

# A high-energy QCD portal to exotic matter

## Heavy-light tetraquarks at the HL-LHC

EXOTICO - ECT\*, Trento, 20<sup>th</sup> October 2022

**Francesco Giovanni Celiberto**

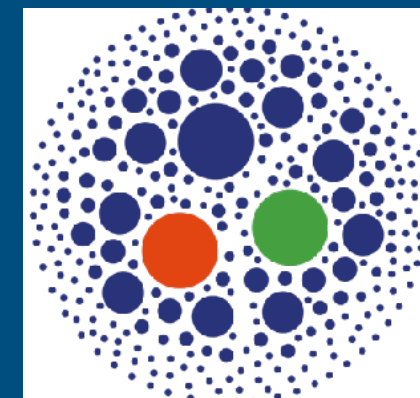
ECT\*/FBK Trento & INFN-TIFPA

**ECT\***

EUROPEAN CENTRE FOR THEORETICAL STUDIES  
IN NUCLEAR PHYSICS AND RELATED AREAS



Trento Institute for  
Fundamental Physics  
and Applications



**HAS QCD**  
HADRONIC STRUCTURE AND  
QUANTUM CHROMODYNAMICS

# High-energy QCD and the proton structure

**High-energy physics**

**Proton structure**



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## High-energy physics

- Precision studies ← SM and beyond
- Fixed-order perturbative calculations...
- ...enhanced by **resummations** ( $p_T$ , energy)
- SM measurements: H, W, Z mass



## Proton structure

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- Inner structure ← intrinsic parton motion
- **Parton densities** → NP nature, global fits
- **Fragmentation functions** → NP nature, dynamics
- Several types: **1D collinear**, 3D TMD

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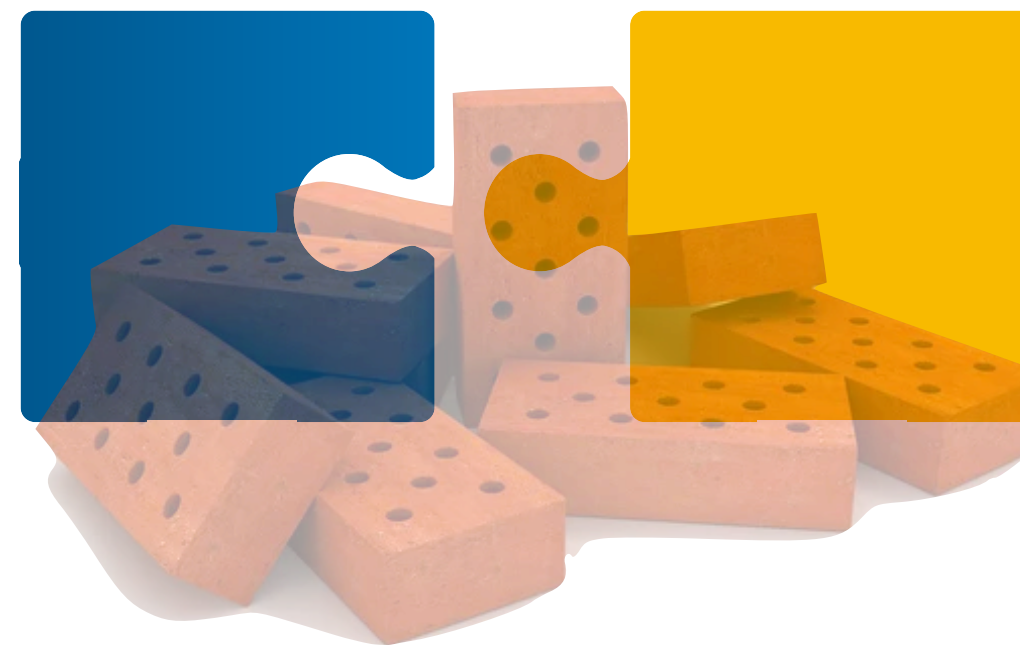
High-energy physics  
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Reduction of uncertainties  
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Perturbative and **nonperturbative** aspects ↔ key ingredients to a joint search for New Physics

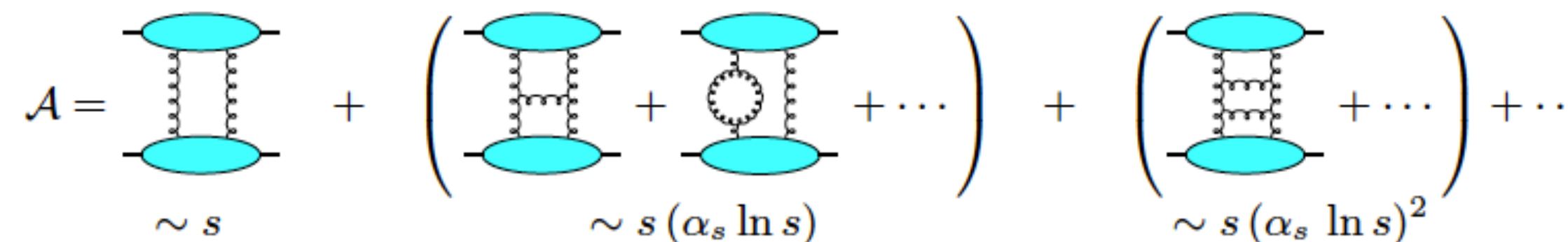
# The high-energy resummation

- **BFKL resummation:** [V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975, 1976, 1977); Y.Y. Balitskii, L.N. Lipatov (1978)]

based on  $\rightarrow$  **gluon Reggeization**

leading logarithmic approximation (LL):

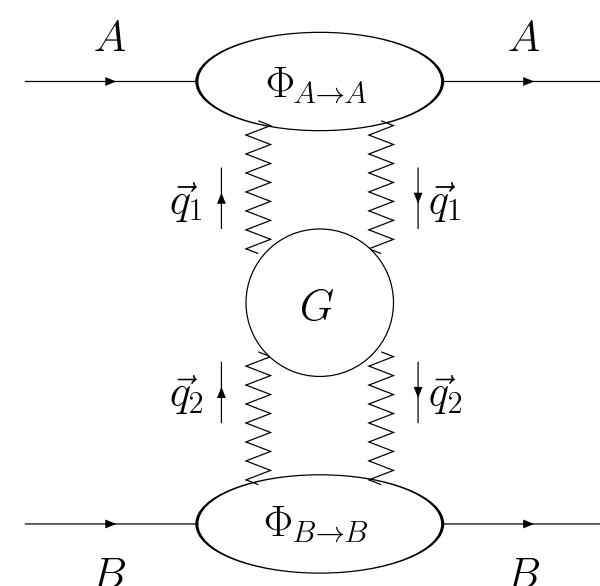
$$\alpha_s^n (\ln s)^n$$



next-to-leading logarithmic approximation (NLL):

$$\alpha_s^{n+1} (\ln s)^n$$

Total cross section for  $A + B \rightarrow X$ :  $\sigma_{AB}(s) = \frac{\text{Im}_s \{ \mathcal{A}_{AB}^{AB} \}}{s} \Leftarrow$  **optical theorem**



►  $\text{Im}_s \{ \mathcal{A}_{AB}^{AB} \}$  factorization:

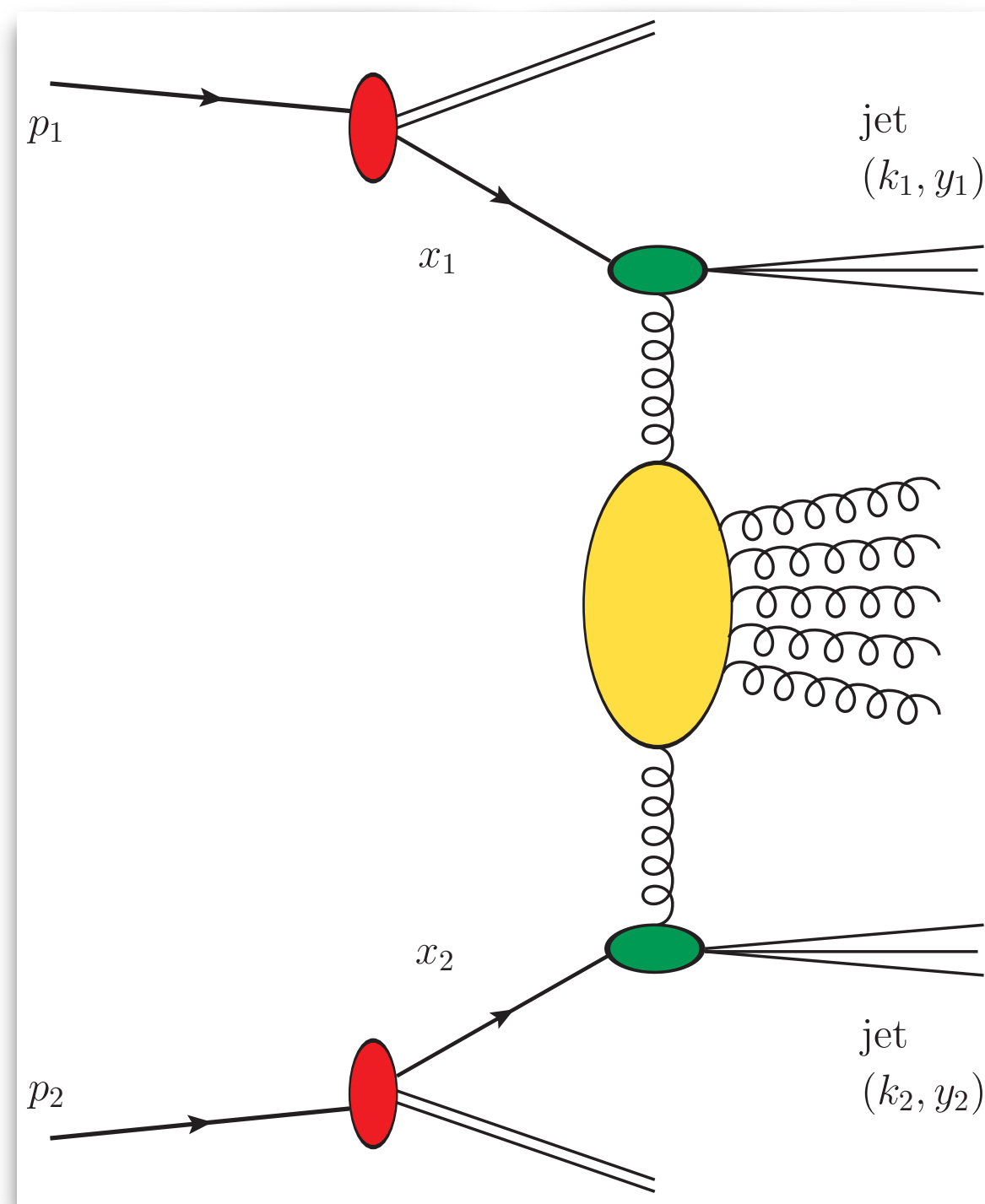
convolution of the **Green's function** of two interacting Reggeized gluons with the **impact factors** of the colliding particles

Green's function is **process-independent**, describes energy dependence and obeys BFKL equation; impact factors are known in the **NLL just for few processes**

# Mueller-Navelet jets at the LHC

- Inclusive hadroproduction of two jets with high  $p_T$  and large rapidity separation,  $\Delta Y$
- Moderate  $x$  (collinear PDFs), but t-channel  $p_T$  (BFKL resummation)  $\rightarrow$  hybrid factorization

$$\frac{d\sigma}{dy_1 dy_2 d^2\vec{k}_1 d^2\vec{k}_2} = \sum_{r,s=q,g} \int_0^1 dx_1 \int_0^1 dx_2 f_r(x_1, \mu_F) f_s(x_2, \mu_F) \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu_F)}{dy_1 dy_2 d^2\vec{k}_1 d^2\vec{k}_2}$$



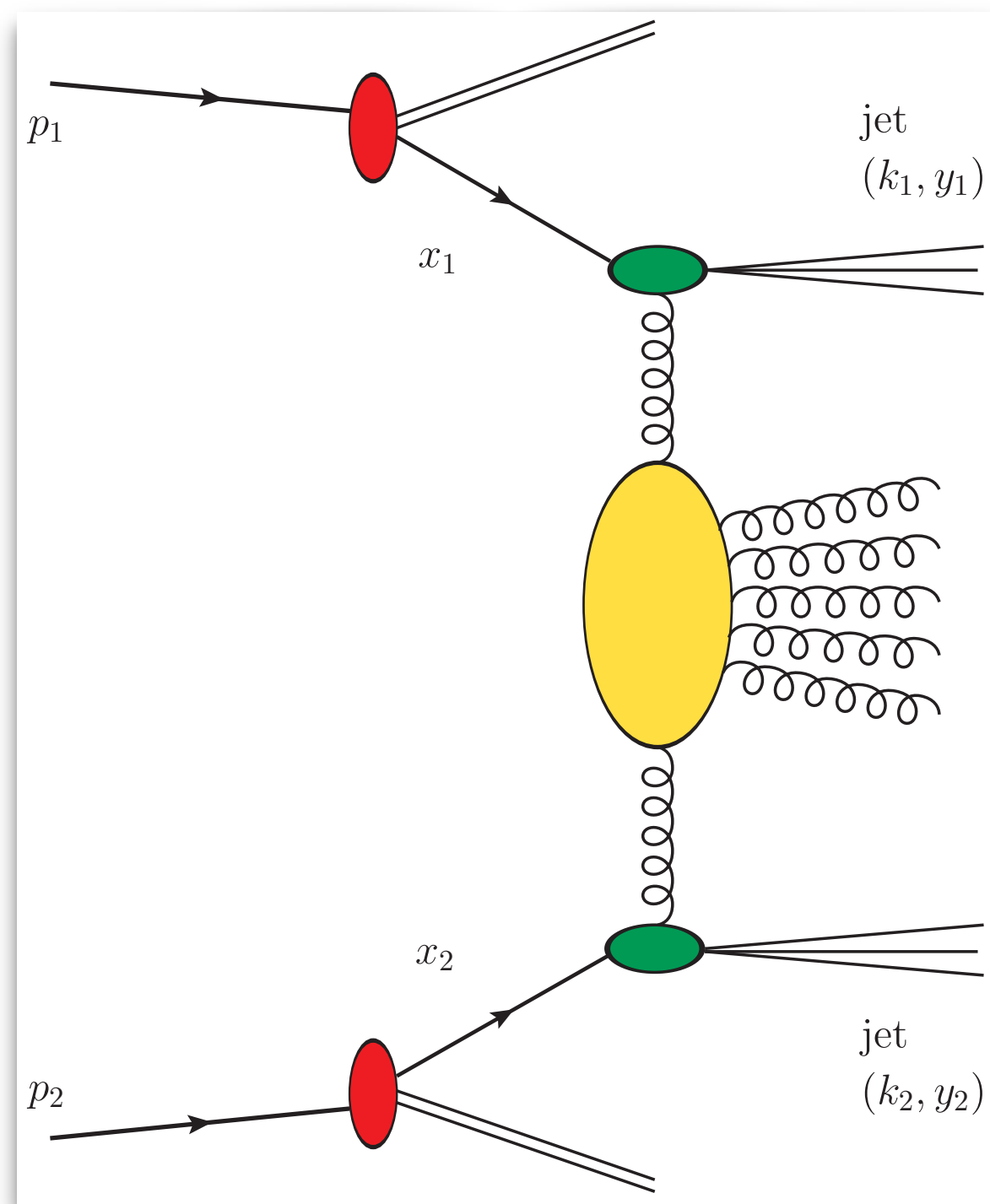


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jet vertices  
(off-shell amplitudes)



NLO

NLL

NLO

$$\frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu)}{dy_1 dy_2 d^2\vec{k}_1 d^2\vec{k}_2} = \frac{1}{(2\pi)^2}$$

$$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^2} \mathcal{V}_J^{(r)}(\vec{q}_1, s_0, x_1, \vec{k}_1)$$

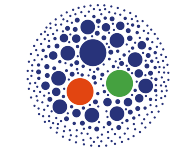
$$\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0}\right)^\omega \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2)$$

$$\times \int \frac{d^2\vec{q}_2}{\vec{q}_2^2} \mathcal{V}_J^{(s)}(\vec{q}_2, s_0, x_2, \vec{k}_2)$$

BFKL Green's function

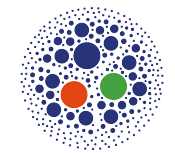
Hybrid NLL/NLO factorization via the **JETHAD** Method

# From Mueller-Navelet to Higgs and heavy flavor



Pheno path: hunt for channels leading to a NLL **stabilization pattern** at **natural scales** (¡!)

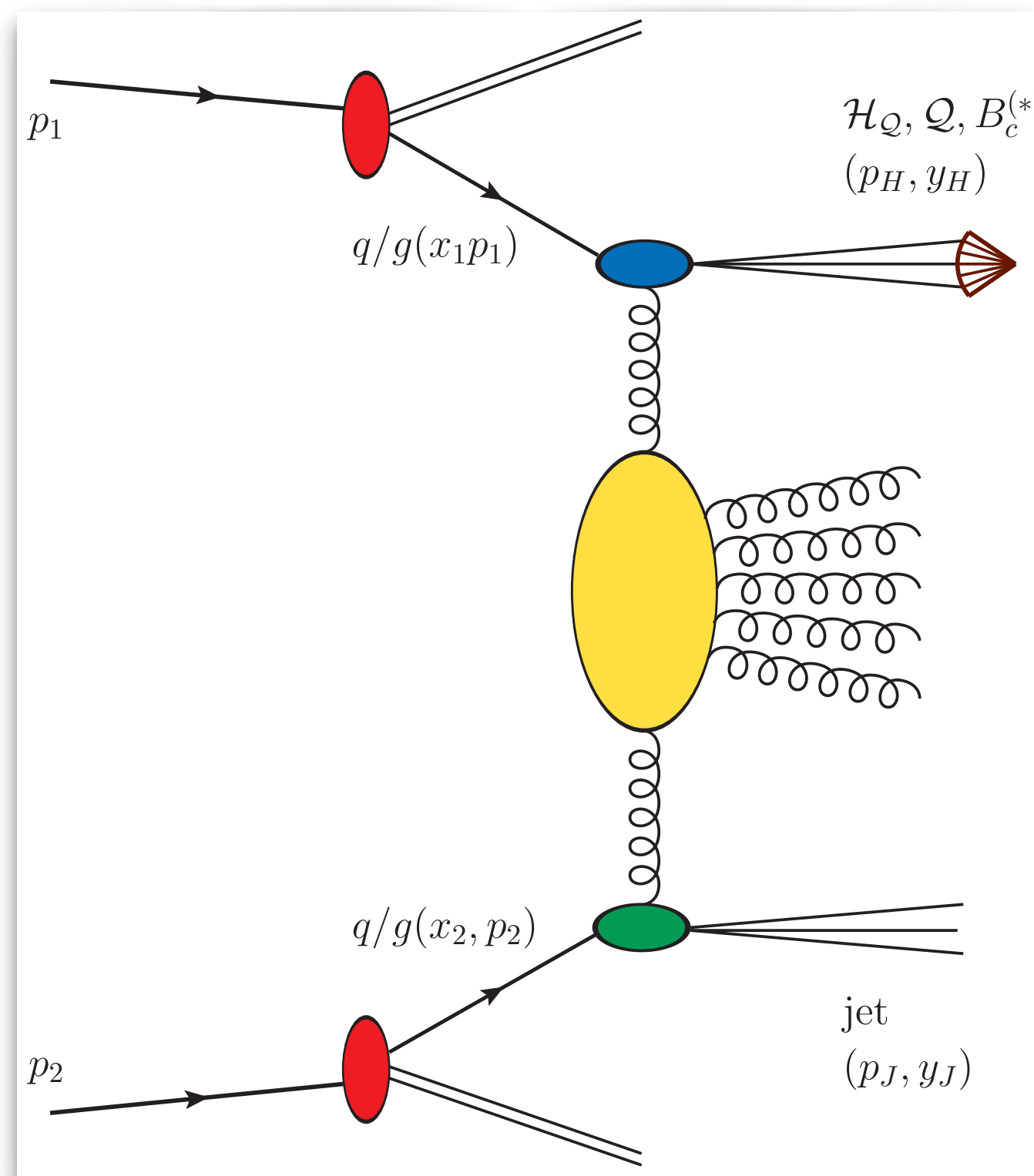
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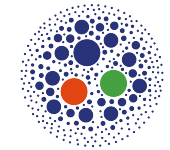
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Heavy flavor at large  $p_T$

Stabilizers  $\Leftrightarrow$  gluon fragmentation channels



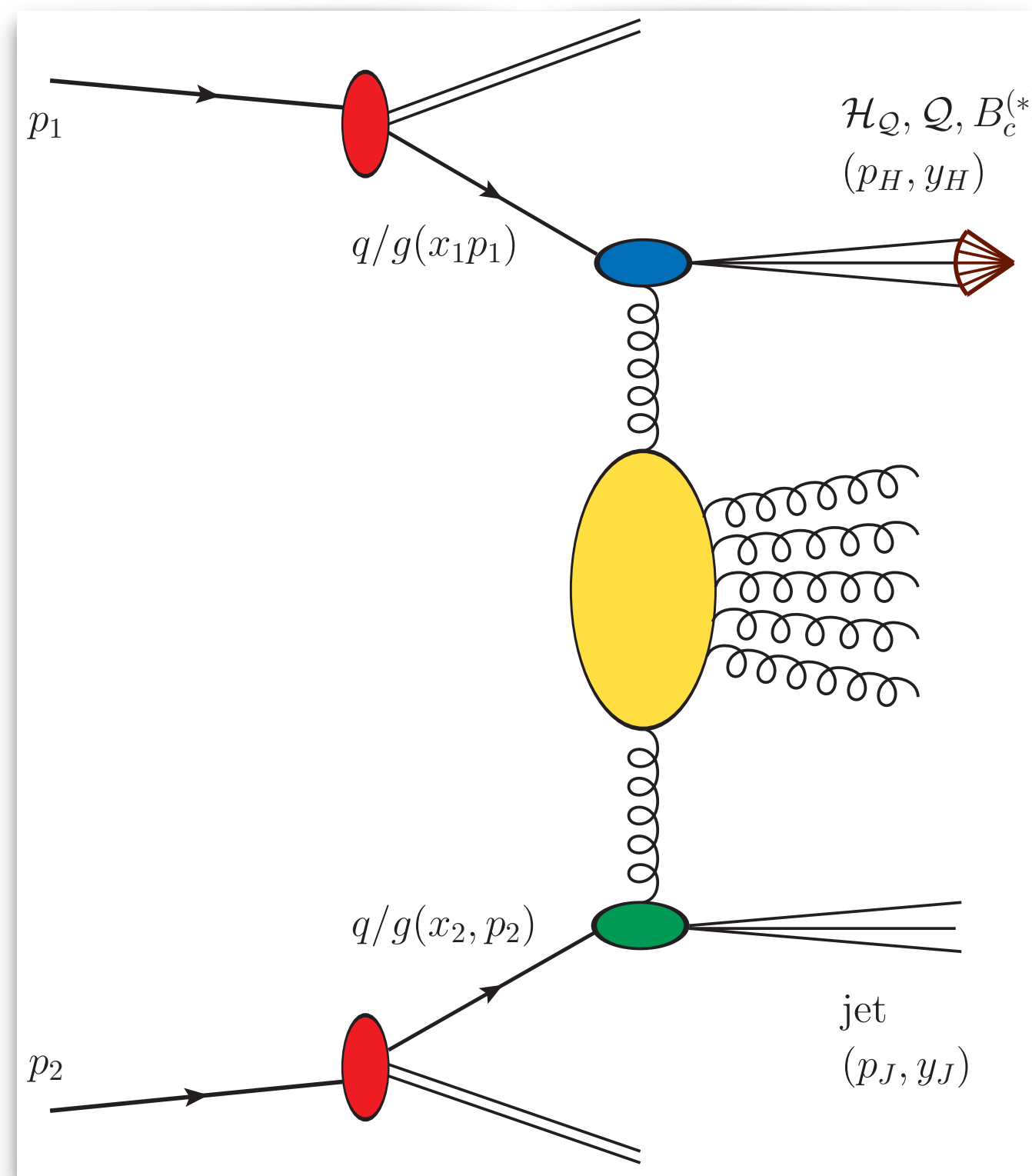
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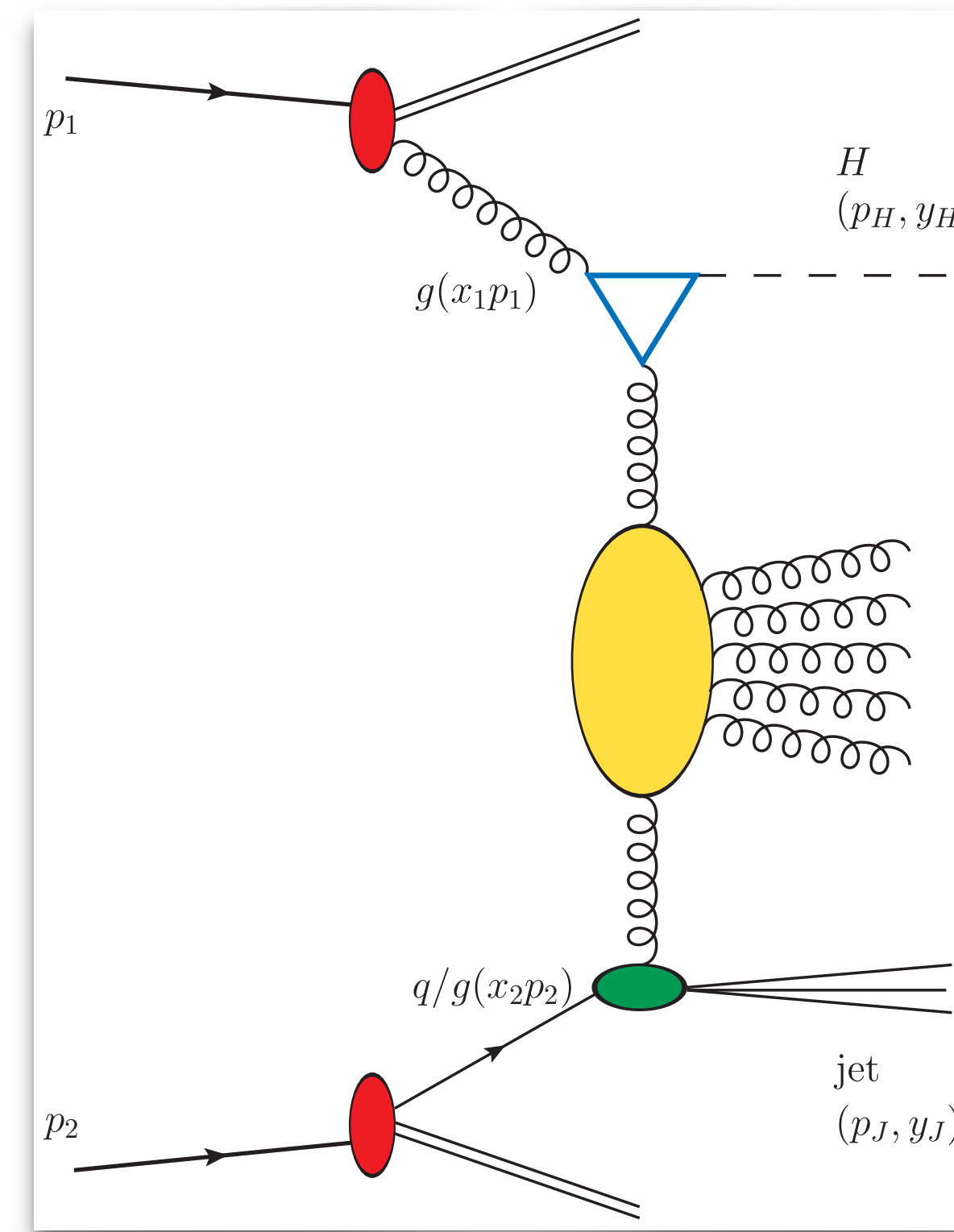
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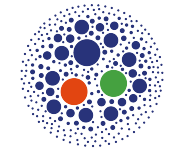


Higgs boson

Stabilizers  $\Leftrightarrow$  large Higgs transverse masses



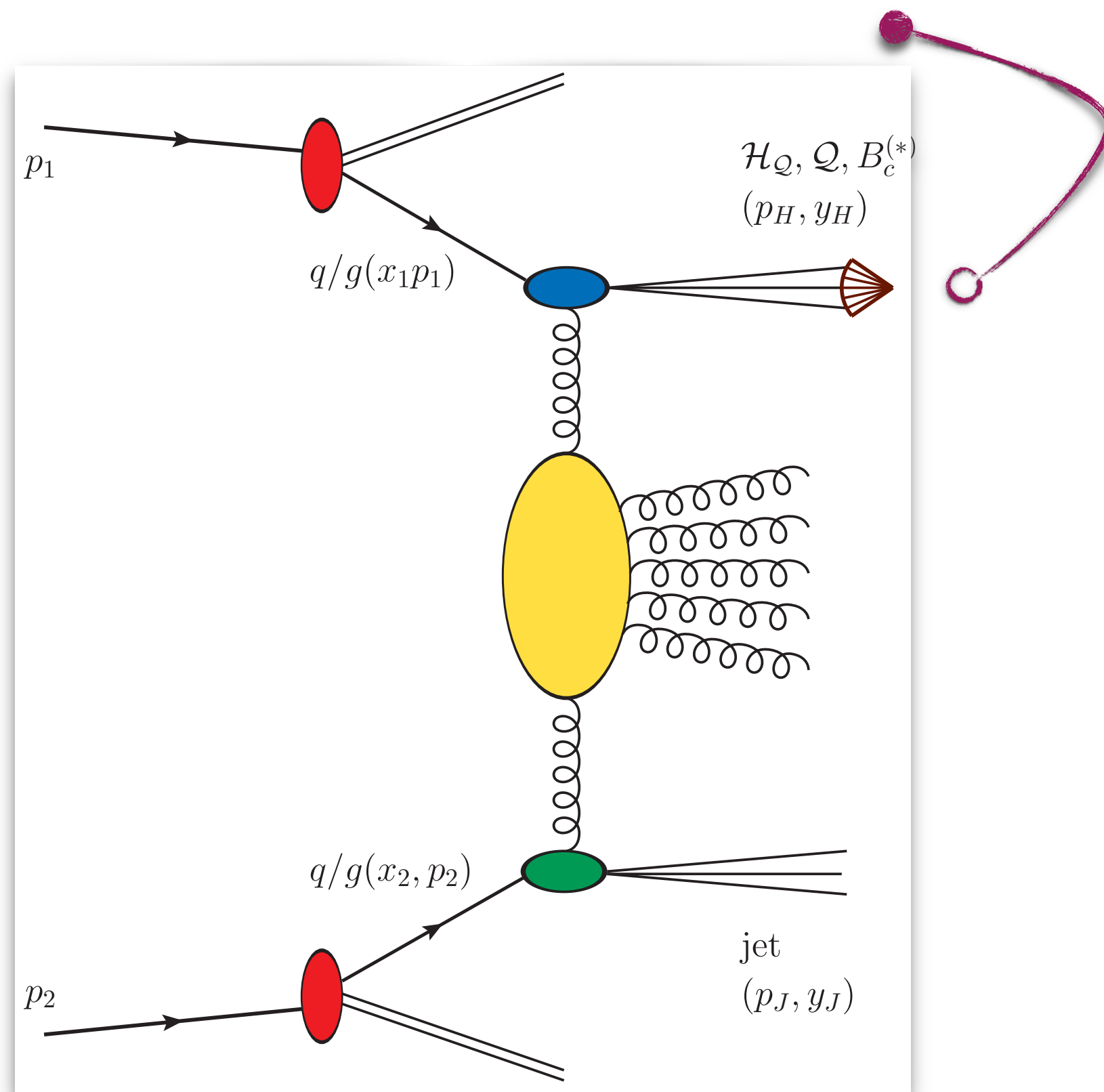
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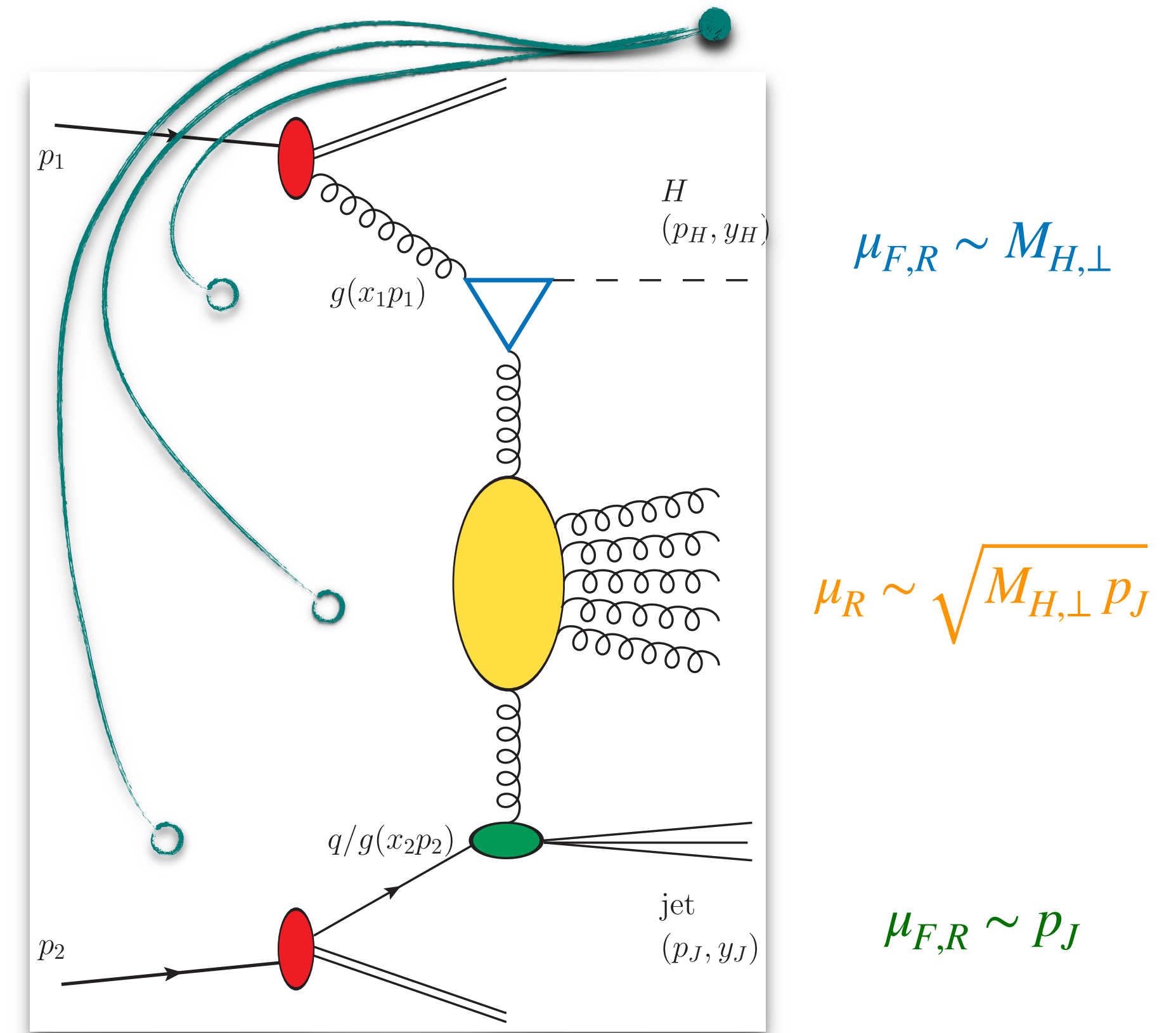
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( $\Lambda_c^\pm$  baryons, NLL/NLO) [\[F. G. C. et al., Phys. Rev. D 104 \(2021\) 11, 114007\]](#)

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$$\mu_{F,R} \sim M_{H,\perp}$$

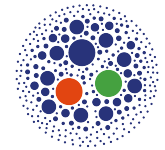
$$\mu_R \sim \sqrt{M_{H,\perp} p_J}$$

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(Higgs + jet, NLL/NLO\*) [\[F. G. C. et al., Eur. Phys. J. C \(2021\) 8, 780\]](#)

(NLO Higgs impact factor) [\[F. G. C. et al., JHEP 08 \(2022\) 092\]](#)

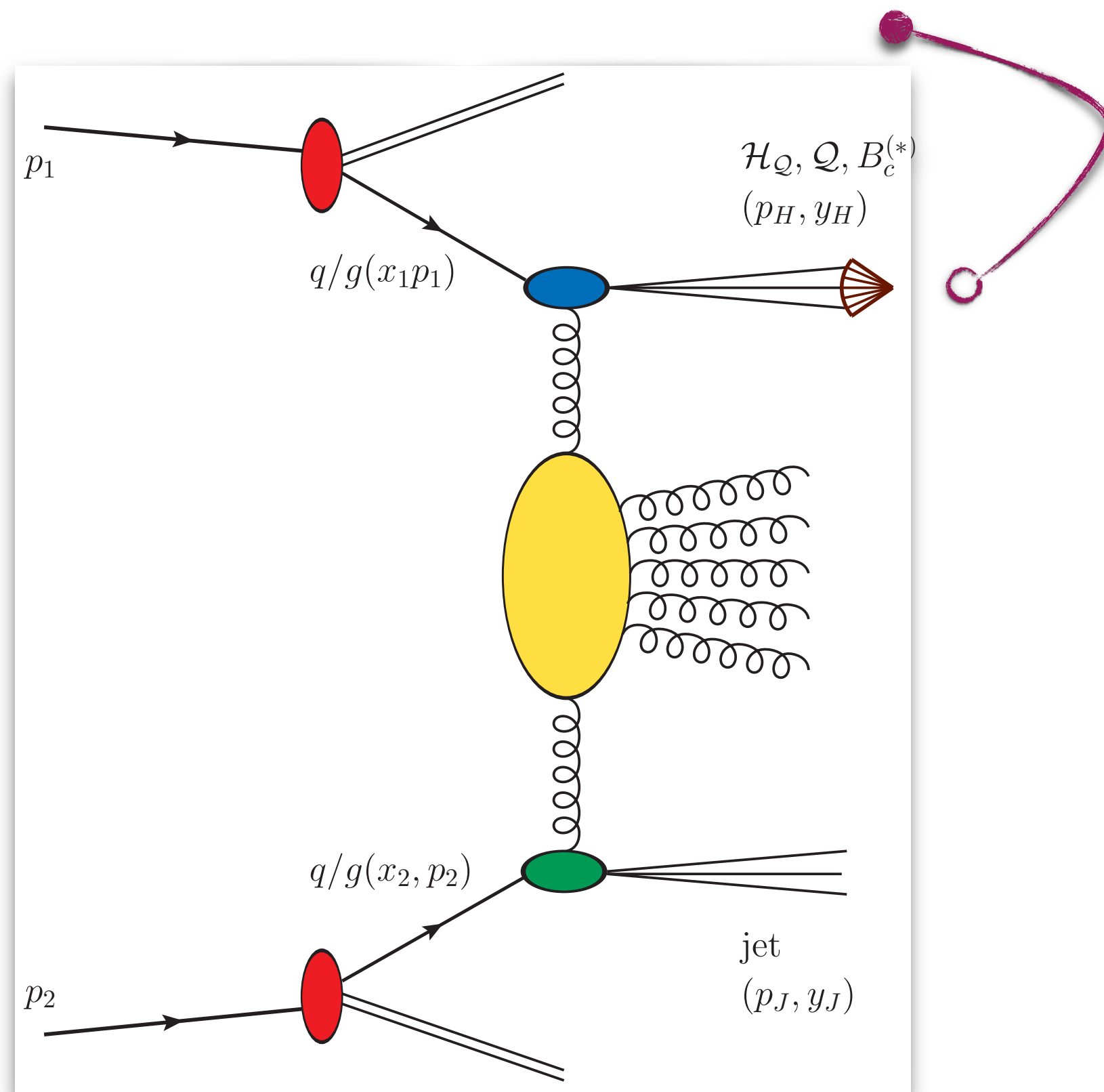
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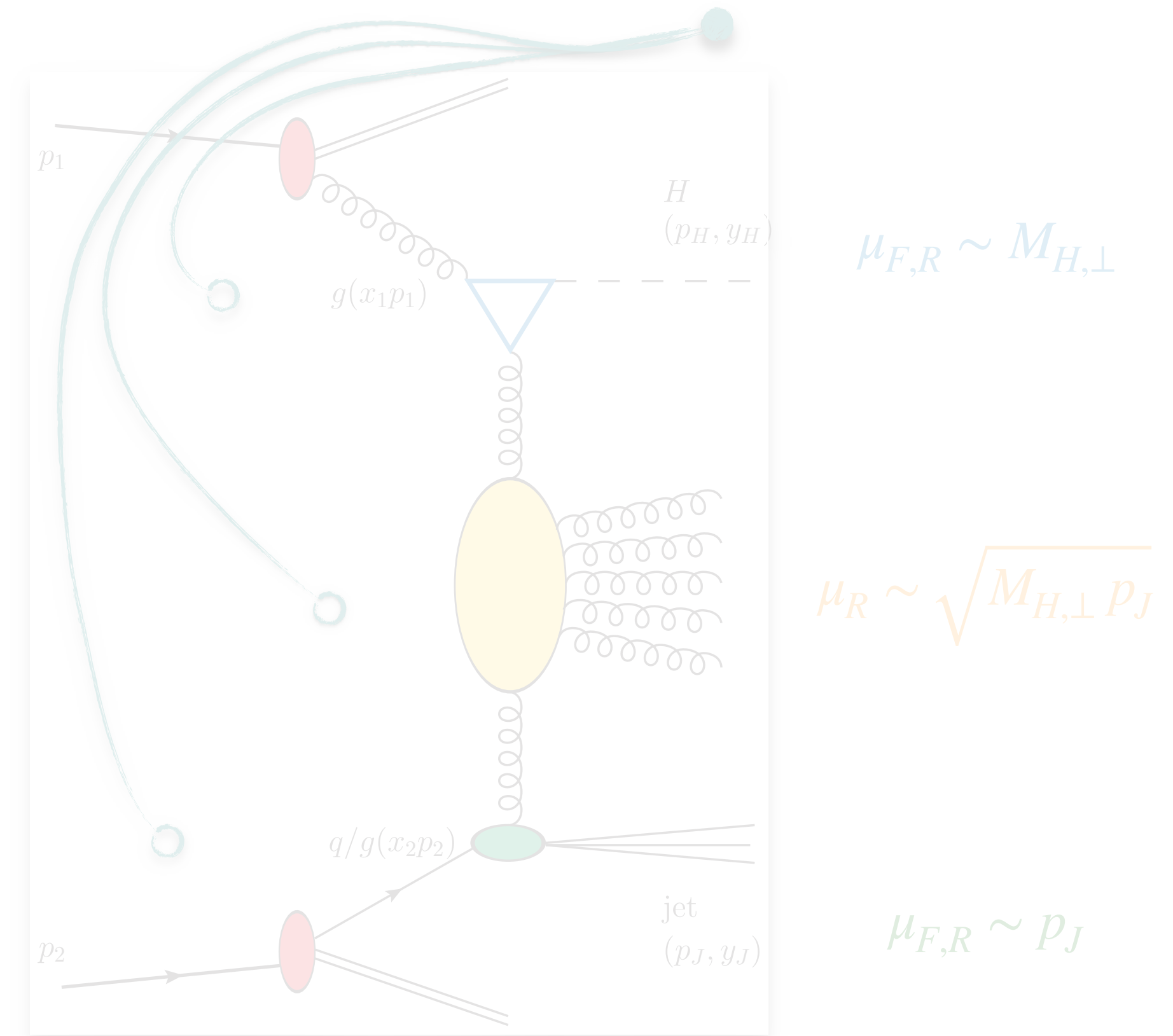
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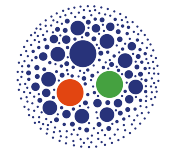
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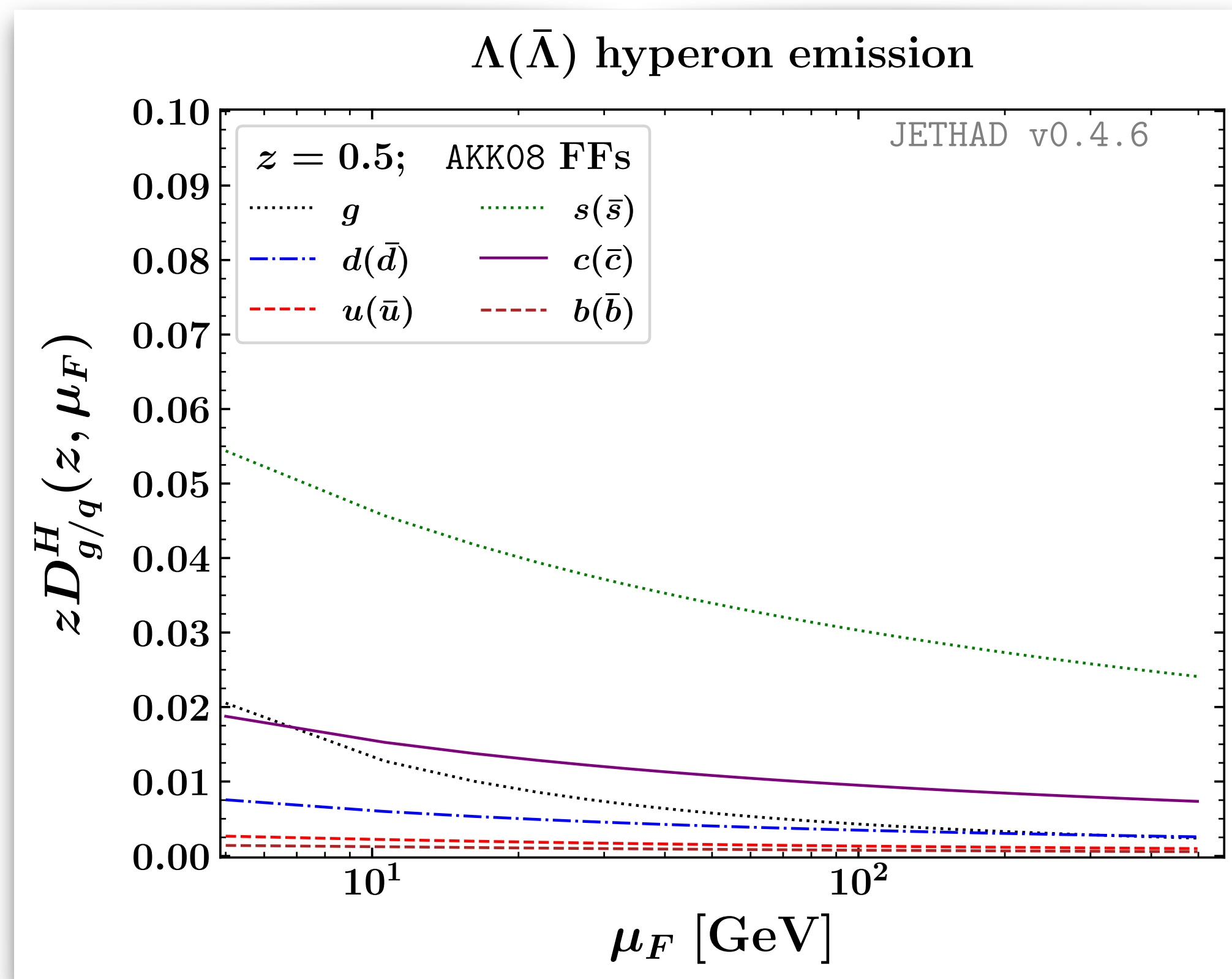
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# Stabilizing effects of heavy-flavor fragmentation



**AKK08** VFNS collinear FFs for  $\Lambda$  hyperons:  $|uds\rangle$

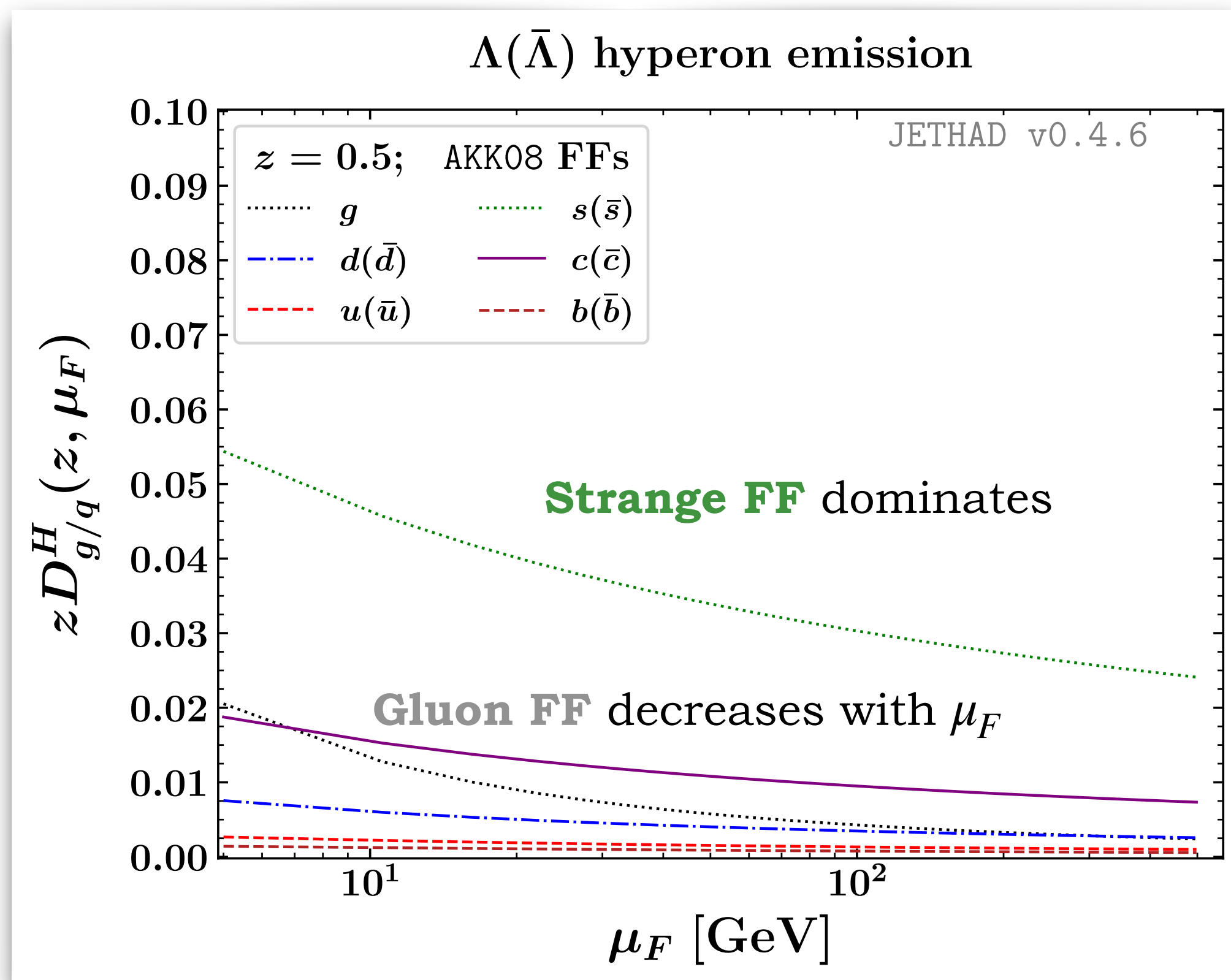
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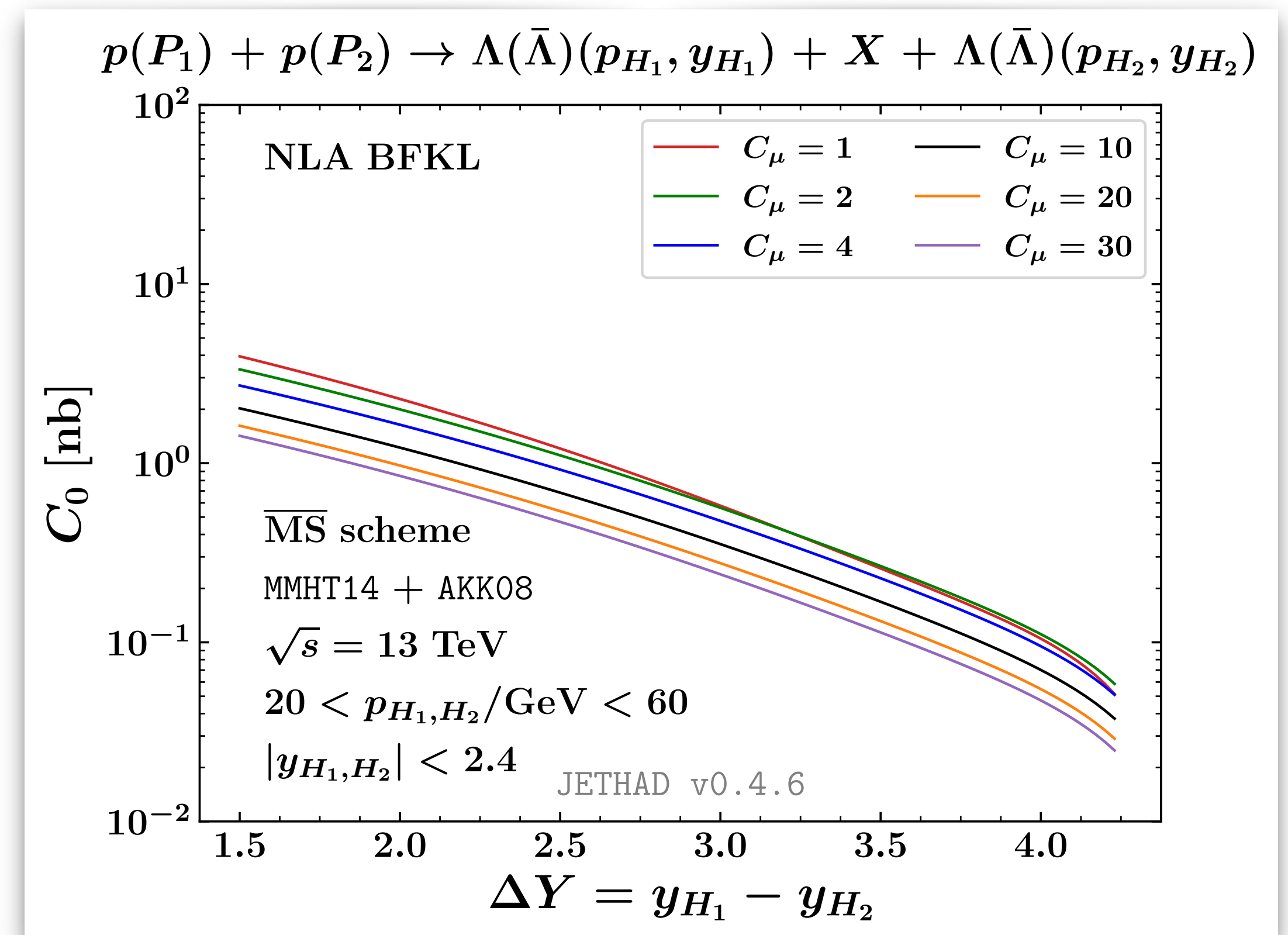
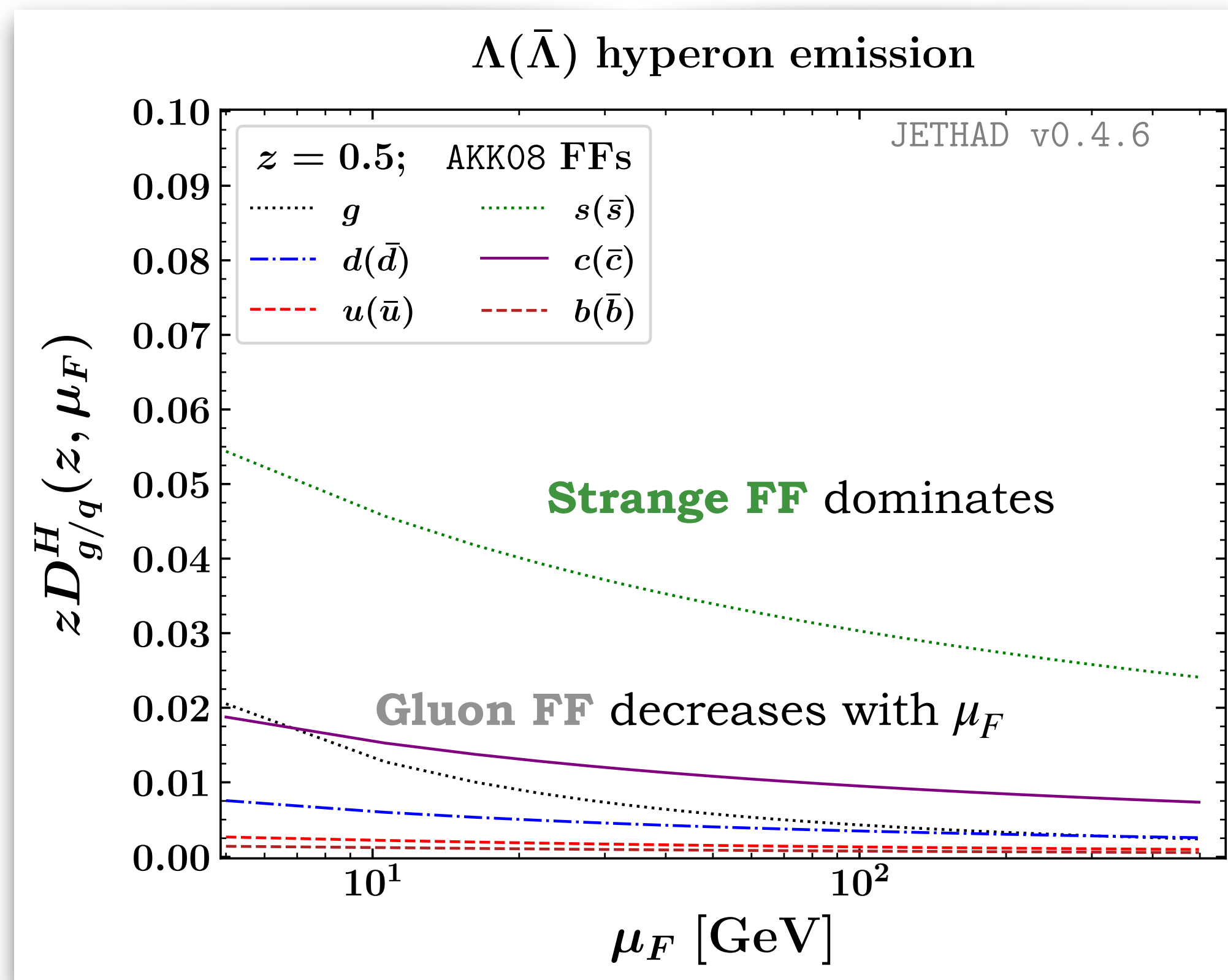




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 Rapidity distribution **sensitive** to scale variations

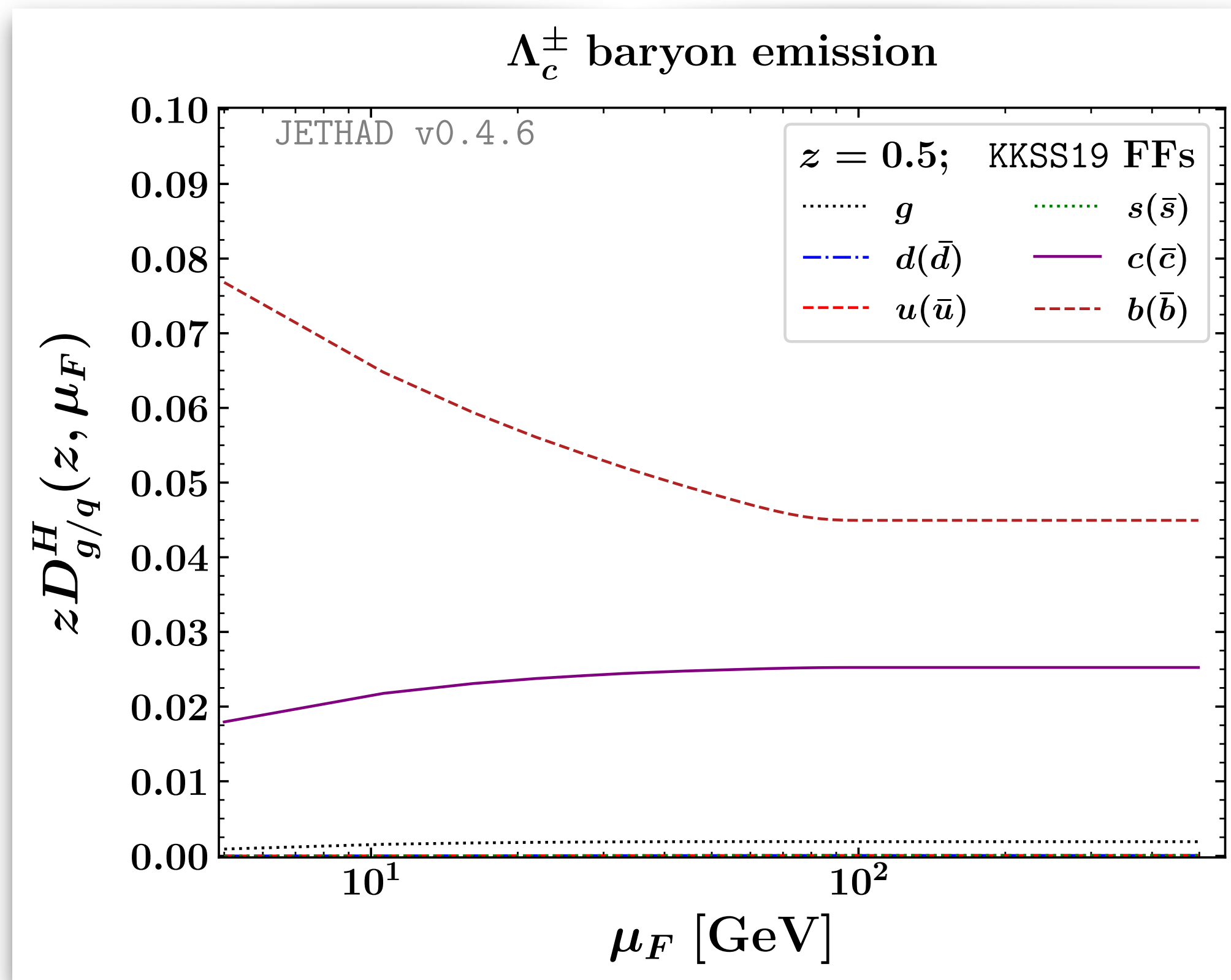
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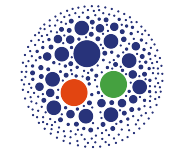


**KKSS19** VFNS collinear FFs for  $\Lambda_c$  baryons:  $|udc\rangle$

[\[B. A. Kniehl et al., Phys. Rev. D 101 \(2020\) 11, 114021\]](#)

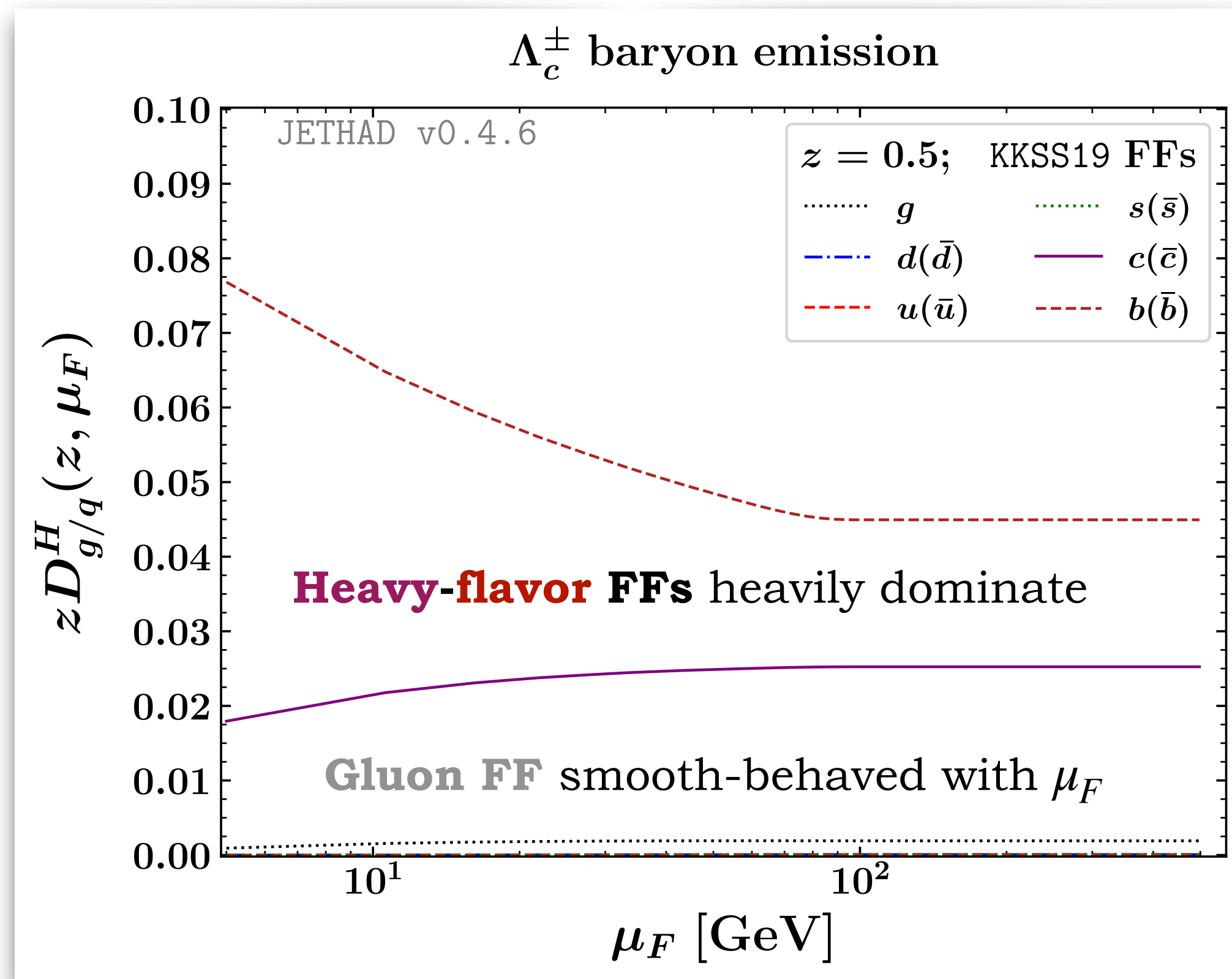


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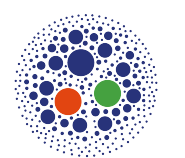
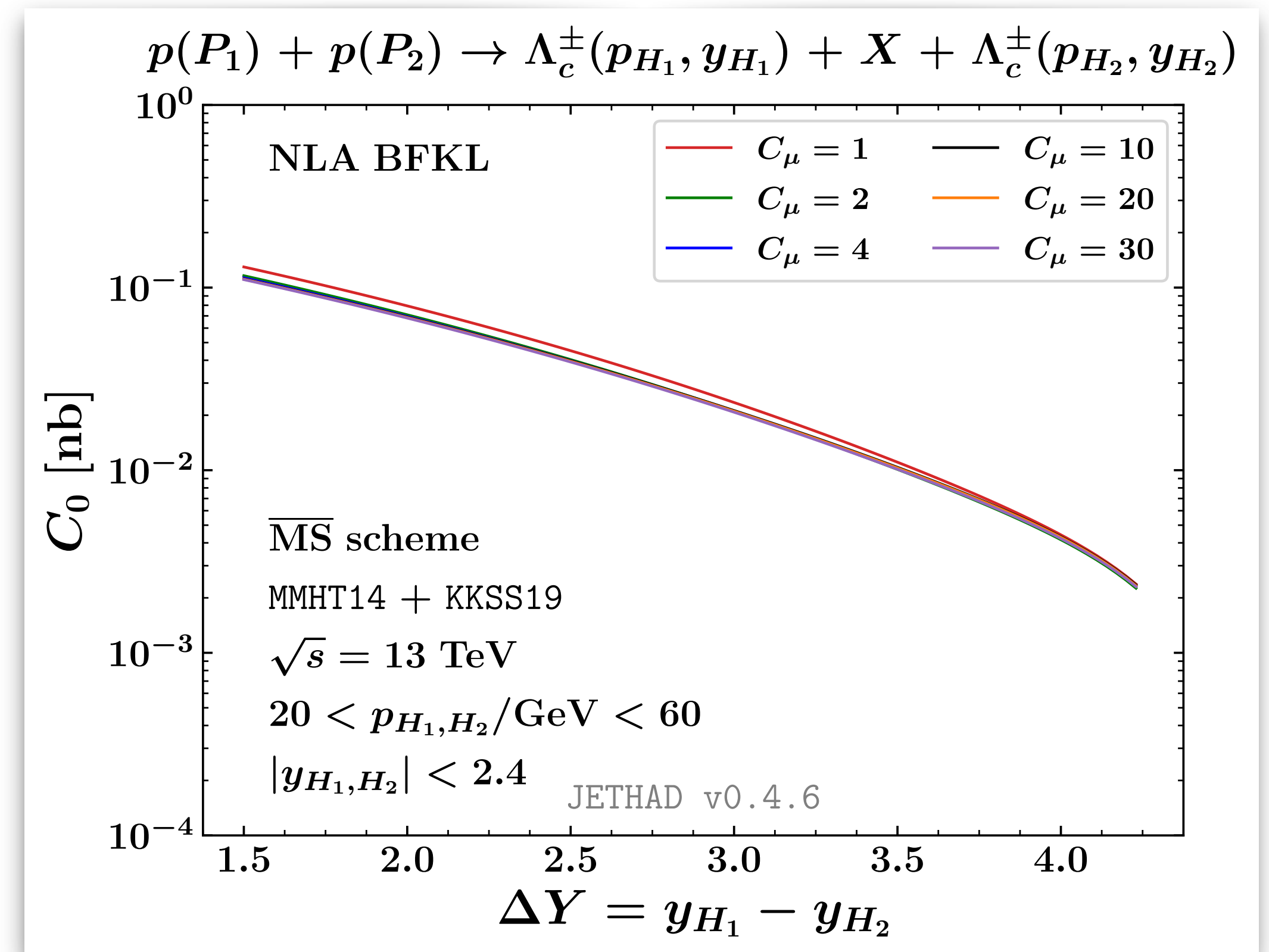
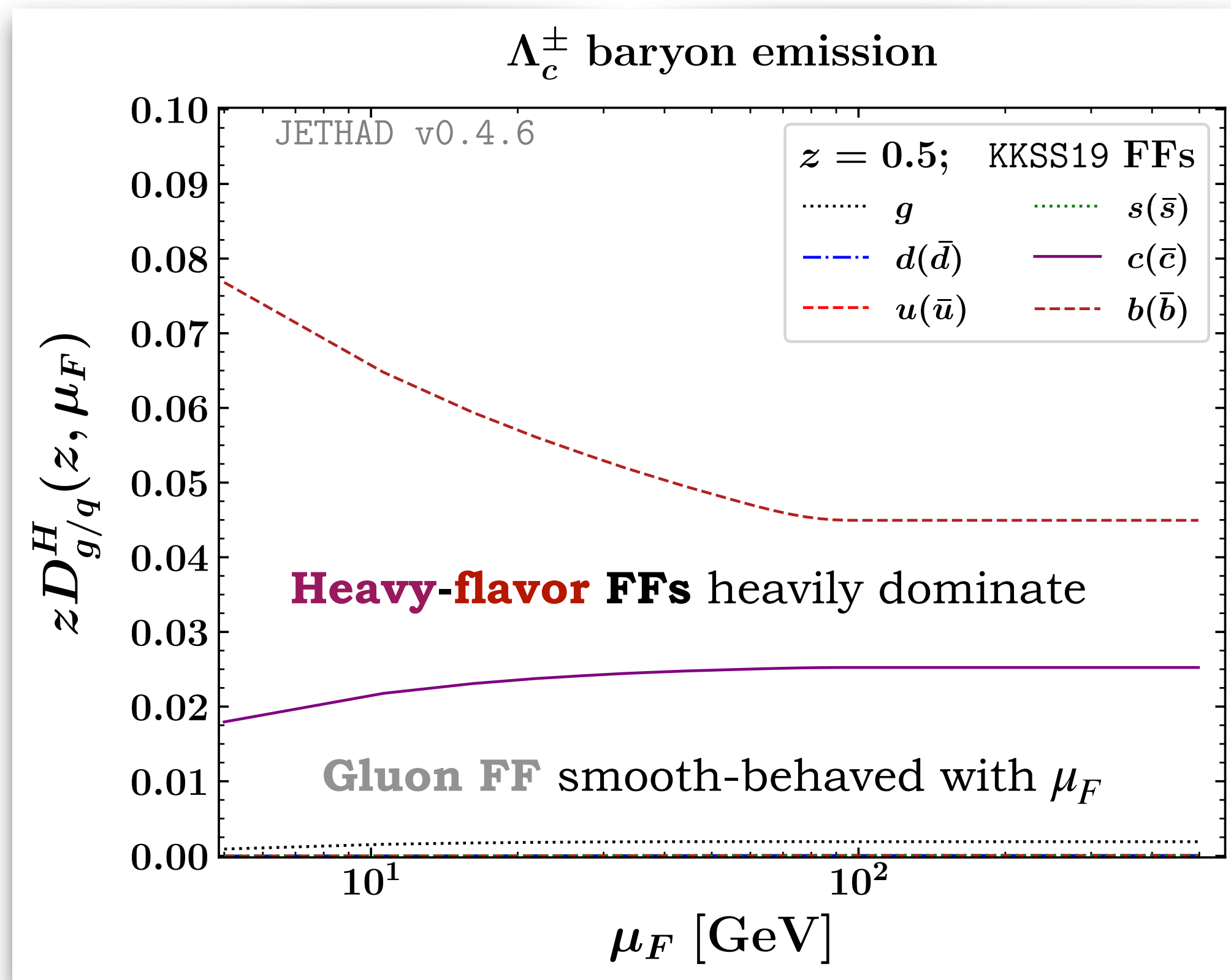


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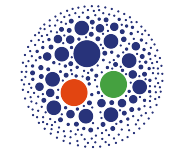


Rapidity distribution **stable** under scale variations

( $\Lambda_c$  baryons, in this slide) [F. G. C. et al., Eur. Phys. J. C 81 (2021) 8, 780]

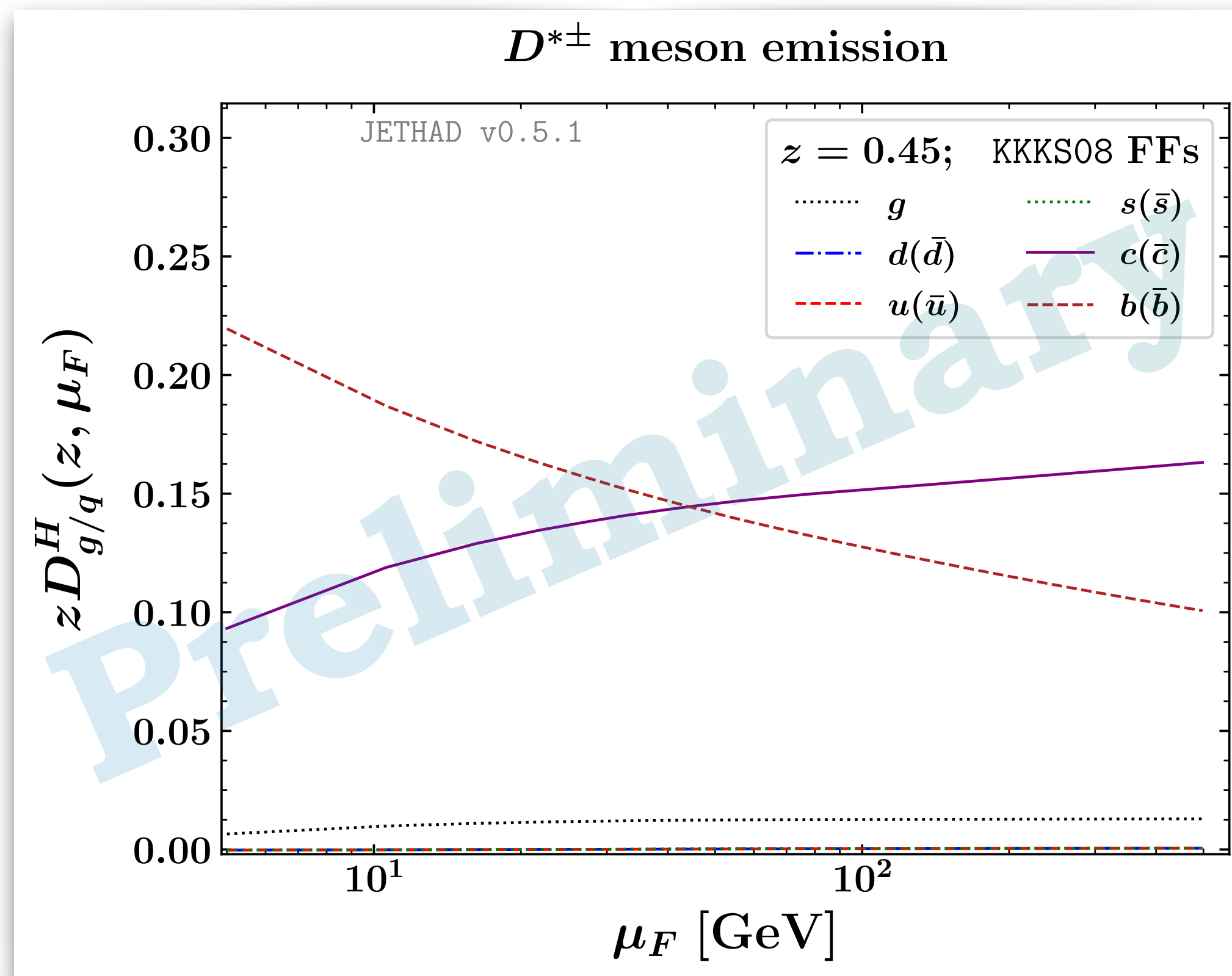
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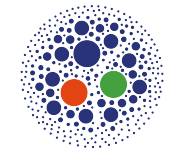


KKKS08 VFNS collinear FFs for  $D^{*\pm}$  mesons:  $|c\bar{d}\rangle$

[T. Kneesch et al., Nucl. Phys. B 799 (2008) 34-59]

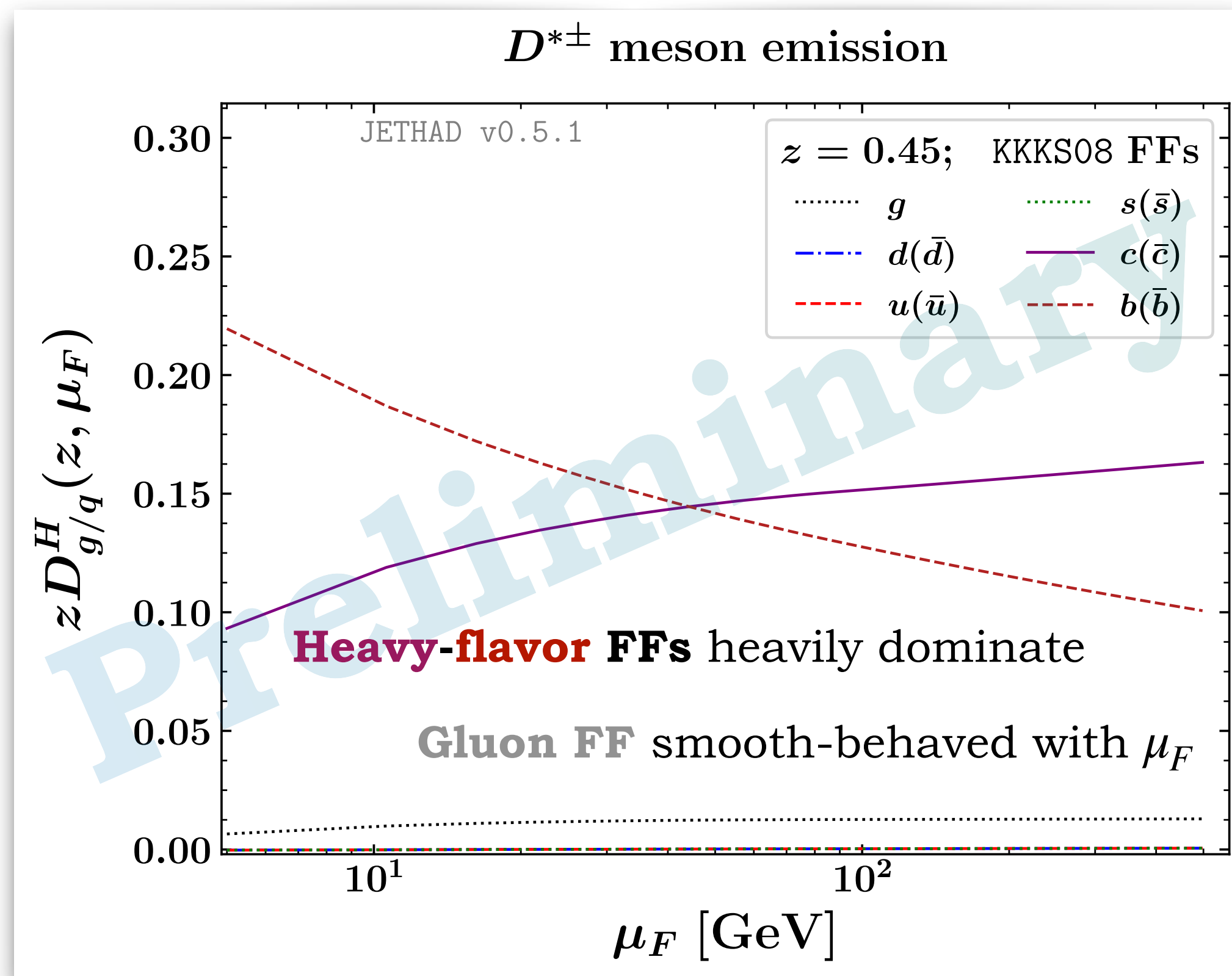


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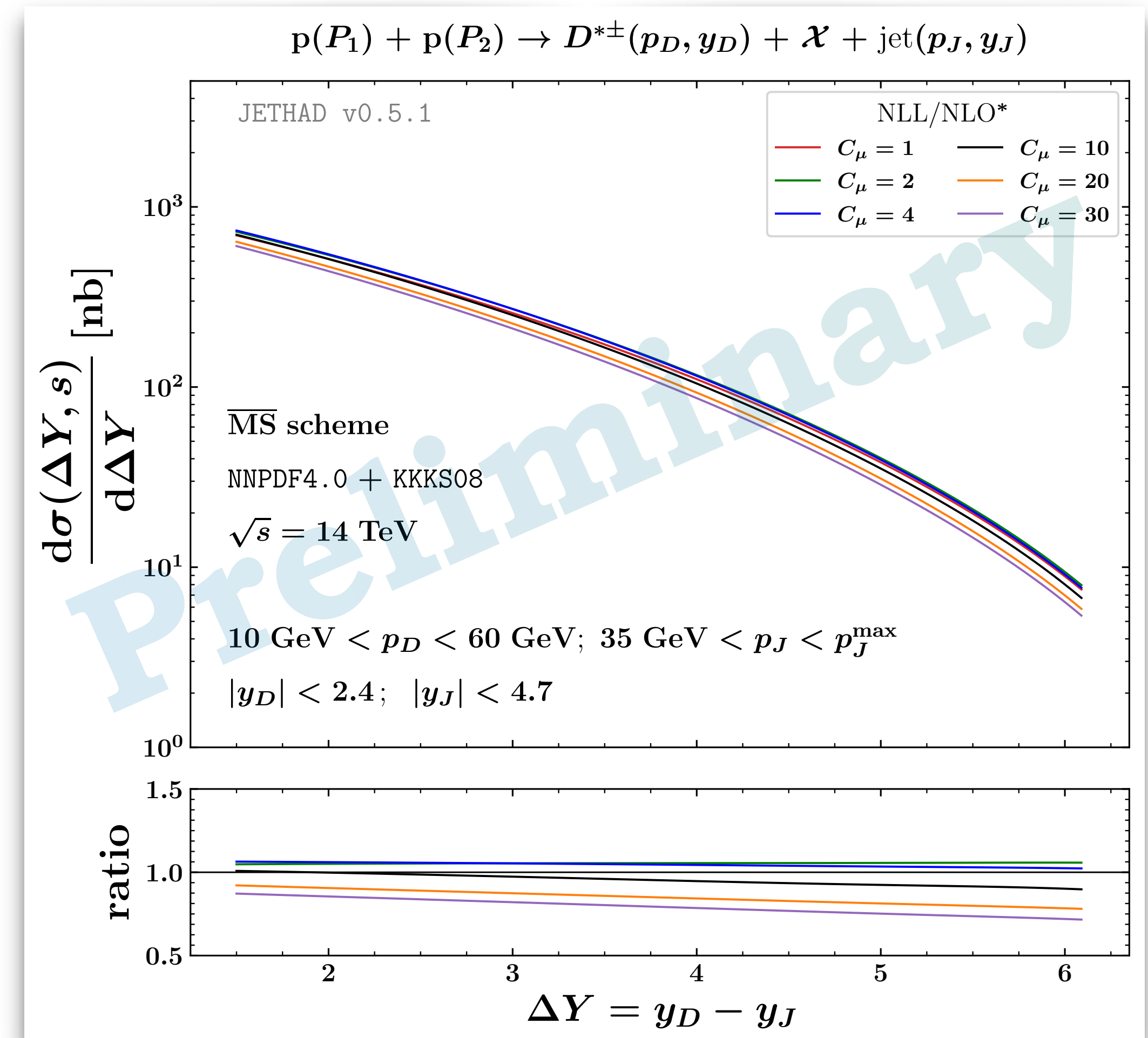
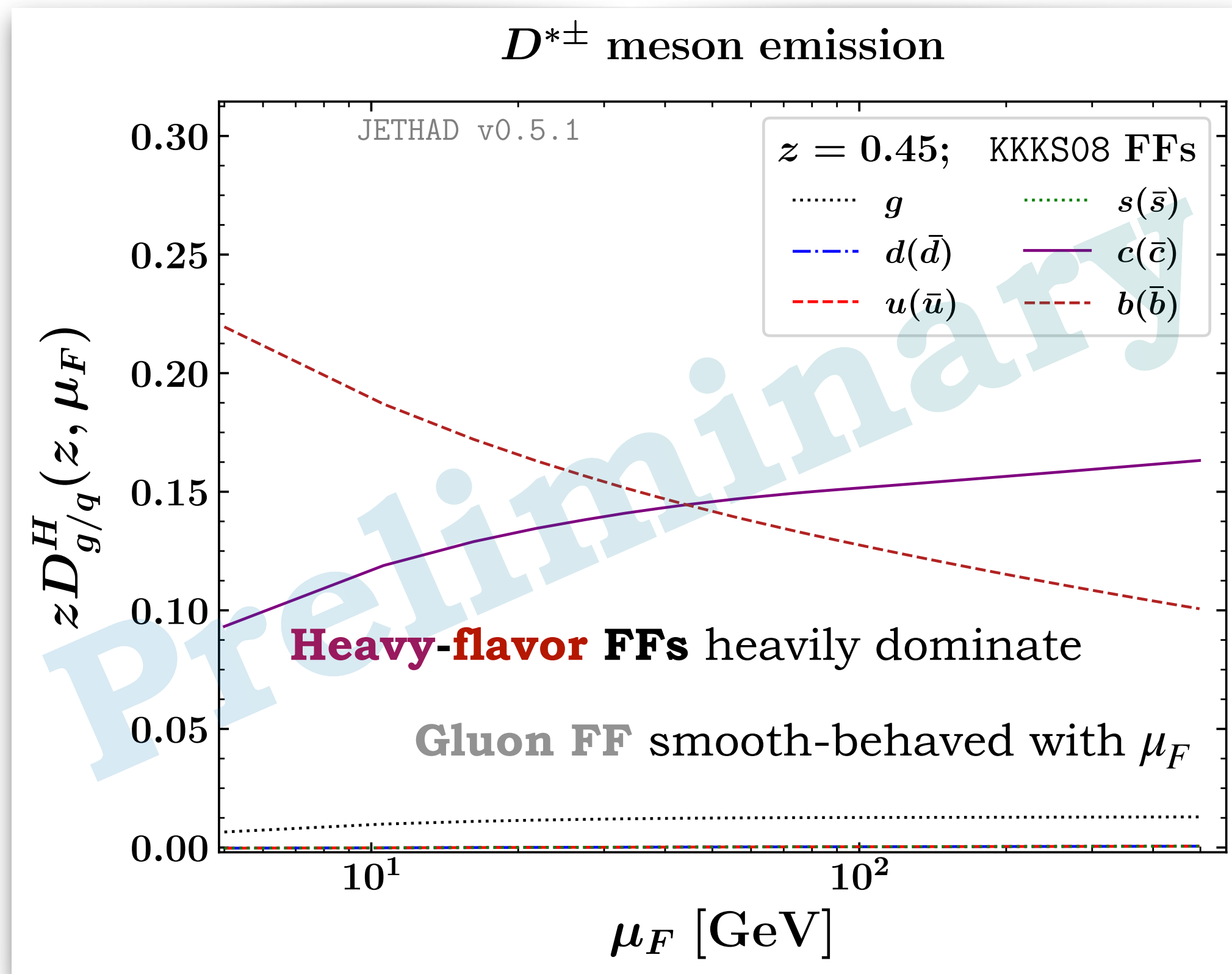
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# Stabilizing effects of heavy-flavor fragmentation

 Stabilization mechanism encoded in the heavy-flavor **gluon FF**

 Forward-hadron LO impact factor  $\Rightarrow$  **gluon FF** enhanced by **gluon PDF** in collinear convolution

$$c_{\Lambda}(n, \nu, |\vec{p}|, x) = 2\sqrt{\frac{C_F}{C_A}} (|\vec{p}|^2)^{i\nu-1/2} \int_x^1 \frac{dz}{z} \left(\frac{z}{x}\right)^{2i\nu-1} \left[ \frac{C_A}{C_F} f_g(z) D_g^{\Lambda}\left(\frac{x}{z}\right) + \sum_{a=q,\bar{q}} f_a(z) D_a^{\Lambda}\left(\frac{x}{z}\right) \right]$$



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Forward-hadron NLO impact factor  $\Rightarrow$  a **non-diagonal heavy-flavor** channel open...

$$c_1^{(1)}(n, \nu, |\vec{k}_1|, \alpha_1) = 2\sqrt{\frac{C_F}{C_A}} (\vec{k}_1^2)^{i\nu-\frac{1}{2}} \frac{1}{2\pi} \int_{\alpha_1}^1 \frac{dx}{x} \int_{\frac{\alpha_1}{x}}^1 \frac{d\zeta}{\zeta} \left(\frac{x\zeta}{\alpha_1}\right)^{2i\nu-1}$$

$$\times \left[ \frac{C_A}{C_F} f_g(x) D_g^h \left(\frac{\alpha_1}{x\zeta}\right) C_{gg}(x, \zeta) + \sum_{a=q,\bar{q}} f_a(x) D_a^h \left(\frac{\alpha_1}{x\zeta}\right) C_{qq}(x, \zeta) \right]$$

$$+ \left[ D_g^h \left(\frac{\alpha_1}{x\zeta}\right) \sum_{a=q,\bar{q}} f_a(x) C_{qg}(x, \zeta) + \frac{C_A}{C_F} f_g(x) \sum_{a=q,\bar{q}} D_a^h \left(\frac{\alpha_1}{x\zeta}\right) C_{gq}(x, \zeta) \right]$$

...but  $|C_{gg}| \sim 50 \div 10^4 |C_{gq}|$

# Stabilizing effects of heavy-flavor fragmentation

Stabilization mechanism encoded in the heavy-flavor gluon FF

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$$+ \left[ D_g^h\left(\frac{\alpha_1}{x\zeta}\right) \sum_{a=q,\bar{q}} f_a(x) C_{qg}(x, \zeta) + \frac{C_A}{C_F} f_g(x) \sum_{a=q,\bar{q}} D_a^h\left(\frac{\alpha_1}{x\zeta}\right) C_{gq}(x, \zeta) \right]$$

Gluon FF rises with energy  $\Rightarrow$  this compensates PDF and BFKL kernel decreasing behavior

# Is the natural stabilization robust?

(1) **KKKS08** and **KKSS19** VFNS collinear FFs share the same extraction technology

⚠ *Might natural stability be related to the given FF determination(s) ?*

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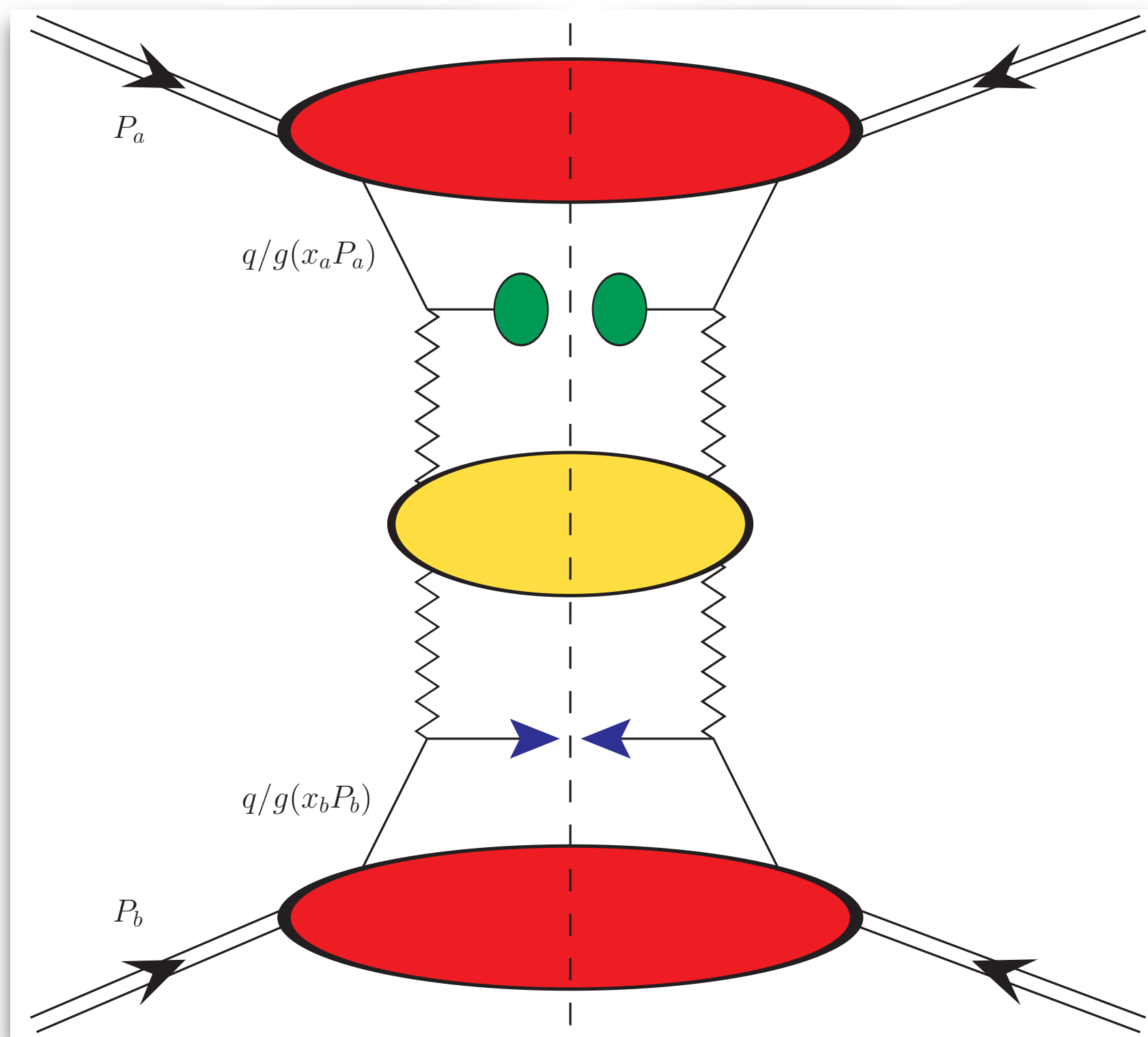
⚠ *Might natural stability be related to the given FF determination(s) ?*

(2) **KKKS08** and **KKSS19** VFNS collinear FFs assume no initial-scale gluon, but evolution-driven

⚠ *Might natural stability be artificially generated by this Ansatz ?*

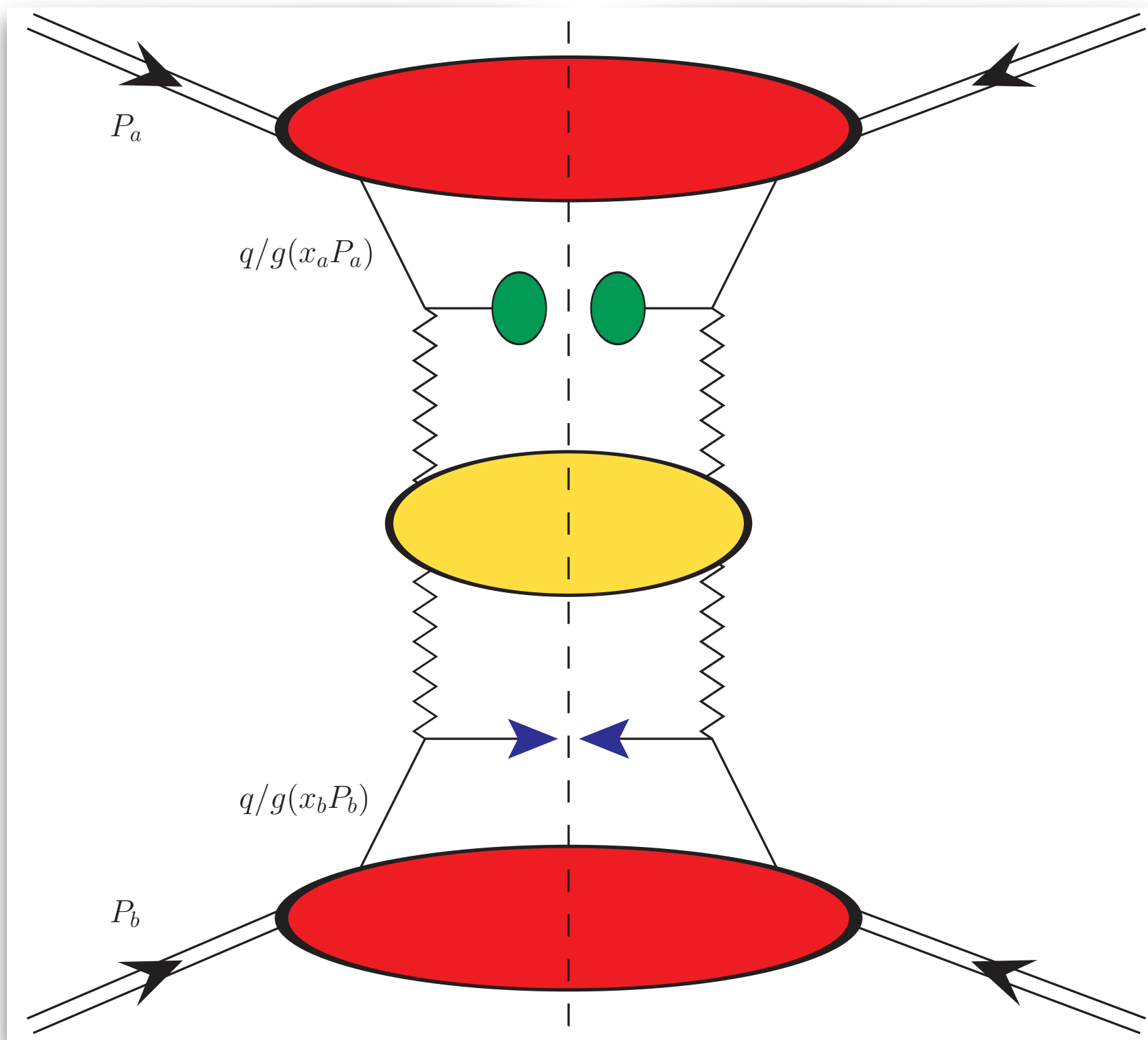
# Vector quarkonium from single-parton fragmentation

- (1) *Let us consider  $J/\psi$  and  $\Upsilon$  at large  $p_T \rightarrow$  single-parton fragmentation from **NRQCD**!*



# Vector quarkonium from single-parton fragmentation

(1) **!** Let us consider  $J/\psi$  and  $\Upsilon$  at large  $p_T \rightarrow$  single-parton fragmentation from **NRQCD**!



**ZCW19** onium FFs:  $(Q \rightarrow \mathcal{Q} Q) \otimes$  **APFEL++**  
 $[\mu_0 = 3m_Q]$

Eur. Phys. J. C (2022) 82:929  
<https://doi.org/10.1140/epjc/s10052-022-10818-8>

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

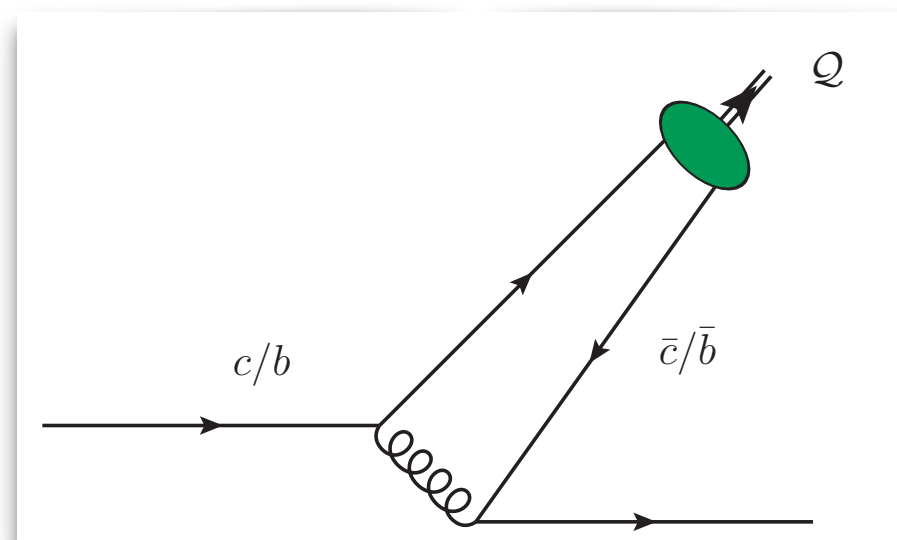
### Diffractive semi-hard production of a $J/\psi$ or a $\Upsilon$ from single-parton fragmentation plus a jet in hybrid factorization

Francesco Giovanni Celiberto<sup>1,2,3,a</sup> , Michael Fucilla<sup>4,5,6,b</sup>

<sup>1</sup> European Centre for Theoretical Studies in Nuclear Physics and Related Areas (ECT\*), 38123 Villazzano, Trento, Italy  
<sup>2</sup> Fondazione Bruno Kessler (FBK), 38123 Povo, Trento, Italy  
<sup>3</sup> INFN-TIFPA Trento Institute of Fundamental Physics and Applications, 38123 Povo, Trento, Italy  
<sup>4</sup> Dipartimento di Fisica, Università della Calabria, 87036 Arcavacata di Rende, Cosenza, Italy  
<sup>5</sup> Istituto Nazionale di Fisica Nucleare, Gruppo collegato di Cosenza, 87036 Arcavacata di Rende, Cosenza, Italy  
<sup>6</sup> Université Paris-Saclay, CNRS, IJCLab, 91405 Orsay, France

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[F. G. C., M. Fucilla, Eur. Phys. J. C 82 (2022) 10, 929]



**Color Singlet (CS)**

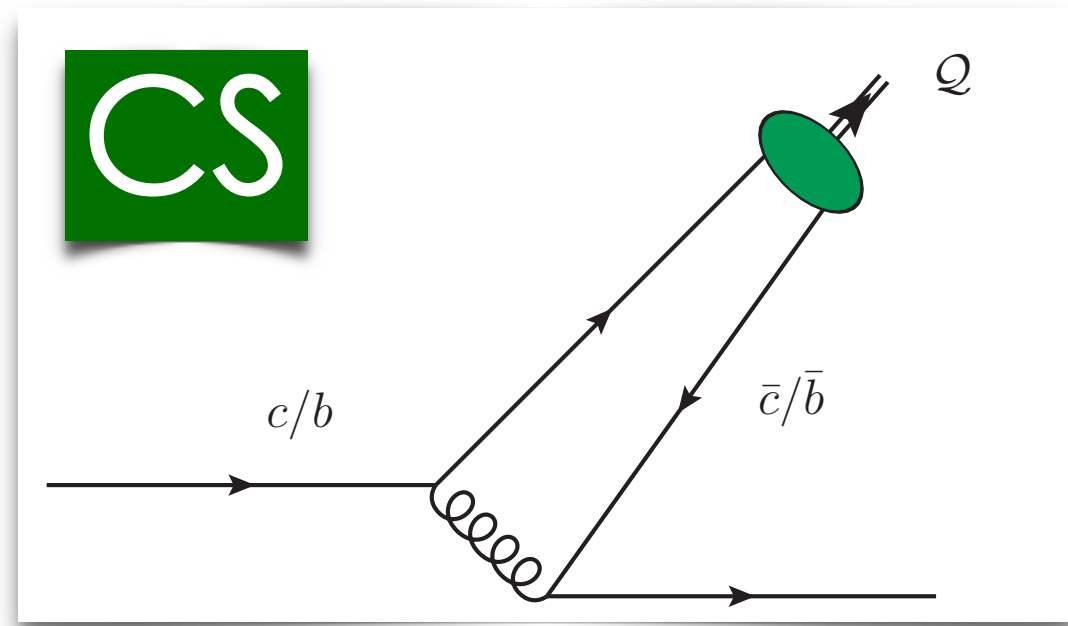
$$D_Q^{\mathcal{Q}}(z, \mu_F \equiv \mu_0) = D_Q^{\mathcal{Q},\text{LO}}(z) + \frac{\alpha_s^3(\mu_R)}{m_Q^3} |\mathcal{R}_{\mathcal{Q}}(0)|^2 \Gamma^{\mathcal{Q},\text{NLO}}(z)$$

(LO) [E. Braaten et al., Phys. Rev. D 48 (1993) 4230-4235]

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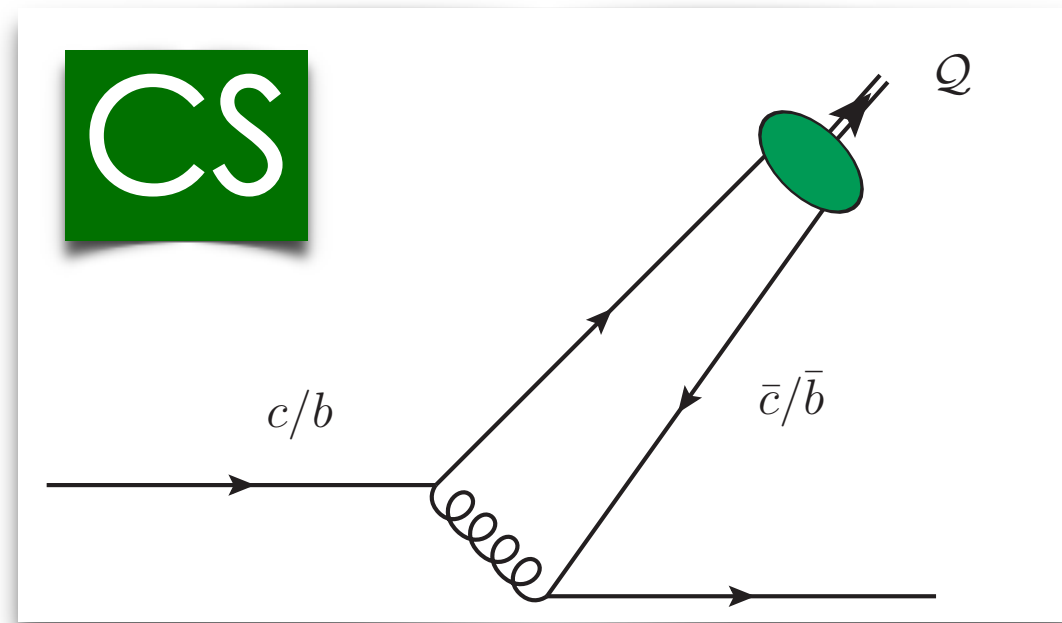
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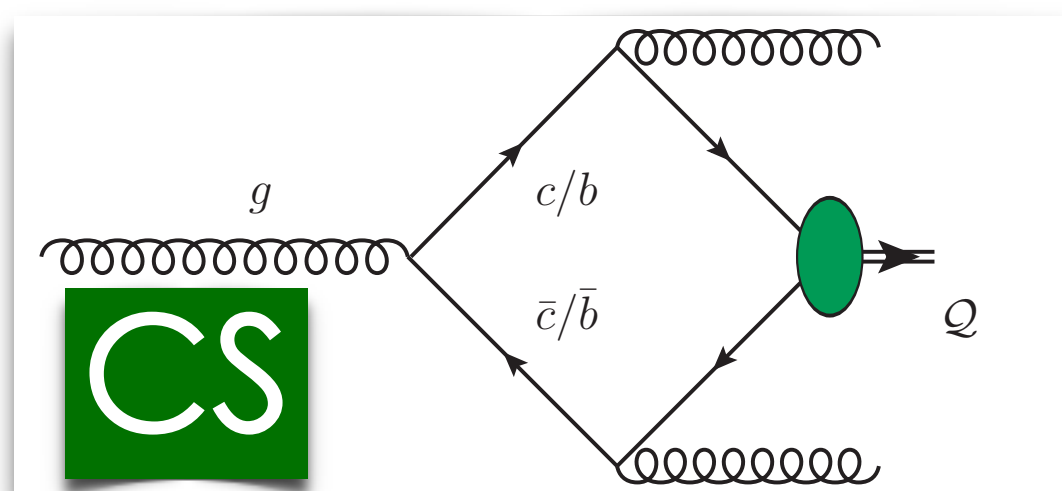
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+



$$D_g^Q(z, 2m_Q) = \frac{5}{36(2\pi)^2} \alpha_s^3(2m_Q) \frac{|\mathcal{R}_Q(0)|^2}{m_Q^3} \int_0^z d\xi \int_{(\xi+z^2)/2z}^{(1+\xi)/2} d\tau \frac{1}{(1-\tau)^2(\tau-\xi)^2(\tau^2-\xi)^2}$$

$(Q \rightarrow Q gg)$  at  $\mu_0 = 2m_Q$

$$\sum_{i=1}^2 z^i \left[ f_i^{(g)}(\xi, \tau) + g_i^{(g)}(\xi, \tau) \frac{1+\xi-2\tau}{2(\tau-\xi)\sqrt{\tau^2-\xi}} \ln \left( \frac{\tau-\xi+\sqrt{\tau^2-\xi}}{\tau-\xi-\sqrt{\tau^2-\xi}} \right) \right],$$

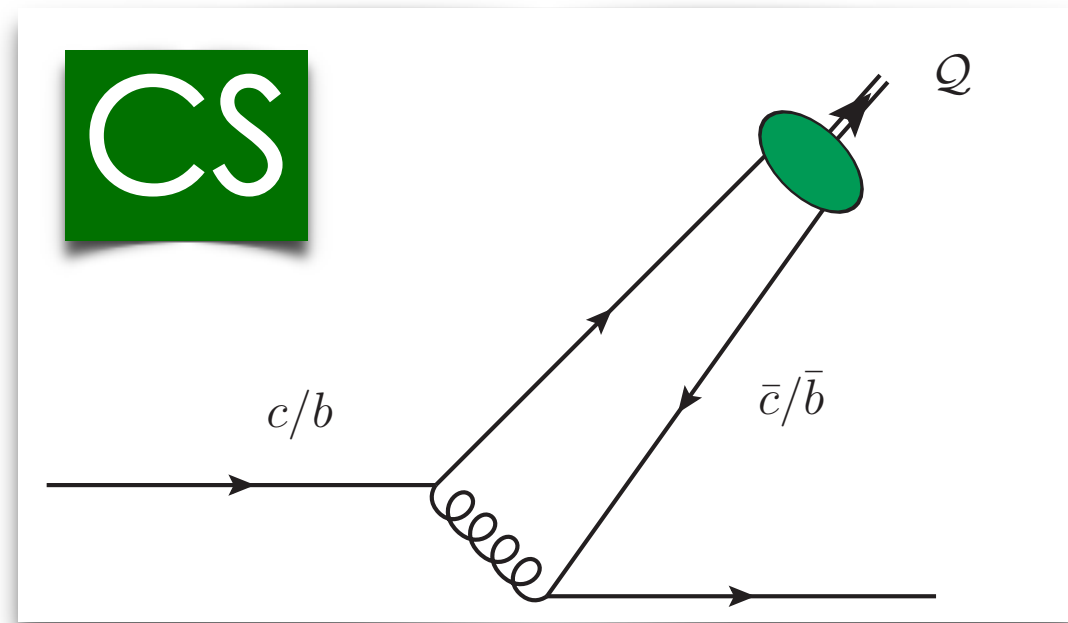
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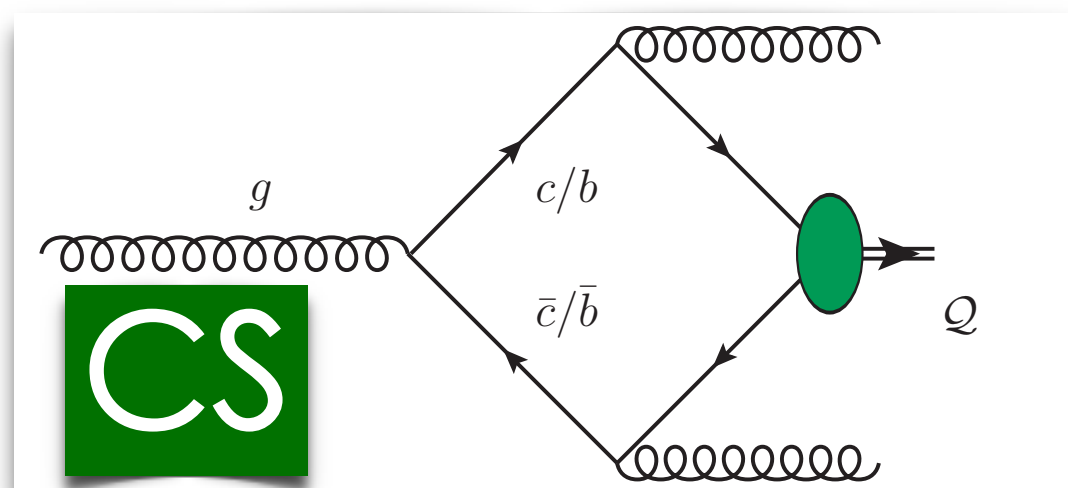
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⊗

[F. G. C., M. Fucilla, Eur. Phys. J. C 82 (2022) 10, 929]

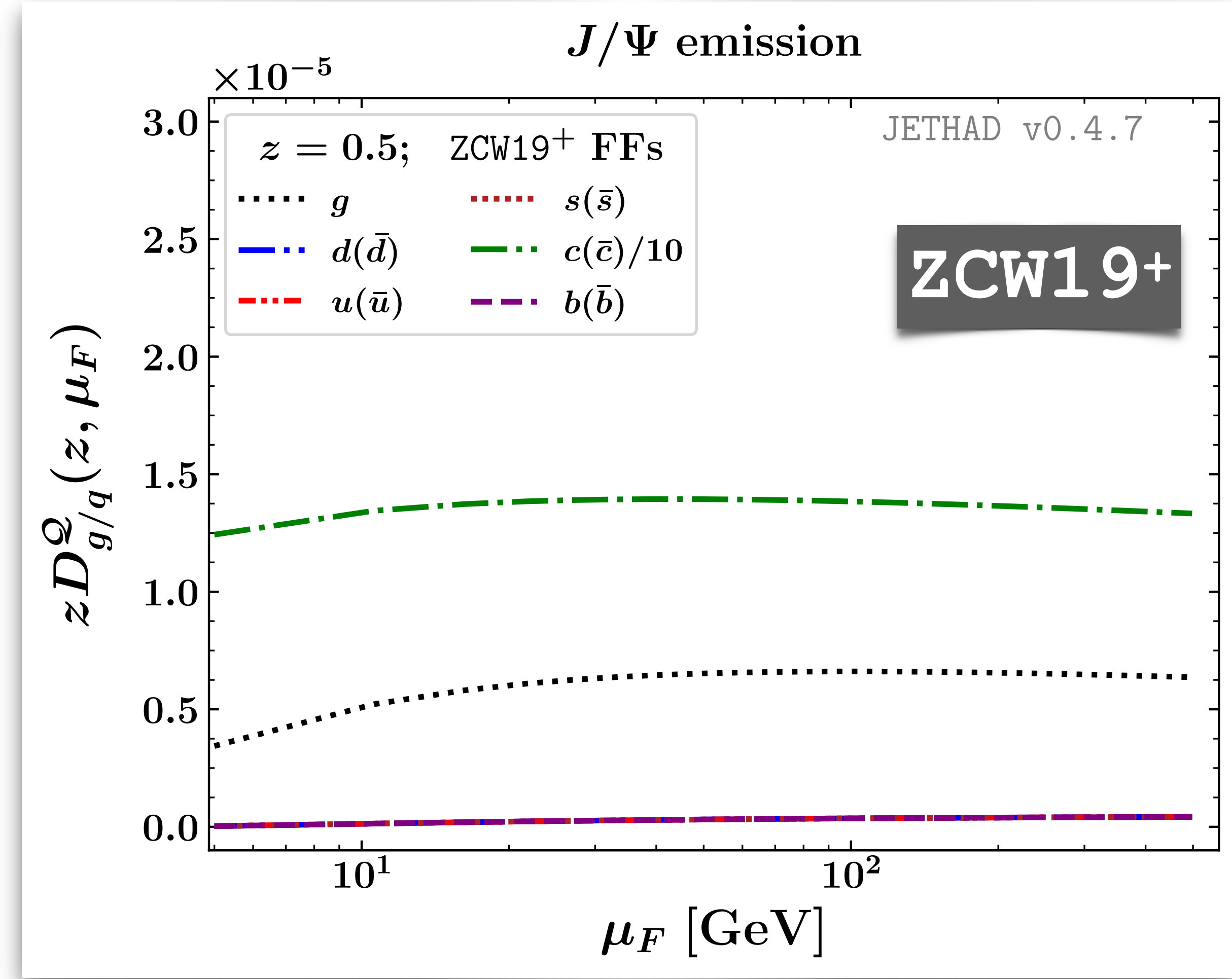
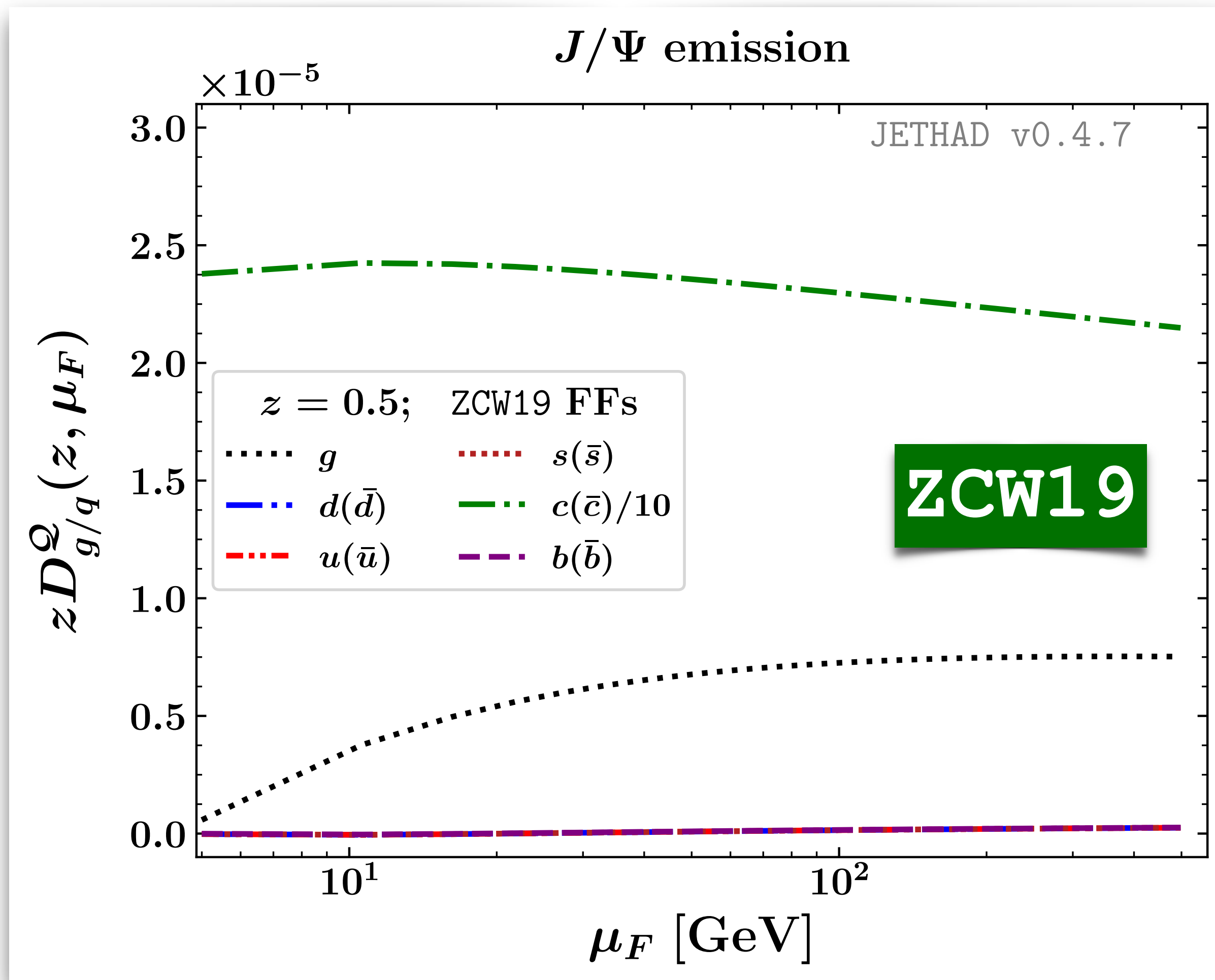
**ZCW19+**  
onium FFs

=

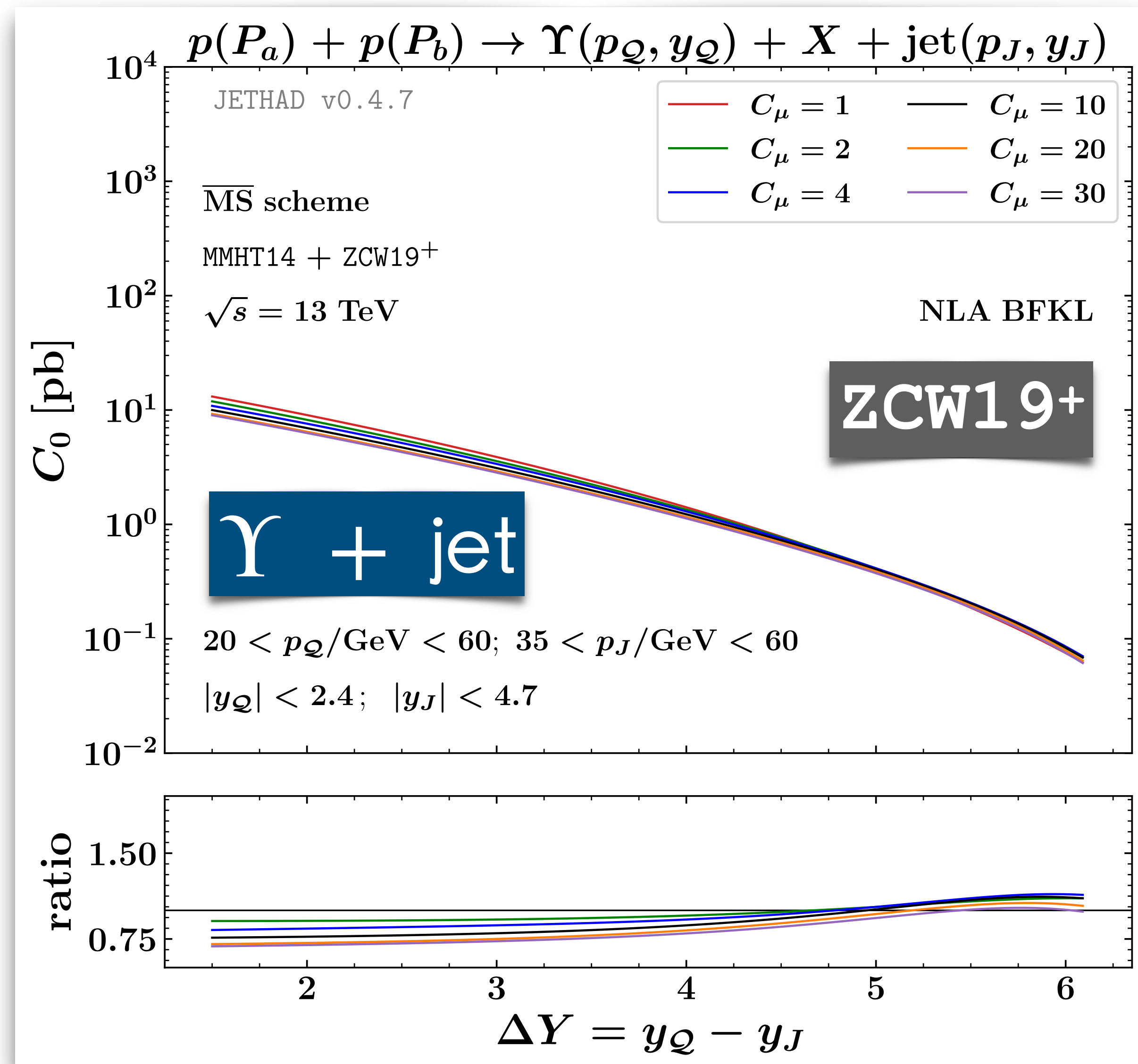
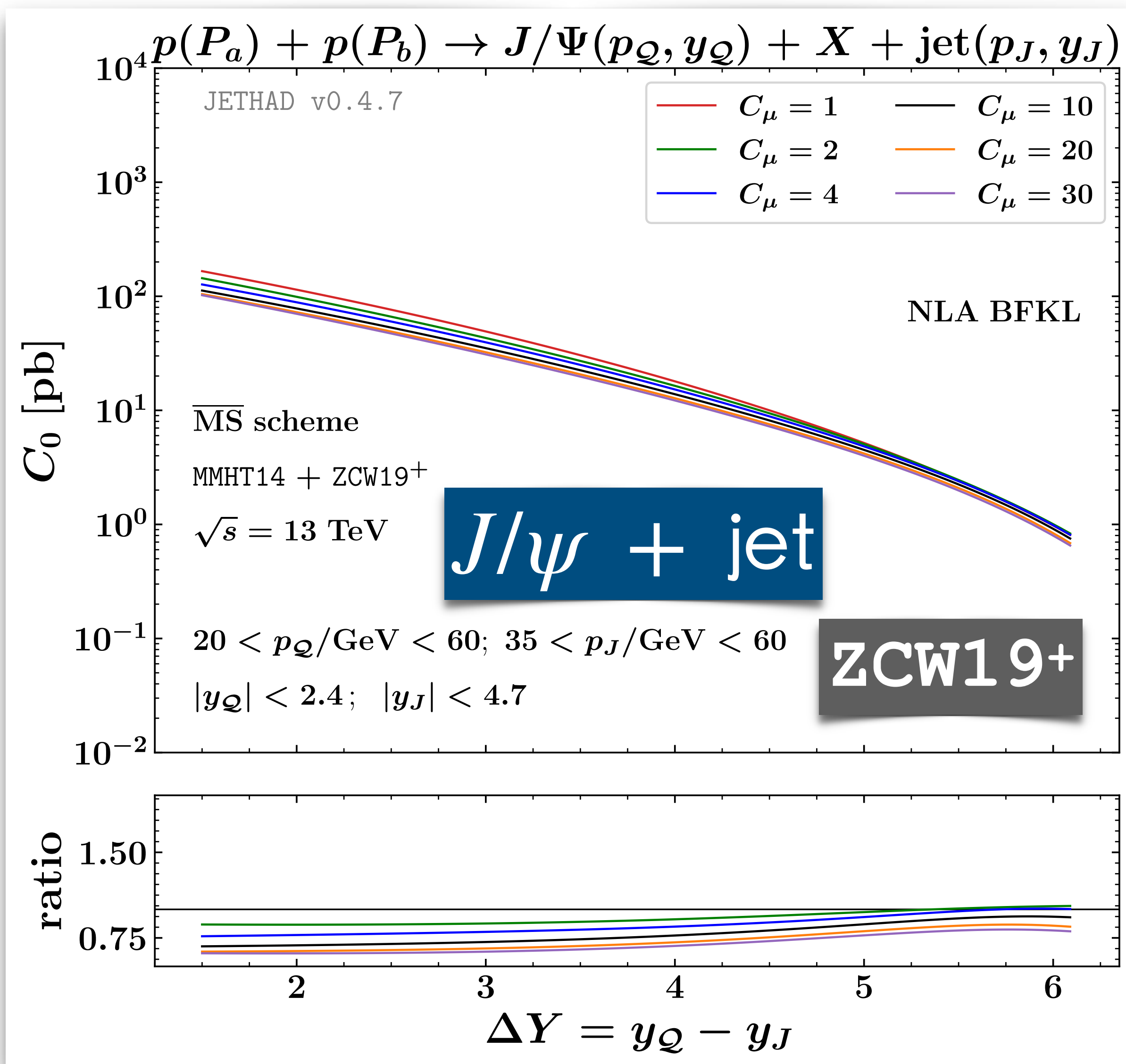
**APFEL++**

# Vector quarkonium + jet at the LHC

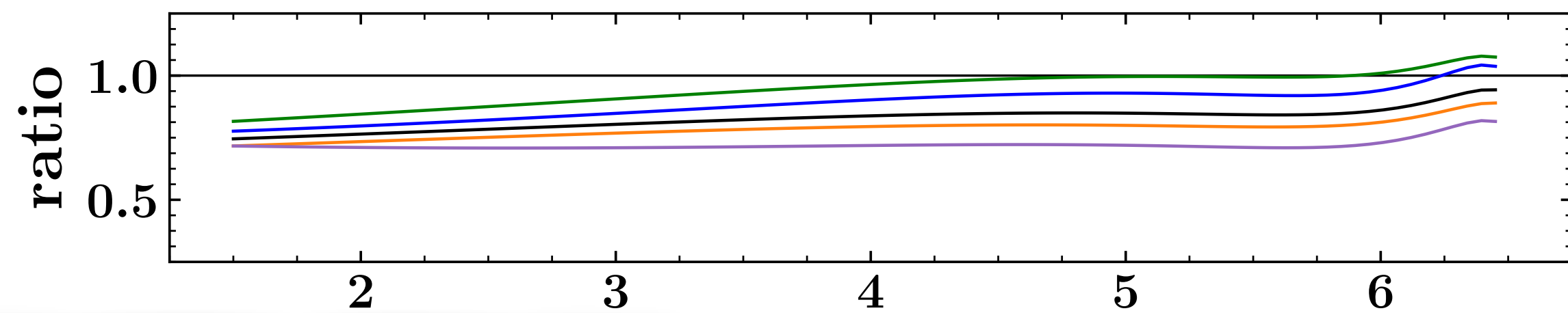
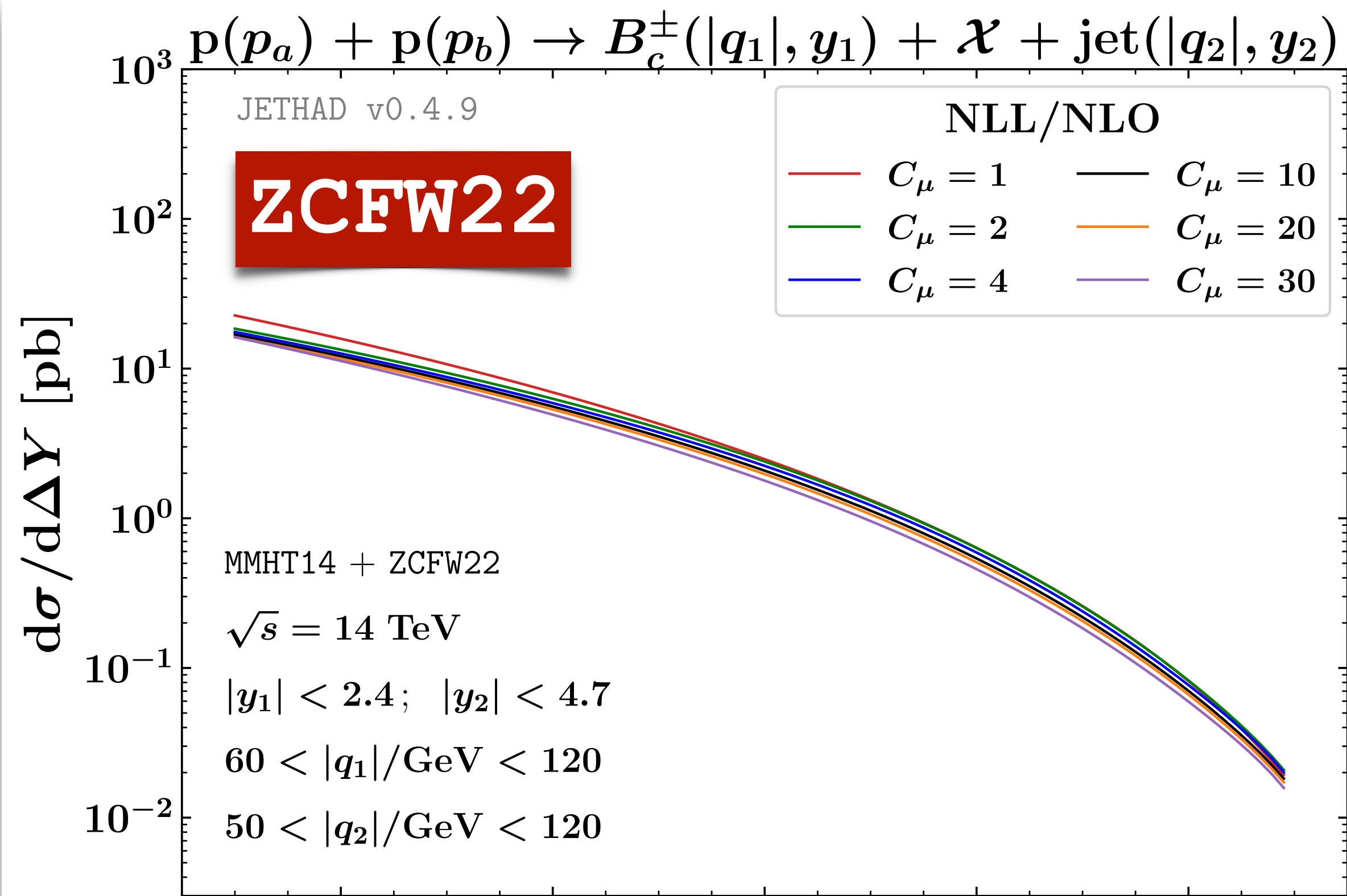
## $J/\psi$ collinear FFs



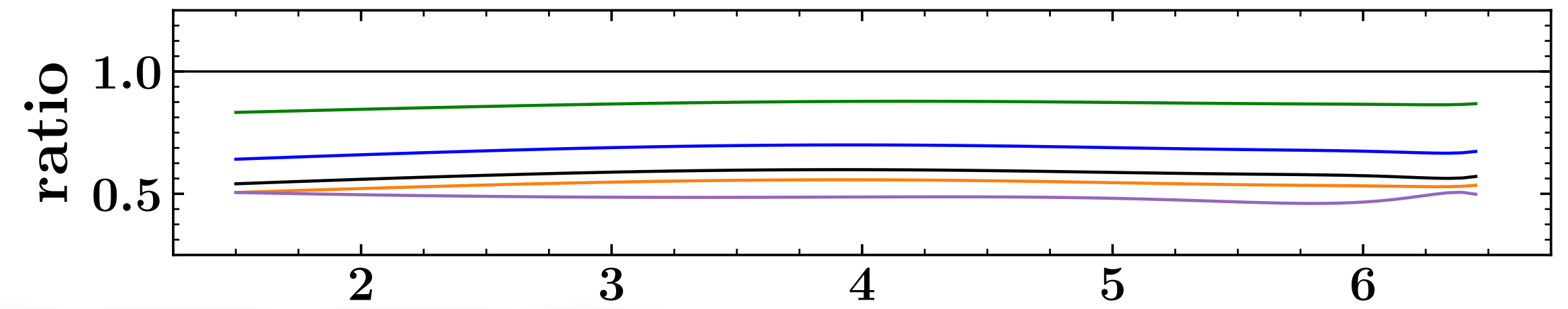
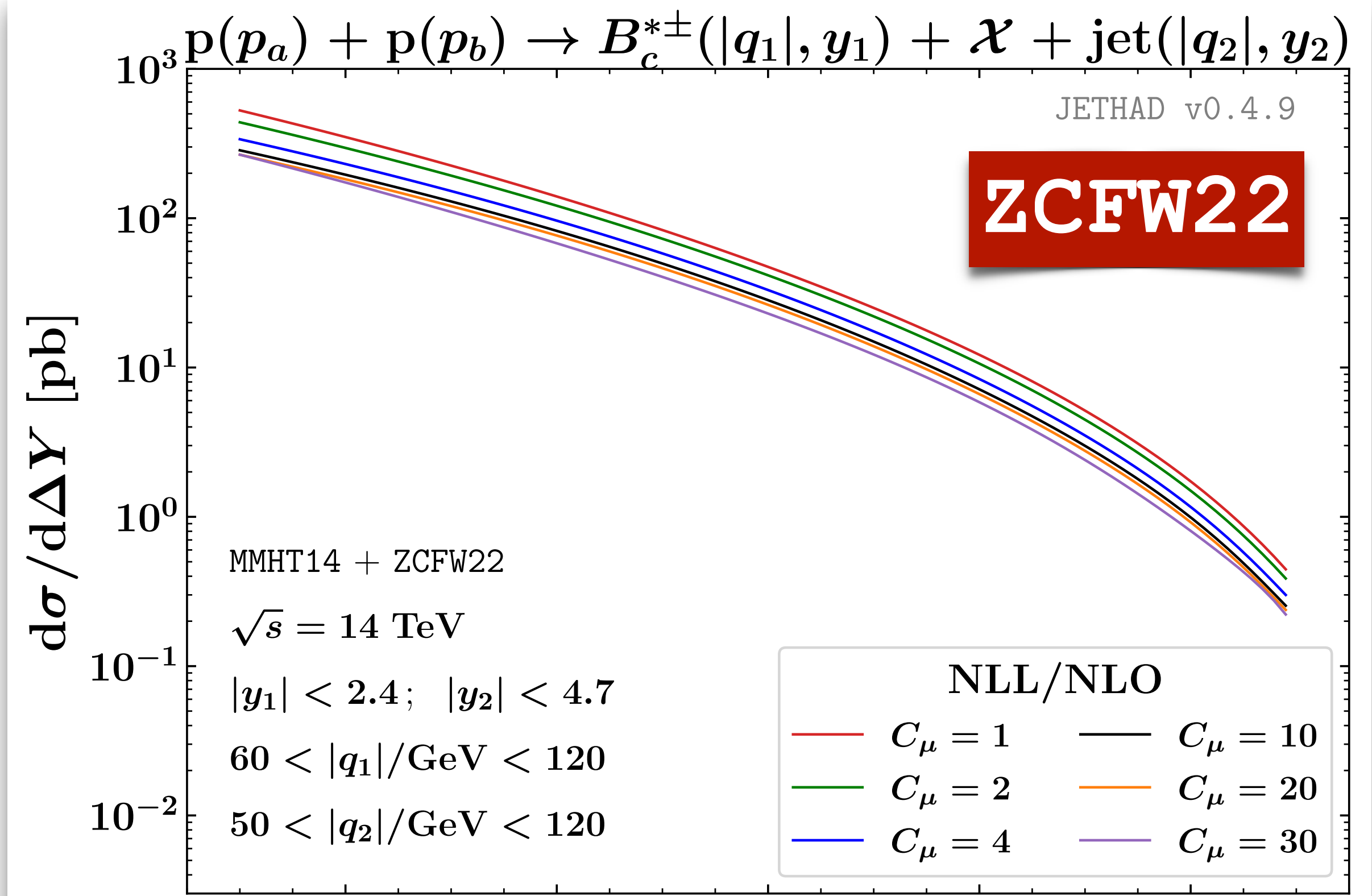
# Vector quarkonium + jet at the LHC



# Charmed $B$ -mesons + jet at the HL-LHC

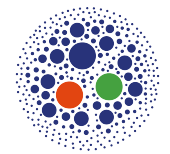


$B_c^\pm(^1S_0) + \text{jet}$



$B_c^\pm(^3S_1) + \text{jet}$

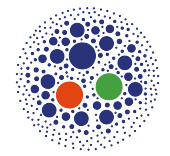
# A high-energy QCD portal to exotic matter



High-energy QCD **precision era** is at the **energy frontier(s)** of new-generation colliders

*; **Key ingredient**: interplay between **resummation(s)** & **production mechanisms** !*

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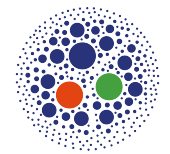
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**High-energy  
physics**



**Hadronic  
structure**

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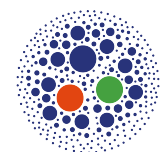
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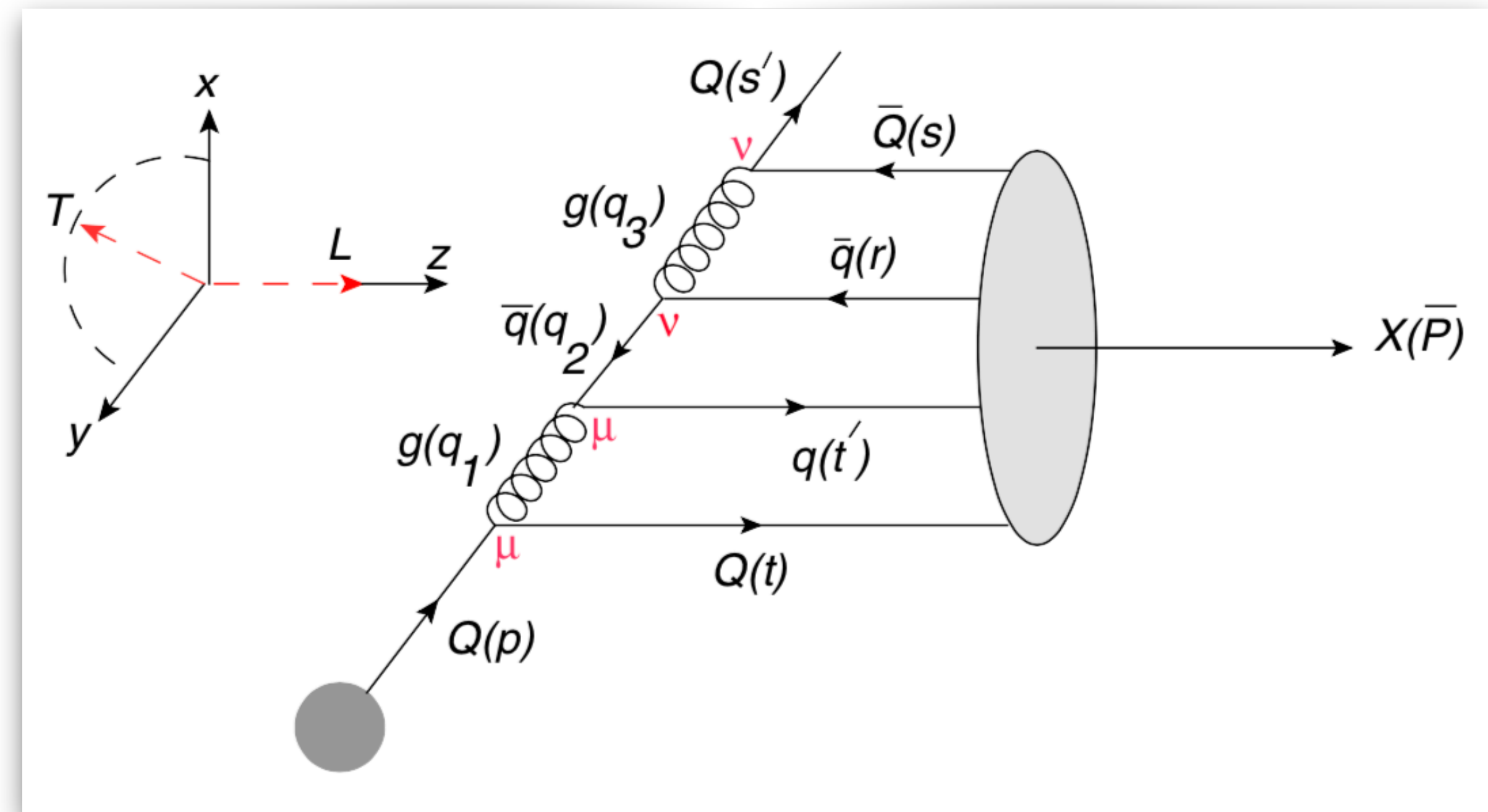
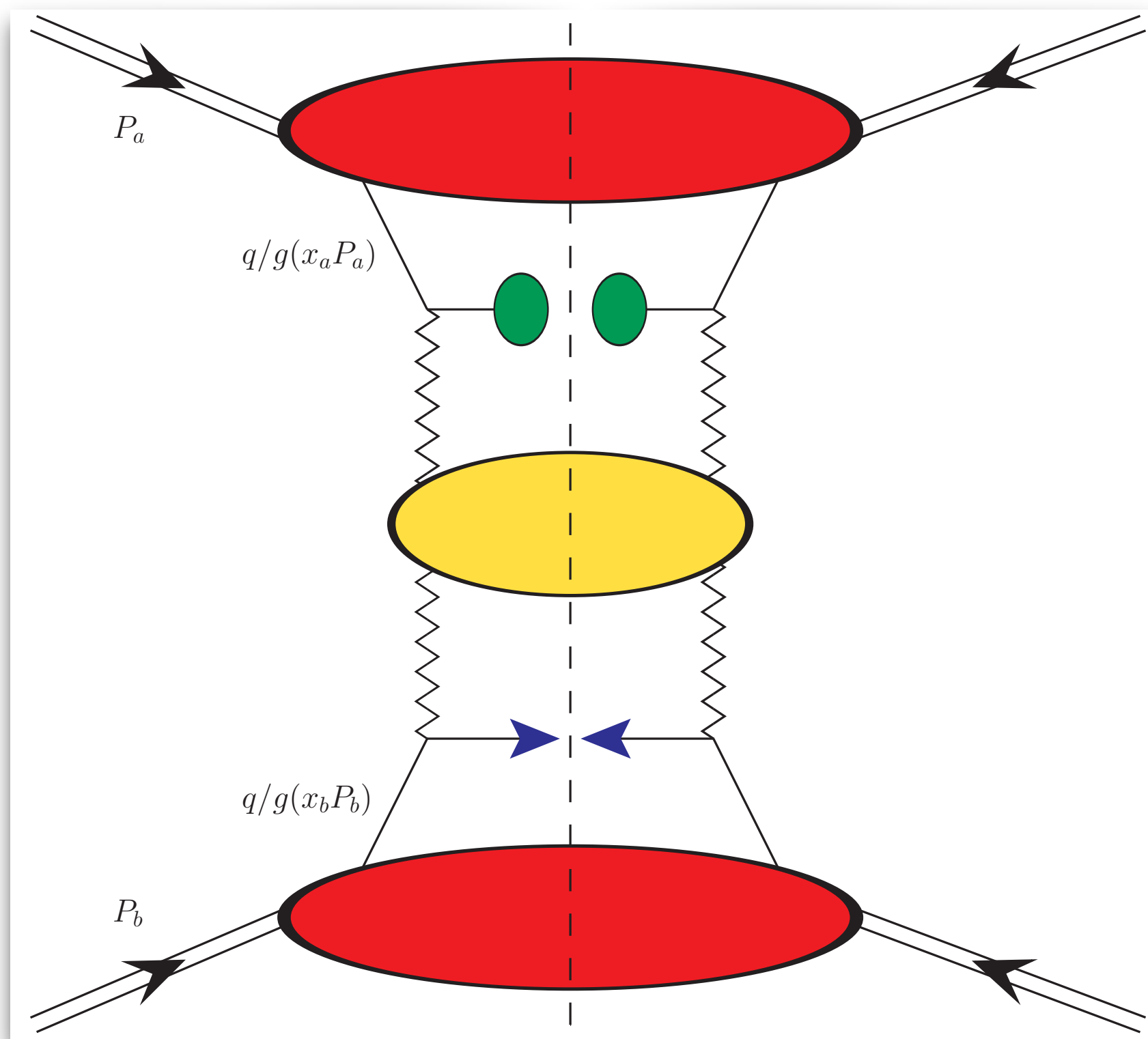
**⚡ Might high-energy QCD give us access to exotic matter ?**



Need for fragmentation mechanisms depicting **exotic** atoms/molecules/hadrons

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! Let us consider heavy-light  $X_{Qq\bar{Q}\bar{q}}$  tetraquarks at large  $p_T \rightarrow$  single-parton fragmentation !

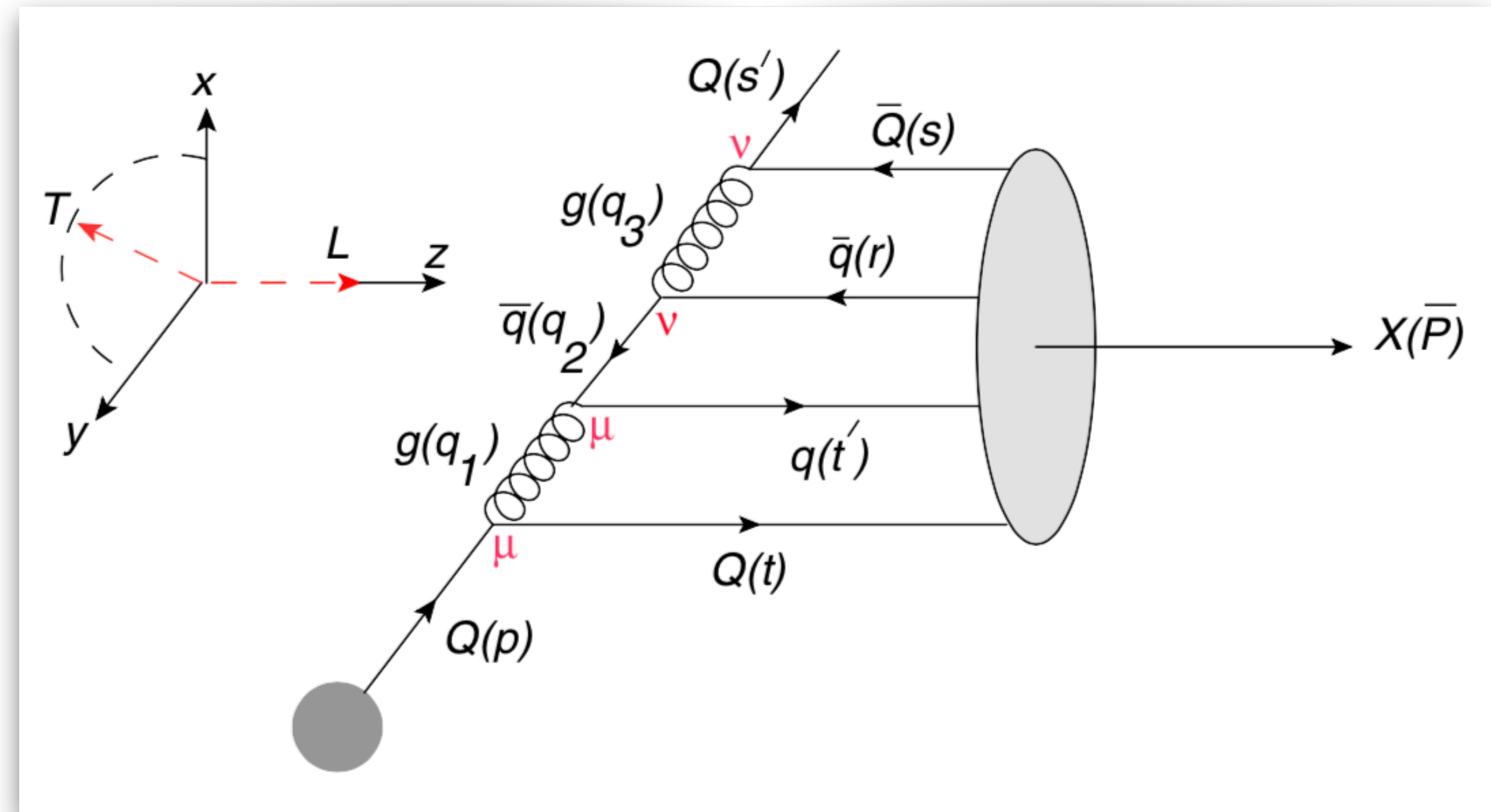
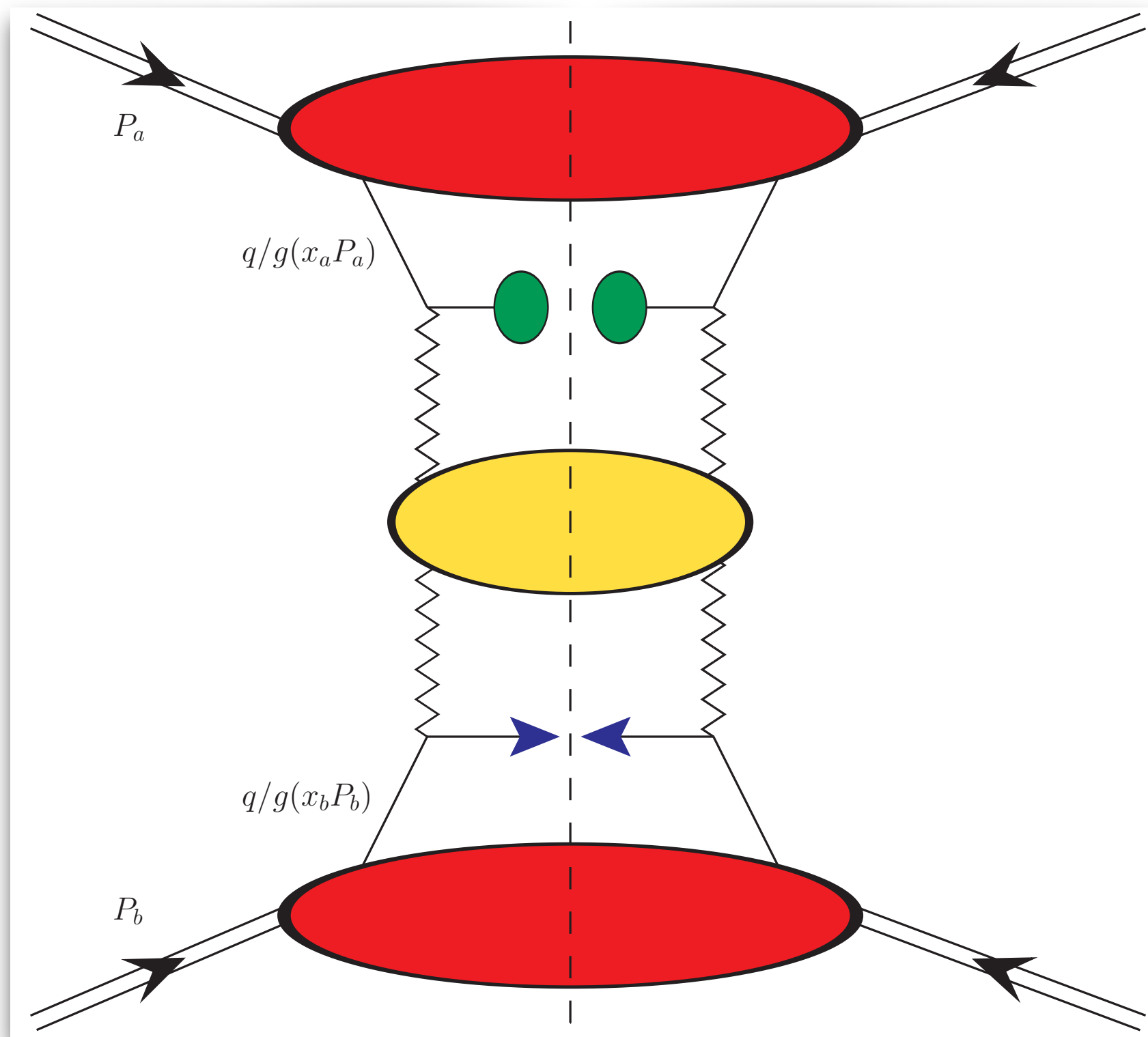


[F. G. C., A. Papa, in preparation]



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S-wave

$$D_Q^X(z, \mu_0) = N \frac{z \times \Sigma_{\text{spin}} \Gamma \bar{\Gamma}}{(m_X^2 - 2m_Q^2 + 2p \cdot s')^2}$$

$$= N \frac{z \times \Sigma_{\text{spin}} \Gamma \bar{\Gamma}}{[m_X^2 - (m_Q^2 + \langle p_T^2 \rangle)(1 + z - \frac{1}{1-z})]^2}$$

TQHL1.0 FFs:  $(Q \rightarrow X_{Qq\bar{Q}\bar{q}}) \otimes$  APFEL++  
 $[\mu_0 = m_X + m_Q]$

(LO) [\[S. M. Moosavi Nejad, Phys. Rev. D 05 \(2022\) 3, 034001\]](#)

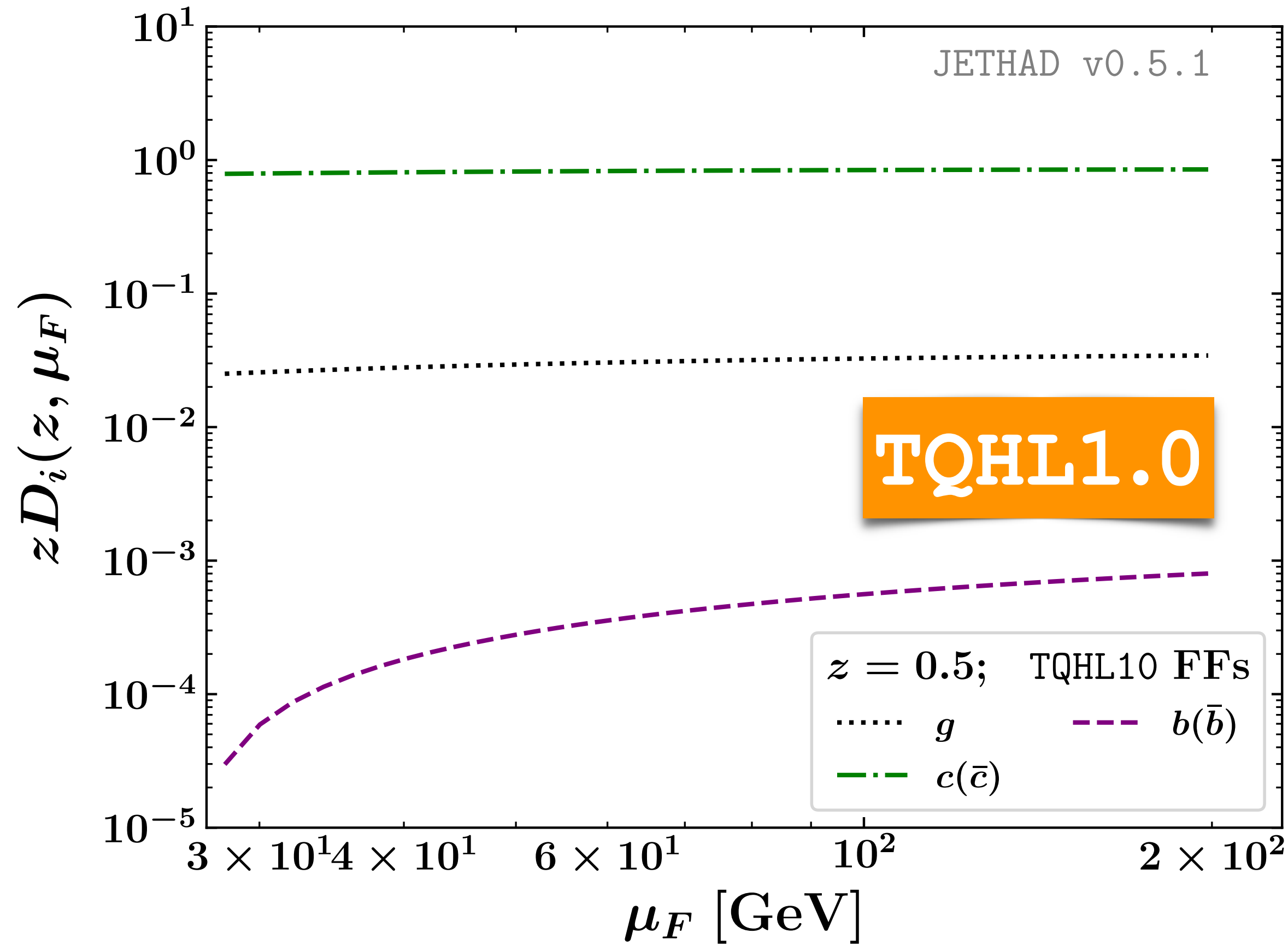
(framework) [\[M. Suzuki, Phys. Rev. D 33 \(1986\) 676\]](#)

# Heavy-light tetraquarks at the HL-LHC

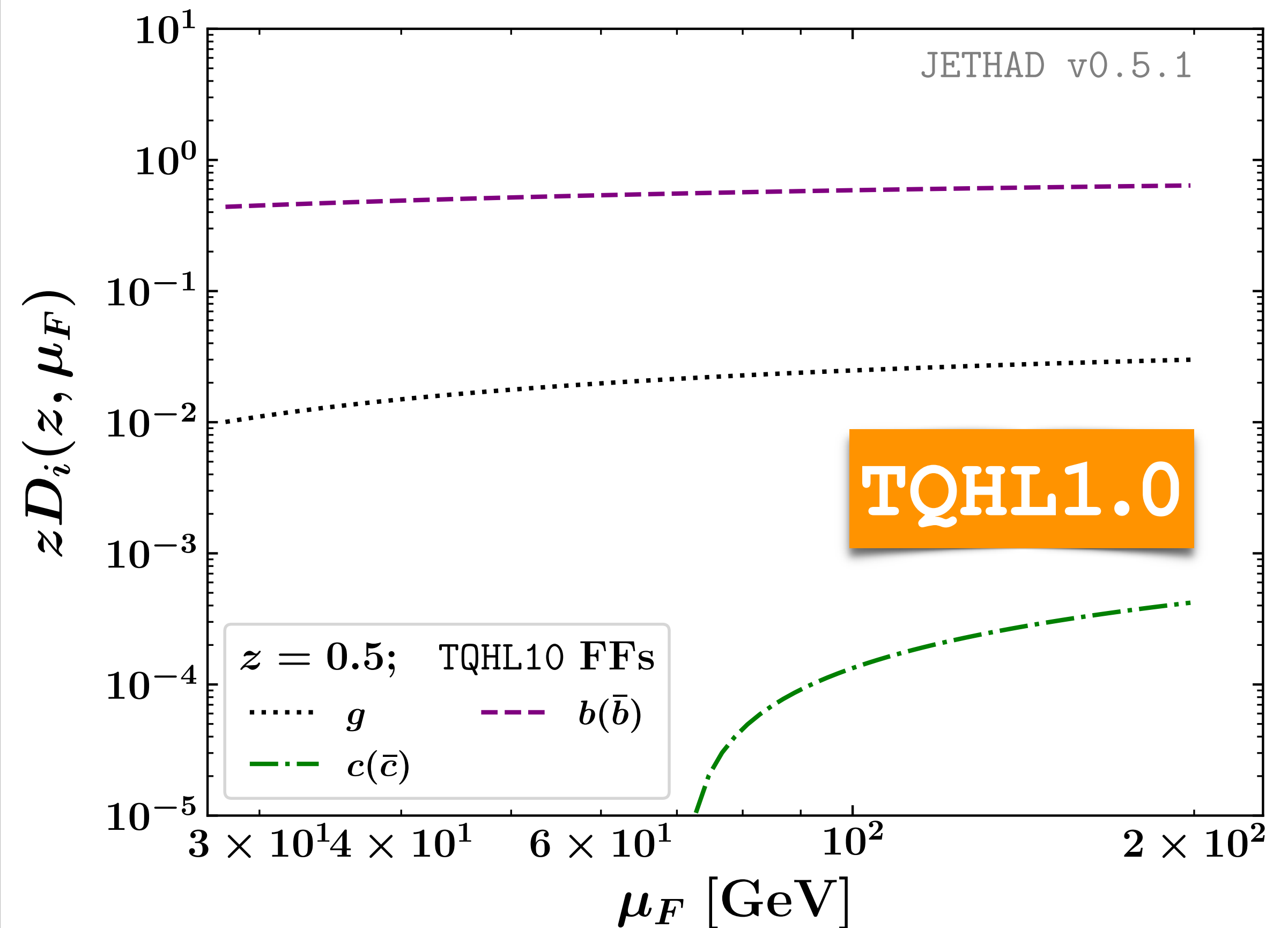
$X_{cu\bar{c}\bar{u}}$  collinear FFs

$X_{bs\bar{b}\bar{s}}$  collinear FFs

$X_{cu\bar{c}\bar{u}}$  tetraquark collinear FFs

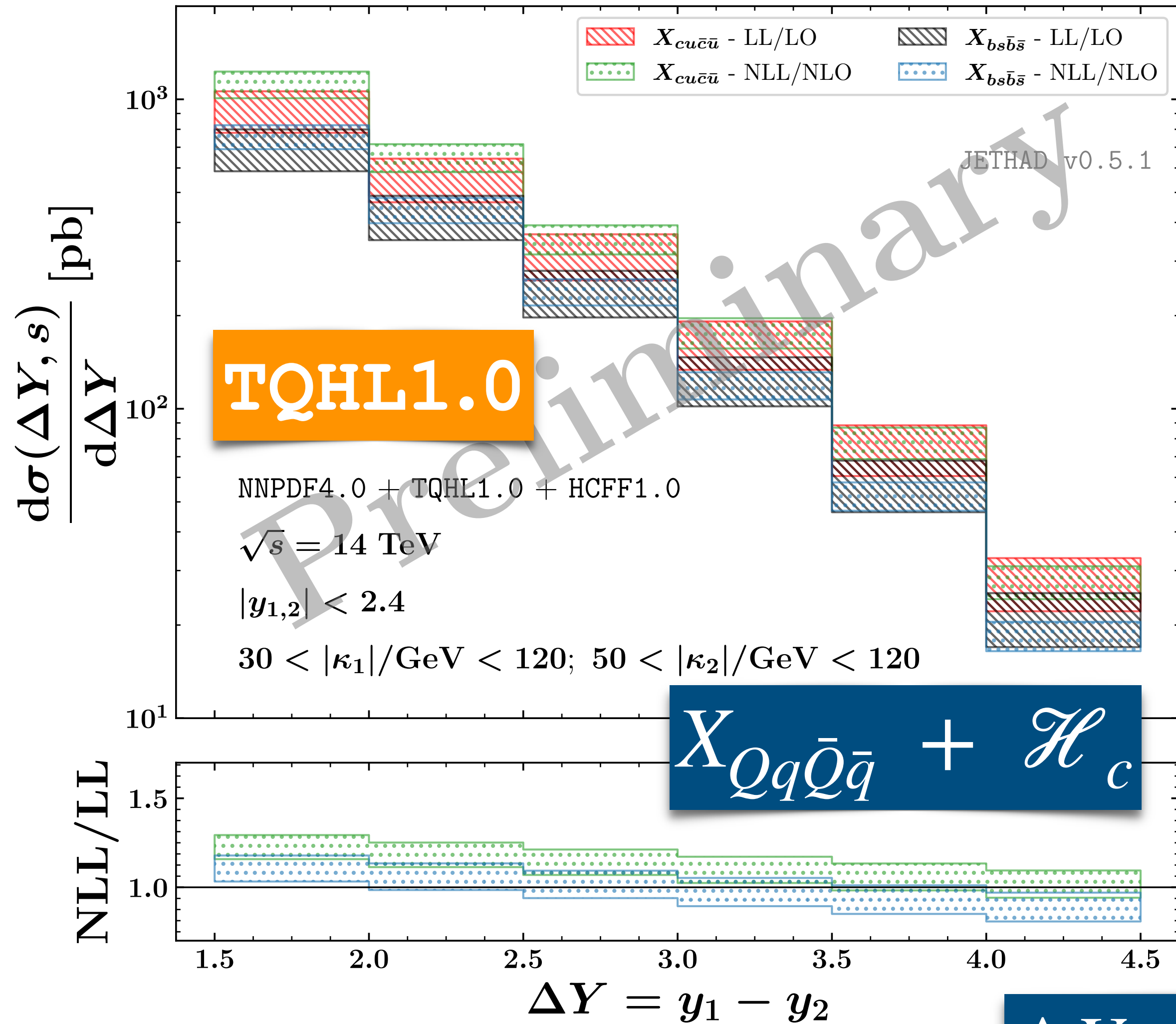


$X_{bs\bar{b}\bar{s}}$  tetraquark collinear FFs



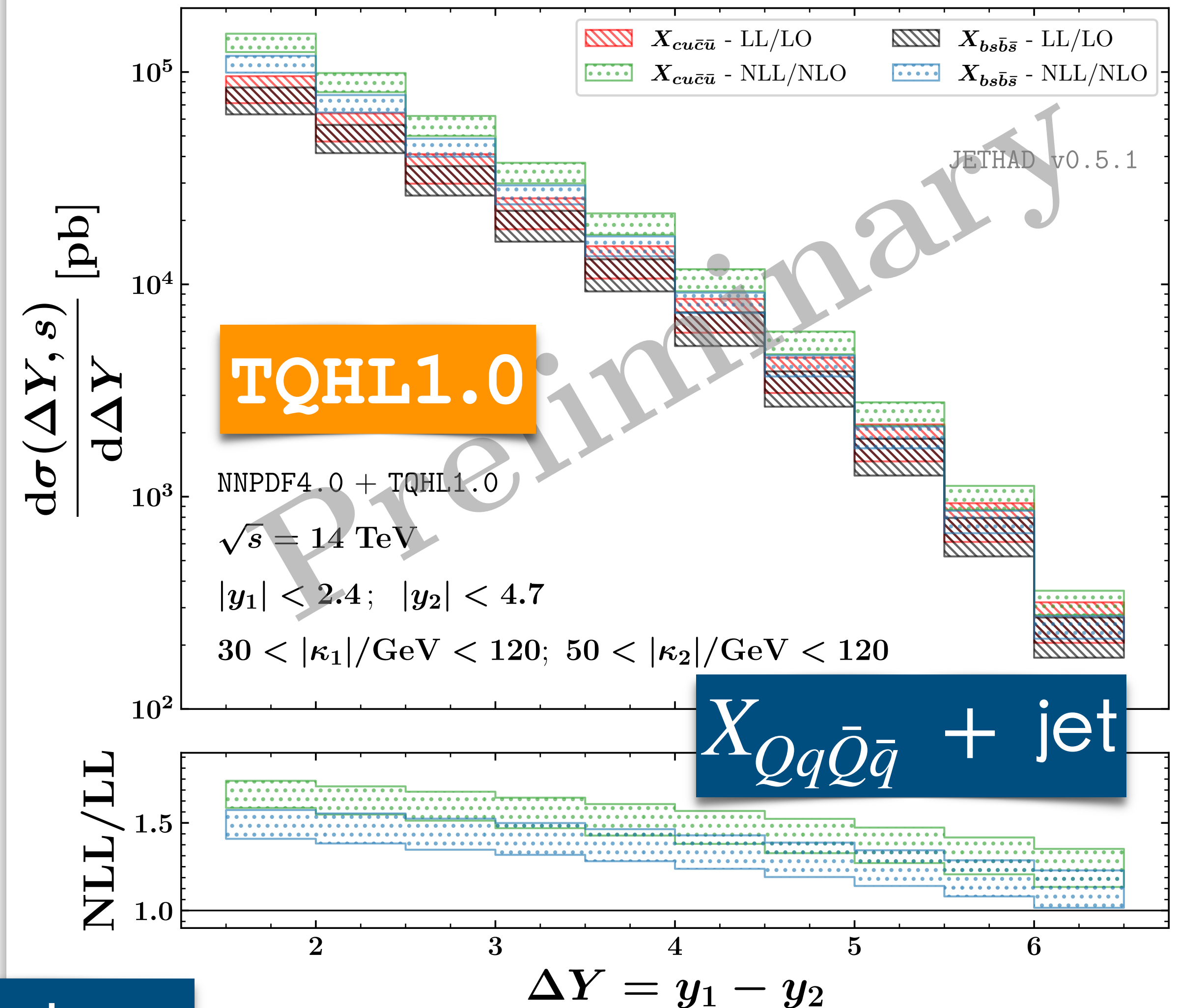
# Heavy-light tetraquarks at the HL-LHC

$$p(p_a) + p(p_b) \rightarrow X_{Qq\bar{Q}\bar{q}}(|\kappa_1|, y_1) + \mathcal{X} + \mathcal{H}_c(|\kappa_2|, y_2)$$

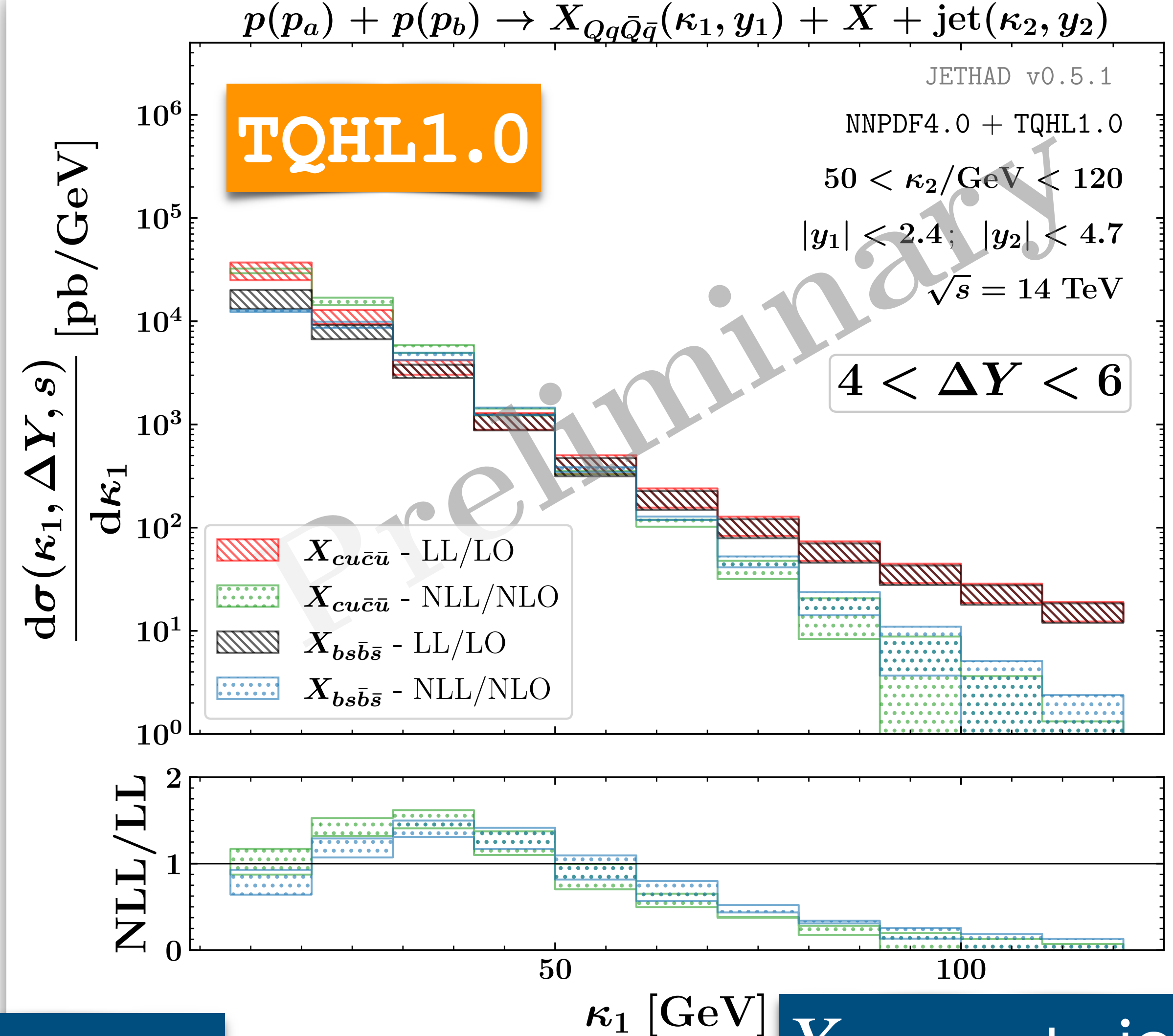
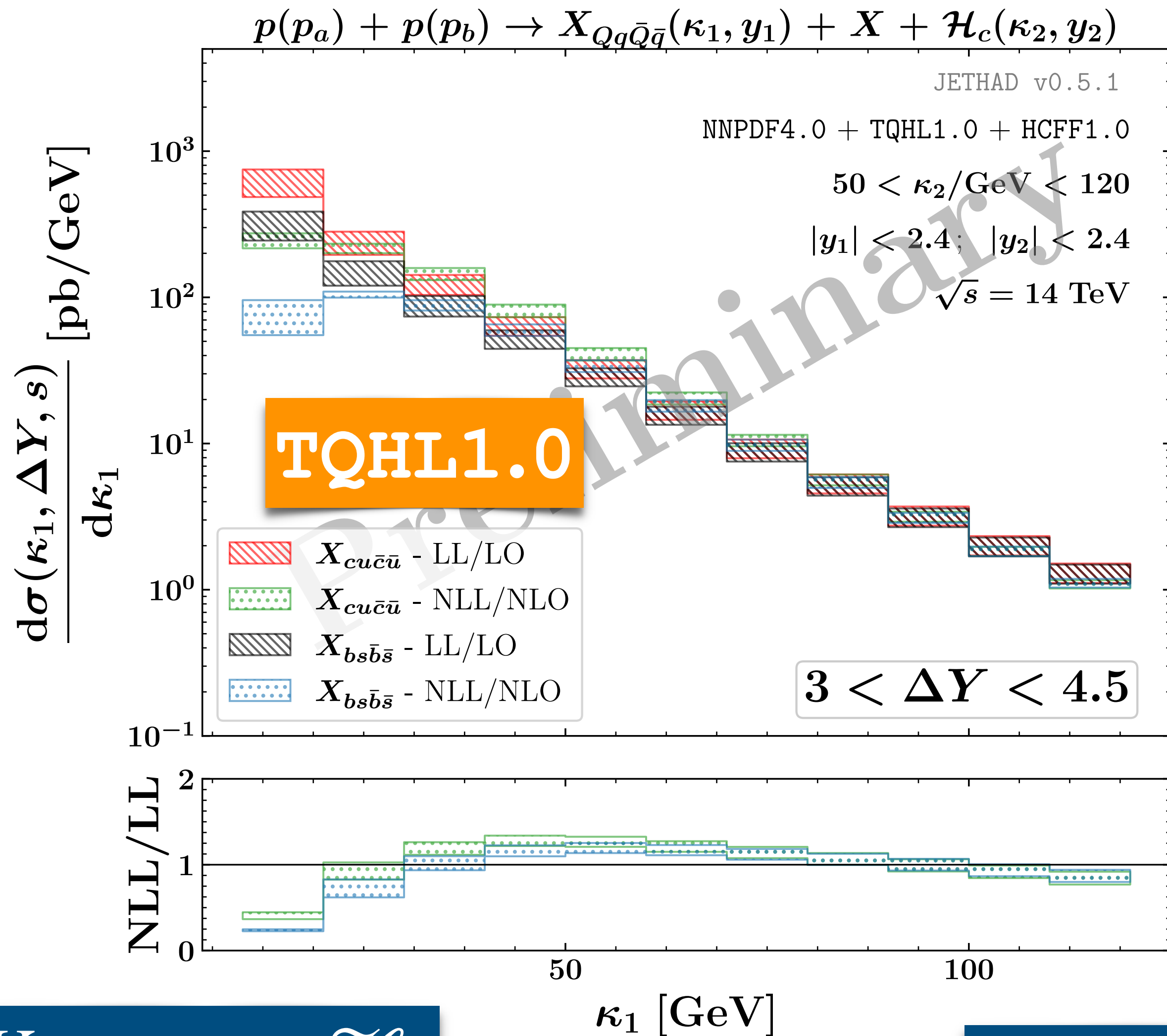


$\Delta Y$  spectrum

$$p(p_a) + p(p_b) \rightarrow X_{Qq\bar{Q}\bar{q}}(|\kappa_1|, y_1) + \mathcal{X} + \text{jet}(|\kappa_2|, y_2)$$



# Heavy-light tetraquarks at the HL-LHC

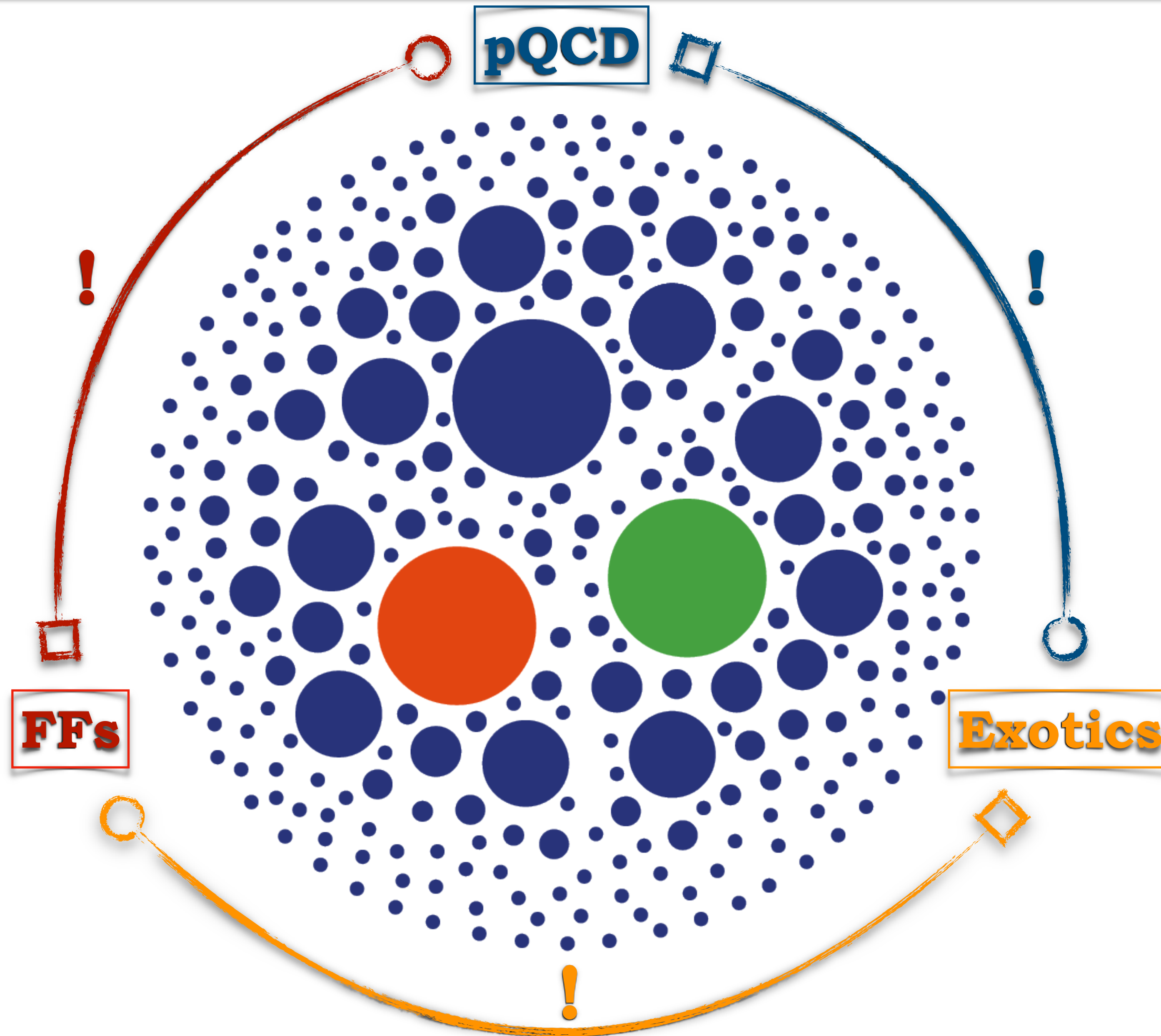


$X_{Qq\bar{Q}\bar{q}} + \mathcal{H}_c$

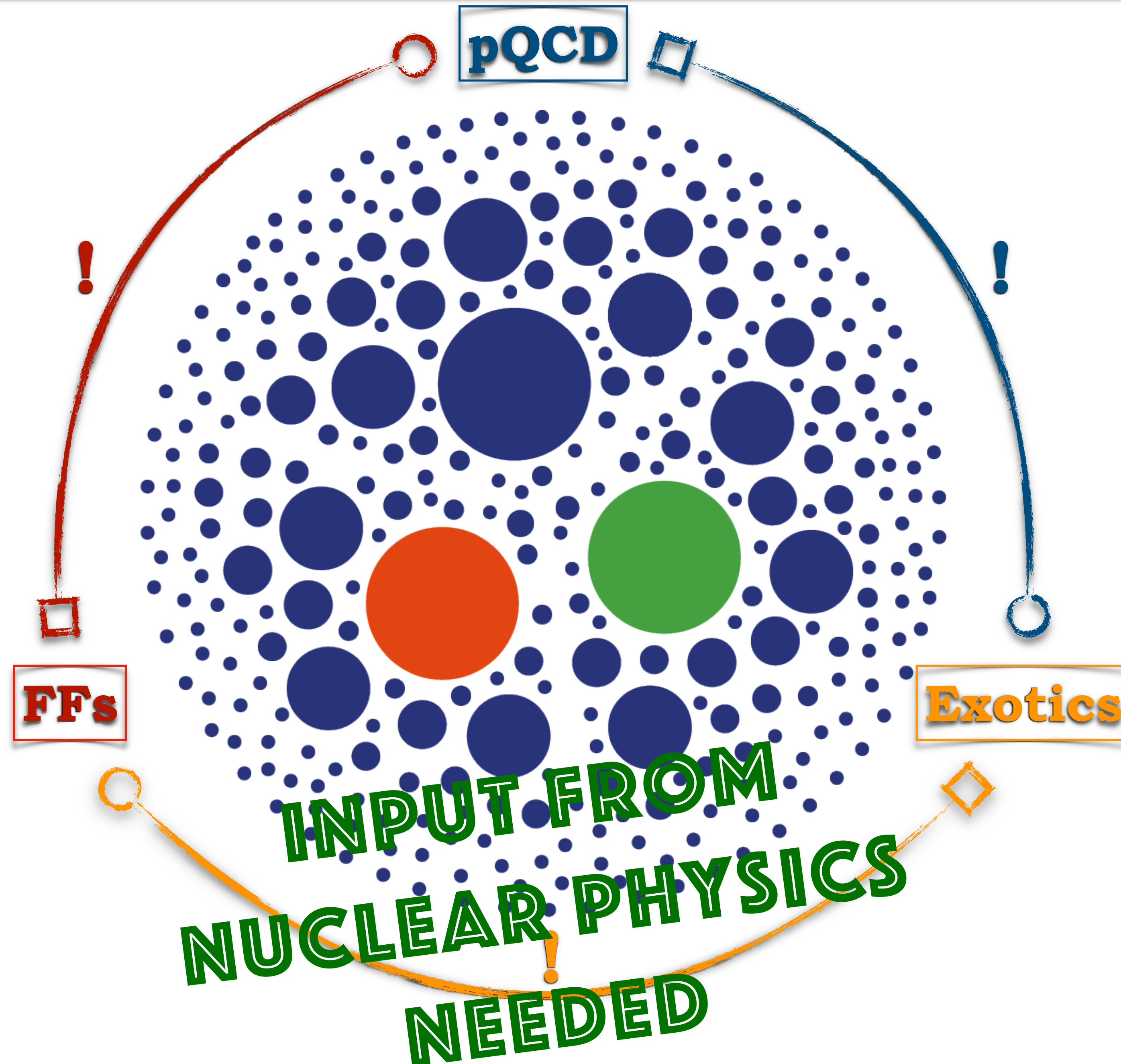
$|\kappa_1|$  spectrum

$X_{Qq\bar{Q}\bar{q}} + \text{jet}$

# Shedding light on exotic matter...



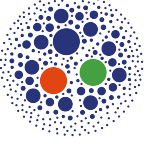

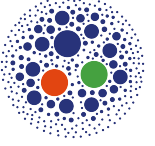
# Shedding light on exotic matter...





# Extras

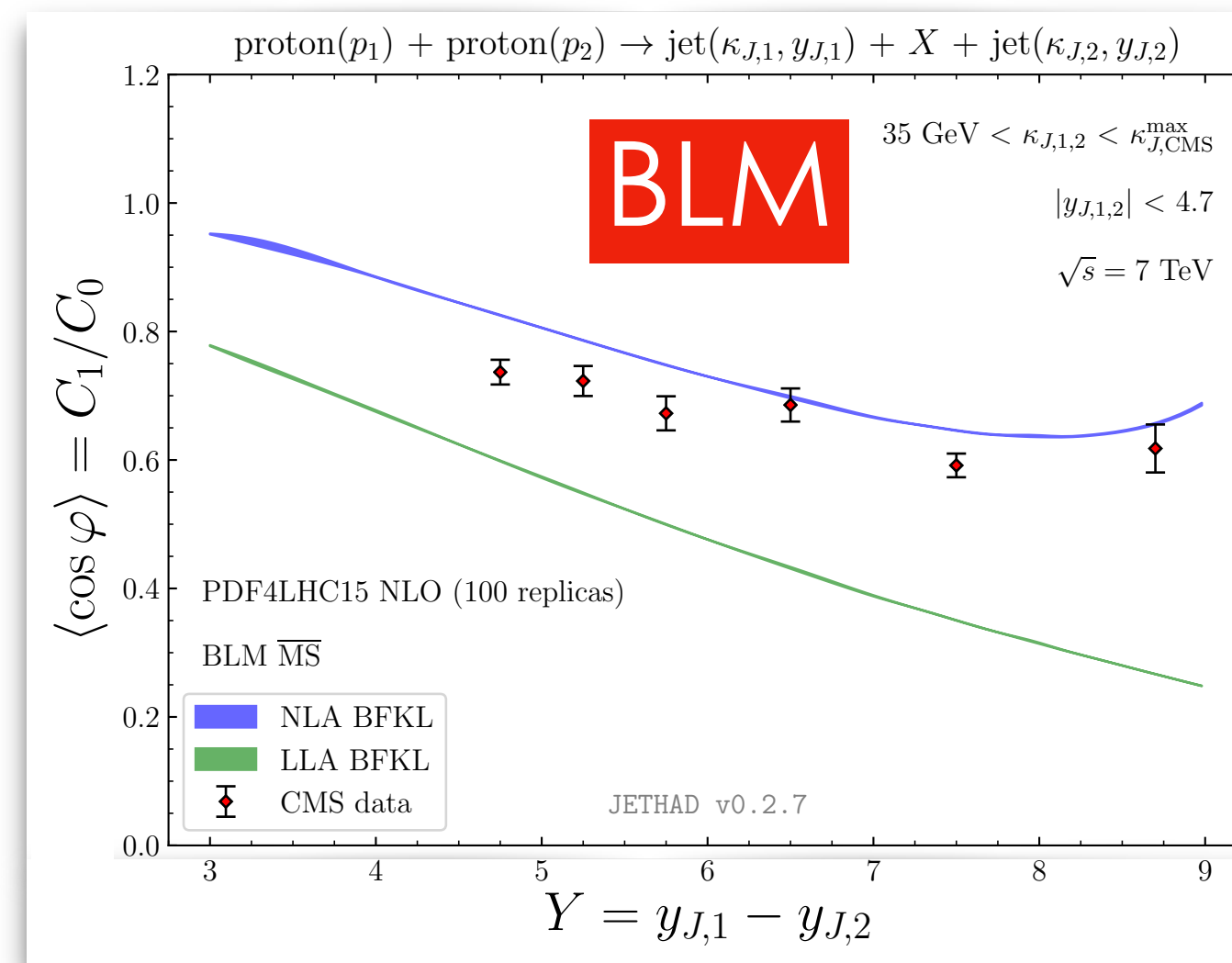
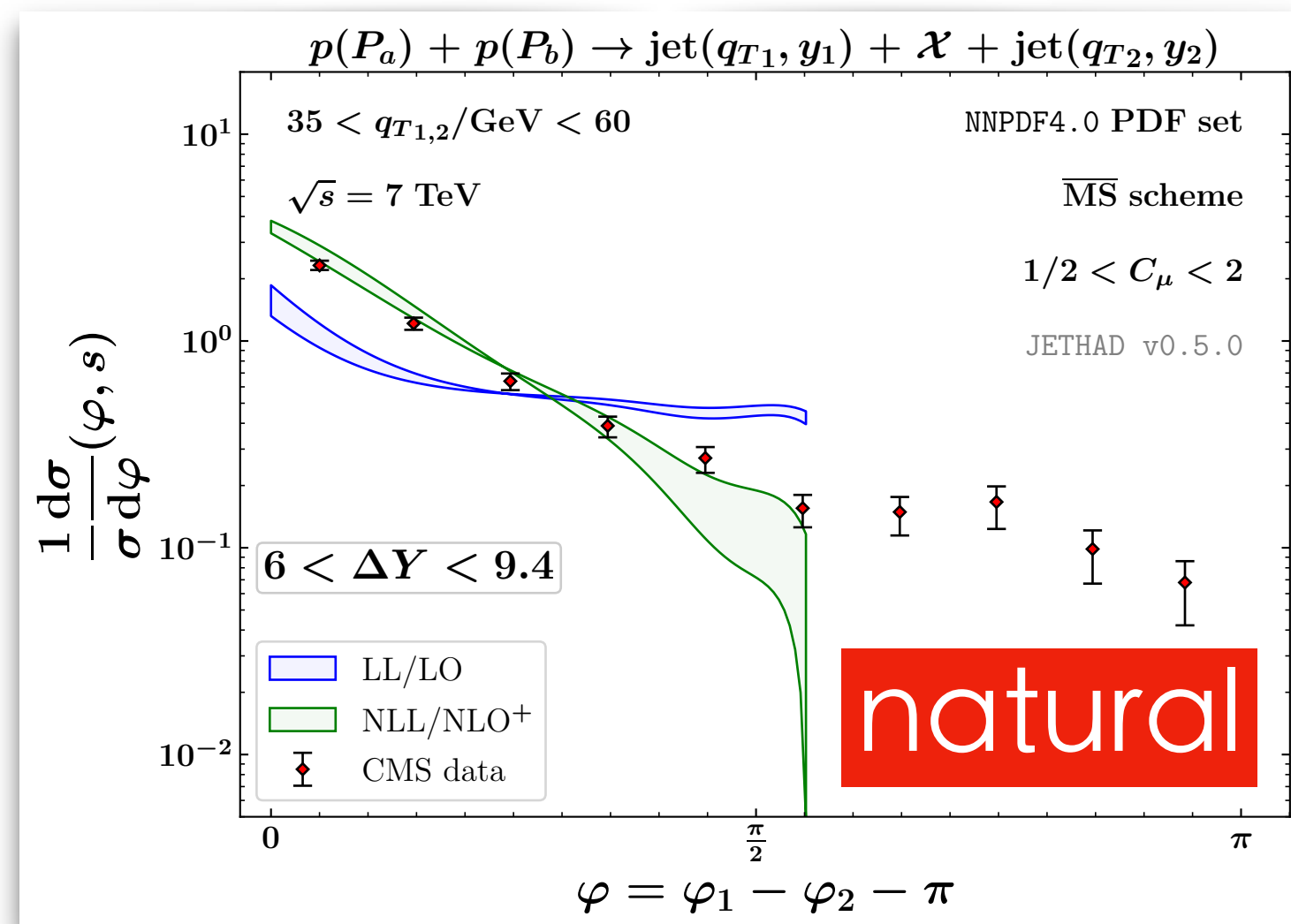
# Mueller-Navelet jets & resummation instabilities

-  Strong manifestation of **higher-order instabilities** via scale variation (i!)
-  i At natural scales: NLL/LL ratio large, no agreement with data, unphysical values !
-  **BLM** scales, theory vs experiment: CMS @7TeV with **symmetric**  $p_T$ -ranges, only



# Mueller-Navelet jets & resummation instabilities

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⌘ [B. Ducloué et al., Phys. Rev. Lett. 112 (2014) 082003]

⌘ [B. Murdaca et al., Eur. Phys. J. C 74 (2014) 10, 3084]

(left figure) ⌘ [F. G. C., A. Papa (2022)]

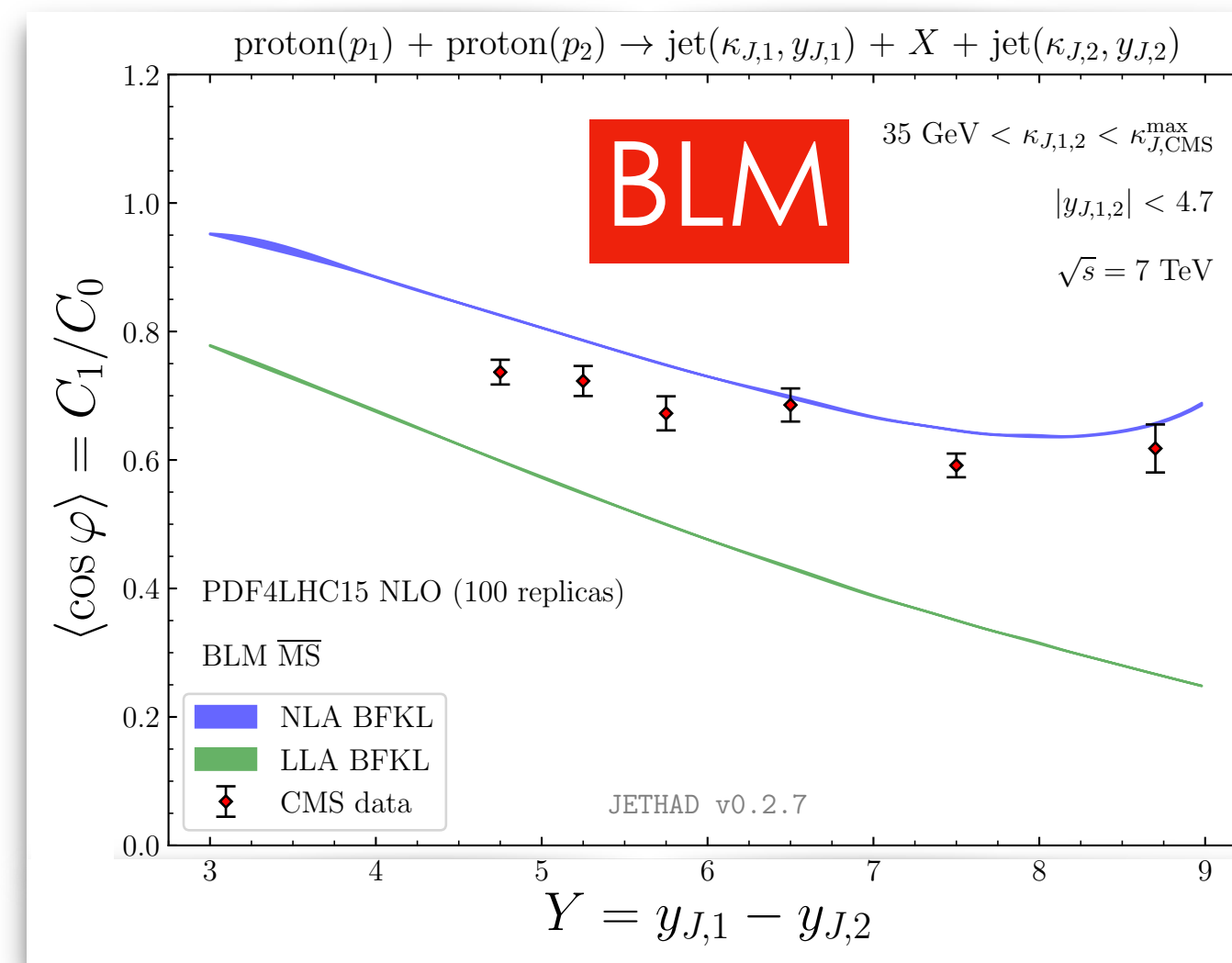
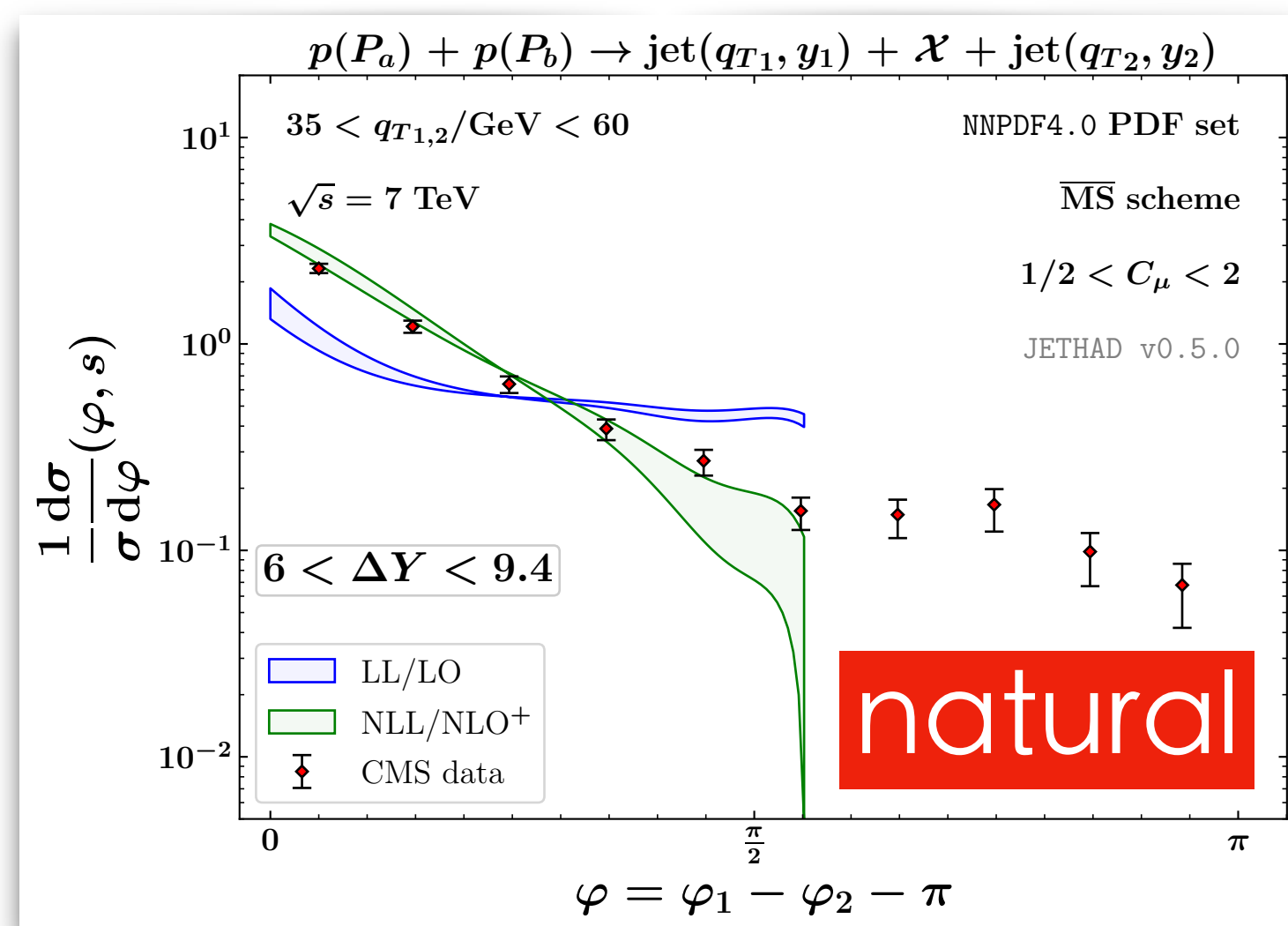
(right figure) ⌘ [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]

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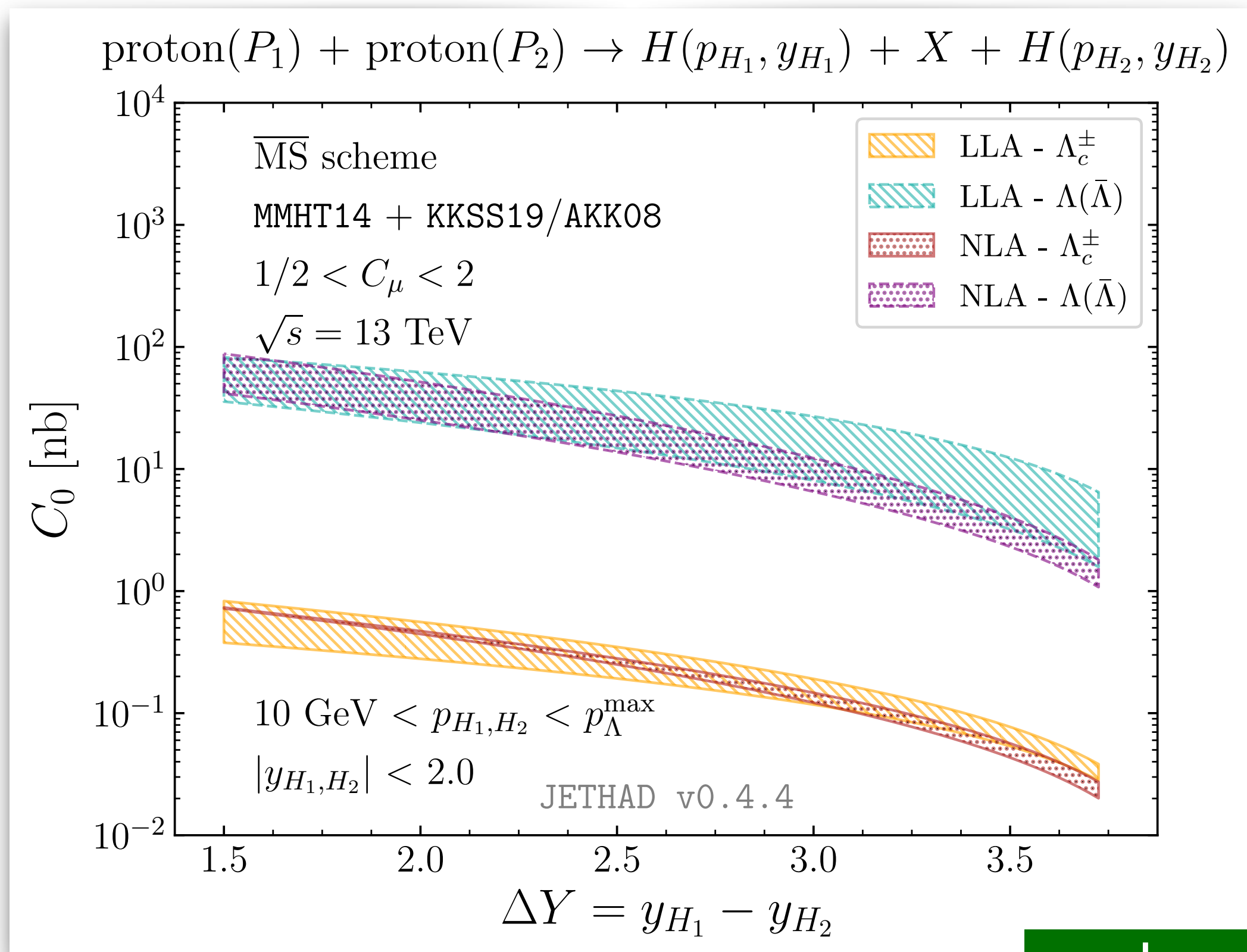
$\mu_R^{\text{BLM}} \gg \mu_R^{\text{nat.}} \Rightarrow d\sigma^{\text{BLM}}/d\sigma^{\text{nat.}} \sim 10^{-(1\div 2)} \Rightarrow$  precision studies hampered



Unsuccessful scale optimization → processes featuring natural stability (⚠️?)

# Stability under scale variations & NLL corrections

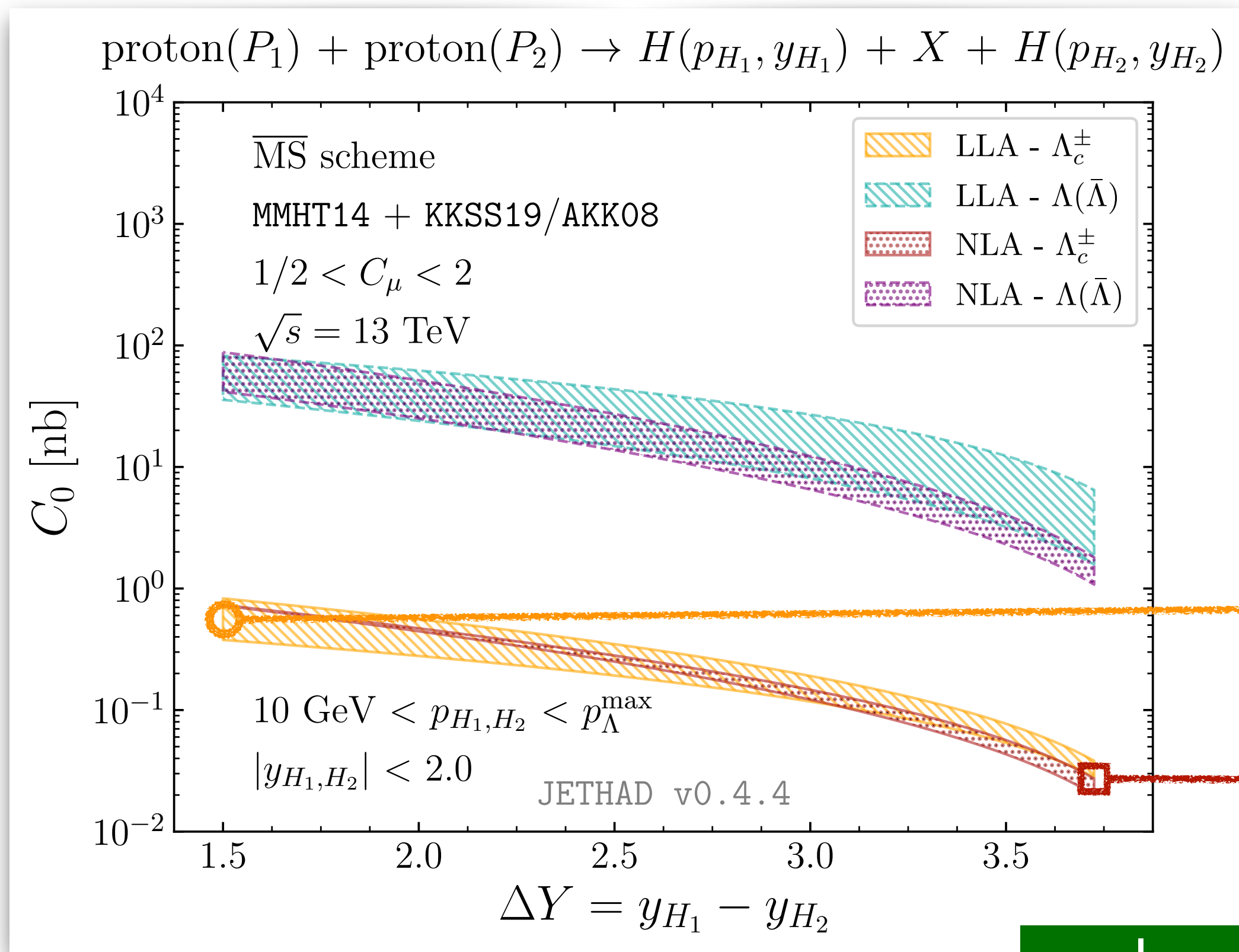
Hybrid factorization @work:  $\Lambda_c$  baryons  $|udc\rangle$  versus  $\Lambda$  hyperons  $|uds\rangle$



natural

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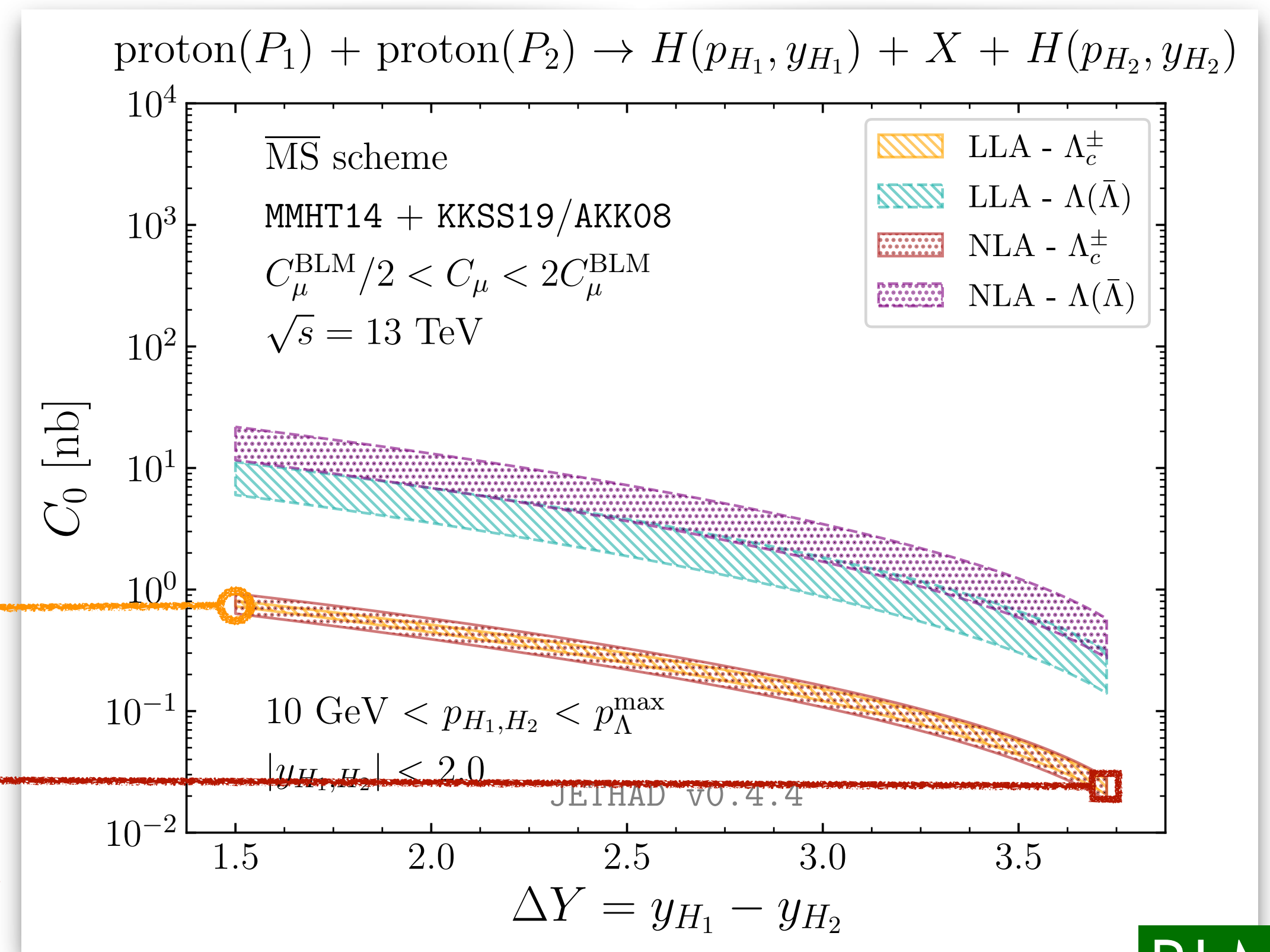
Hybrid factorization @work:  $\Lambda_c$  baryons  $|udc\rangle$  versus  $\Lambda$  hyperons  $|uds\rangle$



LL  $\Lambda_c$

NLL  $\Lambda_c$

natural



BLM

NLL corrections: rapidity distribution **stable** for  $\Lambda_c$

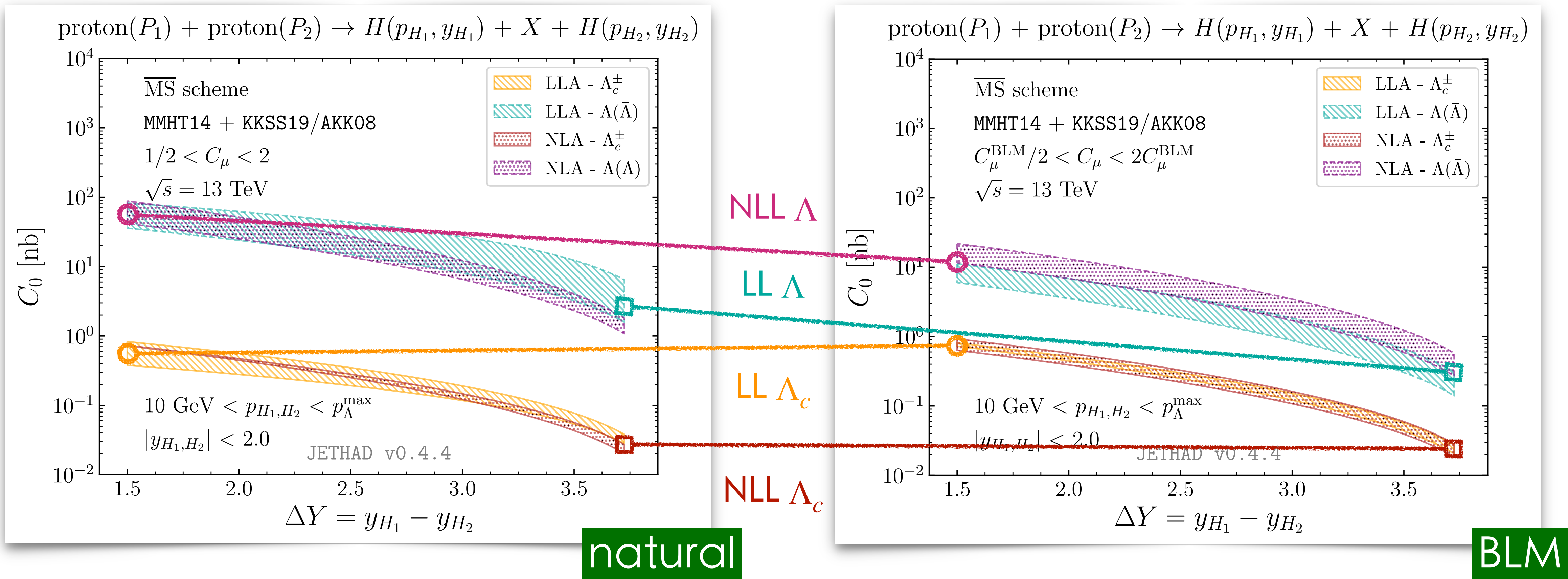
( $\Lambda_c$  baryons, in this slide) [\[F. G. C. et al., Eur. Phys. J. C 81 \(2021\) 8, 780\]](#)

( $H_b$  hadrons) [\[F. G. C. et al., Phys. Rev. D 104 \(2021\) 11, 114007\]](#)

Backup

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Hybrid factorization @work:  $\Lambda_c$  baryons  $|udc\rangle$  versus  $\Lambda$  hyperons  $|uds\rangle$

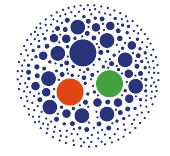


NLL corrections: rapidity distribution **stable** for  $\Lambda_c$ , loses  $\sim 10^1$  magnitude for  $\Lambda$

( $\Lambda_c$  baryons, in this slide) [\[F. G. C. et al., Eur. Phys. J. C 81 \(2021\) 8, 780\]](#)

( $H_b$  hadrons) [\[F. G. C. et al., Phys. Rev. D 104 \(2021\) 11, 114007\]](#)

# Stabilizing effects of heavy-flavor fragmentation

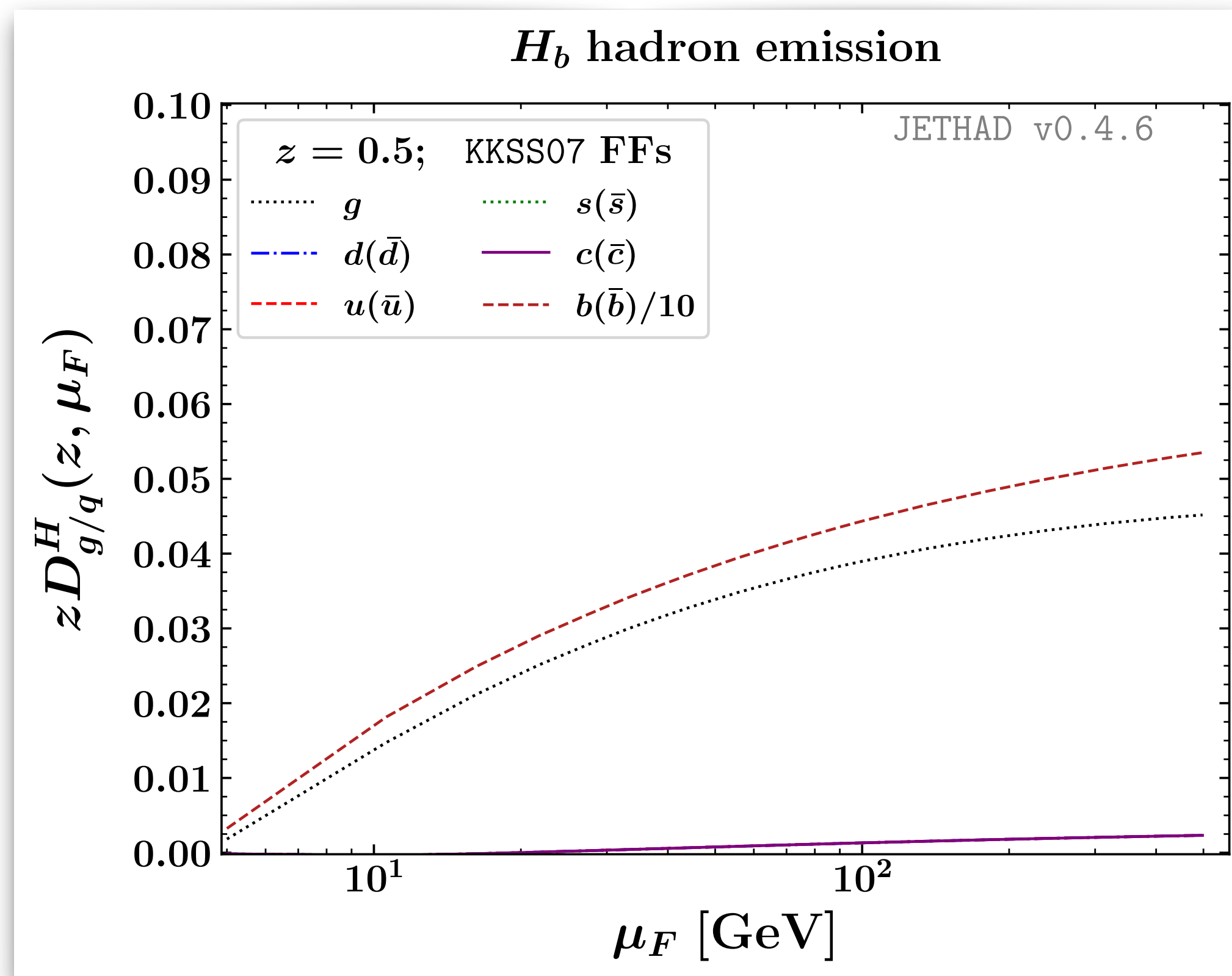


**KKSS07** VFNS collinear FFs for:

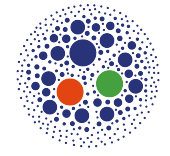
$$H_b = B^\pm, B^0, B_s^0, \Lambda_b$$

[\[B. A. Kniehl, H. Spiesberger, Phys. Rev. D 98 \(2018\) 11, 114010\]](#)

[\[B. A. Kniehl et al., Phys. Rev. D 77 \(2008\) 11, 014011\]](#)



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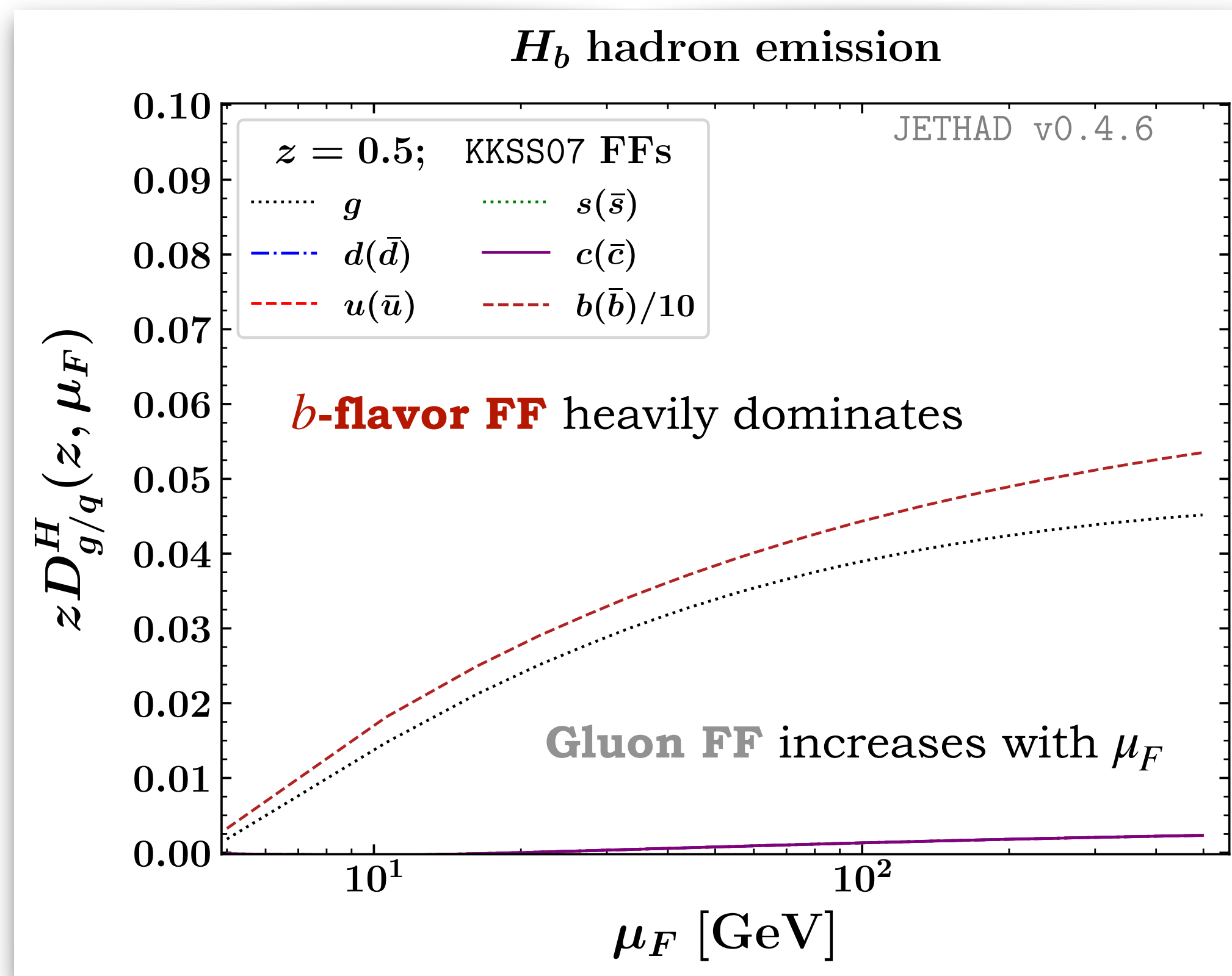


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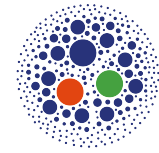
$$H_b = B^\pm, B^0, B_s^0, \Lambda_b$$

[B. A. Kniehl, H. Spiesberger, Phys. Rev. D 98 (2018) 11, 114010]

[B. A. Kniehl et al., Phys. Rev. D 77 (2008) 11, 014011]



# Stabilizing effects of heavy-flavor fragmentation

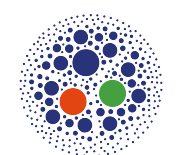
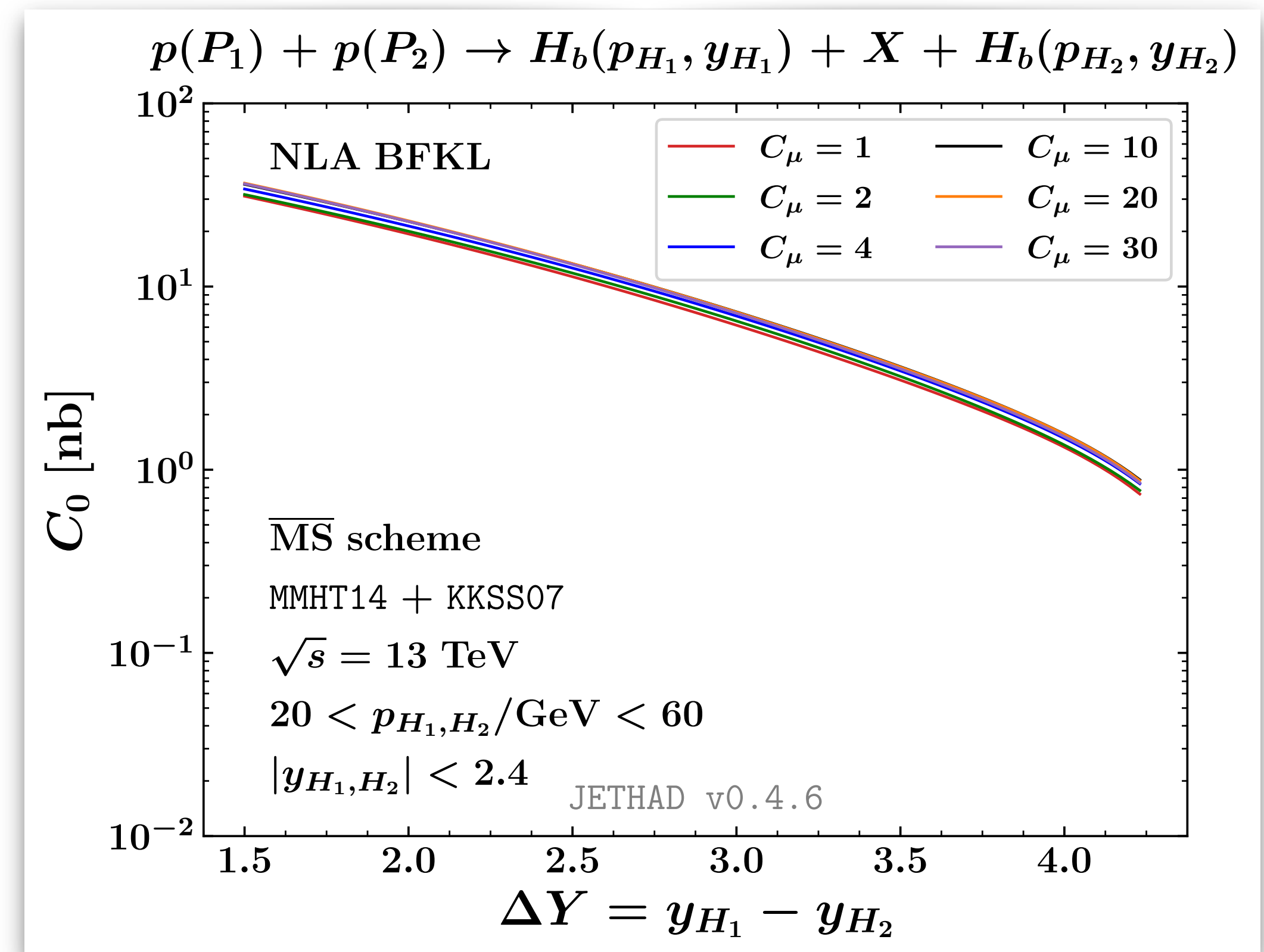
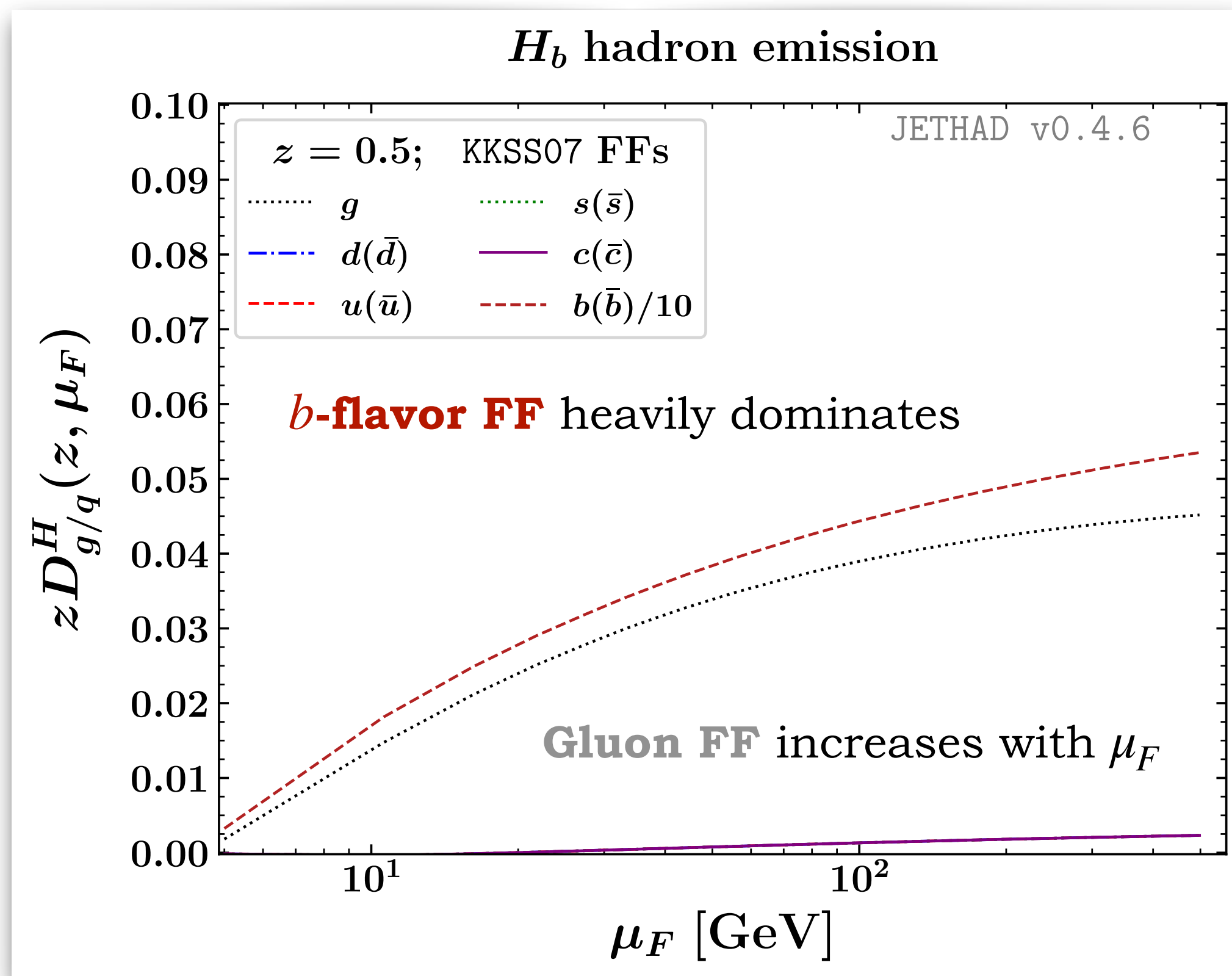


**KKSS07** VFNS collinear FFs for:

$$H_b = B^\pm, B^0, B_s^0, \Lambda_b$$

[\[B. A. Kniehl, H. Spiesberger, Phys. Rev. D 98 \(2018\) 11, 114010\]](#)

[\[B. A. Kniehl et al., Phys. Rev. D 77 \(2008\) 11, 014011\]](#)



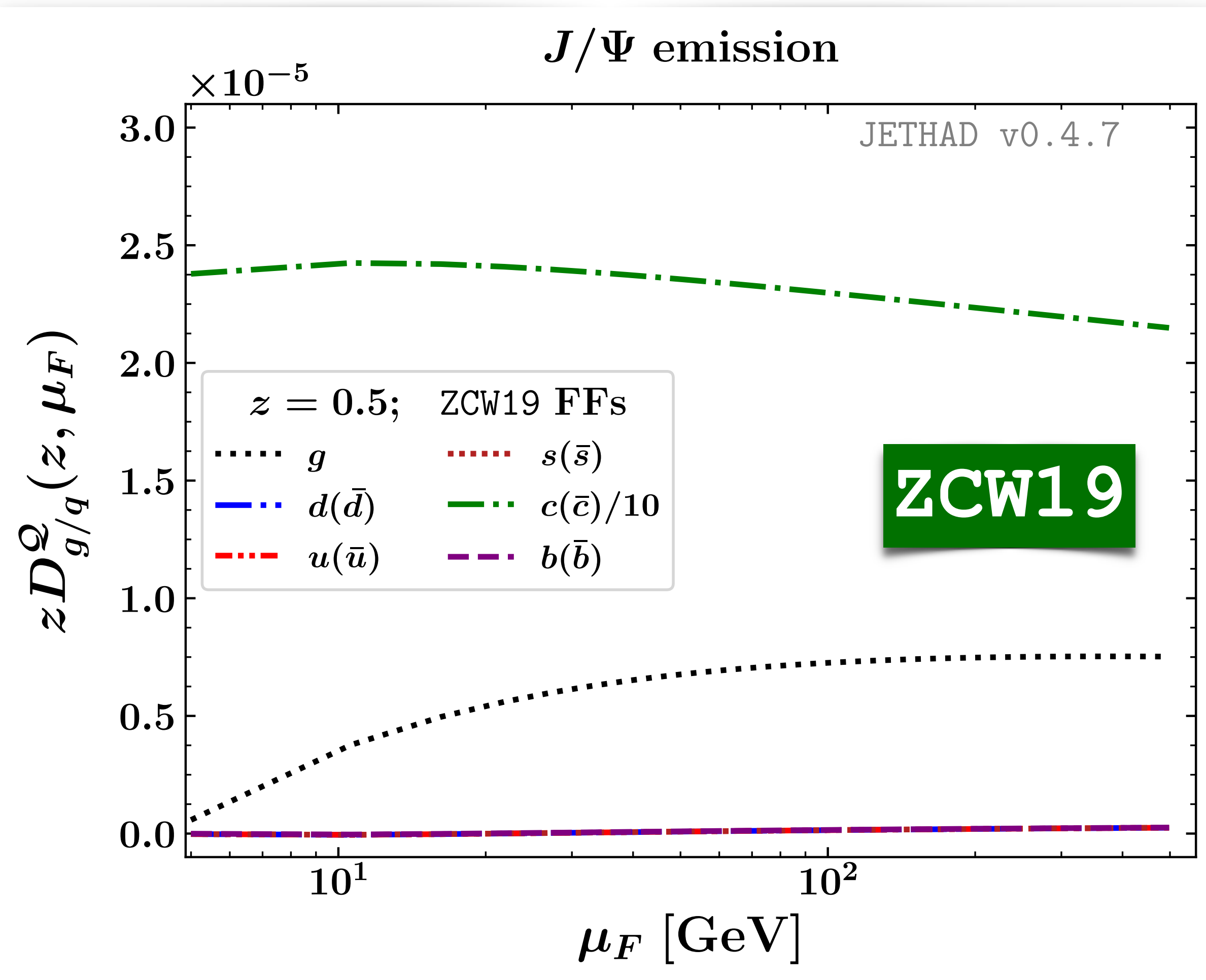
Rapidity distribution **very stable** under scale variations

( $H_b$  hadrons, in this slide) [\[F. G. C. et al., Phys. Rev. D 104 \(2021\) 11, 114007\]](#)

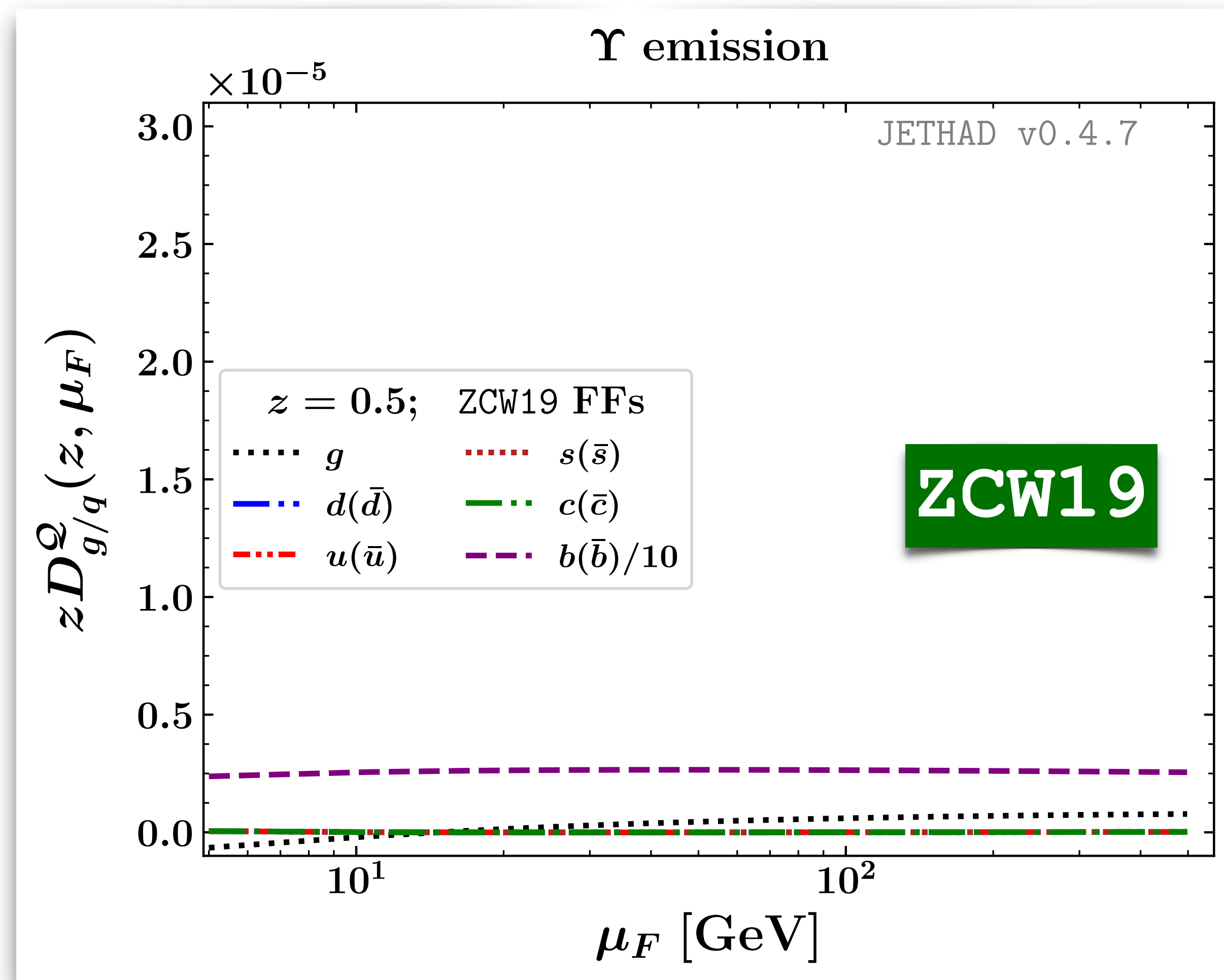


# Vector quarkonium + jet at the LHC

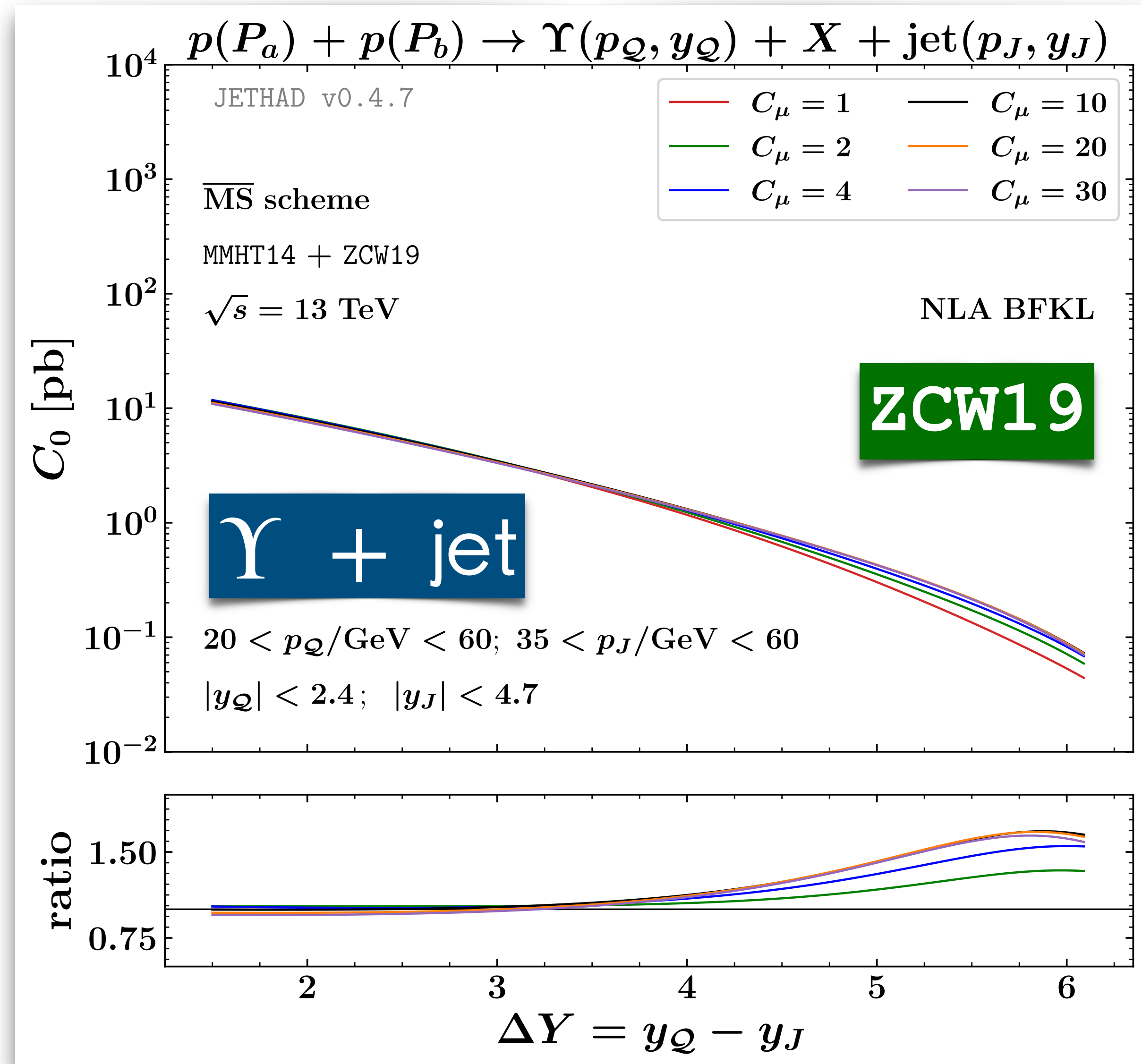
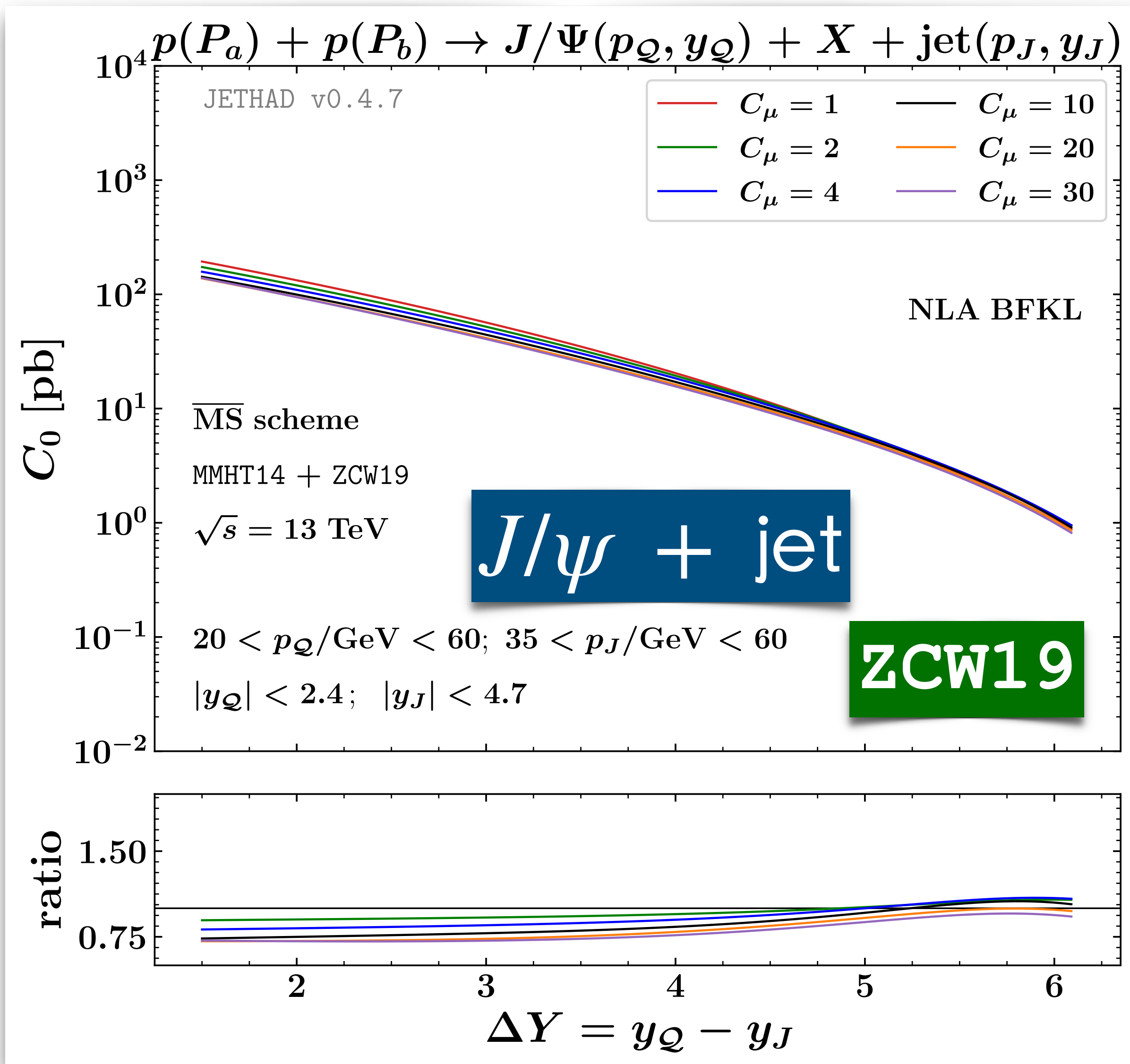
## $J/\psi$ collinear FFs



## $\Upsilon$ collinear FFs



# Vector quarkonium + jet at the LHC



# Heavy-light hadrons

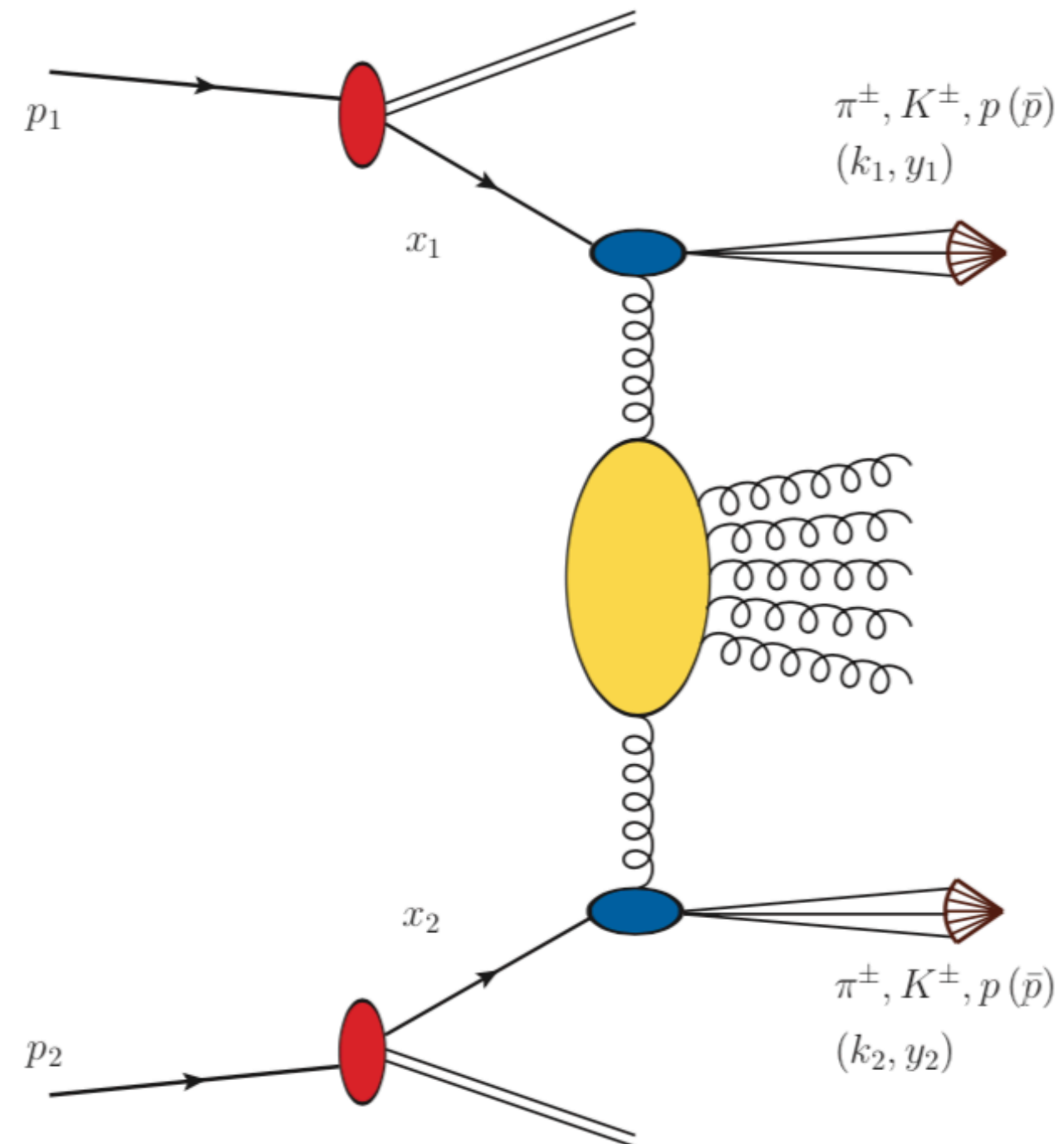
The background features several overlapping, semi-transparent diagrams of hadrons. Each diagram shows a central quark (represented by a red sphere) and a light quark (represented by a blue or green sphere) connected by a wavy line representing a gluon. The diagrams are arranged in a circular pattern, creating a sense of depth and repetition. The overall color palette is soft and pastel, with light blues, greens, and pinks.

# From Higgs+jet to bound states

## Di-hadron and hadron-jet correlations

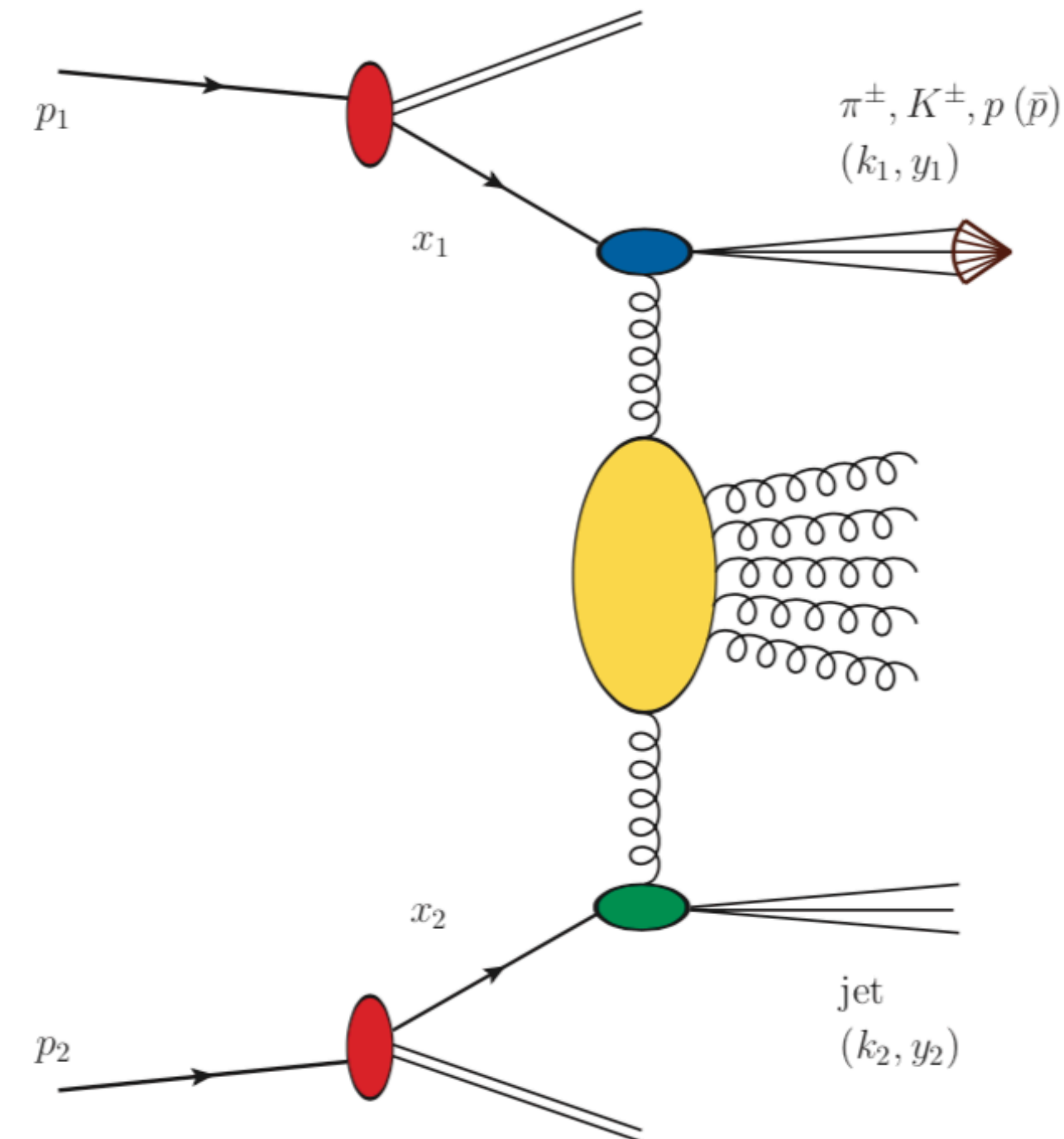
### Inclusive di-hadron production

[D.Yu. Ivanov, A. Papa (2012)] (NLO forward-hadron impact factor)  
[F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017)]



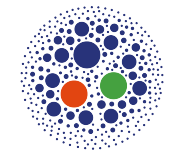
### Inclusive hadron-jet production

[A.D. Bolognino, F.G.C., D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]  
[F.G.C. (in preparation)]



- ◇ NLO impact factors known  $\Rightarrow$  full NLA BFKL analysis feasible
- ◇ PDFs + FFs at work (both), hadrons at smaller rapidities than jets (di-hadron)
- ◇ genuine *asymmetric* cuts in transverse momenta (hadron-jet)

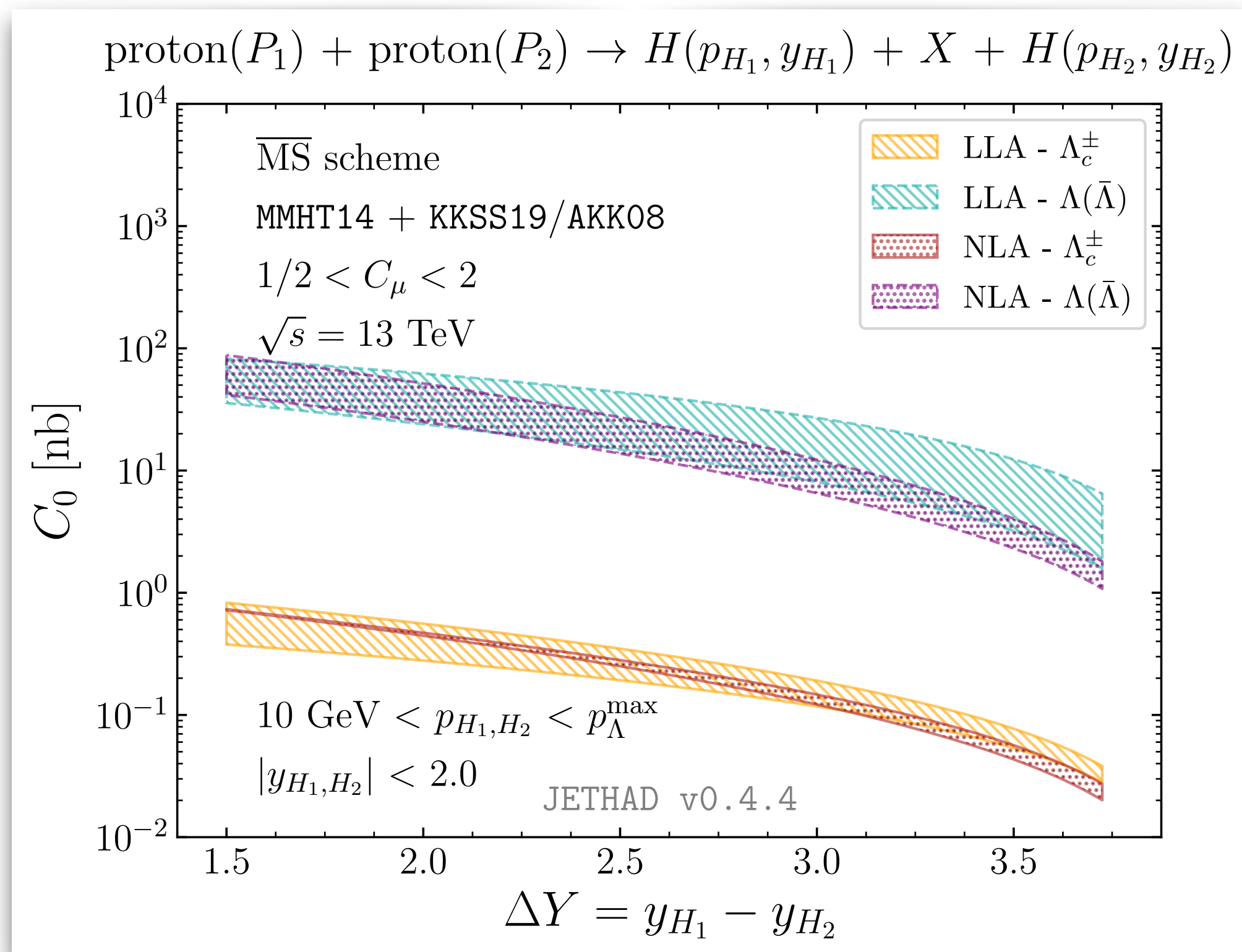
# From light to heavy-light bound states



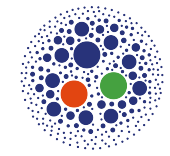
Light-hadron+jet: **higher-order instabilities** via scale variation, as in Mueller-Navelet (i!)

$\Lambda_c$  baryons

[\[F. G. C. et al., Eur. Phys. J. C \(2021\) 8, 780\]](#)



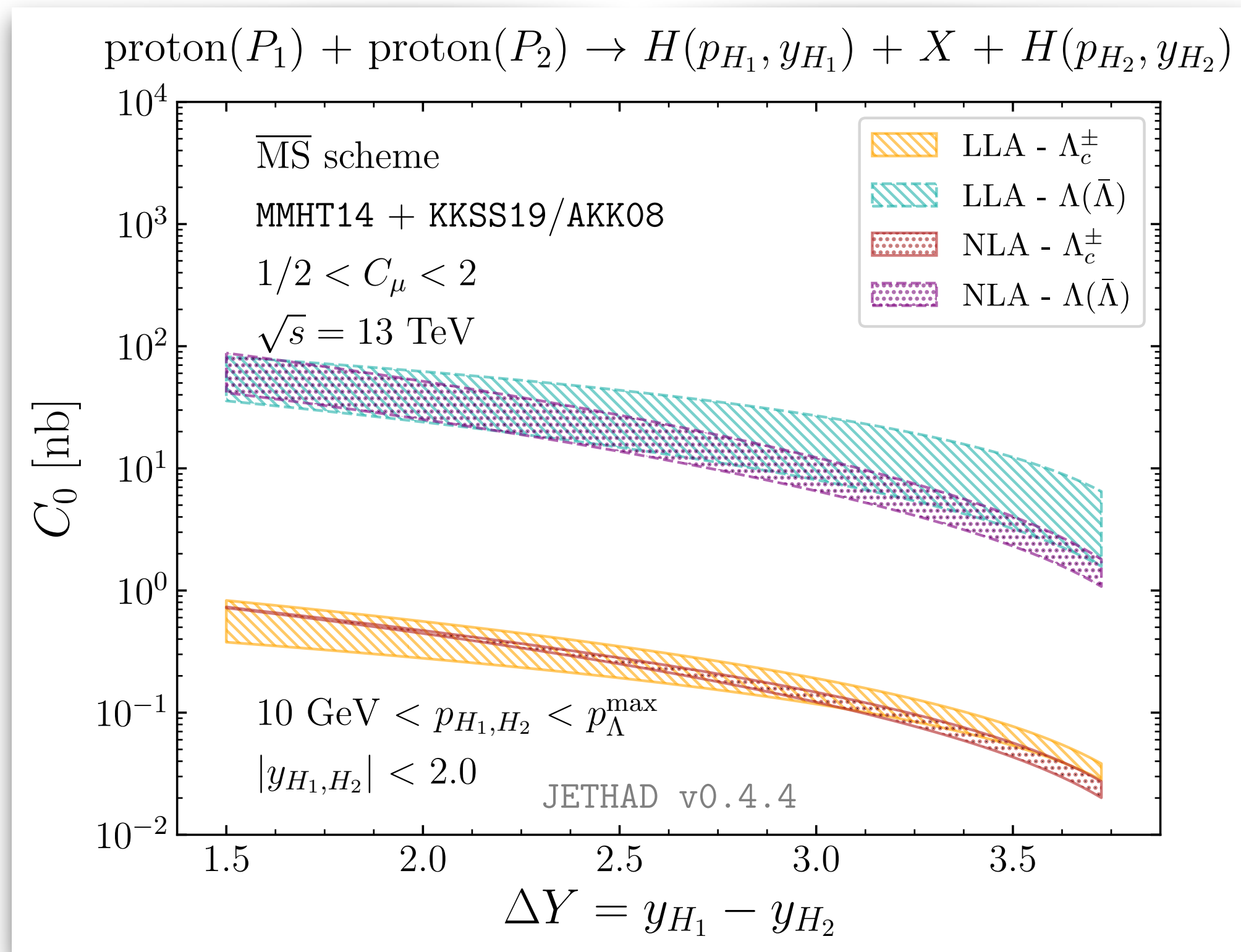
# From light to heavy-light bound states



Light-hadron+jet: higher-order instabilities via scale variation, as in Mueller-Navelet (!)

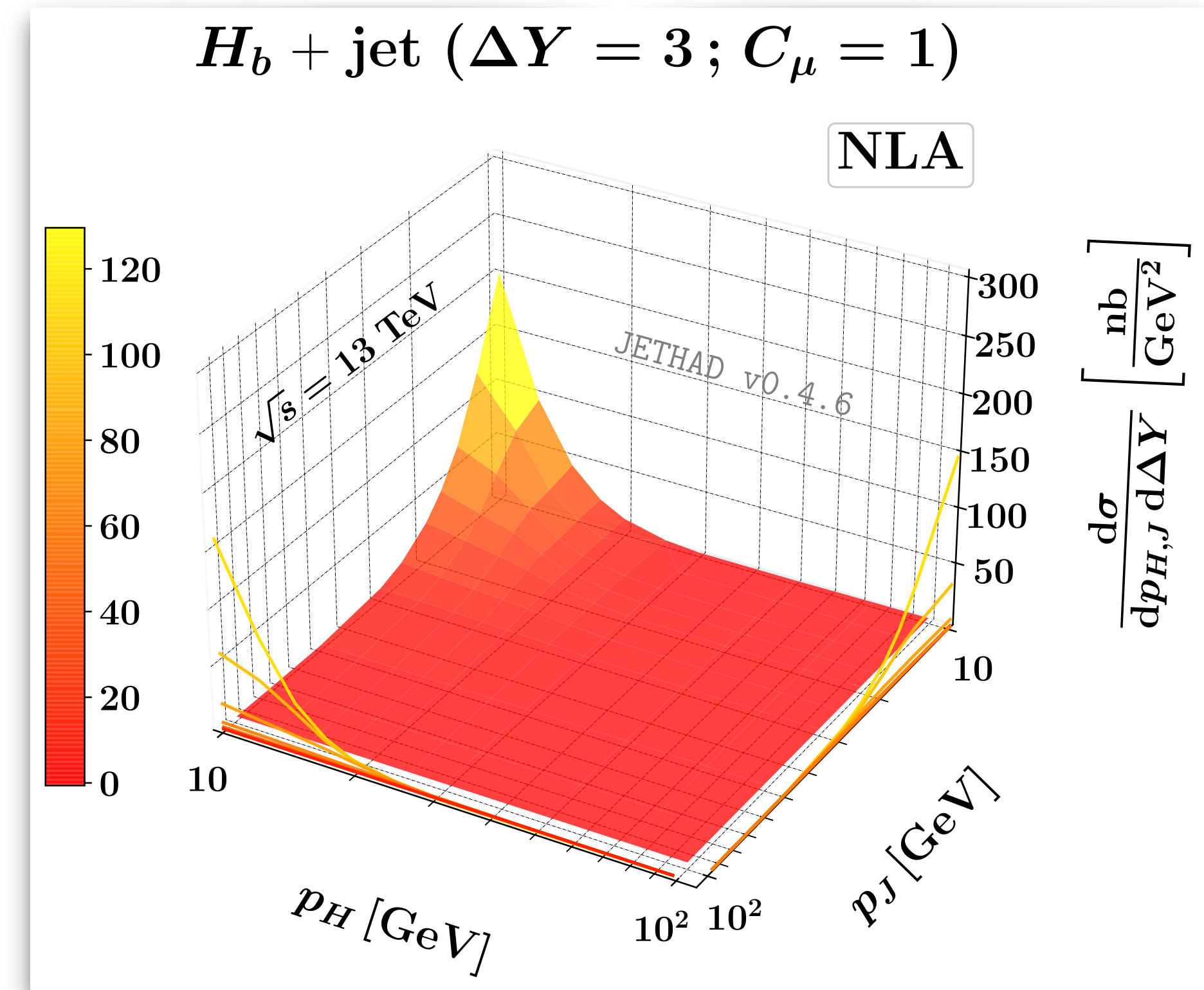
$\Lambda_c$  baryons

[F. G. C. et al., Eur. Phys. J. C (2021) 8, 780]

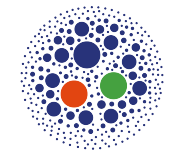


Bottom-flavored hadrons

[F. G. C. et al., Phys. Rev. D 104 (2021) 11, 114007]



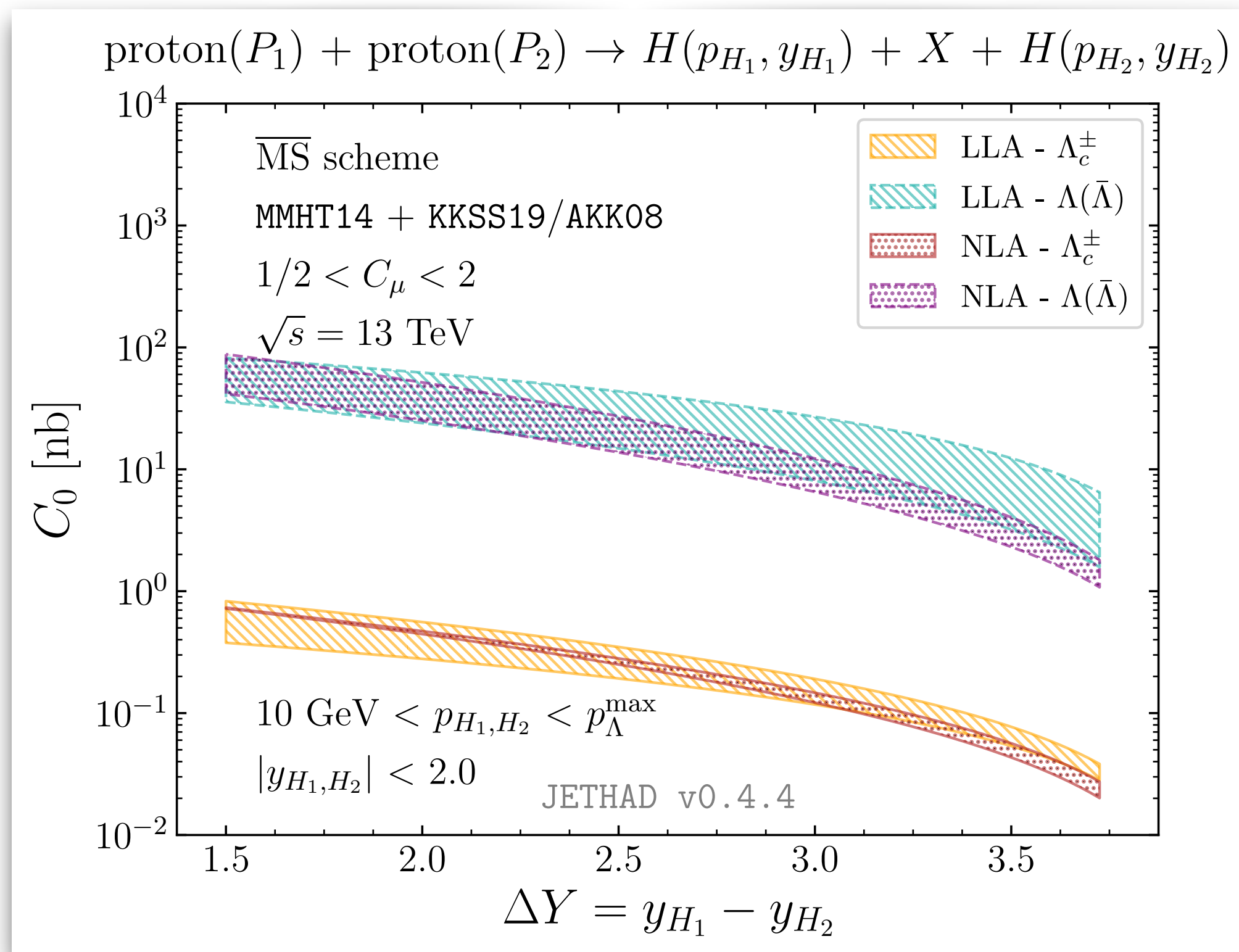
# From light to heavy-light bound states



Light-hadron+jet: higher-order instabilities via scale variation, as in Mueller-Navelet (!)

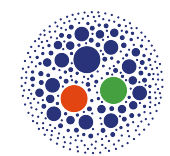
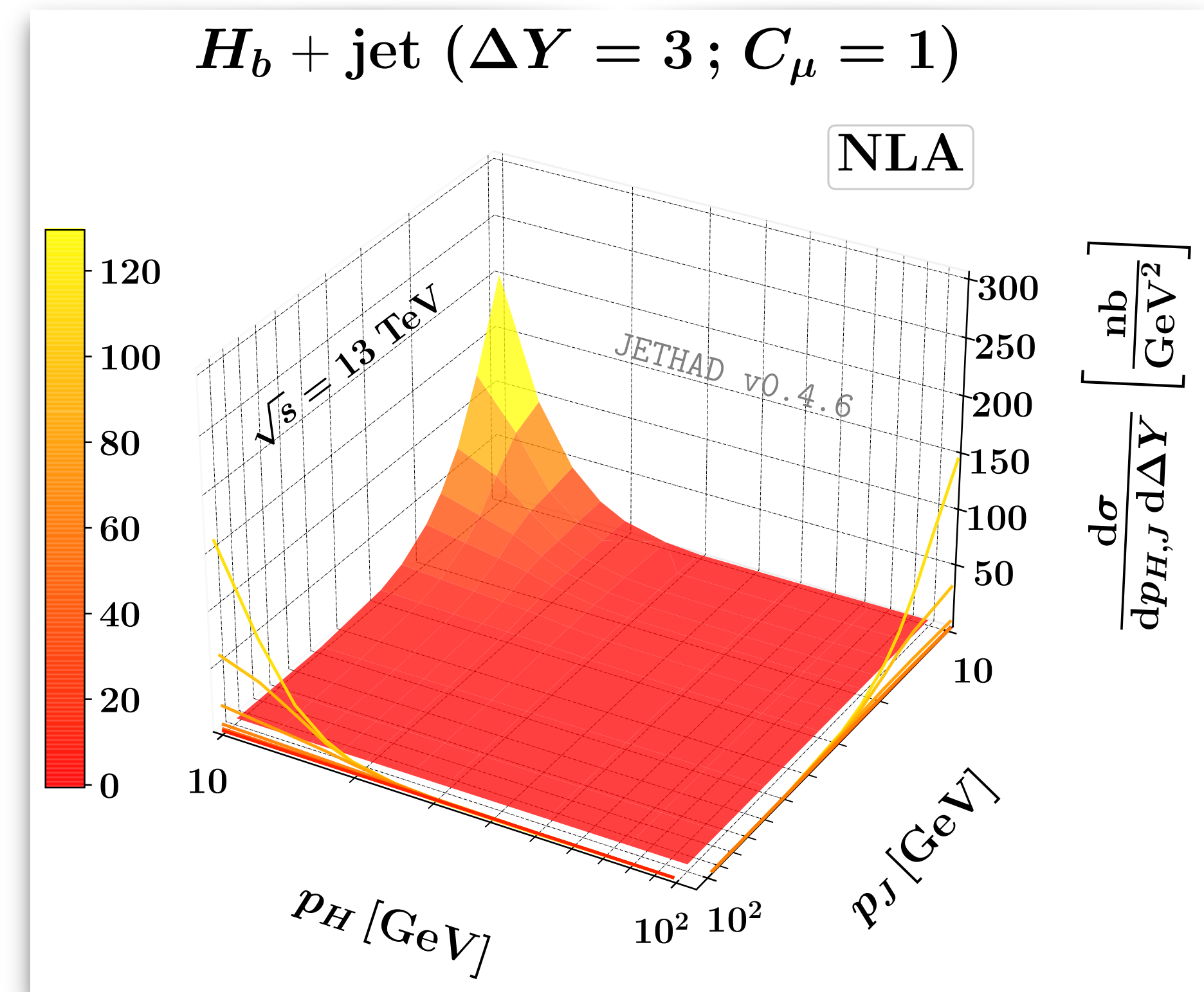
$\Lambda_c$  baryons

[F. G. C. et al., Eur. Phys. J. C (2021) 8, 780]



Bottom-flavored hadrons

[F. G. C. et al., Phys. Rev. D 104 (2021) 11, 114007]



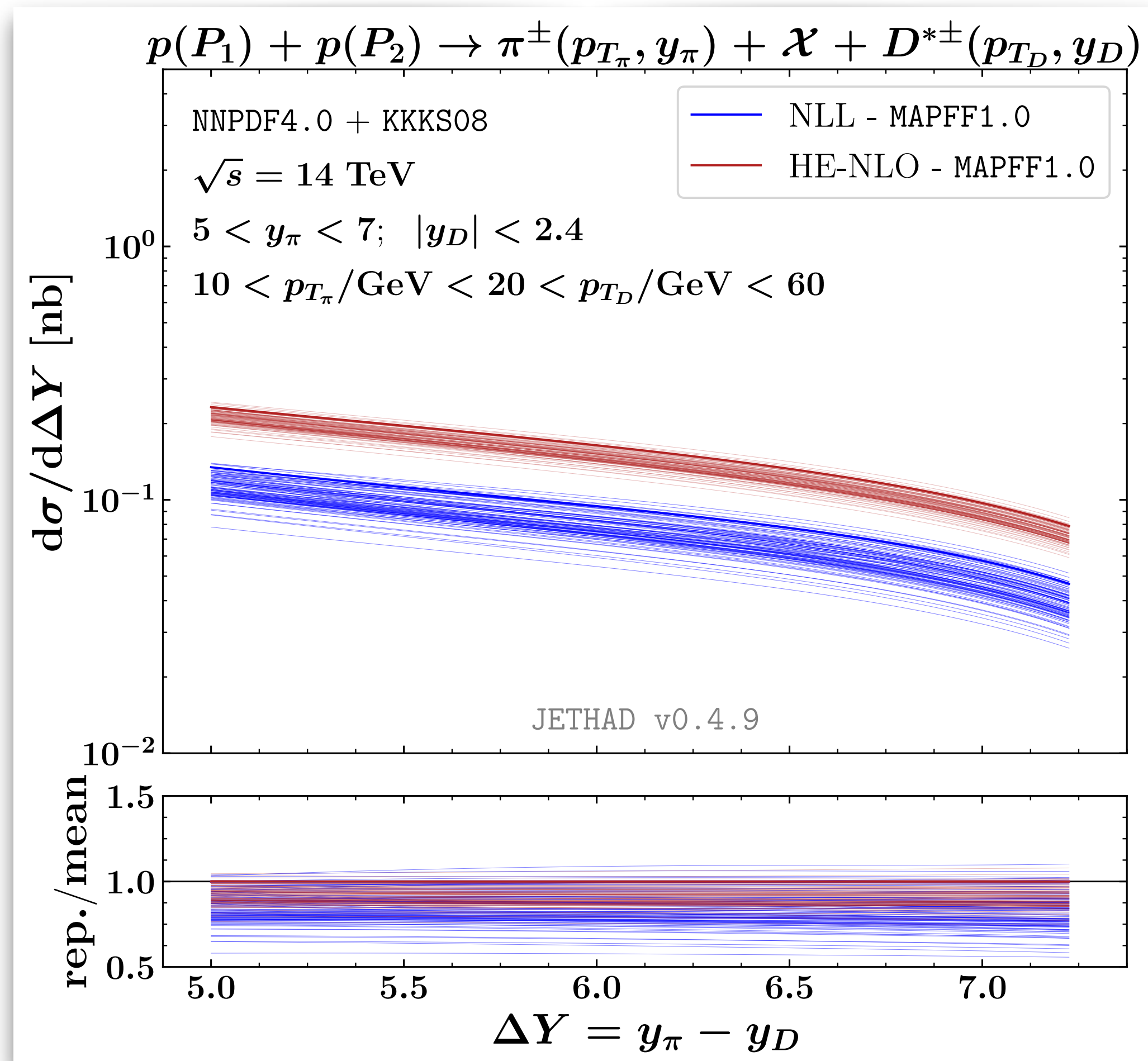
Natural stability as a tool to investigate high-energy dynamics of QCD



# Rapidity distributions @FPF+ATLAS

**Inclusive  $\pi^\pm$  (FPF) +  $D^{*\pm}$  (ATLAS) production**

[FPF Snowmass Whitepaper]

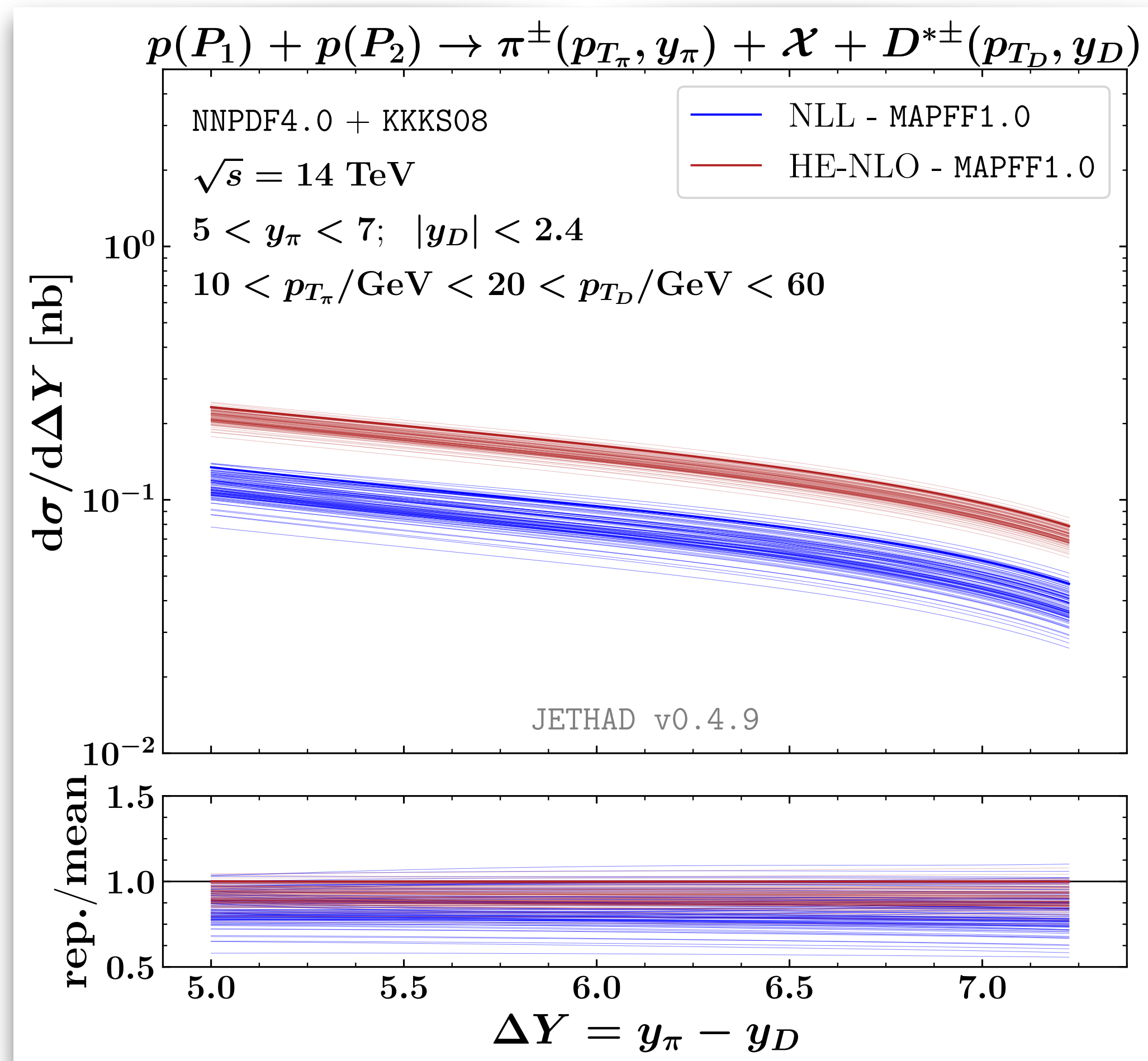




# Rapidity distributions @FPF+ATLAS

**Inclusive  $\pi^\pm$  (FPF) +  $D^{*\pm}$  (ATLAS) production**

[FPF Snowmass Whitepaper]



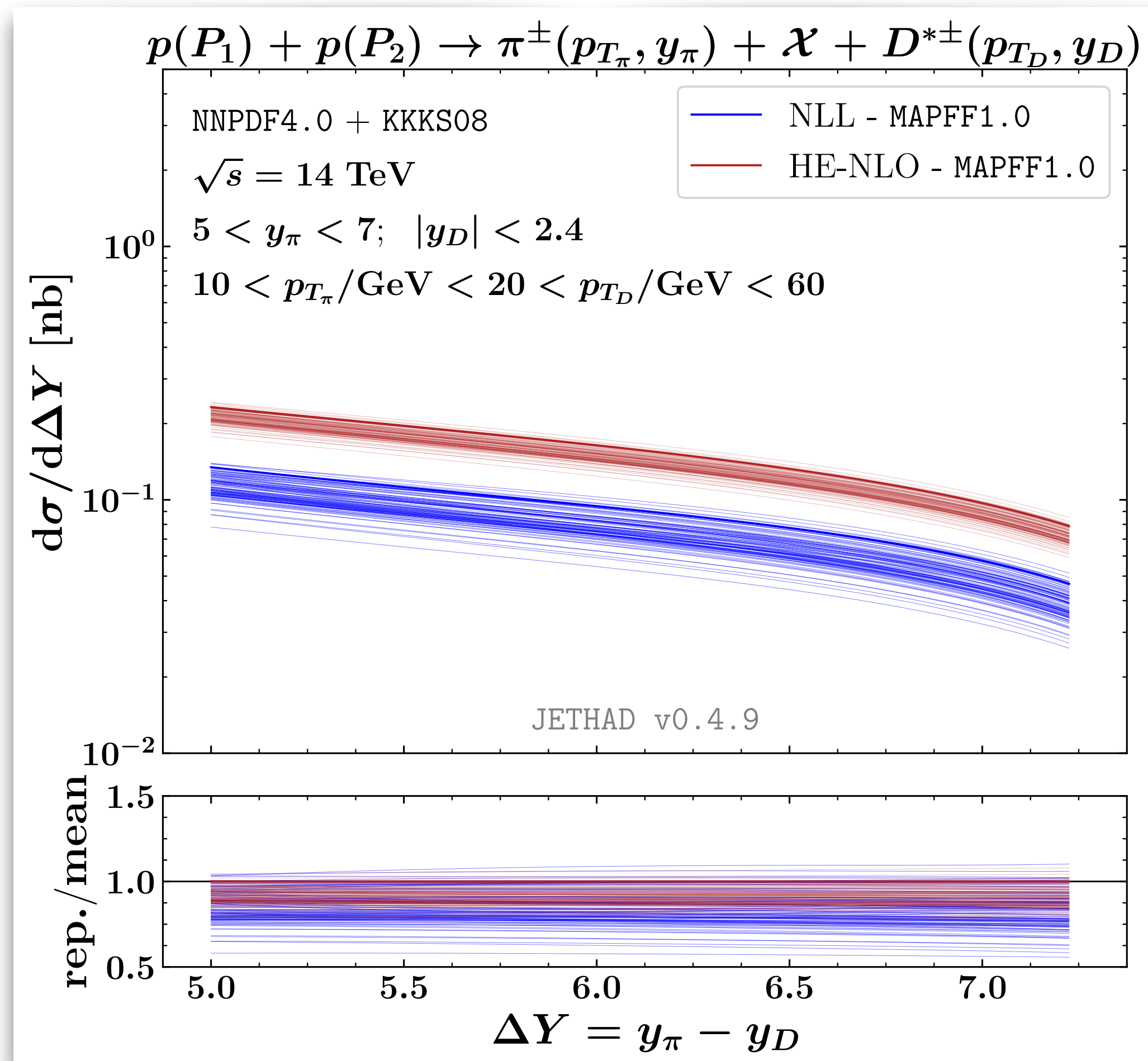
\* Impact of collinear FFs on  $\Delta Y$ -distribution

\* Replica method at work

# Rapidity distributions @FPF+ATLAS

## Inclusive $\pi^\pm$ (FPF) + $D^{*\pm}$ (ATLAS) production

[FPF Snowmass Whitepaper]



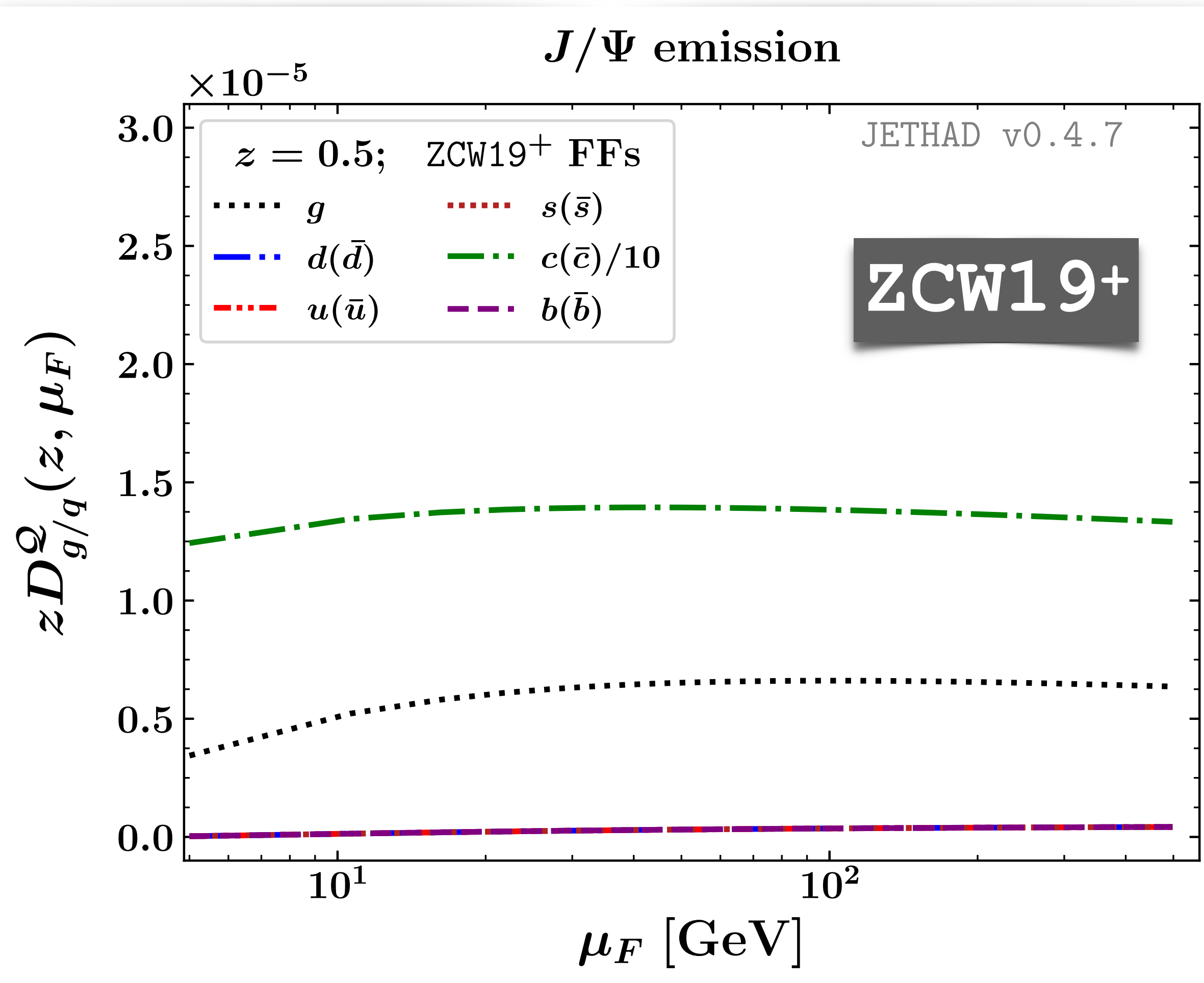
- \* Impact of collinear FFs on  $\Delta Y$ -distribution
- \* Replica method at work
- \* Larger spread of replicas at NLL
- \* Probe FFs in complementary ranges
  - Weight of FF replicas in the same set
  - Different sets via functional correlation?
- \* Complementary studies on FFs



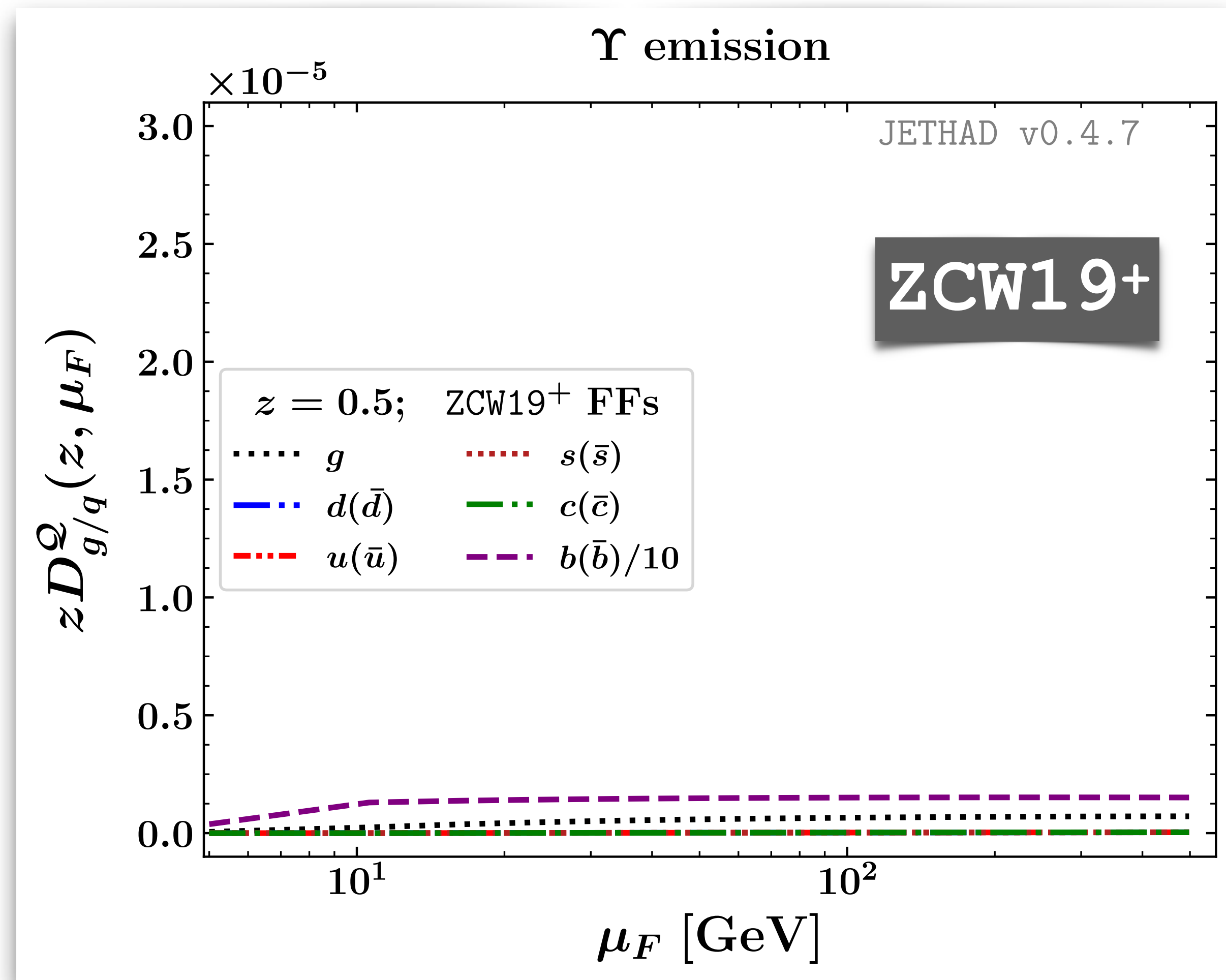
# Quarkonia

# Vector quarkonium + jet at the LHC

## $J/\psi$ collinear FFs

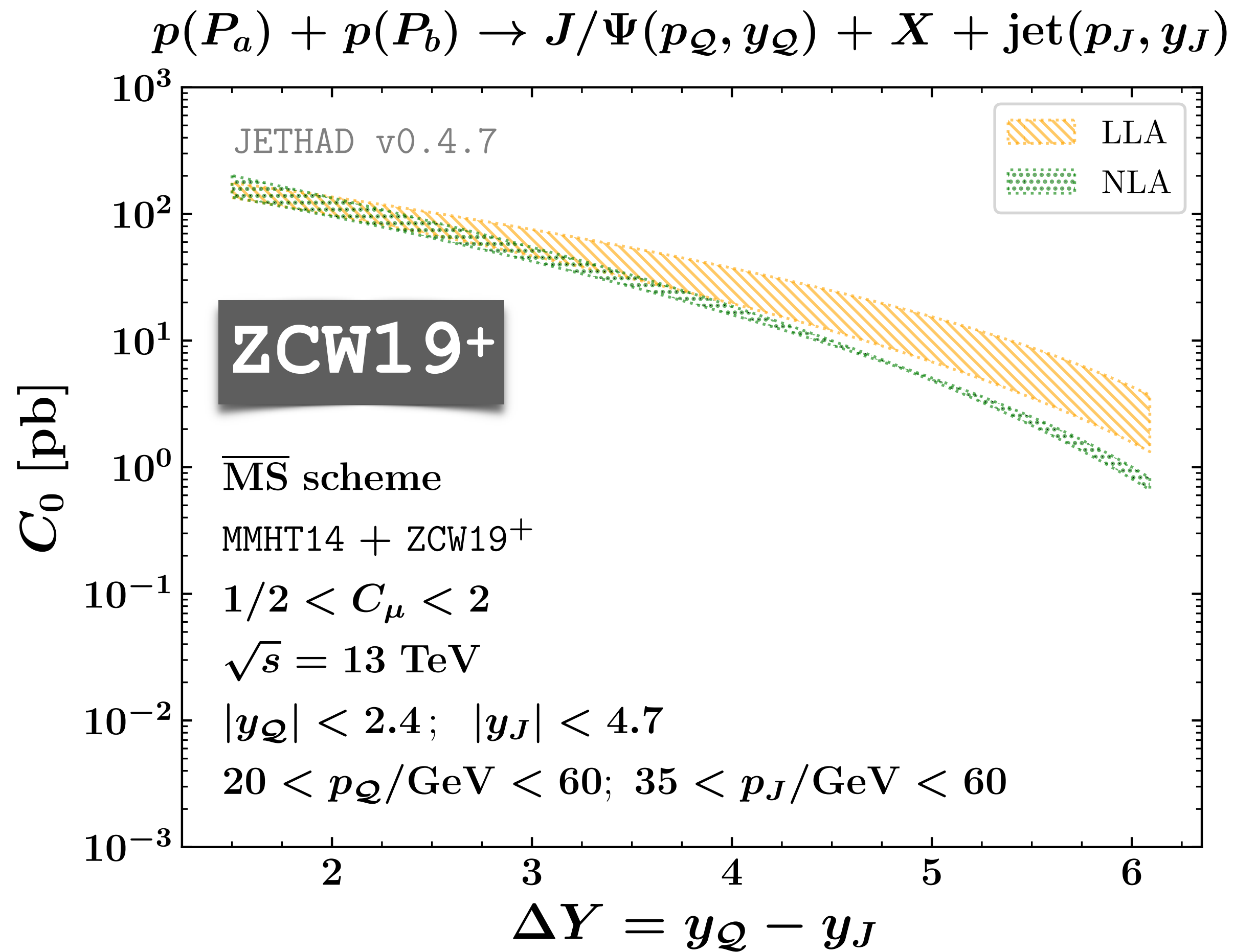


## $\Upsilon$ collinear FFs

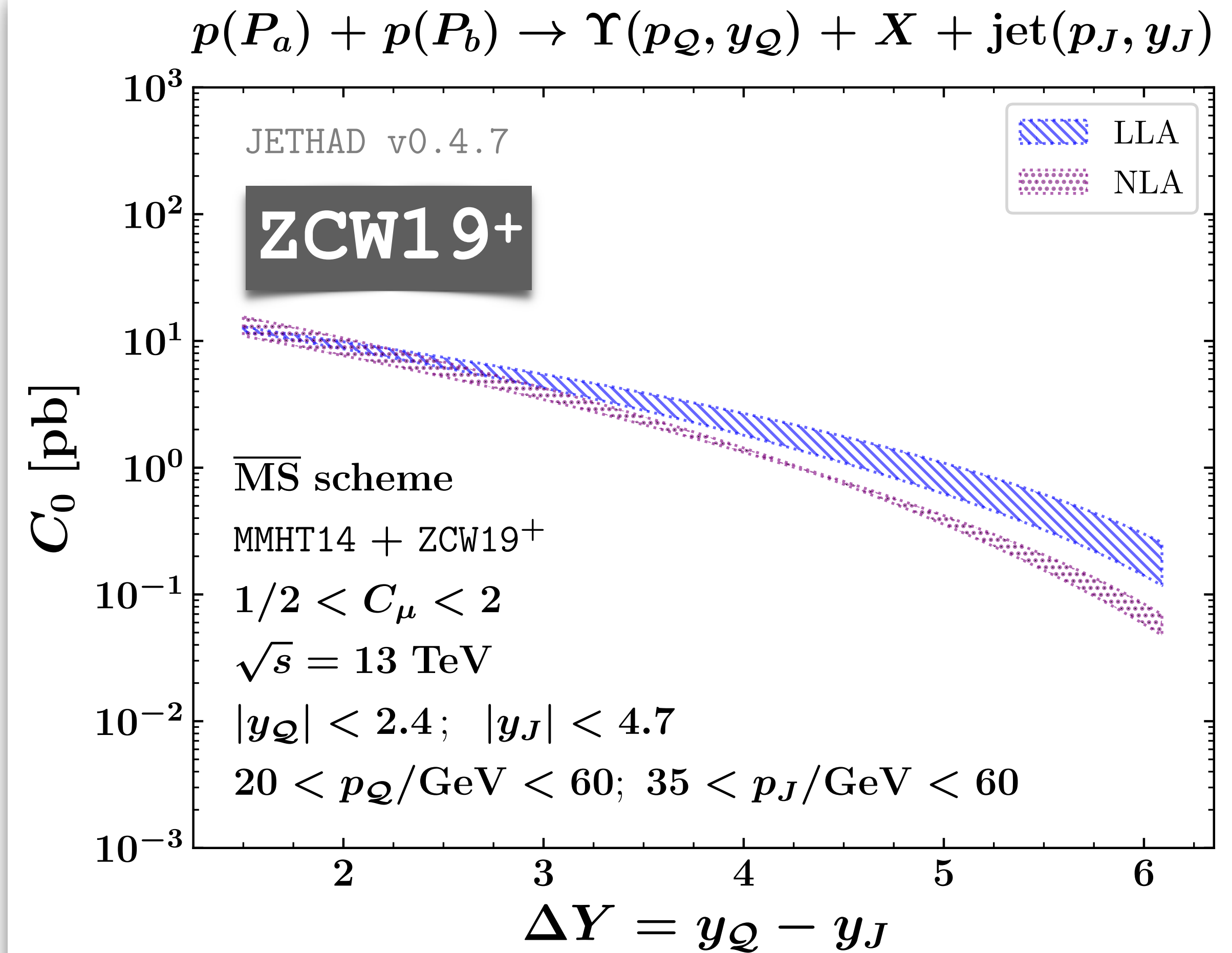


# Vector quarkonium + jet at the LHC

## $J/\psi + \text{jet}$



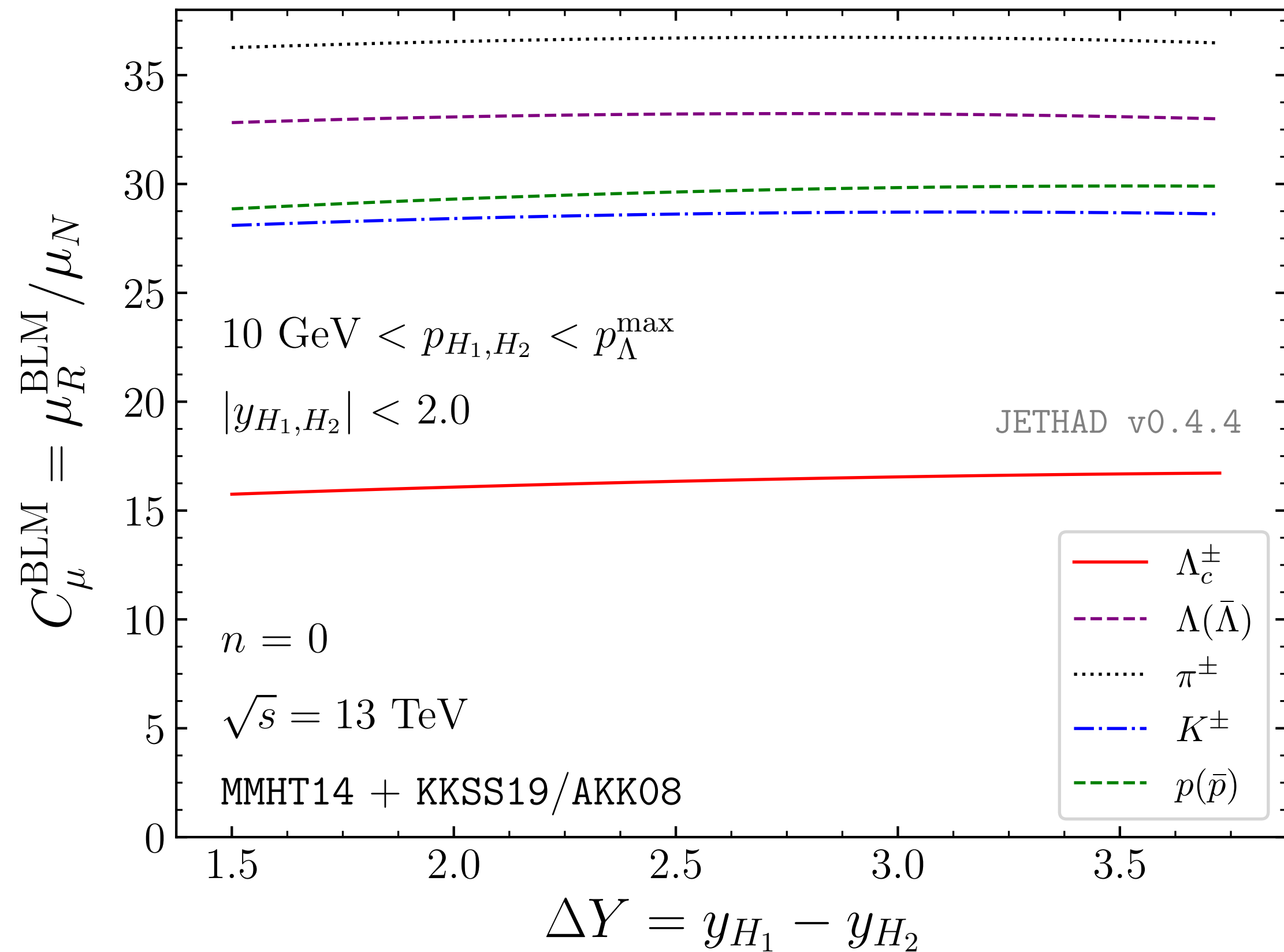
## $\Upsilon + \text{jet}$



# Heavy flavor at the LHC: BLM scales

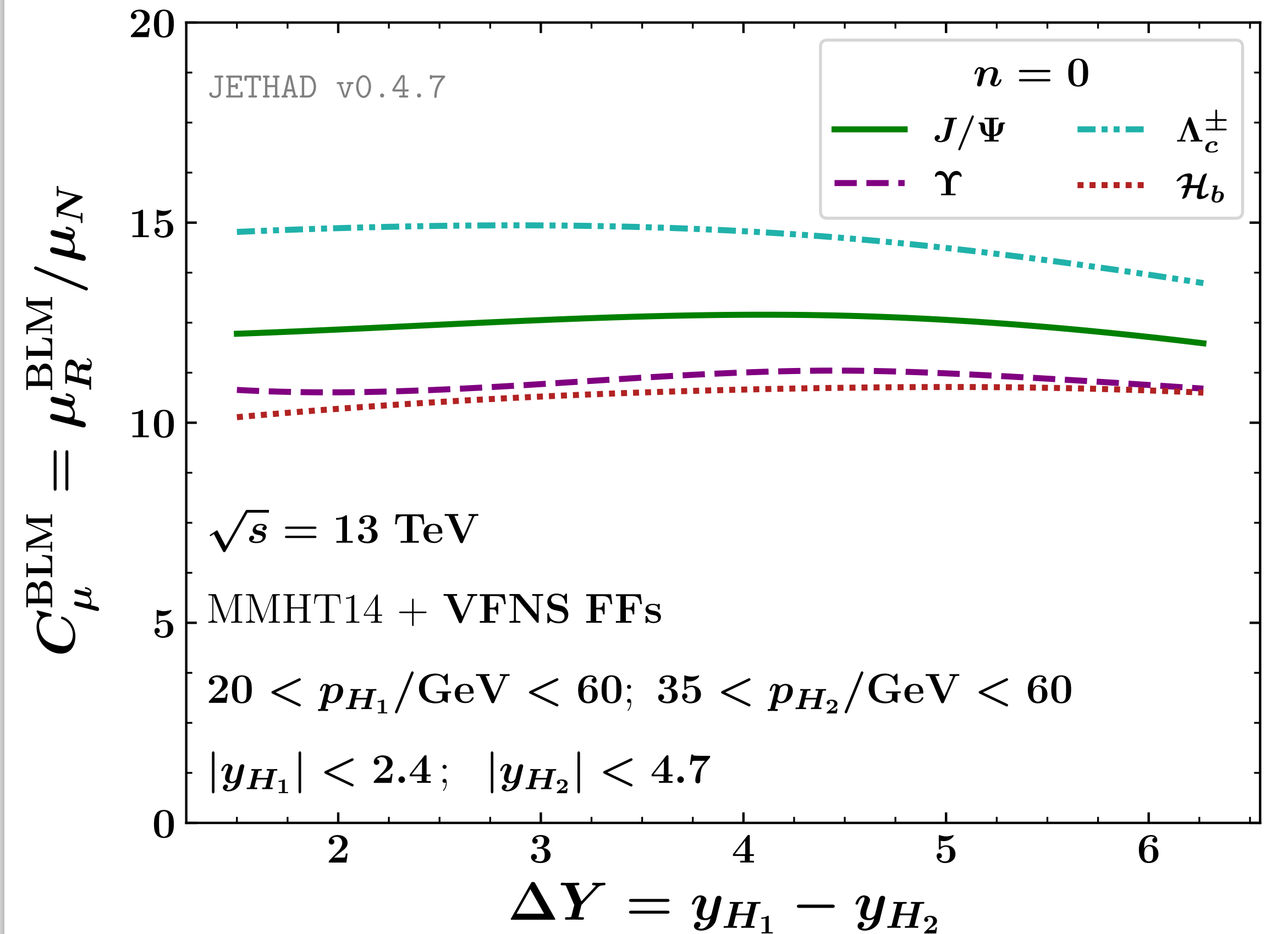
## Heavy-light hadrons

$$\text{proton}(p_1) + \text{proton}(p_2) \rightarrow H(p_{H_1}, y_{H_1}) + X + H(p_{H_2}, y_{H_2})$$

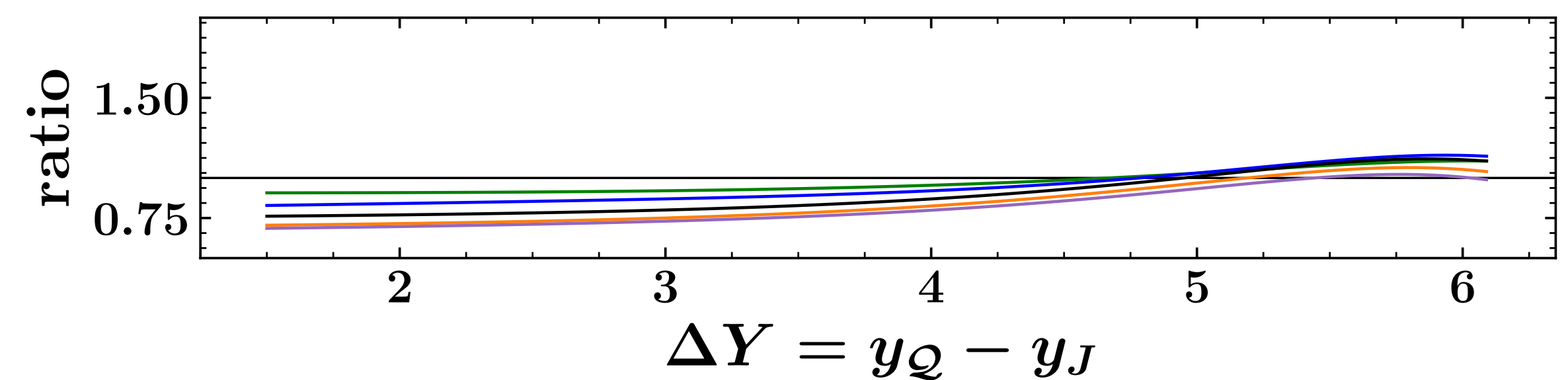
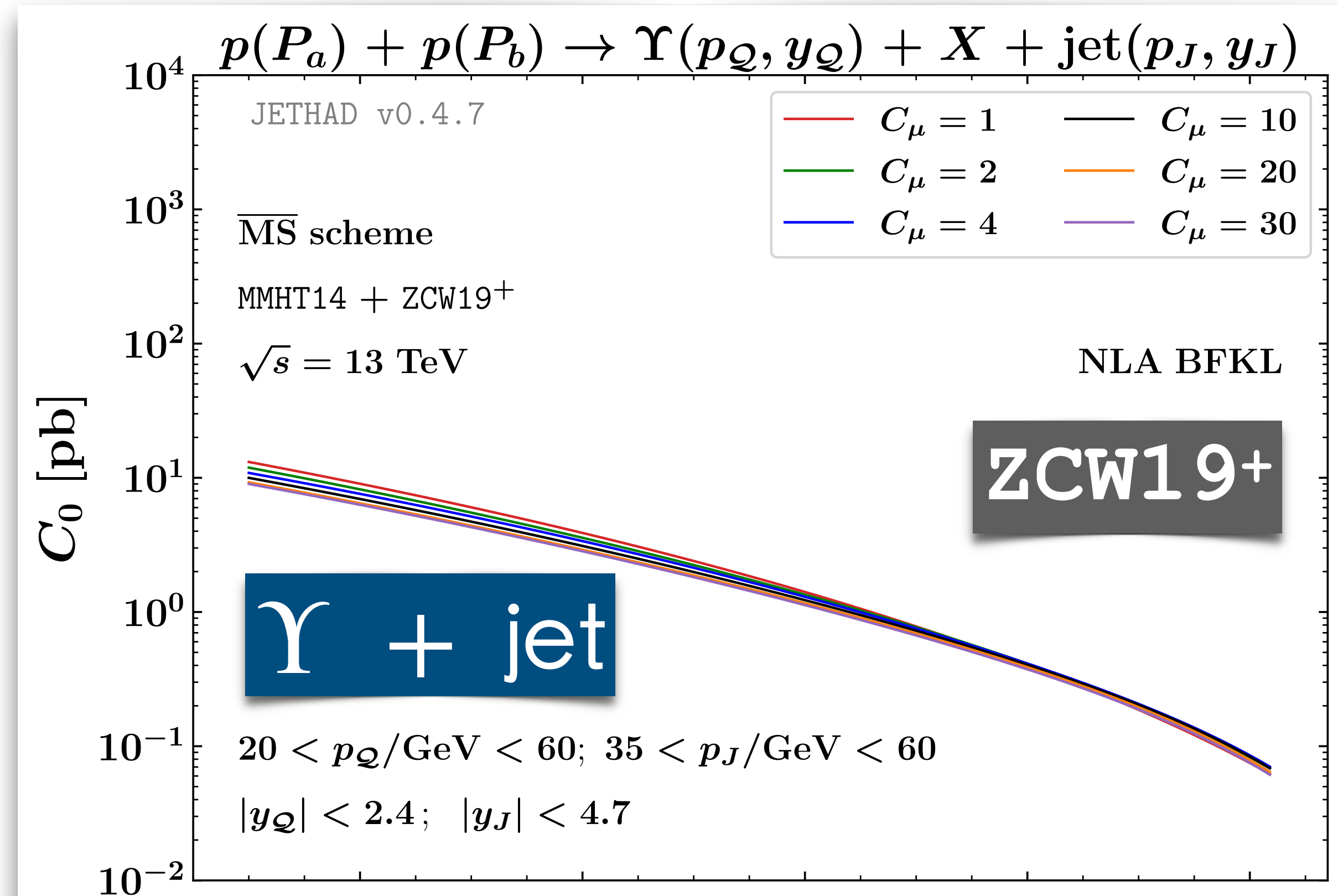
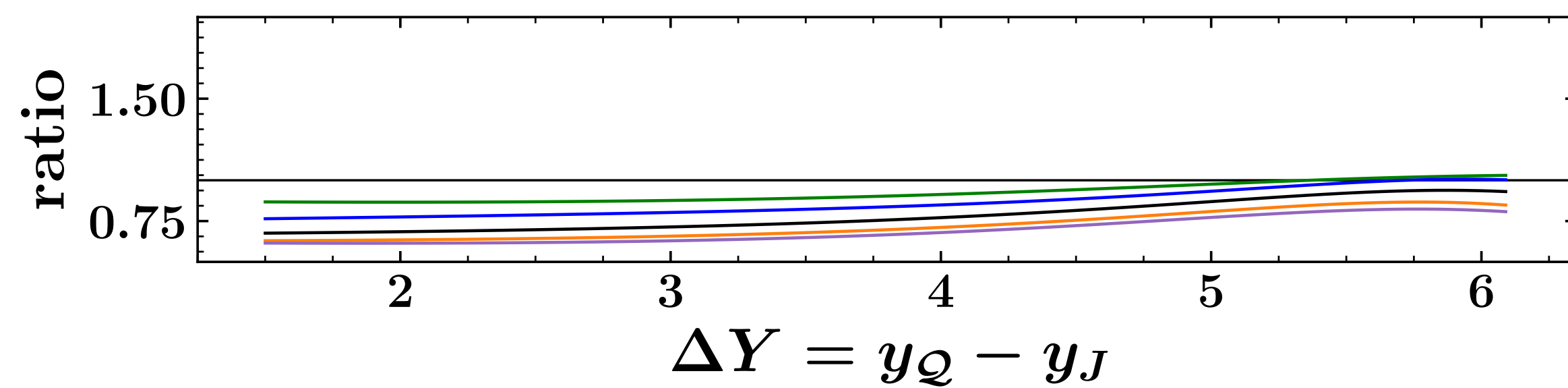
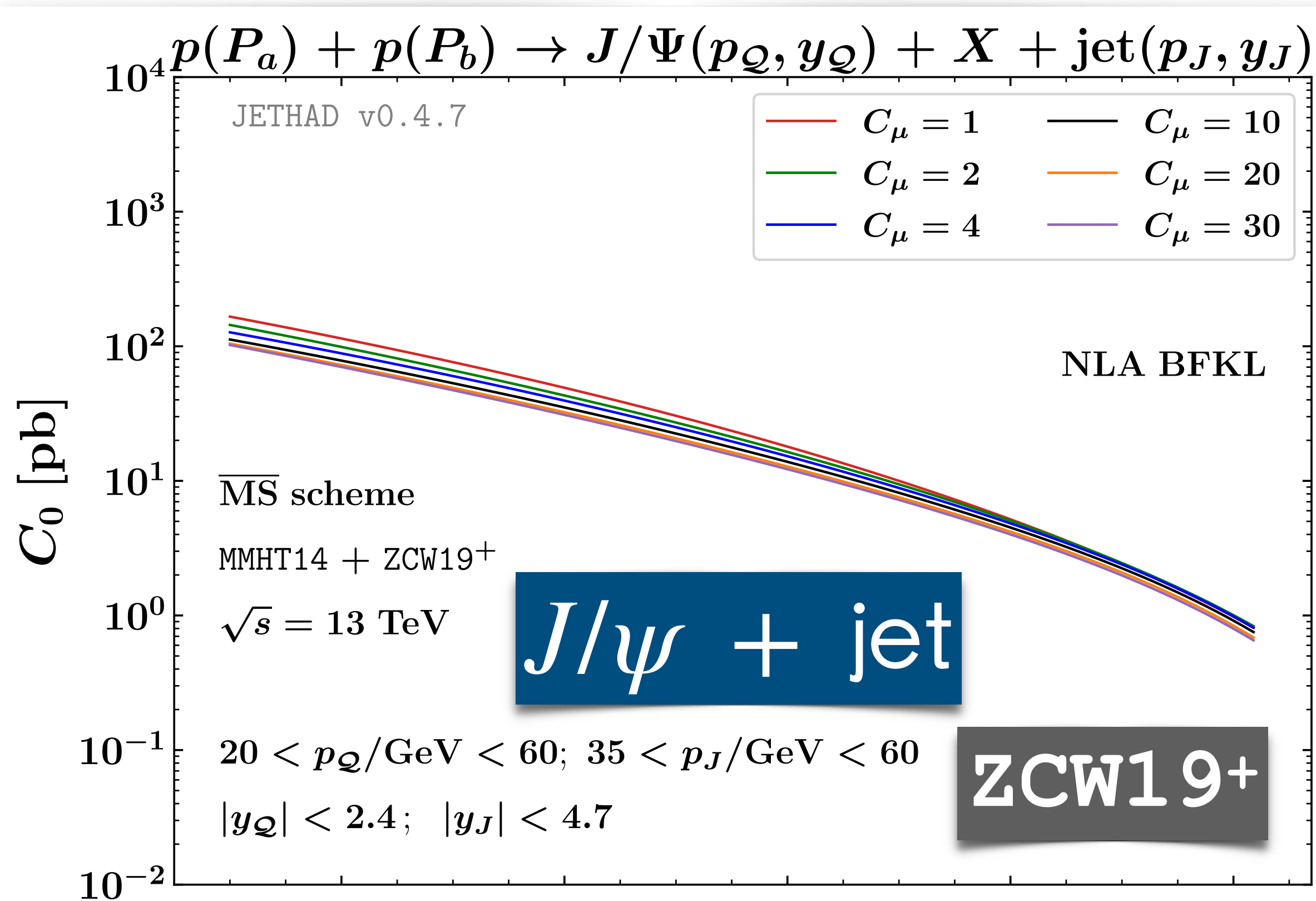


## Vector quarkonia

$$p(P_a) + p(P_b) \rightarrow H(p_{H_1}, y_{H_1}) + X + H(p_{H_2}, y_{H_2})$$



# Vector quarkonium + jet at the LHC

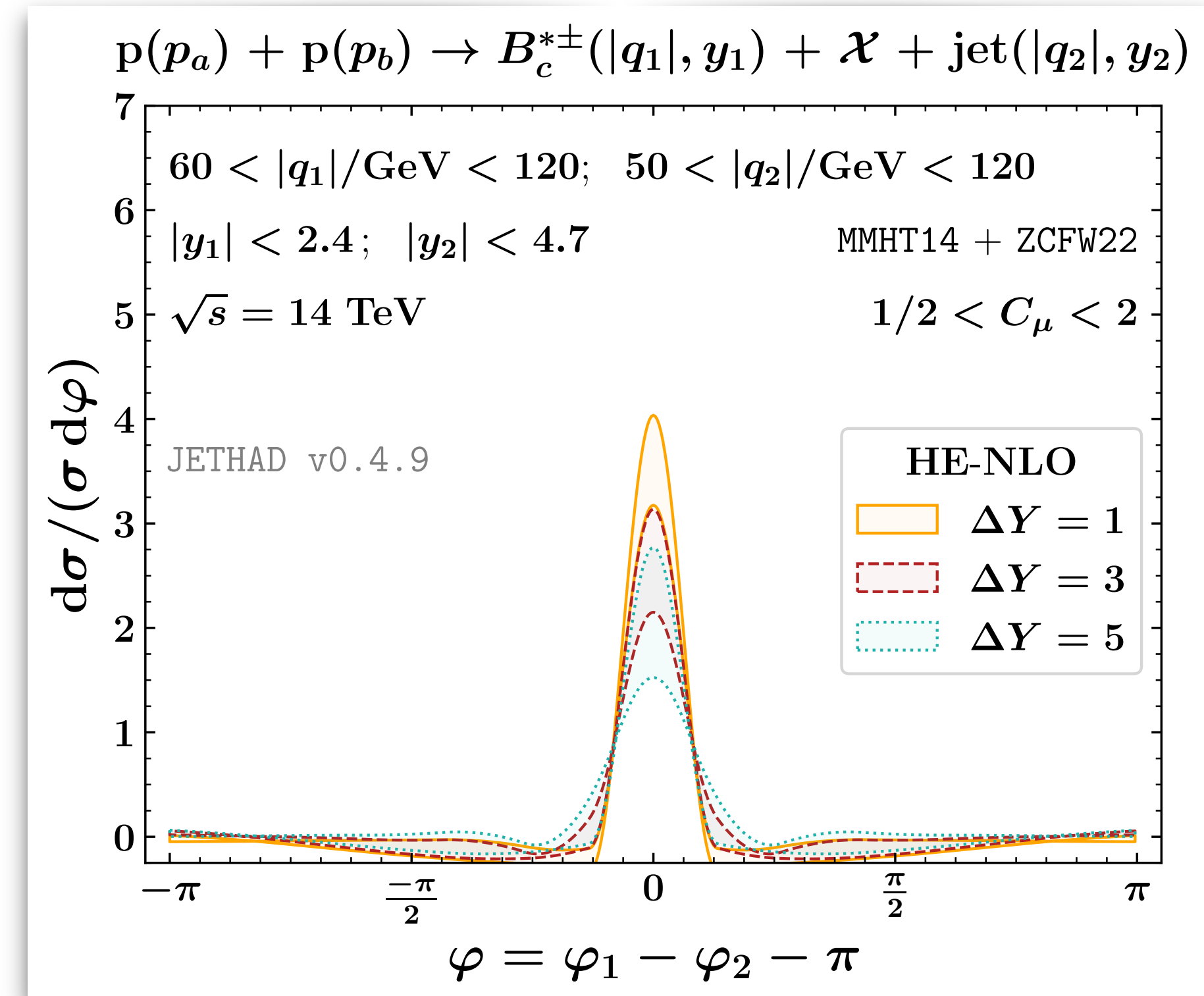
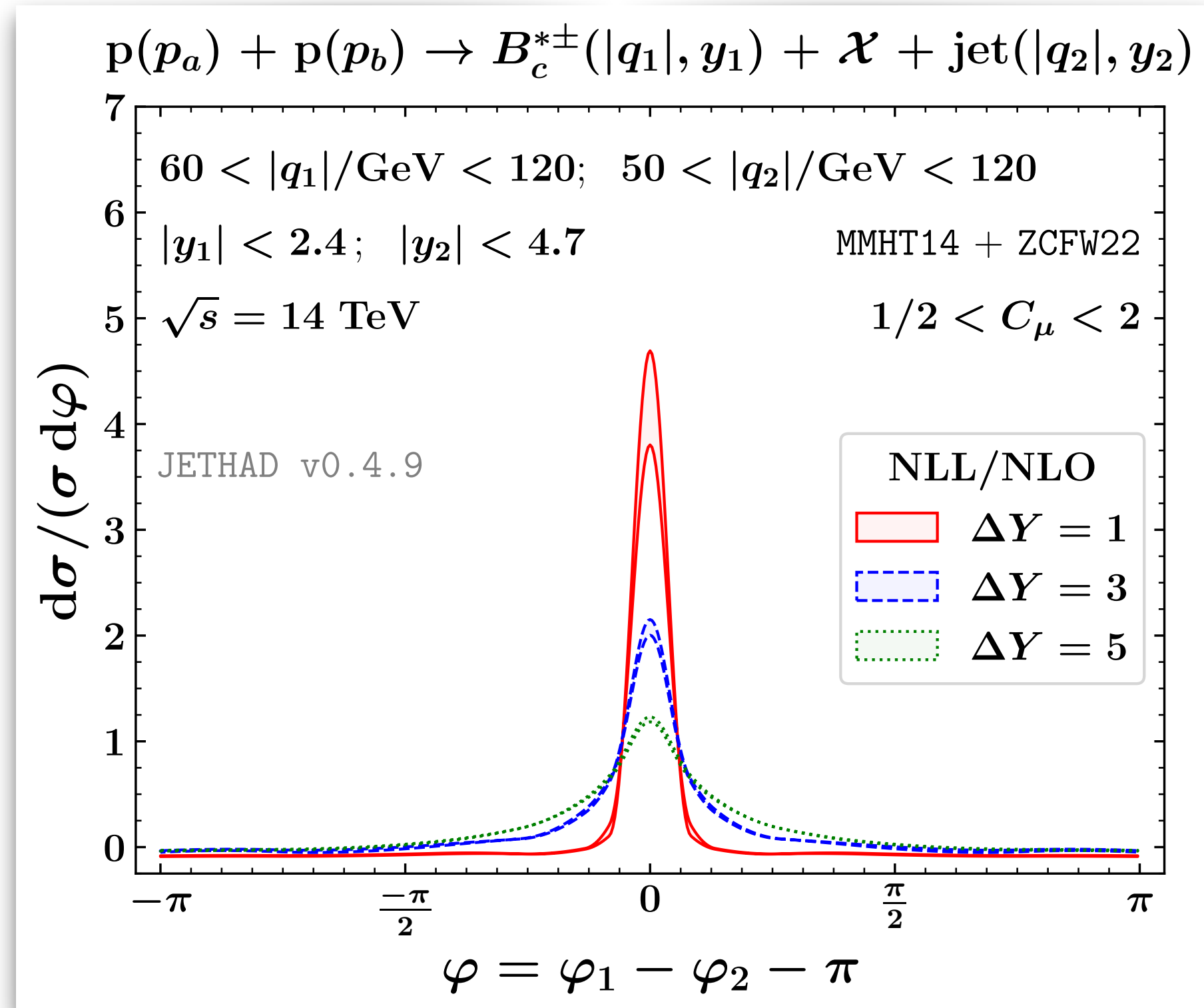


# Charmed $B$ -mesons from single-parton fragmentation

(2) **!** Let us consider  $B_c(^1S_0)$  and  $B_c(^3S_1)$  at large  $p_T \rightarrow$  single-parton fragmentation from **NRQCD** !

(NLO heavy quark) [\[X. Zheng et al., Phys. Rev. D 100 \(2019\) 3, 034004\]](#)

(NLO gluon) [\[X. Zheng et al., JHEP 05 \(2022\) 036\]](#)



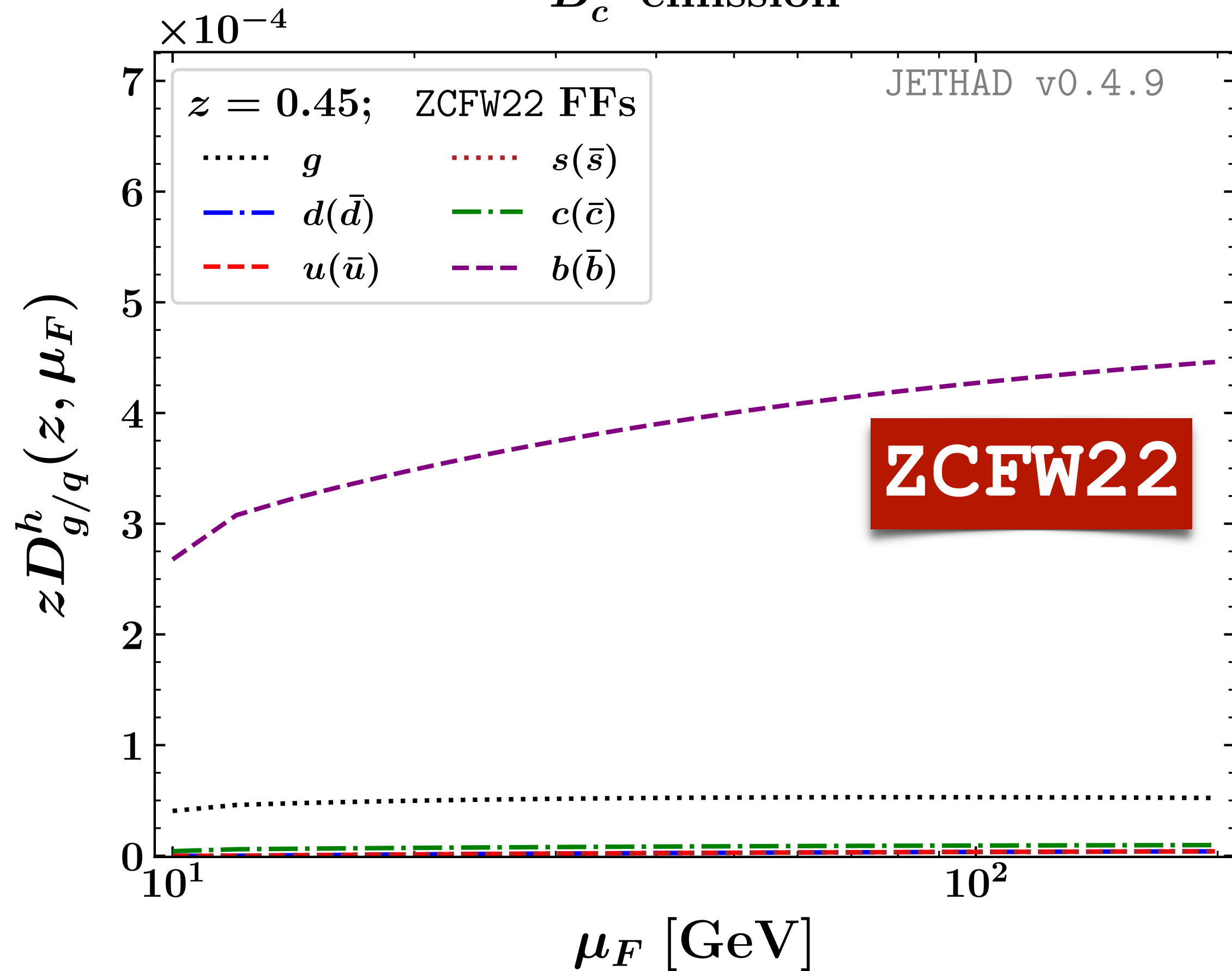


# Charmed $B$ -mesons + jet at the HL-LHC

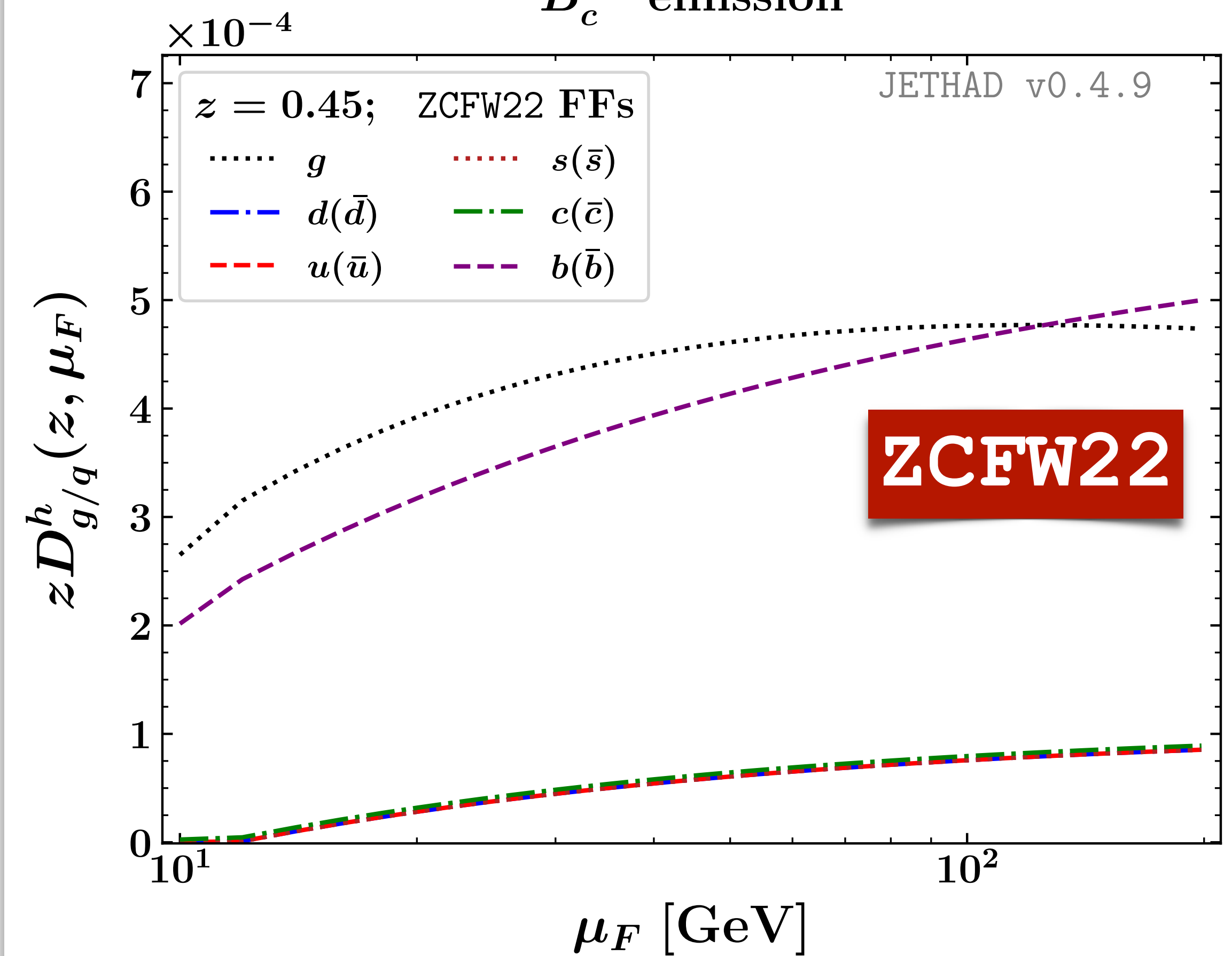
$B_c^\pm(^1S_0)$  collinear FFs

$B_c^\pm(^3S_1)$  collinear FFs

$B_c^\pm$  emission



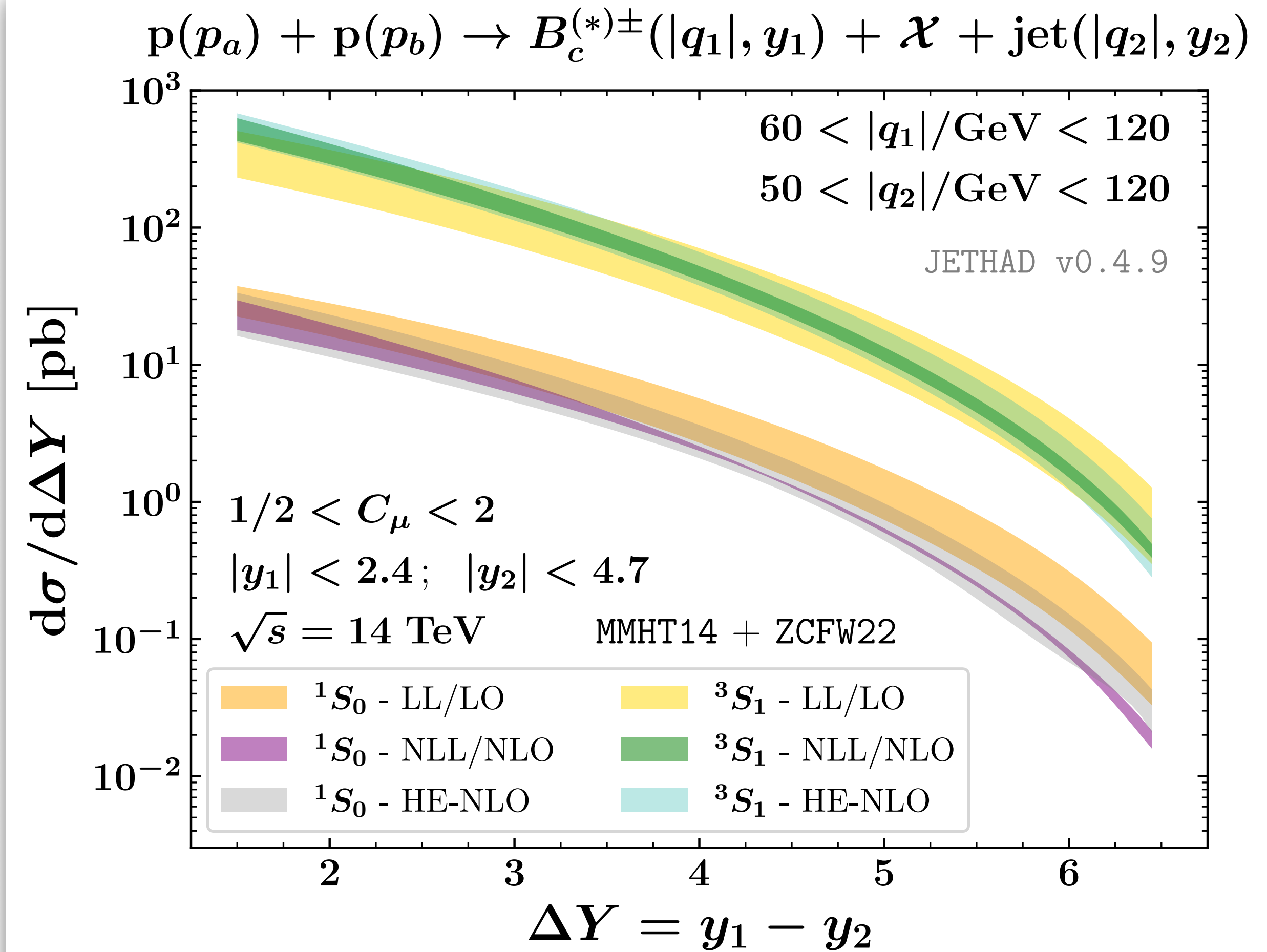
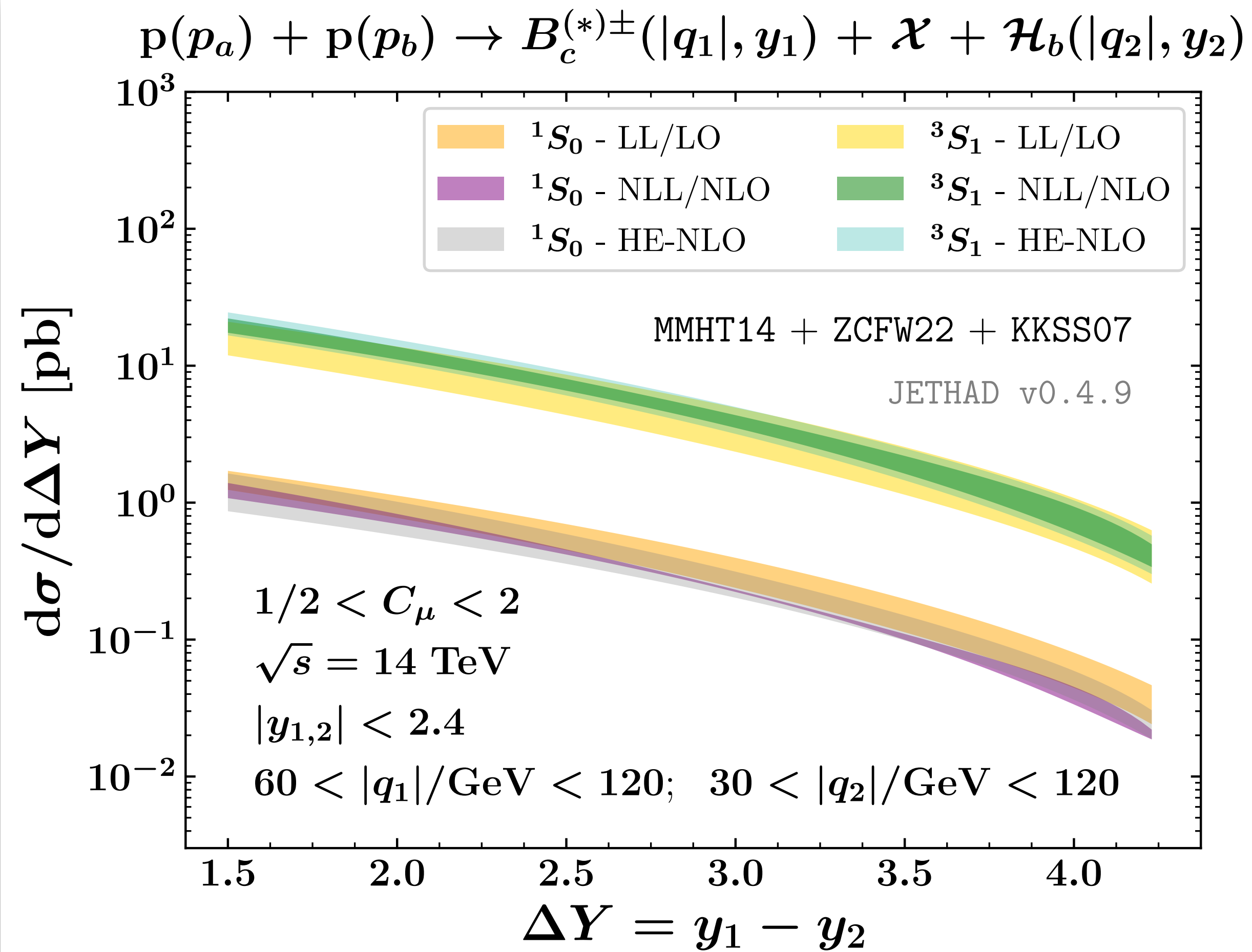
$B_c^{*\pm}$  emission



# Charmed $B$ -mesons + jet at the HL-LHC

$B_c^\pm(^1S_0) + \text{b-hadron}$

$B_c^\pm(^3S_1) + \text{jet}$



The background of the slide is a light blue gradient with several overlapping, semi-transparent Feynman diagrams. These diagrams represent particle interactions, featuring various particles like quarks (colored spheres) and gluons (yellow curly lines).

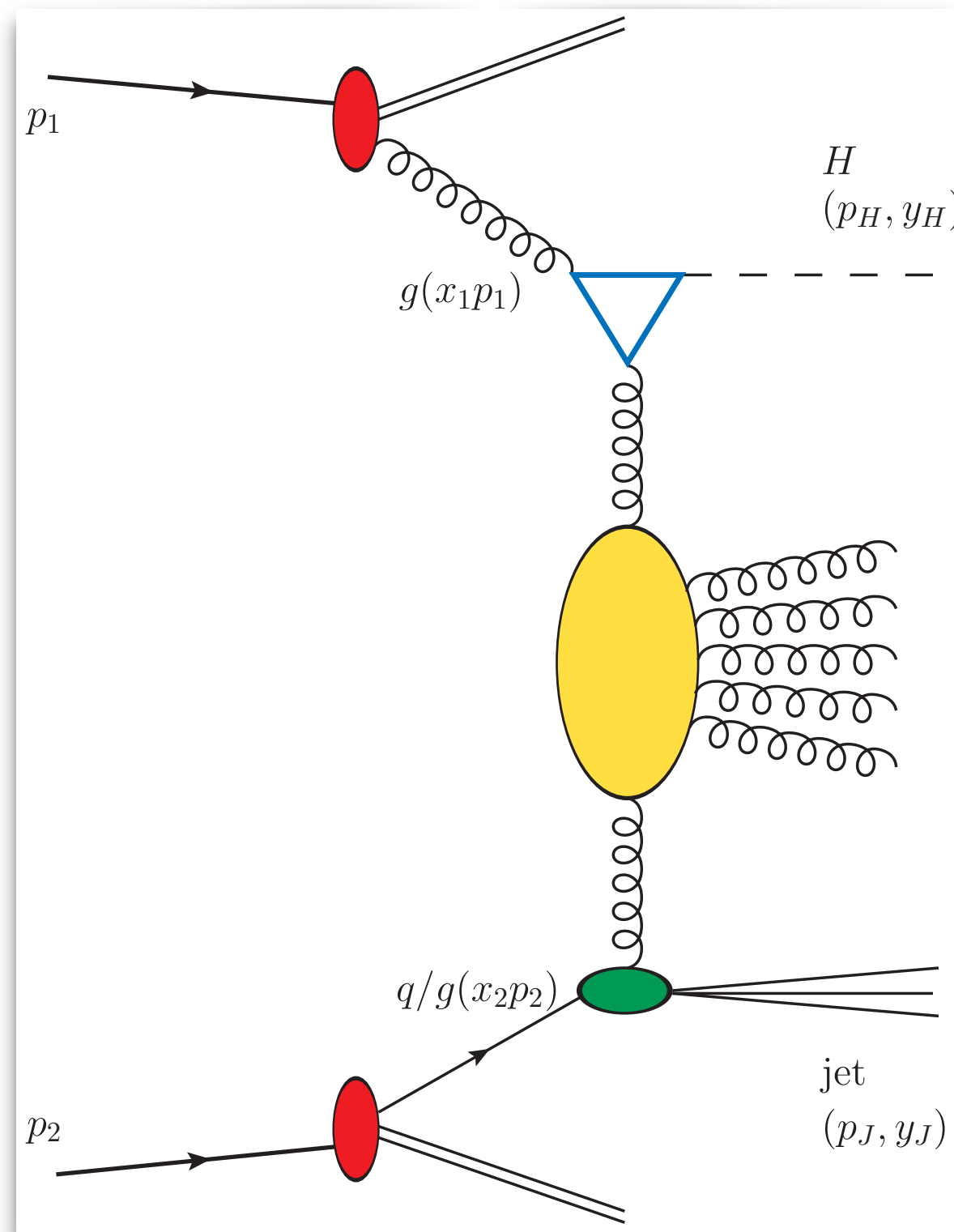
# Higgs+jet distributions

# Inclusive Higgs + jet at the LHC

- Inclusive h.p. of a Higgs + jet system with high  $p_T$  and large rapidity separation,  $\Delta Y$
- Large energy scales expected to **stabilize** the high-energy resummed series

$$\frac{d\sigma}{dx_1 dx_2 d|\vec{p}_H| d|\vec{p}_J| d\varphi_H d\varphi_J} = \frac{1}{(2\pi)^2} \left[ \mathcal{C}_0 + \sum_{n=1}^{\infty} 2 \cos(n\varphi) \mathcal{C}_n \right]$$

$$\varphi = \varphi_H - \varphi_J - \pi$$



# Inclusive Higgs + jet at the LHC

- Inclusive h.p. of a Higgs + jet system with high  $p_T$  and large rapidity separation,  $\Delta Y$
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$$\varphi = \varphi_H - \varphi_J - \pi$$

NLO\*

NLL

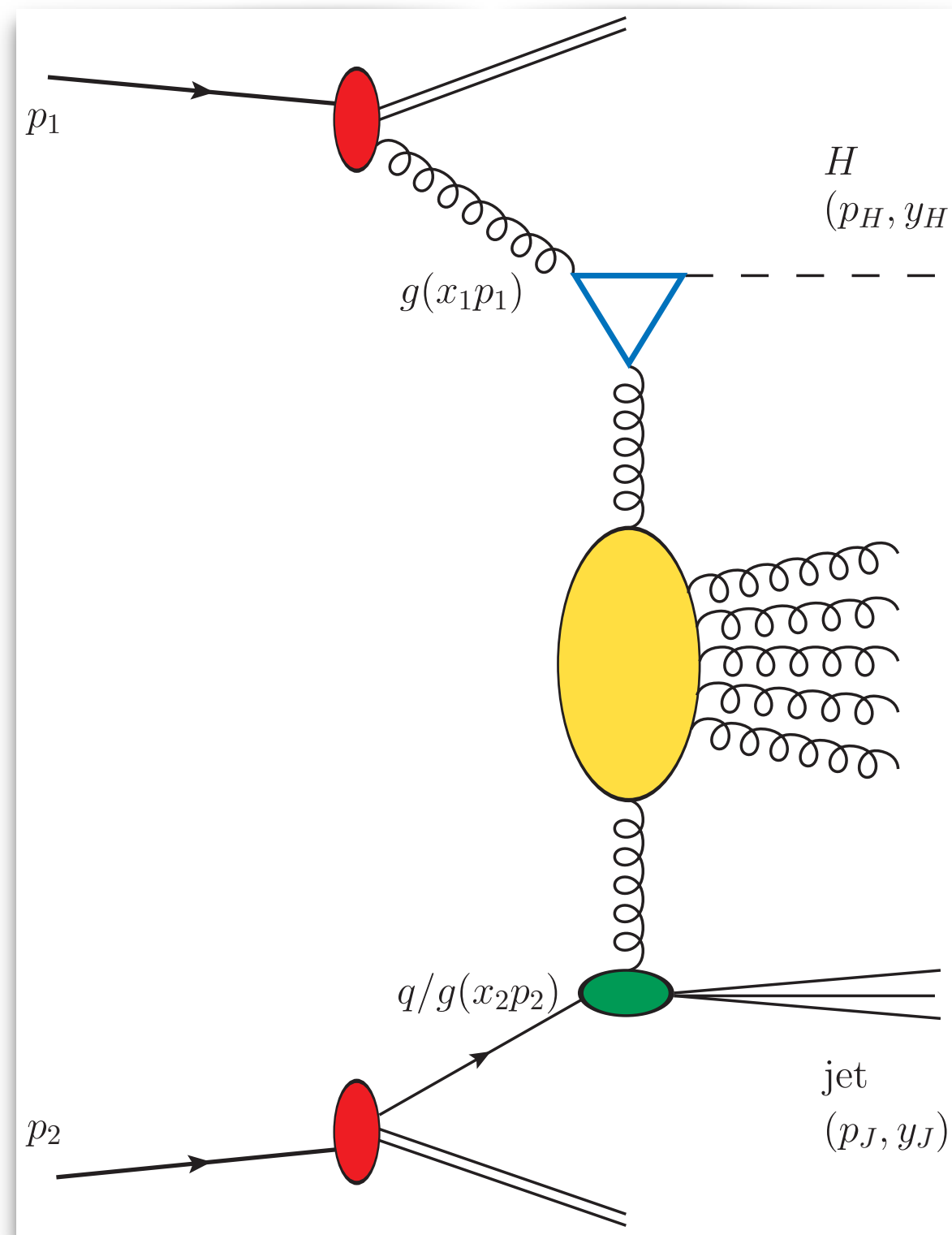
NLO\*

$$\frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu)}{dy_H dy_J d^2\vec{p}_H d^2\vec{p}_J} = \frac{1}{(2\pi)^2} \times \int \frac{d^2\vec{q}_1}{\vec{q}_1^2} \mathcal{V}_H^{(r)}(\vec{q}_1, s_0, x_1, \vec{p}_H) \times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left( \frac{x_1 x_2 s}{s_0} \right)^\omega \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2) \times \int \frac{d^2\vec{q}_2}{\vec{q}_2^2} \mathcal{V}_J^{(s)}(\vec{q}_2, s_0, x_2, \vec{p}_J)$$

Higgs vertex  
(off-shell amplitude)

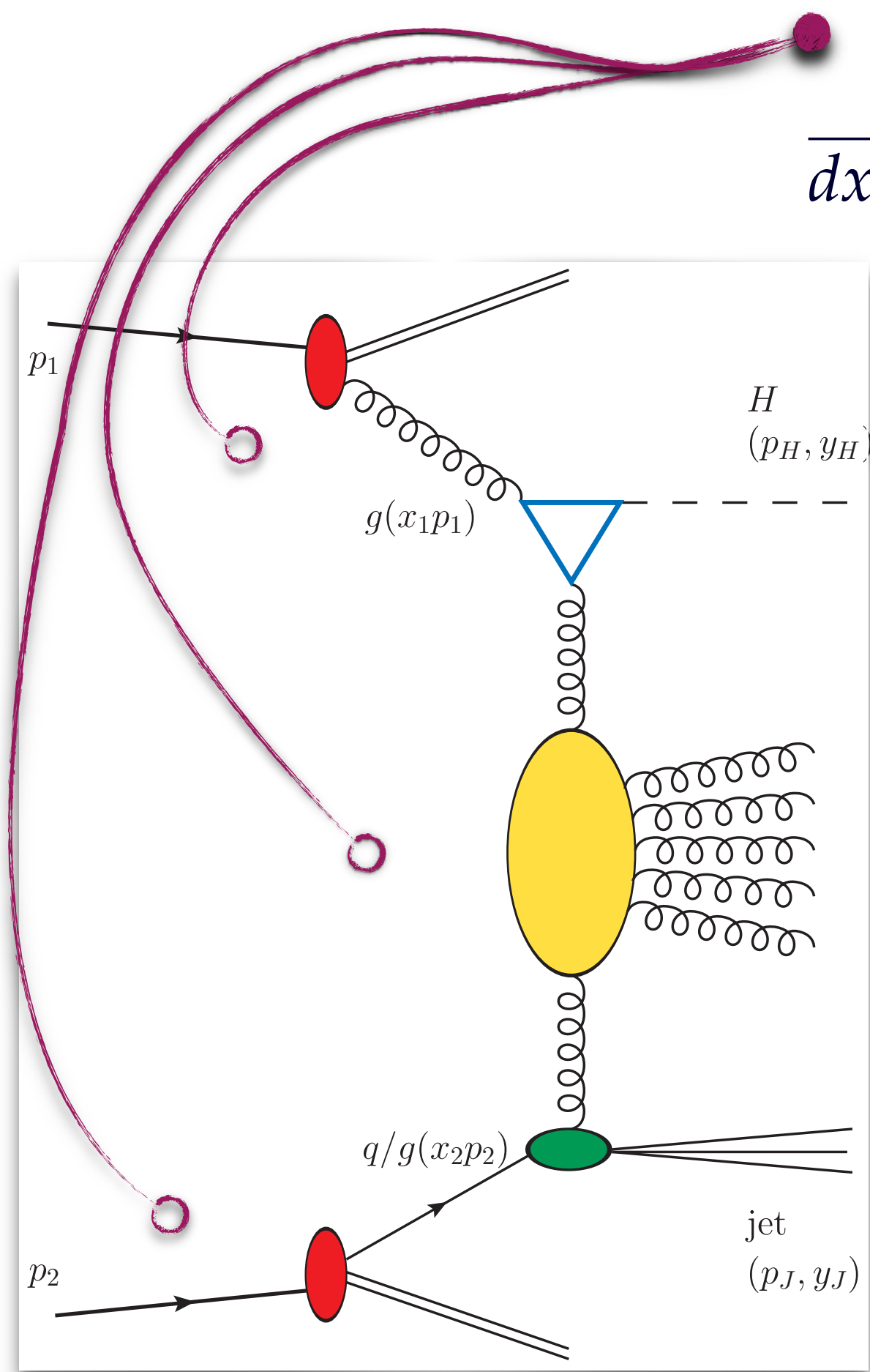
jet vertex  
(off-shell amplitude)

BFKL Green's function



# Inclusive Higgs + jet at the LHC

- Inclusive h.p. of a Higgs + jet system with high  $p_T$  and large rapidity separation,  $\Delta Y$
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$$\frac{d\sigma}{dx_1 dx_2 d|\vec{p}_H| d|\vec{p}_J| d\varphi_H d\varphi_J} = \frac{1}{(2\pi)^2} \left[ \mathcal{C}_0 + \sum_{n=1}^{\infty} 2 \cos(n\varphi) \mathcal{C}_n \right]$$

Higgs vertex  
(off-shell amplitude)

jet vertex  
(off-shell amplitude)

$$\varphi = \varphi_H - \varphi_J - \pi$$

$$\mu_{F,R} \sim M_{H,\perp}$$

$$\mu_R \sim \sqrt{M_{H,\perp} P_J}$$

$$\mu_{F,R} \sim P_J$$

NLO\*

NLL

NLO\*

$$\frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu)}{dy_H dy_J d^2\vec{p}_H d^2\vec{p}_J} = \frac{1}{(2\pi)^2} \times \int \frac{d^2\vec{q}_1}{\vec{q}_1^2} \mathcal{V}_H^{(r)}(\vec{q}_1, s_0, x_1, \vec{p}_H) \times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left( \frac{x_1 x_2 s}{s_0} \right)^\omega \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2) \times \int \frac{d^2\vec{q}_2}{\vec{q}_2^2} \mathcal{V}_J^{(s)}(\vec{q}_2, s_0, x_2, \vec{p}_J)$$

BFKL Green's function

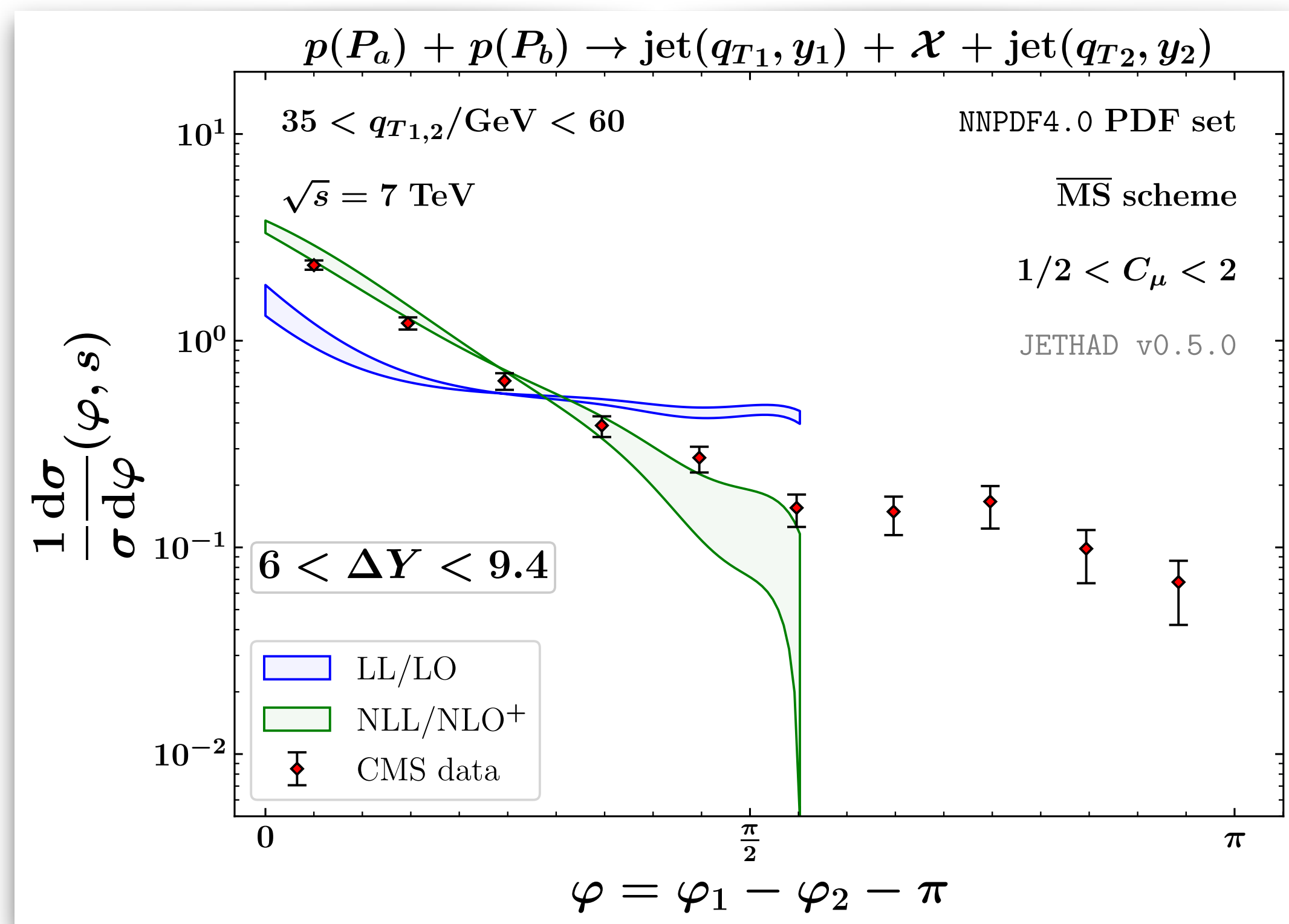
# Azimuthal-angle multiplicity

$$\frac{1}{\sigma} \frac{d\sigma(\Delta Y, s)}{d\varphi} = \frac{1}{2\pi} \left\{ 1 + 2 \sum_{n=1}^{\infty} \cos(n\varphi) \langle \cos(n\varphi) \rangle \right\}$$

## Mueller-Navelet jets

[\[B. Ducloué, L. Szymanowski, S. Wallon, Phys.Rev.Lett. 112 \(2014\) 082003\]](#)

(figure below) [\[F. G. C., A. Papa \(2022\)\]](#)



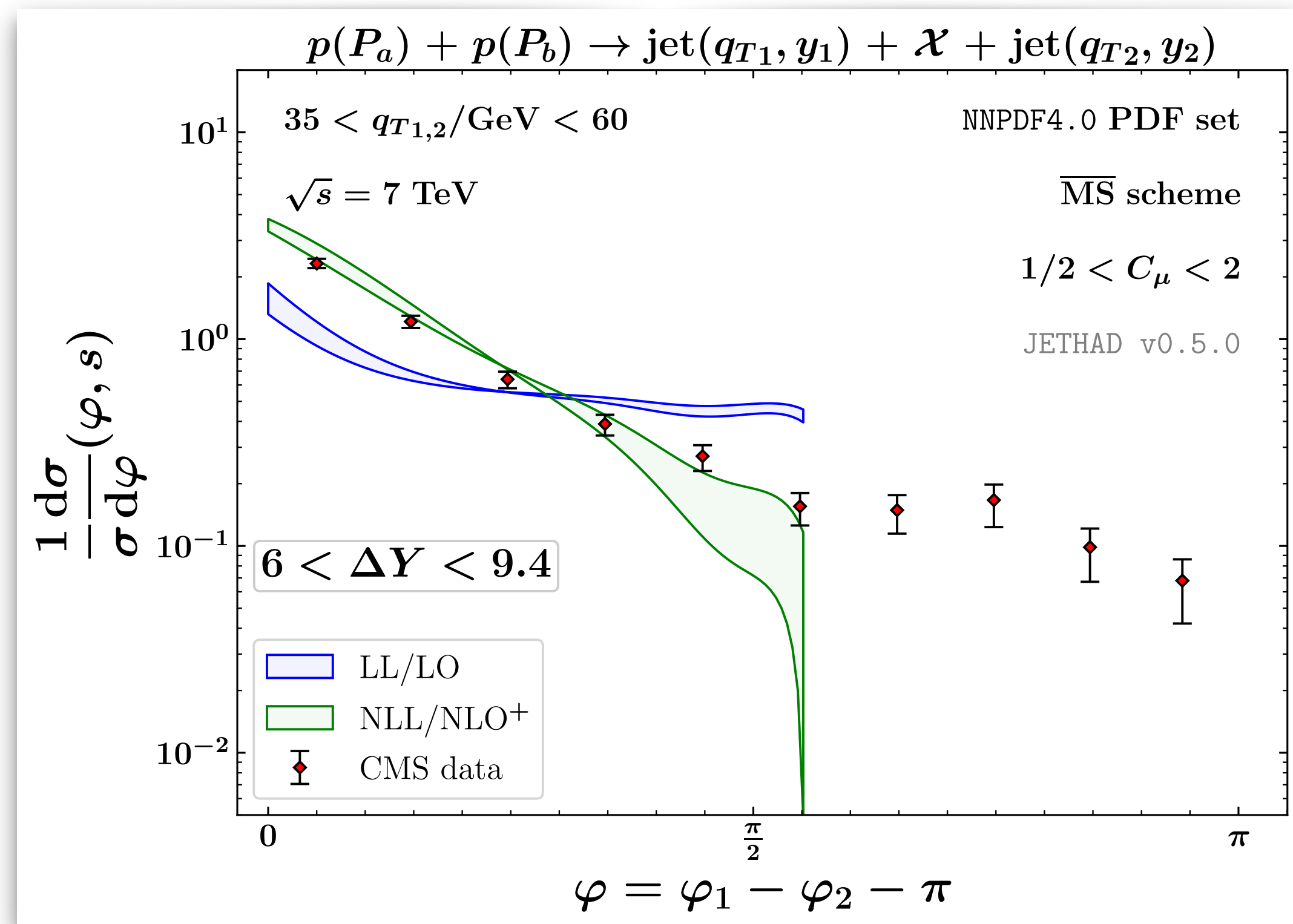
# Azimuthal-angle multiplicity

$$\frac{1}{\sigma} \frac{d\sigma(\Delta Y, s)}{d\varphi} = \frac{1}{2\pi} \left\{ 1 + 2 \sum_{n=1}^{\infty} \cos(n\varphi) \langle \cos(n\varphi) \rangle \right\}$$

## Mueller-Navelet jets

[B. Ducloué, L. Szymanowski, S. Wallon, Phys.Rev.Lett. 112 (2014) 082003]

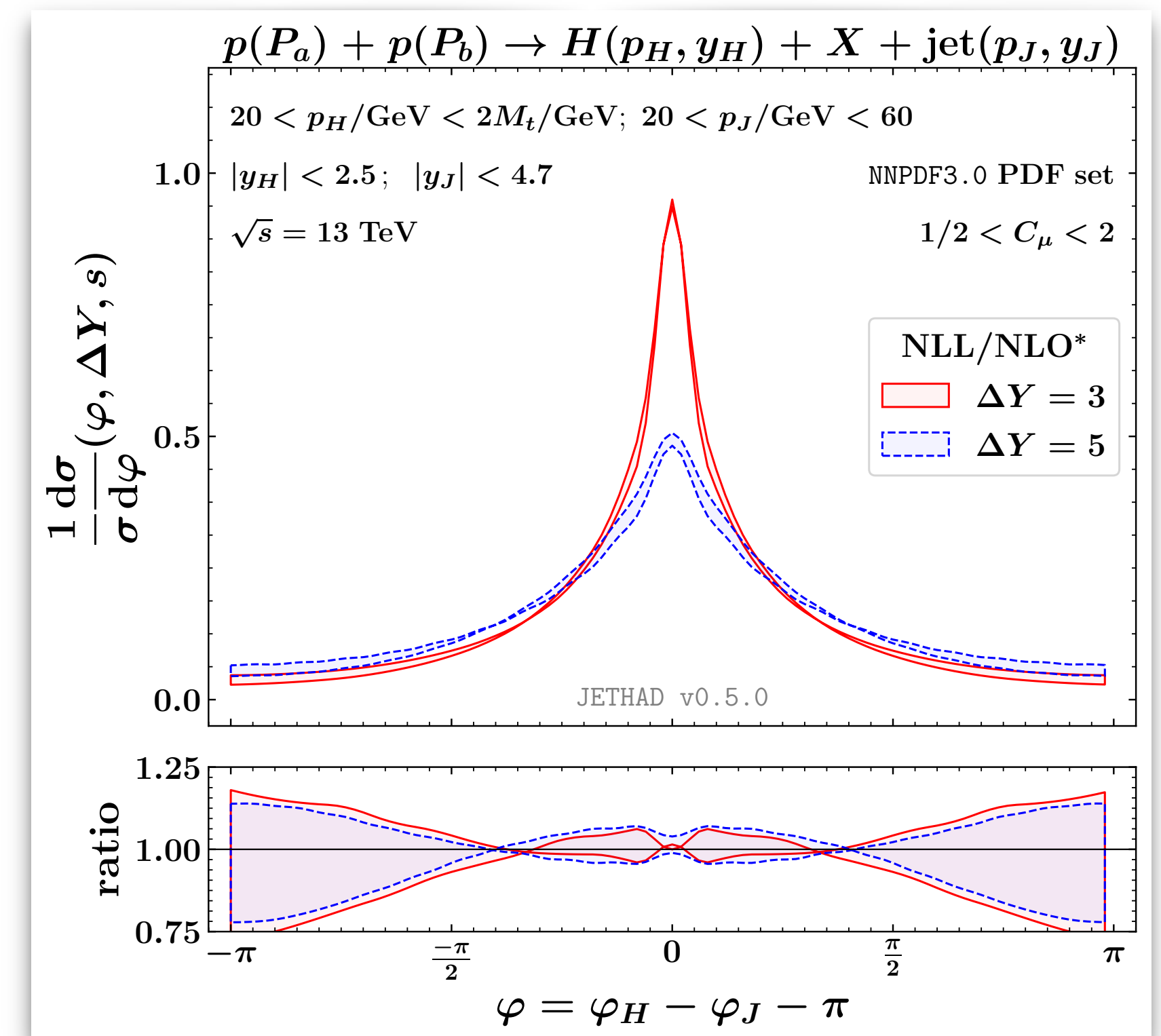
(figure below) [F. G. C., A. Papa (2022)]



## Higgs + jet

(figure below) [F. G. C. et al., Eur. Phys. J. C 81 (2021) 4, 293]

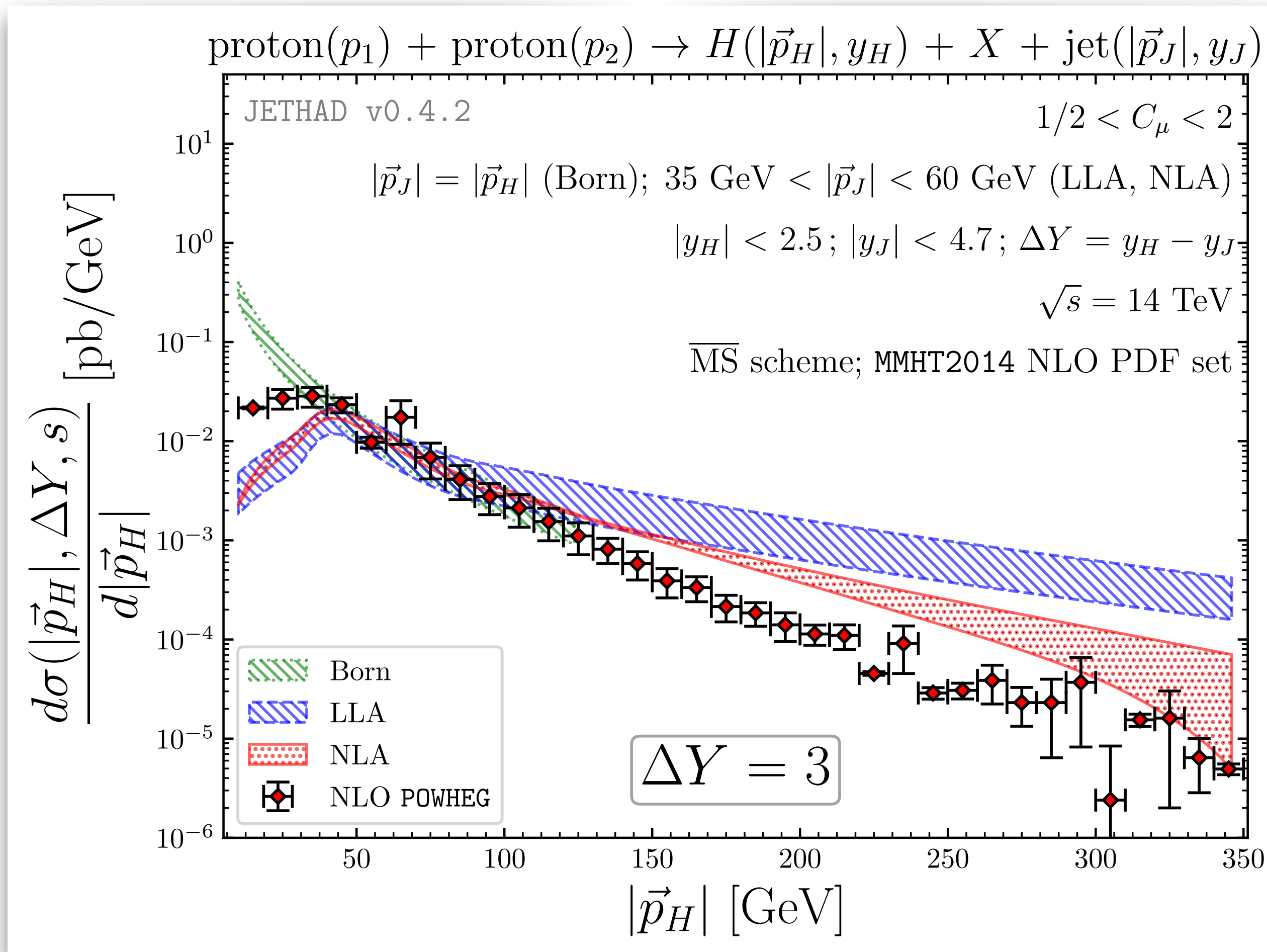
(NLO Higgs) [F. G. C. et al., JHEP 08 (2022) 092]





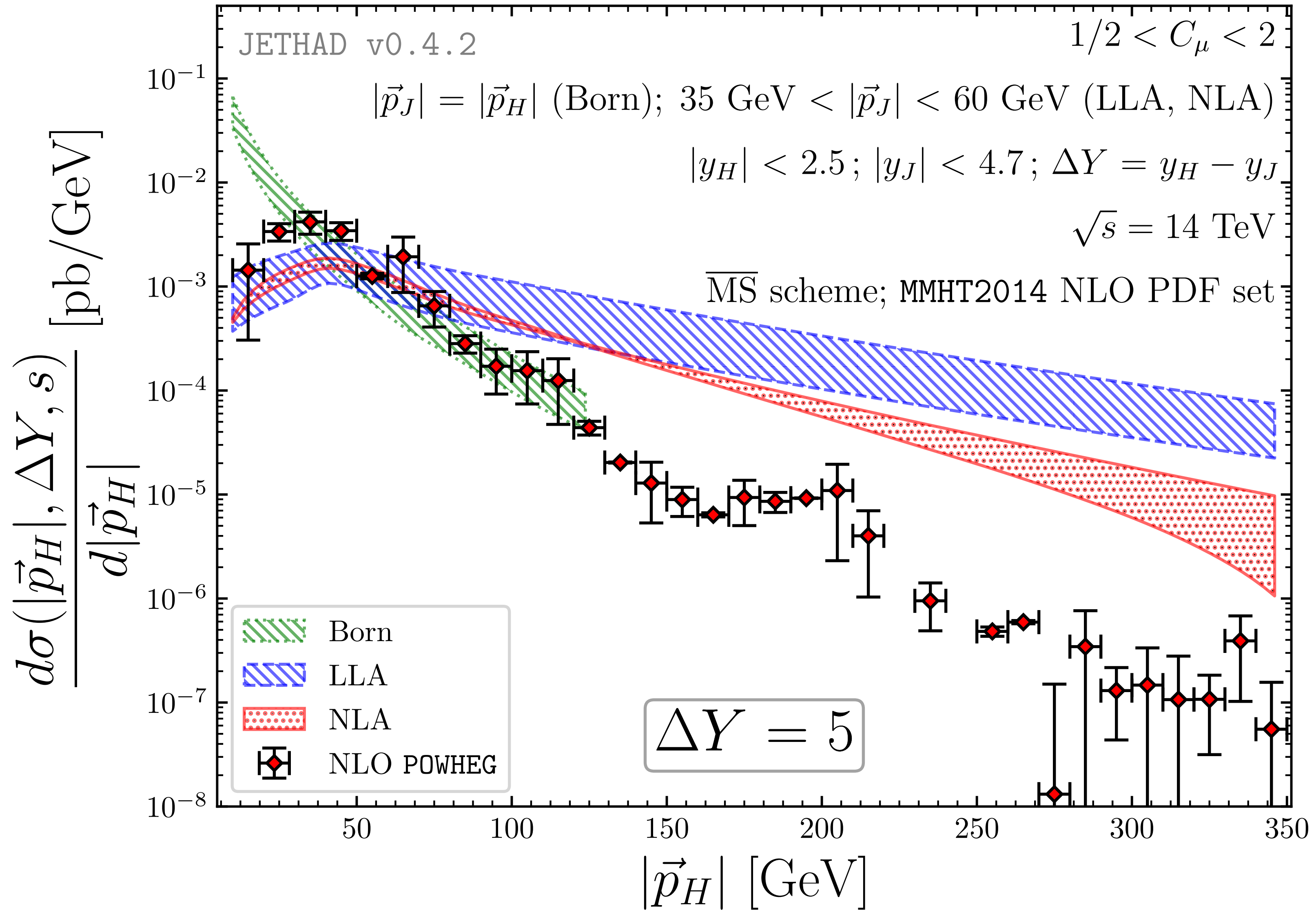
# Higgs transverse-momentum distribution

$$\frac{d\sigma(|\vec{p}_H|, \Delta Y, s)}{d|\vec{p}_H|d\Delta Y} = \int_{p_J^{\min}}^{p_J^{\max}} d|\vec{p}_J| \int_{y_H^{\min}}^{y_H^{\max}} dy_H \int_{y_J^{\min}}^{y_J^{\max}} dy_J \delta(y_H - y_J - \Delta Y) \mathcal{C}_0$$



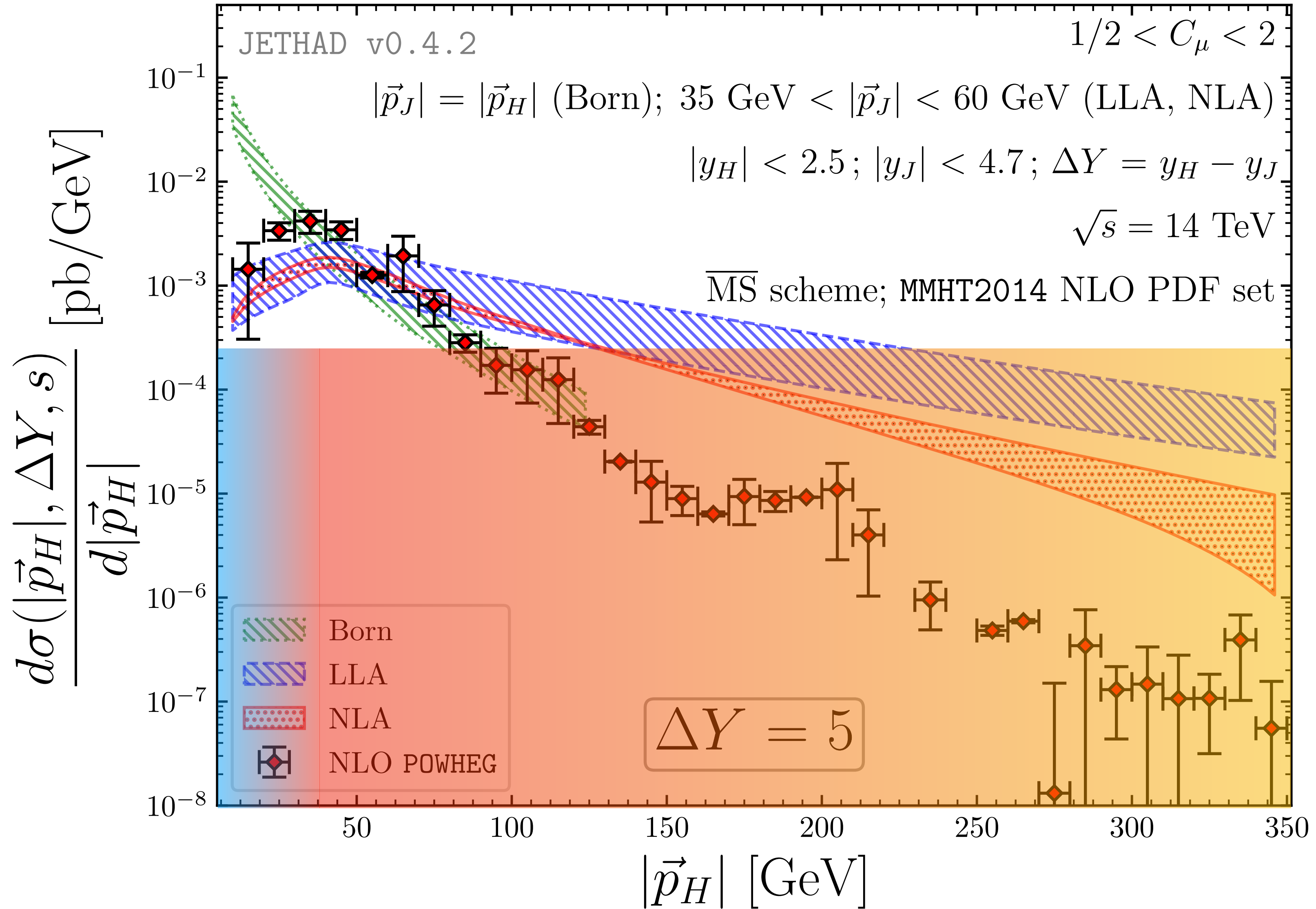
- HE resummation from **JETHAD**
- Comparison with fixed-order **POWHEG**
- Distributions stable under NLL corrections

proton( $p_1$ ) + proton( $p_2$ )  $\rightarrow$   $H(|\vec{p}_H|, y_H) + X + \text{jet}(|\vec{p}_J|, y_J)$



Backup

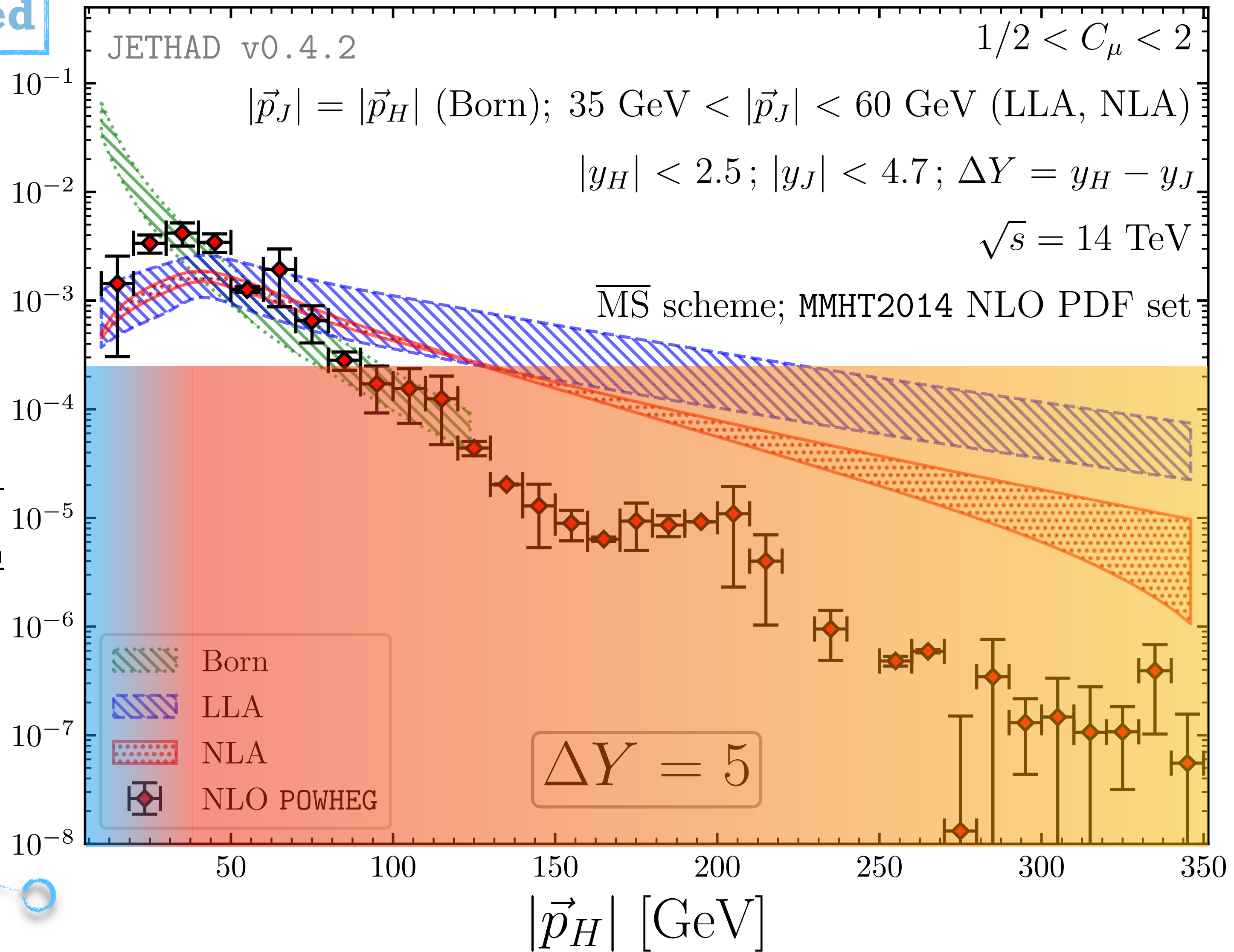
proton( $p_1$ ) + proton( $p_2$ )  $\rightarrow$   $H(|\vec{p}_H|, y_H) + X + \text{jet}(|\vec{p}_J|, y_J)$



Backup

large  $p_T$  logs  
 $p_T$ -resum. needed

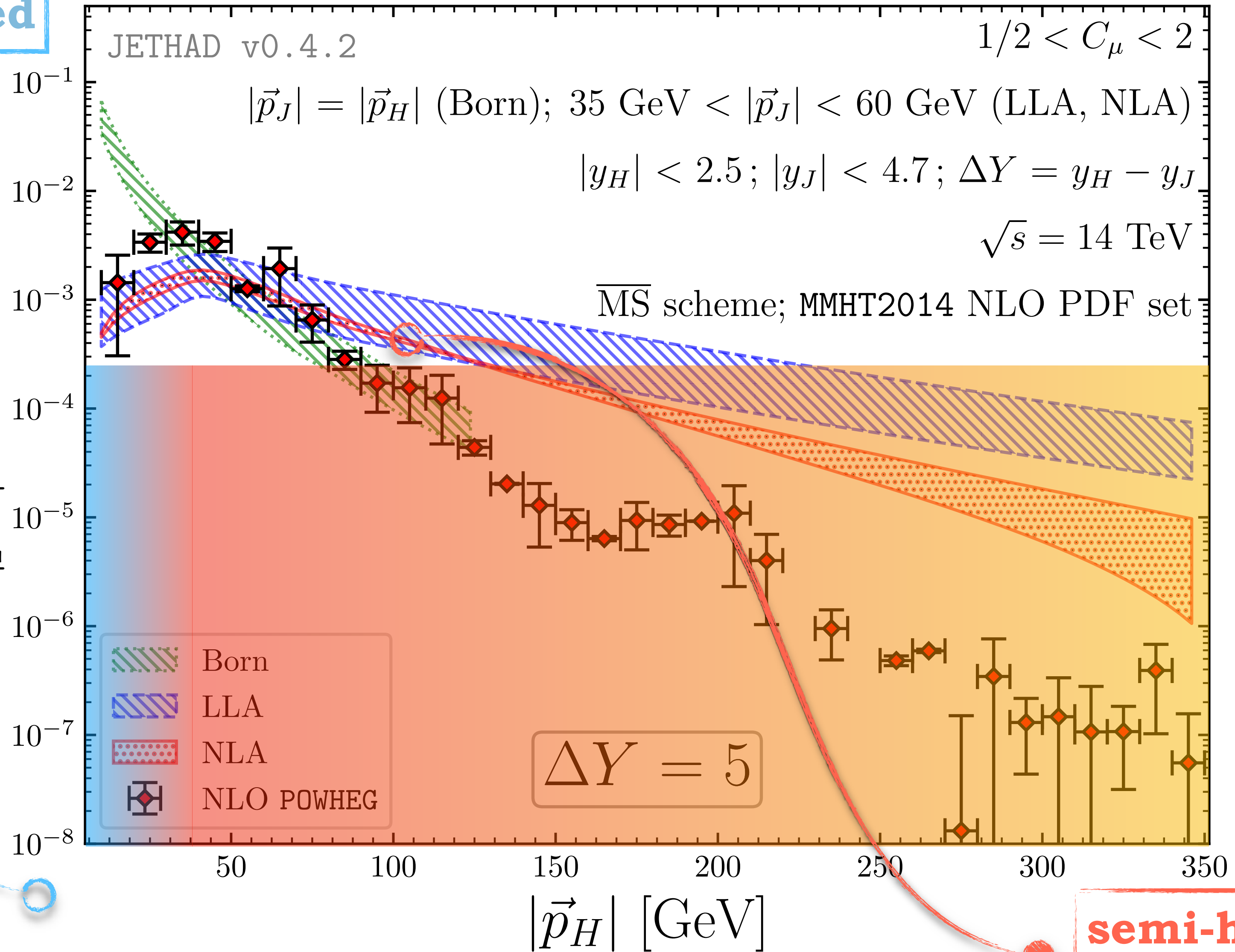
$$\text{proton}(p_1) + \text{proton}(p_2) \rightarrow H(|\vec{p}_H|, y_H) + X + \text{jet}(|\vec{p}_J|, y_J)$$



Backup

large  $p_T$  logs  
 $p_T$ -resum. needed

$$\text{proton}(p_1) + \text{proton}(p_2) \rightarrow H(|\vec{p}_H|, y_H) + X + \text{jet}(|\vec{p}_J|, y_J)$$



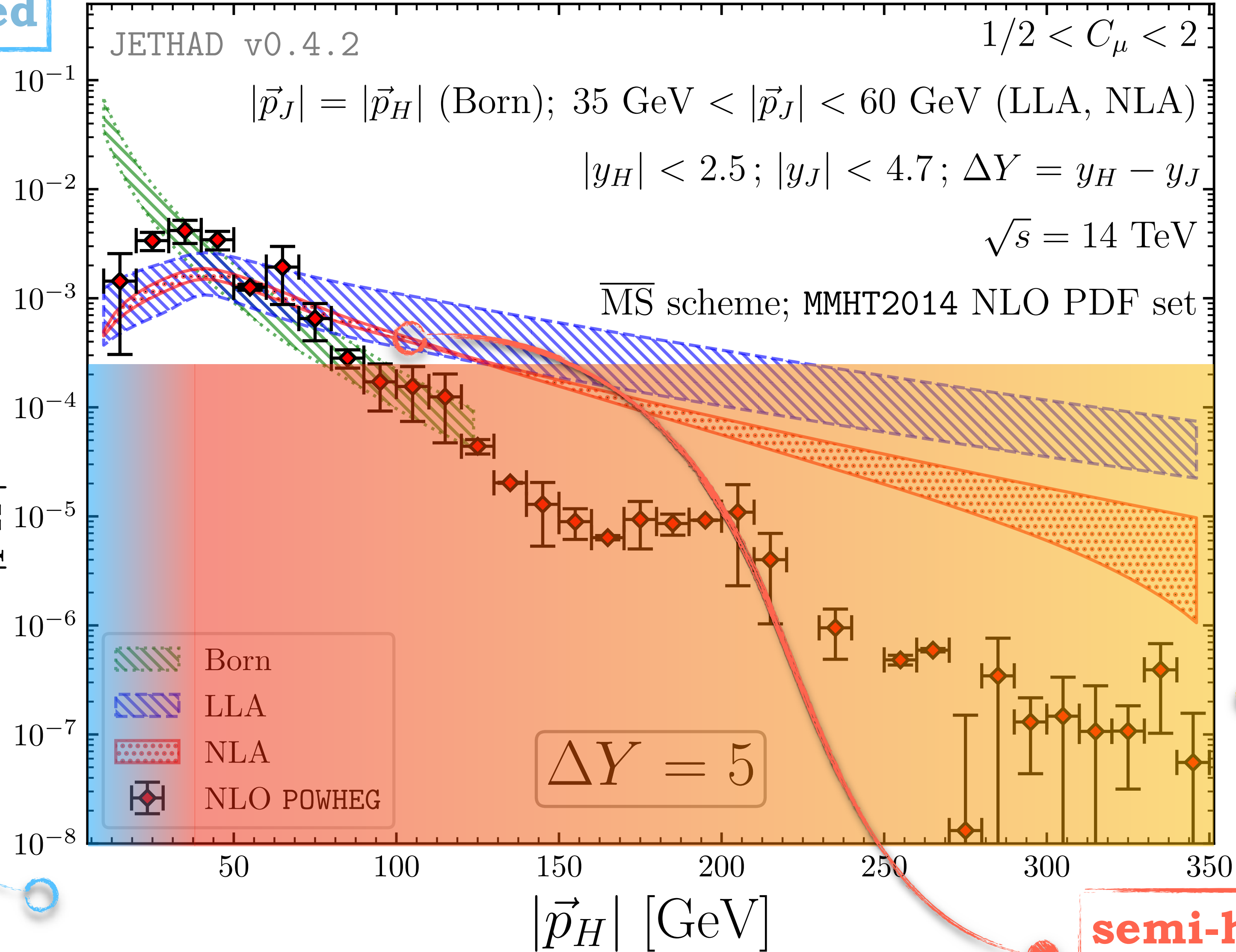
semi-hard regime  
 BFKL expected

Backup

**DGLAP-type + large- $x$  threshold logs  $\rightarrow$  BFKL decoupling**

large  $p_T$  logs  
 $p_T$ -resum. needed

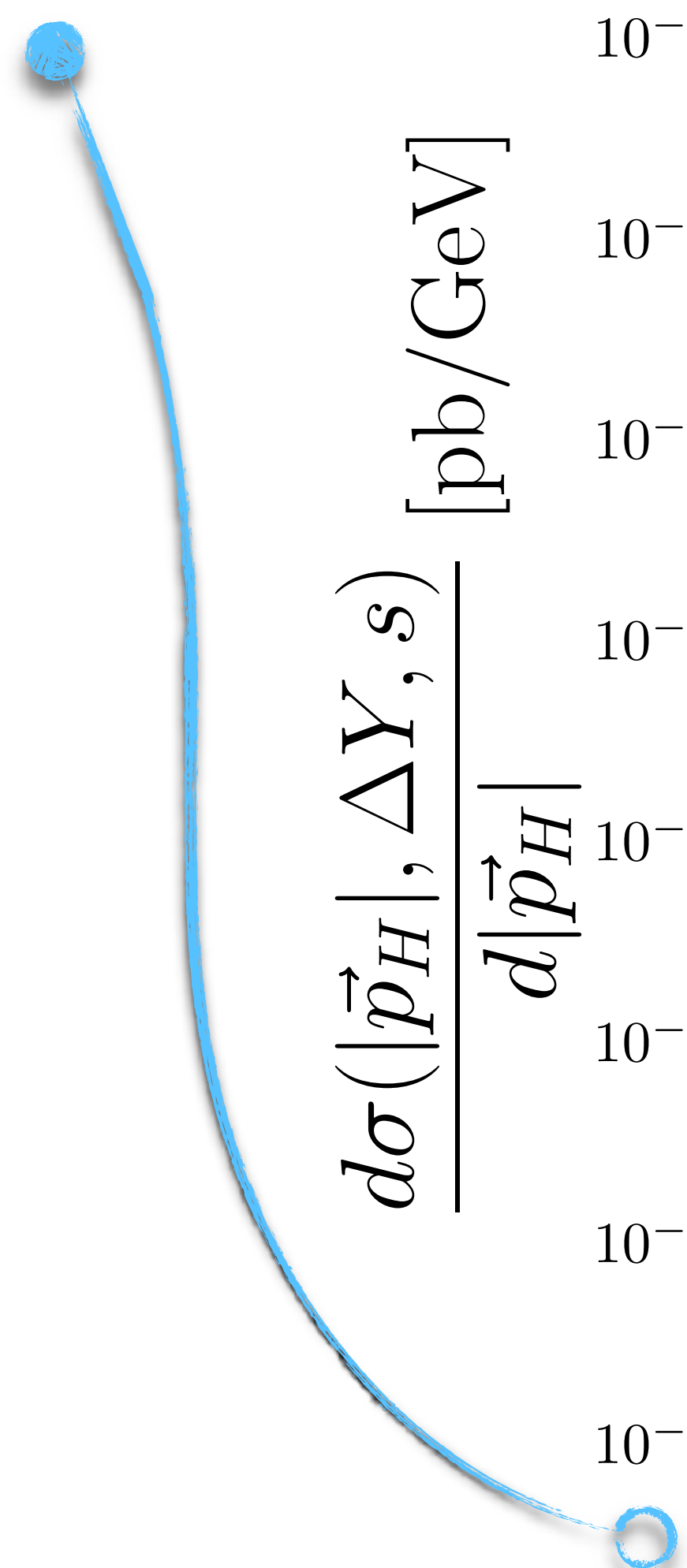
$$\text{proton}(p_1) + \text{proton}(p_2) \rightarrow H(|\vec{p}_H|, y_H) + X + \text{jet}(|\vec{p}_J|, y_J)$$



**semi-hard regime  
 BFKL expected**

**Backup**

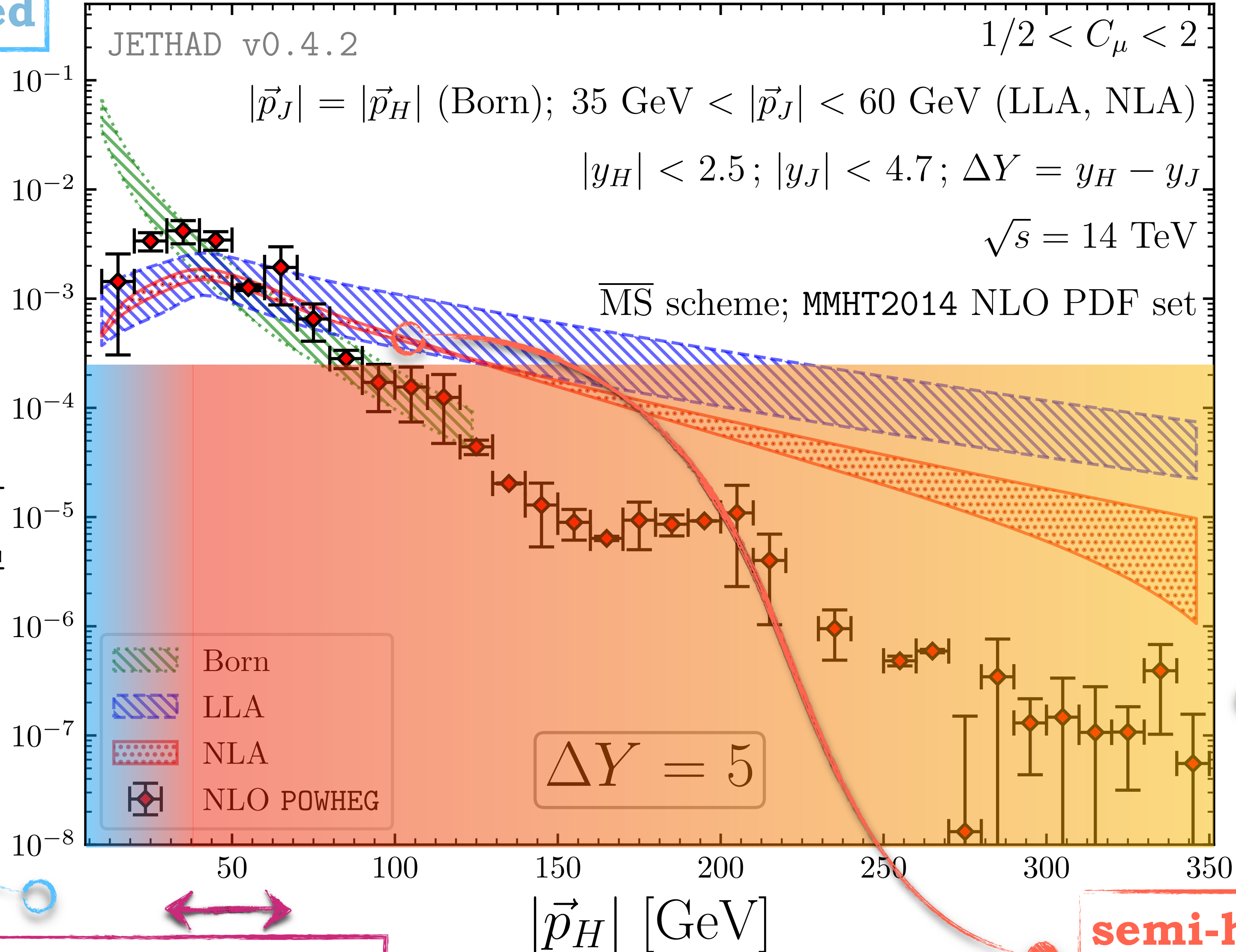
$$\frac{d\sigma(|\vec{p}_H|, \Delta Y, s)}{d|\vec{p}_H|} \text{ [pb/GeV]}$$



**DGLAP-type + large- $x$  threshold logs  $\rightarrow$  BFKL decoupling**

large  $p_T$  logs  
 $p_T$ -resum. needed

$$\text{proton}(p_1) + \text{proton}(p_2) \rightarrow H(|\vec{p}_H|, y_H) + X + \text{jet}(|\vec{p}_J|, y_J)$$



Backup

$$\frac{d\sigma(|\vec{p}_H|, \Delta Y, s)}{d|\vec{p}_H|} \text{ [pb/GeV]}$$

almost back-to-back emissions  
 Sudakov-type double logs

semi-hard regime  
 BFKL expected

# Higgs + jet highlights from the FCC Week 2022

The high-energy QCD dynamics from Higgs+jet correlations at FCC

Francesco G. Celiberto<sup>1,2,3</sup> and Alessandro Papa<sup>4,5</sup>

FCC Week 2022, Sorbonne Université, France

## Hors d'œuvre

- Higgs sector → SM benchmarks, BSM portals
- Gluon fusion → key ingredient for precision QCD
- Fixed-order ← improved by resummations
- FCC energies ↔ high-energy (HE) resummation
- Higgs+jet → golden channel to hunt for HE signals

## NLL/NLO differential cross section

$$\frac{d\sigma}{dy_1 dy_2 d^2k_1 d^2k_2} = \sum_{r,s=q,g} \int_0^1 dx_1 \int_0^1 dx_2 f_r(x_1, \mu_F) f_s(x_2, \mu_F) \frac{d\hat{\sigma}_{rs}(x_1, x_2, s, \mu_F)}{dy_1 dy_2 d^2k_1 d^2k_2}$$

$$\frac{d\hat{\sigma}_{rs}(x_1, x_2, s, \mu)}{dy_1 dy_2 d^2\vec{p}_{T1} d^2\vec{p}_{T2}} = \frac{1}{(2\pi)^2} \times \int \frac{d^2\vec{q}_1}{q_1^2} V_H^{(r)}(\vec{q}_1, s_0, x_1, \vec{p}_{T1}) \times \int_{s-i\infty}^{s+i\infty} \frac{d\omega}{2\pi i} \left( \frac{x_1 x_2 s}{s_0} \right)^\omega \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2) \times \int \frac{d^2\vec{q}_2}{q_2^2} V_J^{(s)}(\vec{q}_2, s_0, x_2, \vec{p}_{T2})$$

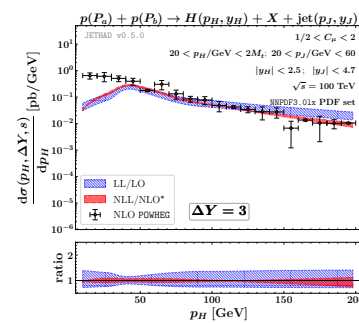
PDFs with threshold

NLO Higgs vertex

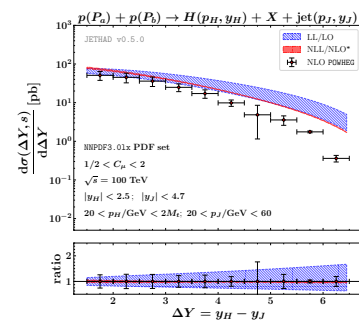
NLL BFKL kernel

NLO Jet vertex

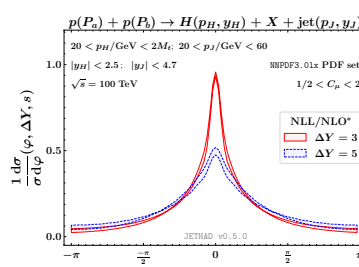
## Hybrid high-energy and collinear factorization at work



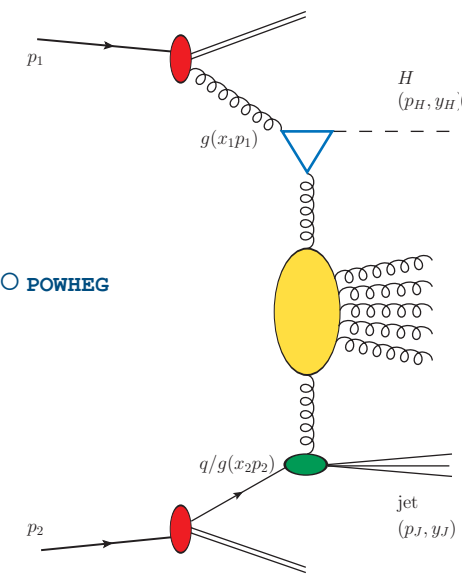
Higgs  $p_T$  distribution: NLL/NLO JETHAD vs NLO POWHEG



Rapidity distribution: NLL/NLO JETHAD vs NLO POWHEG



Azimuthal distribution at NLL/NLO



HE resummation from JETHAD

Large-x NNPDF3.0Lx PDFs with threshold

Comparison with fixed-order from POWHEG

Distributions stable under NLL corrections

## A path towards precision

- ✓ NLL bands nested inside LL ones → solid stability
- ✓ HE signal clearly disengaged from NLO background
- ✓ Way toward precision studies of HE QCD (1!)
- Multilateral formalism → encode other resummations
- A window on proton structure at small-x (2?)

## Further information

- ECT\*, I-38123 Villazzano, Trento, Italy
- Fondazione Bruno Kessler (FBK), I-38123 Povo, Trento, Italy
- INFN-TIFPA, I-38123 Povo, Trento, Italy
- Università della Calabria, I-87036 Rende, Cosenza, Italy
- INFN-Cosenza, I-87036 Rende, Cosenza, Italy

Contact: fceliberto@ectstar.eu

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PDFs with threshold

NLO Higgs vertex

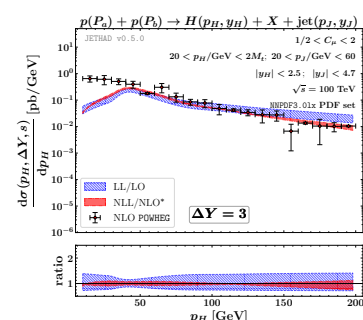
NLL BFKL kernel

NLO Jet vertex

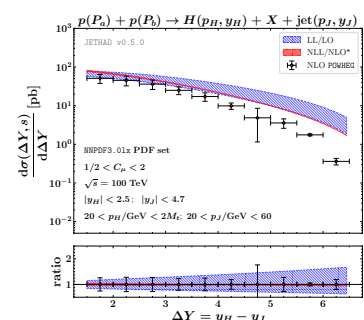
$$C_n(\Delta Y, s) = \int_{p_H^{\min}}^{p_H^{\max}} d|\vec{p}_H| \int_{p_J^{\min}}^{p_J^{\max}} d|\vec{p}_J| \int_{y_H^{\min}}^{y_H^{\max}} dy_H \int_{y_J^{\min}}^{y_J^{\max}} dy_J \delta(y_H - y_J - \Delta Y) C_n$$

## Rapidity distribution: NLL/NLO\* JETHAD vs NLO POWHEG

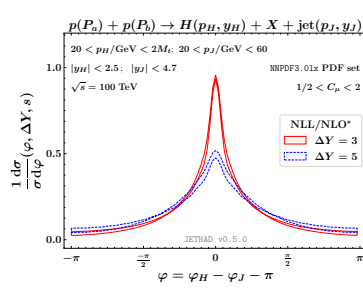
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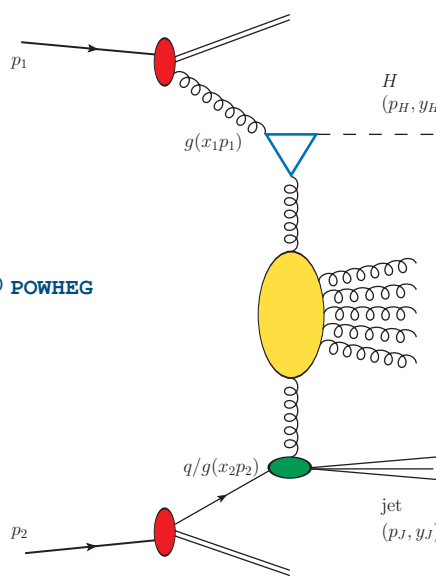
Higgs  $p_T$  distribution: NLL/NLO JETHAD vs NLO POWHEG



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Azimuthal distribution at NLL/NLO



## HE resummation from JETHAD

- Large-x NNPDF3.01x PDFs with threshold
- Comparison with fixed-order from POWHEG
- Distributions stable under NLL corrections

## A path towards precision

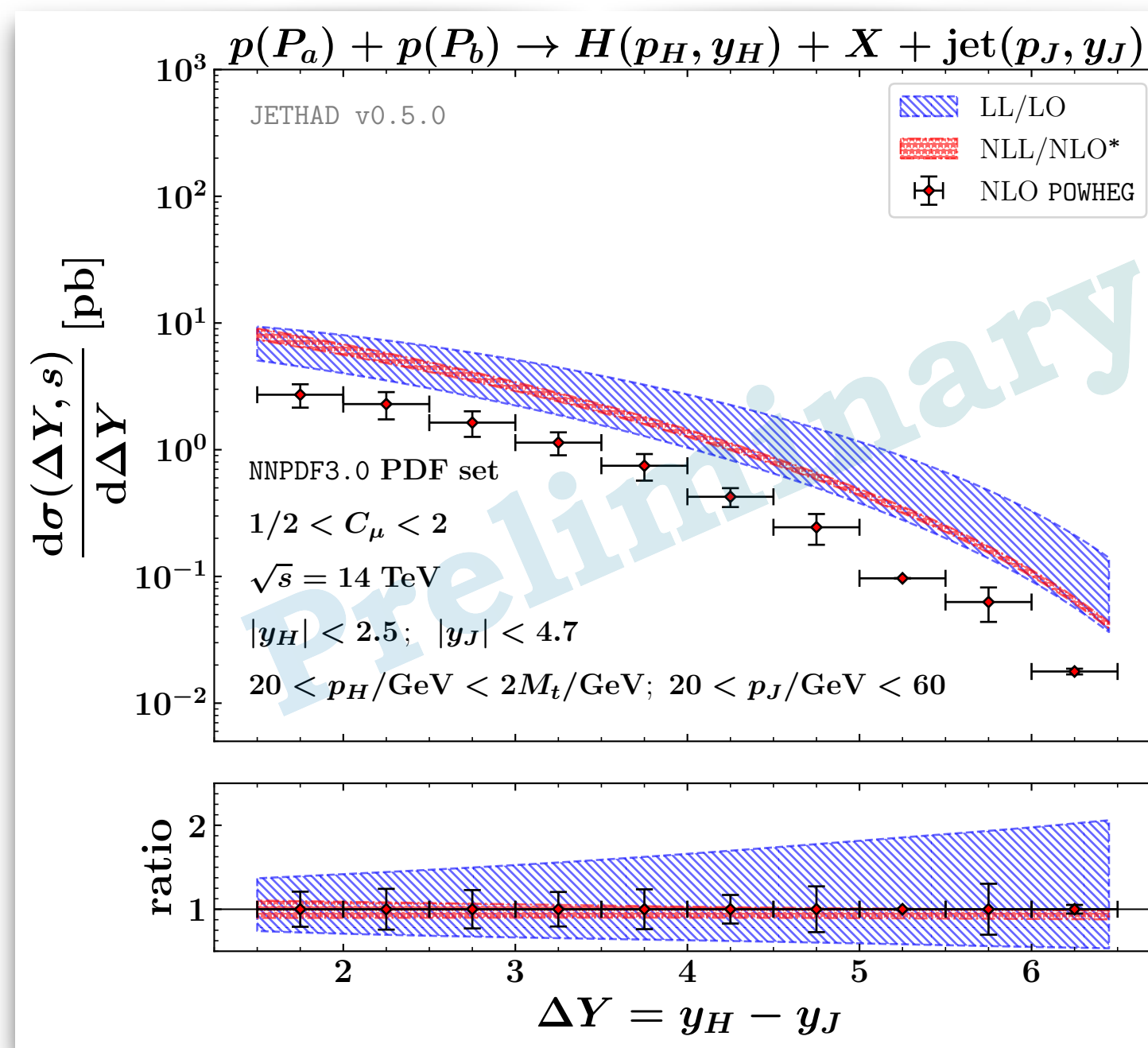
- ✓ NLL bands nested inside LL ones → solid stability
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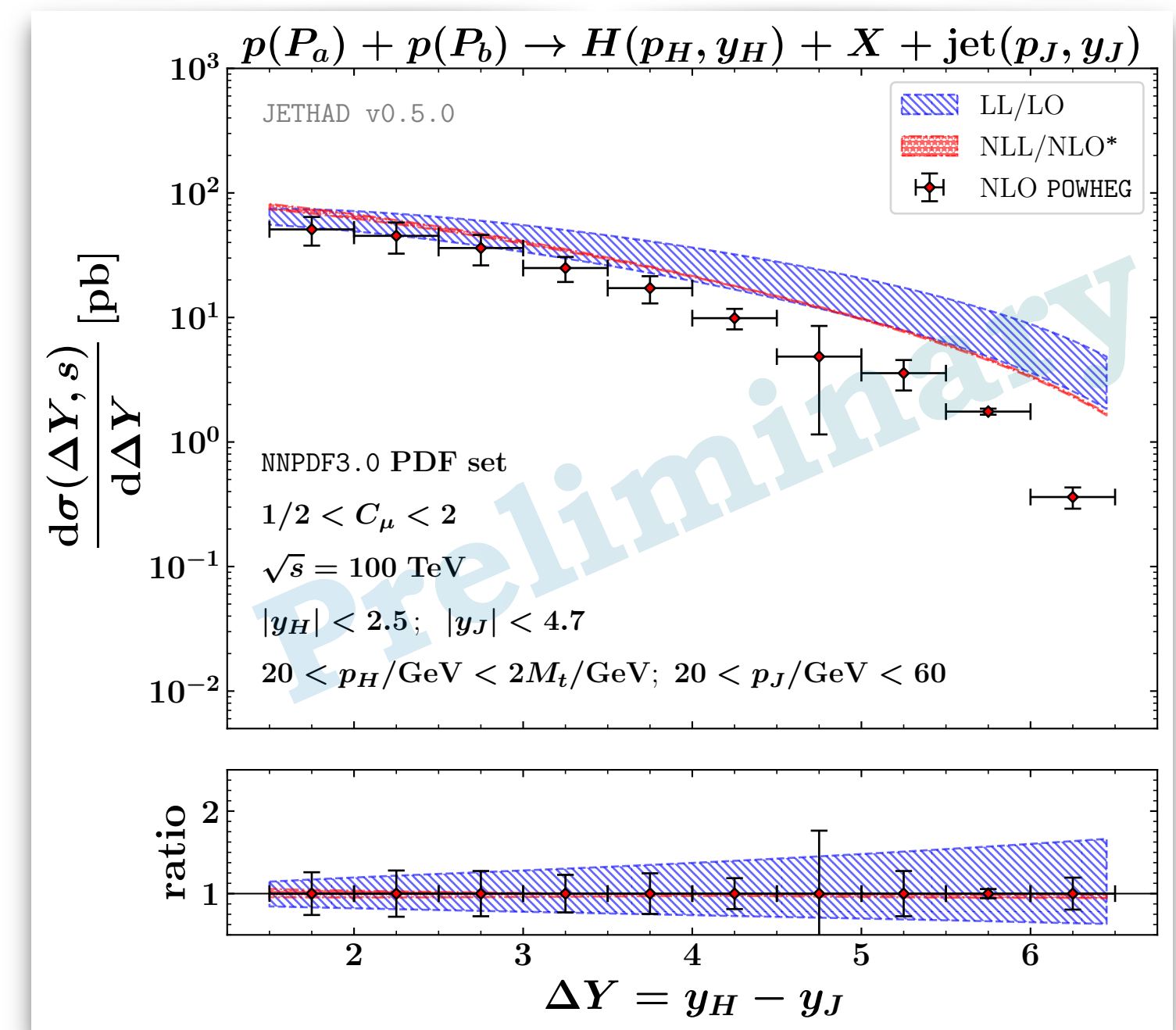
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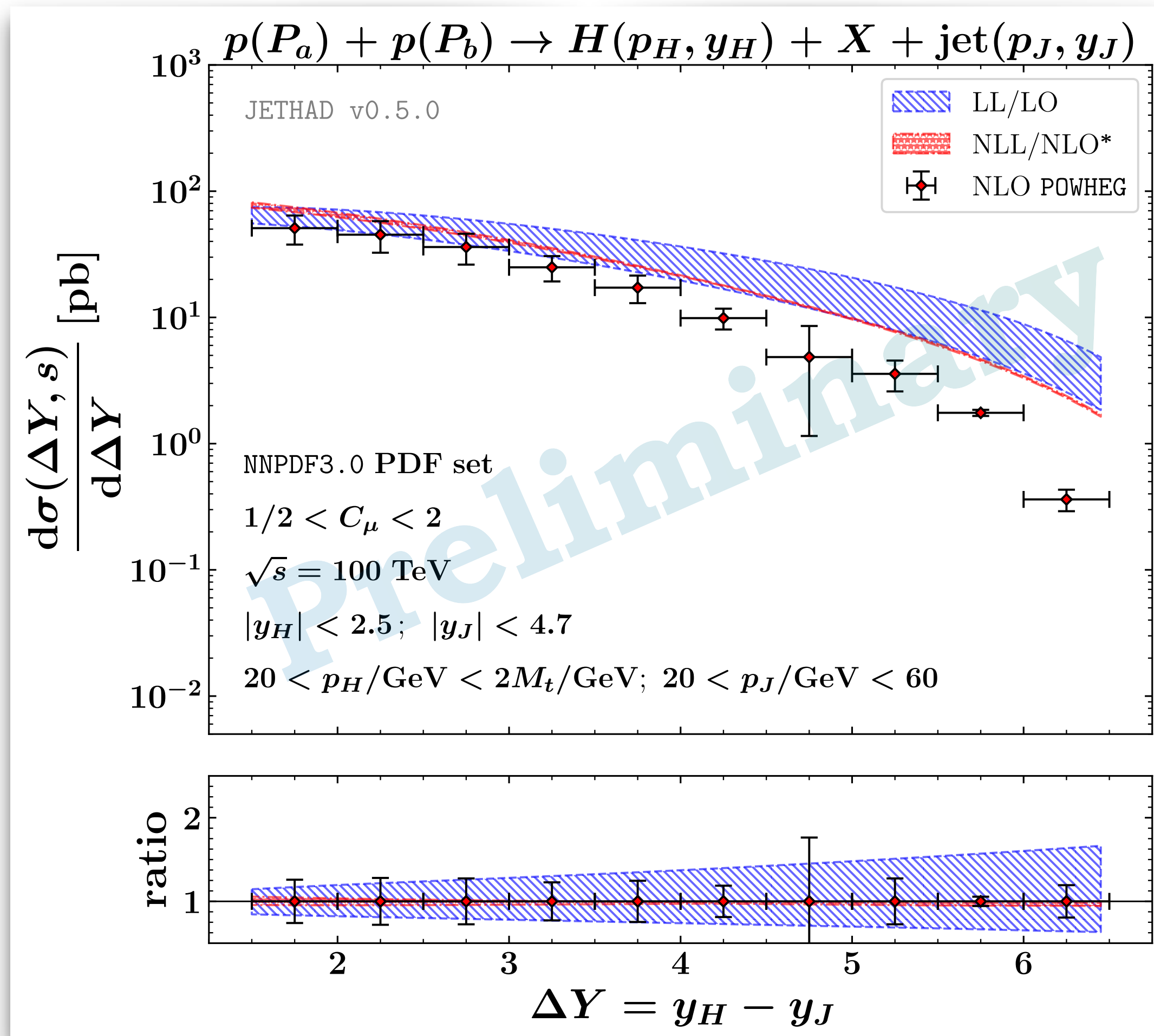
14 TeV LHC



100 TeV FCC

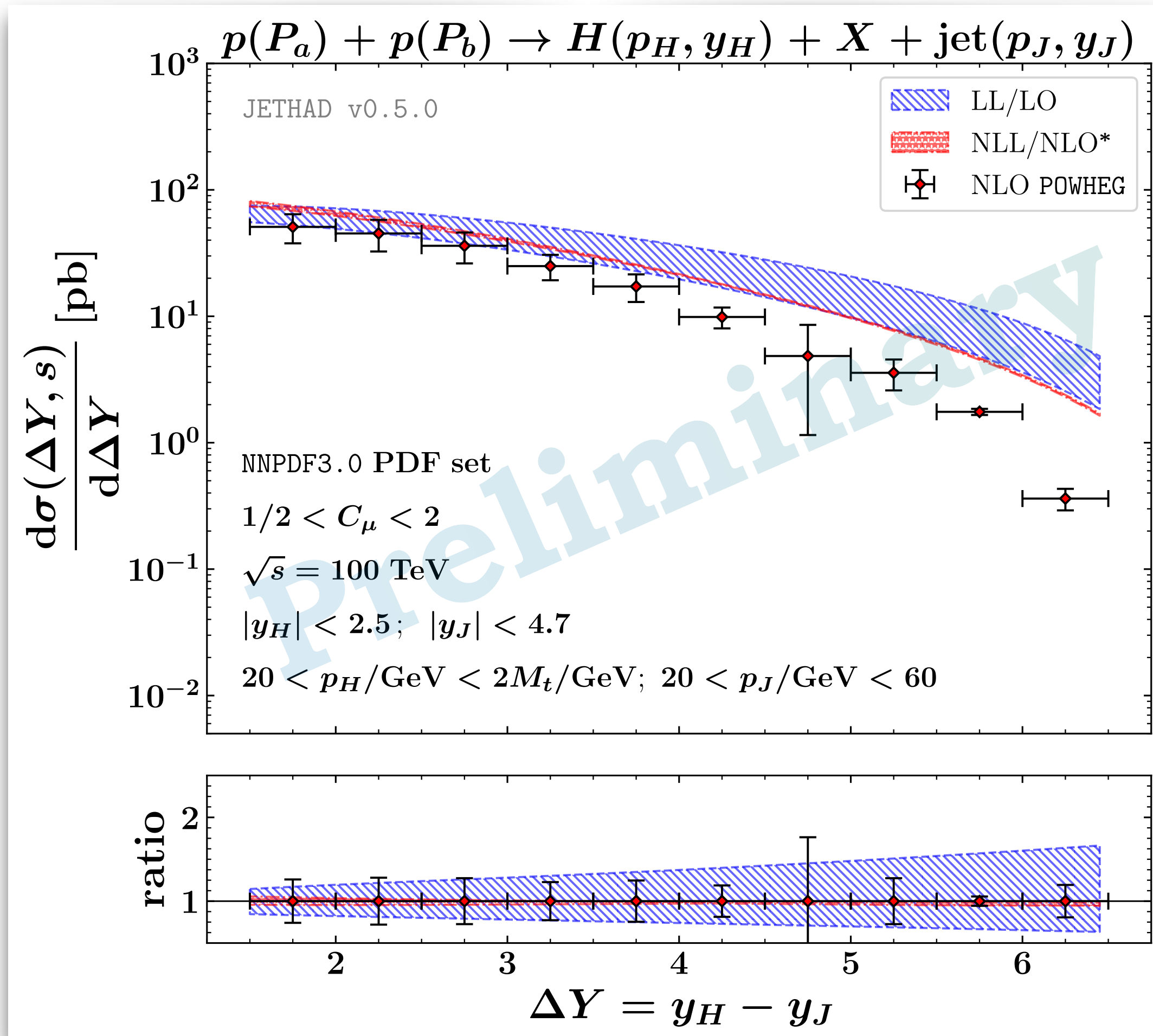
# Higgs + jet at @FCC: small- $\chi$ enhancement from PDFs

High-energy resummation + NNPDF3.0 [link](#)

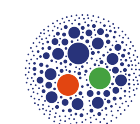
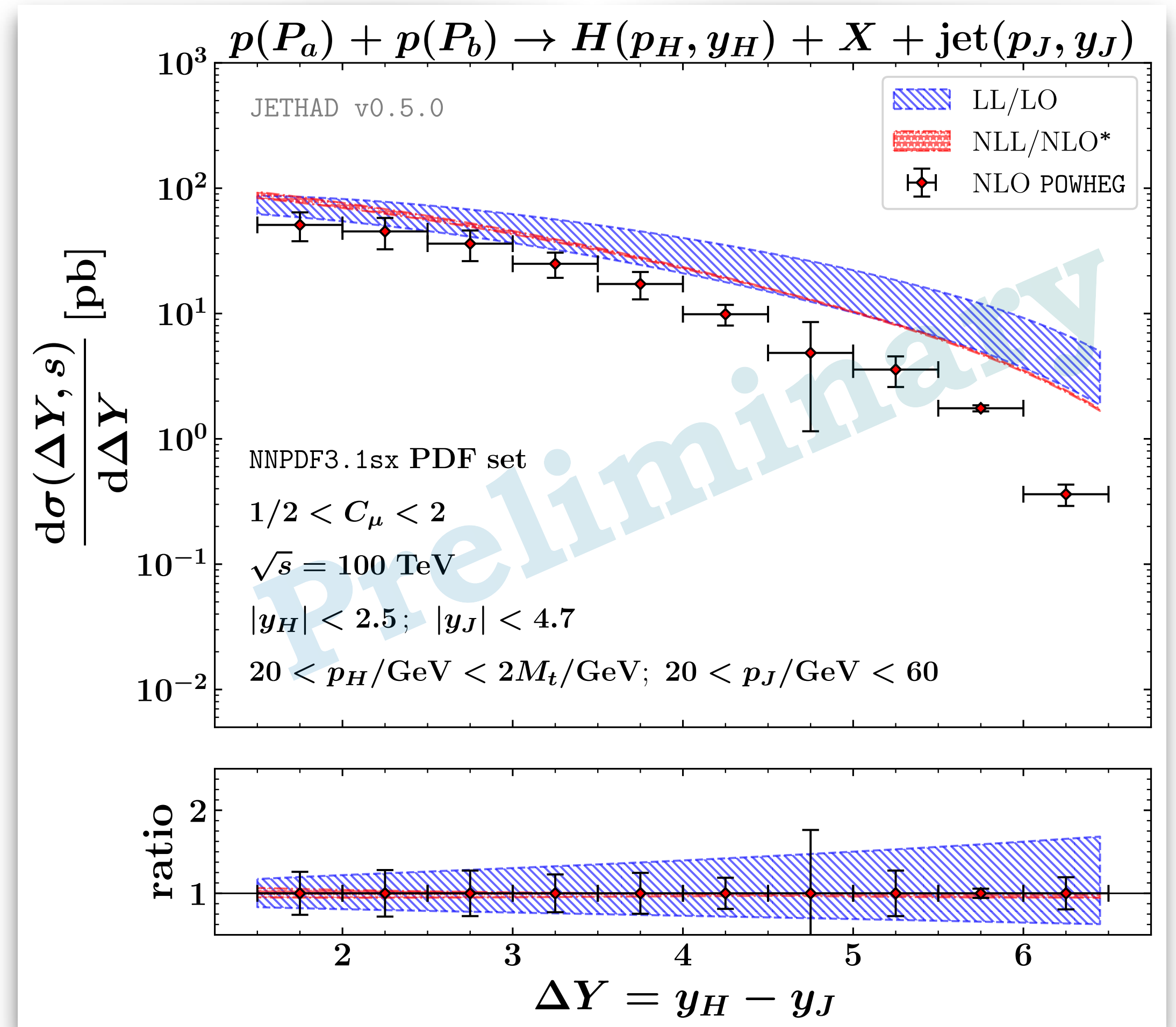


# Higgs + jet at @FCC: small- $x$ enhancement from PDFs

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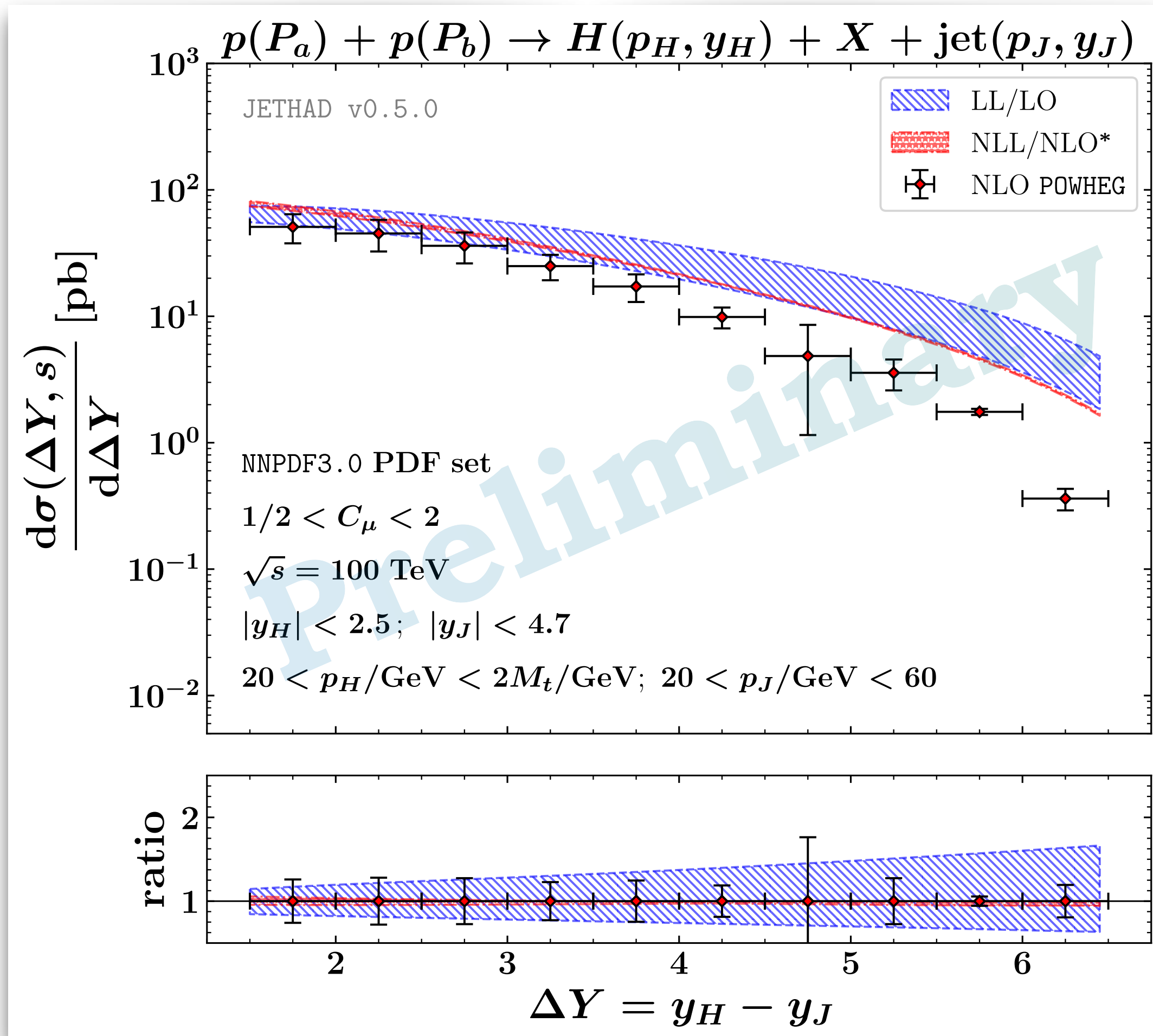
High-energy resummation + NNPDF3.1sx 



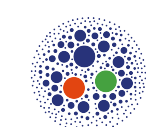
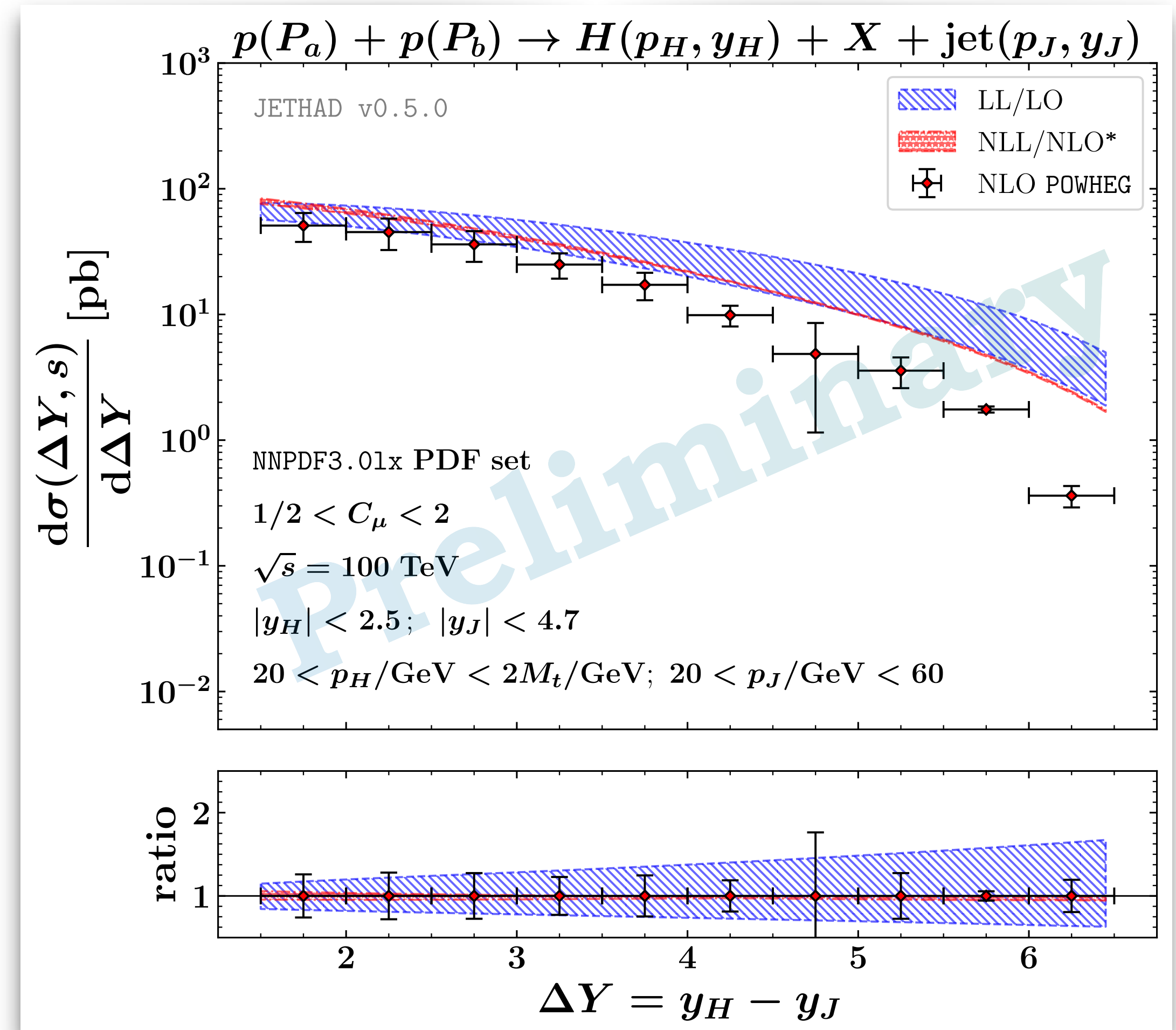
Small- $x$  resummation on PDFs  $\Rightarrow$   $+(13.5 \div 2.10) \%$  @NLL/NLO\*

# Higgs + jet at @FCC: large- $x$ enhancement from PDFs

High-energy resummation + NNPDF3.0 



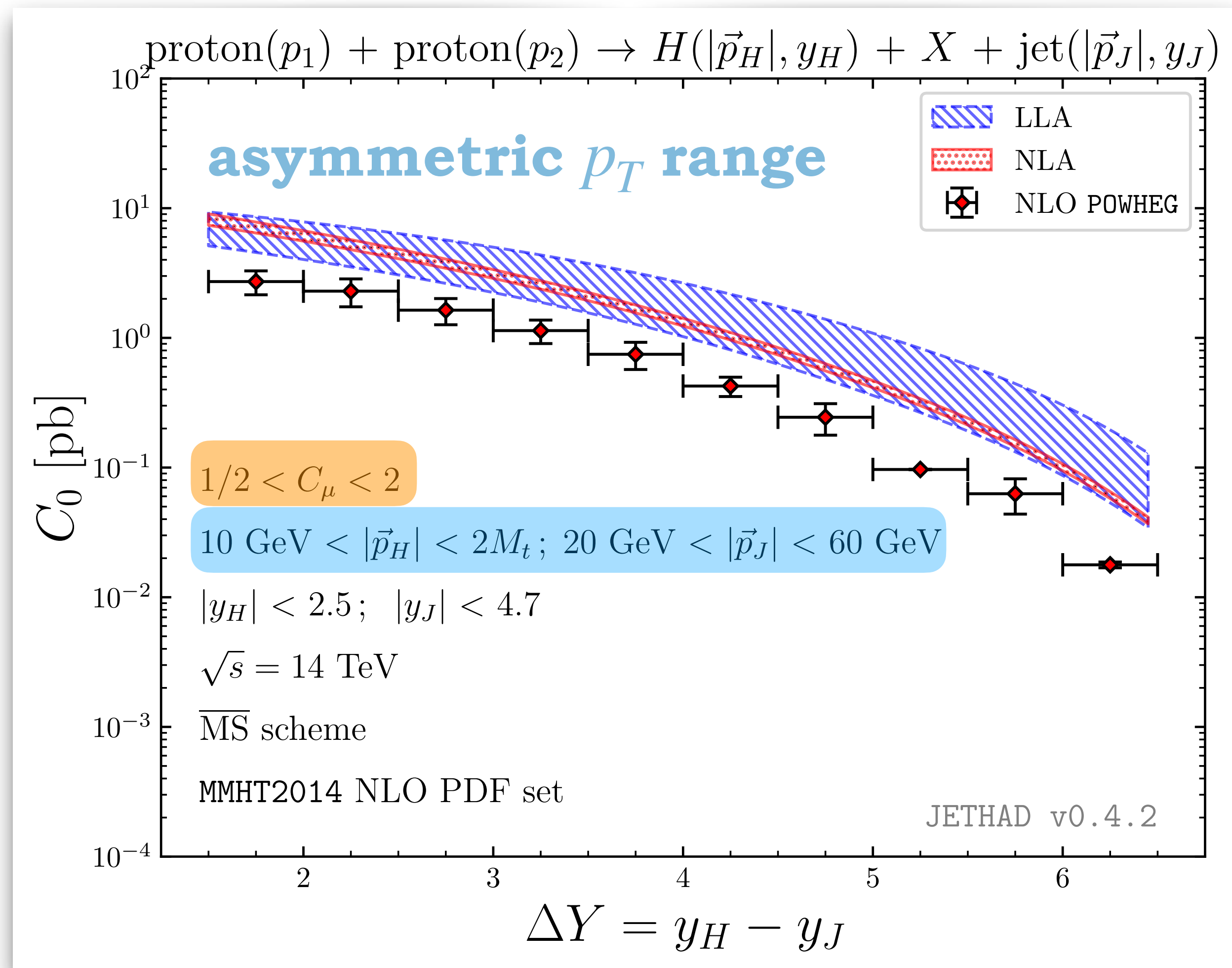
High-energy resummation + NNPDF3.01x 



Threshold resummation on PDFs  $\Rightarrow$   $+(10.7 \div 2.15)\%$  @NLL/NLO\*

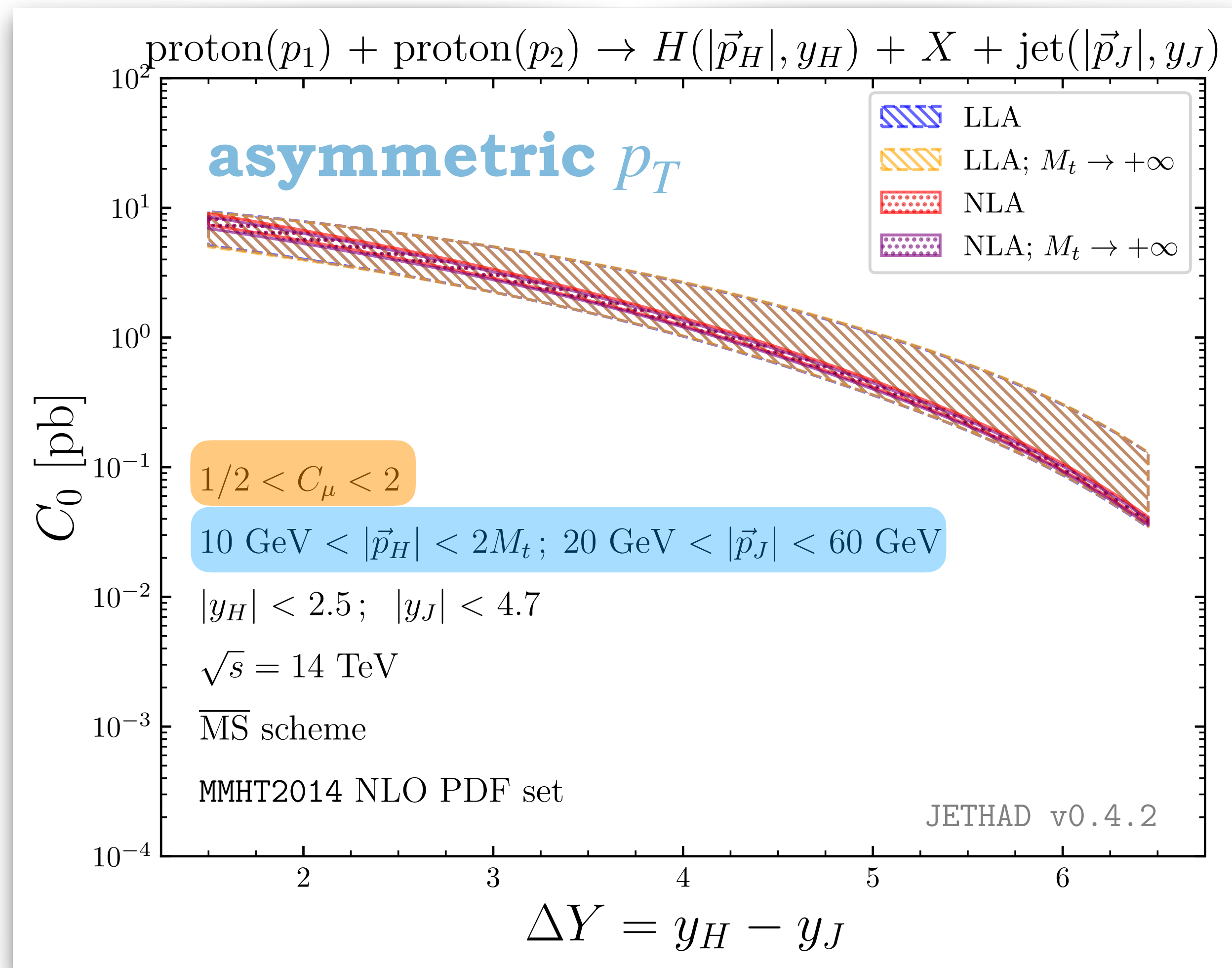
# $\Delta Y$ -distribution

$$C_n(\Delta Y, s) = \int_{p_H^{\min}}^{p_H^{\max}} d|\vec{p}_H| \int_{p_J^{\min}}^{p_J^{\max}} d|\vec{p}_J| \int_{y_H^{\min}}^{y_H^{\max}} dy_H \int_{y_J^{\min}}^{y_J^{\max}} dy_J \delta(y_H - y_J - \Delta Y) C_n$$



# $\Delta Y$ -distribution in the infinite top-mass limit

$$C_n(\Delta Y, s) = \int_{p_H^{\min}}^{p_H^{\max}} d|\vec{p}_H| \int_{p_J^{\min}}^{p_J^{\max}} d|\vec{p}_J| \int_{y_H^{\min}}^{y_H^{\max}} dy_H \int_{y_J^{\min}}^{y_J^{\max}} dy_J \delta(y_H - y_J - \Delta Y) C_n$$



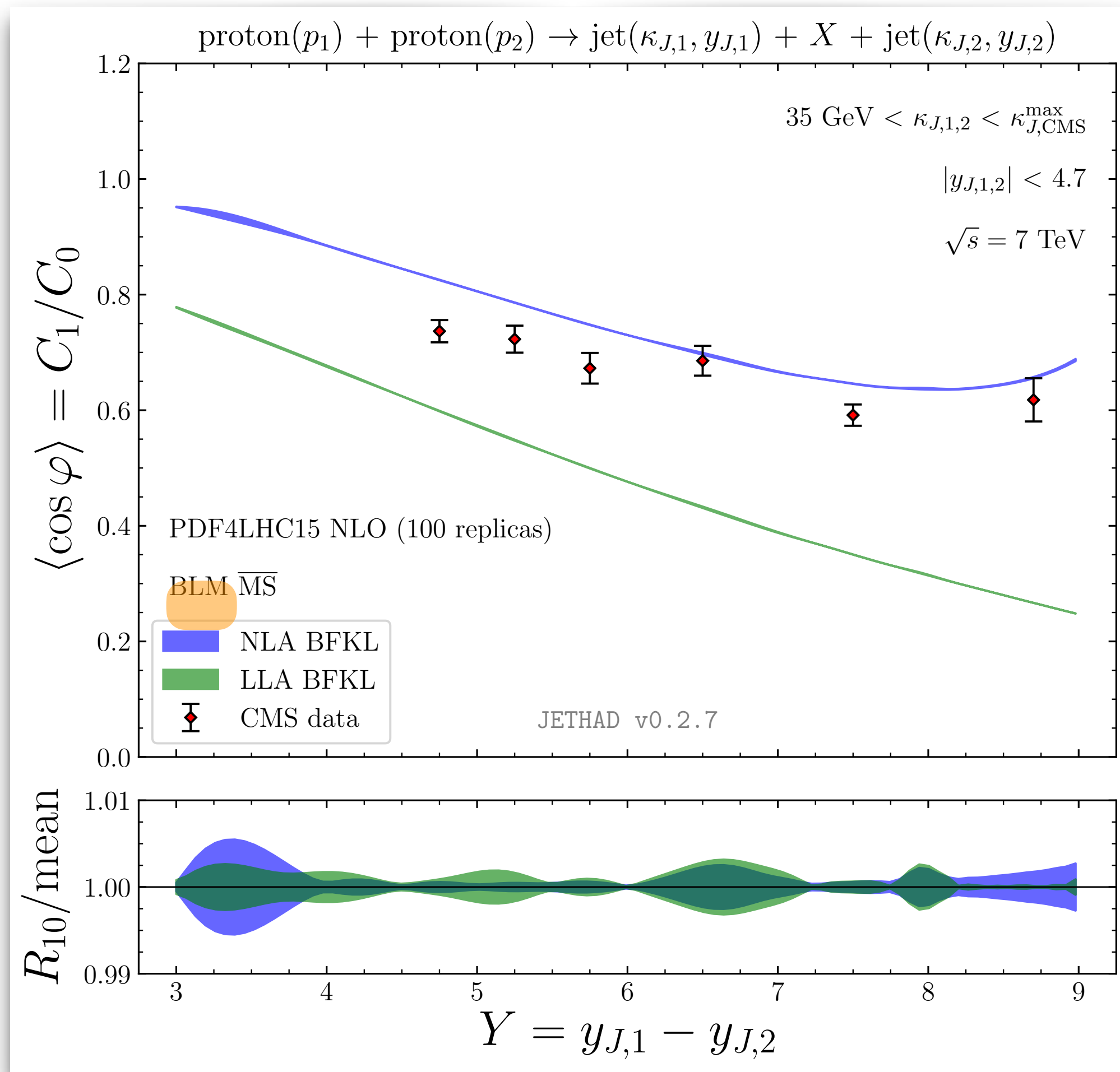
# Angular correlations in the infinite top-mass limit

$$R_{n0}(\Delta Y, s) = C_n/C_0 \equiv \langle \cos n\varphi \rangle$$

## Mueller-Navelet jets

[\[B. Ducloué, L. Szymanowski, S. Wallon, Phys.Rev.Lett. 112 \(2014\) 082003\]](#)

(figure below) [\[F. G. C., Eur. Phys. J. C 81 \(2021\) 8, 691\]](#)



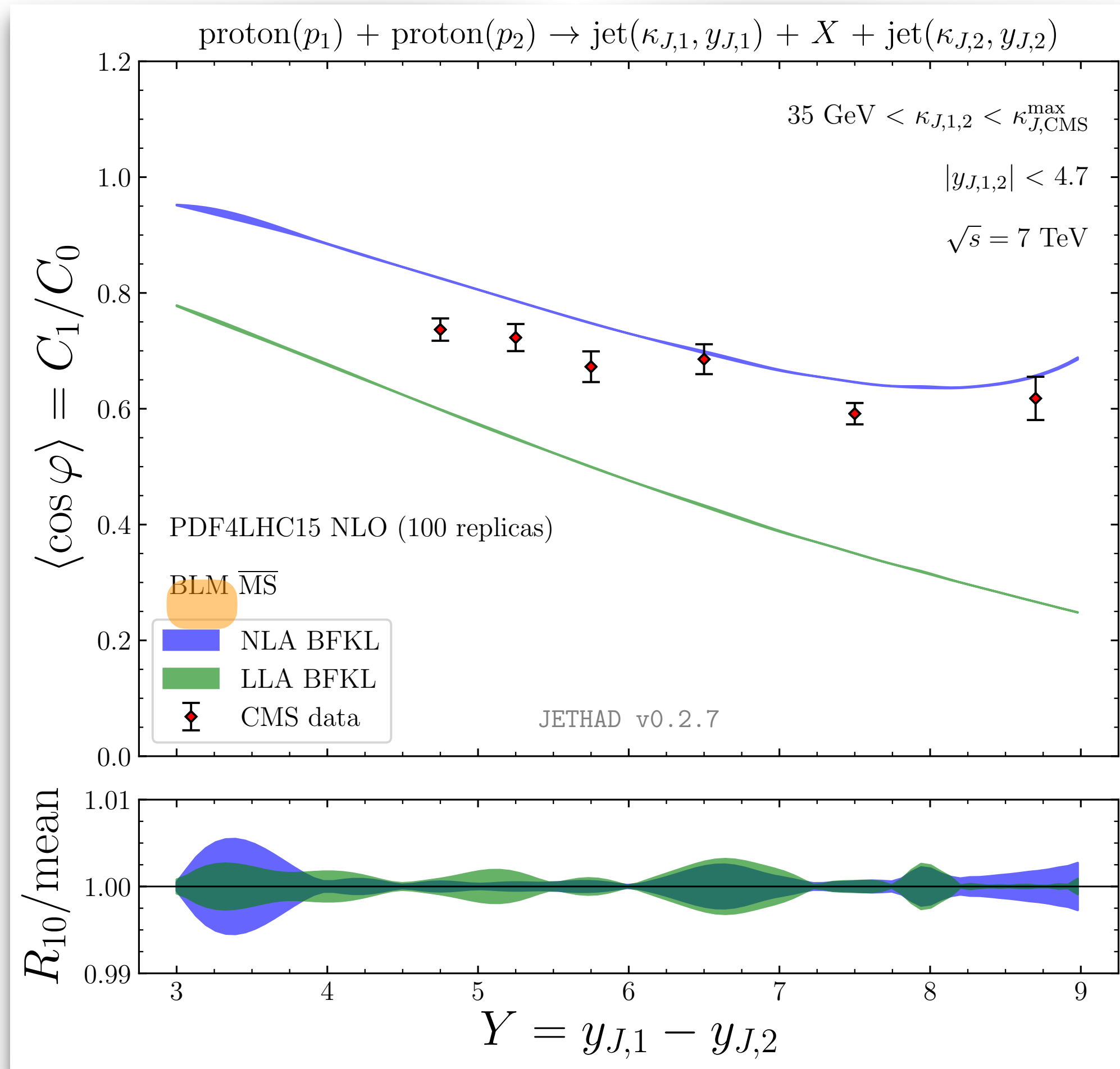
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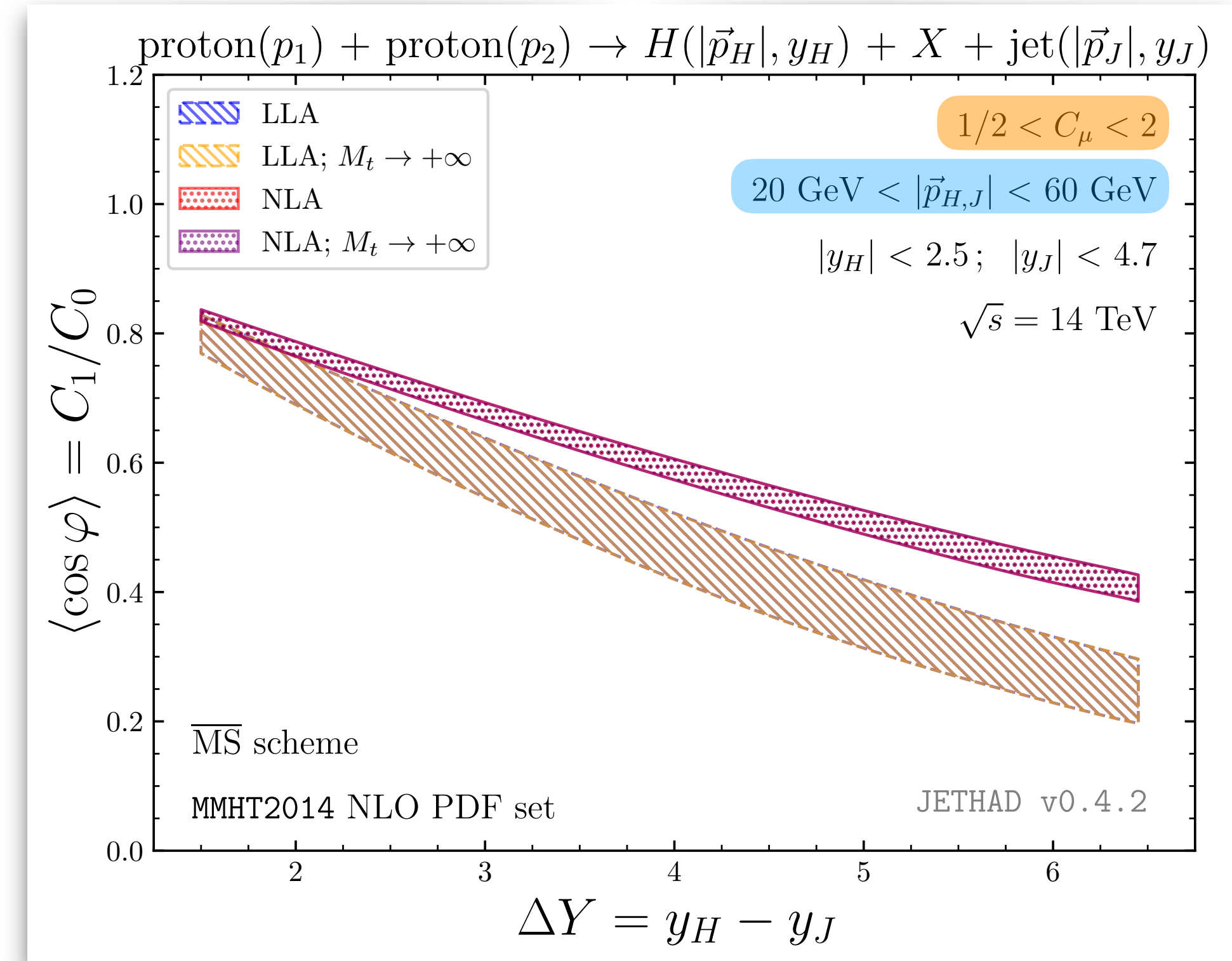
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## Higgs + jet

(figure below) [F. G. C. et al., Eur. Phys. J. C 81 (2021) 4, 293]

(NLO Higgs impact factor) [F. G. C. et al., under review (2022)]

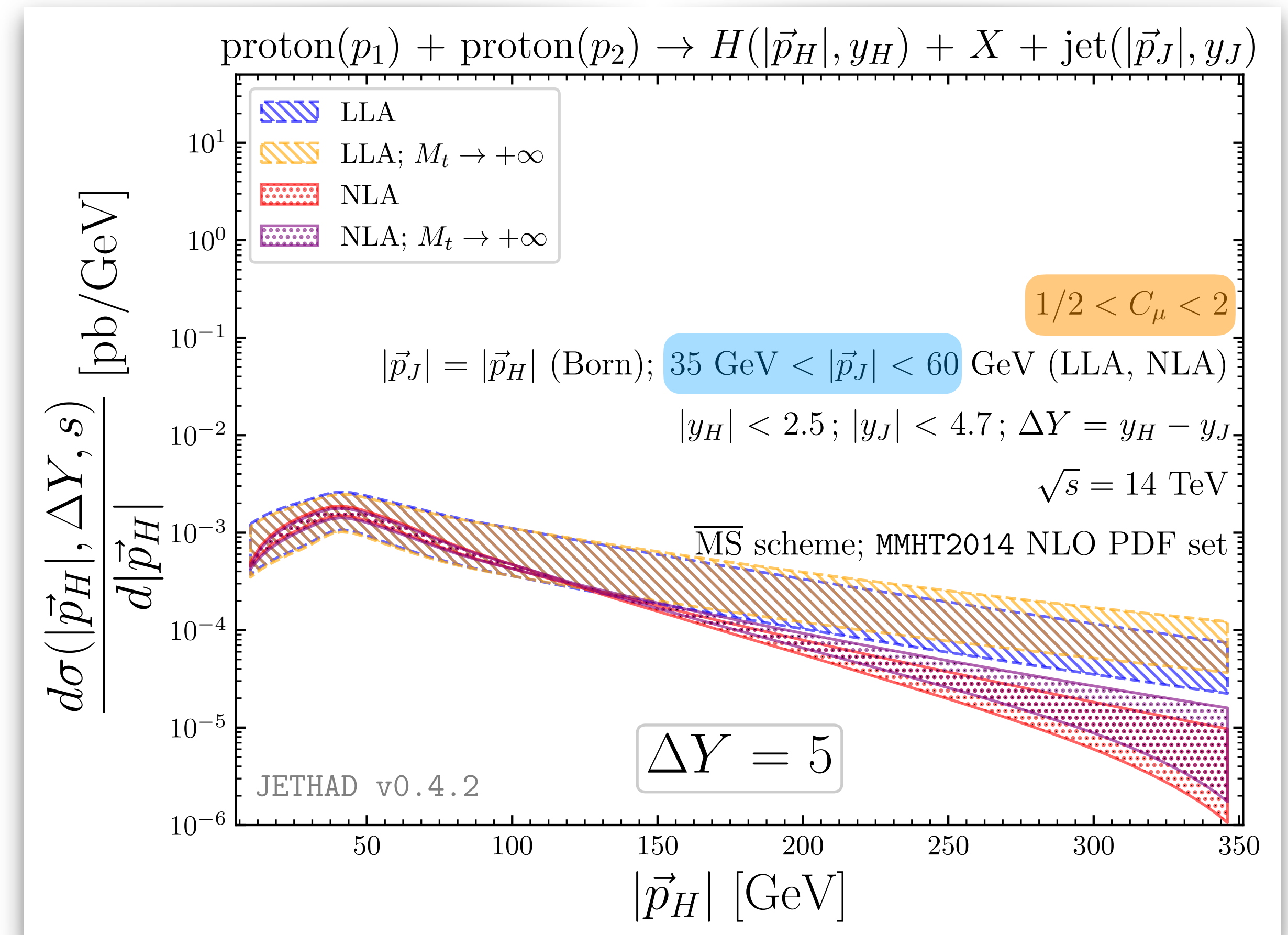
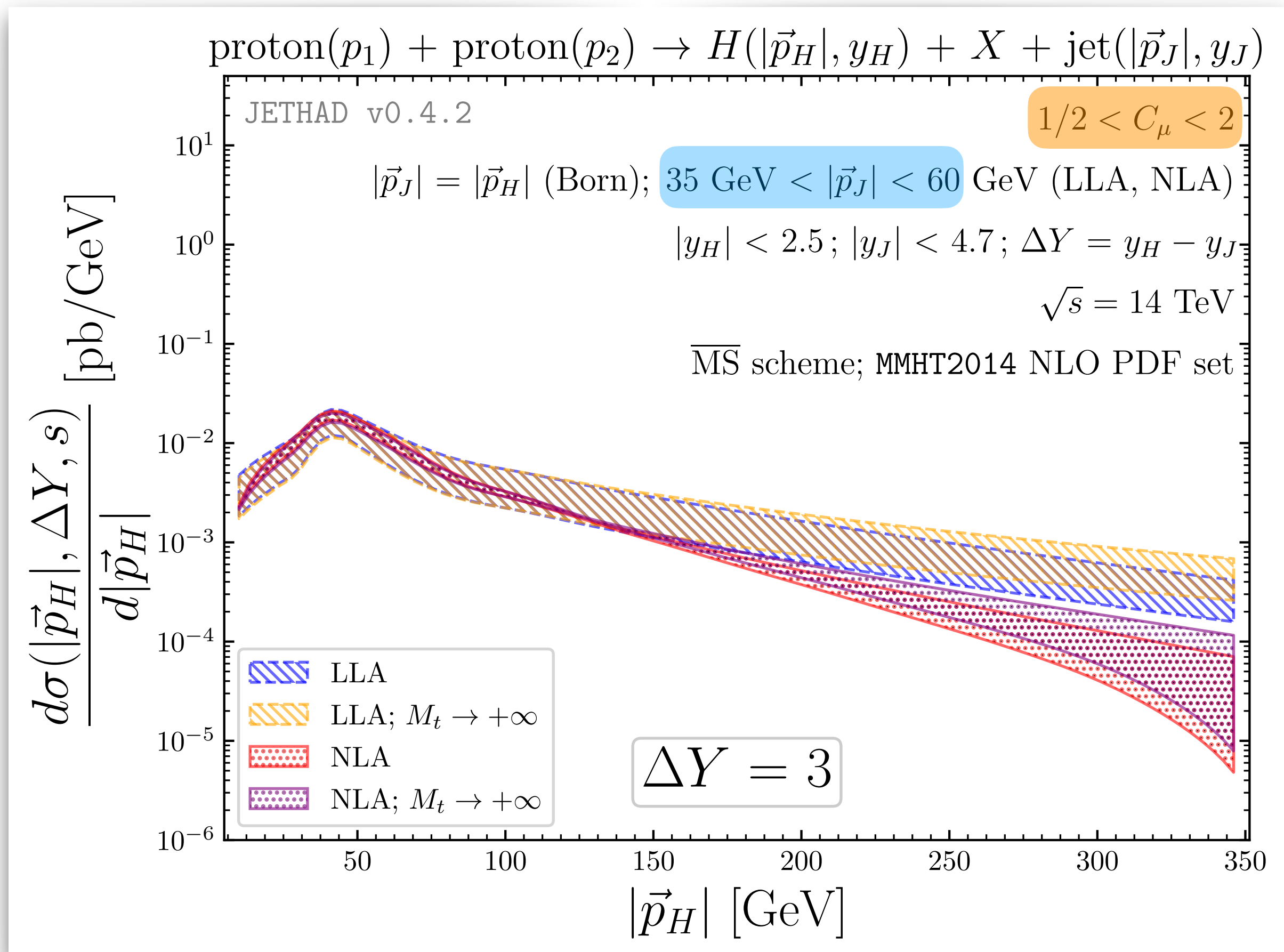


natural scales

symmetric  $p_T$  range



# Higgs transverse-momentum distribution for $(M_t \rightarrow +\infty)$



# Basics of BFKL

The background features a complex, multi-layered illustration of particle physics concepts. It includes several overlapping circular diagrams, each containing a network of yellow wavy lines representing gluons. These lines are connected to various colored spheres (red, blue, green) and arrows, likely representing quarks and their interactions. The overall aesthetic is scientific and abstract, with a light blue and green color palette and a subtle grid pattern.

# The high-energy resummation

## Glue Reggeization in perturbative QCD

◇ Glue quantum numbers in the  $t$ -channel:  $8^-$  representation

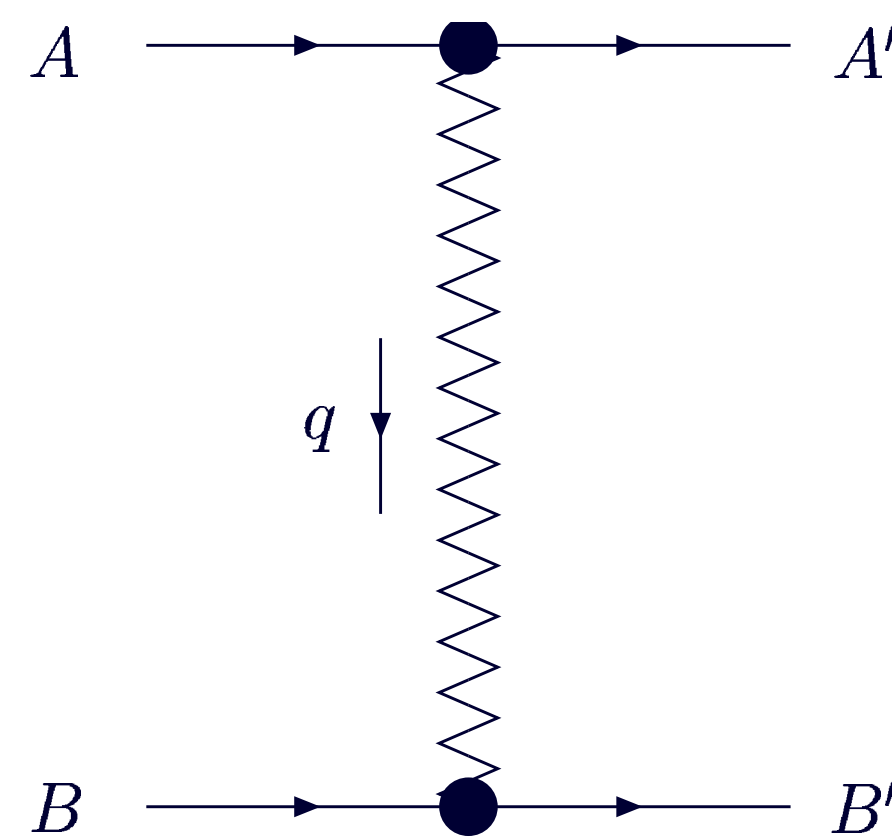
◇ Regge limit:  $s \simeq -u \rightarrow \infty$ ,  $t$  not growing with  $s$

→ amplitudes governed by **glue Reggeization**  $\rightarrow D_{\mu\nu} = -i \frac{g_{\mu\nu}}{q^2} \left(\frac{s}{s_0}\right)^{\alpha_g(q^2)-1}$

$\xrightarrow{\text{feature}}$  all-order resummation: **LLA** [ $\alpha_s^n (\ln s)^n$ ] + **NLA** [ $\alpha_s^{n+1} (\ln s)^n$ ]

$\xrightarrow{\text{consequence}}$  factorization of elastic and real part of inelastic amplitudes

$\xrightarrow{\text{example}}$  Elastic scattering process:  $A + B \rightarrow A' + B'$



$$(\mathcal{A}_8^-)_{AB}^{A'B'} = \Gamma_{A'A}^c \left[ \left(\frac{-s}{-t}\right)^{j(t)} - \left(\frac{s}{-t}\right)^{j(t)} \right] \Gamma_{B'B}^c$$

$$j(t) = 1 + \omega(t), \quad j(0) = 1$$

$\omega(t) \rightarrow$  Reggeized gluon trajectory

$$\Gamma_{A'A}^c = g \langle A' | T^c | A \rangle \Gamma_{A'A} \rightarrow \text{PPR vertex}$$

$T^c \rightarrow$  fundamental ( $q$ ) or adjoint ( $g$ )

- QCD is the unique SM theory where all elementary particles reggeize
- Possible extensions: N=4 SYM, AdS/CFT,...

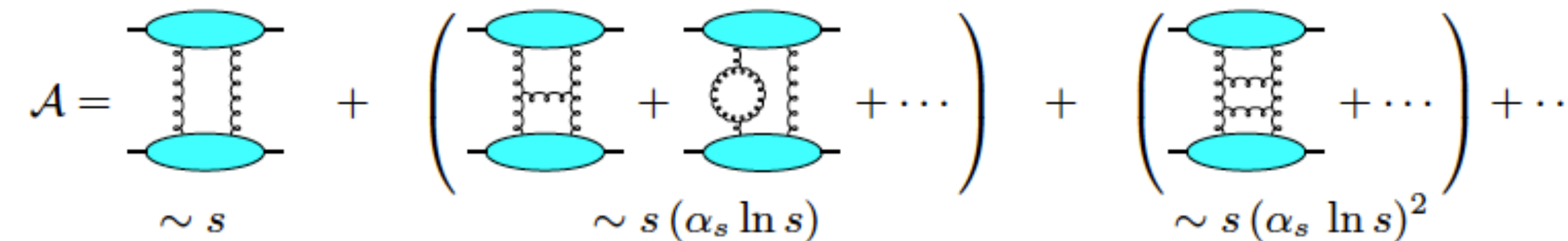
# The high-energy resummation

- **BFKL resummation:** [V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975, 1976, 1977); Y.Y. Balitskii, L.N. Lipatov (1978)]

based on  $\longrightarrow$  **gluon Reggeization**

leading logarithmic approximation (LL):

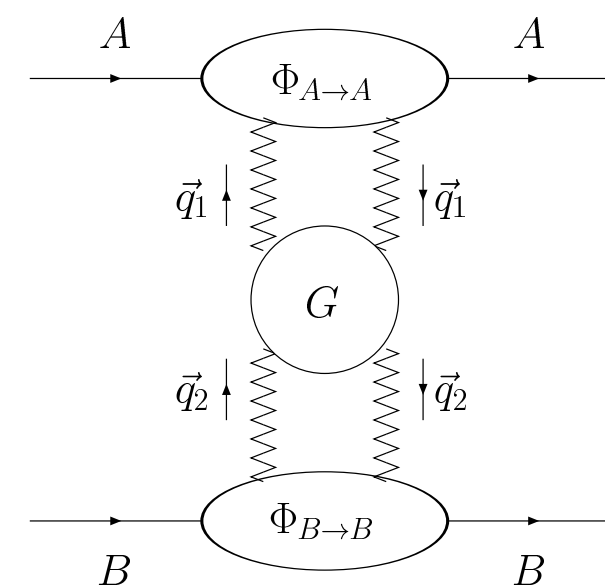
$$\alpha_s^n (\ln s)^n$$



next-to-leading logarithmic approximation (NLL):

$$\alpha_s^{n+1} (\ln s)^n$$

Total cross section for  $A + B \rightarrow X$ :  $\sigma_{AB}(s) = \frac{\text{Im}_s \{ \mathcal{A}_{AB}^{AB} \}}{s} \Leftarrow$  **optical theorem**



►  $\text{Im}_s \{ \mathcal{A}_{AB}^{AB} \}$  factorization:

convolution of the **Green's function** of two interacting Reggeized gluons with the **impact factors** of the colliding particles

Green's function is **process-independent**, describes energy dependence and obeys BFKL equation; impact factors are known in the **NLL just for few processes**

# The high-energy resummation

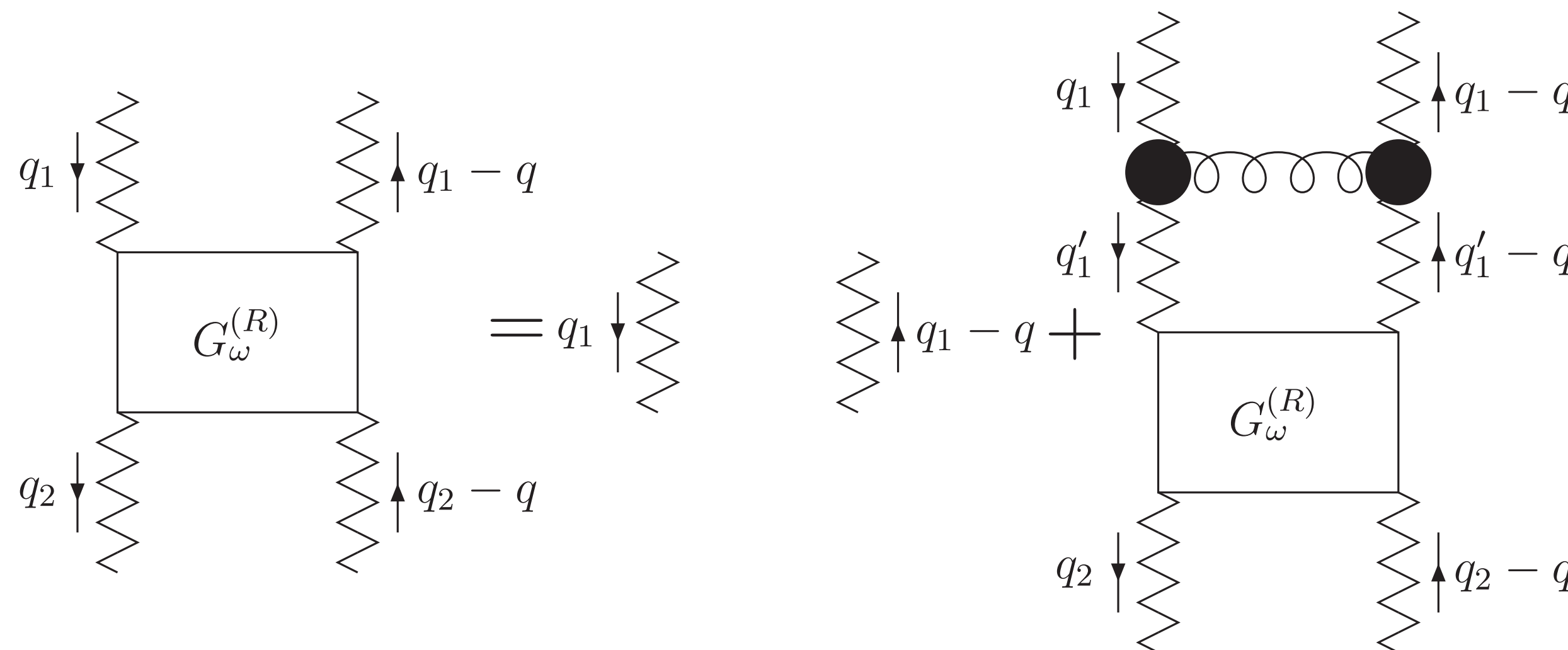
$$\text{Im}_s \{ \mathcal{A} \} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2} q_1}{\vec{q}_1^2} \Phi_A(\vec{q}_1, \mathbf{s}_0) \int \frac{d^{D-2} q_2}{\vec{q}_2^2} \Phi_B(-\vec{q}_2, \mathbf{s}_0) \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left( \frac{s}{s_0} \right)^\omega G_\omega(\vec{q}_1, \vec{q}_2)$$

- **Green's function** is **process-independent** and takes care of the **energy dependence**

→ determined through the **BFKL equation**

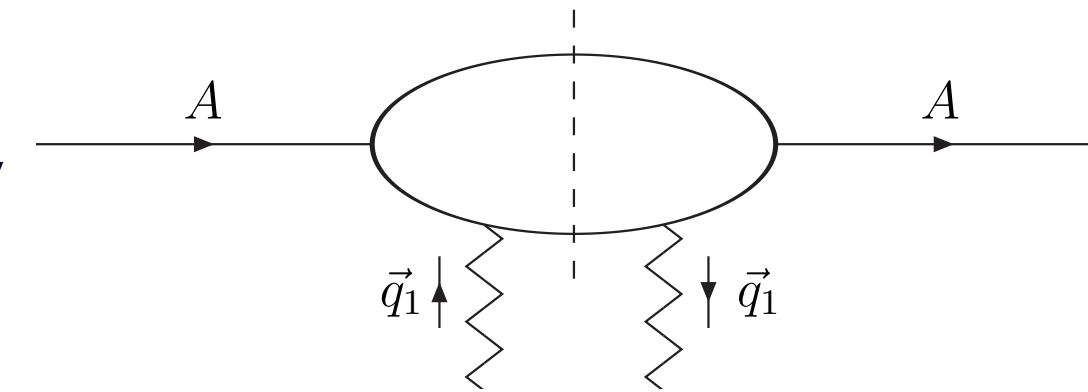
[Ya.Ya. Balitskii, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

$$\omega G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{D-2}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2} q K(\vec{q}_1, \vec{q}) G_\omega(\vec{q}, \vec{q}_1).$$



# The high-energy resummation

- **Impact factors** are **process-dependent** and depend on the hard scale, but not on the energy  
→ known in the NLA just for few processes



[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]  
[M. Ciafaloni, G. Rodrigo (2000)]

- ◇  $\gamma^* \longrightarrow V$ , with  $V = \rho^0, \omega, \phi$ , forward case

[D.Yu. Ivanov, M.I. Kotsky, A. Papa (2004)]

- ◇ forward jet production

[J. Bartels, D. Colferai, G.P. Vacca (2003)]  
(exact IF) [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, A. Perri (2012)]  
(small-cone IF) [D.Yu. Ivanov, A. Papa (2012)]  
(several jet algorithms discussed) [D. Colferai, A. Niccoli (2015)]

- ◇ forward identified hadron production

[D.Yu. Ivanov, A. Papa (2012)]

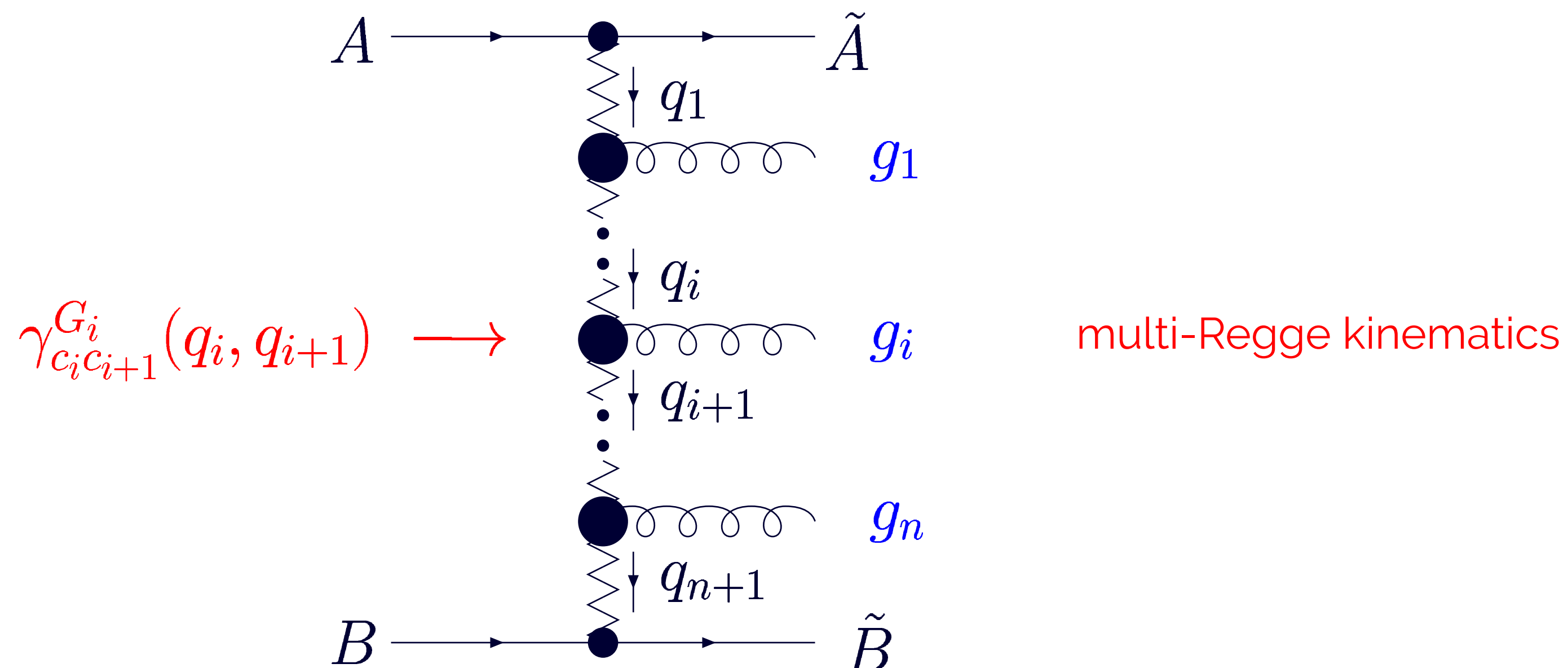
- ◇  $\gamma^* \longrightarrow \gamma^*$

[J. Bartels *et al.* (2001), I. Balitsky, G.A. Chirilli (2011, 2013)]

# The high-energy resummation

## BFKL in the LLA (I)

Inelastic scattering process  $A + B \rightarrow \tilde{A} + \tilde{B} + n$  in the LLA



$$\text{Re} \mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} = 2s \Gamma_{\tilde{A}A}^{c_1} \left( \prod_{i=1}^n \gamma_{c_i c_{i+1}}^{P_i}(q_i, q_{i+1}) \left( \frac{s_i}{s_R} \right)^{\omega(t_i)} \frac{1}{t_i} \right) \frac{1}{t_{n+1}} \left( \frac{s_{n+1}}{s_R} \right)^{\omega(t_{n+1})} \Gamma_{\tilde{B}B}^{c_{n+1}}$$

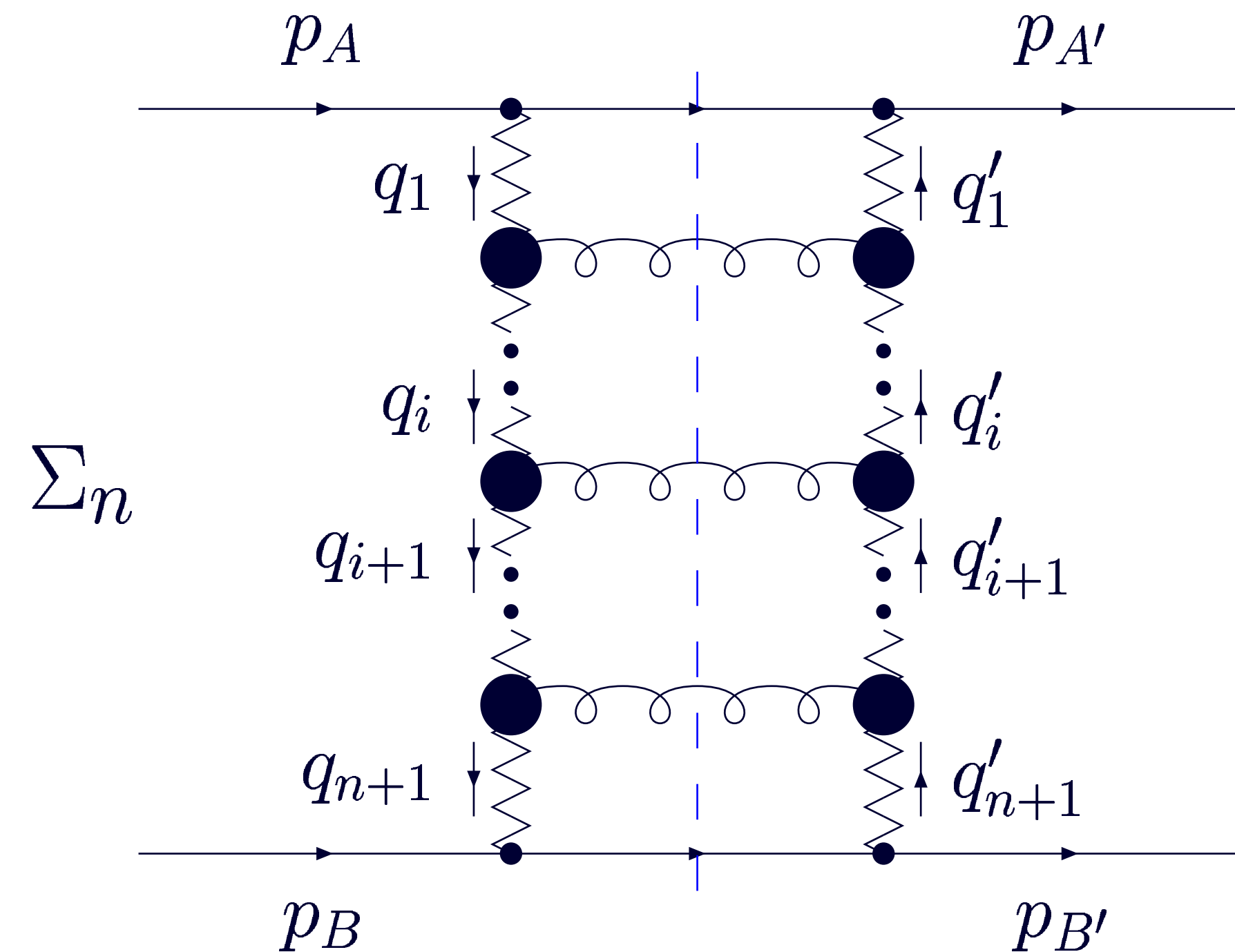
$\gamma_{c_i c_{i+1}}^{P_i}(q_i, q_{i+1}) \rightarrow$  RRG vertex

$s_R \rightarrow$  energy scale, irrelevant in the LLA

# The high-energy resummation

## BFKL in the LLA (II)

Elastic amplitude  $A + B \longrightarrow A' + B'$  in the LLA via  $s$ -channel unitarity



$$\mathcal{A}_{AB}^{A'B'} = \sum_{\mathcal{R}} (\mathcal{A}_{\mathcal{R}})_{AB}^{A'B'}, \quad \mathcal{R} = 1 \text{ (singlet), } 8^- \text{ (octet), } \dots$$

The  $8^-$  color representation is important for the **bootstrap**, i.e. the consistency between the above amplitude and that with one Reggeized gluon exchange



# The high-energy resummation

$$\text{Im}_s (\mathcal{A}) = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^2} \Phi_A(\vec{q}_1, \mathbf{s}_0) \int \frac{d^{D-2}q_2}{\vec{q}_2^2} \Phi_B(-\vec{q}_2, \mathbf{s}_0) \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left( \frac{s}{\mathbf{s}_0} \right)^\omega G_\omega(\vec{q}_1, \vec{q}_2)$$

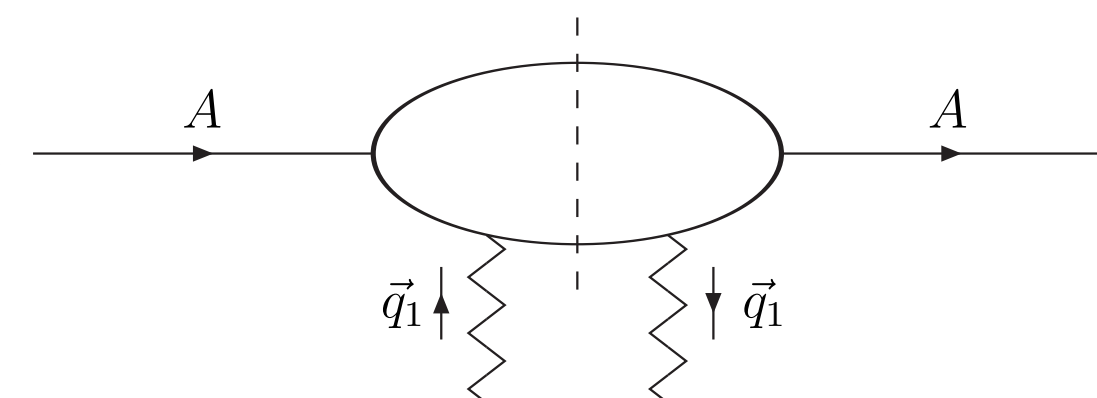
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[Ya.Ya. Balitskii, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

- **Impact factors** are **process-dependent** and depend on the hard scale, but not on the energy

→ known in the NLA just for few processes



- Successful tests of NLA BFKL in the **Mueller–Navelet** channel with the advent of the LHC; nevertheless, *new BFKL-sensitive observables* as well as *more exclusive final-state reactions* are needed (**di-hadron**, **hadron-jet**, **heavy-quark pair**, **multi-jet**, production processes,...)

(**MN jets**) [B. Ducloué, L. Szymanowski, S. Wallon (2014); F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2015, 2016)]

(**di-hadron**) [F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017)]

(**four-jet**) [F. Caporale, F.G.C., G. Chachamis, A. Sabio Vera (2016)]

(**multi-jet**) F. Caporale, F.G.C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016, 2017, 2017)]

(**heavy-quark pair**) [F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2018); A.D. Bolognino, F.G.C., D.Yu. Ivanov, M. Fucilla, A. Papa (2018)]

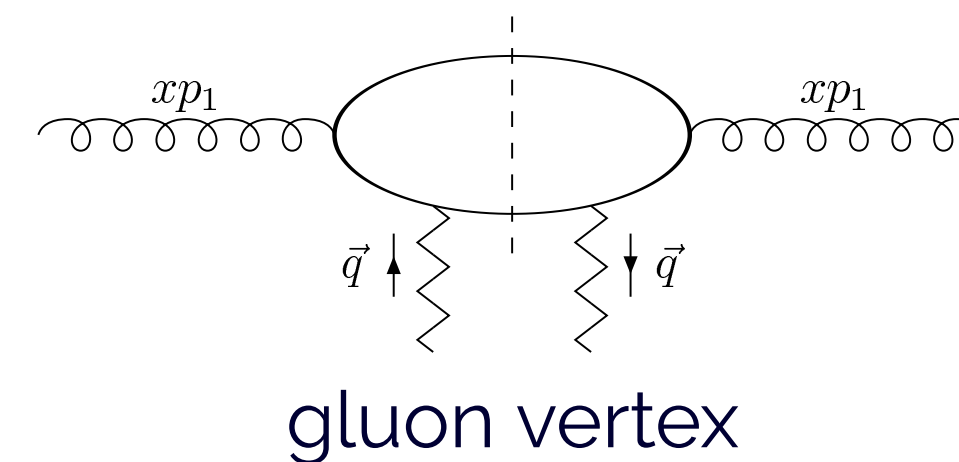
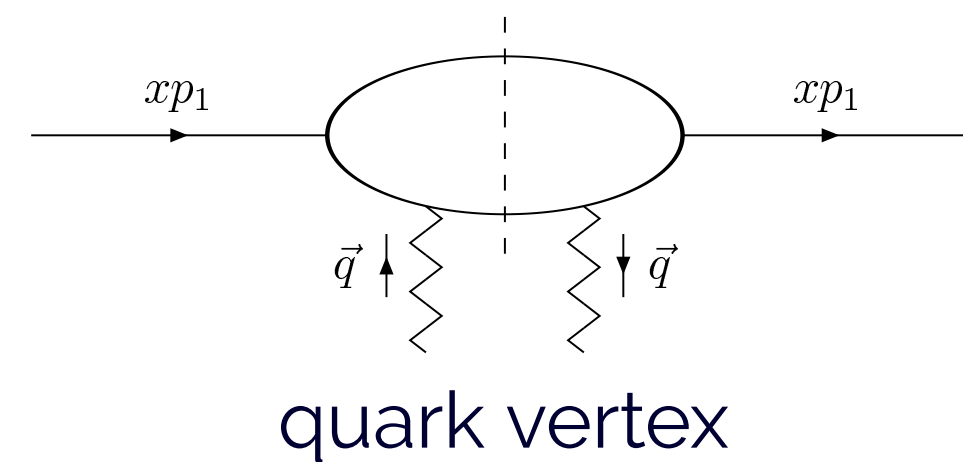
(**hadron-jet**) [M.M.A. Mohammed, MD thesis (2018); A.D. Bolognino, F.G.C., D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

## Forward-jet impact factor

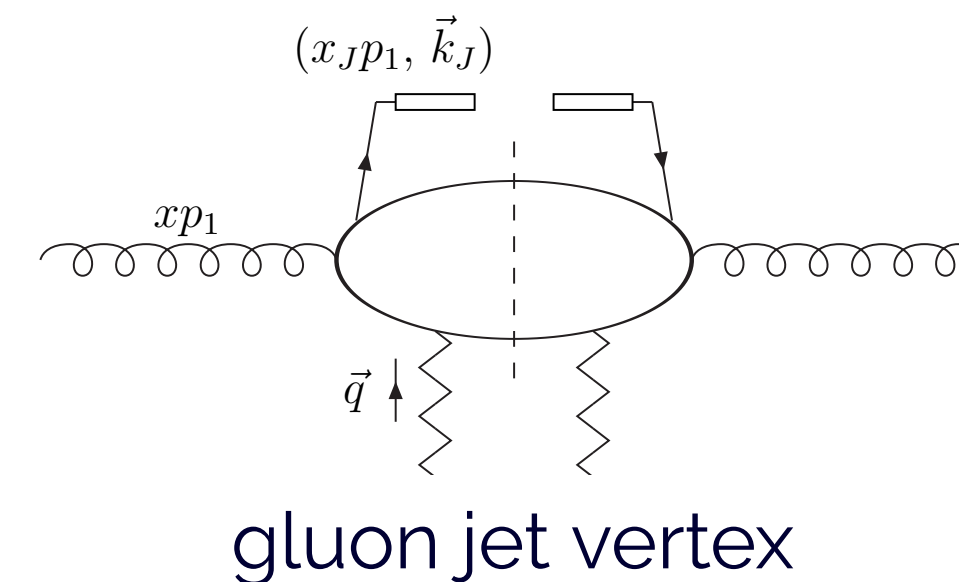
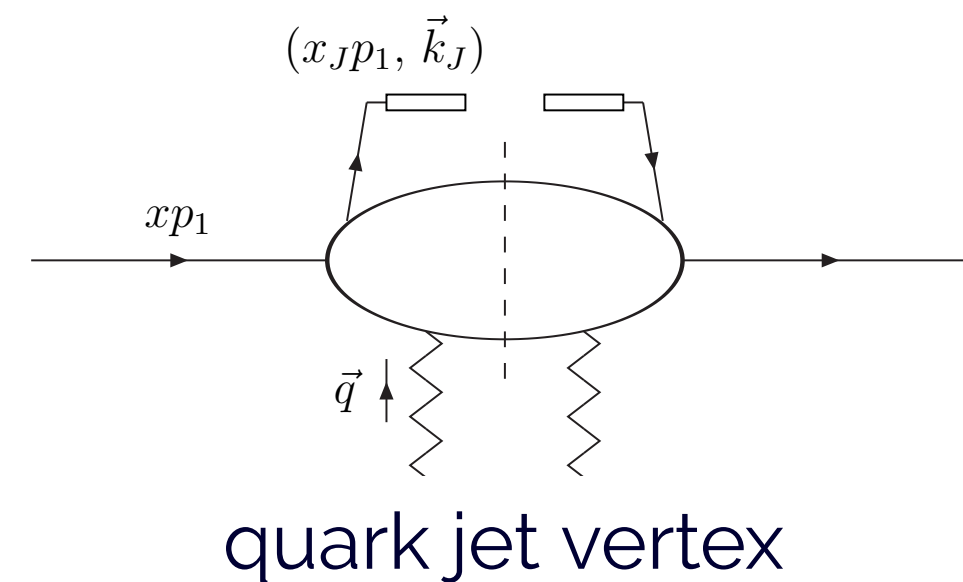
- take the impact factors for **colliding partons**

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]

[M. Ciafaloni and G. Rodrigo (2000)]

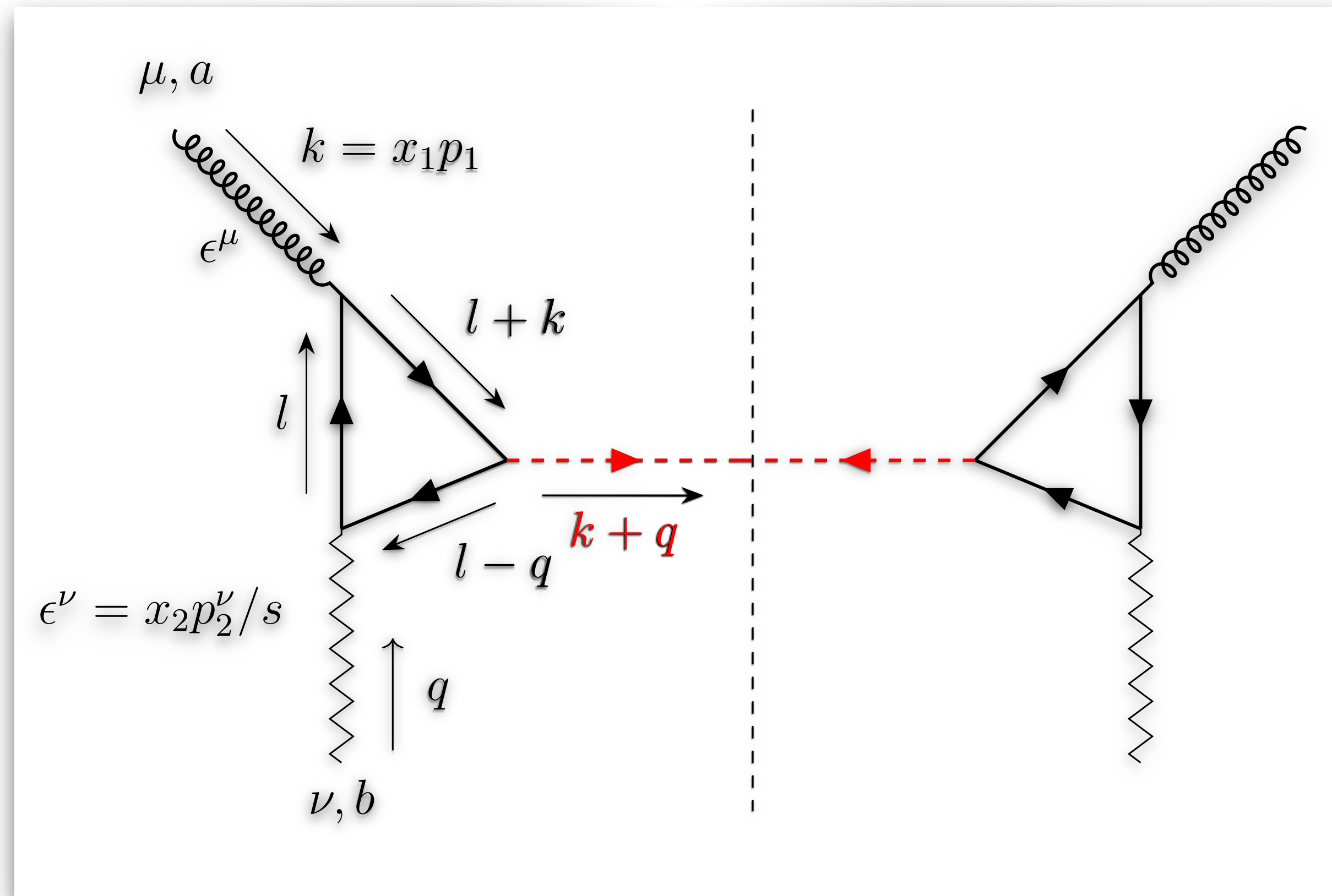


- “open” one of the integrations over the phase space of the intermediate state to allow one parton to generate the jet



- use QCD collinear factoriz.:  $\sum_{s=q,\bar{q}} f_s \otimes [\text{quark vertex}] + f_g \otimes [\text{gluon vertex}]$

# Forward-Higgs LO impact factor



$$\frac{d\Phi_J^{(0)}(\nu, n)}{dx_J d^2\vec{p}_J} = 2\alpha_s \sqrt{\frac{C_F}{C_A}} (\vec{p}_J^2)^{i\nu-3/2} \left( \frac{C_A}{C_F} f_g(x_J) + \sum_{a=q\bar{q}} f_a(x_J) \right) e^{in\phi_J}$$

# Forward-Higgs NLO-RG impact factor

$$\begin{aligned} \tilde{c}_H^{(1)}(n, \nu, |\vec{p}_H|, x_H) = c_H(n, \nu, |\vec{p}_H|, x_H) & \left\{ \frac{\beta_0}{4N_c} \left( 2 \ln \frac{\mu_{R_1}}{|\vec{p}_H|} + \frac{5}{3} \right) + \frac{\chi(n, \nu)}{2} \ln \left( \frac{s_0}{M_{H,\perp}^2} \right) \right. \\ & + \frac{\beta_0}{4N_c} \left( 2 \ln \frac{\mu_{R_1}}{M_{H,\perp}} \right) \\ & \left. - \frac{1}{2N_c f_g(x_H, \mu_{F_1})} \ln \frac{\mu_{F_1}^2}{M_{H,\perp}^2} \int_{x_H}^1 \frac{dz}{z} \left[ P_{gg}(z) f_g \left( \frac{x_H}{z}, \mu_{F_1} \right) + \sum_{a=q, \bar{q}} P_{ga}(z) f_a \left( \frac{x_H}{z}, \mu_{F_1} \right) \right] \right\} \end{aligned}$$

# Forward-jet NLO-RG impact factor

$$\begin{aligned}
 \tilde{c}_J^{(1)}(n, \nu, |\vec{p}_J|, x_J) = & c_J(n, \nu, |\vec{p}_J|, x_J) \left\{ \frac{\beta_0}{4N_c} \left( 2 \ln \frac{\mu_{R_2}}{|\vec{p}_J|} + \frac{5}{3} \right) + \frac{\chi(n, \nu)}{2} \ln \left( \frac{s_0}{|\vec{p}_J|^2} \right) \right. \\
 & - \frac{1}{2N_c \left( \frac{C_A}{C_F} f_g(x_J, \mu_{F_2}) + \sum_{a=q, \bar{q}} f_a(x_J, \mu_{F_2}) \right)} \ln \frac{\mu_{F_2}^2}{|\vec{p}_J|^2} \\
 & \times \left( \frac{C_A}{C_F} \int_{x_J}^1 \frac{dz}{z} \left[ P_{gg}(z) f_g \left( \frac{x_J}{z}, \mu_{F_2} \right) + \sum_{a=q, \bar{q}} P_{ga}(z) f_a \left( \frac{x_J}{z}, \mu_{F_2} \right) \right] \right. \\
 & \left. \left. + \sum_{a=q, \bar{q}} \int_{x_J}^1 \frac{dz}{z} \left[ P_{ag}(z) f_g \left( \frac{x_J}{z}, \mu_{F_2} \right) + P_{aa}(z) f_a \left( \frac{x_J}{z}, \mu_{F_2} \right) \right] \right) \right\} .
 \end{aligned}$$

# Inclusive Higgs+jet: NLL/NLO\* azimuthal coefficients

$$\begin{aligned}
 C_n &= \frac{e^{\Delta Y}}{s} \frac{M_{H,\perp}}{|\vec{p}_H|} \\
 &\times \int_{-\infty}^{+\infty} d\nu \left( \frac{x_J x_H s}{s_0} \right)^{\bar{\alpha}_s(\mu_{R_c})} \left\{ \chi(n, \nu) + \bar{\alpha}_s(\mu_{R_c}) \left[ \bar{\chi}(n, \nu) + \frac{\beta_0}{8N_c} \chi(n, \nu) \left[ -\chi(n, \nu) + \frac{10}{3} + 4 \ln \left( \frac{\mu_{R_c}}{\sqrt{|\vec{p}_H \vec{p}_J|}} \right) \right] \right] \right\} \\
 &\quad \times \left\{ \alpha_s^2(\mu_{R_1}) c_H(n, \nu, |\vec{p}_H|, x_H) \right\} \left\{ \alpha_s(\mu_{R_2}) [c_J(n, \nu, |\vec{p}_J|, x_J)]^* \right\} \\
 &\quad \times \left\{ 1 + \bar{\alpha}_s(\mu_{R_1}) \frac{\tilde{c}_H^{(1)}(n, \nu, |\vec{p}_H|, x_H)}{c_H(n, \nu, |\vec{p}_H|, x_H)} + \bar{\alpha}_s(\mu_{R_2}) \left[ \frac{\tilde{c}_J^{(1)}(n, \nu, |\vec{p}_J|, x_J)}{c_J(n, \nu, |\vec{p}_J|, x_J)} \right]^* \right\} .
 \end{aligned}$$