$\bar{K}NN$ and $\bar{K}NNN$ sates: Faddeev calculations

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Interest to antikaon-nucleon systems: quasi-bound state in the K^-pp system
$\rightarrow$ experimental and theoretical studies

Our contribution: series of dynamically exact calculations of three-body antikaon-nucleon systems

- Quasi-bound states in $\bar{K}NN$, spin 0 (K^-pp) and $\bar{K}\bar{K}N$ systems - binding energy and width, no quasi-bound state in $\bar{K}NN$, spin 1 (K^-np)
- Near-threshold scattering in $\bar{K}NN$, spin 1 (K^-d)
(Faddeev-type AGS equations with coupled $\bar{K}NN - \pi\Sigma N$ channels)
- $1s$ level shift and width of kaoninc deuterium (Faddeev-type equations with directly included Coulomb interaction)

[N.V. S., Three-Body Antikaon–Nucleon Systems. Few Body Syst. 58, 6 (2017)]

In the present talk:

- Four-body calculations of $\bar{K}NNN$ system (Faddeev-type equations)
- Checks of Deser formulas
- New results for $\bar{K}NN$ with different spins

Four-body equations

The four-body Faddeev-type AGS equations, written for separable potentials [*P. Grassberger, W. Sandhas, Nucl. Phys. B 2, 181-206 (1967)*]

$$\bar{U}^{\sigma\rho}_{\alpha\beta}(z) = (1-\delta_{\sigma\rho})(\bar{G}_0{}^{-1})_{\alpha\beta}(z) + \sum_{\tau,\gamma,\delta}(1-\delta_{\sigma\tau})\bar{T}^{\tau}_{\alpha\gamma}(z)(\bar{G}_0)_{\gamma\delta}(z)\bar{U}^{\tau\rho}_{\delta\beta}(z),$$

$$\bar{U}^{\sigma\rho}_{\alpha\beta}(z) = \langle g_\alpha | G_0(z) U^{\sigma\rho}_{\alpha\beta}(z) G_0(z) | g_\beta \rangle,$$

$$\bar{T}^{\tau}_{\alpha\beta}(z) = \langle g_\alpha | G_0(z) U^{\tau}_{\alpha\beta}(z) G_0(z) | g_\beta \rangle,$$

$$(\bar{G}_0)_{\alpha\beta}(z) = \delta_{\alpha\beta}\tau_\alpha(z).$$

Operators $\bar{U}^{\sigma\rho}_{\alpha\beta}$ and $\bar{T}^{\tau}_{\alpha\beta}$ contain four-body $U^{\sigma\rho}_{\alpha\beta}(z)$ and three-body $U^{\tau}_{\alpha\beta}(z)$ transition operators of the general form, correspondingly.

$\bar{K}NNN$ system:

- two partitions of $3+1$ type: $|\bar{K}+(NNN)\rangle$, $|N+(\bar{K}NN)\rangle$,
- one of the $2+2$ type: $|(\bar{K}N)+(NN)\rangle$

Four-body equations

Separable form of the "effective three-body potentials" :

$$\bar{T}^{\tau}_{\alpha\beta}(z) = |\bar{g}^{\tau}_{\alpha}\rangle \bar{\tau}^{\tau}_{\alpha\beta}(z) \langle \bar{g}^{\tau}_{\beta}|$$

$\rightarrow$ the four-body equations can be rewritten as
[A. Casel, H. Haberzettl, W. Sandhas, Phys. Rev. C 25, 1738 (1982)]

$$\bar{X}^{\sigma\rho}_{\alpha\beta}(z) = \bar{Z}^{\sigma\rho}_{\alpha\beta}(z) + \sum_{\tau,\gamma,\delta} \bar{Z}^{\sigma\tau}_{\alpha\gamma}(z) \bar{\tau}^{\tau}_{\gamma\delta}(z) \bar{X}^{\tau\rho}_{\delta\beta}(z)$$

with new four-body transition $\bar{X}^{\sigma\rho}$ and kernel $\bar{Z}^{\sigma\rho}$ operators

$$\begin{aligned} \bar{X}^{\sigma\rho}_{\alpha\beta}(z) &= \langle \bar{g}^{\sigma}_{\alpha}| \bar{G}_0(z)_{\alpha\alpha} \bar{U}^{\sigma\rho}_{\alpha\beta}(z) \bar{G}_0(z)_{\beta\beta} |\bar{g}^{\rho}_{\beta}\rangle, \\ \bar{Z}^{\sigma\rho}_{\alpha\beta}(z) &= (1-\delta_{\sigma\rho}) \langle \bar{g}^{\sigma}_{\alpha}| \bar{G}_0(z)_{\alpha\beta} |\bar{g}^{\rho}_{\beta}\rangle. \end{aligned}$$

Full system of equations with:

- 1-term separable $\bar{K}N$ potential, 2-term separable NN potential (input)
- 1-term separabelized 3-body NNN, $\bar{K}NN$ "T-matrices" and "3-body" $\bar{K}N + NN$ "T-matrices"

$\rightarrow$ system of 18×18 coupled equations

EDPE/EDPA separabelization

$\bar{K}N$ and NN potentials: separable by construction, separabelization of $3+1$ and 2+2: Energy Dependent Pole Expansion/Approximation (EDPE/ EDPA)
[S. Sofianos, N.J. McGurk, H. Fiedeldeldey, Nucl. Phys. A 318, 295 (1979)]

EDPE/EDPA method: solution of the eigenequations for a fixed energy z, usually $z = E_B$. After that energy dependent form-factors and propagators are calculated. The separable version of a three-body amplitude:

$$X_{\alpha\beta}(p,p';z) = \sum_{m,n=1}^{\infty} g_{m\alpha}(p;z)\,\Theta_{mn}(z)\,g_{n\beta}(p';z).$$

The method needs only one solution of the eigenvalue equations and calculations of the integrals after that. The method is accurate already with one term (i.e. EDPA), and it converges faster than Hilbert-Schmidt expansion.

Two-term Separable New potential (TSN) of nucleon-nucleon interaction

$$V_{NN}^{\rm TSN}(k,k') = \sum_{m=1}^{2} g_m(k)\,\lambda_m\,g_m(k')\,,$$

$$g_m(k) = \sum_{n=1}^{3} \frac{\gamma_{mn}}{(\beta_{mn})^2 + k^2}, \quad \text{for } m = 1,2$$

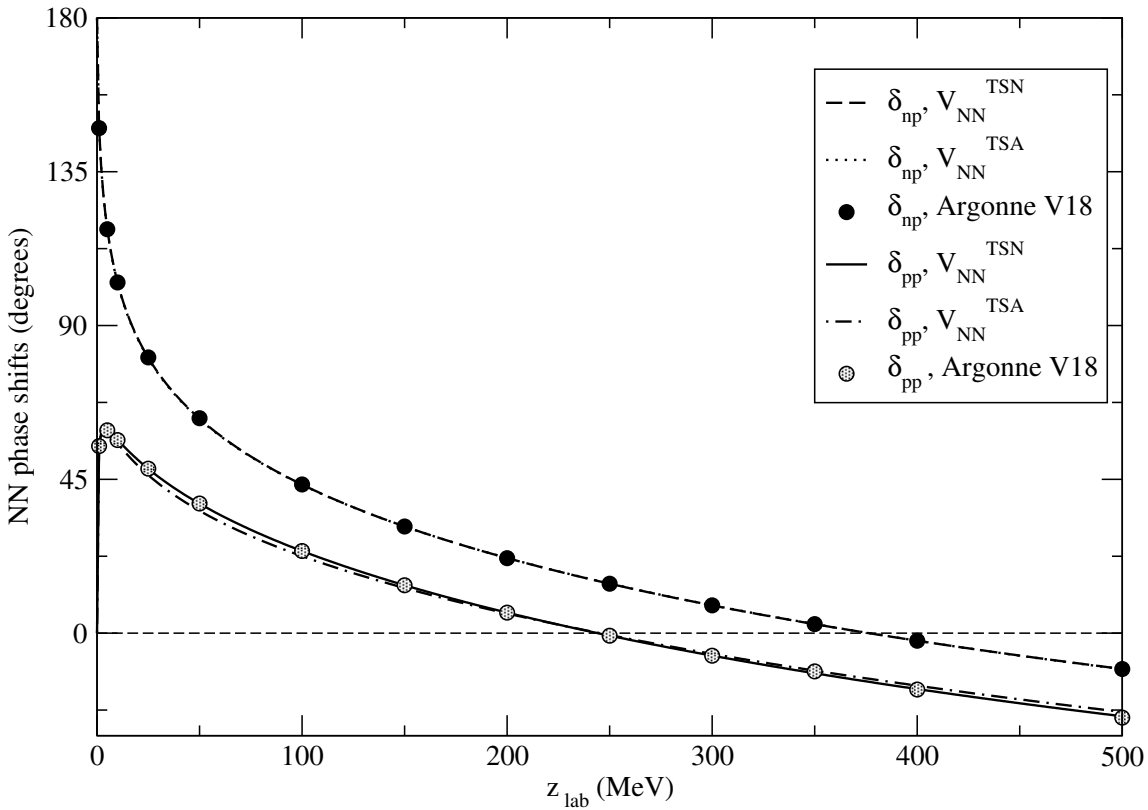
fitted to Argonne $V18$ potential *[R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, Phys. Rev. C 51, 38 (1995)]* phase shifts

Triplet and singlet scattering lengths a and effective ranges $r_{\rm eff}$

$$a_{np}^{\rm TSN} = -5.400\,{\rm fm}, \qquad r_{{\rm eff},np}^{\rm TSN} = 1.744\,{\rm fm}$$
$$a_{pp}^{\rm TSN} = 16.325\,{\rm fm}, \qquad r_{{\rm eff},pp}^{\rm TSN} = 2.792\,{\rm fm},$$

deuteron binding energy $E_{\rm deu} = 2.2246$ MeV.

New NN potential



Phase shifts of np and pp scattering calculated using the new V_{NN}^{TSN} and old $V_{NN}^{\mathrm{TSA-B}}$ potentials plus phase shifts of Argonne $V18$

Antikaon-nucleon interaction

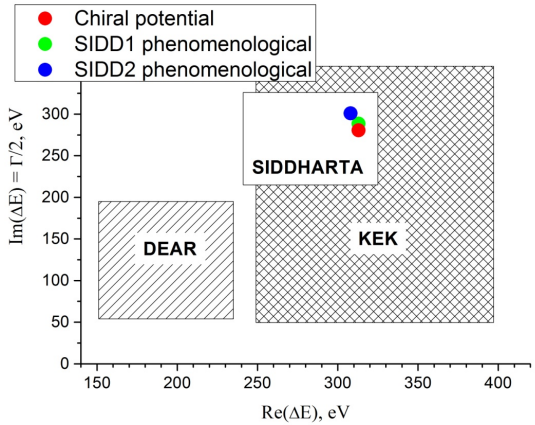
Three our potentials:

- phenomenological $\bar{K}N - \pi\Sigma$ with one-pole $\Lambda(1405)$ resonance
- phenomenological $\bar{K}N - \pi\Sigma$ with two-pole $\Lambda(1405)$ resonance
- chirally motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potential, two-pole $\Lambda(1405)$

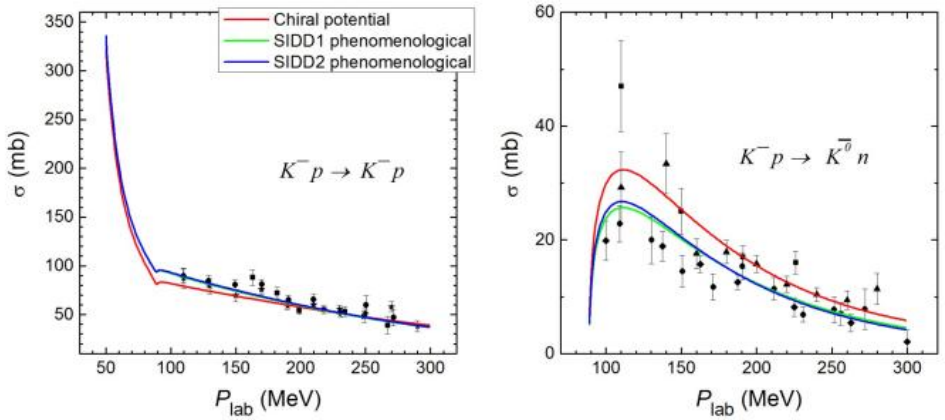
reproduce (with the same level of accuracy):

- $1s$ level shift and width of kaonic hydrogen *(SIDDHARTA)*
  direct inclusion of Coulomb interaction, no Deser-type formula used
- Cross-sections of $K^-p \to K^-p$ and $K^-p \to MB$ reactions
- Threshold branching ratios γ, R_c and R_n
- $\Lambda(1405)$ resonance (*one- or two-pole structure*)
  $M^{PDG}_{\Lambda(1405)} = 1405.1^{+1.3}_{-1.0}$ MeV, $\Gamma^{PDG}_{\Lambda(1405)} = 50.5 \pm 2.0$ MeV [*PDG (2016)*]

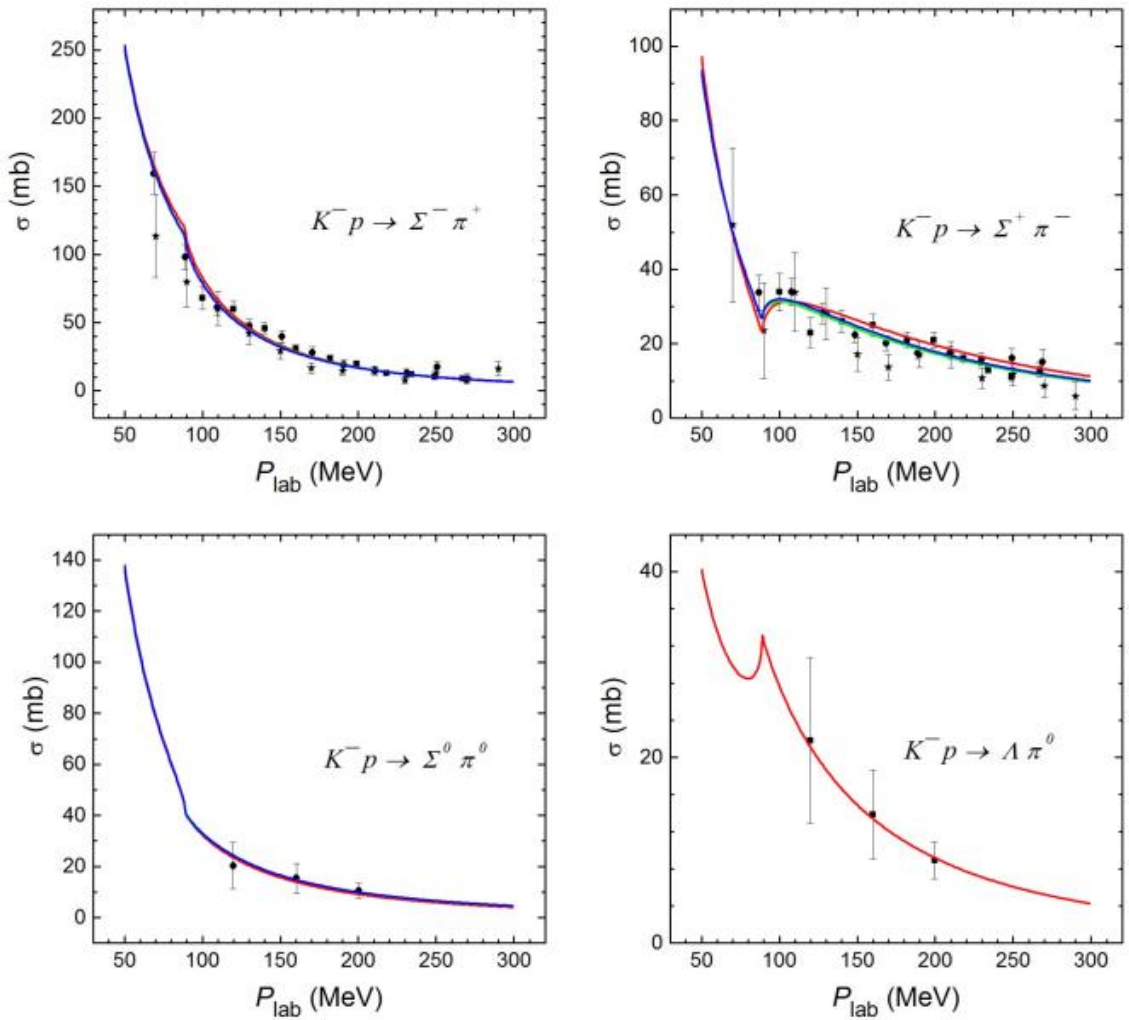
$\bar{K}N$ interaction



$1s$ level shift and width of kaonic hydrogen for three $\bar{K}N$ potentials



Cross-sections of $K^- p$ scattering for three $\bar{K}N$ potentials



Cross-sections of $K^- p$ scattering for three $\bar{K}N$ potentials, cont.

1s level shift and width of kaonic hydrogen calculated exactly – possibility to check Deser-type formulas

[N.V.S., Few-Body Syst. 63, 22 (2022)]

"Corrected Deser"

[U.-G. Meißner, U. Raha, A. Rusetsky, Eur. Phys. J. C 35, 349 (2004)]

$$\Delta E_{1s}^{K^-p} - i\frac{\Gamma_{1s}^{K^-p}}{2} = -2\alpha^3\mu^2 a_{K^-p}[1 - 2\alpha\mu a_{K^-p}(\ln\alpha - 1)]$$

"Summed-up Deser"

[V. Baru, E. Epelbaum, A. Rusetsky, Eur. Phys. J. A 42, 111 (2009)], a footnote

$$\Delta E_{1s}^{K^-p} - i\,\frac{\Gamma_{1s}^{K^-p}}{2} = -2\alpha^3\mu^2 a_{K^-p}/[1 + 2\alpha\mu a_{K^-p}(\ln\alpha - 1)]. \qquad (1)$$

Also studied in *[T. Hoshino, S. Ohnishi, W. Horiuchi, T. Hyodo, W. Weise, Phys. Rev. C 96, 045204 (2017)]*, talk T. Hyodo at EXOTICO

Kaonic hydrogen

	Corrected Deser		Summed up Deser		Exact	
	$\Delta E_{1s}^{K^-p}$	$\Gamma_{1s}^{K^-p}$	$\Delta E_{1s}^{K^-p}$	$\Gamma_{1s}^{K^-p}$	$\Delta E_{1s}^{K^-p}$	$\Gamma_{1s}^{K^-p}$
$V_{\bar{K}N}^{1,\mathrm{SIDD}}$	-328	579	-318	593	-313	597
$V_{\bar{K}N}^{2,\mathrm{SIDD}}$	-322	589	-312	603	-308	602
$V_{\bar{K}N}^{\mathrm{Chiral}}$	-326	544	-318	559	-313	561

$1s$ level shift $\Delta E_{1s}^{K^-p}$ (eV) and width $\Gamma_{1s}^{K^-p}$ (eV) of kaonic hydrogen calculated using the "corrected Deser" and "summed up Deser" formulas together with the exact results.

The accuracy of the summed up Deser formula for kaonic hydrogen is better than 2 %

Kaonic deuterium

	Corrected Deser		Summed up Deser		Exact		K^-d optical potential	
	$\Delta E_{1s}^{K^-d}$	$\Gamma_{1s}^{K^-d}$	$\Delta E_{1s}^{K^-d}$	$\Gamma_{1s}^{K^-d}$	$\Delta E_{1s}^{K^-d}$	$\Gamma_{1s}^{K^-d}$	$\Delta E_{1s}^{K^-d}$	$\Gamma_{1s}^{K^-d}$
$V_{\bar{K}N}^{1,\mathrm{SIDD}}$	-826	731	-792	921	-767	928	-785	1018
$V_{\bar{K}N}^{2,\mathrm{SIDD}}$	-835	727	-800	923	-782	938	-797	1025
$V_{\bar{K}N}^{\mathrm{Chiral}}$	-876	724	-836	951	-835	1004	-828	1055

$1s$ level shift $\Delta E_{1s}^{K^-d}$ (eV) and width $\Gamma_{1s}^{K^-d}$ (eV) of kaonic deuterium calculated using the "corrected Deser", "summed up Deser" formulas and using K^-d optical potentials together with the exact results.
Exact values are from *[J. Révai, Phys. Rev. C 94, 054001 (2016)]*

The accuracy of the summed up Deser formula for kaonic deuterium is better than 6 %

$\bar{K}NN$ systems, coupled-channel $\bar{K}NN - \pi\Sigma N$ calculations

New NN potential → by-product result:

quasi-bound state in K^-np system ($\bar{K}NN$, $S = 1$), caused by strong interactions (additional to the atomic state – kaonic deuterium)

	B_{K^-pp}	Γ_{K^-pp}	B_{K^-np}	Γ_{K^-np}
$V_{\bar{K}N}^{1,\mathrm{SIDD}}$	52.2	67.1	–	–
$V_{\bar{K}N}^{2,\mathrm{SIDD}}$	46.6	51.2	0.9	59.4
$V_{\bar{K}N}^{\mathrm{Chiral}}$	29.4	46.4	1.3	41.8

Binding energy B (MeV) and width Γ (MeV) of the quasi-bound state in the K^-pp and K^-np systems; $V_{\bar{K}N}$ and V_{NN}^{TSN}

[N.V.S., Few-Body Syst. 61, 27 (2020)]

Subsystems of the $\bar{K}NNN$ system

- $\bar{K}NN\,(I^{(3)}=1/2, S^{(3)}=0 \text{ or } 1)$

	B_{K^-pp}	Γ_{K^-pp}	B_{K^-np}	Γ_{K^-np}
$V_{\bar{K}N}^{1,\mathrm{SIDD}}$	53.3	64.7	1.9	68.7
$V_{\bar{K}N}^{2,\mathrm{SIDD}}$	46.7	48.4	5.6	62.7
$V_{\bar{K}N}^{\mathrm{Chiral}}$	29.9	48.2	2.3	45.5

Binding energy B (MeV) and width Γ (MeV) of the quasi-bound state in the K^-pp and K^-np (sub)systems; exact optical $V_{\bar{K}N}^{\mathrm{Opt}}$ and V_{NN}^{TSN}

- $NNN\,(I^{(3)}=1/2, S^{(3)}=1/2)$ $\qquad B_{NNN}=9.52$ MeV (V_{NN}^{TSN})
- $\bar{K}N+NN\,(I^{(4)}=0, S^{(4)}=1/2)$ – a special system with two non--interacting pairs of particles; 3-body system of equations to be solved

$K^- ppn - \bar{K}^0 nnp$ system: $\bar{K}NNN$ with $I^{(4)} = 0, S^{(4)} = 1/2, L^{(4)} = 0$

Dependence of the binding energy B (MeV) and width Γ (MeV) of the quasi-bound state in the $K^- ppn - \bar{K}^0 nnp$ system on three $\bar{K}N$ and three NN interaction models.

	$V_{NN}^{\mathrm{TSA-A}}$		$V_{NN}^{\mathrm{TSA-B}}$		V_{NN}^{TSN}	
	B	Γ	B	Γ	B	Γ
$V_{\bar{K}N}^{1,\mathrm{SIDD}}$	52.0	50.4	50.3	49.6	51.2	50.8
$V_{\bar{K}N}^{2,\mathrm{SIDD}}$	47.0	39.6	46.4	38.2	46.4	39.9
$V_{\bar{K}N}^{\mathrm{Chiral}}$	32.6	39.7	34.5	50.9	30.5	42.8

Other results: $\bar{K}NNN$ quasi-bound state

Binding energy B (MeV) and width Γ (MeV) of the quasi-bound state in the $K^-ppn - \bar{K}^0nnp$ system, other results

		B	Γ
AY		108.0	20.0
BGL		29.3	32.9
OHHMH,	$V_{\bar{K}N}^{\text{Kyoto}-\text{I}}$	45.3	25.5
OHHMH,	$V_{\bar{K}N}^{\text{Kyoto}-\text{II}}$	49.7	69.4
ME,	$V_{\bar{K}N}^{1,\text{SIDD}}$	73.5	22.0
ME,	$V_{\bar{K}N}^{2,\text{SIDD}}$	58.5	27.0
ME,	$V_{\bar{K}N}^{\text{IKS chiral}}$	41.4	31.5

[AY: Y. Akaishi, T. Yamazaki, Phys. Rev. C 65, 044005 (2002)]
[BGL: N. Barnea, A. Gal, E.Z. Liverts, Phys. Lett. B 712, 132 (2012)]
[S. Ohnishi et.al, Phys. Rev C 95, 065202 (2017)]
[ME: S. Marri, J. Esmaili, Eur. Phys. J. A 55, 43 (2019)]

All results: $\bar{K}NNN$ quasi-bound state

Binding energy B (MeV) and width Γ (MeV) of the quasi-bound state in the $K^-ppn - \bar{K}^0nnp$ system ($\bar{K}NNN$ with $I^{(4)} = 0, S^{(4)} = 1/2, L^{(4)} = 0$)

	$V_{NN}^{\rm TSA-A}$		$V_{NN}^{\rm TSA-B}$		$V_{NN}^{\rm TSN}$		Other results	
	B	Γ	B	Γ	B	Γ	B	Γ
$V_{\bar{K}N}^{\rm 1,SIDD}$	52.0	50.4	50.3	49.6	51.2	50.8		
$V_{\bar{K}N}^{\rm 2,SIDD}$	47.0	39.6	46.4	38.2	46.4	39.9		
$V_{\bar{K}N}^{\rm Chiral}$	32.6	39.7	34.5	50.9	30.5	42.8		
AY							108.0	20.0
BGL							29.3	32.9
OHHMH, $V_{\bar{K}N}^{\rm Kyoto-I}$							45.3	25.5
OHHMH, $V_{\bar{K}N}^{\rm Kyoto-II}$							49.7	69.4
ME, $V_{\bar{K}N}^{\rm 1,SIDD}$							73.5	22.0
ME, $V_{\bar{K}N}^{\rm 2,SIDD}$							58.5	27.0
ME, $V_{\bar{K}N}^{\rm IKS\ chiral}$							41.4	31.5

Summary

- summed up Deser formula is much more accurate than the corrected Deser one, and should be used for estimations of $1s$ level shifts of kaonic hydrogen and deuterium
- a quasi-bound state can exist in $K^- np$ system
- four-body binding energy and width of the $\bar{K}NNN$ system strongly depend on $\bar{K}N$ potential and noticeably - on NN potential, especially together with the chirally motivated antikaon-nucleon interaction model
- quasi-bound $K^- ppn$ state: binding energies $B^{\text{Chiral}}_{K^- ppn} \sim 30.5 - 34.5$ MeV obtained with chirally motivated and $B^{\text{SIDD}}_{K^- ppn} \sim 46.4 - 52.0$ MeV obtained with phenomenological antikaon-nucleon potentials are close to those for the $K^- pp$ system, calculated with the same $V_{\bar{K}N}$ and V_{NN} potentials
- quasi-bound $K^- ppn$ state: widths of the four-body states $\Gamma_{K^- ppn} \sim 38.2 - 50.9$ MeV are smaller than the three-body widths of $K^- pp$. Therefore, the neutron added to the $K^- pp$ system slightly influences the binding, but sufficiently reduces the width