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LATTICE QCD AND EFT FOR NUCLEAR PDFS

ECT* Workshop “Tomography of Light Nuclei at an EIC”, November 10th, 2022

NUCLEAR PDFS IN LQCD AND EFT

- How does partonic nucleon structure change in a nucleus?
    - EMC effect: Modification of per-nucleon DIS cross section of nucleons bound in nuclei [EMC 1983]
    - Recent connections to experimental results on short-range correlations
- Many EMC effects accessible at EIC
    - Polarised EMC (polarised light ions)
    - Isovector EMC (SIDIS)
    - Gluon EMC (quarkonium production)
- LQCD can demonstrate QCD origins and make new predictions
- Some old EFT, some new LQCD and some new EFT

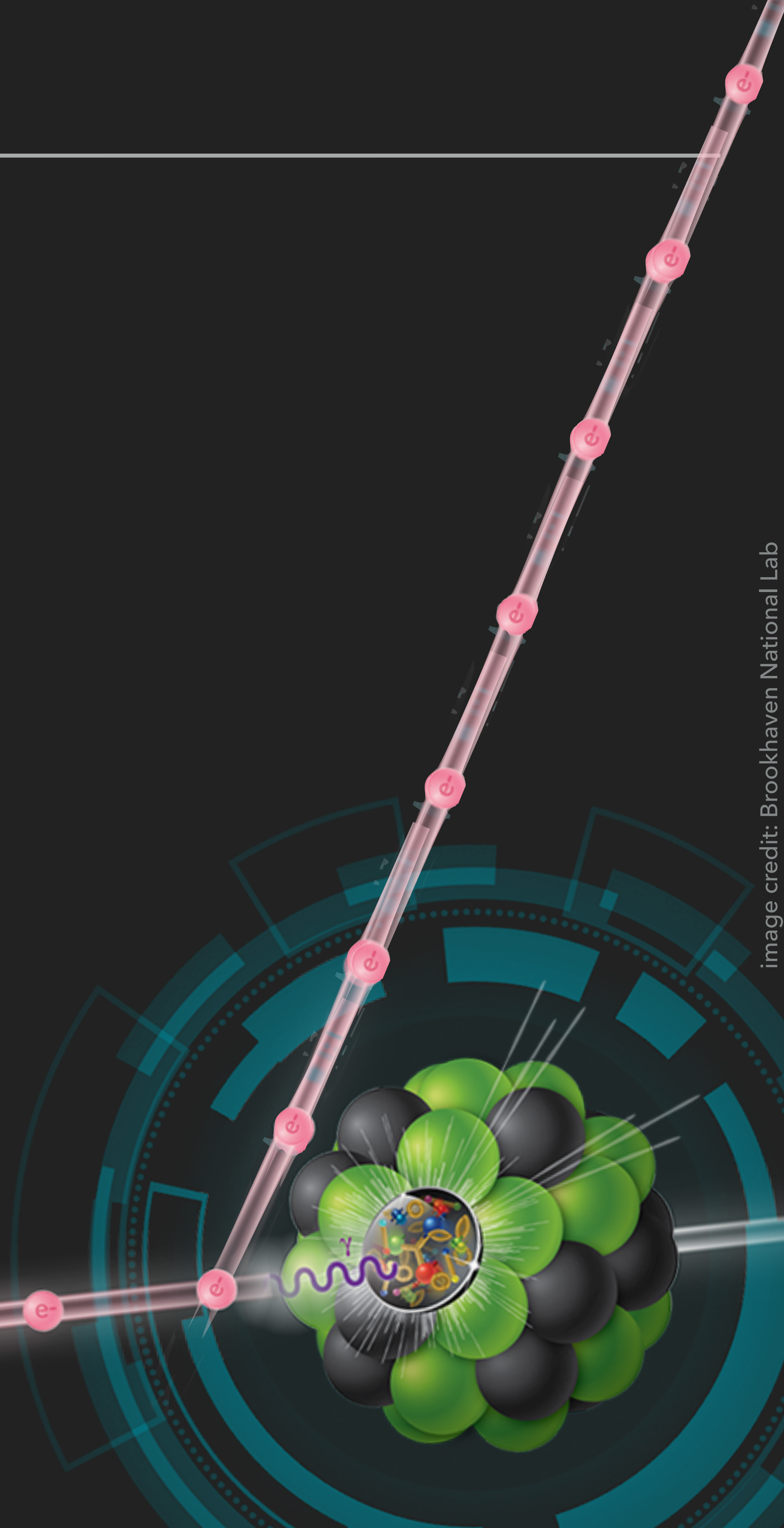


image credit: Brookhaven National Lab

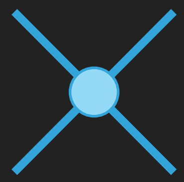
NUCLEAR EFFECTIVE FIELD THEORY

- Hadron level description of low energy properties and interactions of nuclei
- Based on separation of scales and power counting: spontaneously broken chiral symmetry
- Nuclear EFT Lagrangian (pionless for simplicity)

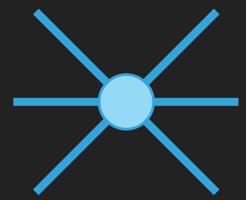
$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \dots$$

$$\mathcal{L}_1 = N^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2M_N} \right) N + \dots$$

$$\mathcal{L}_2 = -\frac{1}{2}\left[C_0 \left(N^\dagger N\right)^2 + C_1 \left(N^\dagger \vec{\sigma} N\right)^2 \right] + \dots,$$



$$\mathcal{L}_3 = -\frac{D_0}{6}(N^\dagger N)^3 + \dots$$



PARTON PHYSICS IN EFT

- EFT provides rigorous description of low-energy QCD with quantifiable uncertainties
- Hard partonic processes not naturally described
- Operator product expansion: moments of PDFs are matrix elements of local twist-2 operators

$$\mathcal{O}^{\mu_0\cdots\mu_n} = \bar{q}\gamma^{(\mu_0} iD^{\mu_1} \dots iD^{\mu_n)} q$$

$$\langle A; p | \mathcal{O}^{\mu_0\cdots\mu_n} | A; p \rangle = \langle x^n \rangle_A (Q) p^{(\mu_0} \dots p^{\mu_n)}$$

$$\langle x^n \rangle_A (Q) = \int_{-A}^{A} x^n q_A(x, Q) dx$$

Evaluate in rest frame, EFT methods applicable

TWIST-2 OPERATORS

- EFT: match QCD operators to all possible hadronic operators with same symmetries
- Used in pion and N sectors to connect lattice PDF moments to experiment [Arndt & Savage; Chen & Ji; Detmold et al.,…]
- Isoscalar, spin independent operator matching:

$$\bar{q}\gamma^{\{\mu_1} D^{\mu_2} \ldots D^{\mu_n\}} q \longrightarrow a_n \frac{1}{\Lambda^n} \mathrm{tr}\left[\Sigma^\dagger D^{\mu_1} \ldots D^{\mu_n} \Sigma + h.c.\right]$$

$$+ c_n N^\dagger \mathcal{V}^{\mu_1 \cdots \mu_n} N + c_n' N^\dagger S^{\{\mu_1} A^{\mu_2} \mathcal{V}^{\mu_3 \cdots \mu_n\}} N + \ldots$$

$$+ \alpha_n N^\dagger \mathcal{V}^{\mu_1 \cdots \mu_n} N N^\dagger N + \beta_n N^\dagger \mathcal{V}^{\mu_1 \cdots \mu_n} \tau_j^{\xi+} N N^\dagger \tau_j^{\xi+} N + \ldots$$

Two body counterterms

- where

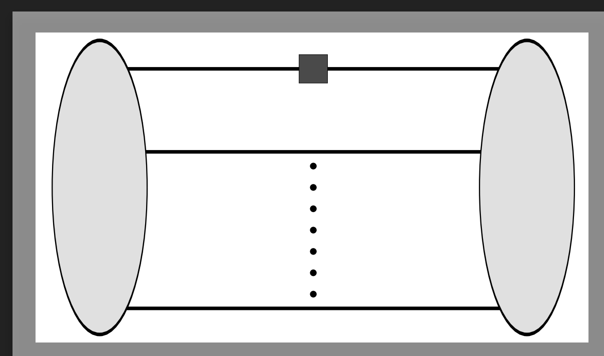
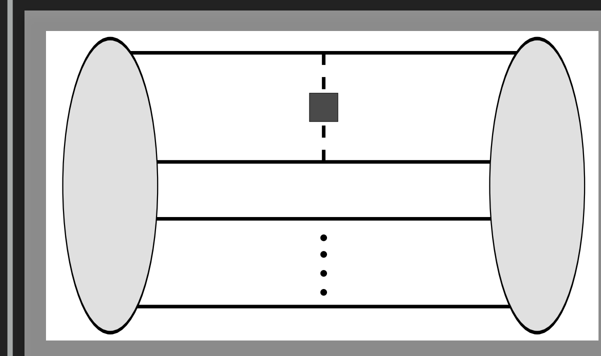
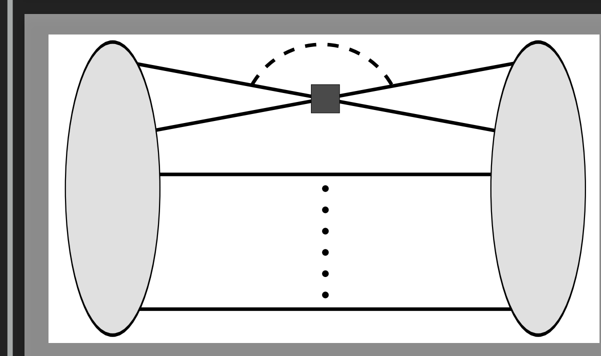
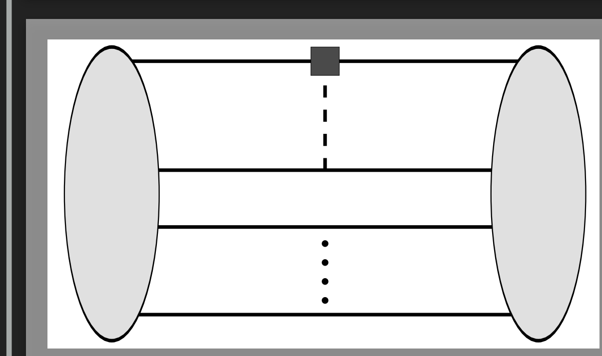
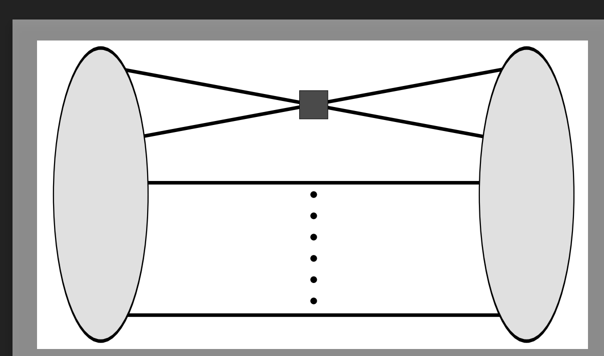
$$\mathcal{V}^{\mu_1 \cdots \mu_n} = \left(v + i\frac{D}{M}\right)^{\mu_1} \ldots \left(v + i\frac{D}{M}\right)^{\mu_n}$$

$$\tau_j^{\xi\pm} = \frac{1}{2}\left(\xi^\dagger \tau_j \xi \pm \xi \tau_j \xi^\dagger\right)$$

[J W Chen, WD PLB 2005]

POWER COUNTING

- Power counting in KSW scheme (Weinberg scheme similar) for small ε~p/M

ε^{-3}		higher orders	   ... ...
ε^{0}			

NUCLEAR PDF MOMENTS

- Nucleon matrix elements (includes pion loop effects)

$$v_{\mu_1} \ldots v_{\mu_n} \langle N | \mathcal{O}^{\mu_1 \cdots \mu_n} | N \rangle = \langle x^n \rangle_q$$

- Nuclear matrix elements

$$\begin{aligned} \langle x^n \rangle_{q|A} &\equiv v_{\mu_1} \ldots v_{\mu_n} \langle A | \mathcal{O}^{\mu_1 \cdots \mu_n} | A \rangle \\ &= \langle x^n \rangle_q \left[A + \alpha_n \left\langle A \middle| \left(N^\dagger N\right)^2 \middle| A \right\rangle + \beta_n \left\langle A \middle| \left(N^\dagger \tau N\right)^2 \middle| A \right\rangle \right] + \ldots \end{aligned}$$

Dominant term

- β_n term suppressed by N_c^2 [Kaplan & Savage 96; K & Manohar 97]
- Ellipsis includes higher-body operators, terms with derivatives: higher-order in power-counting

[J W Chen, WD PLB 2005]

FACTORISATION AND UNIVERSALITY

- Inverse Mellin transform

$$\langle x^n \rangle_{q|A} = \langle x^n \rangle_q \left[A + \alpha_n \left\langle A \middle| \left(N^\dagger N\right)^2 \middle| A \right\rangle \right]$$

$$\Rightarrow \quad \frac{f^A(x)}{A} = f^N(x) + g_2(A) f_2(x)$$

with

$$g_2(A,\Lambda) = \frac{1}{A} \left\langle A \middle| \left(N^\dagger N\right)^2 \middle| A \right\rangle_\Lambda \qquad \alpha_n = \frac{1}{\langle x^n \rangle_q} \int dx x^n f_2(x)$$

$f_2(x)$ describes two-body contributions

- Factorisation of (x,Q^2) and A dependence: universality
- Observed in data [Daté et al. 84,..., Frankfurt & Strikman 87, Gomez et al. 95]
- Requires there be only a single relevant non-trivial source of A dependence in EFT operator
- Factorisation breaks: holds to O(ε) or N_c^2: expect ~20%

[J W Chen, WD PLB 2005]

FACTORISATION AND UNIVERSALITY

- Factorised form (also holds for QE cross section)

$$\frac{f^A(x)}{A} = f^N(x) + g_2(A) f_2(x)$$

- Simple manipulations imply

$$R(x,A) = \frac{2}{A}\frac{f^A(x)}{f^d(x)} = 1 + (a_2(A)-1)\left(1 - \frac{f^p(x)+f^n(x)}{f^d(x)}\right)$$

where (scheme independent)

$$a_2(A) = \frac{g_2(A,\Lambda)}{g_2(2,\Lambda)} = \frac{\left\langle A\middle|\left(N^\dagger N\right)^2\middle|A\right\rangle_\Lambda}{\left\langle d\middle|\left(N^\dagger N\right)^2\middle|d\right\rangle_\Lambda}$$

- Consequently $R(1<x<2,A) = a_2(A)$ as $f_p(x>1) = f_n(x>1) = 0$
- EMC-SRC relation: $dR(x,A)/dx = (a_2(A)-1)\,h(x)$

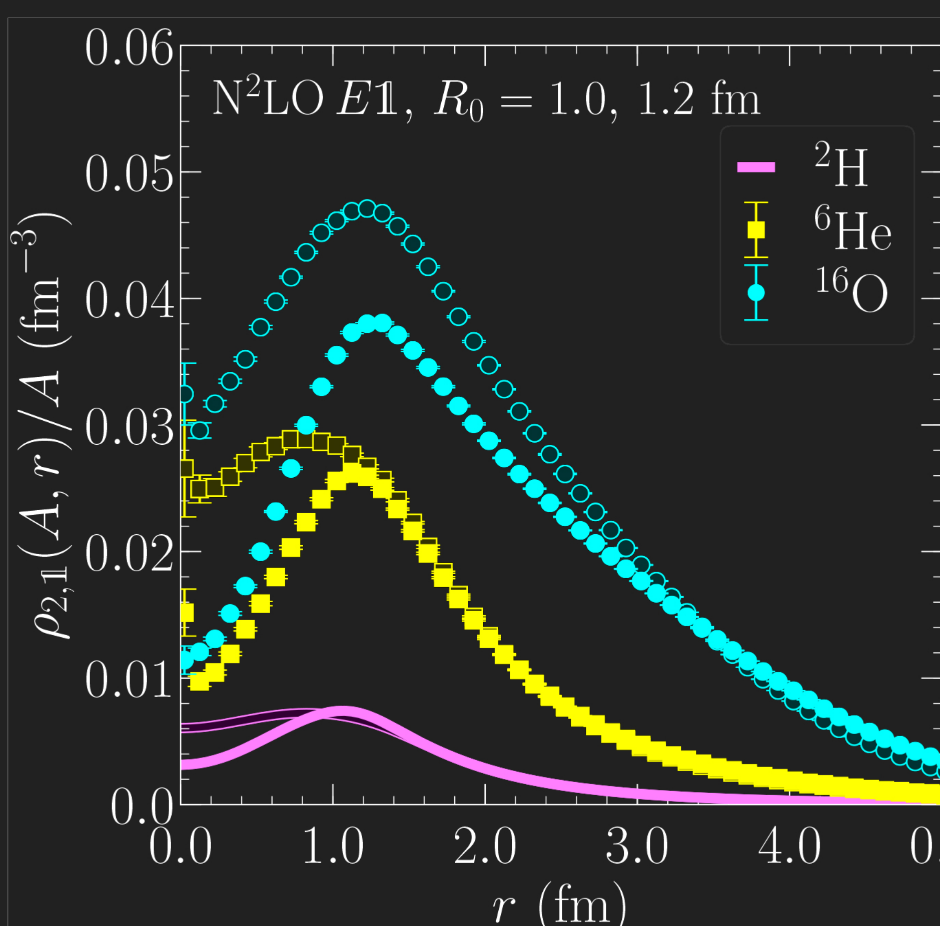
[J W Chen, WD, J Lynn, A Schwenk PRL 2017]

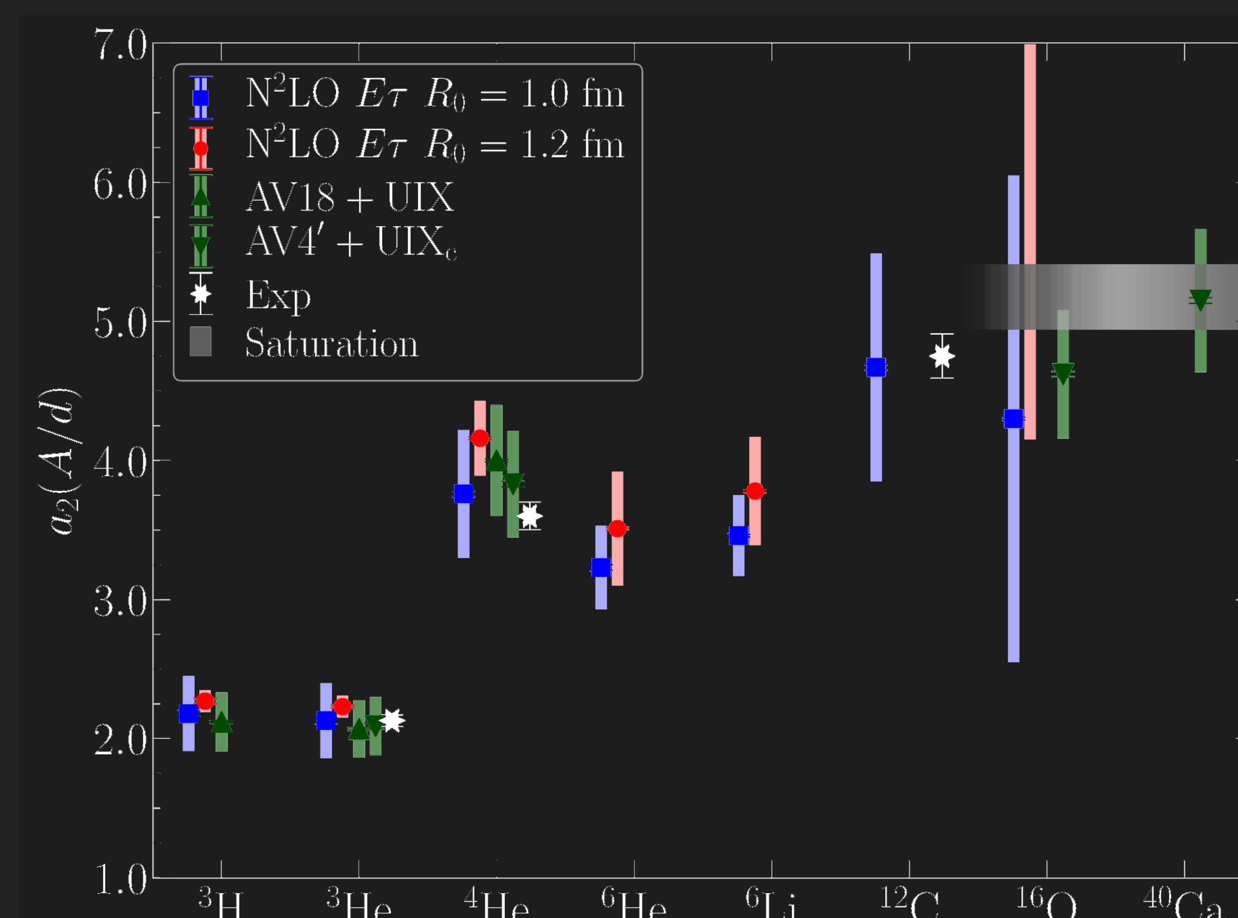
QUANTUM MONTE CARLO CALCULATIONS

- QMC calculations of two-nucleon distributions
- Postdict/predict SRC scaling factors

$$\rho_{2,1}(A,r) = \frac{1}{4\pi r^2}\left\langle \Psi \left| \sum_{i<j}^{A} \delta\left(r - r_{ij}\right) \right| \Psi \right\rangle$$

$$a_2(A/d) = \lim_{r\to 0} \frac{2}{A}\frac{\rho_{2,1}(A,r)}{\rho_{2,1}(d,r)}$$

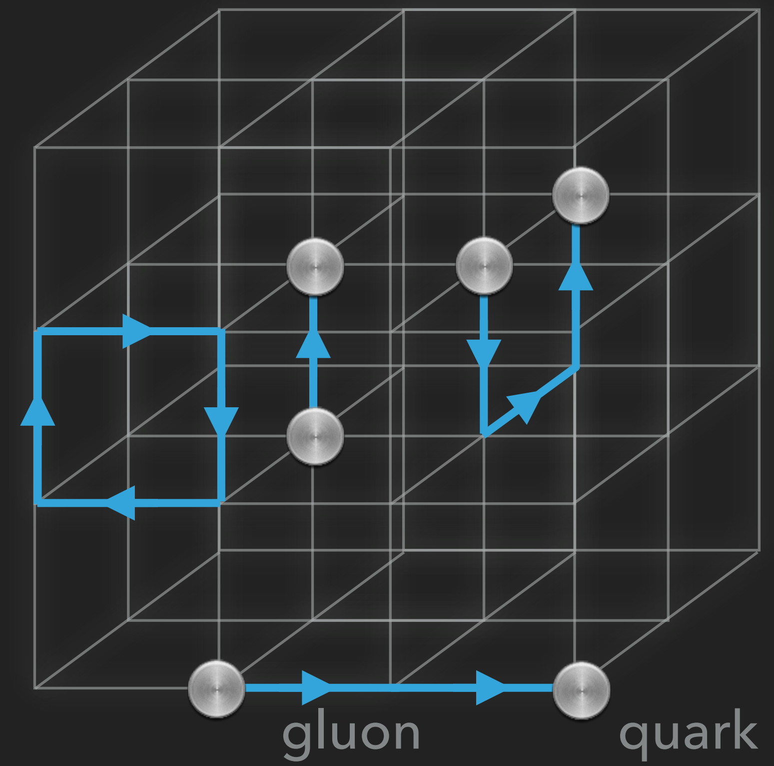


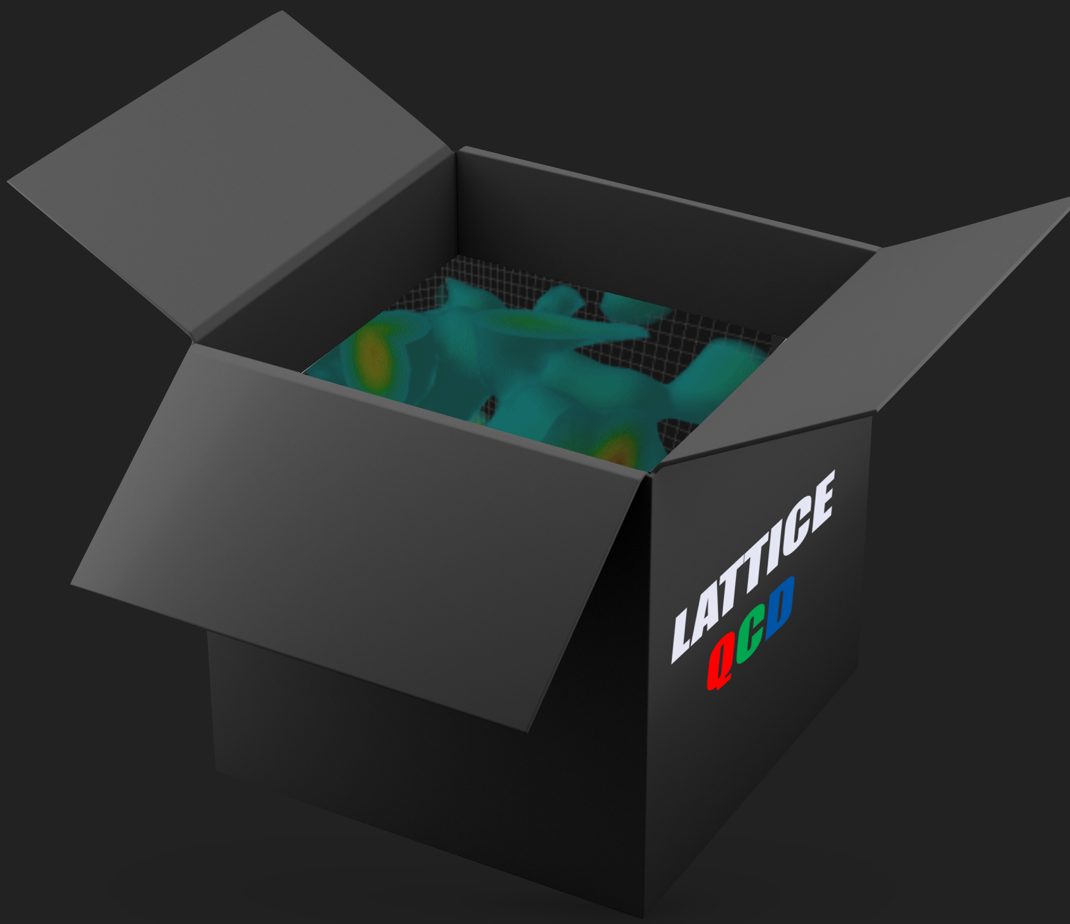


[J Lynn, D Lonardoni, WD et al JPG 2020]

LATTICE QCD

- Strong coupling definition of QCD
- Numerical tool for nonperturbative QCD calculations
  - Discretise and compactify spacetime
  - Integration over 10^{12} degrees of freedom in current calculations using importance sampling Monte Carlo
  - Understand effects of discretisation and compactification and finite statistics





LQCD MOMENTUM FRACTIONS

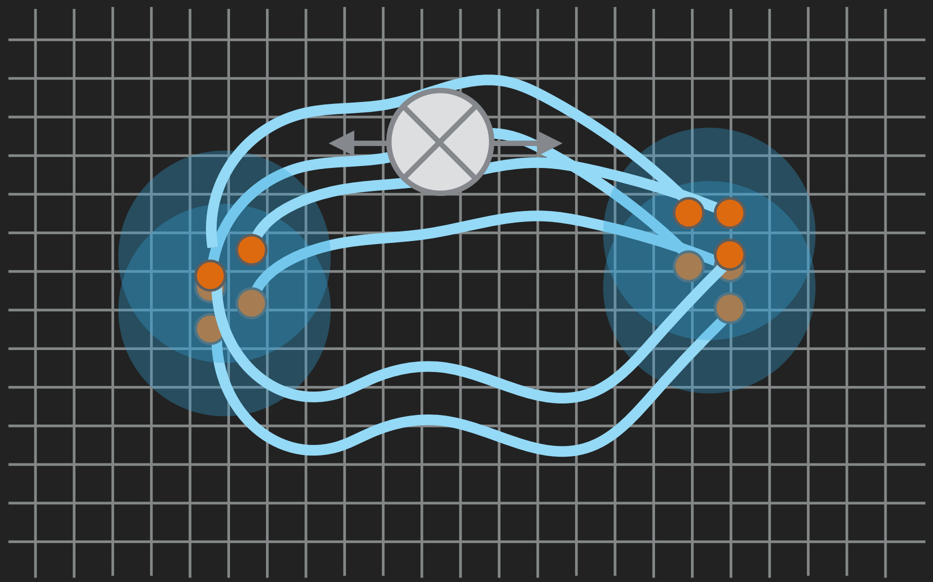
- NPLQCD calculation of quark momentum fractions
- Local operator matrix element (n=1)

$$\mathcal{O}^{\mu_0\cdots\mu_n} = \bar{q}\gamma^{(\mu_0} iD^{\mu_1}\cdots iD^{\mu_n)}q$$

$$\mathcal{O}_g^{\mu_0\ldots\mu_n} = F^{\mu(\mu_0} iD^{\mu_1}\ldots iD^{\mu_{n-1}} F^{\mu_n)\mu}$$

$$\langle A;p\,|\mathcal{O}^{\mu_0\cdots\mu_n}|\,A;p\rangle = \langle x^n\rangle_A(Q)p^{(\mu_0}\ldots p^{\mu_n)}$$

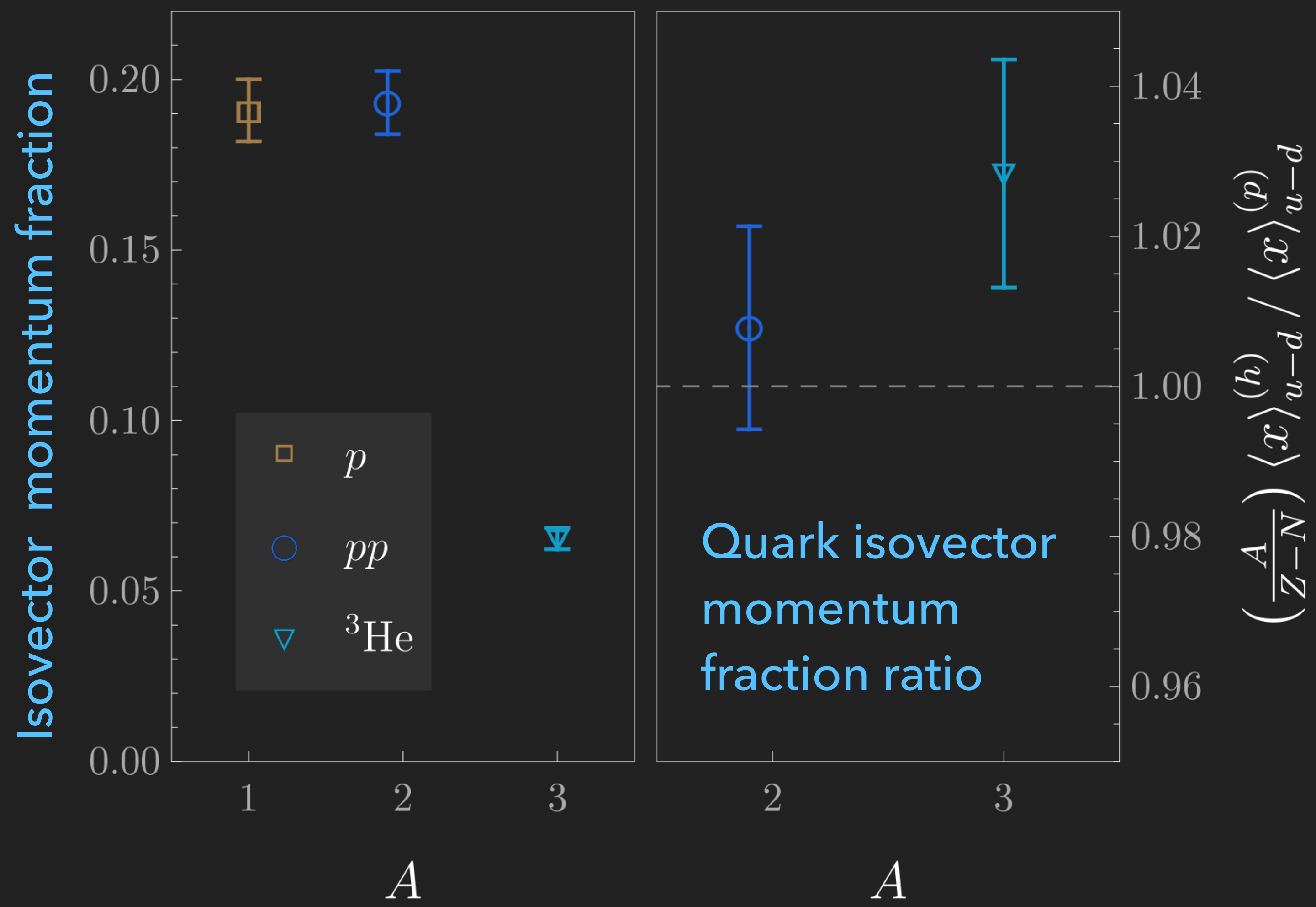
- *pp*, *d* and ^{3}*He* systems
- Unphysical quark masses for which pion mass is 806 MeV
- Single lattice volume (EFT critical for extrapolation to infinite volume - see backup slides)

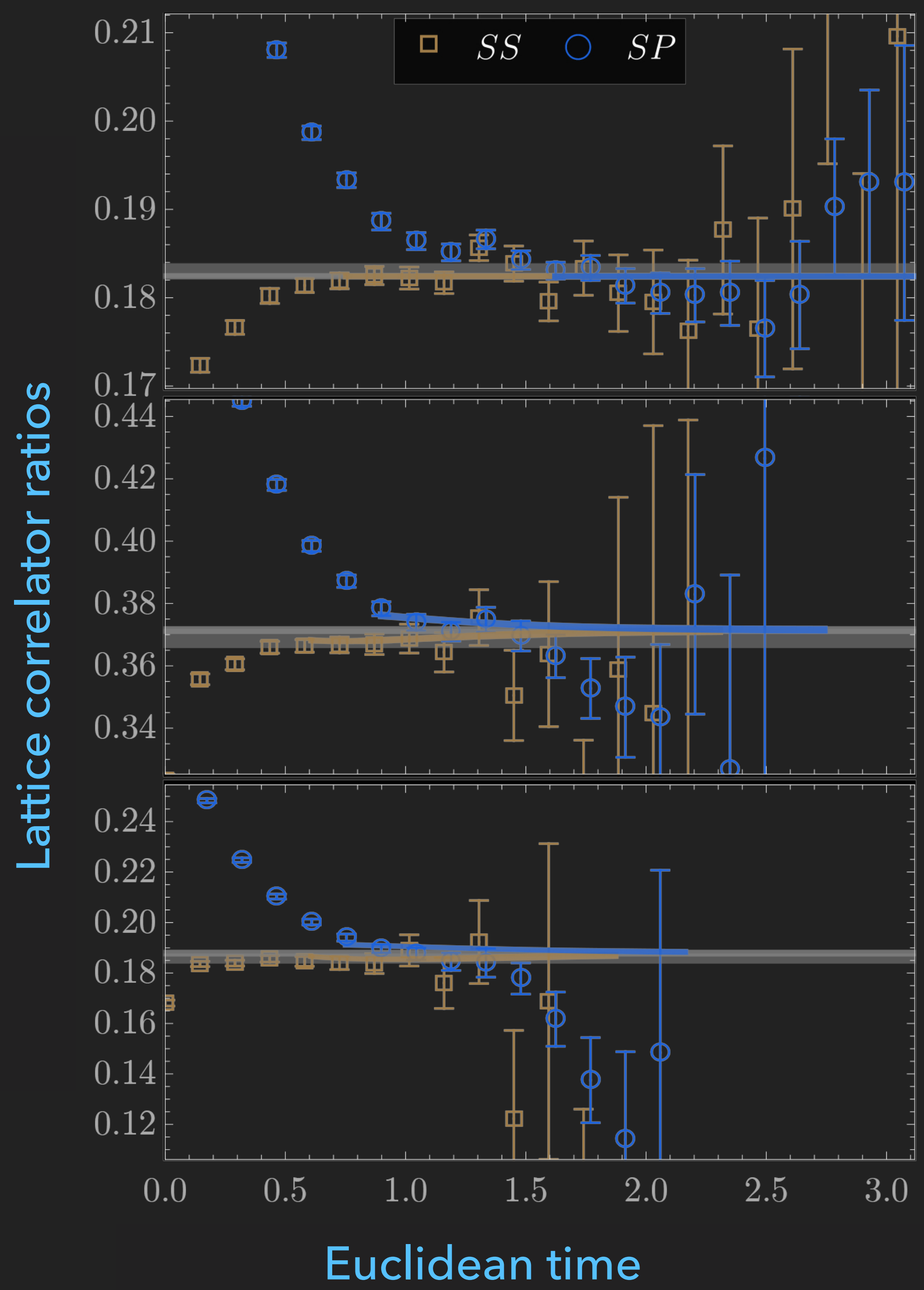


[WD et al. PRL 2021]

QUARK MOMENTUM FRACTION

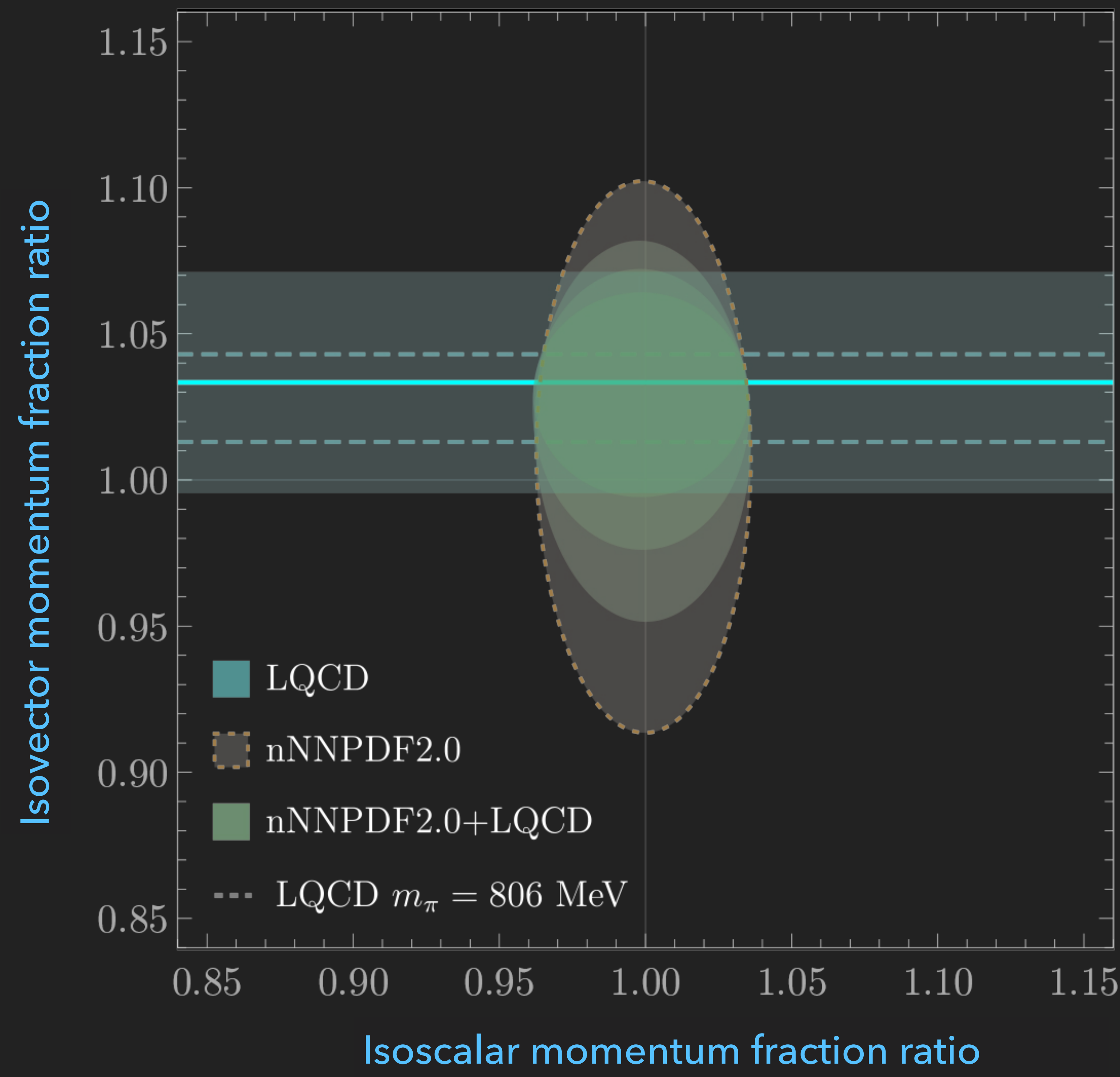
- Isovector quark momentum fraction [NPLQCD PRL 2021]
  - 800 MeV pion mass
  - p, pp, ^{3}He





QUARK MOMENTUM FRACTION

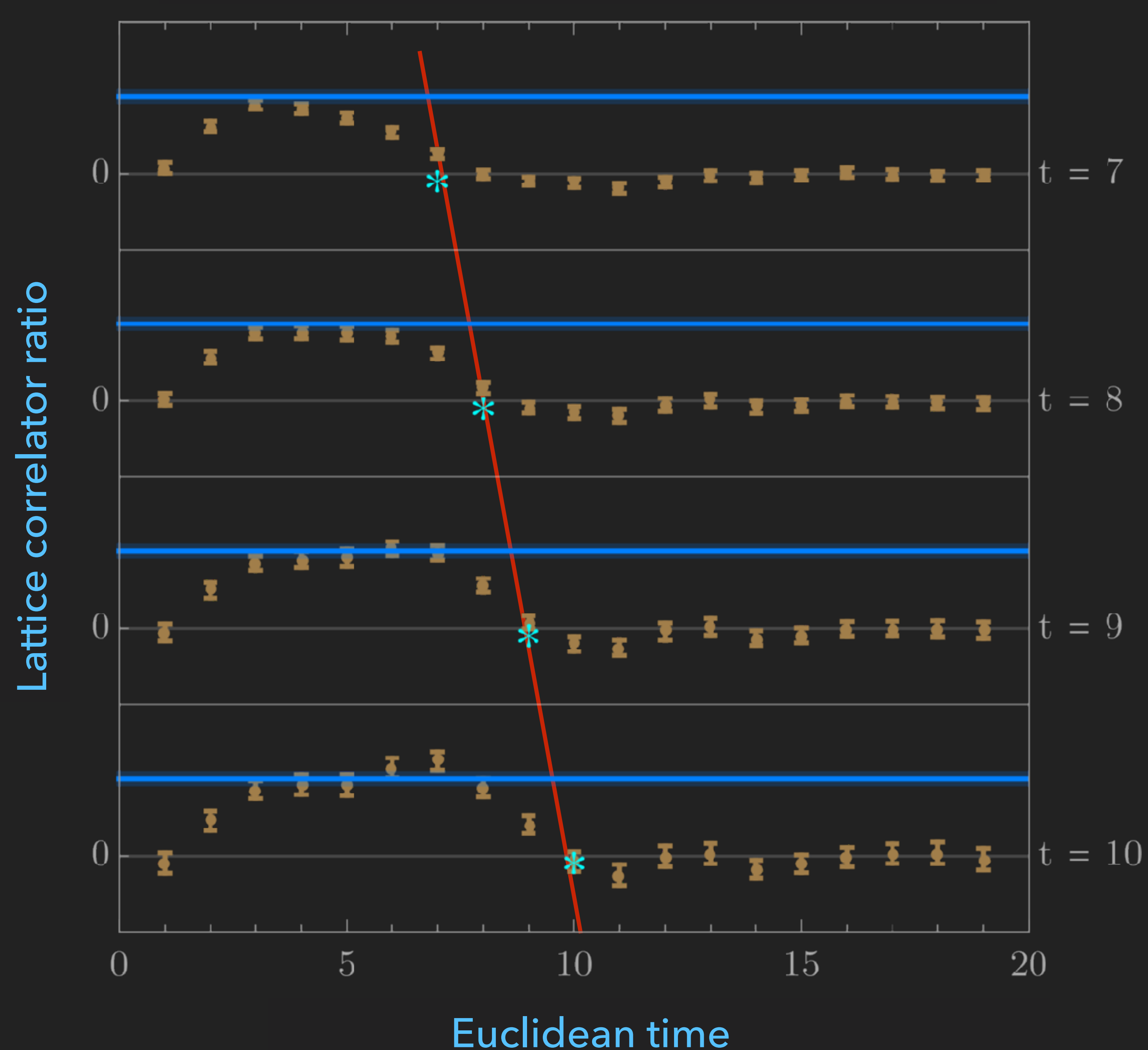
- With mild assumptions, this constrains phenomenology [NPLQCD PRL 2021]
- Isovector vs isoscalar momentum fraction
  - Ratios to naive expectation
  - Ellipses: different uncertainties from quark mass extrapolation
  - Input LQCD result into nNNPDF analysis framework
- Physical quark mass calculations ongoing



GLUON MOMENTUM FRACTION

- First LQCD attempt: Look for nuclear effects in the first moments of the gluon PDF
- Doubly challenging
  - Nuclear matrix element
  - Gluon observable (suffer from poor signal-to-noise)
- Study for A=2,3 systems at unphysical quark masses
- Ratios of matrix elements gives deviation from nucleon
- Suggestive of gluon EMC effect but more precision and physical parameters required

Deuteron: ratio matrix element for 0<<τ<<t



[NPLQCD Collaboration PRD 2017]

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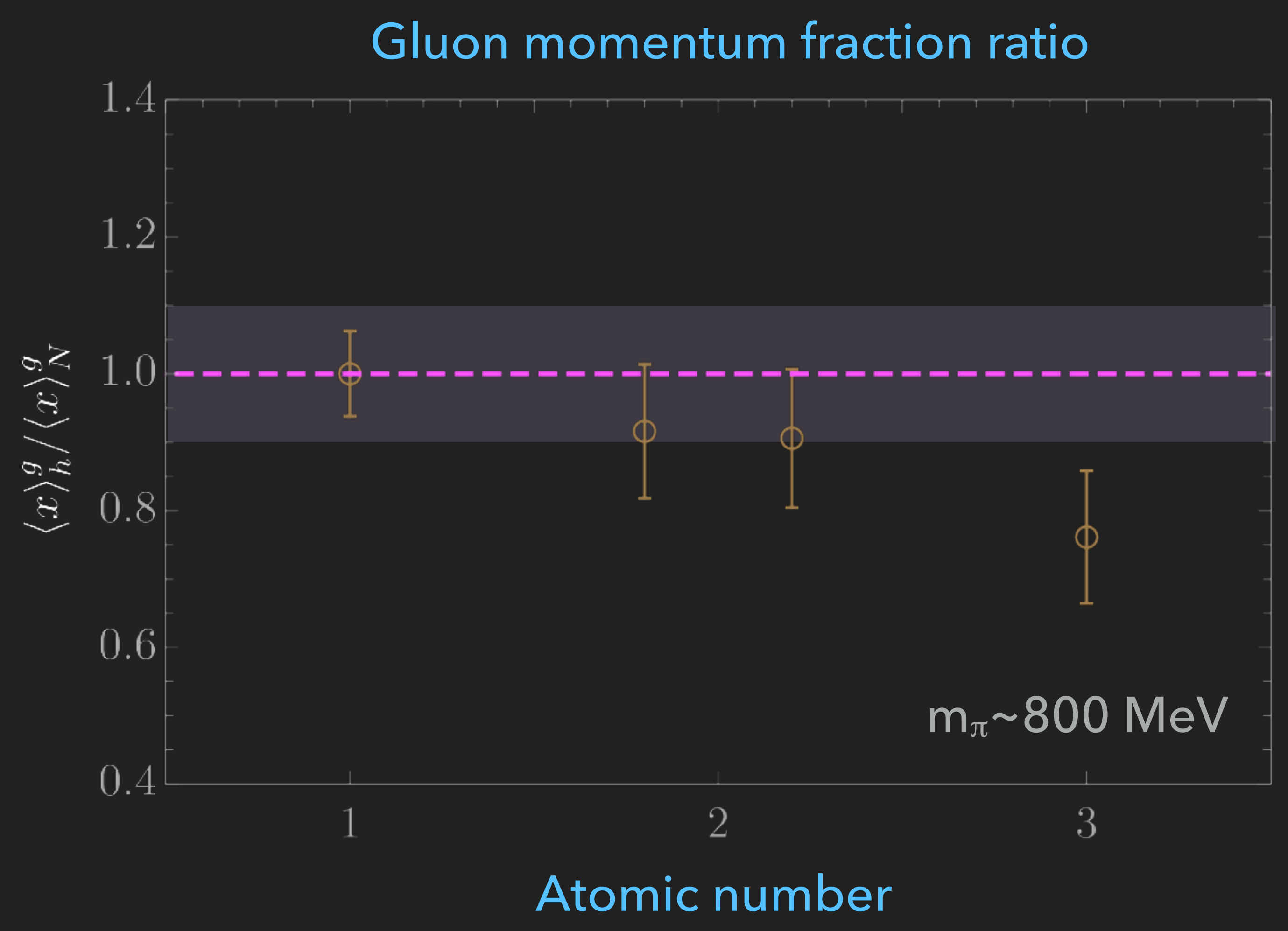
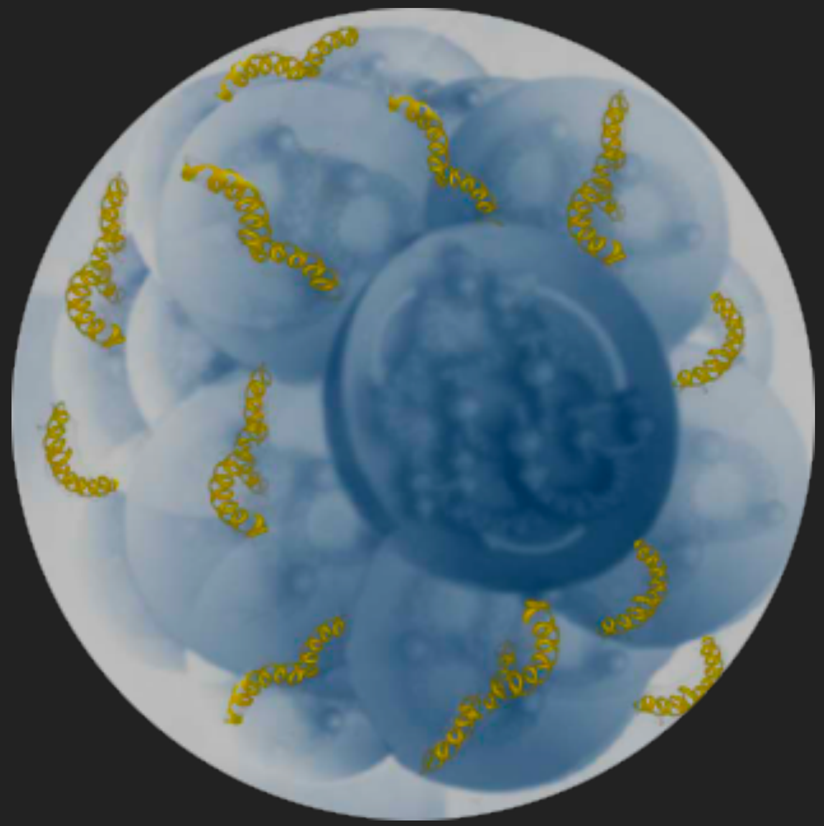


[NPLQCD Collaboration PRD (2017)]

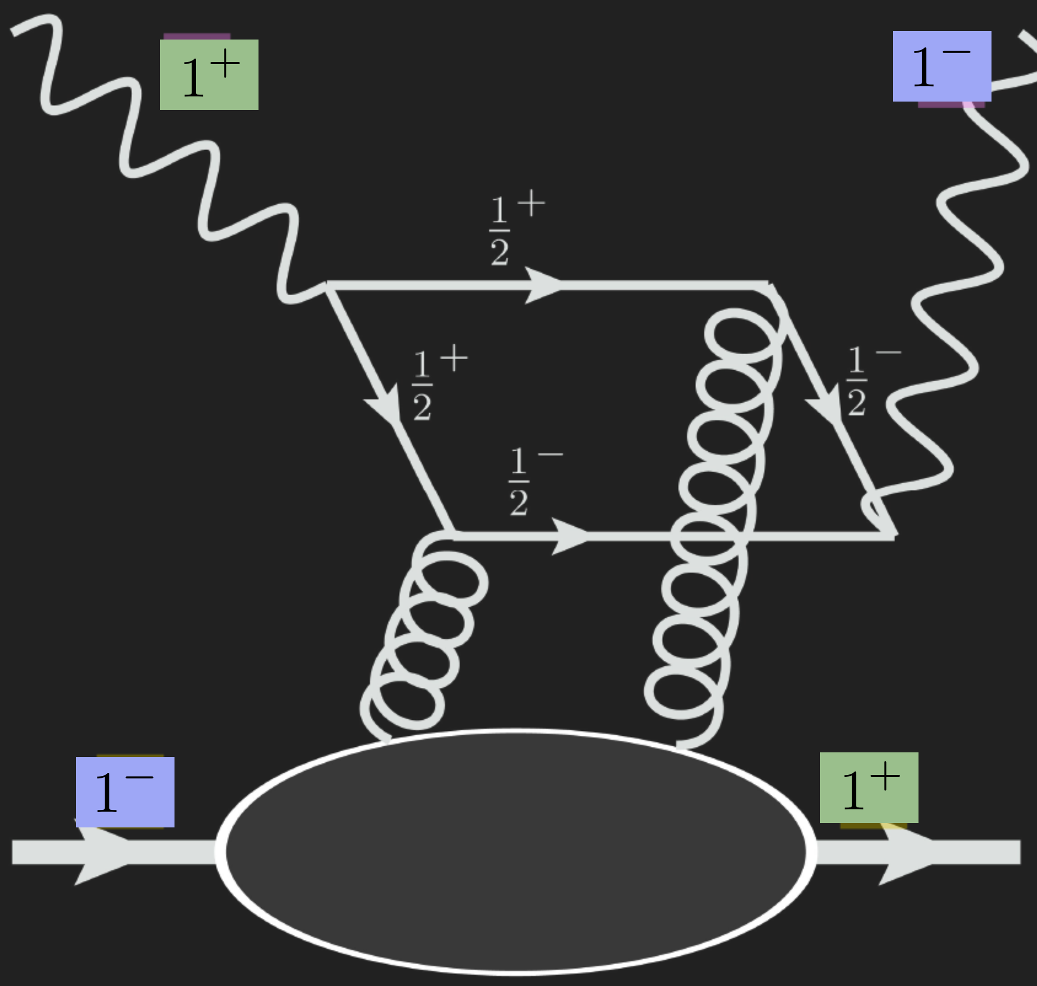
EXOTIC GLUE

- Contributions to nuclear structure from gluons not associated with individual nucleons in nucleus
- Exotic glue operator:
  - Nucleon: $\langle p|\mathcal{O}|p\rangle = 0$
  - Nuclear: $\langle A, Z|\mathcal{O}|A, Z\rangle \neq 0$
- The ultimate EMC effect!
- Example is gluon transversity



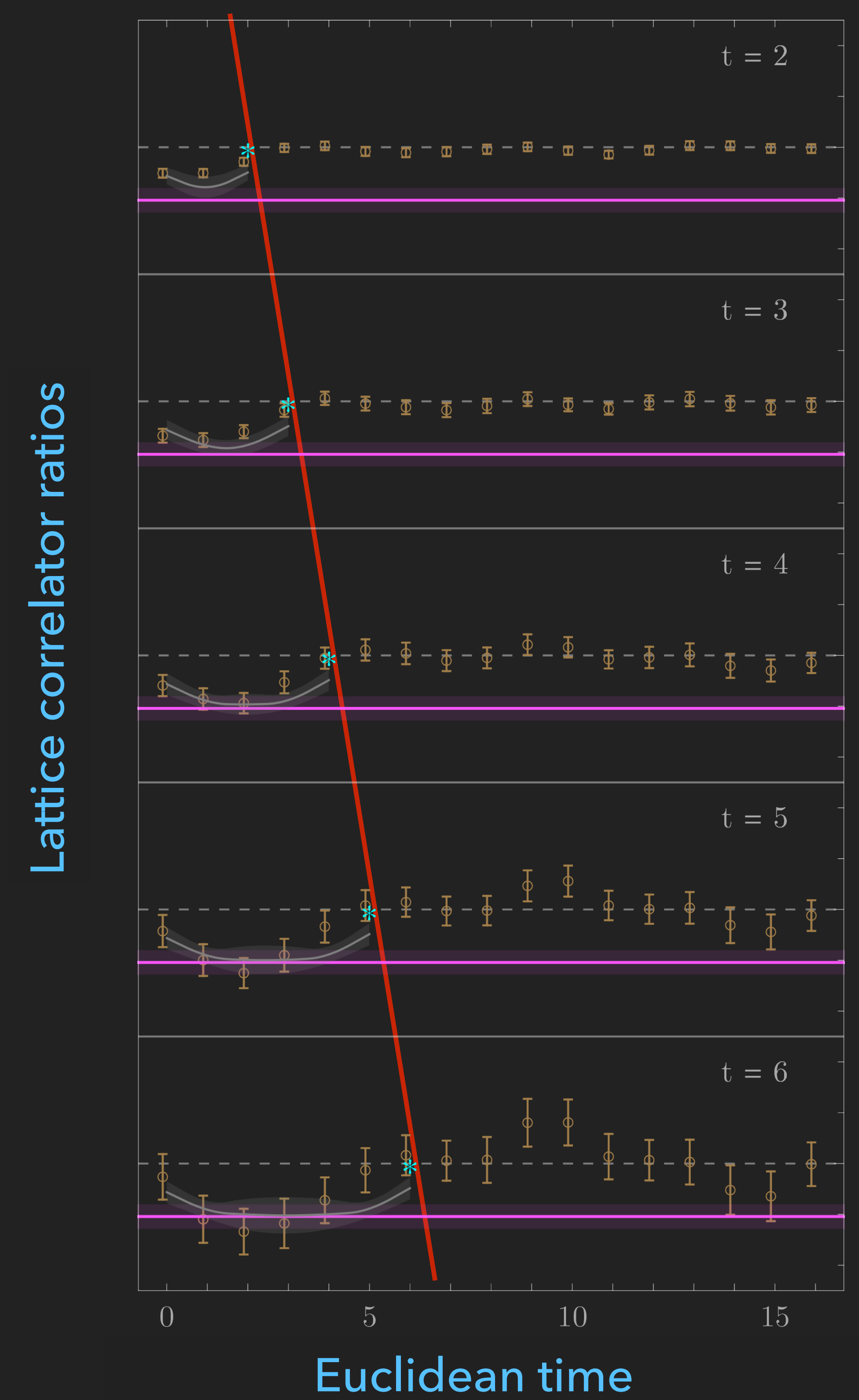
GLUON TRANSVERSITY

- Double helicity flip: non-vanishing in forward limit for targets with spin≥1 [Jaffe & Manohar, PLB223 (1989) 218]
- Purely gluonic: No mixing with quarks at leading twist
- Experimentally measurable in unpolarised electron DIS on polarised target
  - Nitrogen target: JLab LoI
  - Polarised nuclei at EIC



GLUON TRANSVERSITY

- First moment of gluon transversity distribution in the deuteron, m_π~800 MeV
- First evidence for non-nucleonic gluon contributions to nuclear structure
- Clear signal but not sufficient systematic control to determine moment
- Magnitude relative to momentum fraction as expected from large-N_c
- Great opportunity for prediction for EIC



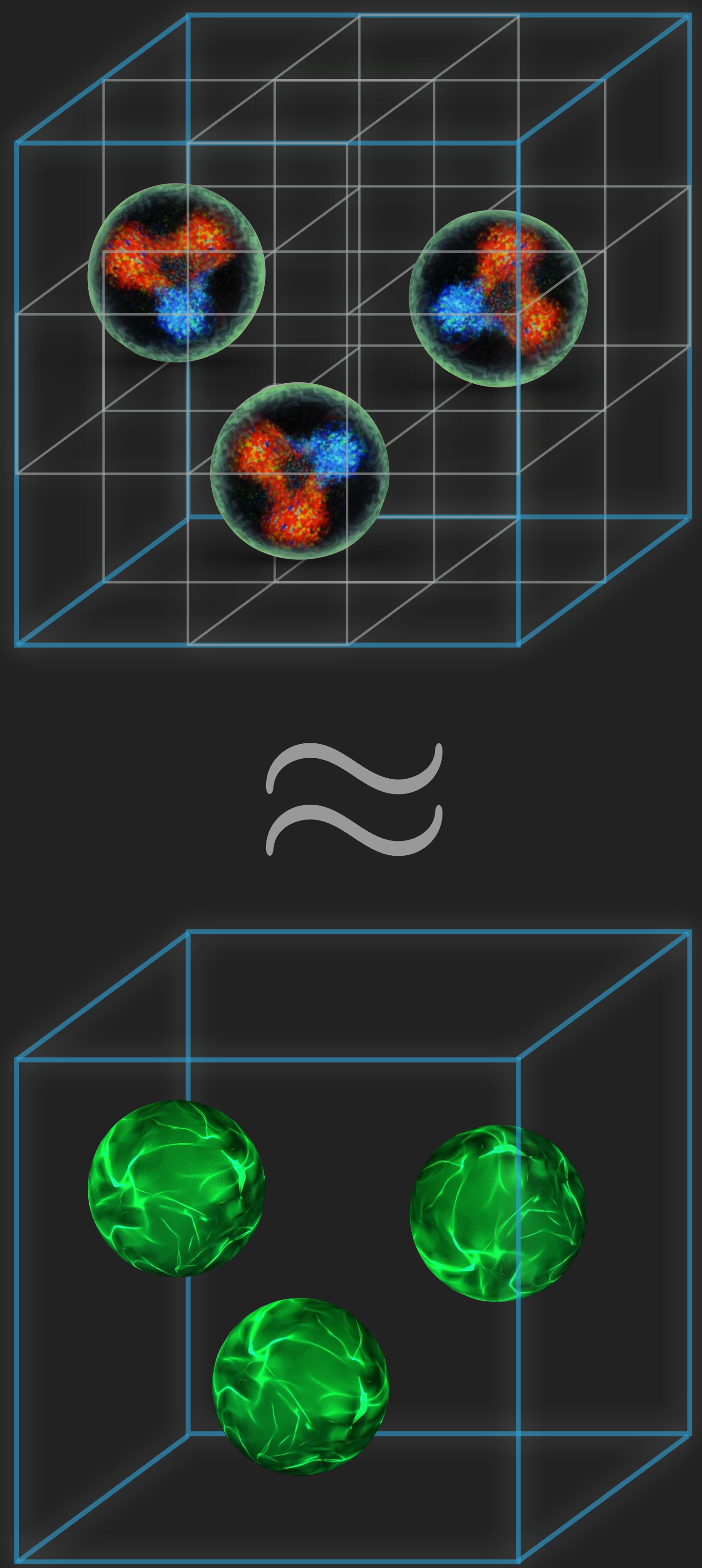
[NPLQCD Collaboration PRD 2017]

OUTLOOK

- First LQCD constraints on nuclear PDFs via EFT matching
  - Isovector EMC ratios
  - Exotic glue
- Other flavour and spin structures, higher moments underway
- Larger nuclei: might require different many-body techniques
  - Nuclear lattice EFT
- x-dependence of nuclear PDFs
  - Reconstruct from enough moments
  - Use new lattice methods based on large momentum effective theory (LaMET)

EFT MATCHING

- Major issue for LQCD nuclei is finite volume (FV) effects
- Well-known Lüscher method to understand FV effect in spectrum
- Less well-developed technologies for matrix elements
- Direct matching between LQCD and EFT in same FV
  - Use EFT to extrapolate to infinite volume
  - Pionless EFT for simplicity



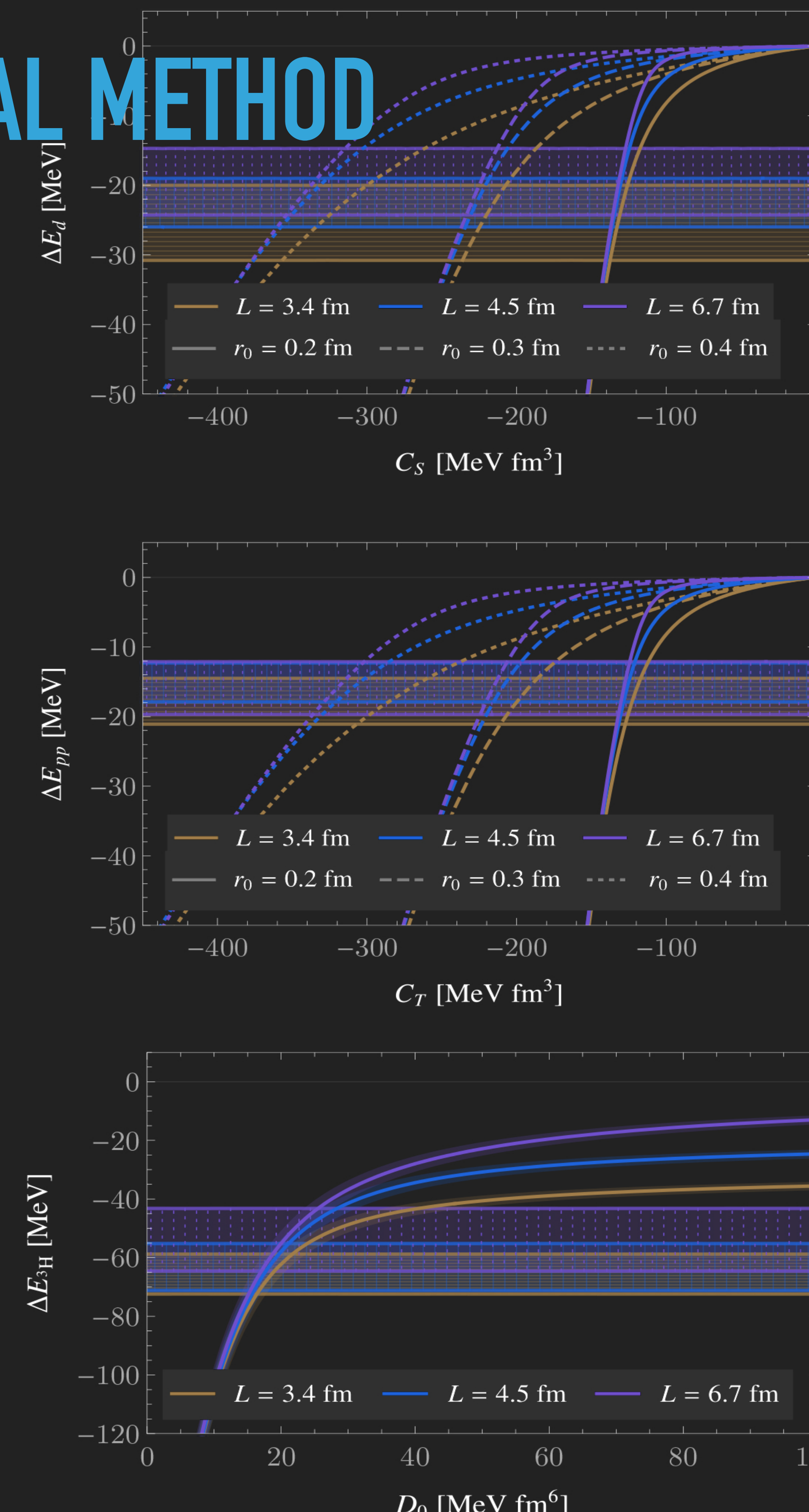
FVEFT USING THE STOCHASTIC VARIATIONAL METHOD

- SVM: variational approach using an expanding set of correlated (shifted) Gaussian trial states

  [Eliyahu, Bazak, Barnea 2019]

$$E_0^h \leq \frac{\int \Psi_h^{(N)}(\mathbf{x})^* H \Psi_h^{(N)}(\mathbf{x}) d\mathbf{x}}{\int \Psi_h^{(N)}(\mathbf{x})^* \Psi_h^{(N)}(\mathbf{x}) d\mathbf{x}}$$

- Match onto LQCD FV energies to determine nuclear wave functions
  - 2 and 3-body energies fix NN and NNN contact interactions
- Determines infinite volume bindings



[WD, P Shanahan. 2102.04329]

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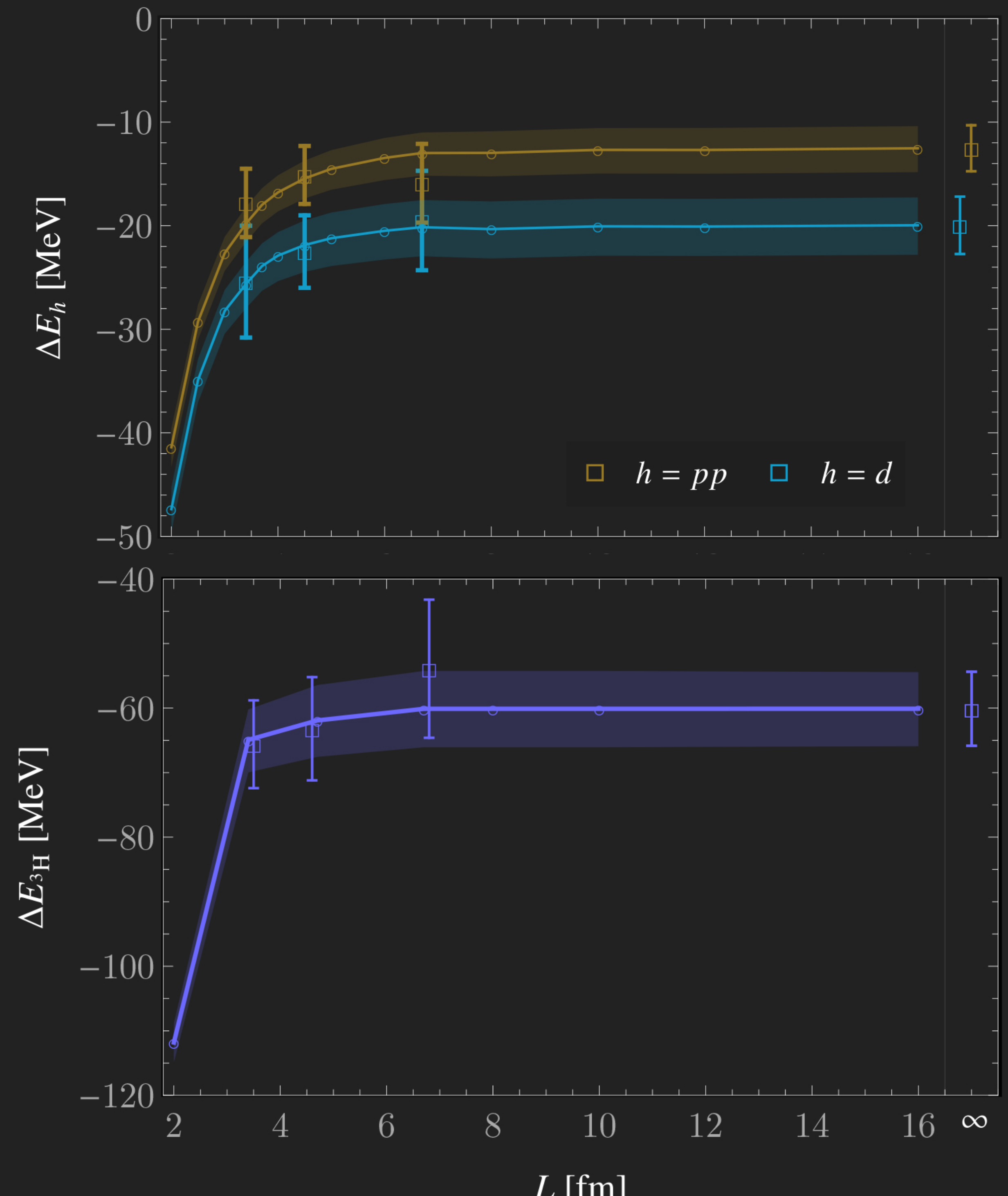

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FVEFT USING THE STOCHASTIC VARIATIONAL METHOD

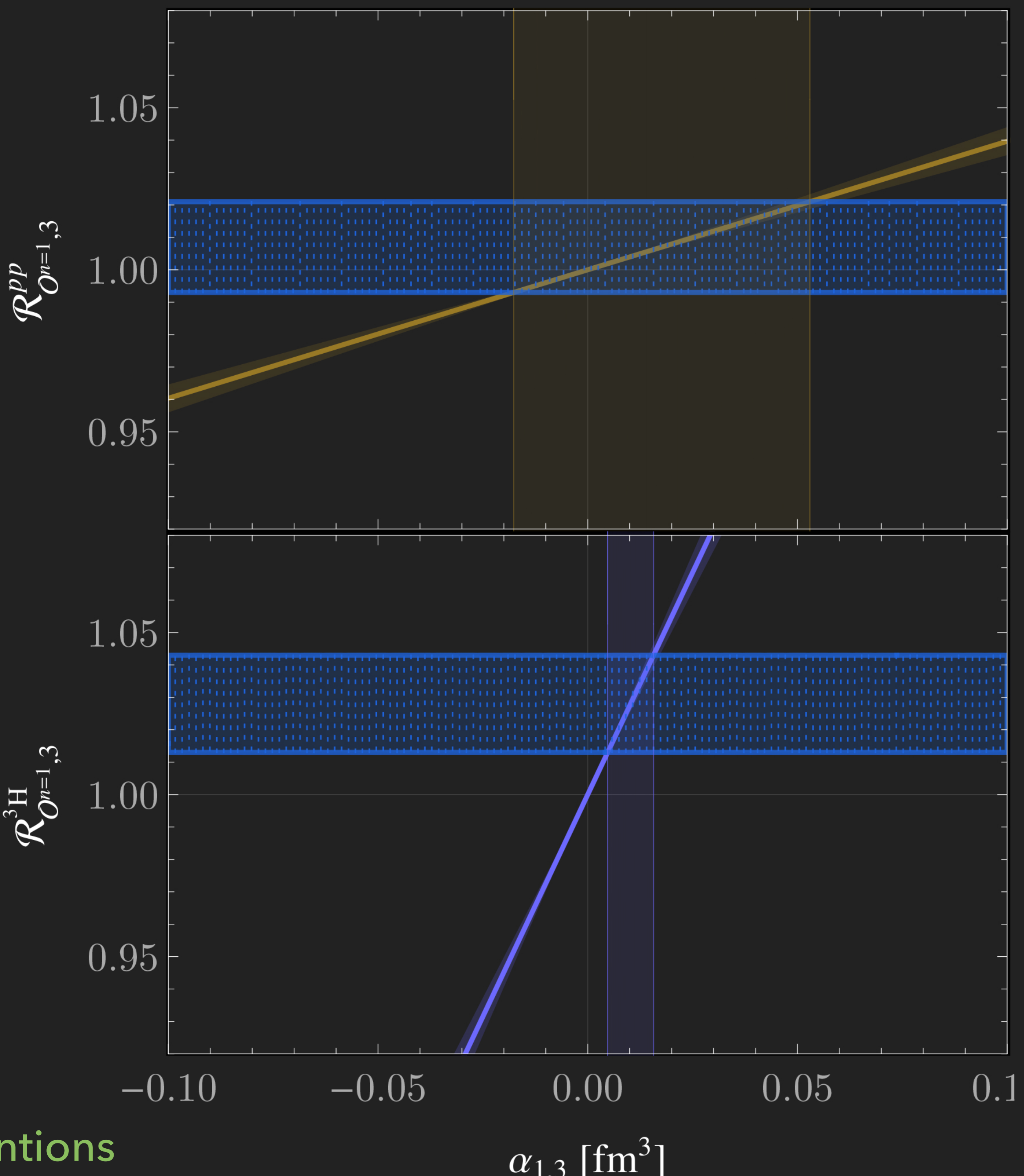
- Given EFT wavefunctions, matrix elements are easily computed
- LQCD matching determines EFT counterterms
  - Enables infinite volume prediction for matrix elements
  - Example: isovector momentum fraction

$$\mathcal{R}^{h}_{\mathcal{O}^n,3} \equiv \frac{A^h}{(Z^h - N^h)\langle x^n \rangle_3} \frac{\left\langle \Psi_h \middle| \mathcal{O}_3^{(n)} \middle| \Psi_h \right\rangle}{\langle \Psi_h \mid \Psi_h \rangle}$$

$$= \left(1 + \frac{\alpha_{n,3}}{(Z^h - N^h)\langle x^n \rangle_3} h_h(\Lambda, L)\right)$$

From wavefucntions

Two body counterterm



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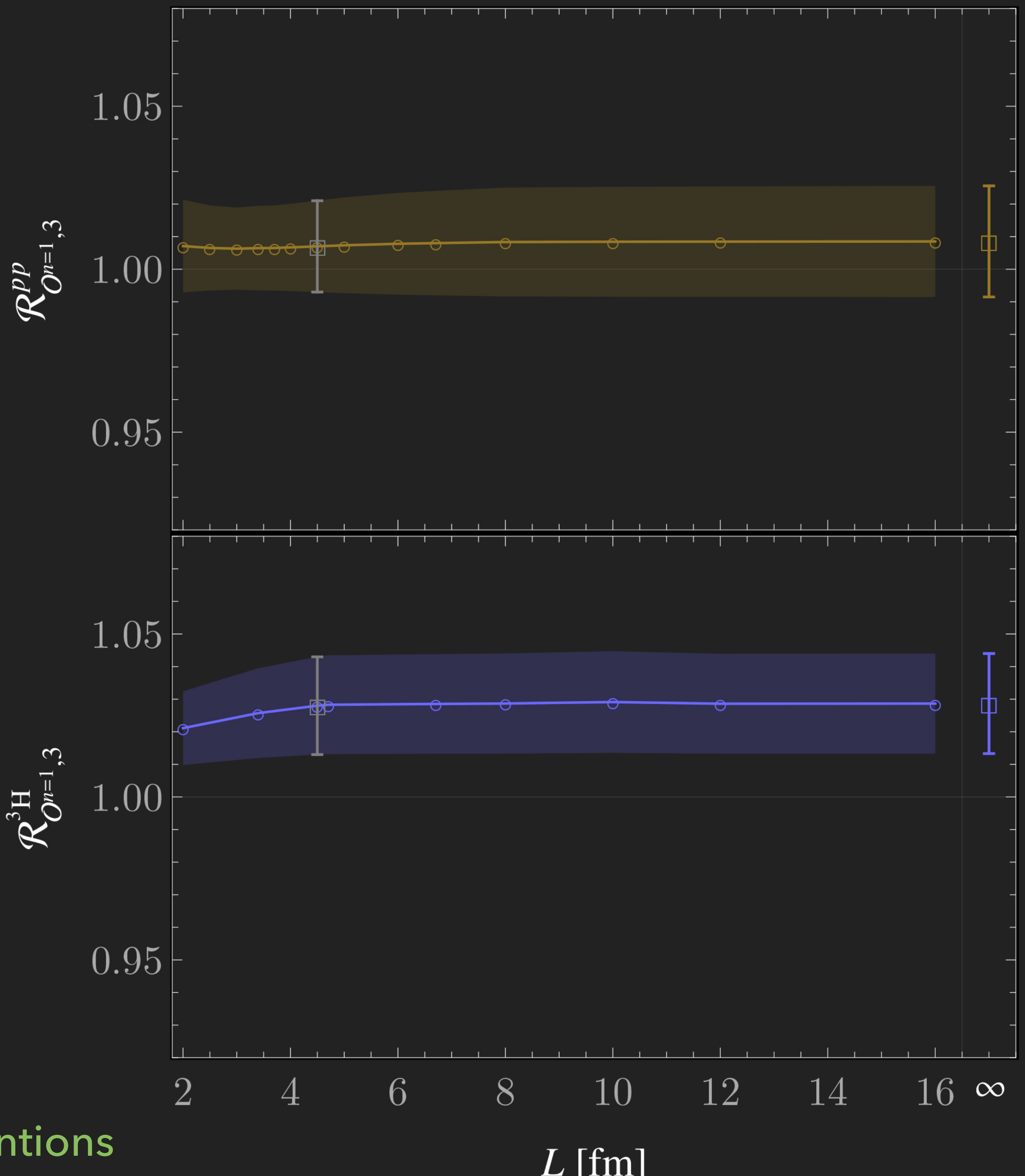

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