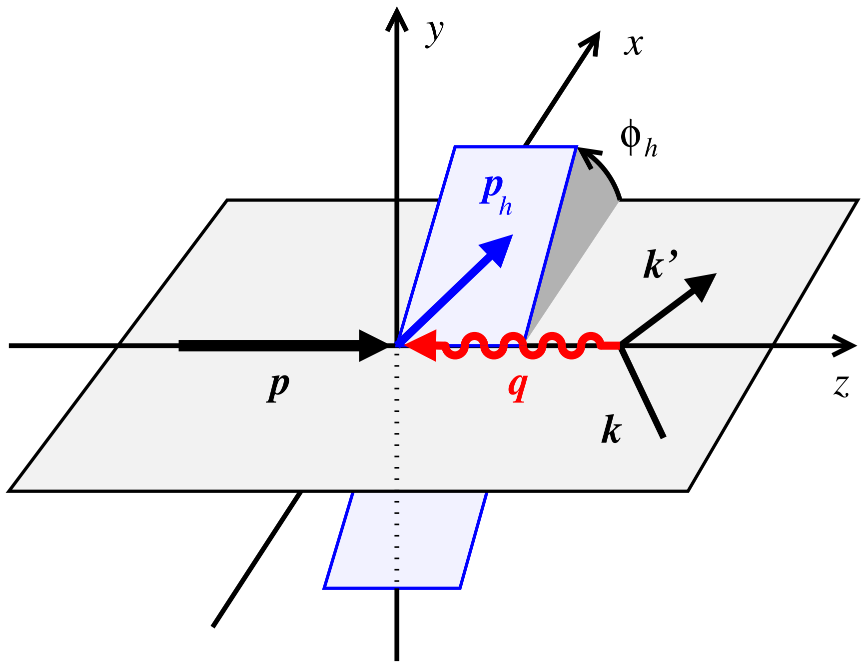
Crossing angle

Central detector aligned with
electron beam axis

Far-forward detector around outgoing
ion beam axis

Uncertainties: Beam momentum,
detector resolution

Physics frames: Collinear frames



$\mathbf{p}, \mathbf{q}$ collinear $\rightarrow$ z-axis

$\mathbf{k}, \mathbf{k}'$ plane $\rightarrow$ xz-plane

$a^\pm \equiv a^0 \pm a^z$ Light-cone components

$a^\mu = [a^+, a^-, \mathbf{a}_T]$ Notation

$ab = \frac{1}{2}(a^+b^- + a^-b^+) - \mathbf{a}_T\mathbf{b}_T$

Initial state

$$p = \left[p^+, \frac{m^2}{p^+}, \mathbf{0}_T\right] \qquad q = \left[-\xi p^+, \frac{Q^2}{\xi p^+}, \mathbf{0}_T\right] \qquad k, k' = [\ldots] \text{ (involves } y)$$

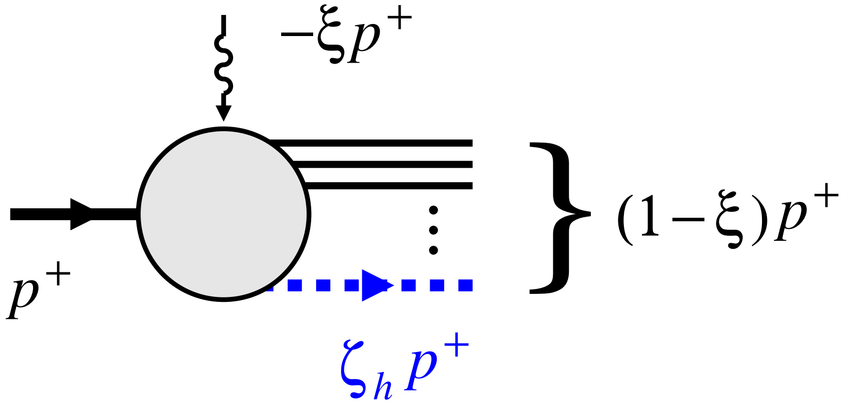
$$\xi = \frac{2x}{1 + \sqrt{1+\gamma^2}} = x + \mathcal{O}(\gamma^2)$$

Light-cone fraction “removed” by virtual photon

$$\gamma = \frac{2mx}{Q}$$

Parameter governing power corrections

Physics frames: Collinear frames



Final state

$$p_h = \left[\zeta_h p^+, \frac{M_h^2}{\zeta_h p^+}, \mathbf{p}_{hT}\right]$$

LC momentum of observed hadron

$$0 < \zeta_h < 1 - \xi$$

LC momentum conservation

Class of frames

Collinear frames are not single frame, but class of frames related by longitudinal boosts (along z-axis)

Momentum components are expressed thru p^+. Its value selects a frame, acts as boost parameter

$p^+ = m$ Target rest frame

$p^+ = Q/\xi$ Breit frame ($q^0 = 0$)

$p^+ = \dfrac{\sqrt{Q^2 + \xi^2 m^2}}{\sqrt{\xi(1-\xi)}}$ Photon-target CM frame (x_F def)

$p^+ \to \infty$ "Infinite-mom frame"

Most physics frames of interest contained in class of collinear frames

Transitions between them can be effected simply by changing the value of p^+

Natural basis vectors

Orthonormal basis 4-vectors constructed from physical momenta

$$e_0^\mu = \frac{p^\mu}{m} \qquad e_3^\mu = \frac{1}{\sqrt{1+\gamma^2}}\left(-\frac{\gamma q^\mu}{Q} + \frac{p^\mu}{m}\right) \qquad e_0^2 = 1,\ e_3^2 = -1$$

Collinear space (0, 3)

$$e_1^\mu = \frac{1}{\sqrt{..}}\left[k^\mu - (e_0 k)e_0^\mu + (e_3 k)e_3^\mu\right] \qquad e_2^\mu = -\frac{1}{\sqrt{..}}\epsilon^{\mu\alpha\beta\gamma}e_{0\alpha}e_{3\beta}e_{1\gamma} \qquad e_{1,2}^2 = -1$$

Transverse space (1, 2)

Light-like basis vectors

$$n_+^\mu = e_0^\mu + e_3^\mu \qquad n_-^\mu = e_0^\mu - e_3^\mu \qquad n_+^2 = n_-^2 = 0 \qquad n_+ n_- = 2$$

$$n_+ = \left[\frac{2p^+}{m}, 0, \mathbf{0}_T\right] \qquad n_- = \left[0, \frac{2m}{p^+}, \mathbf{0}_T\right]$$

Components in collinear frame

n_+ has only ”plus” component, n_- only ”minus”

Natural basis vectors

Expansion of 4-vector a

$$n_- a = \frac{m}{p^+}\, a^+ \qquad n_+ a = \frac{p^+}{m}\, a^- \qquad -e_1 a = a^x \qquad -e_2 a = a^y$$

(1) Collinear frame components of a as contractions with basis vectors

$$a^\mu \;=\; \frac{m}{2p^+}\, a^+ n_+^\mu \;+\; \frac{p^+}{2m}\, a^- n_-^\mu \;+\; a^x e_1^\mu \;+\; a^y e_2^\mu$$

(2) Expansion of a in basis vectors

Transition collinear frame → detector frame

- Given collinear-frame components a^+, a^-, a_T (in frame with given p^+)
- Take momenta p, q, k in detector frame and form basis vectors $\{n_+, n_-, e_1, e_2\}$
- Obtain detector-frame components of a from expansion (2)

Transition detector frame → collinear frame

- Given detector-frame components a^μ
- Take momenta p, q, k in detector frame and form basis vectors $\{n_+, n_-, e_1, e_2\}$
- Obtain collinear-frame components of a from scalar products (1) evaluated using detector-frame components

Example: Hadron LC momenta $\leftrightarrow$ pseudorapidity

Given final-state hadron with collinear-frame momentum $p_h^+ = \zeta_h p^+$ and p_{hT}

Compute pseudorapidity in detector frame $\eta \equiv -\ln\tan\frac{\theta_h \text{ (det)}}{2} \approx -\ln\frac{|p_{hT}| \text{ (det)}}{2p_h^z \text{ (det)}}$

[Here: zero crossing angle = head-on collision, can be generalized]

Take 4-momenta p, q, k in detector frame (ordinary components)

$p = (E_p, 0, 0, p_p)$ proton beam in $+z$ direction

$k = (k_e, 0, 0, -k_e)$ electron beam in $-z$ direction

$q = (q^0, q^x, 0, q^z)$ q-vector

Express components in terms of $x, y, Q^2 \ldots$ simple!

Example: Hadron LC momenta ↔ pseudorapidity

Construct basis vectors in detector frame

$$e_0 = \left(\frac{E_p}{m}, 0, 0, \frac{p_p}{m} \right) \qquad e_3 = \left(\cos\alpha \frac{p_p}{m}, \; -\sin\alpha, 0, \; \cos\alpha \frac{E_p}{m} \right)$$

from p, q

$$e_1 = \left(\sin\alpha \frac{p_p}{m}, \; \cos\alpha, 0, \; \sin\alpha \frac{E_p}{m} \right) \qquad e_2 = (0, 0, 1, 0)$$

from k, p, q

$$\sin\alpha = \frac{\gamma\sqrt{1 - y - \gamma^2 y^2/4}}{\sqrt{1+\gamma^2}}$$

"rotation angle" = $\mathcal{O}(\gamma)$

Represent p_h as expansion in basis vectors

$$p_h^\mu = \frac{m}{2}\frac{p_h^+}{p^+}(e_0 + e_3)^\mu + \frac{p^+ p_h^-}{2m}(e_0 - e_3)^\mu + p_h^x e_1^\mu + p_h^y e_2^\mu$$

Basis vectors given in detector frame

$$\frac{p_h^+}{p^+} = \zeta_h \qquad p^+ p_h^- = \frac{M_h^2 + p_{hT}^2}{\zeta_h}$$

Expansion coefficents given in physics frame

Example: Hadron LC momenta ↔ pseudorapidity

Read off x, y, z components of p_h in detector frame

$$p_h^z\,(\text{det}) \;=\; \zeta_h\, p_p \;+\; \mathcal{O}(\gamma^2)$$

$$p_h^x\,(\text{det}) \;=\; p_h^x \;+\; \frac{M_h^2 + p_{hT}^2 - \zeta_h^2 m^2}{2\zeta_h m}\gamma \;+\; \mathcal{O}(\gamma^2)$$

$$p_h^y\,(\text{det}) \;=\; p_h^y$$

Here simplifications:

I) $p_p \gg m$ ultrarelativistic beam

II) $Q \gg m$ leading powers

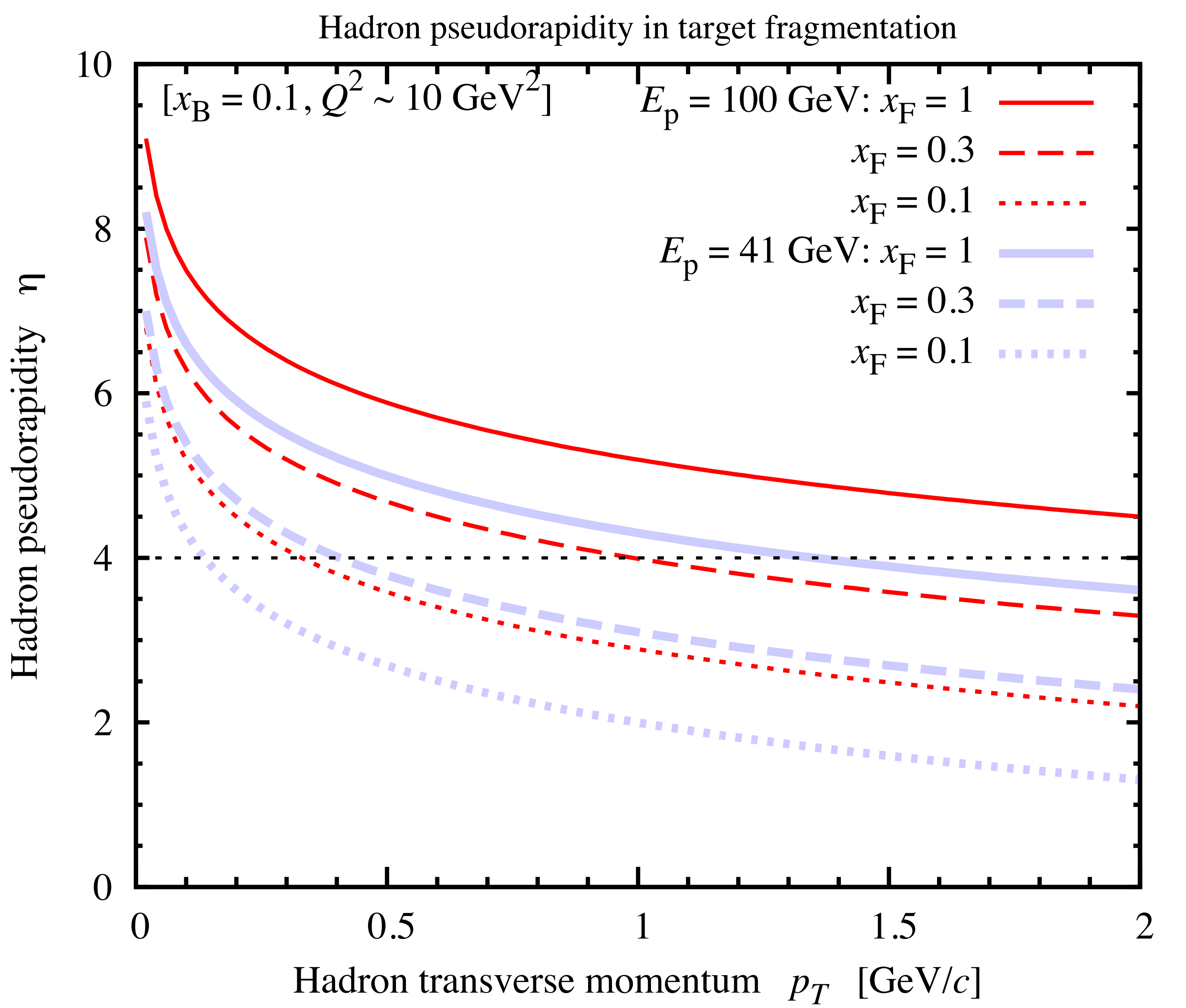
Compute pseudorapidity

$$|p_{hT}|\,(\text{det}) \;=\; |p_{hT}| \;+\; A\gamma\,\cos\phi_h \qquad A = \frac{M_h^2 + p_{hT}^2 - \zeta_h^2 m^2}{2\zeta_h m}$$

$$\eta = -\ln\frac{|p_{hT}|\,(\text{det})}{2p_h^z\,(\text{det})} = -\ln\frac{|p_{hT}| + A\gamma\cos\phi_h}{2\zeta_h p_p} + \mathcal{O}(\gamma^2)$$

ϕ_h dependent modulation caused by angle of q-vector relative to beam axis, power-suppressed $\mathcal{O}(\gamma)$

Example: $x_F \leftrightarrow$ pseudorapidity



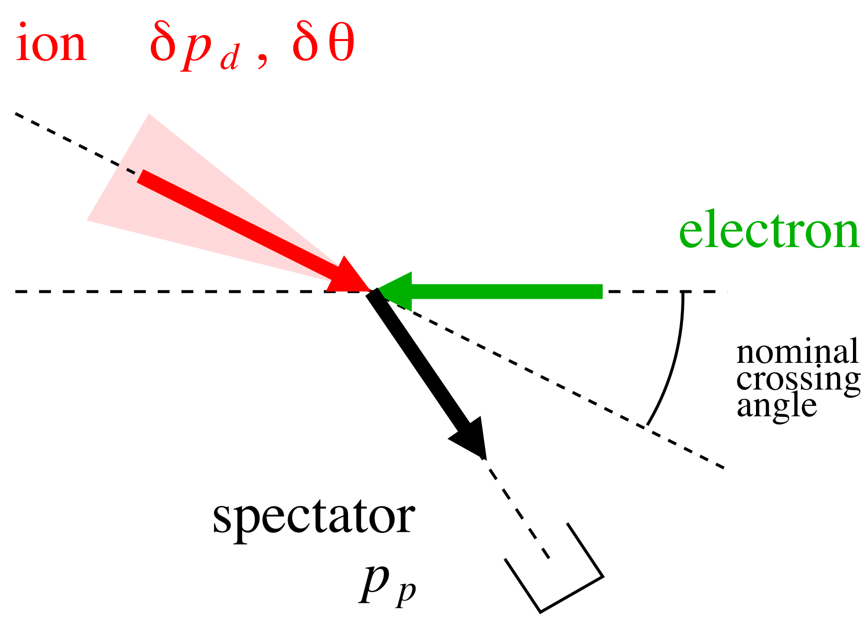
EIC Yellow Report (2021)

Similarly: $x_F, p_{hT} \leftrightarrow$ pseudorapidity η

$$x_F = \frac{p_h^z}{p_h(\text{max})} \text{ in collinear frame } \mathbf{q} = -\mathbf{p}$$

Use same basis vector technique

Far-forward processes



Far-forward processes with nuclei:
Breakup/spectator tagging, coherent scattering

Exact geometry essential: Crossing angle,
x-y asymmetry in detector and beam

Use variables centered on ion beam, e.g. θ_h, p_h

[Pseudorapidity centered on central detector axis becomes ambiguous]

Initial ion beam momentum uncertain:
Beam divergence (optics - focusing)
Beam emittance (beam - cooling)
Crabbing kick uncertainty (longitudal position)

→ this workshop, EIC Yellow Report

Basis vector method very useful for:
Analytic relations between variables
Uncertainty propagation

First studies: JLab LDRD 2014/15

Summary

- Natural basis vectors provide simple method for connecting variables in different frames without use of Lorentz transformations

- Many applications

    Final-state hadrons in central region

    Far-forward processes, esp. nuclear breakup, coherent scattering

    Polarization in initial state

- Description to be provided in updated “Beam conditions” note