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ALICE

Three-body femtoscopy with ALICE

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EXOTICO: EXOTIc atoms meet nuclear COllisions for a new frontier precision era in low-energy strangeness nuclear physics

ECT*, Trento, 21 October 2022

*raffaele.del-grande@tum.de

Many-body systems with baryons

- Properties of nuclei and hypernuclei cannot be described satisfactorily with two-body forces only.
  L.E. Marcucci et al., Front. Phys. 8:69 (2020), see talk by A. Kievsky @ EXOTICO

- Many-body scattering requires three-body calculations (e.g. **neutron-deuteron**).
  L. Girlanda et al., PRC 102, 064003 (2020)

^{3}H and ^{4}He Binding Energies and n-d scattering length

Potential(NN)	^{3}H[MeV]	^{4}He[MeV]	$^2a_{nd}$[fm]
AV18	7.624	24.22	1.258
CDBonn	7.998	26.13	
N3LO-Idaho	7.854	25.38	1.100
Potential(NN+NNN)			
AV18/UIX	8.479	28.47	0.590
CDBonn/TM	8.474	29.00	
N3LO-Idaho/N2LO	8.474	28.37	0.675
Exp.	8.48	28.30	0.645±0.010

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- Many-body scattering requires three-body calculations (e.g. **neutron-deuteron**).
  L. Girlanda et al., PRC 102, 064003 (2020)

- **N-N-N and N-N-Λ interactions**: fundamental ingredients for the Equation of State (EoS) of neutron stars.
  see talk by D. Logoteta @ EXOTICO

^{3}H and ^{4}He Binding Energies and n-d scattering length

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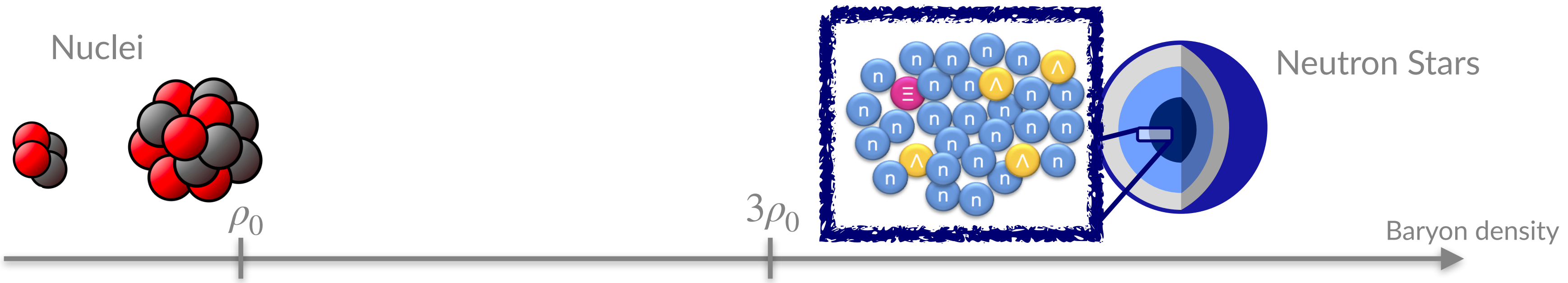
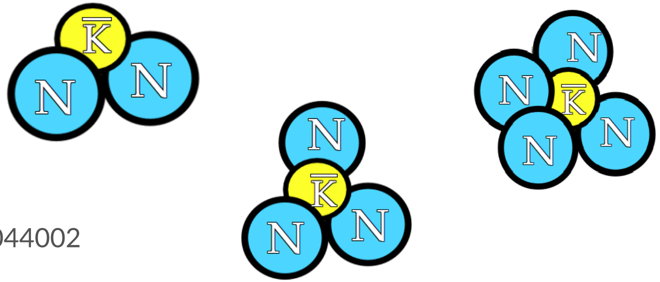
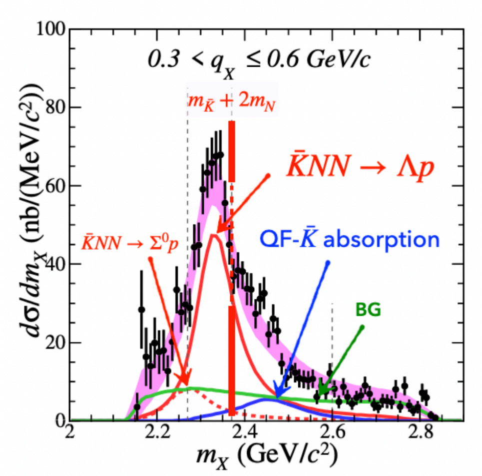


Many-body systems with mesons

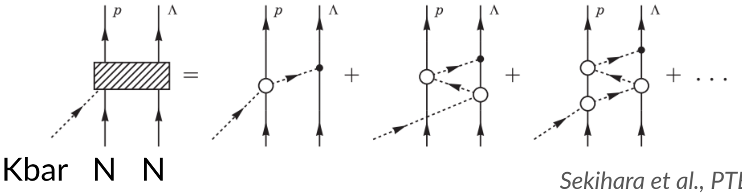
- Strongly attractive $\bar{K}N$ interaction in I = 0 channel
  —> Exotic bound states of antikaons with nucleons
  S. Wycech, NPA 450 (1986) 399c; Y. Akaishi, T. Yamazaki, PRC 65 (2002) 044005



- First positive experimental evidence of the **p-p-K⁻ bound state** by the E15 Collaboration. *E15 Coll., PLB 789 (2019) 620, Phys.Rev.C 102 (2020) 4, 044002*



Kaonic bound state formation mechanism



Sekihara et al., PTEP 2016 no. 12, (2016)

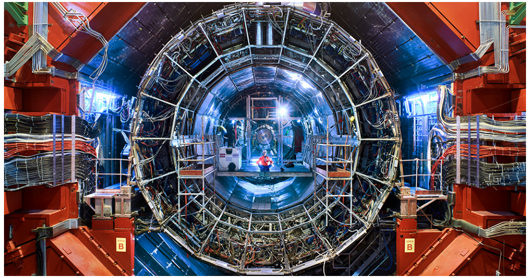
	B. E. (MeV)	Width (MeV)
Exp. (E15)	42	100
Theo. (Sekihara et al.)	16	72

Next challenge: explore many-body systems dynamics using femtoscopy!

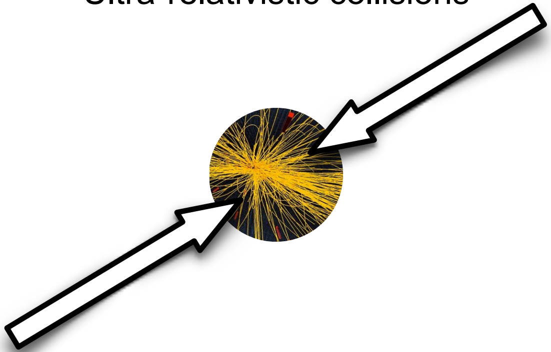
Three-body femtoscopy

Data set: high-multiplicity pp collisions @ $\sqrt{s}$ = 13 TeV

ALICE at the LHC



Ultra-relativistic collisions



Three-body femtoscopy

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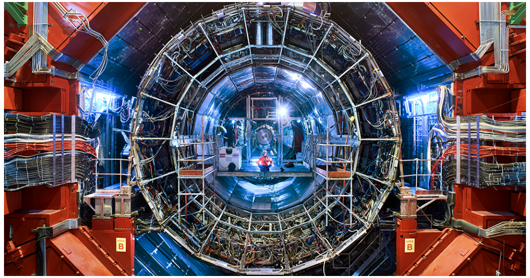
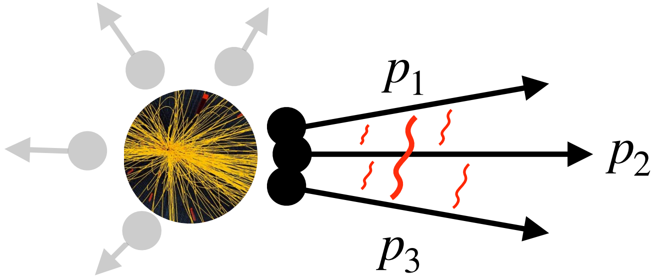


Correlation of **three hadrons**



Three-particle correlation function:

$$C(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = \frac{P(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)}{P(\mathbf{p}_1)\ P(\mathbf{p}_2)\ P(\mathbf{p}_3)}$$

Three-body femtoscopy

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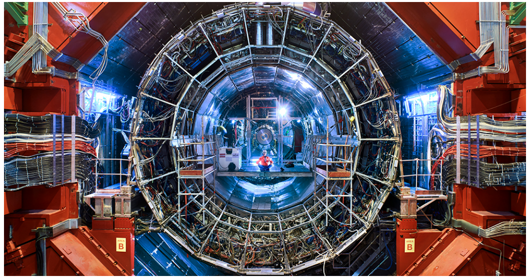

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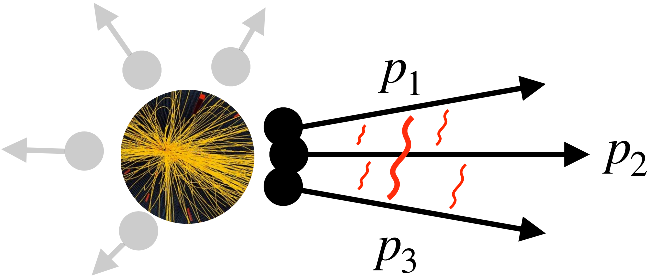


Three-particle correlation function:

$$C(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = \iiint S_3\left(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\right) \left|\psi_{\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3}\left(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\right)\right|^2 d^3x_1 d^3x_2 d^3x_3 = \mathcal{N} \cdot \frac{N_{\mathrm{same}}(Q_3)}{N_{\mathrm{mixed}}(Q_3)}$$

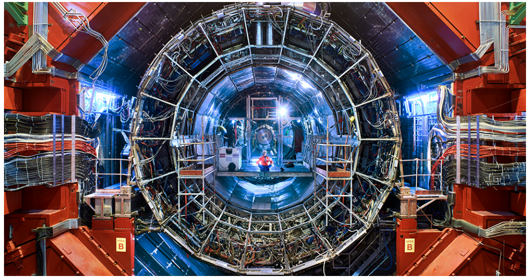
Lorentz-invariant Q_3 is defined as: $Q_3 = \sqrt{-q_{12}^2 - q_{23}^2 - q_{31}^2}$

$$q_{ij}^{\mu} = 2\left(\frac{m_j E_i}{m_i + m_j} - \frac{m_i E_j}{m_i + m_j}, \frac{m_j}{m_i + m_j}\mathbf{p}_i - \frac{m_i}{m_i + m_j}\mathbf{p}_j\right)$$

Three-body femtoscopy

<u>Data set</u>: high-multiplicity pp collisions @ $\sqrt{s}$ = 13 TeV

ALICE at the LHC



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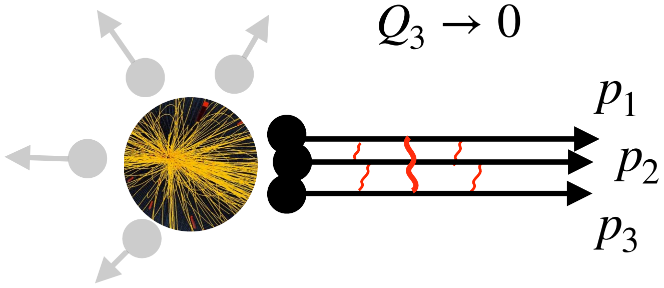

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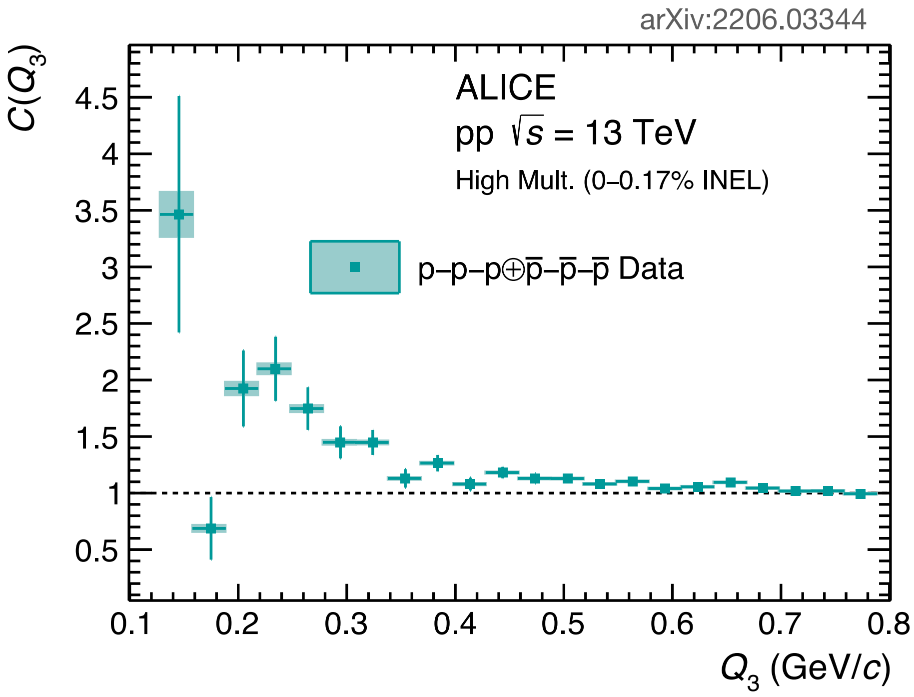
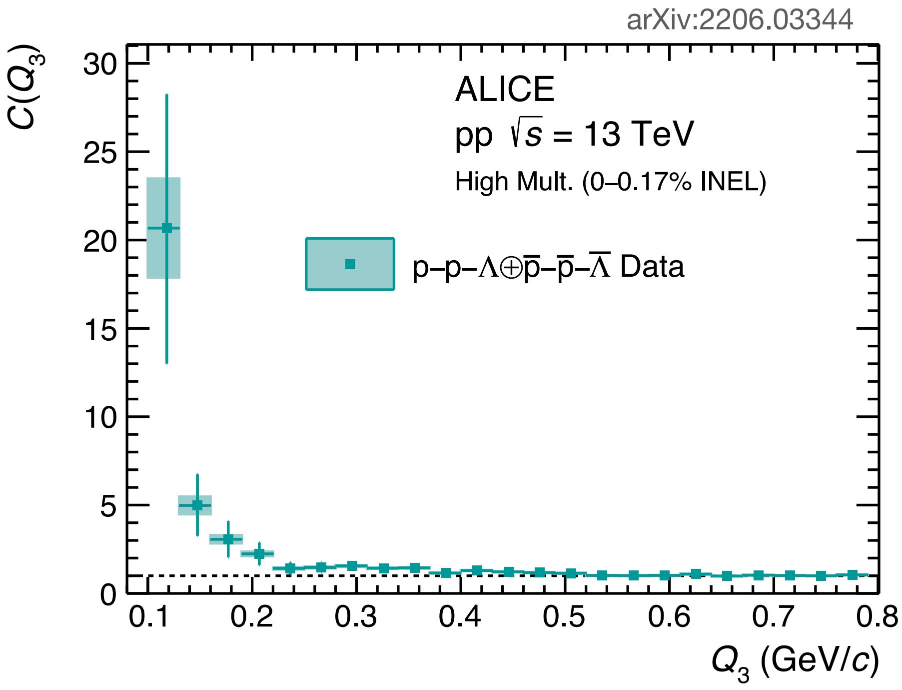
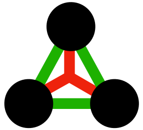
p-p-p and p-p-Λ correlation functions



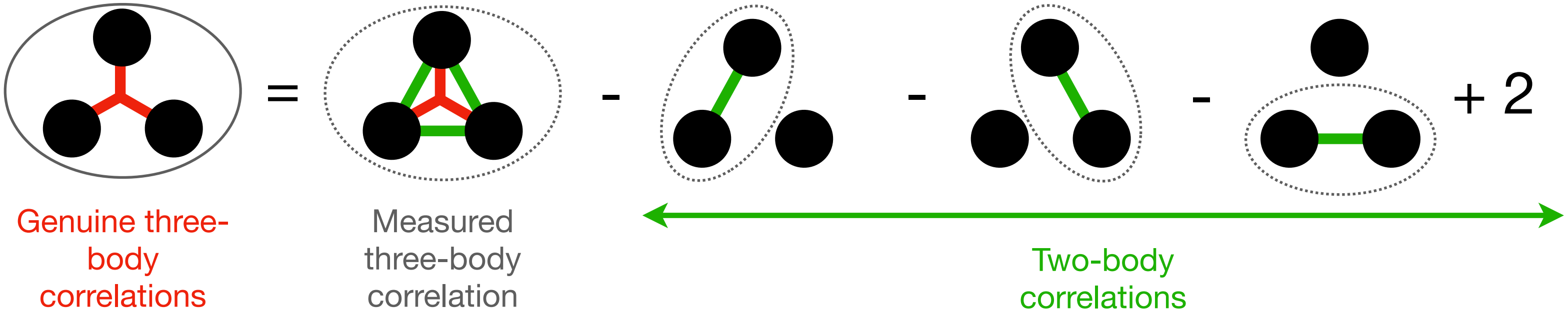


<u>Correlation functions include two- and three-particle correlations</u>



Cumulants in femtoscopy

The total three-particle correlations can be expressed as a sum of genuine three-body correlation and the lower-order contribution employing Kubo's cumulants [1]:



In terms of correlation functions:

$$c_3\left(Q_3\right) = C\left(Q_3\right) - C_{12}\left(Q_3\right) - C_{23}\left(Q_3\right) - C_{31}\left(Q_3\right) + 2$$

How to estimate lower-order contributions?

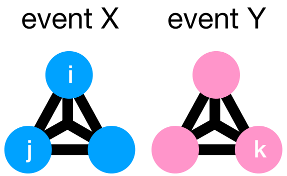
[1] J. Phys. Soc. Jpn. 17, pp. 1100-1120 (1962)

Lower-order contributions

Data-driven method

- Use event mixing

- Two particles from the same event and one particle from another:



$$C_{ij}\left(\left[\mathbf{p}_i, \mathbf{p}_j\right], \mathbf{p}_k\right) = \frac{N_2\left(\mathbf{p}_i, \mathbf{p}_j\right) N_1\left(\mathbf{p}_k\right)}{N_1\left(\mathbf{p}_i\right) N_1\left(\mathbf{p}_j\right) N_1\left(\mathbf{p}_k\right)}$$

- Calculate Lorentz-invariant scalar Q_3 for every triplet $\mathbf{p}_i, \mathbf{p}_j, \mathbf{p}_k$ to obtain $C_{ij}(Q_3)$

Lower-order contributions

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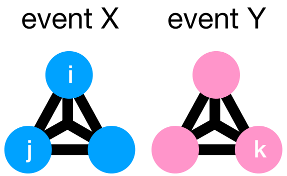
- Two particles from the same event and one particle from another:



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- Calculate Lorentz-invariant scalar Q_3 for every triplet $\mathbf{p}_i, \mathbf{p}_j, \mathbf{p}_k$ to obtain $C_{ij}(Q_3)$

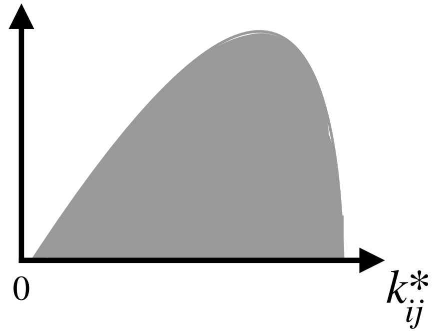
Projector method

- Use two-particle measured or theoretical correlation function $C([\mathbf{p}_i, \mathbf{p}_j])$
- Perform kinematic transformation:

$$C_2\left(k^*_{ij}\right) \rightarrow C_{ij}\left(Q_3\right)$$

k^*_{ij} (pair) $\rightarrow$ Q_3 (triplet)

For one Q_3 value

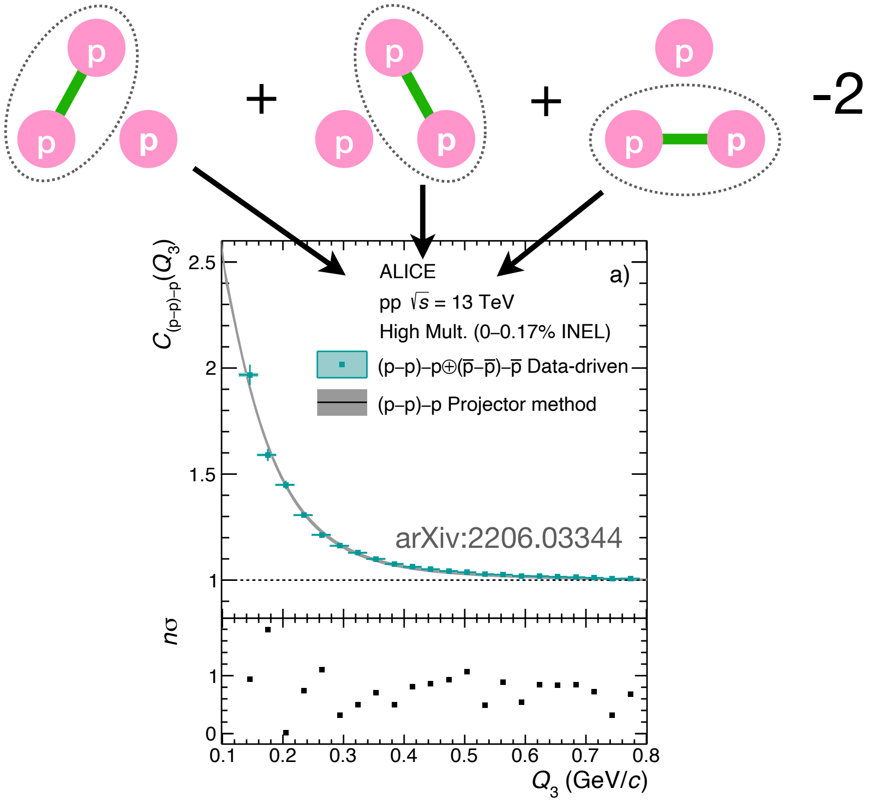


- To obtain the correlation function:

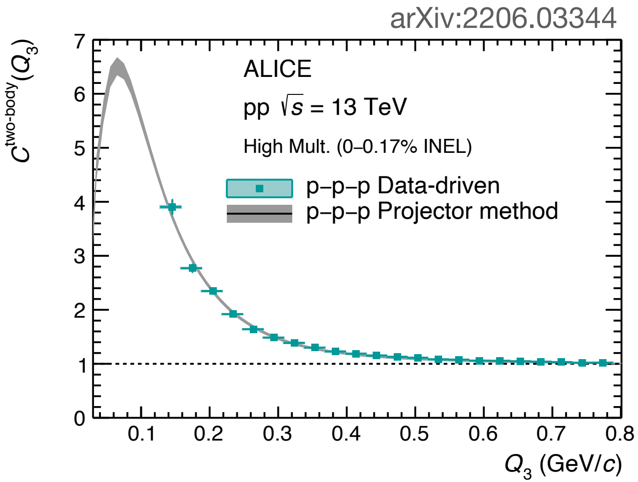
$$C_{ij}(Q_3) = \int C(k^*_{ij}) W_{ij}(k^*_{ij}, Q_3) dk^*_{ij}$$

R.D.G., L. Šerkšnytė et al. EPJC 82 (2022) 244

Lower-order contributions : p-p-p



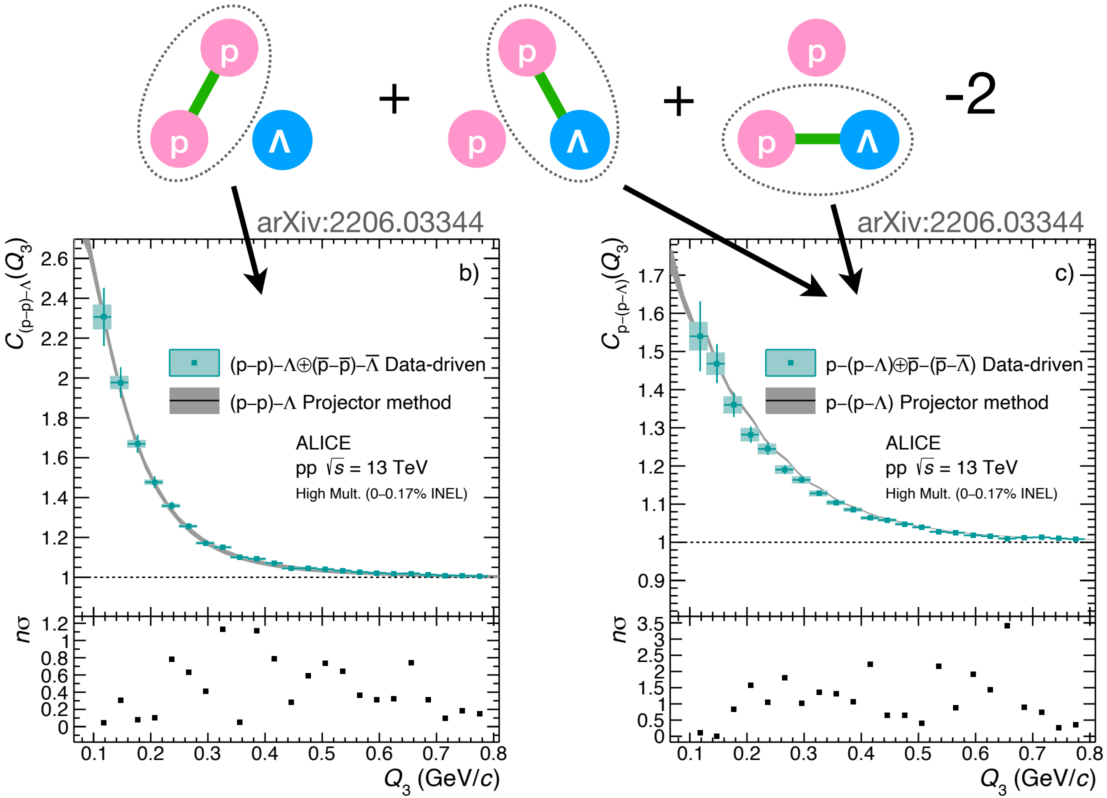
Total lower-order contributions



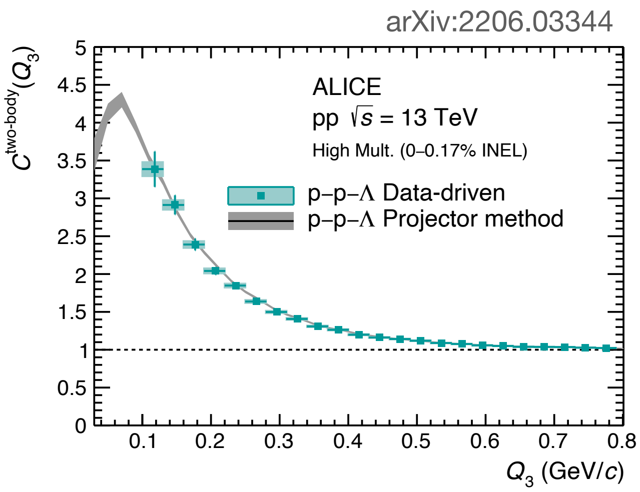
Already measured p-p [1] correlation function used for projection.

[1] PLB 805 (2020) 135419

Lower-order contributions : p-p-Λ



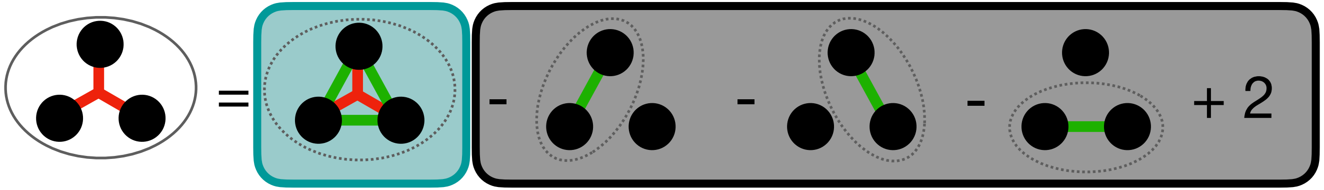
Total lower-order contributions

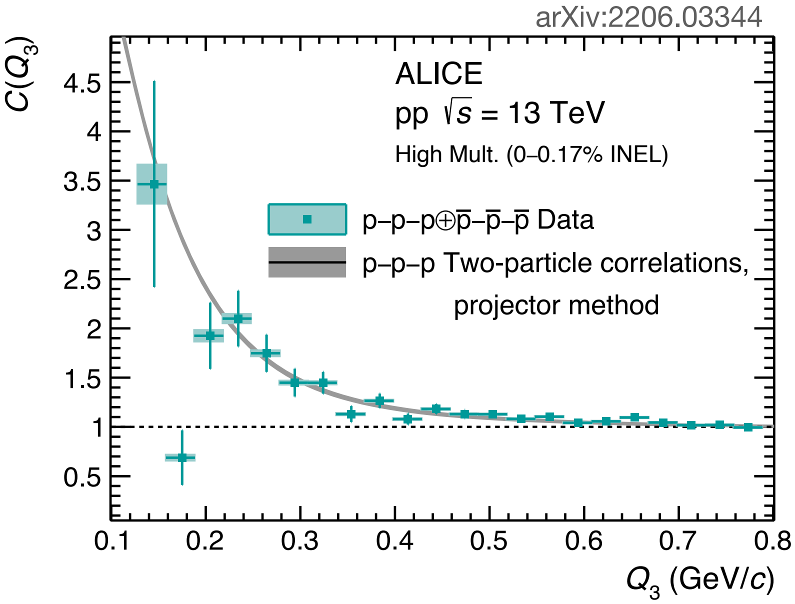


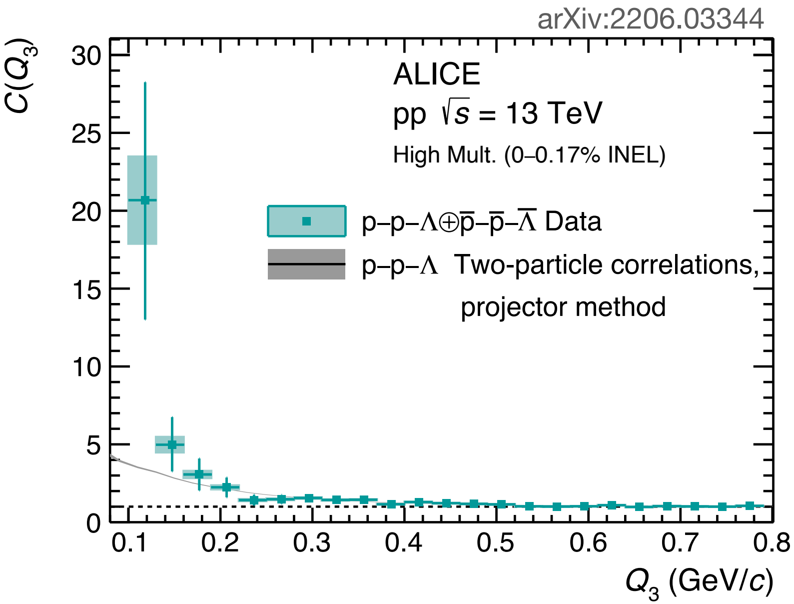
Already measured p-p [1] and p-Λ [2] correlation functions used for projection.

[1] PLB 805 (2020) 135419; [2] arXiv:2104.04427

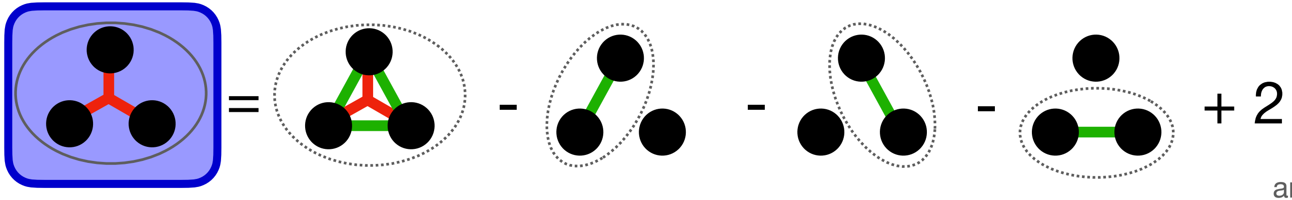
p-p-Λ and p-p-p correlation functions







p-p-p cumulant



Negative cumulant for p-p-p

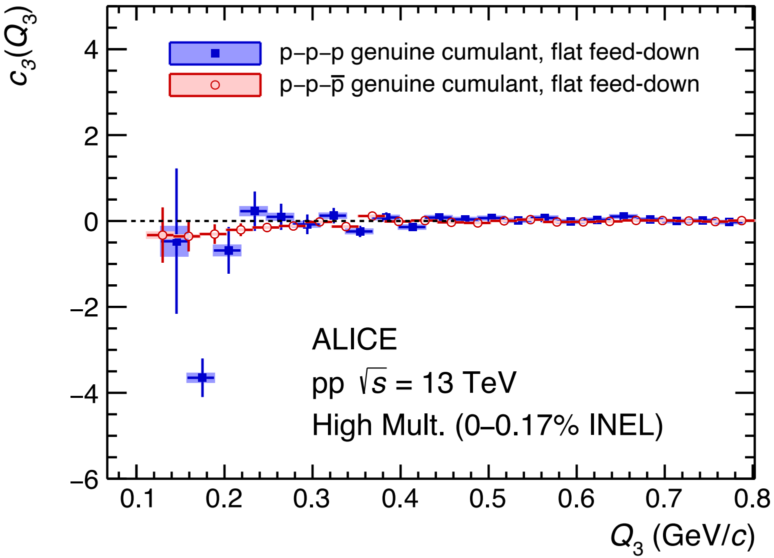
Possible forces at play:

- Pauli blocking at the three-particle level
- three-body strong interaction

Statistical significance:

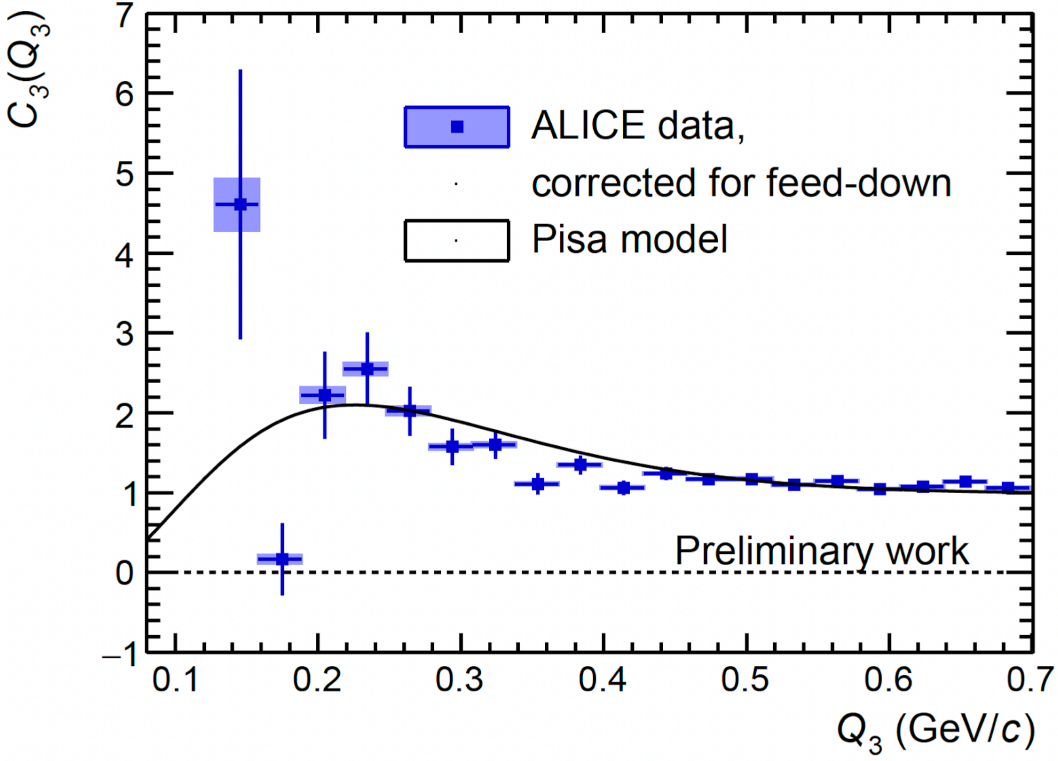
n_σ= 6.7 for Q_3 < 0.4 GeV/c

arXiv:2206.03344



Test with mixed-charge particles, cumulant negligible.

p-p-p correlation function



Comparison with the theory

$$C_{ppp}(Q) = \int \rho^5 d\rho d\Omega \; S_{\rho_0}(\rho) |\Psi_{ppp}|^2$$

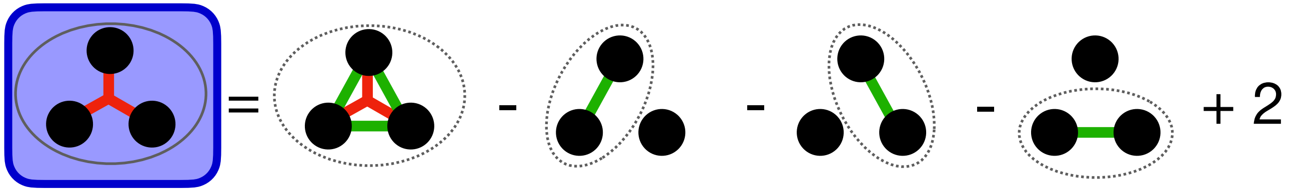
Wave function calculated using the Hyperspherical Harmonics (HH) method *(see A. Kievsky talk)*

The source function is

$$S_{\rho_0}(\rho) = \frac{1}{\pi^3 \rho_0^6} e^{-(\rho/\rho_0)^2}$$

Preliminary study with ρ_0 = 2 fm

p-p-Λ cumulant



Positive cumulant for p-p-Λ

- Only two identical and charged particles
  - ✓ Main expected contribution from three-body strong interaction

- Relevant measurement for equation of state of neutron stars.

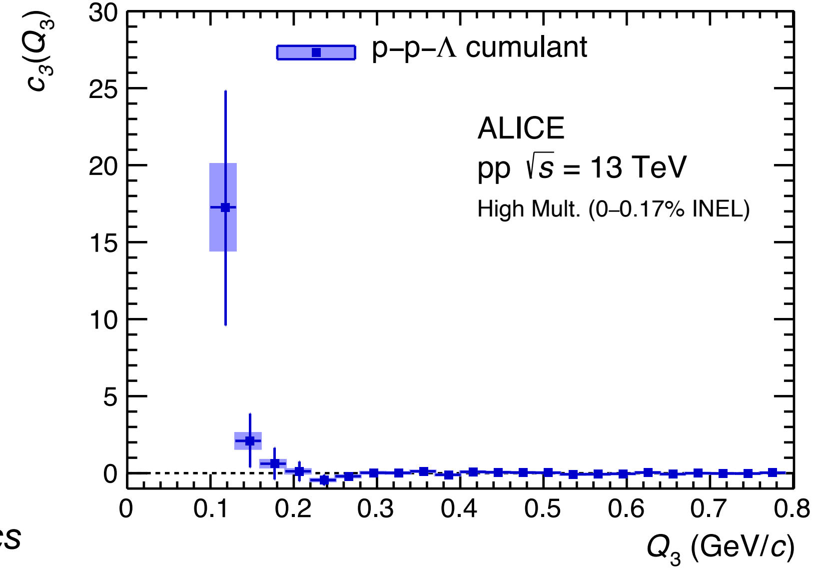
Statistical significance:

n_σ= 0.8 for Q_3 < 0.4 GeV/c

Conclusion: no significant deviation from null hypothesis.

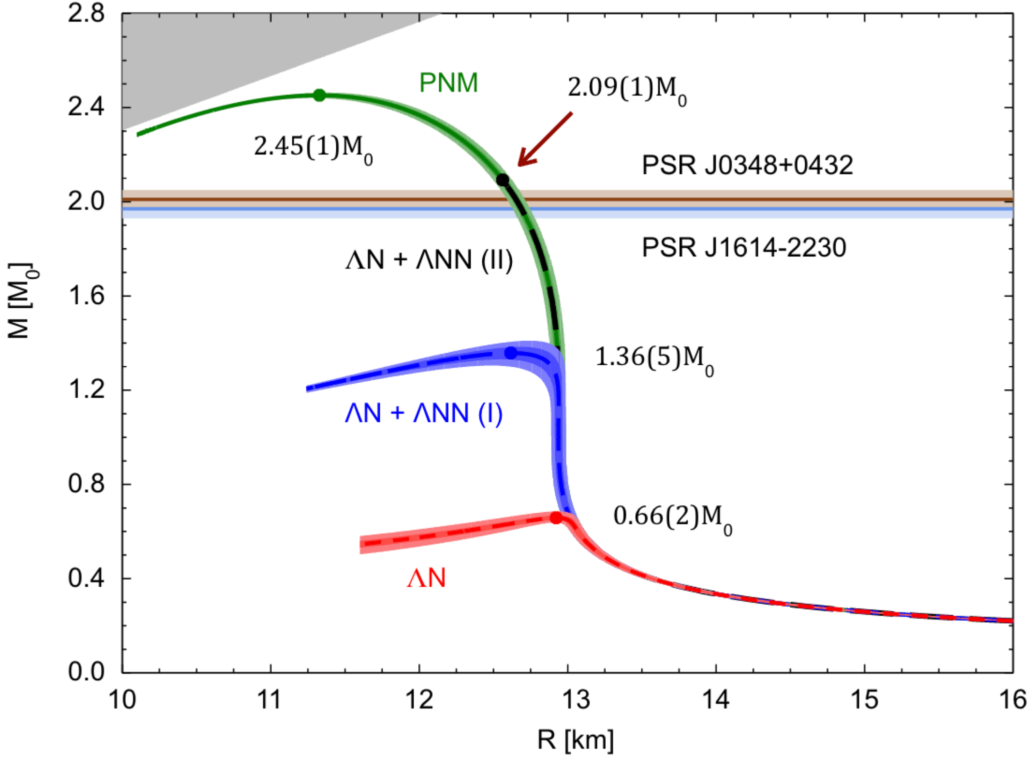
In upcoming Run 3, two orders of magnitude gain in statistics expected!

arXiv:2206.03344



Equation of State of neutron stars

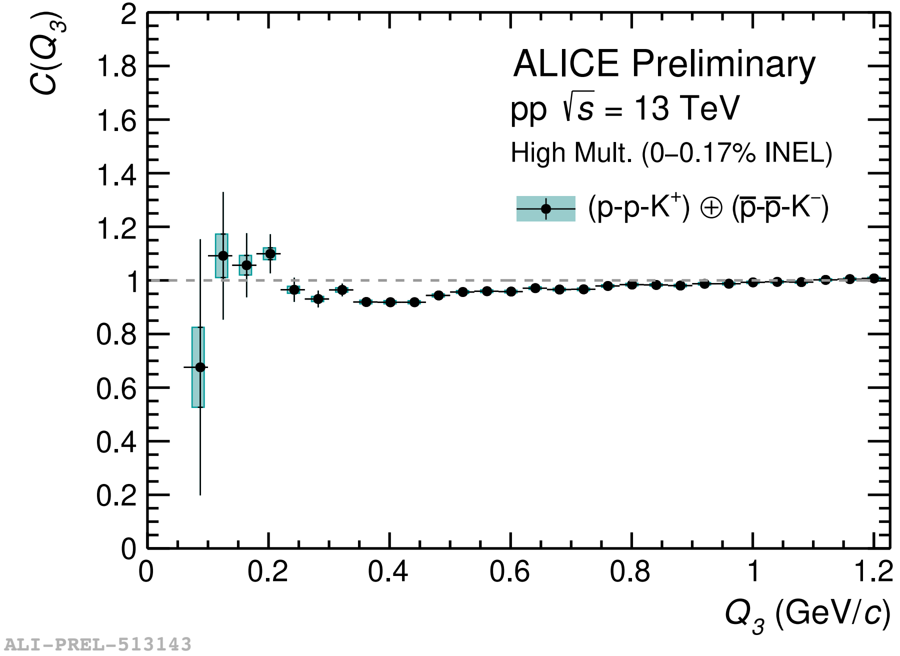
Adapted from D. Lonardoni et al., PRL 114, 092301 (2015)



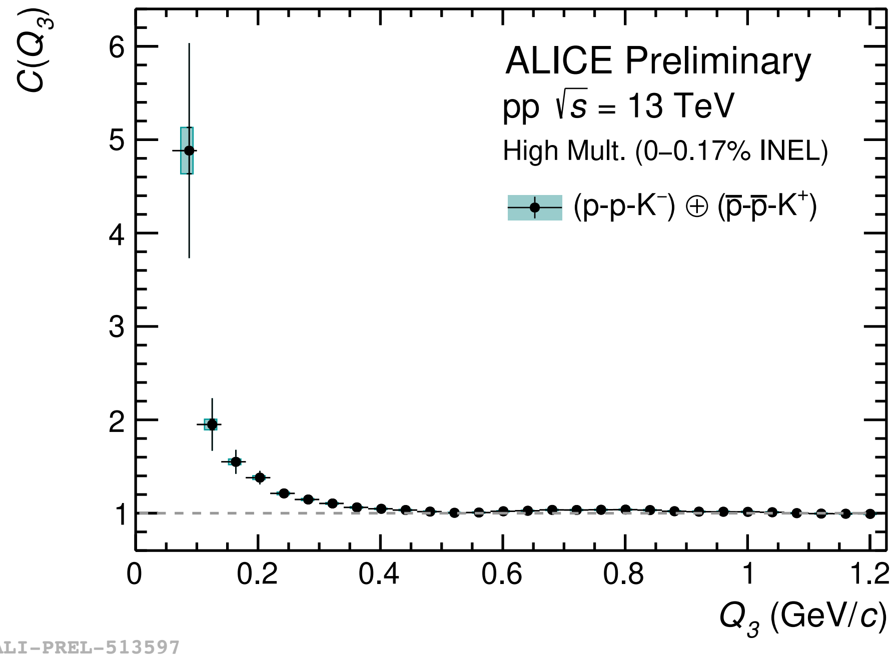
- Hyperons might appear in neutron stars since it is energetically favourable
- But the resulting equation of state might be too soft to explain heavy neutron stars
- Possible solution: repulsive three-body interaction

p-p-K+ and p-p-K⁻ correlation functions

p-p-K+

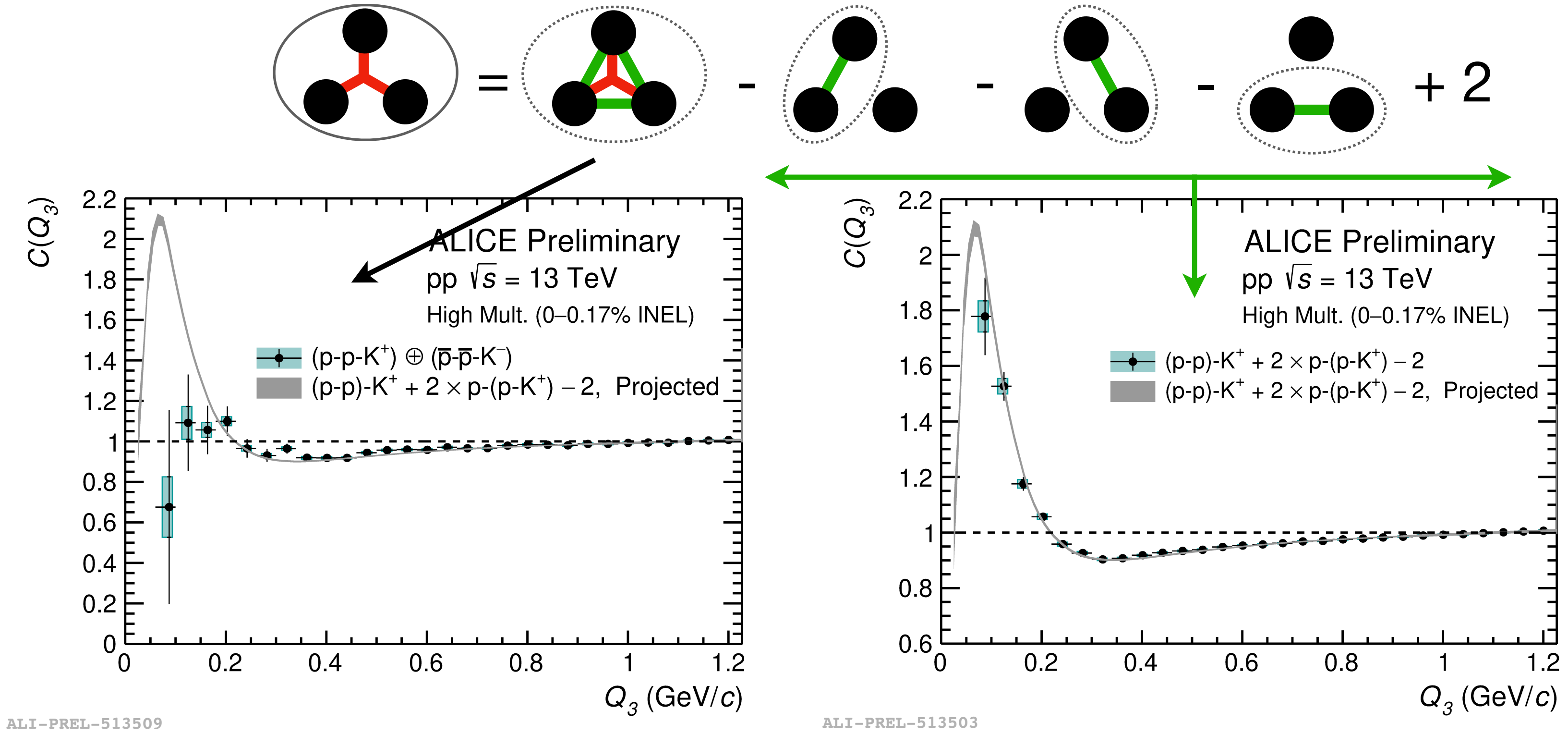


p-p-K⁻

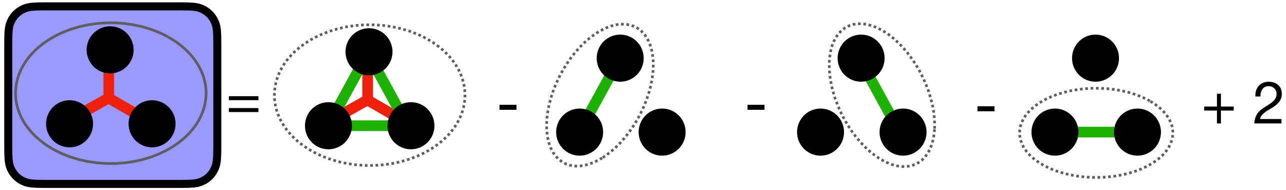


<u>Correlation functions include two- and three-particle correlations</u>

p-p-K+ correlation function



p-p-K^+ cumulant

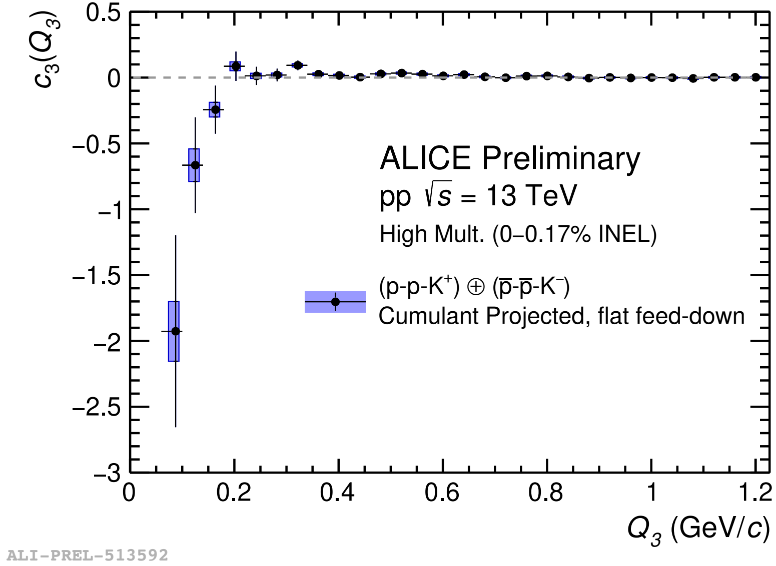


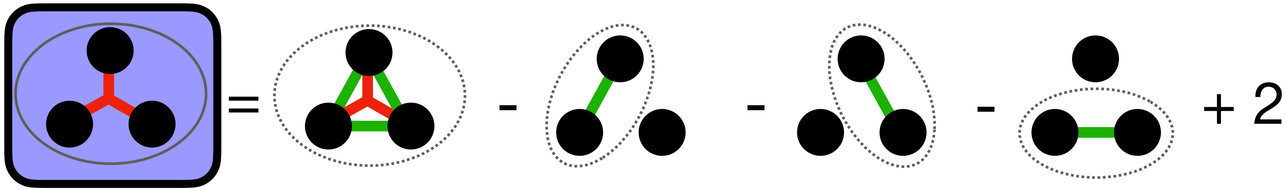
Negative cumulant for p-p-K^+

Statistical significance:

n_σ **= 2.3 for** Q_3 **< 0.4 GeV/**c

Conclusion: the measured cumulant is compatible with zero within the uncertainties.

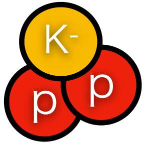




Zero cumulant for p-p-K$^-$

Statistical significance:

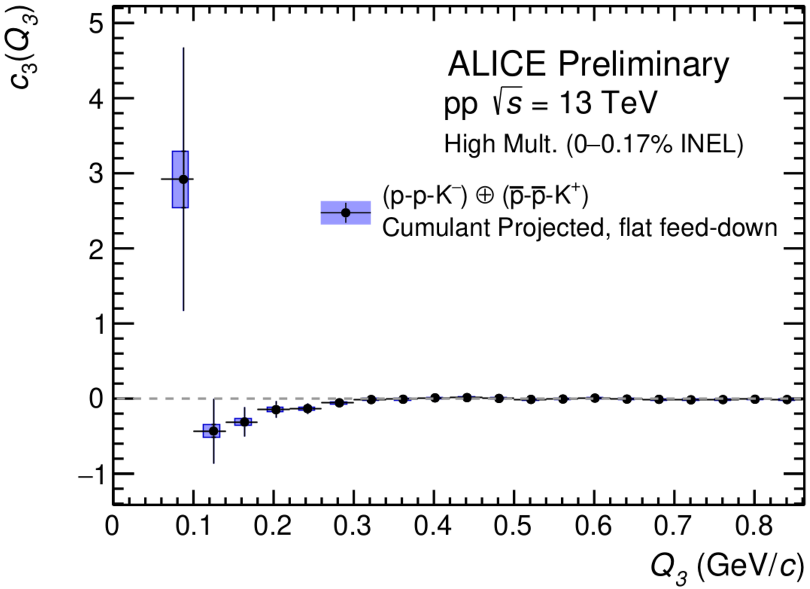
n_σ **= 1.5 for** Q_3 **< 0.4 GeV/**c



Conclusion: the measured cumulant is compatible with zero within the uncertainties.

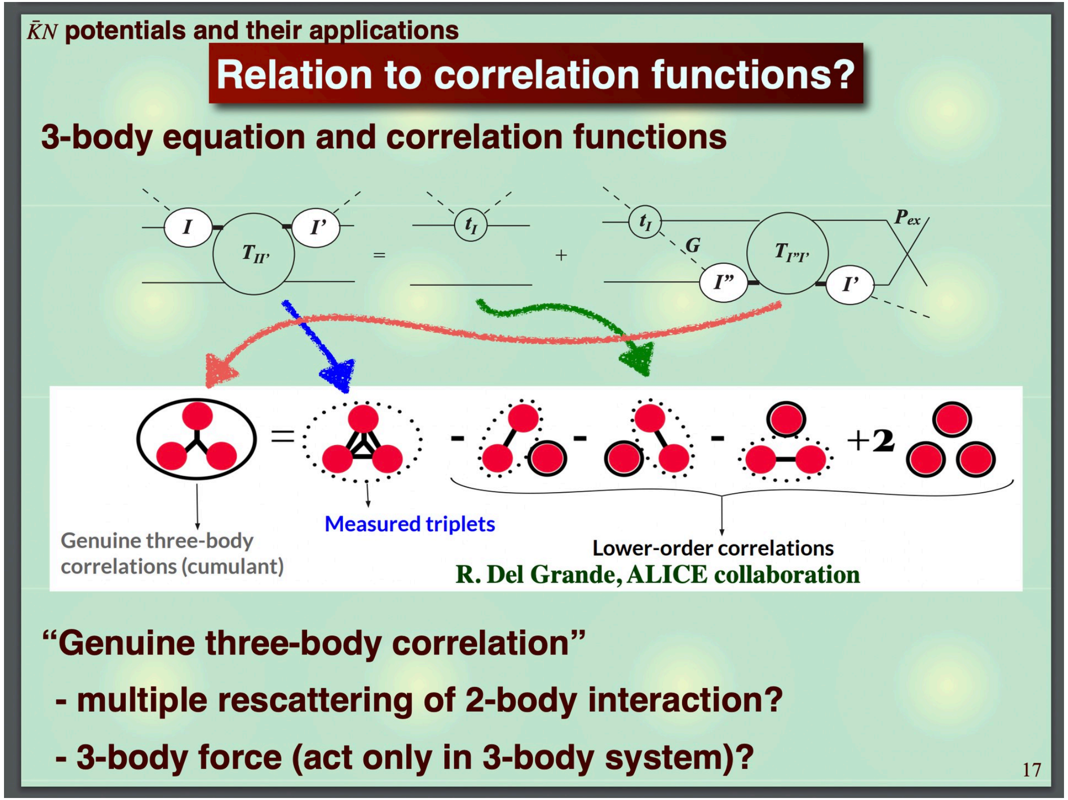
p-p-K$^-$ system shows only two-body interactions.

✓No evidence of genuine three-body effects and bound state formation

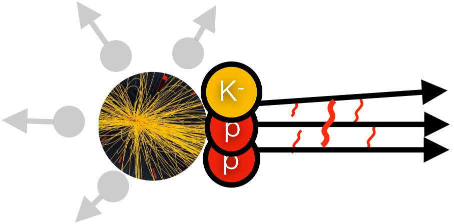


p-p-K⁻ cumulant

From Prof. Hyodo:



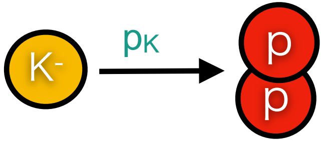
p-p-K$^-$ cumulant



$$Q_3 = \sqrt{-q_{12}^2 - q_{23}^2 - q_{31}^2}$$

$$q_{ij}^{\mu} = 2\left(\frac{m_j E_i}{m_i + m_j} - \frac{m_i E_j}{m_i + m_j}, \frac{m_j}{m_i + m_j}\mathbf{p}_i - \frac{m_i}{m_i + m_j}\mathbf{p}_j\right)$$

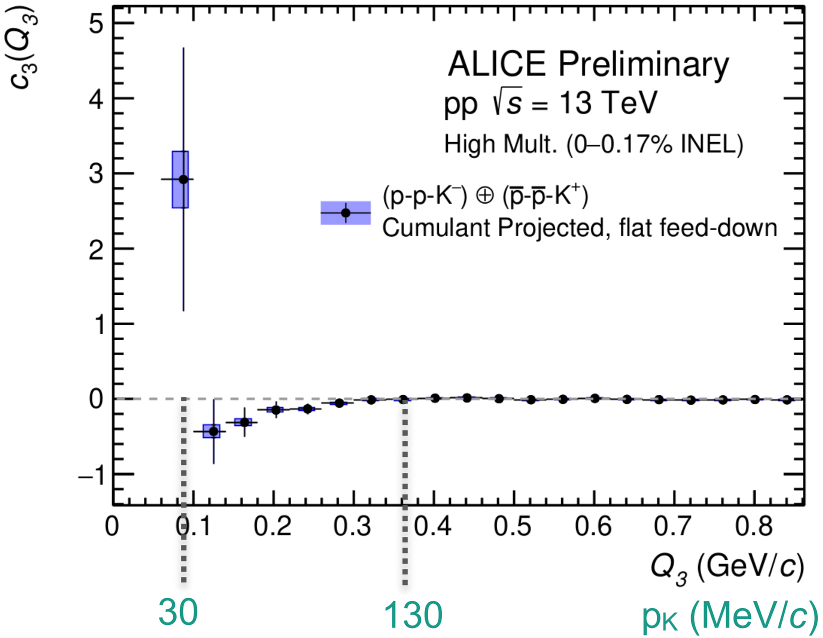
Comparison with K$^-$ interaction studies in nuclei: kaon momentum evaluated in the p-p rest frame



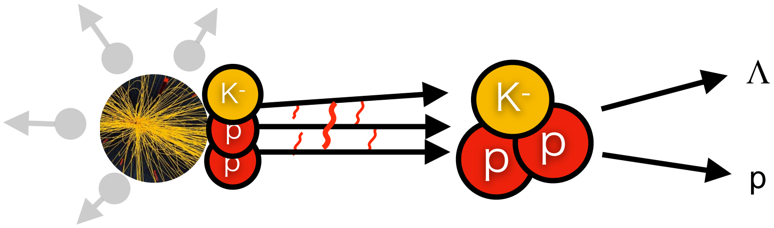
k_{pp} < 180 MeV/c

Compatible with Fermi momentum of nucleons in nuclei

K$^-$ momentum in the range (30-130) MeV/c



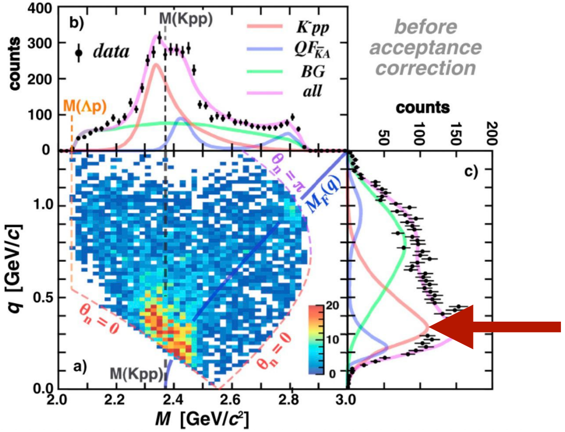
p-p-K⁻ cumulant



Which is the Q_3 of the p-p-K⁻ triplets?

If we believe in the measurement by E15, the bound state is compact (R ~ 0.6 fm) and the transfer momentum by the K⁻ on the two rest protons is q_X ~ 0.3 GeV/c.

Transfer momentum
$q_X = p_K$ ~ 0.3 GeV/c

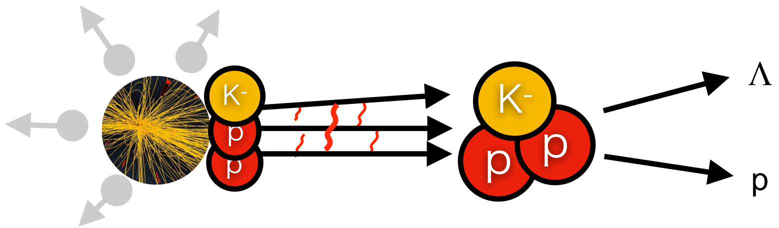


Q_3 is Lorentz-invariant —> we can choose the rest frame of the two-protons

$$Q_3 = \sqrt{-q_{12}^2 - q_{23}^2 - q_{31}^2}$$

$$q_{ij}^{\mu} = 2\left(\frac{m_j E_i}{m_i + m_j} - \frac{m_i E_j}{m_i + m_j}, \frac{m_j}{m_i + m_j}\mathbf{p}_i - \frac{m_i}{m_i + m_j}\mathbf{p}_j\right)$$

p-p-K⁻ cumulant



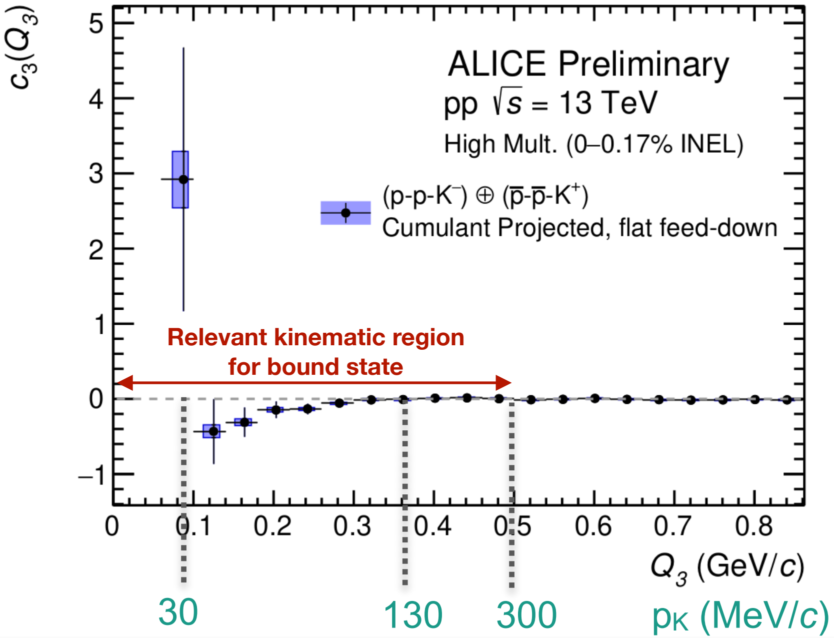
Comparison with K⁻ interaction studies in nuclei: kaon momentum evaluated in the p-p rest frame

K-
p_K
p
p

k_{pp} < 180 MeV/c

Compatible with Fermi momentum of nucleons in nuclei

K⁻ momentum in the range (30-130) MeV/c



Conclusions and Outlooks

Femtoscopy technique applied in pp collisions at the LHC to study many-body systems dynamics:

— Genuine three-body effects isolated for the first time using the Kubo's rule.

- **p-p-p:** negative cumulant with a significance of 6.7 σ
  —> first comparison of the correlation function with theoretical calculations

- **p-p-Λ:** no significant deviation from 0 in Run 2 data.

- **p-p-K^{+}** and **p-p-K^{-}:** cumulants compatible with 0, no evidence of a genuine three-body effects and bound state

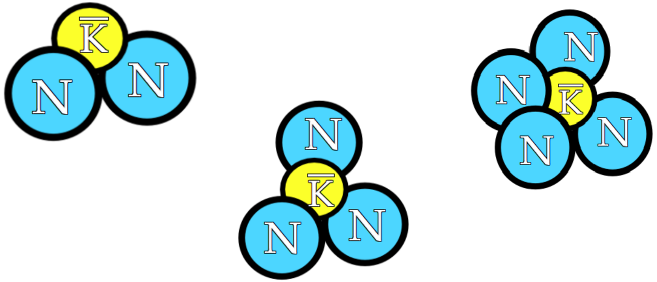
— More precision studies within reach with the large data samples collected in Run 3 & 4.

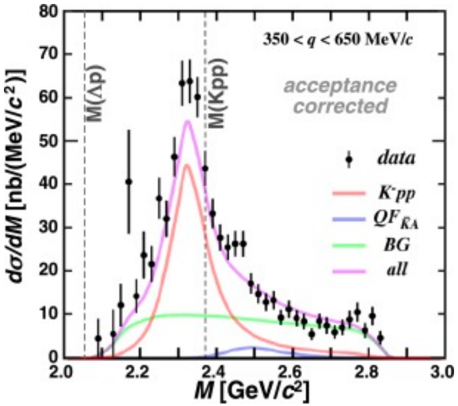
Thank You

Many-body systems with mesons

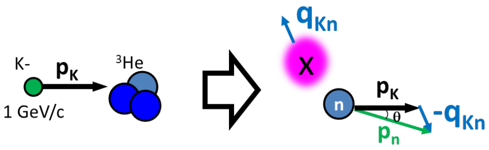
- Strongly attractive K̄N interaction in I = 0 channel
  —> Exotic bound states of antikaons with nucleons
  S. Wycech, NPA 450 (1986) 399c; Y. Akaishi, T. Yamazaki, PRC 65 (2002) 044005

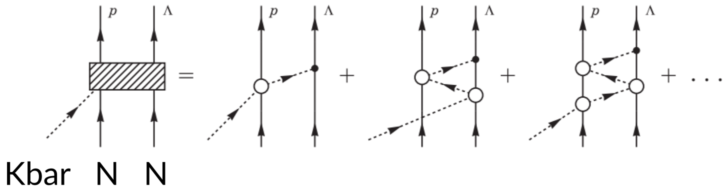
- First positive experimental evidence of the **p-p-K⁻ bound state** by the E15 Collaboration. *E15 Coll., PLB 789 (2019) 620*





Kaonic bound state formation mechanism



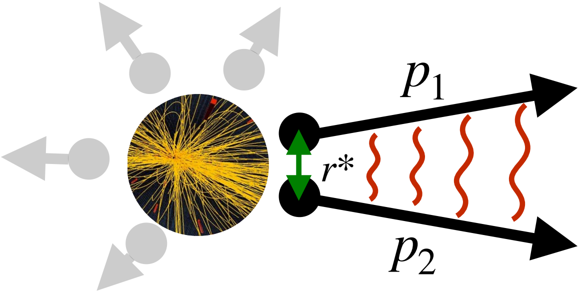


Sekihara et al., PTEP 2016 no. 12, (2016)

	B. E. (MeV)	Width (MeV)
Exp. (E15)	42	100
Theo.	16	72

Next challenge: explore many-body systems dynamics using femtoscopy!

Two-body femtoscopy

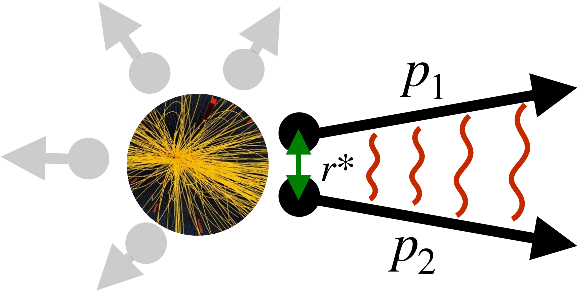


Emission source *S(r*)*

Two-particle correlation function:

$$C\left(k^*\right) = \mathcal{N}\frac{N_{\text{same}}\left(k^*\right)}{N_{\text{mixed}}\left(k^*\right)} = \int S\left(r^*\right)\left|\psi\left(\mathbf{k}^*, \mathbf{r}^*\right)\right|^2 \mathbf{d}^3 r^*$$

Two-body femtoscopy



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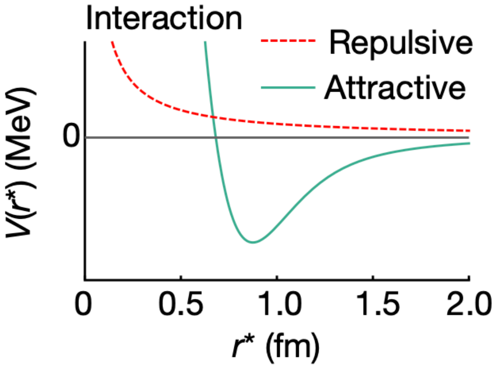
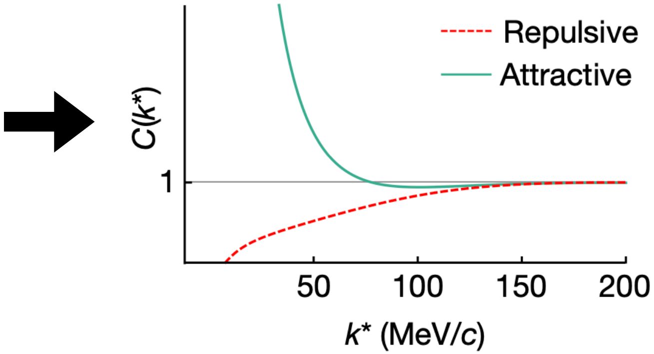


Schrödinger equation

Two-particle wave function

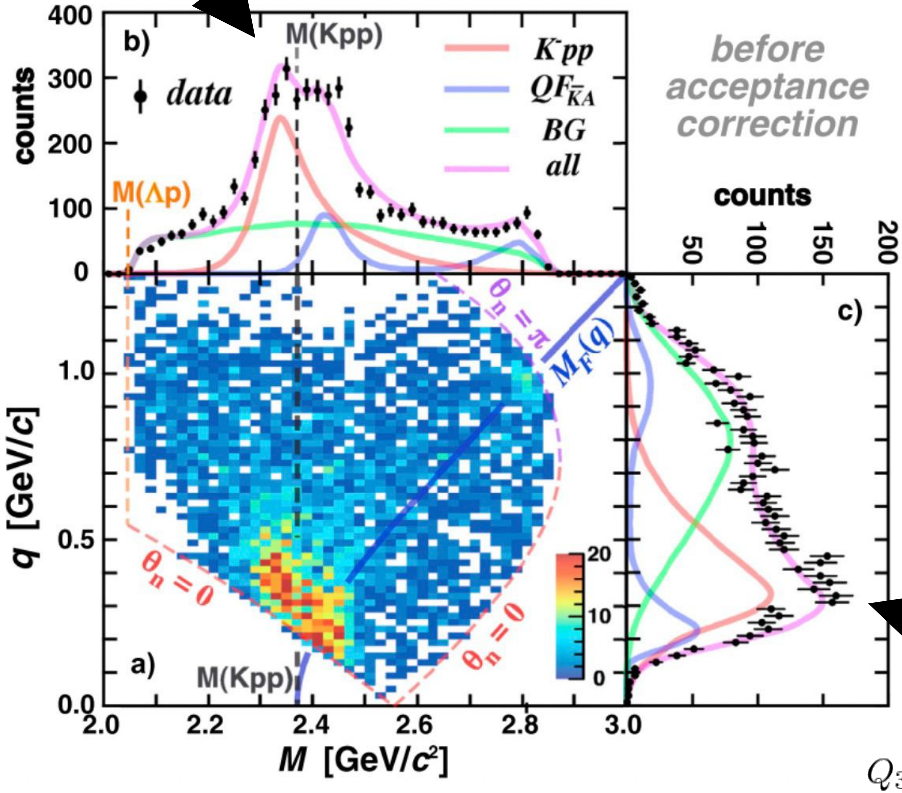
$$\left|\psi(\mathbf{k}^*, \mathbf{r}^*)\right|$$



Correlation function *C(k*)*

Kaonic bound state measured by E15

E15 Coll., PLB 789 (2019) 620



The E15 collaboration measured the bound state via the following decay:

$$K^- + p + p \rightarrow \Lambda + p$$

The Λp momentum distribution has a peak at

$$q = p_\Lambda + p_p \approx 0.35\ \mathrm{GeV}/c$$

Using the momentum conservation:

$$p_{K^-} + p_p + p_p \approx 0.35\ \mathrm{GeV}/c$$

The protons are at-rest → $p_K \approx 0.35\ \mathrm{GeV}/c$

In terms of Q_3 we have

$$Q_3 = 2\sqrt{k_{\mathrm{pK}}^2 + k_{\mathrm{pK}}^2 + k_{\mathrm{pp}}^2} = 2\sqrt{2}\,k_{\mathrm{pK}} = 4/3\sqrt{2}\,p_K < 0.5\,\mathrm{GeV}/c$$

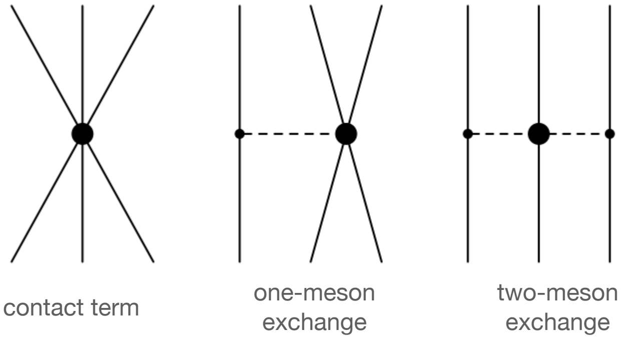
Many-body systems

- Properties of nuclei and hypernuclei cannot be described satisfactorily with two-body forces only.
  L.E. Marcucci et al., Front. Phys. 8:69 (2020)

- **N-N-N and N-N-Λ interactions**: fundamental ingredients for the Equation of State (EoS) of neutron stars.
  D. Lonardoni et al., PRL 114, 092301 (2015)

- Many-body scatterings (e.g. **proton-deuteron**) and formation mechanisms of light nuclei. *L. Girlanda et al., PRC 102, 064003 (2020)*

Three-body interaction diagrams in χEFTs



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- $\bar{K}$NN: exotic bound states of antikaons with nucleons predicted twenty years ago due to the strongly attractive $\bar{K}$N interaction in I = 0 channel. *S. Wycech, NPA 450 (1986) 399c; Y. Akaishi, T. Yamazaki, PRC 65 (2002) 044005*

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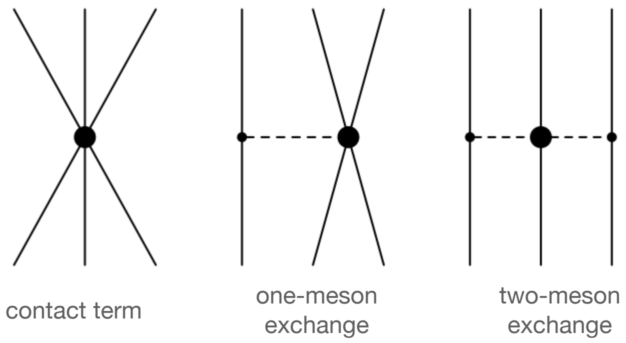

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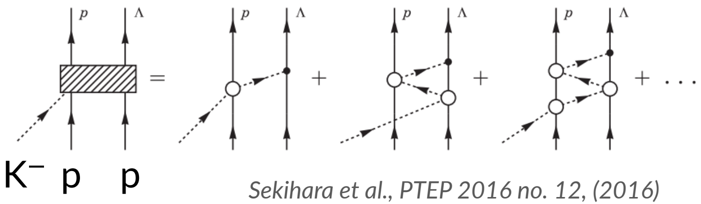


Sekihara et al., PTEP 2016 no. 12, (2016)

Next experimental challenge: genuine three-body interaction measurements

Source determination

➡ Femtoscopy used in the "traditional" way:
known interaction —> source determination

➡ Determined using:

- p-p interaction: **Argonne v18**
- crosscheck using p-Λ (χEFT LO and NLO)

➡ Exponential tail is included to account for the effect due to the short lived strongly-decaying resonances

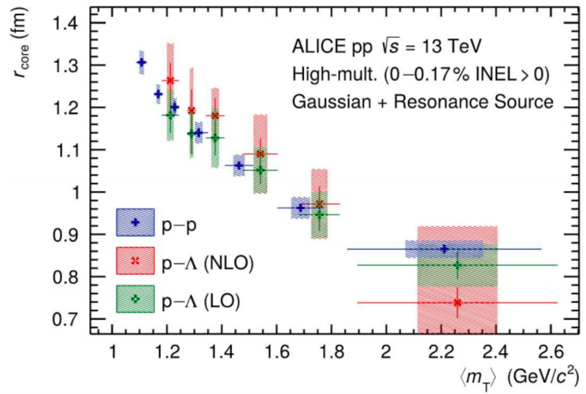
$$S(r) = \frac{1}{(4\pi r_{\mathrm{core}}^2)^{3/2}} \cdot \exp\left(-\frac{r^2}{4r_{\mathrm{core}}^2}\right) \otimes \frac{1}{s}\exp\left(-\frac{r}{s}\right)$$

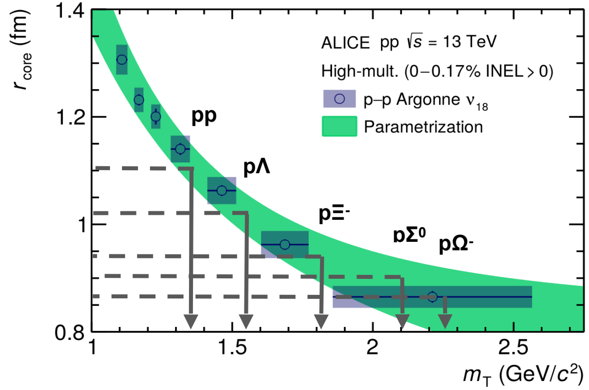
Gaussian source profile

Exponential tail
(for the resonances)

➡ Common universal core source for baryons

➡ Fix the source at $< m_{\mathrm{T}} >$ of the hadron-hadron pair under study





ALICE Coll. Physics Lett. B, 811 (2020) 135849

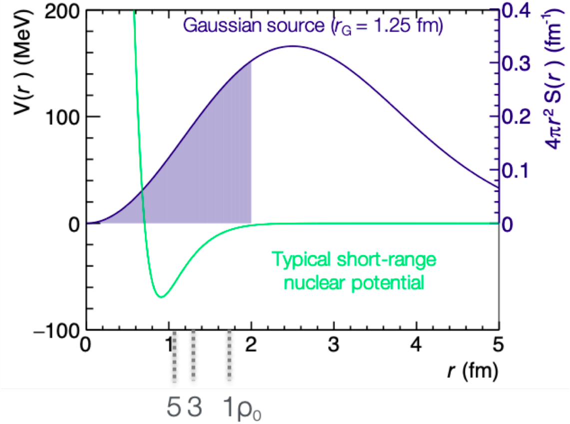
Two-body femtoscopy

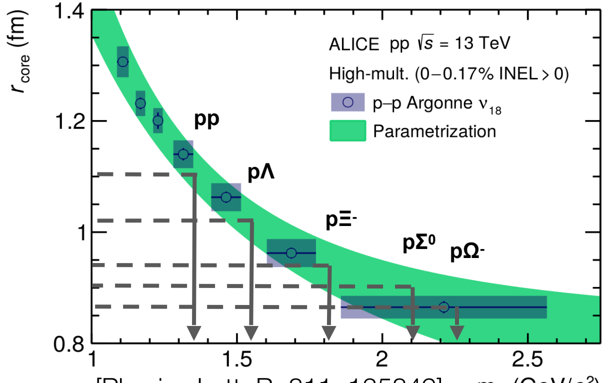
$$S(r) = \left(4\pi r_{core}^2\right)^{-3/2} \cdot \exp\left(-\frac{r^2}{4r_{core}^2}\right) \otimes \frac{1}{s}\exp\left(-\frac{r}{s}\right), \quad s = \beta\gamma\tau_{res} = \frac{p_{res}}{M_{res}}\tau_{res}$$

Gaussian source profile

Exponential tail added to account for the effect due to strong short-lived resonances

Small particle-emitting source created in pp and p–Pb collisions at the LHC.





[Physics Lett. B, 811, 135849]

$$C(k^*) = \mathcal{N} \cdot \frac{N_{same}(k^*)}{N_{mixed}(k^*)} = \int S(r)\left|\psi(k^*, r)\right|^2 d^3r$$

Emission source Two-particle wave function

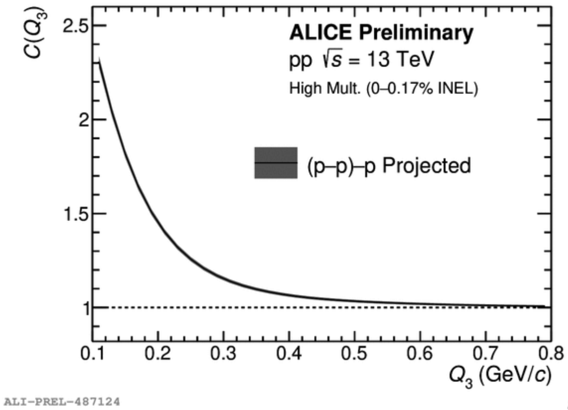
Projector method

$$\boxed{C_3^{ij}(Q_3)} = \int \boxed{C_2(k^*_{ij})}\, W_{ij}(k^*_{ij}, Q_3)\; dk^*_{ij}$$

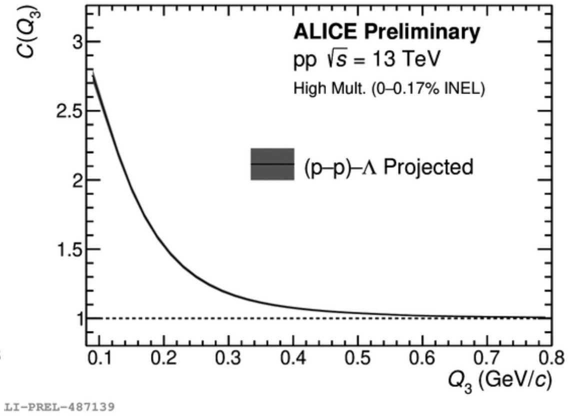
Outputs:

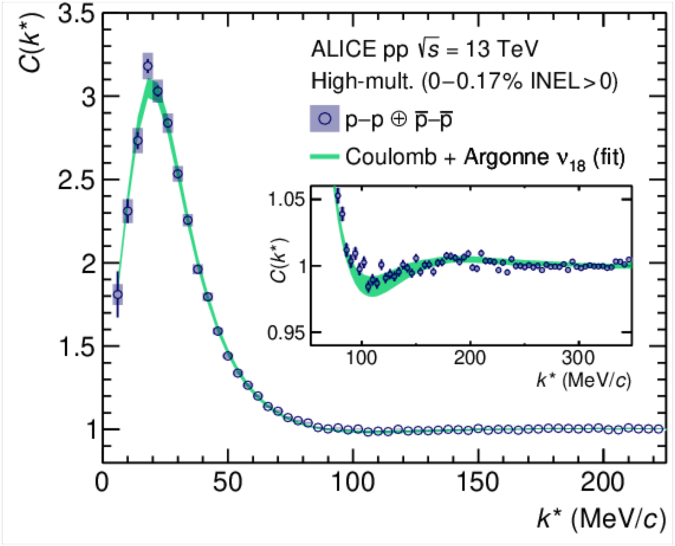
Input:
proton-proton

(proton-proton)-proton



(proton-proton)-Λ





[ALICE Collaboration / Physics Letters B 805 (2020) 135419]

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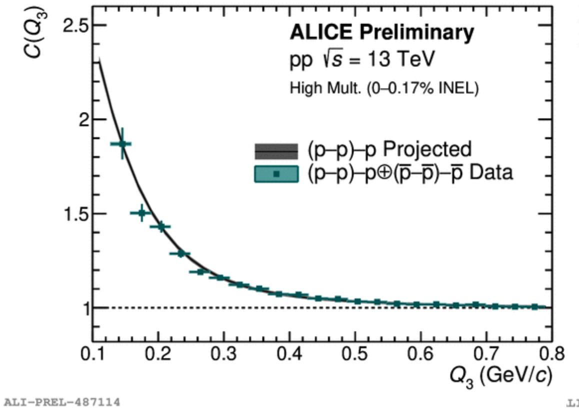
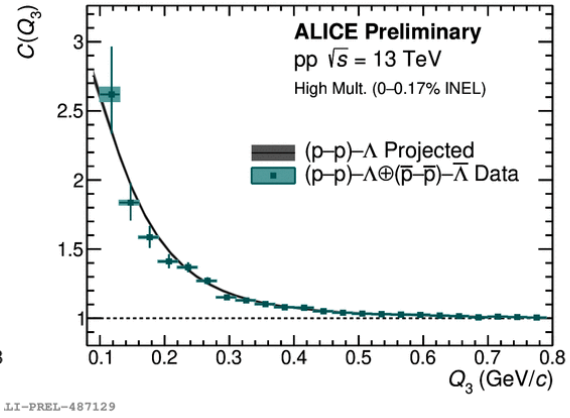
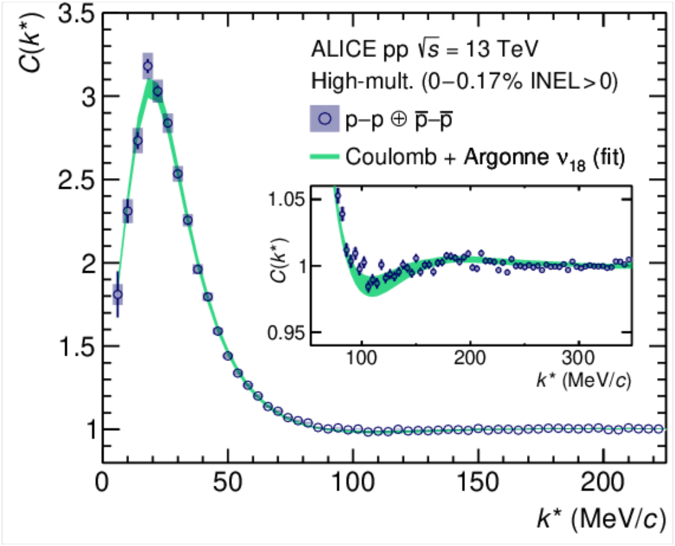


(proton-proton)-Λ





Data-driven approach VS **Projector method**

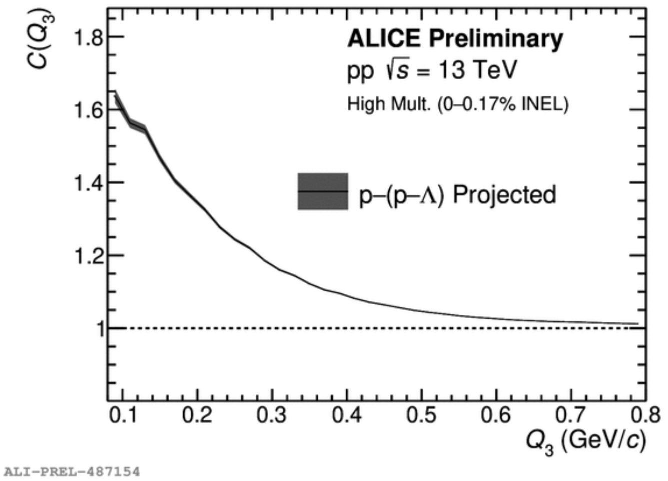
[ALICE Collaboration / Physics Letters B 805 (2020) 135419]

Projector method

$$C_3^{ij}(Q_3) = \int C_2(k_{ij}^*)\, W_{ij}(k_{ij}^*, Q_3)\, dk_{ij}^*$$

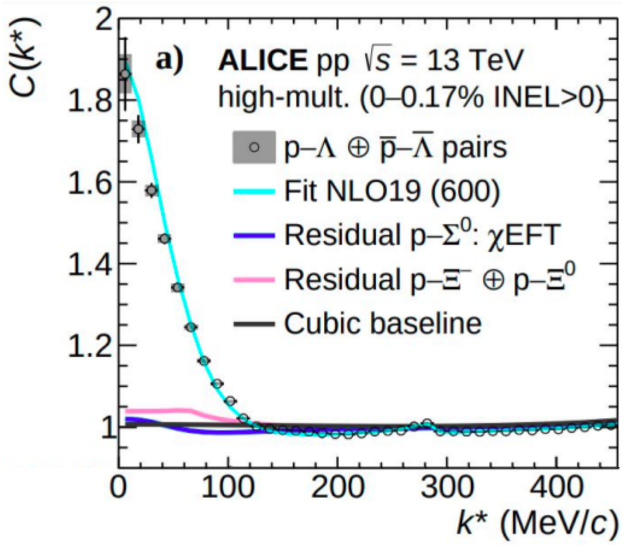
Outputs:

(proton-Λ)-proton



Input:

proton-Λ



[ALICE Collaboration / arXiv:2104.04427]

Projector method

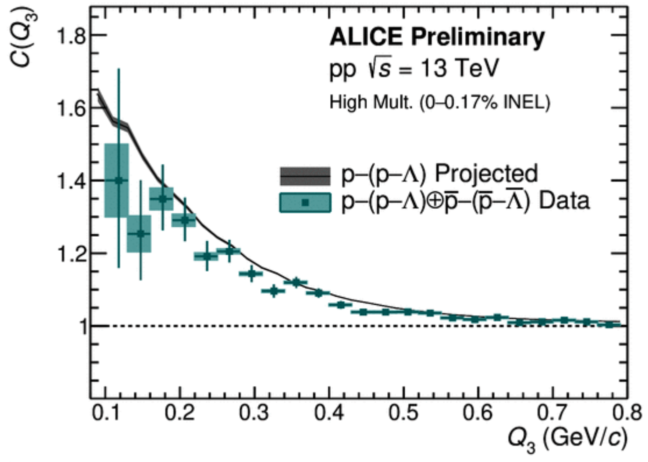
$$C_3^{ij}(Q_3) = \int C_2(k_{ij}^*)\, W_{ij}(k_{ij}^*, Q_3)\, dk_{ij}^*$$

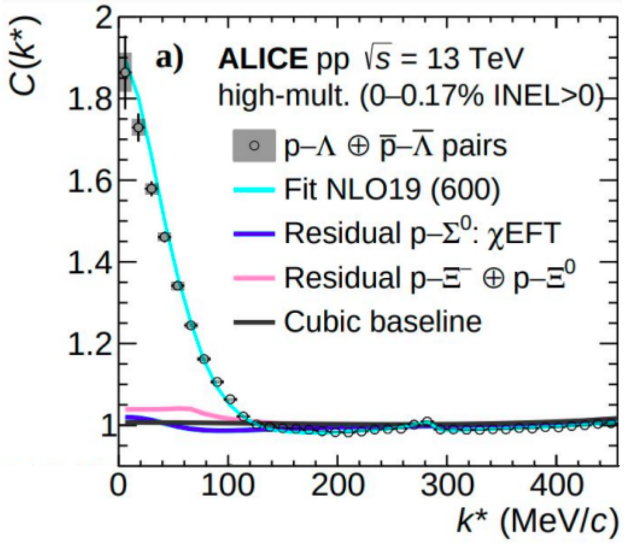
Outputs:

(proton-Λ)-proton

Input:

proton-Λ





Data-driven approach VS **Projector method**

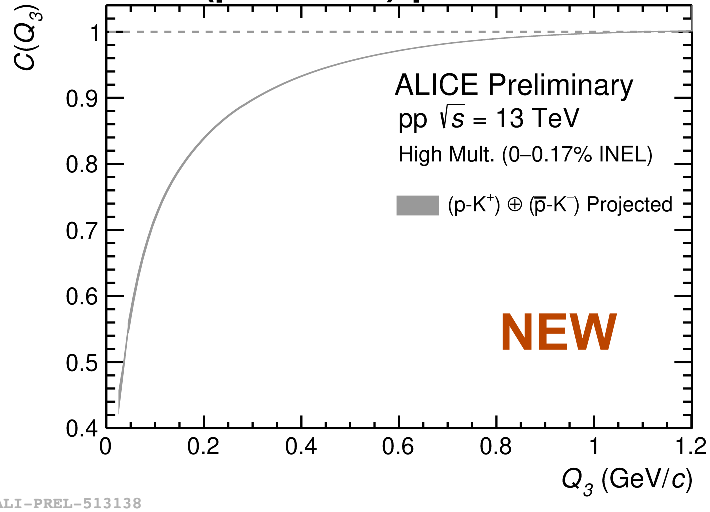
[ALICE Collaboration / arXiv:2104.04427]

Projector method

$$C_3^{ij}(Q_3) = \int C_2(k^*_{ij})\; W_{ij}(k^*_{ij}, Q_3)\; dk^*_{ij}$$

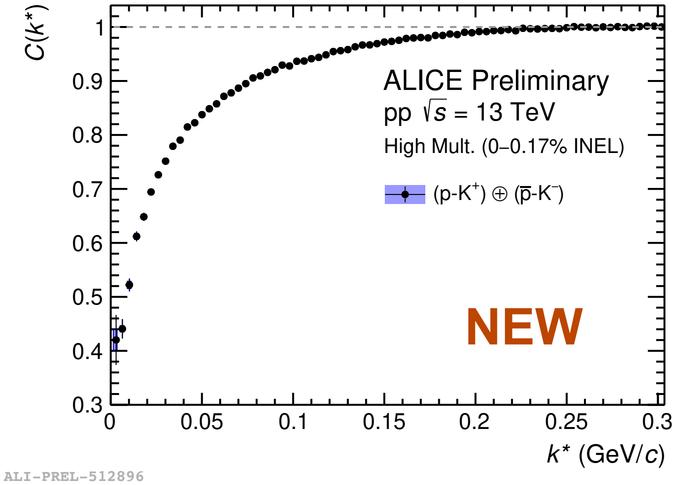
Output:

(proton-K+)-proton



Input:

proton-K+

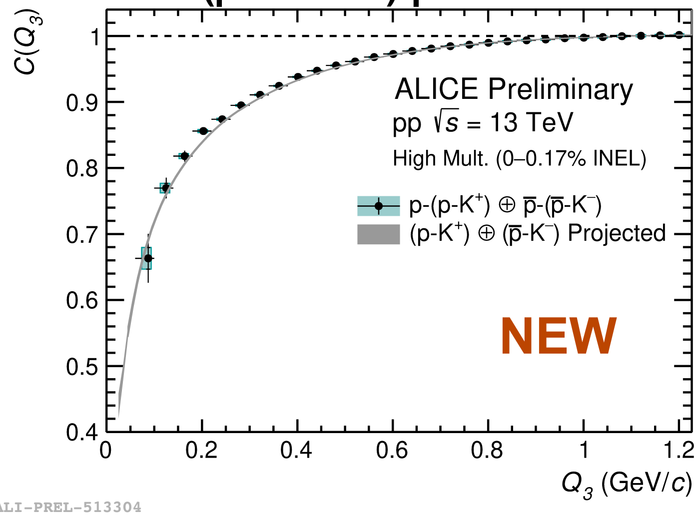


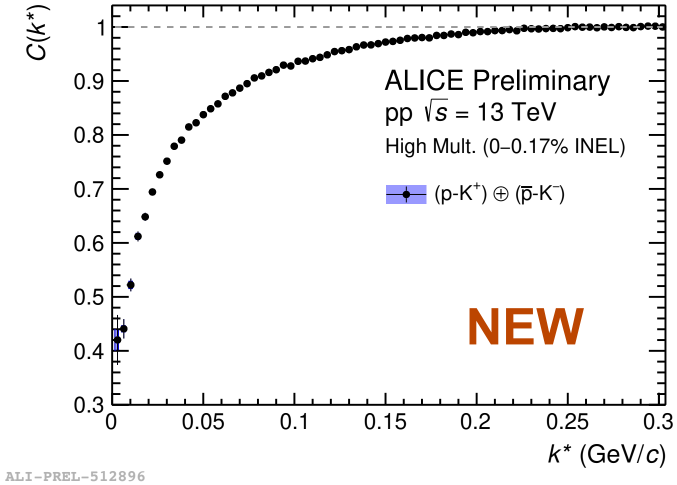
Projector method

$$C_3^{ij}(Q_3) = \int C_2(k_{ij}^*)\, W_{ij}(k_{ij}^*, Q_3)\, dk_{ij}^*$$

Output:
(proton-K+)-proton

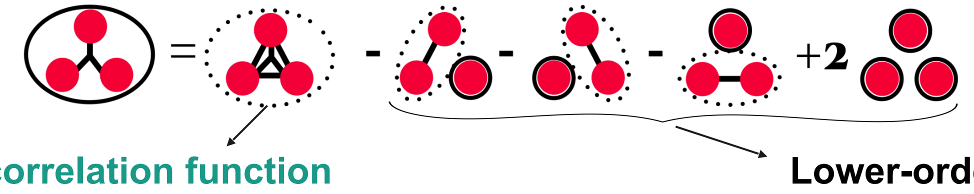
Input:
proton-K+

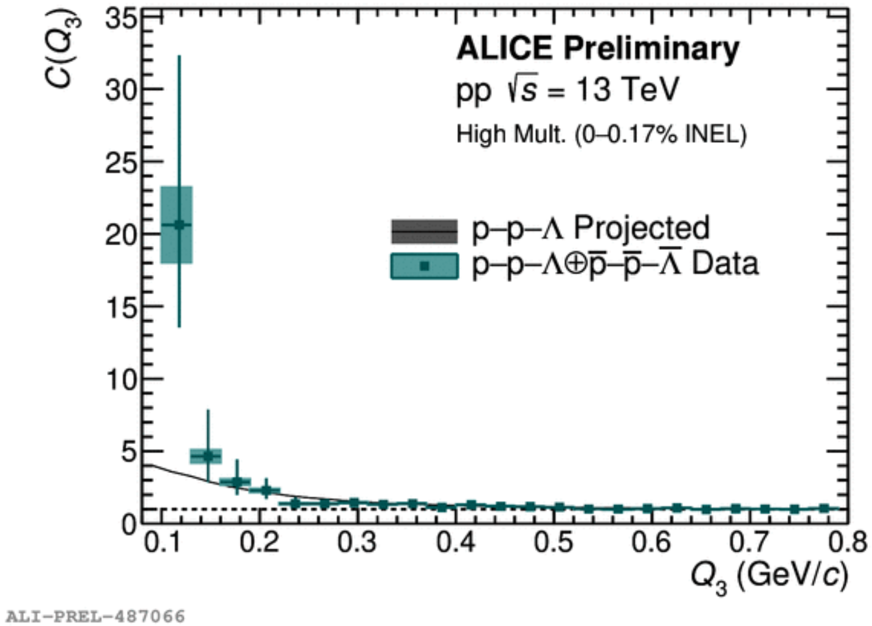


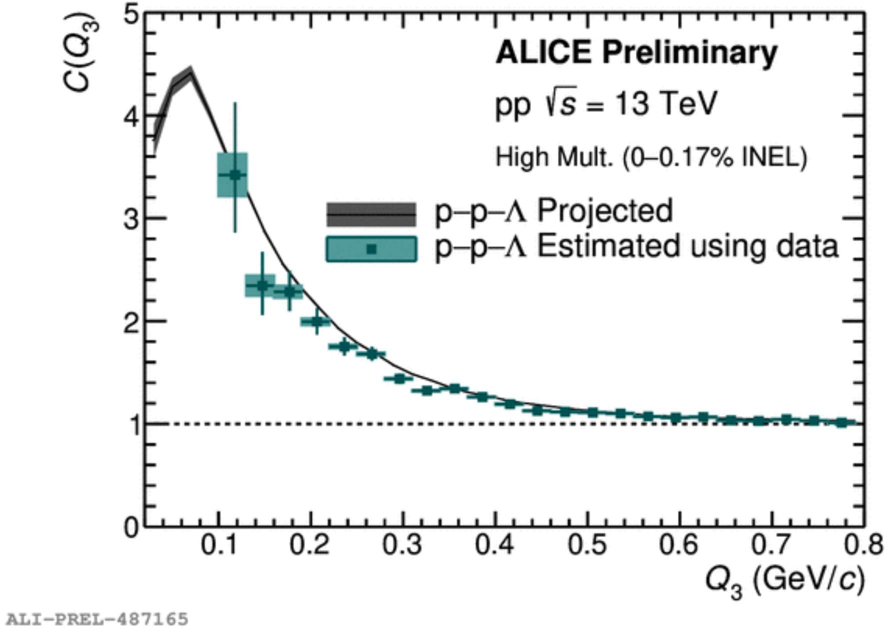


Data-driven approach VS **Projector method**

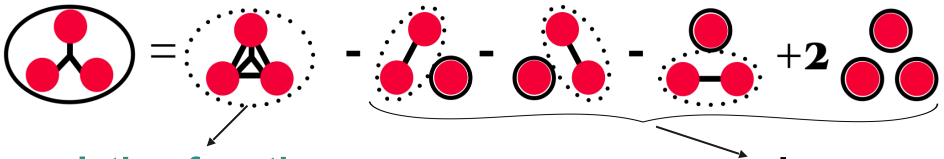
p-p-Λ Correlation Function

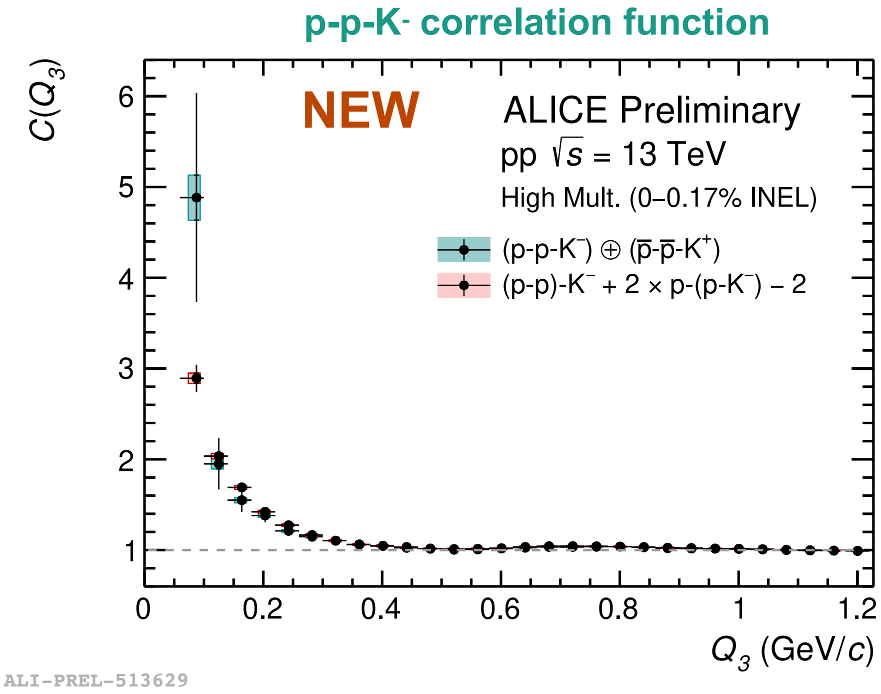


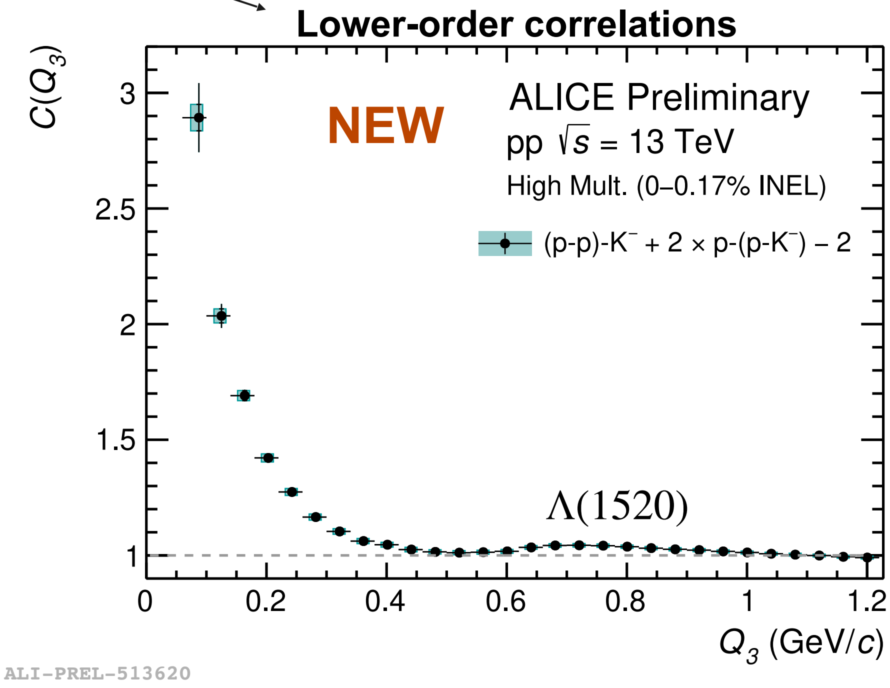




p-p-K- Correlation Function







λ parameters

The measured correlation function includes also misidentified particles and and feed-down particles coming from decays of resonances. Total **measured** function thus is:

$$C(XYZ) = \sum_{i,j,k} \lambda_{i,j,k}(XYZ) C_{i,j,k}(XYZ) = \boxed{\lambda_{X_0,Y_0,Z_0}(XYZ) C_{X_0,Y_0,Z_0}(XYZ)} + \sum_{ijk!=X_0Y_0Z_0} \lambda_{i,j,k}(XYZ) C_{i,j,k}(XYZ)$$

Correctly identified primary particles

- The cumulant is calculated with the measured correlation functions not accounting for the λ parameters.

$$\lambda_{i,j,k}(XYZ) = \mathscr{P}(X_i) f(X_i) \mathscr{P}(Y_j) f(Y_j) \mathscr{P}(Z_k) f(Z_k)$$

Extracted from measurement | **What we are interested in** | **Feed-down and misidentified particle contribution**

$$\boxed{c(XYZ)} = \sum_{i,j,k} \lambda_{i,j,k}(XYZ) c(X_iY_jZ_k) = \boxed{\lambda_{X_0Y_0Z_0}(XYZ) c(X_0Y_0Z_0)} + \boxed{\sum_{i,j,k \neq (X_0Y_0Z_0)} \lambda_{i,j,k}(XYZ) c(X_iY_jZ_k)}$$

- The genuine three body interaction for the feed-down and misidentified particle contributions is currently not known.

λ parameters

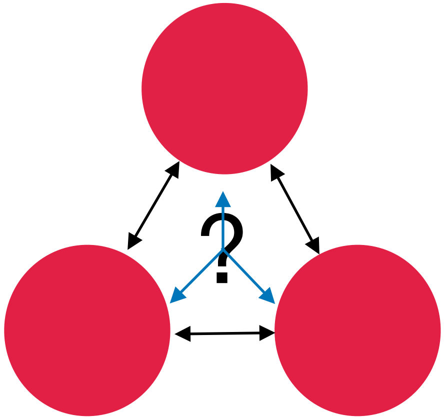
- The λ parameters requires purity and the secondary fraction evaluation.
- The average Λ purity is 95.57% and for protons the purity is 98.34%.
- The fractions of secondaries are estimated using Monte Carlo simulations.

Some of the contributions with highest lambda parameters:

p-p-p	61.8%
p-p-p_{Λ} x3	19.6%
p-p-p_{Σ^+} x3	8.5%
p-p_{Λ}-p_{Λ} x3	0.69%
p-p_{Λ}-p_{Σ^+} x3	0.3 %
p-p_{Σ^+}-p_{Σ^+} x3	0.13%

p-p-Λ	40.5%
p-p-Λ_{Σ^0}	13.5%
p-p-Λ_{Ξ^0}	7.56%
p-p-Λ_{Ξ^-}	7.56%
p-p_{Λ}-Λ x2	8.56%
p-p_{Σ^+}-Λ x2	3.7%

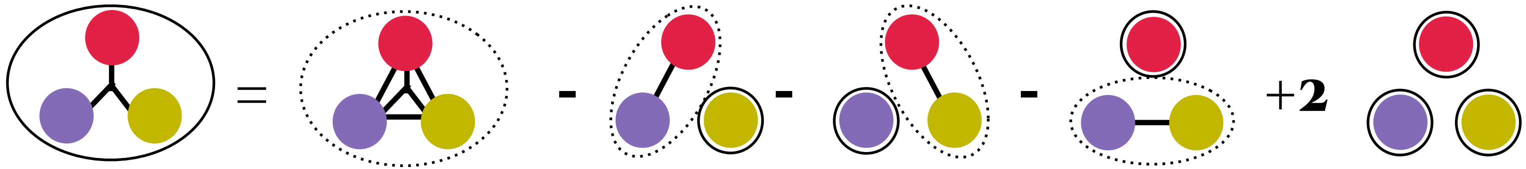
Accessing genuine three-body interaction



Measured correlation function includes:

- pairwise particle interactions,
- genuine three-particle interaction.

Use Kubo's cumulant expansion method [1] to extract the genuine three-body interaction.



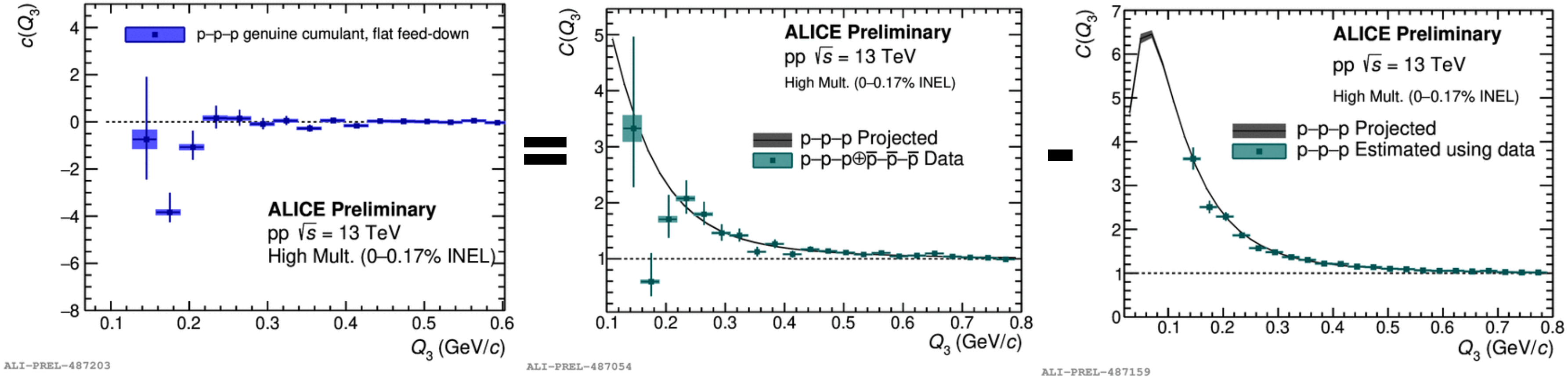
$$\mathbf{c}_3(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = \frac{[N_3(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) - N_2(\mathbf{p}_1, \mathbf{p}_2)\, N_1(\mathbf{p}_3) - N_2(\mathbf{p}_2, \mathbf{p}_3)\, N_1(\mathbf{p}_1) - N_2(\mathbf{p}_1, \mathbf{p}_3)\, N_1(\mathbf{p}_2) + 2\, N_1(\mathbf{p}_1)\, N_1(\mathbf{p}_2)\, N_1(\mathbf{p}_3)]}{N_1(\mathbf{p}_1)\, N_1(\mathbf{p}_2)\, N_1(\mathbf{p}_3)}$$

$$\mathbf{c}_3\left(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\right) = C([\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3]) - C([\mathbf{p}_1, \mathbf{p}_2], \mathbf{p}_3) - C(\mathbf{p}_1, [\mathbf{p}_2, \mathbf{p}_3]) - C([\mathbf{p}_1, \mathbf{p}_3], \mathbf{p}_2) + 2$$

[1] J. Phys. Soc. Jpn. 17, pp. 1100–1120 (1962)

Accessing genuine three-body interaction

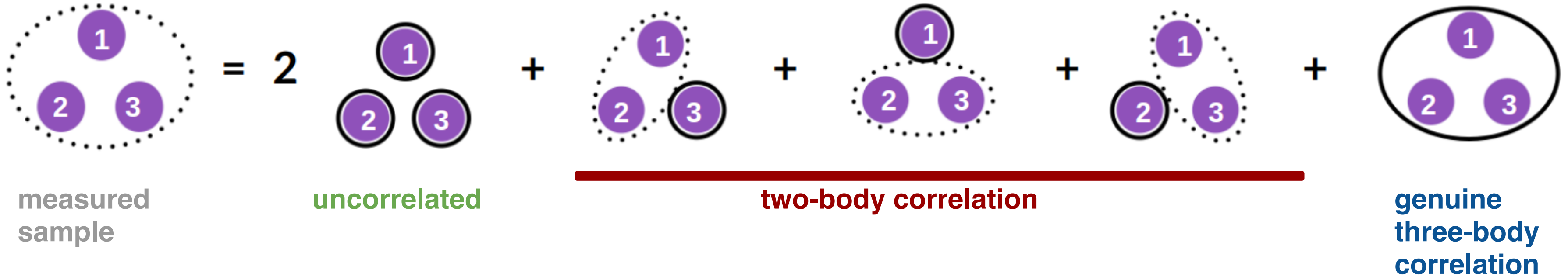
ppp results: cumulant, three-body correlation function, two-body correlation projected on Q_3



Work of Laura Šerkšnytė (TUM)

Kubo's cumulant expansion method

The measured triplet sample can be decomposed in the following sub-samples



In terms of the correlation functions

$$C_3(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3) = C_3([\mathbf{p}_1,\mathbf{p}_2],\mathbf{p}_3) + C_3(\mathbf{p}_1,[\mathbf{p}_2,\mathbf{p}_3]) + C_3(\mathbf{p}_2,[\mathbf{p}_3,\mathbf{p}_1]) + \mathbf{c}_3([\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3]) - 2$$

X

The pairs in the square brackets are correlated, the particle outside is not correlated.

Two-body correlation in the three-body systems

Each term $C_3([p_i, p_j], p_k)$ can be evaluated using two-body correlation functions.

three-body source

$$C_3(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = \int_{V_1}\int_{V_2}\int_{V_3} S_3(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)\ |\psi_{\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)|^2\ d^3\mathbf{x}_1 d^3\mathbf{x}_2 d^3\mathbf{x}_3$$

wave function of the system

solution of the Schrödinger equation

$$\mathcal{H} = \left[\frac{\mathbf{p}_1^2}{2m_1} + \frac{\mathbf{p}_2^2}{2m_2} + V_2(\mathbf{x}_1 - \mathbf{x}_2)\right] + \frac{\mathbf{p}_3^2}{2m_3}$$

$$\mathcal{H} = \mathcal{H}_{CM} + \mathcal{H}_1 + \mathcal{H}_2 = \frac{\mathbf{P}^2}{2\ M} + \left[\frac{\mathbf{k}_1^2}{2\ \mu_1} + V(\mathbf{r}_1)\right] + \frac{\mathbf{k}_2^2}{2\ \mu_2}$$

The three hamiltonian operators commute, plane waves are assumed as solutions for the Schrödinger equations with H_{CM} and H_2

Jacobi coordinates

$$\mathbf{r}_i = \mathbf{x}_{i+1} - \frac{\sum_{j=1}^{i} m_j\ \mathbf{x}_j}{\sum_{j=1}^{i} m_j}$$

Conjugate momenta

$$\mathbf{k}_i = \frac{\sum_{j=1}^{i} m_j}{\sum_{j=1}^{i+1} m_j}\mathbf{p}_{i+1} - \frac{m_{i+1}}{\sum_{j=1}^{i+1} m_j}\sum_{j=1}^{i}\mathbf{p}_j$$

Two-body correlation in the three-body systems

Each term $C_3([p_i, p_j], p_k)$ can be evaluated using two-body correlation functions.

$$C_3(\mathbf{P}, \mathbf{k}_1, \mathbf{k}_2) = \int_{V_{\mathbf{r}_1}} S_3(\mathbf{r}_1)\ |\psi_{\mathbf{k}_1}(\mathbf{r}_1)|^2\ d^3\mathbf{r}_1$$

solution of the Schrödinger equation for the hamiltonian H_1

$$\mathcal{H}_1 = \frac{\mathbf{k}_1^2}{2\ \mu_1} + V(\mathbf{r}_1)$$

The Koonin-Pratt formula for the two-body correlation function is obtained.

$$C_3(\mathbf{P}, \mathbf{k}_1, \mathbf{k}_2) = C_2(\mathbf{k}_1)$$

We can use the **experimental** or the **theoretical** two-body correlation functions to evaluate the lower order contributions in the three-body correlation functions

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Projection on Q3

- The projection onto Q_3 is performed as follows

$$C_3(Q_3) = \iiint_{(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)\in\mathcal{D}} C_3(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)\ \mathcal{N}\ d^3\mathbf{p}_1\ d^3\mathbf{p}_2\ d^3\mathbf{p}_3$$

$$\mathcal{D} = \{(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3) \in \mathcal{S} \mid Q_3 = \text{constant}\}$$

density of states in the phase space (uniform)

- In the case of two-body correlations, the projections turns to be

two-body correlation function

projector $W(k_1, Q_3)$ ----> phase space density at Q_3 = constant

$$C_3(Q_3) = \int_0^{\sqrt{\frac{\gamma}{\alpha\gamma-\beta^2}}\,Q_3} C_2(k_1) \left[\frac{16(\alpha\gamma-\beta^2)^{3/2}k_1^2}{\pi Q_3^4\gamma^2}\sqrt{\gamma Q_3^2-(\alpha\gamma-\beta^2)k_1^2}\right] dk_1$$

where α, β and γ are constants depending on the particles mass.

Projection on Q3

If we project onto Q_3 all the two-body contributions

$$C_3(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = C_3([\mathbf{p}_1, \mathbf{p}_2], \mathbf{p}_3) + C_3(\mathbf{p}_1, [\mathbf{p}_2, \mathbf{p}_3]) + C_3(\mathbf{p}_2, [\mathbf{p}_3, \mathbf{p}_1]) - 2$$

we have

$$C_3(Q_3) = C_3^{12}(Q_3) + C_3^{23}(Q_3) + C_3^{31}(Q_3) - 2$$

where

$$C_3^{ij}(Q_3) = \int C_2(k_1^{ij})\ W^{ij}(k_1^{ij}, Q_3)\ dk_1^{ij}$$

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