

ECT*

EUROPEAN CENTRE FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS



FONDAZIONE
BRUNO KESSLER



Equation of state for hot hyperonic neutron star matter



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Laura Tolos
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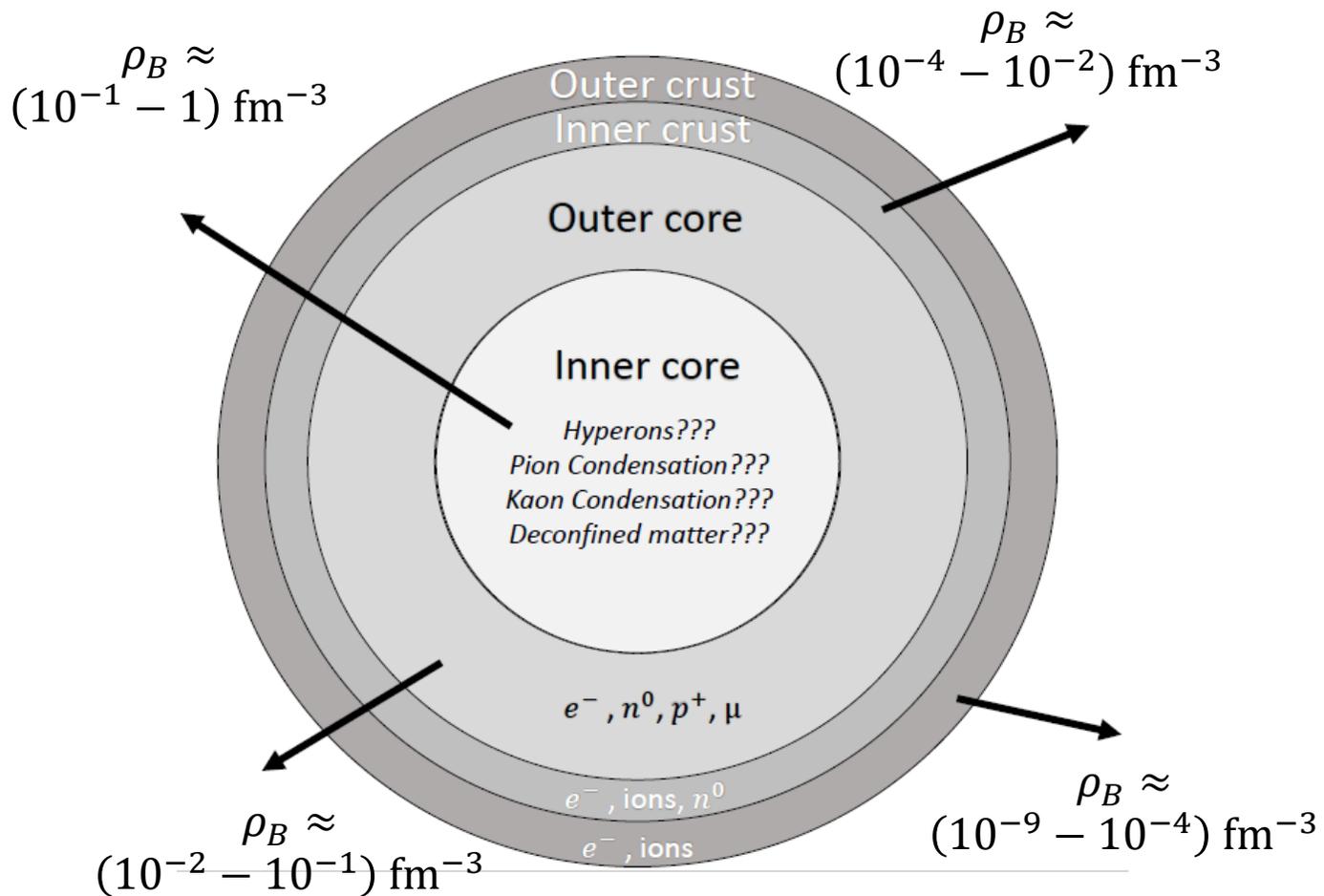
EXOTICO workshop 2022
Trento, Italy

17-21 October 2022

OUTLINE

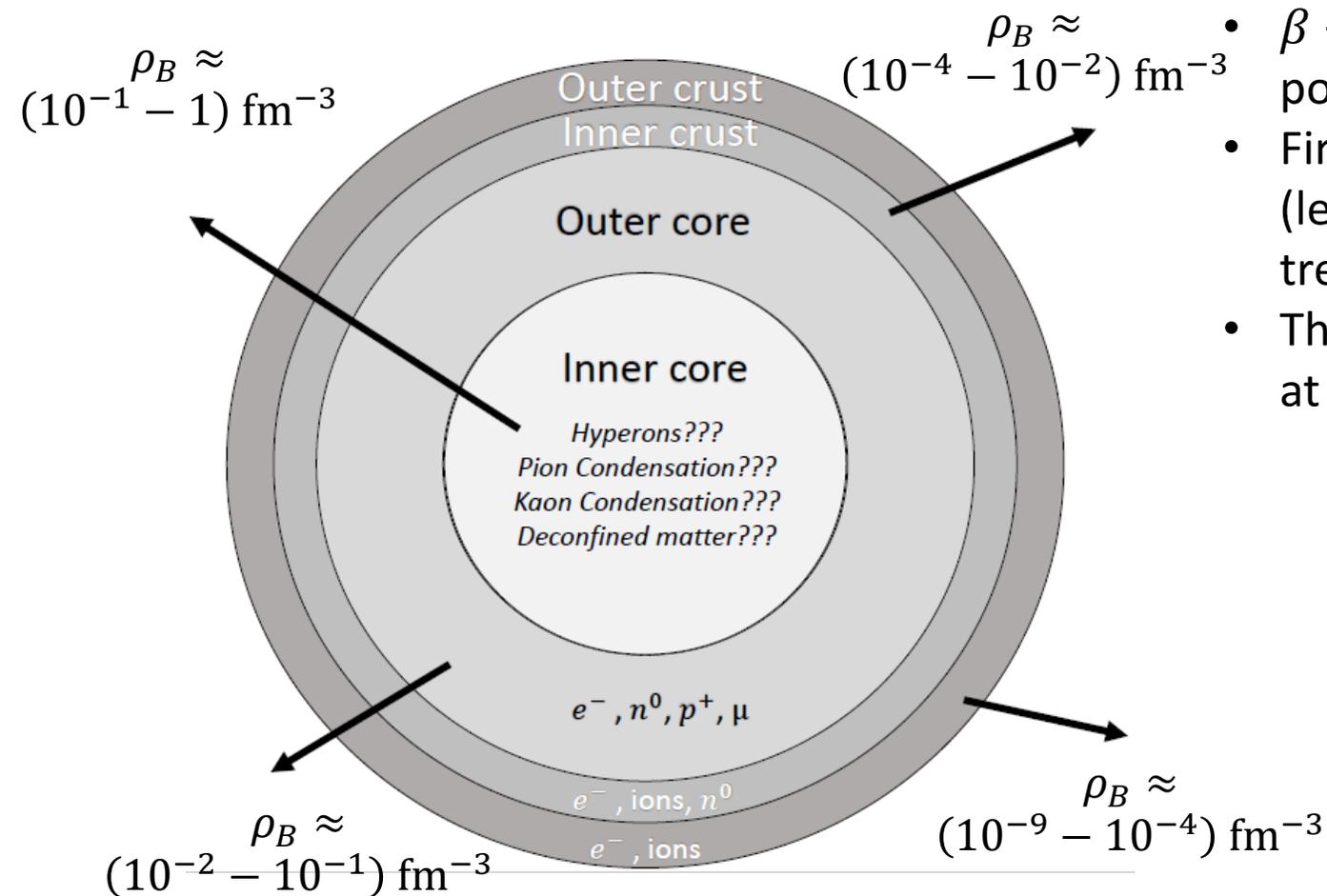
- ***Motivation. Structure of Neutron Stars (NS)***
- Brief introduction to FSU2H* model
- Equation of State (EoS) and composition of hot neutron star core
- Thermal index of neutron star core
- Summary

Motivation. Structure of NS



- Remnant of supernovae processes: **high density – several times ρ_0**

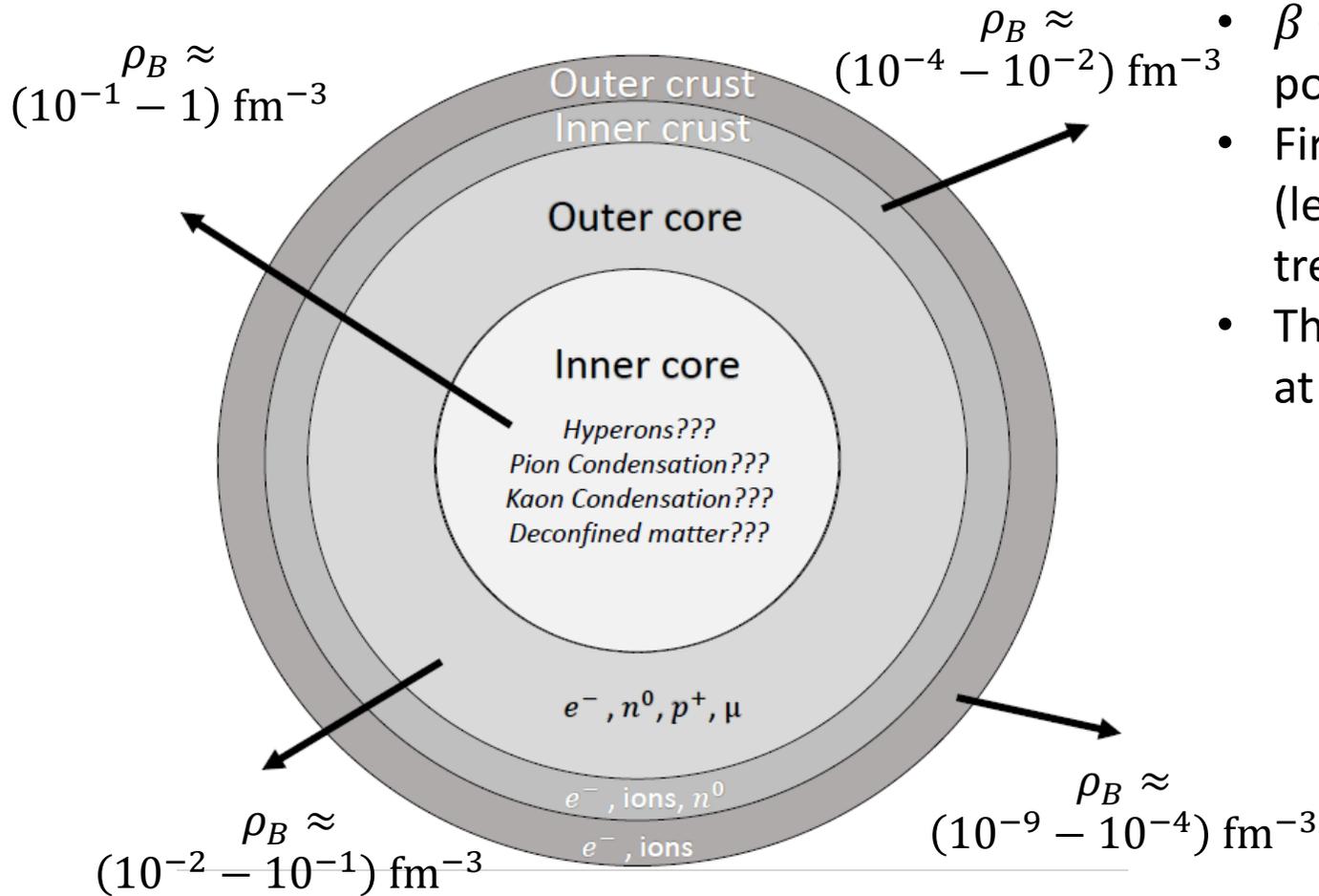
Motivation. Structure of NS



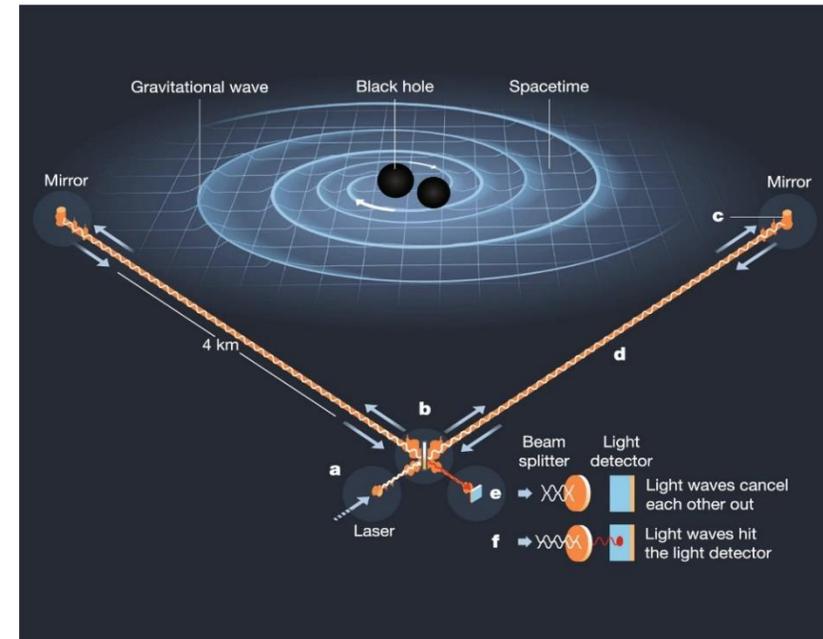
- β – stable matter made of nucleons, leptons and possibly exotic particles (**HYPERONS**)
- First stage of the evolution: **proto neutron star** (lepton rich and hot object) – Finite temperature treatment is needed
- There is no experimental data of the nuclear matter at high densities ($\rho > 3\rho_0$)

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Miller, M.C., Yunes, N.
 The new frontier of gravitational waves. *Nature*

- Remnant of supernovae processes: **high density – several times ρ_0**

- **But there are new astrophysical measurements!**

OUTLINE

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core

<https://arxiv.org/abs/2206.11266>

**QR code and link of the paper
where the results are published**

Brief introduction to FSU2H* model

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- Belongs to the broad group of relativistic mean field approaches

$$\begin{aligned}
 \mathcal{L} &= \sum_b \mathcal{L}_b + \mathcal{L}_m + \sum_l \mathcal{L}_l, \\
 \mathcal{L}_b &= \bar{\Psi}_b (i\gamma_\mu \partial^\mu - q_b \gamma_\mu A^\mu - m_b \\
 &\quad + g_{\sigma b} \sigma + g_{\sigma^* b} \sigma^* - g_{\omega b} \gamma_\mu \omega^\mu - g_{\rho, b} \gamma_\mu \vec{I}_b \vec{\rho}^\mu) \Psi_b, \\
 \mathcal{L}_m &= \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{\kappa}{3!} (g_{\sigma b} \sigma)^3 - \frac{\lambda}{4!} (g_{\sigma b} \sigma)^4 \\
 &\quad + \frac{1}{2} \partial_\mu \sigma^* \partial^\mu \sigma^* - \frac{1}{2} m_{\sigma^*}^2 \sigma^{*2} \\
 &\quad - \frac{1}{4} \Omega^{\mu\nu} \Omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{\zeta}{4!} (g_{\omega b} \omega_\mu \omega^\mu)^4 \\
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- Euler eqs. of motion
- RMF approximation
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$$\epsilon_{tot}, S, P, f$$

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Why FSU2H* model? I

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Values of parameters in the model

m_σ (MeV)	m_ω (MeV)	m_ρ (MeV)	m_{σ^*} (MeV)	m_ϕ (MeV)	$g_{\sigma N}^2$	$g_{\omega N}^2$	$g_{\rho N}^2$	κ (MeV)	λ	ζ	Λ_ω
497.479	782.500	763.000	980.000	1020.000	102.72	169.53	197.27	4.00014	-0.0133	0.008	0.045

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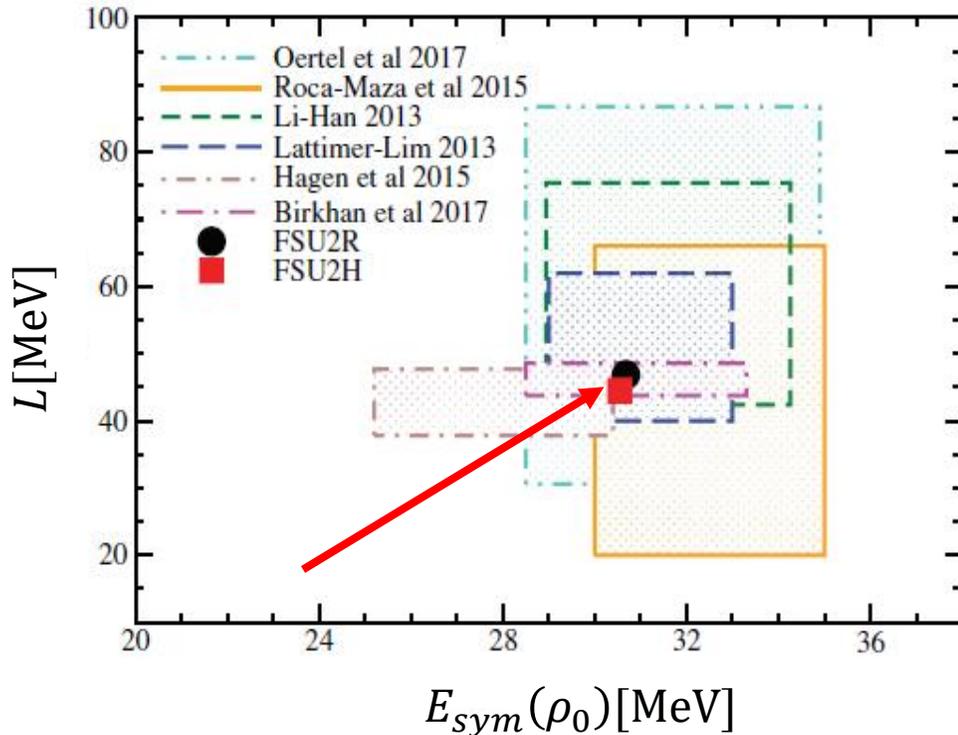
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Low density region constrains

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ρ_0 (fm^{-3})	E/A (MeV)	K (MeV)	m_N^*/m_N (ρ_0)	$E_{sym}(\rho_0)$ (MeV)	L (MeV)	K_{sym} (MeV)
0.1505	-16.28	238.0	0.593	30.5	44.5	86.4

Consistent with the majority of the calculations, variety of nuclear data from terrestrial experiments, astrophysical observations...

PASA, 34, e065 (2017)

Why FSU2H* model? II

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Values of the parameters in the model
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Y	$R_{\sigma Y}$	$R_{\omega Y}$	$R_{\rho Y}$	$R_{\sigma^* Y}$	$R_{\phi Y}$
Λ	0.6613	2/3	0	0.2812	$-\sqrt{2}/3$
Σ	0.4673	2/3	2	0.2812	$-\sqrt{2}/3$
Ξ	0.3305	1/3	1	0.5624	$-2\sqrt{2}/3$

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$$U_i = -g_{\sigma i} \bar{\sigma} - g_{\sigma i} \bar{\sigma}^* + g_{\omega i} \bar{\omega} + g_{\rho i} I_{3i} \bar{\rho} + g_{\phi i} \bar{\phi}$$

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$$U_{\Lambda}^{(N)}(\rho_0) = -28 \text{ MeV};$$

$$U_{\Sigma}^{(N)}(\rho_0) = 30 \text{ MeV};$$

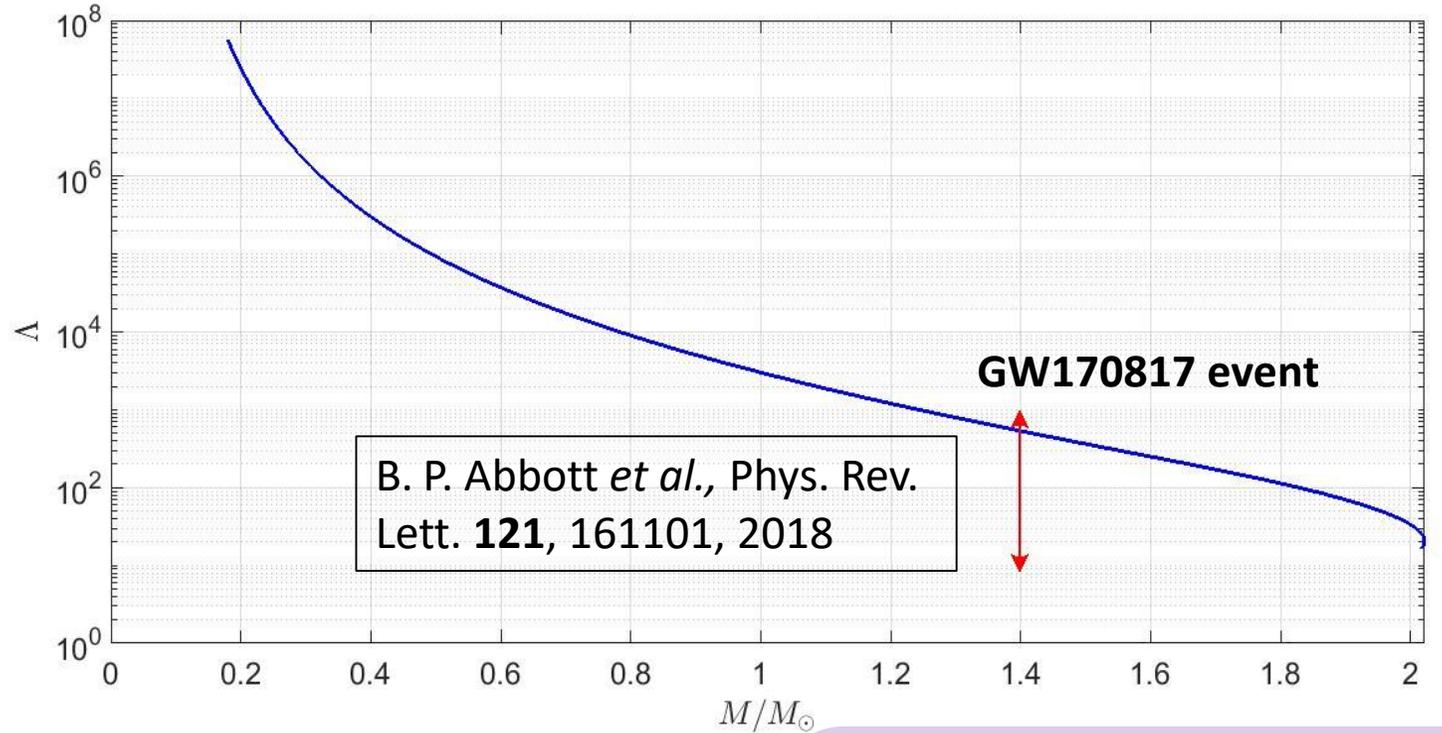
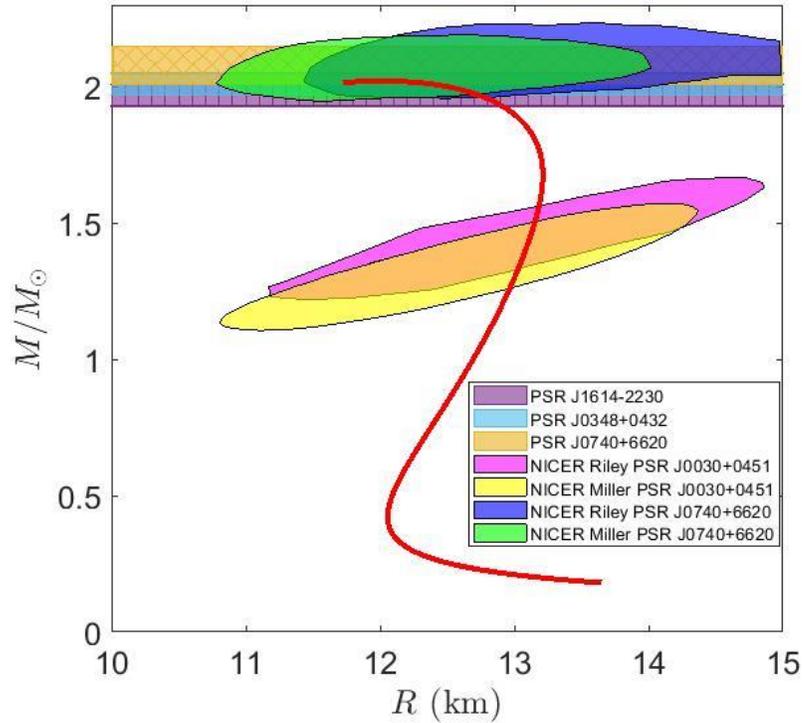
$$U_{\Xi}^{(N)}(\rho_0) = -24 \text{ MeV};$$

Hyperon potentials in SNM

$$\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^6\text{He}) = 0.67 \text{ MeV}$$

$\Lambda\Lambda$ interaction energy

Why FSU2H* model? III



Intermediate and high density regions constrains

M_{\max} (M_{\odot})	$R(M_{\max})$ (km)	$R(1.4M_{\odot})$ (km)	$\Lambda(1.4M_{\odot})$
2.03	12.02	13.08	526.3

In agreement with:

- $M_{\max} > 2M_{\odot}$
- $70 < \Lambda(1.4M_{\odot}) < 580$
- $10.5 \text{ km} < R(1.4M_{\odot}) < 13.3 \text{ km}$

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Composition and EoS of hot neutron star core I

- The finite temperature EoS depends on three parameters (ρ_B, T, Y_e)

Wide range of values to **account for conditions in PNS and NS mergers:**

$$T = (0 - 100) \text{ MeV}$$

$$\rho_B = (0.5 - 10)\rho_0$$

$$Y_e = (0 - 0.4); \nu \text{ free case}$$

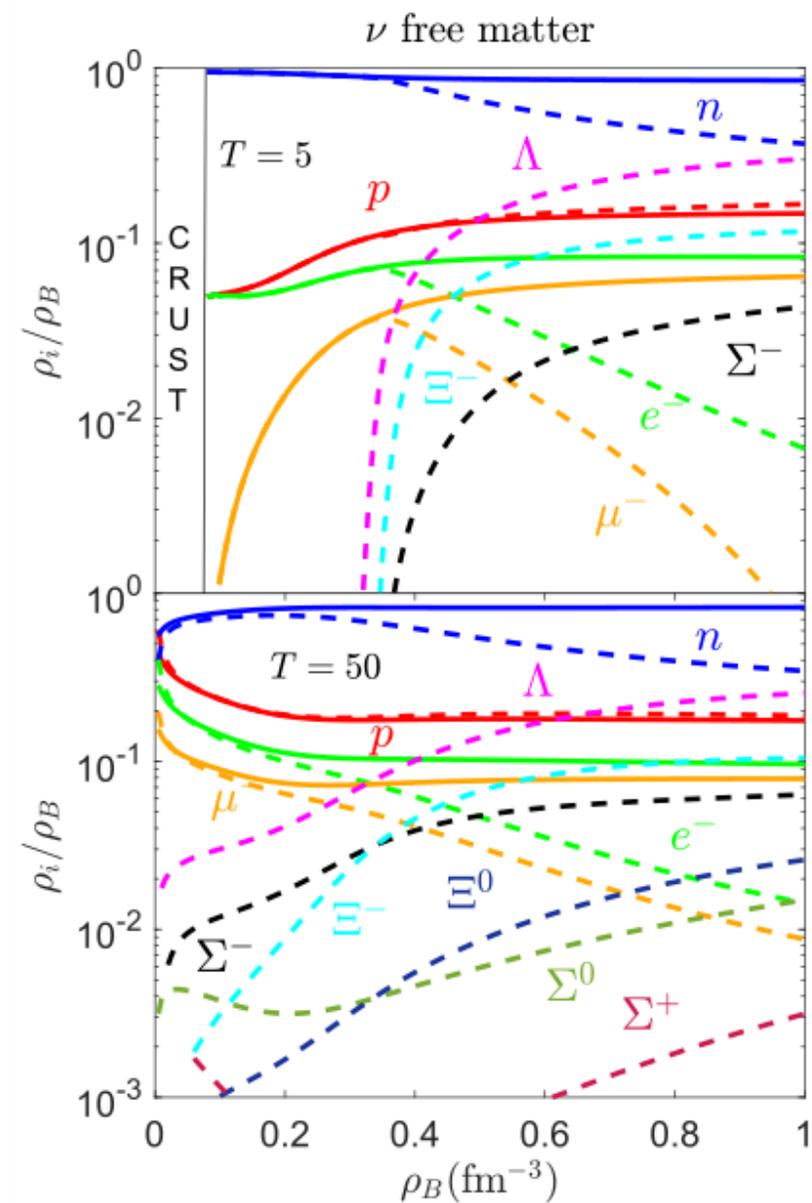
- We focus on:

$$T = 5 \text{ MeV and } T = 50 \text{ MeV}$$

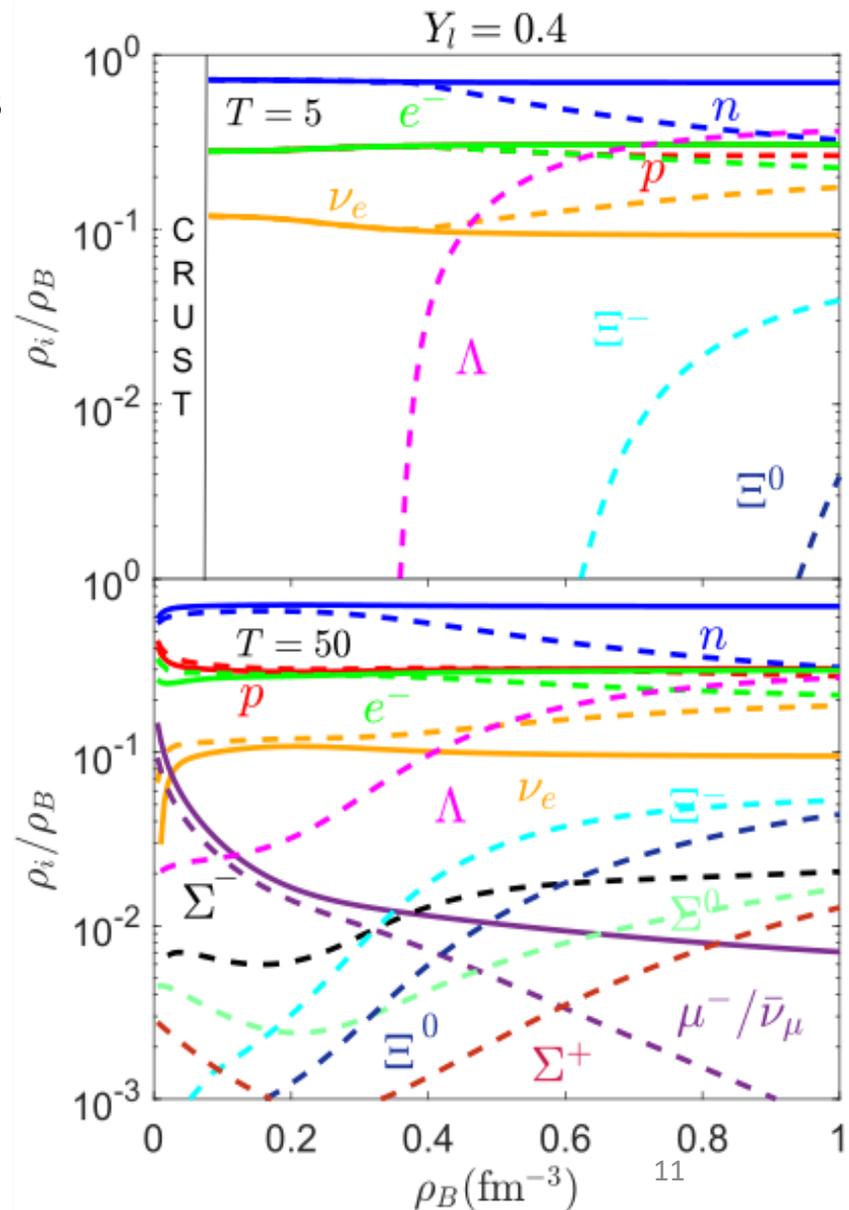
and two different lepton situations:

$$Y_e = 0.4 \text{ and } \nu \text{ free matter}$$

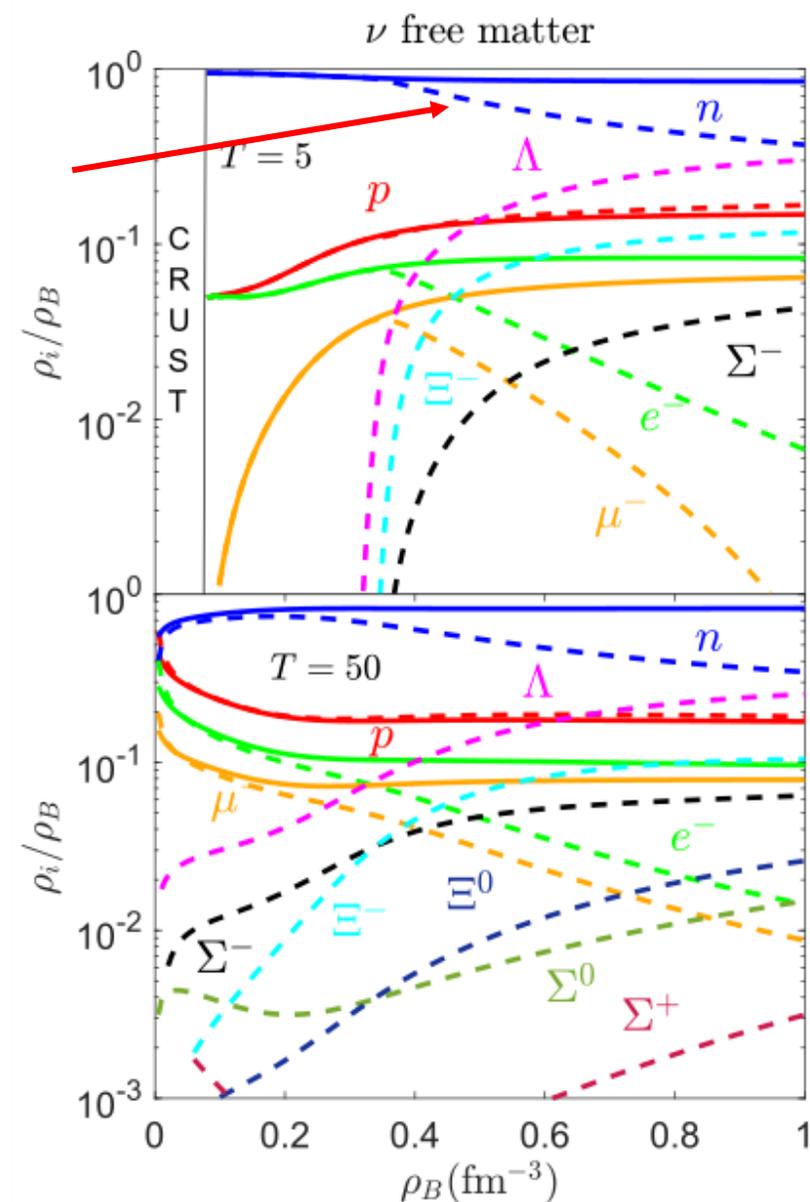
Composition and EoS of hot neutron star core II



Solid lines – core without hyperons
Dashed lines – core with hyperons



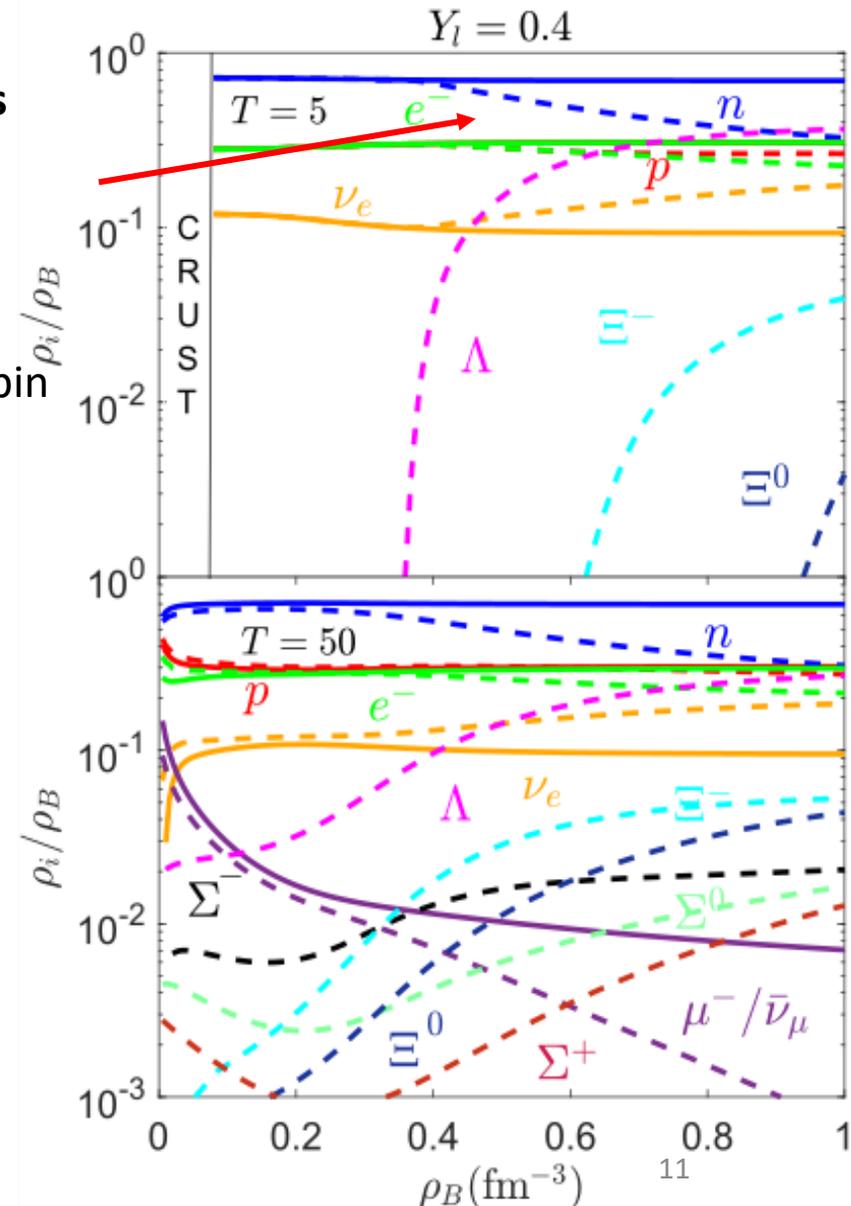
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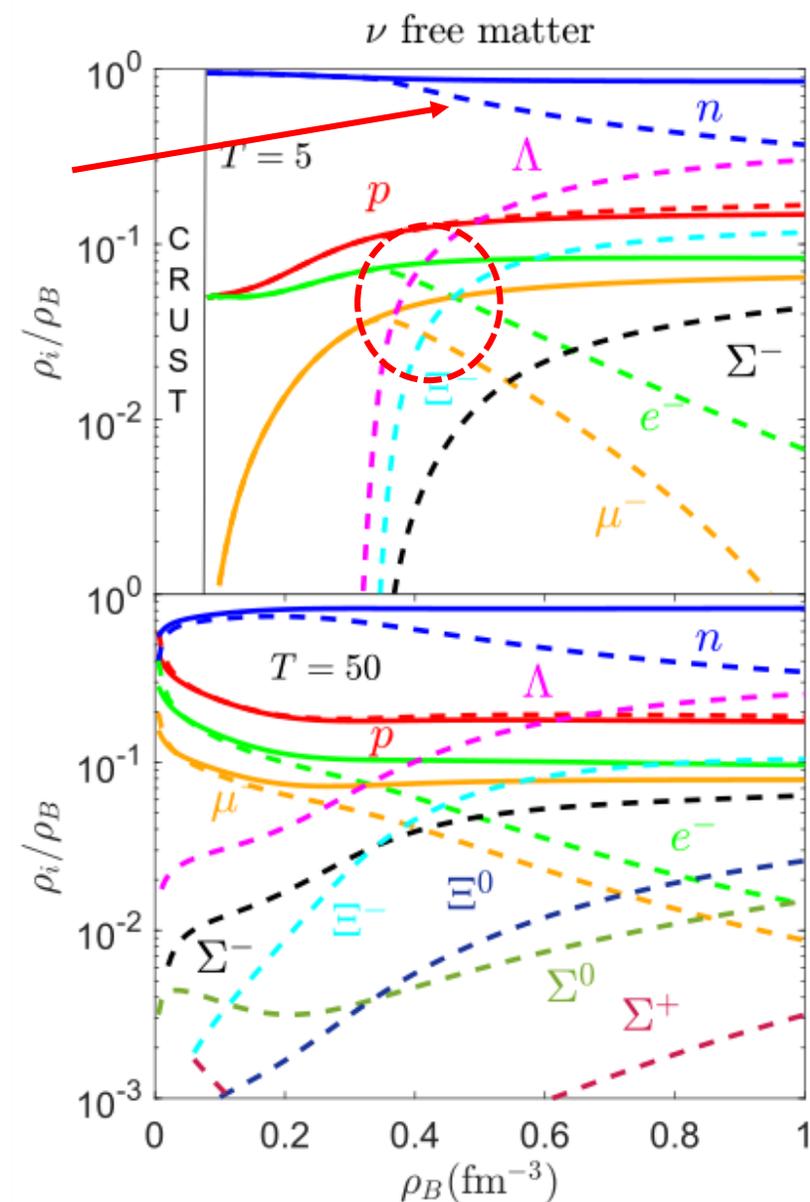
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Main effects:

Hyperons make the matter more isospin symmetric



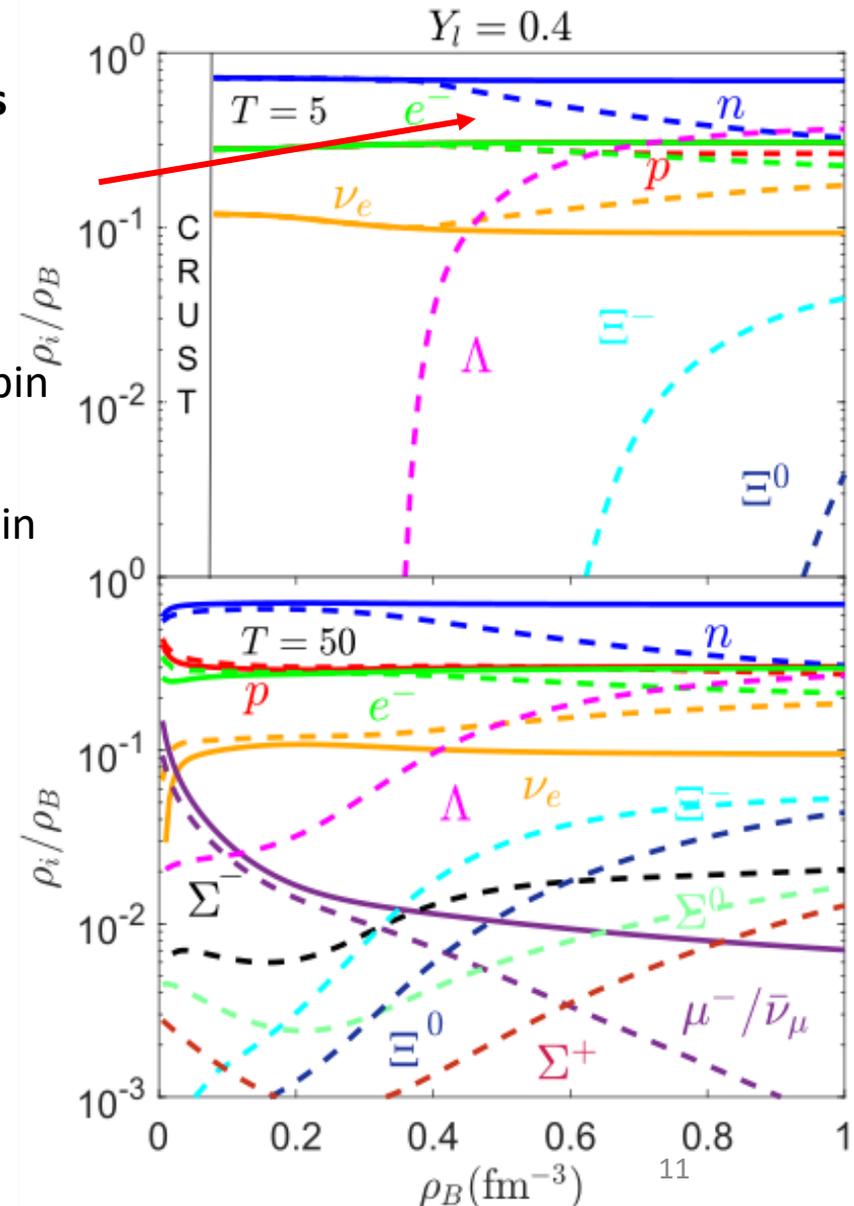
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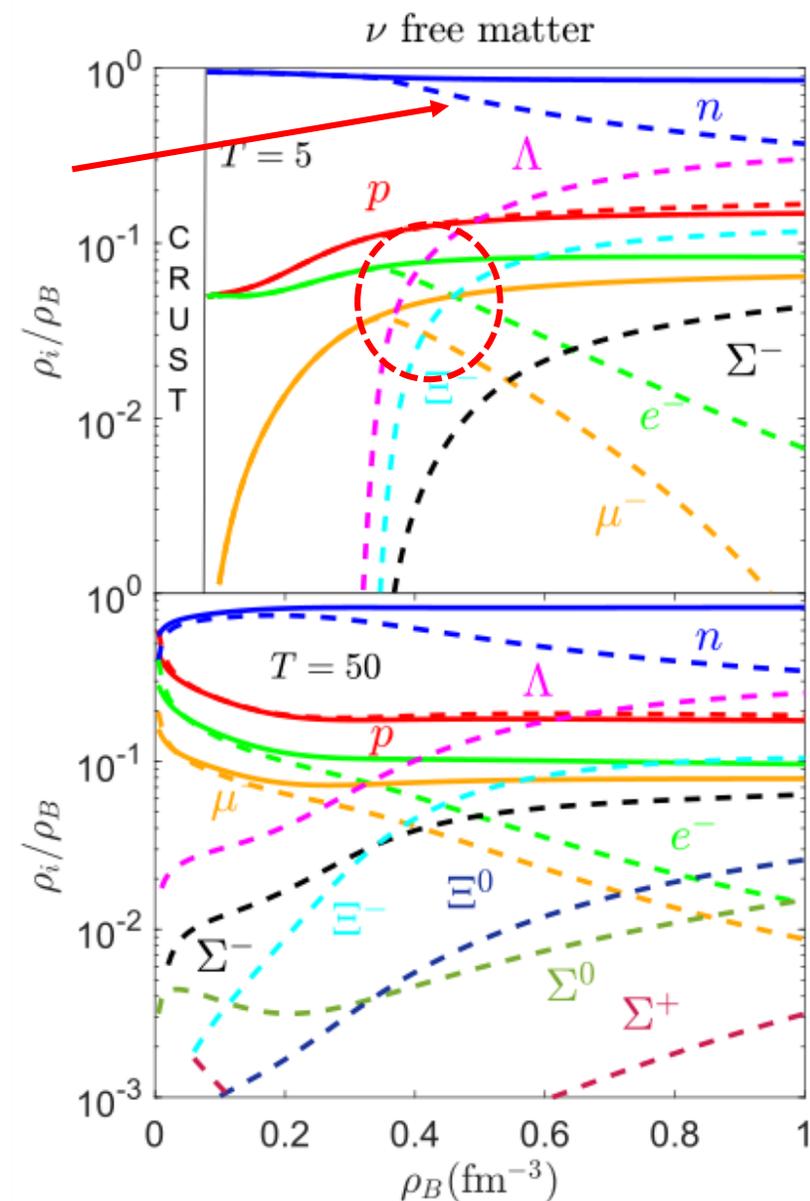
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Main effects:

- Hyperons make the matter more isospin symmetric
- Hyperons replace the negative leptons in order charge neutrality to be fulfilled.



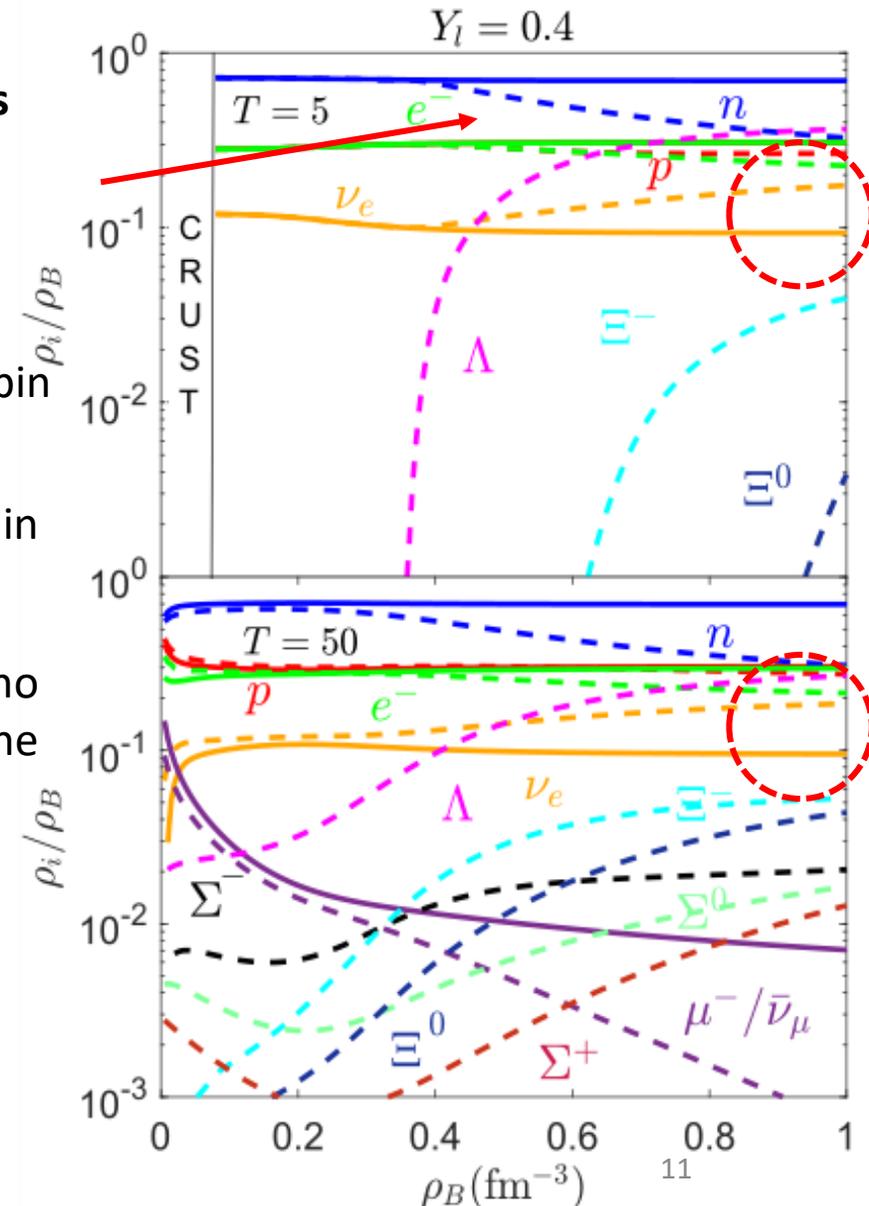
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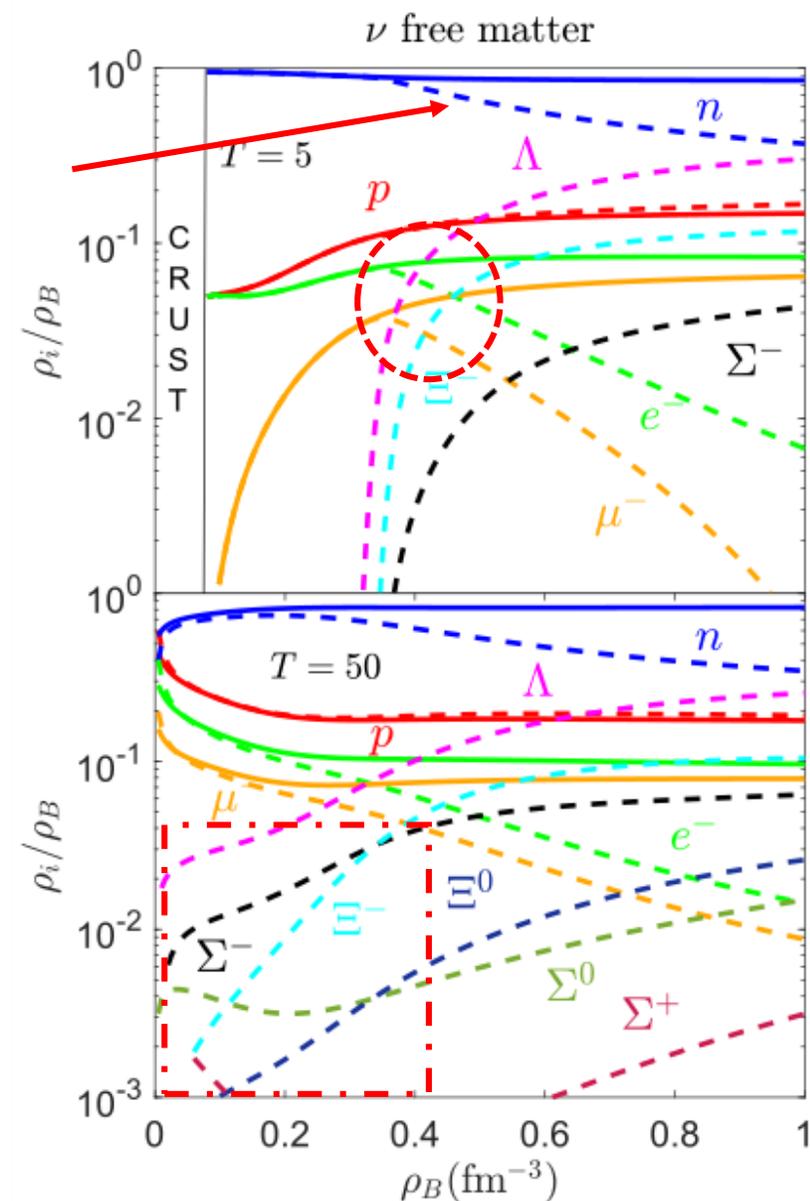
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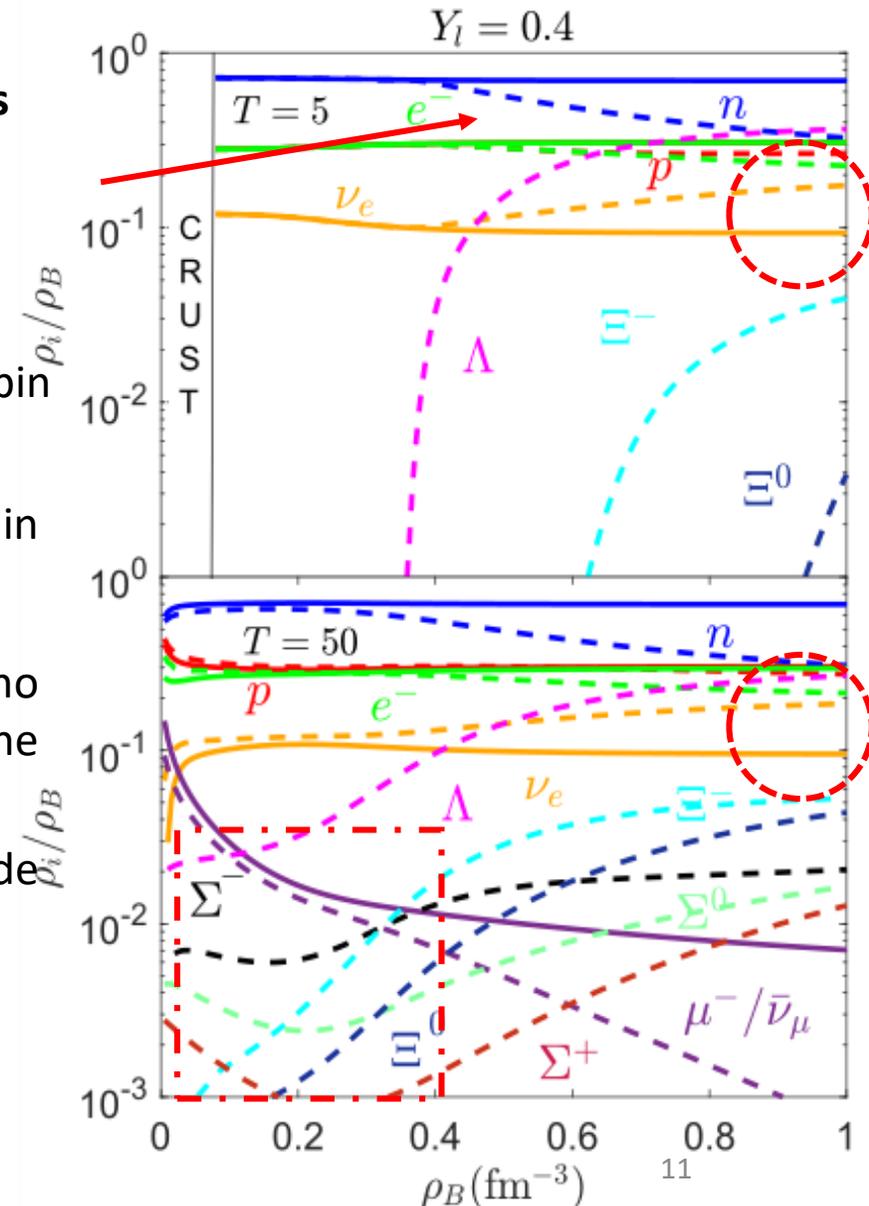
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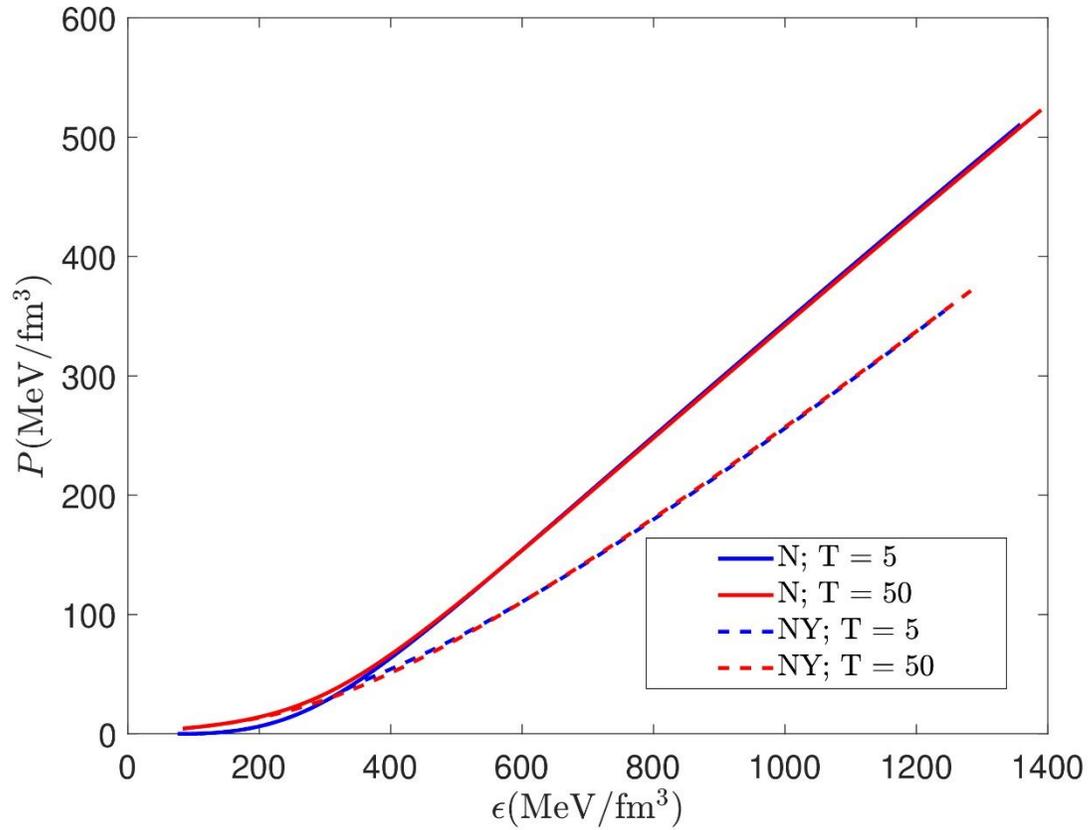
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- Hyperons increase the neutrino abundance when they are trapped in the core
- At high temperature hyperons are inside the core at any density



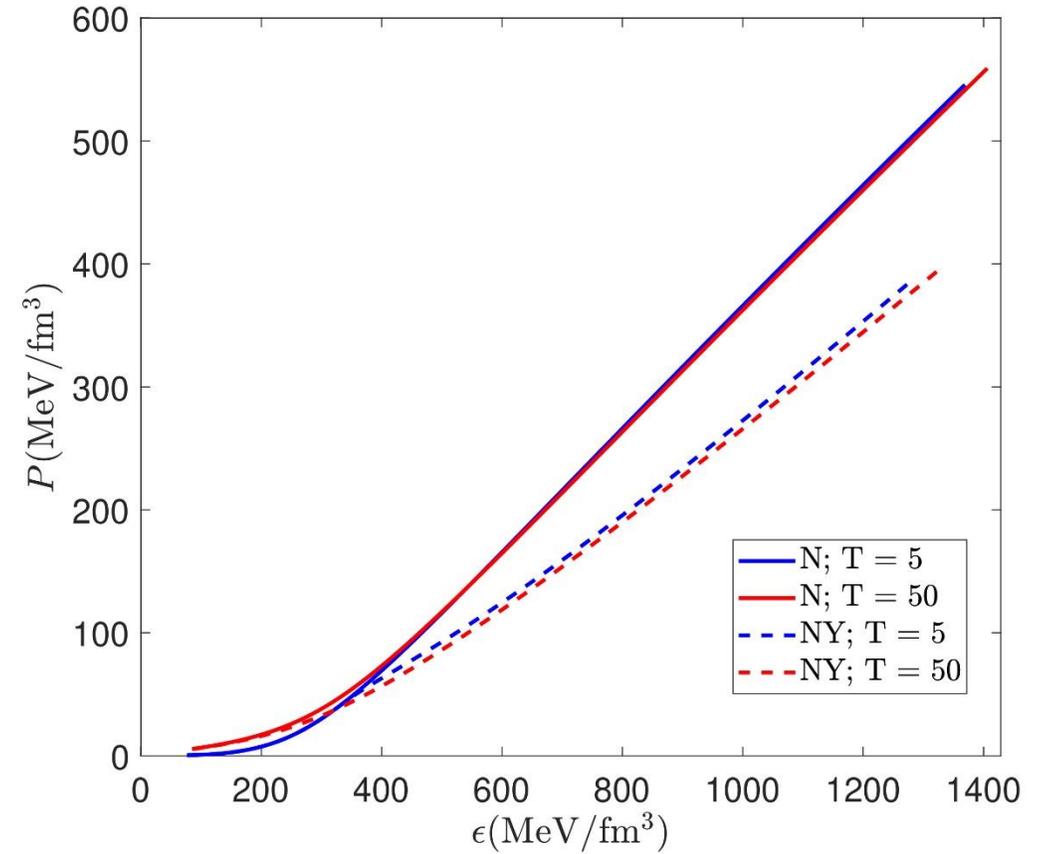
Composition and EoS of hot neutron star core III

Composition and EoS of hot neutron star core III

ν free matter

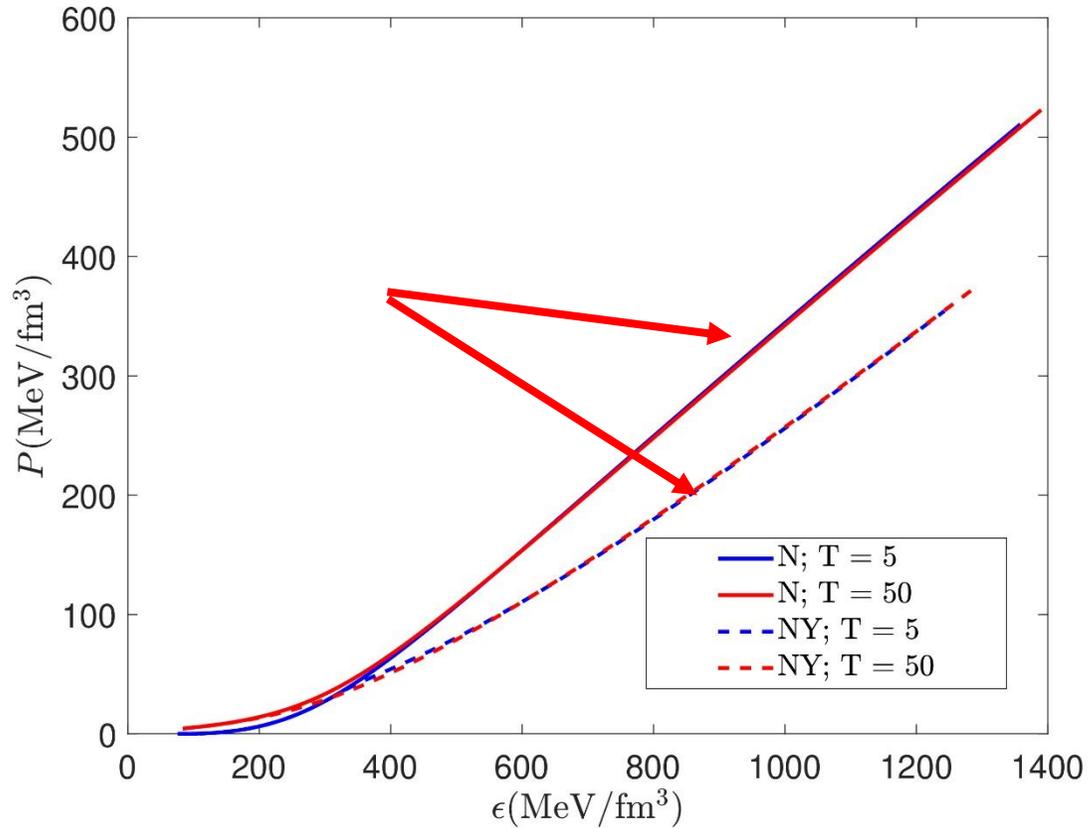


$Y_e = 0.4$

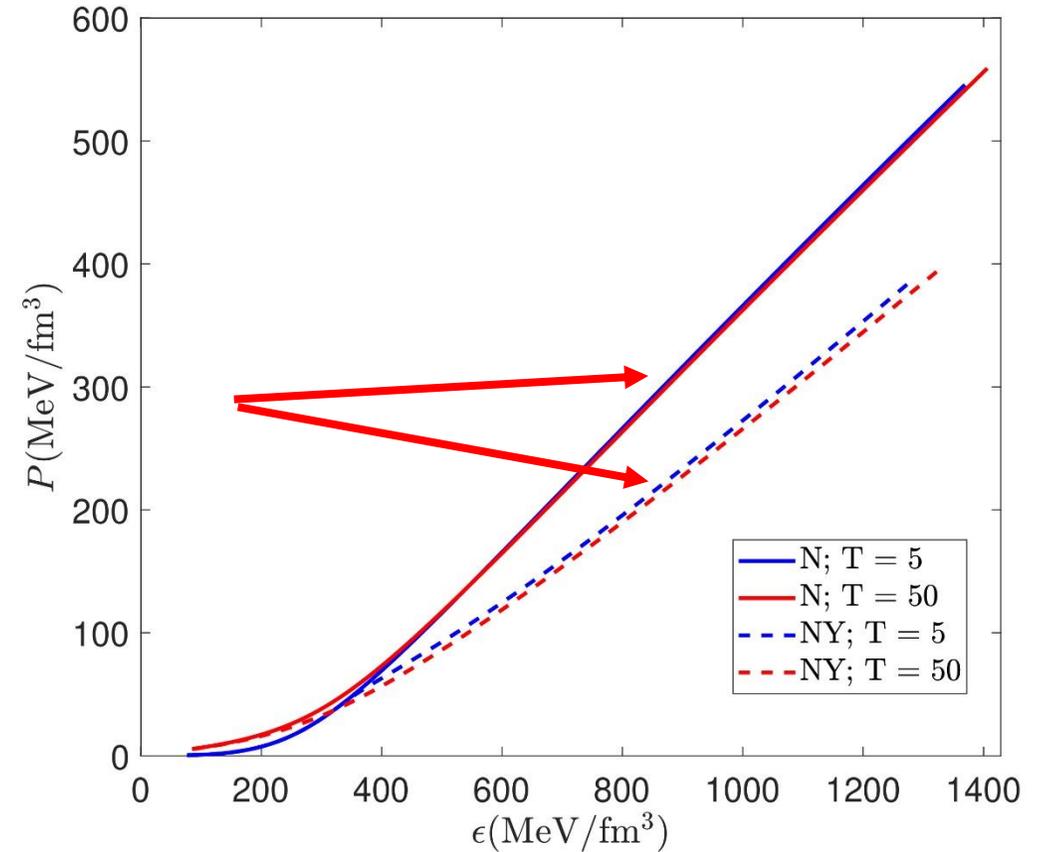


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ν free matter

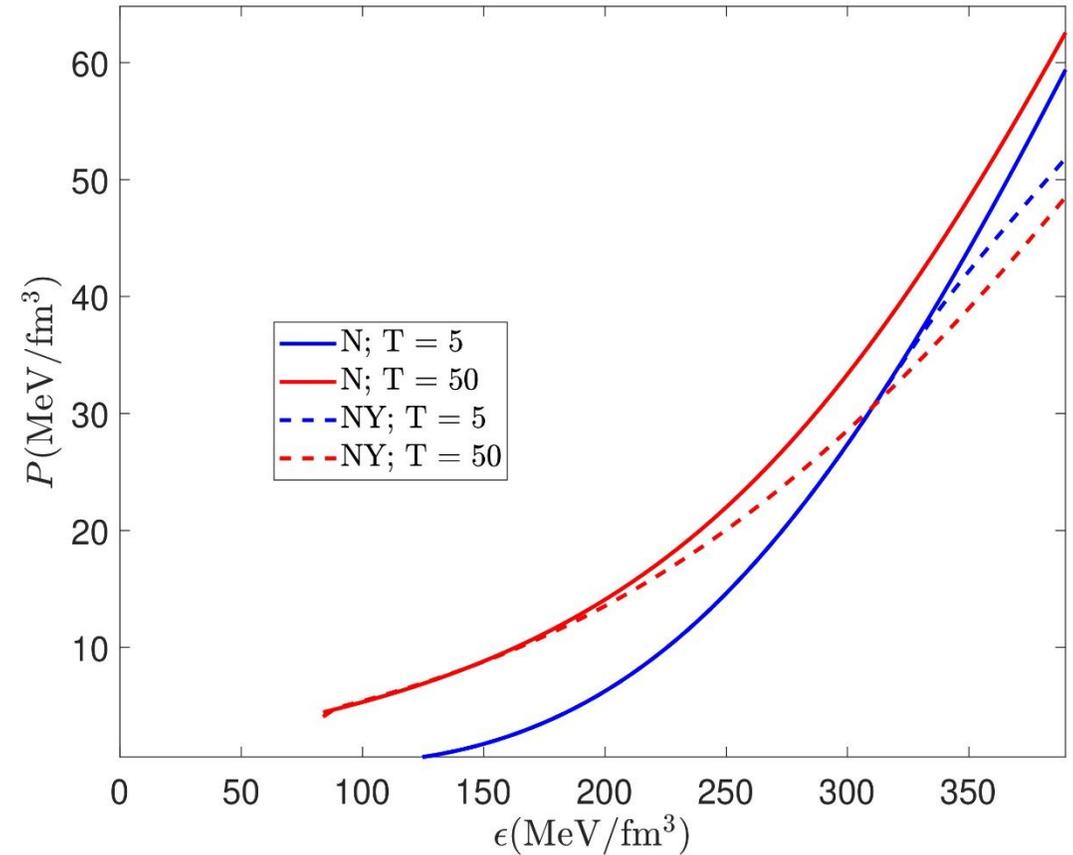
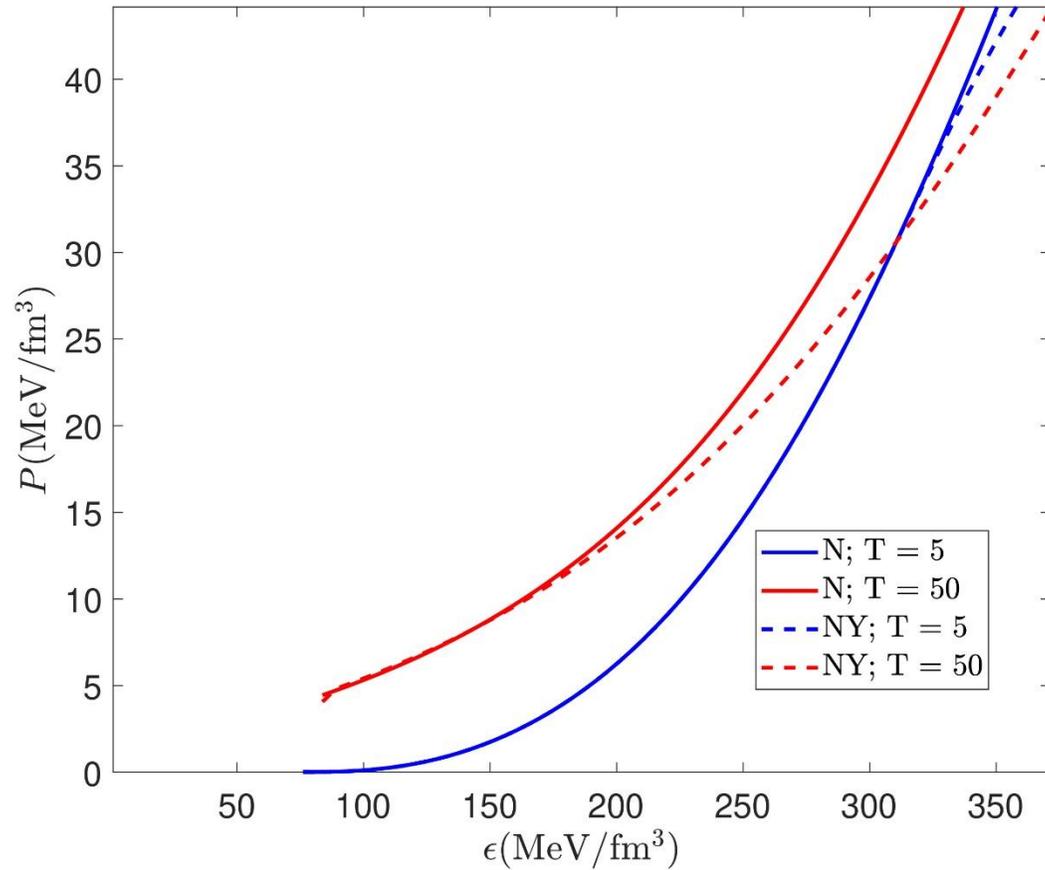


$Y_e = 0.4$



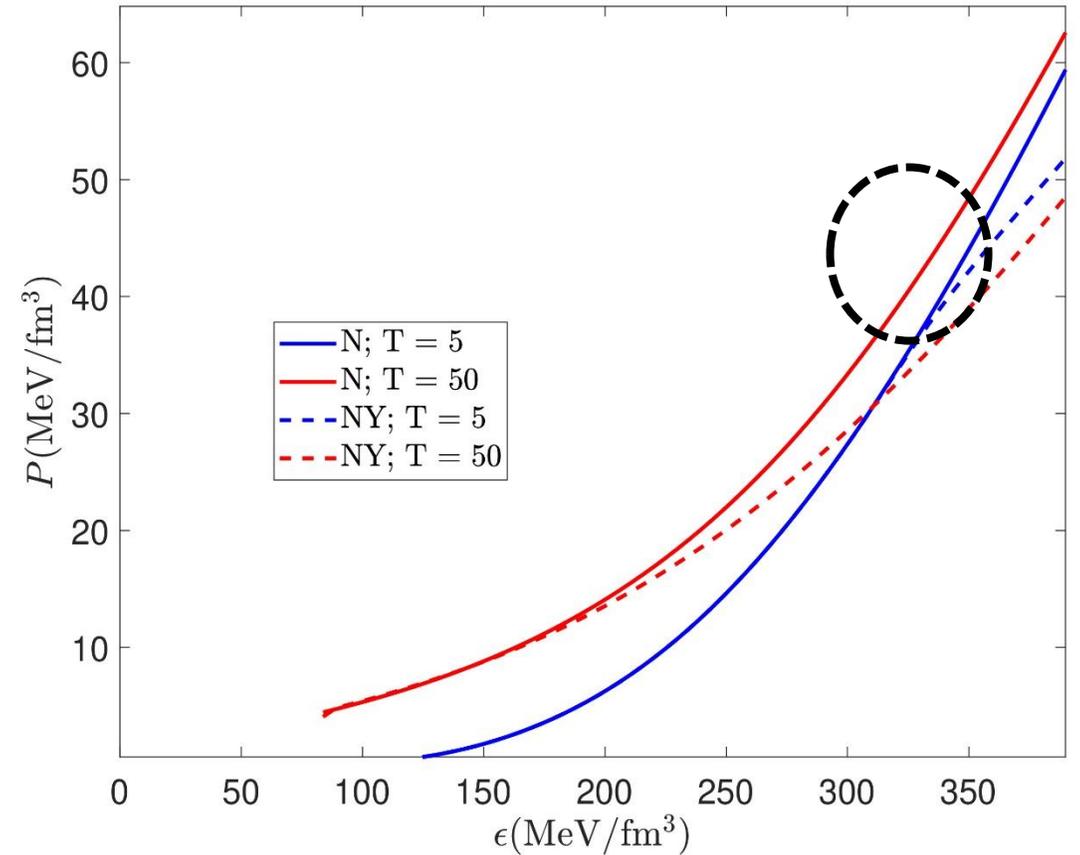
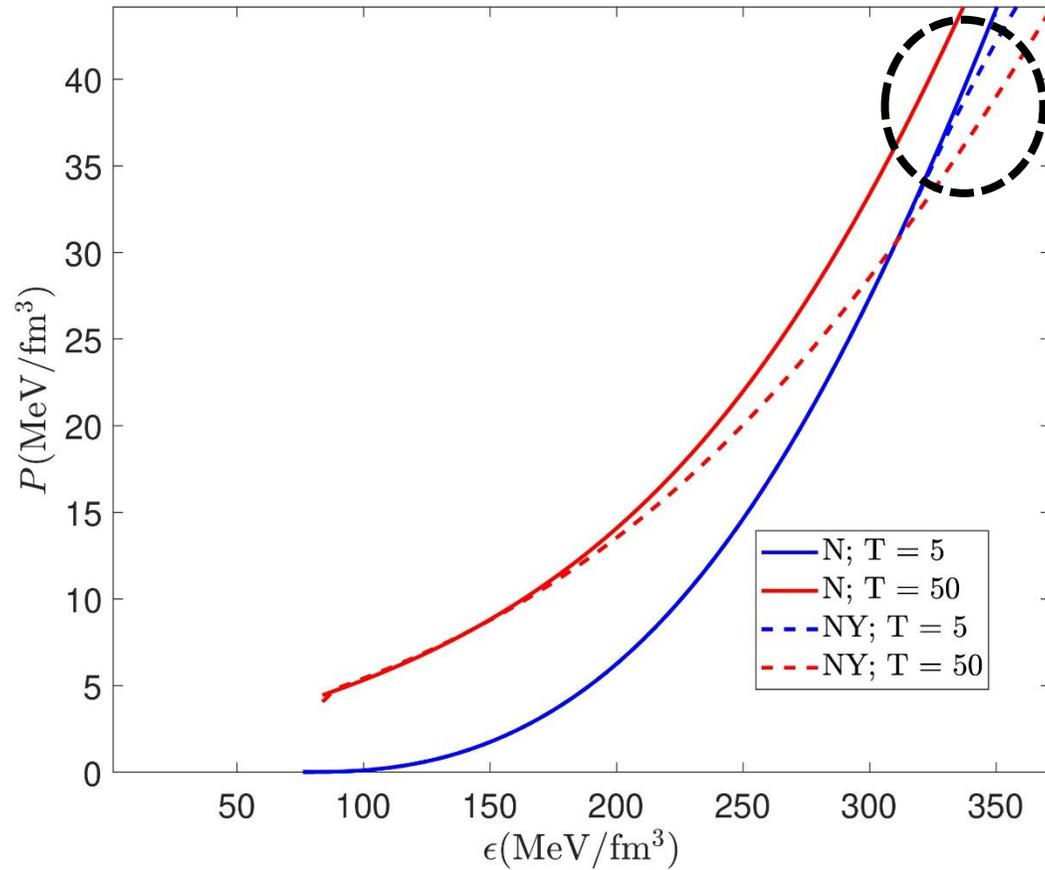
- Hyperons induce significant softening of the EoS
- When neutrinos are trapped, the EoS becomes stiffer

Composition and EoS of hot neutron star core III



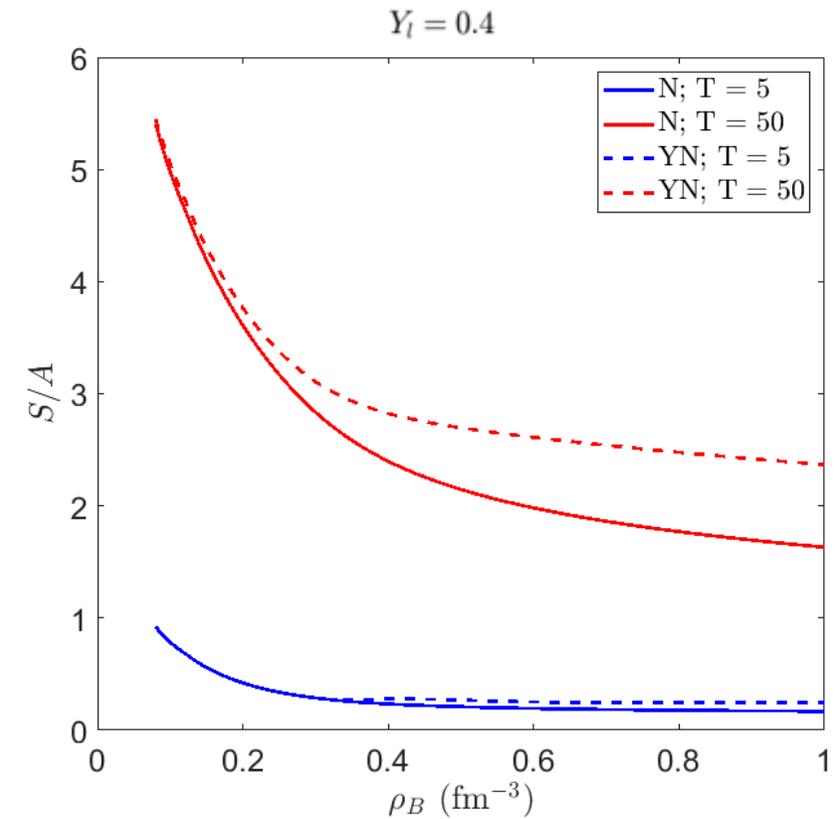
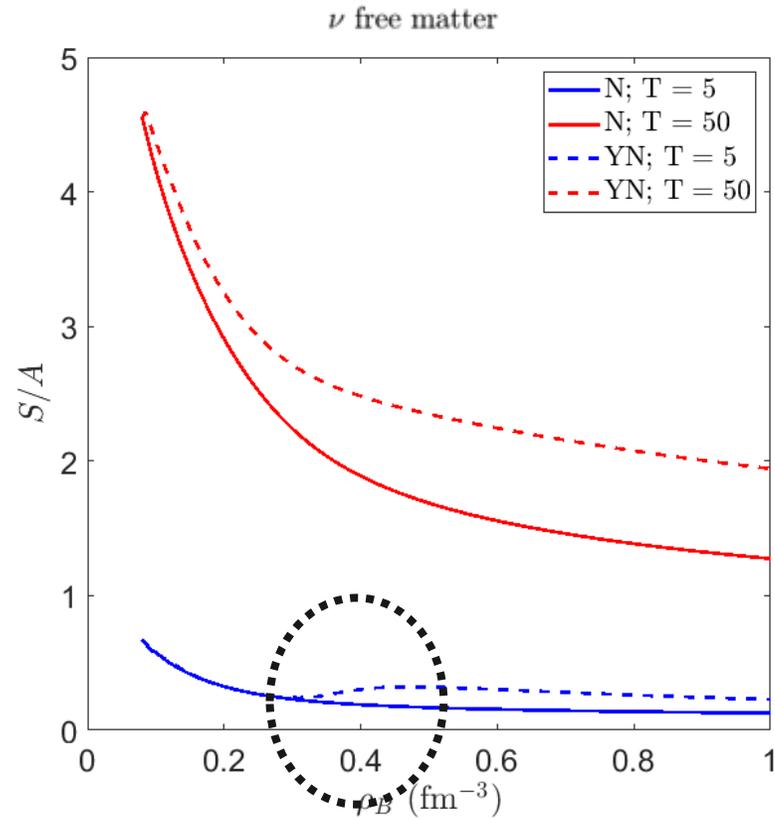
- Hyperons induce significant softening of the EoS
- When neutrinos are trapped, the EoS becomes stiffer
- At low temperatures their production changes the slope of the curve

Composition and EoS of hot neutron star core III



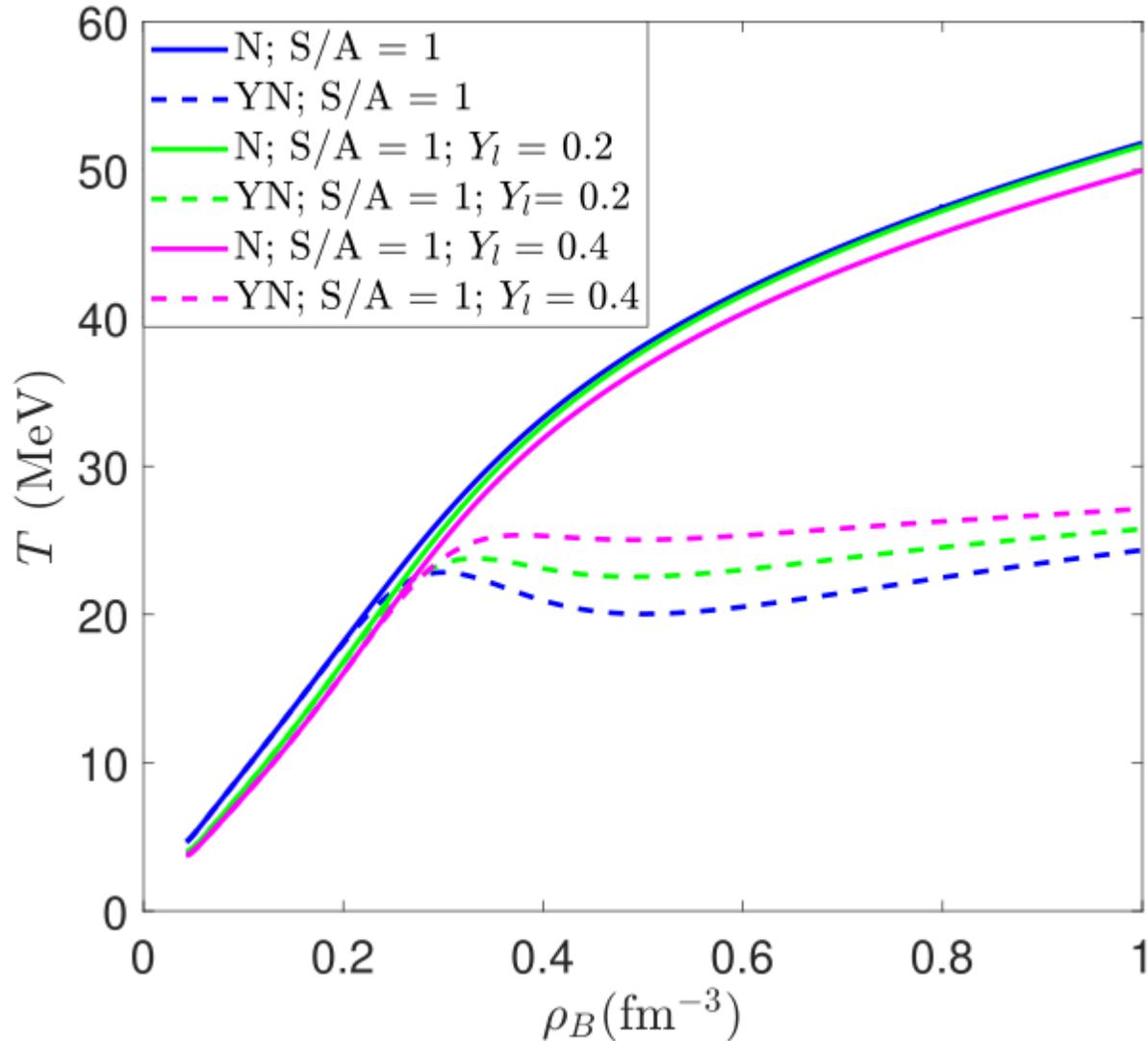
- Hyperons induce significant softening of the EoS
- When neutrinos are trapped, the EoS becomes stiffer
- At low temperatures their production changes the slope of the curve

Composition and EoS of hot neutron star core IV



- Hyperons have strong influence on the entropy per particle
- The effect is more important in ν free matter
- At low temperatures can even break the monotonous behavior of the curve

Composition and EoS of hot neutron star core IV



- Important for stars with isentropic profile
- Flattening the temperature profile in wide range of the core

OUTLINE

- Motivation. Structure of Neutron Stars (NS)
- Brief introduction to FSU2H* model
- Equation of State (EoS) and Composition of hot neutron star core
- *Thermal index of neutron star core*
- Summary

Thermal index - introduction

- Useful in complicated simulations in order to reproduce thermal effects on the EoS

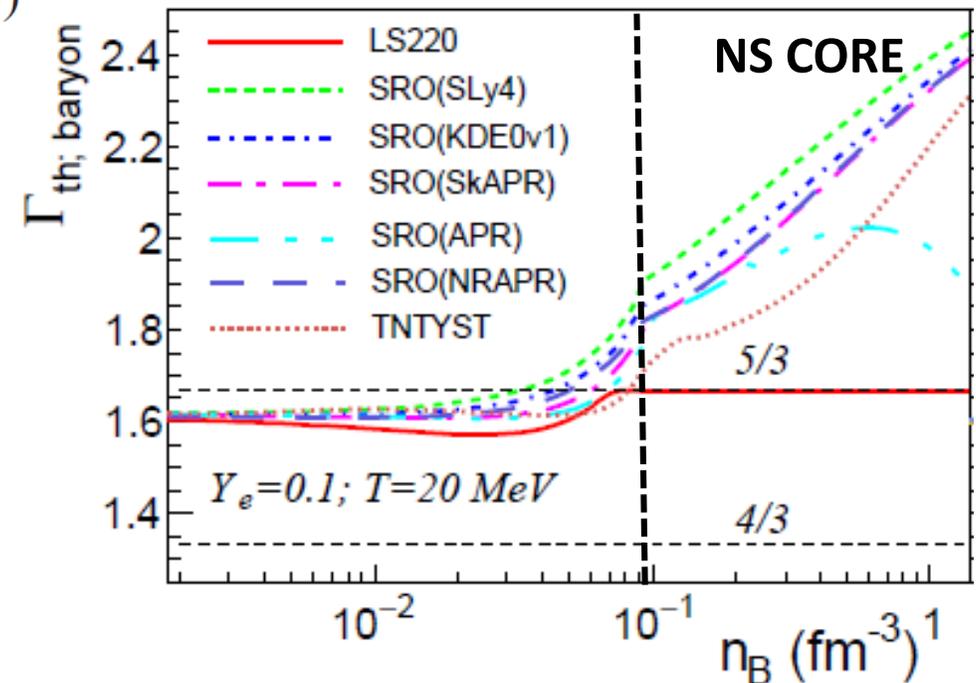
$$\Gamma(\rho_B, T) \equiv 1 + \frac{P_{\text{th}}}{\epsilon_{\text{th}}} \quad \begin{aligned} P_{\text{th}} &= P(\rho_B, T) - P(\rho_B, T = 0) \\ \epsilon_{\text{th}} &= \epsilon(\rho_B, T) - \epsilon(\rho_B, T = 0) \end{aligned}$$

- One decomposes the energy density and the pressure to a zero-temperature contribution and a thermal correction
- Simulation uses Γ that is constant, so $P(\epsilon)$ relation can be found only knowing $T = 0$ EoS:

$$\mathbf{P} = P(T = 0) + (\Gamma - 1)(\epsilon - \epsilon(T = 0))$$

- The parameter that is evolving with the time in simulations is the energy density, so the equation above is from a special interest

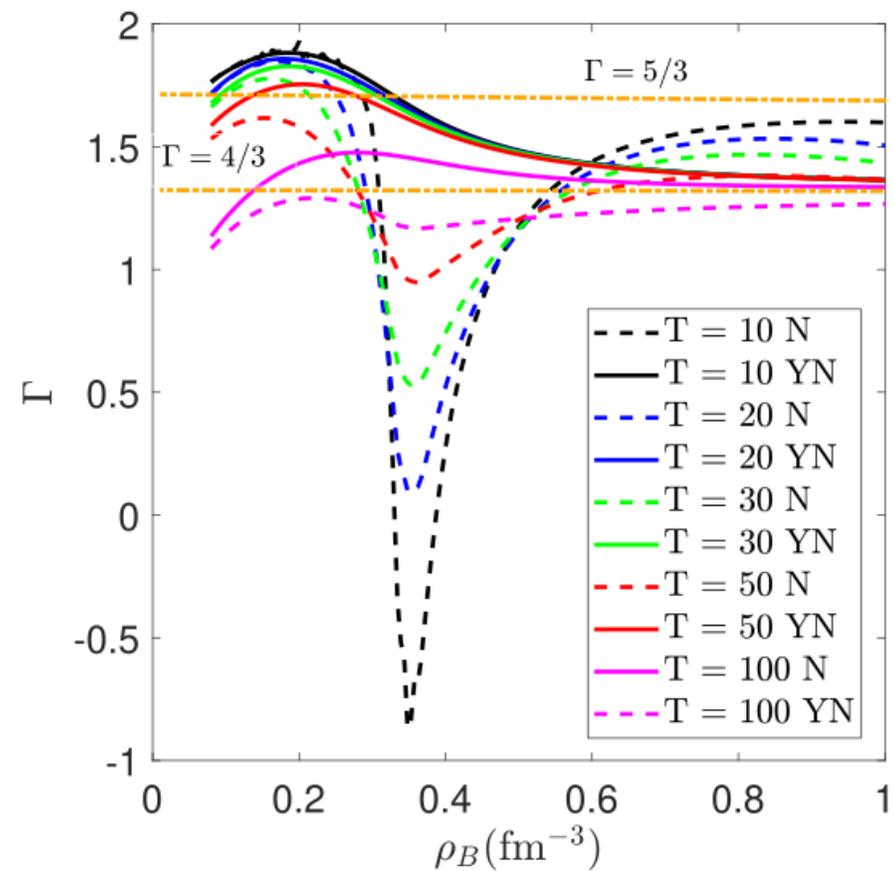
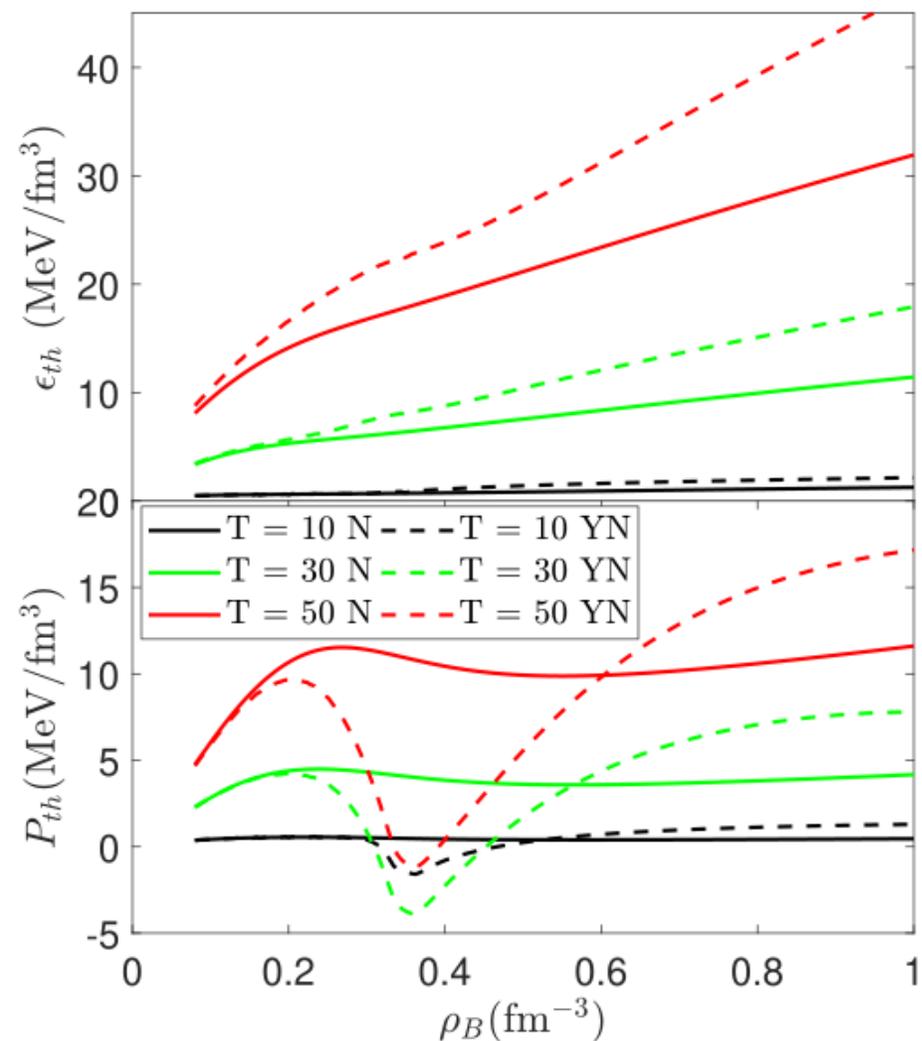
However, this approach can be inaccurate!



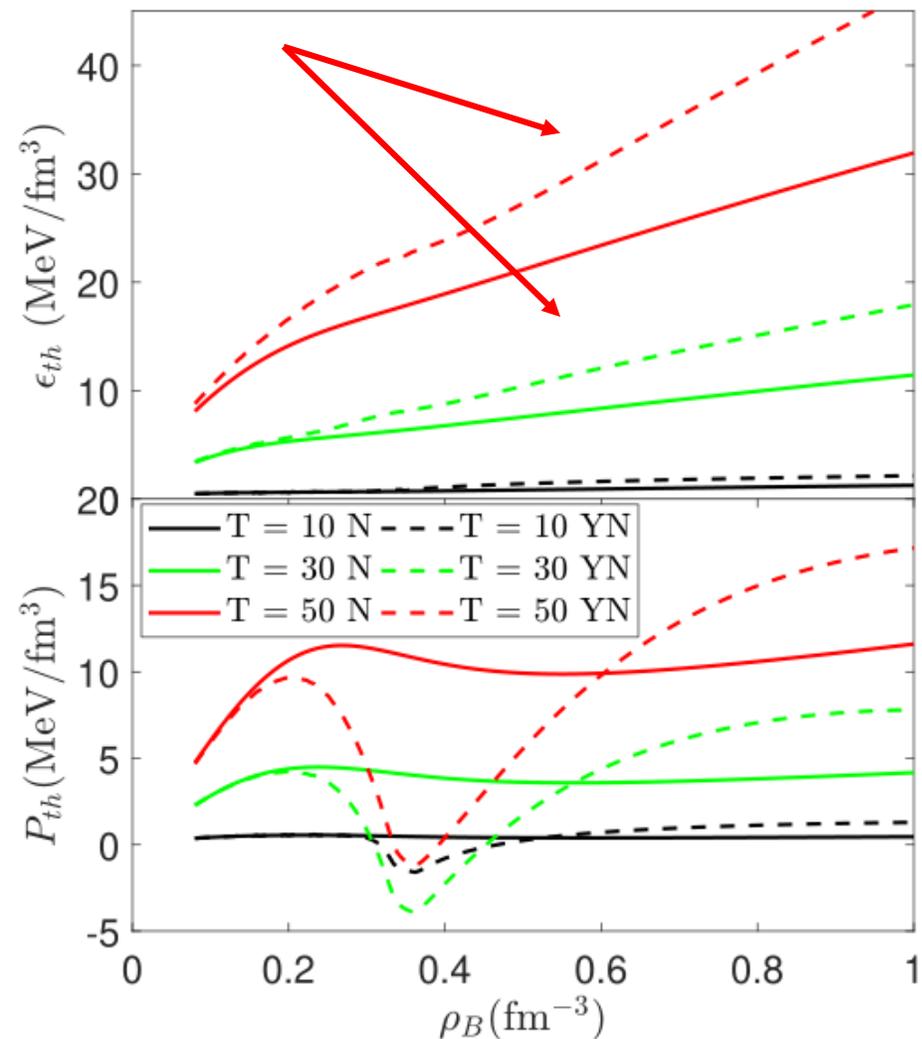
arXiv:2109.00251v1

In all models on the graph, nucleons are the only baryons considered

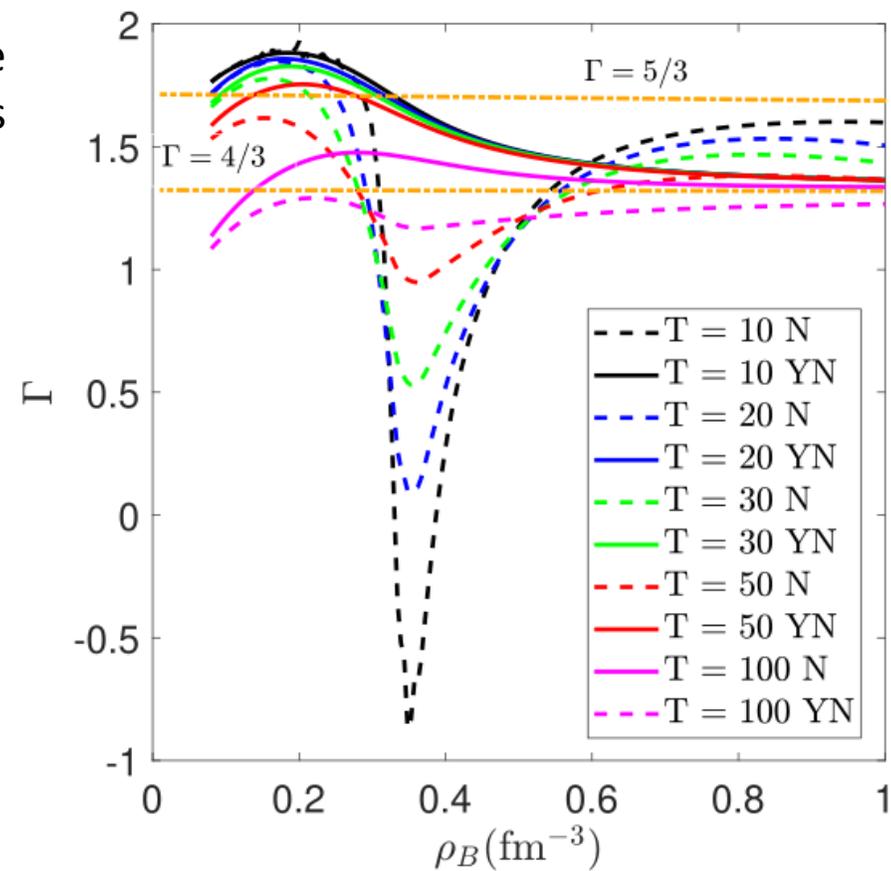
Thermal index – β - stable ν free matter in FSU2H* I



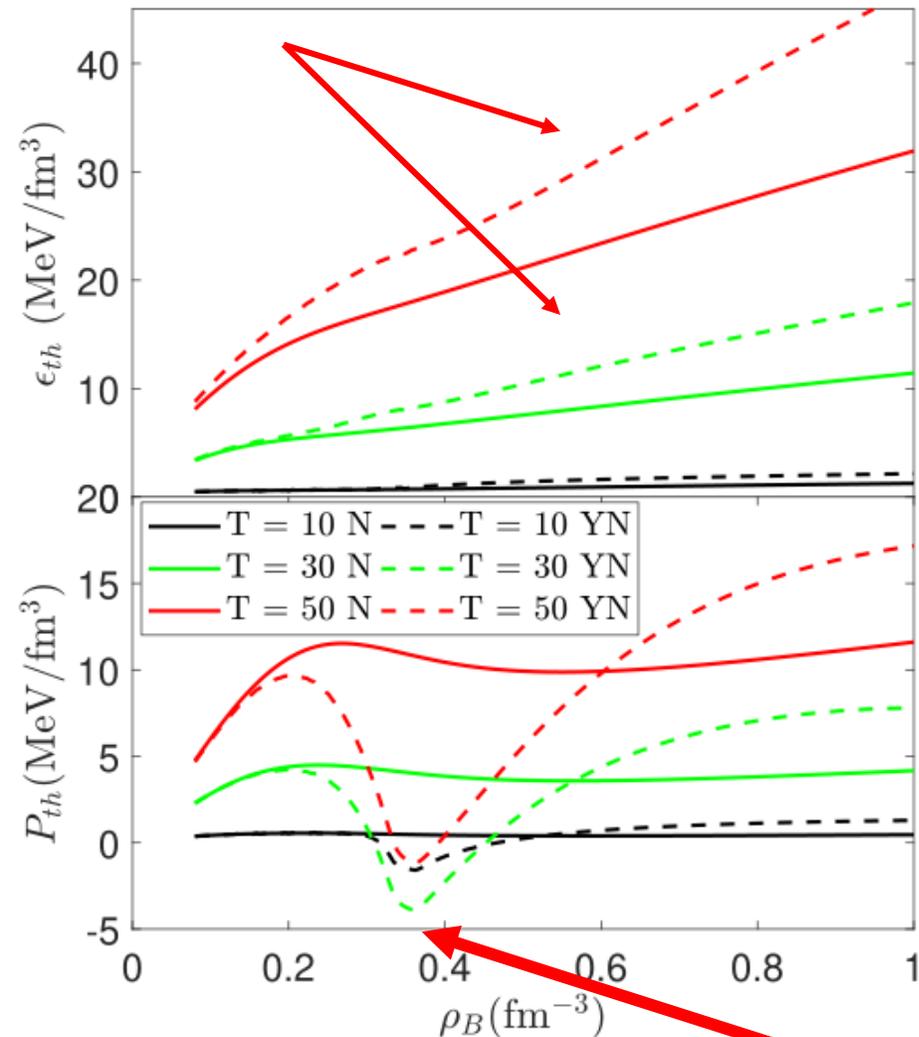
Thermal index – β - stable ν free matter in FSU2H* I



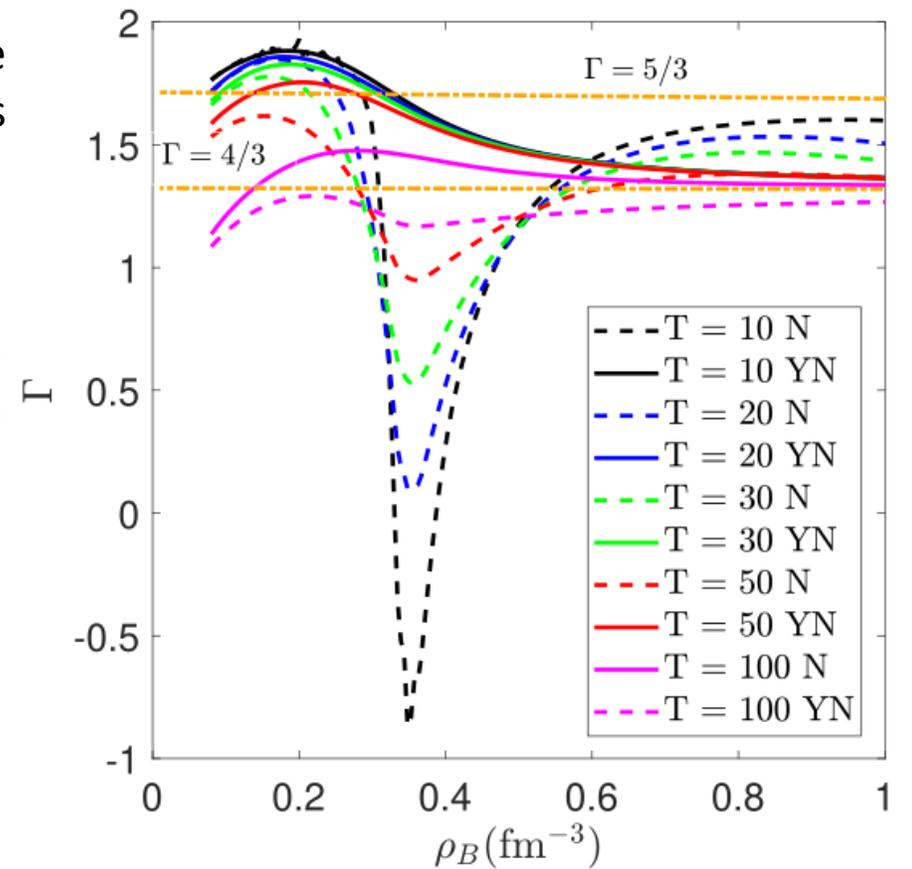
- Thermal effects are more emphasized when hyperons are included



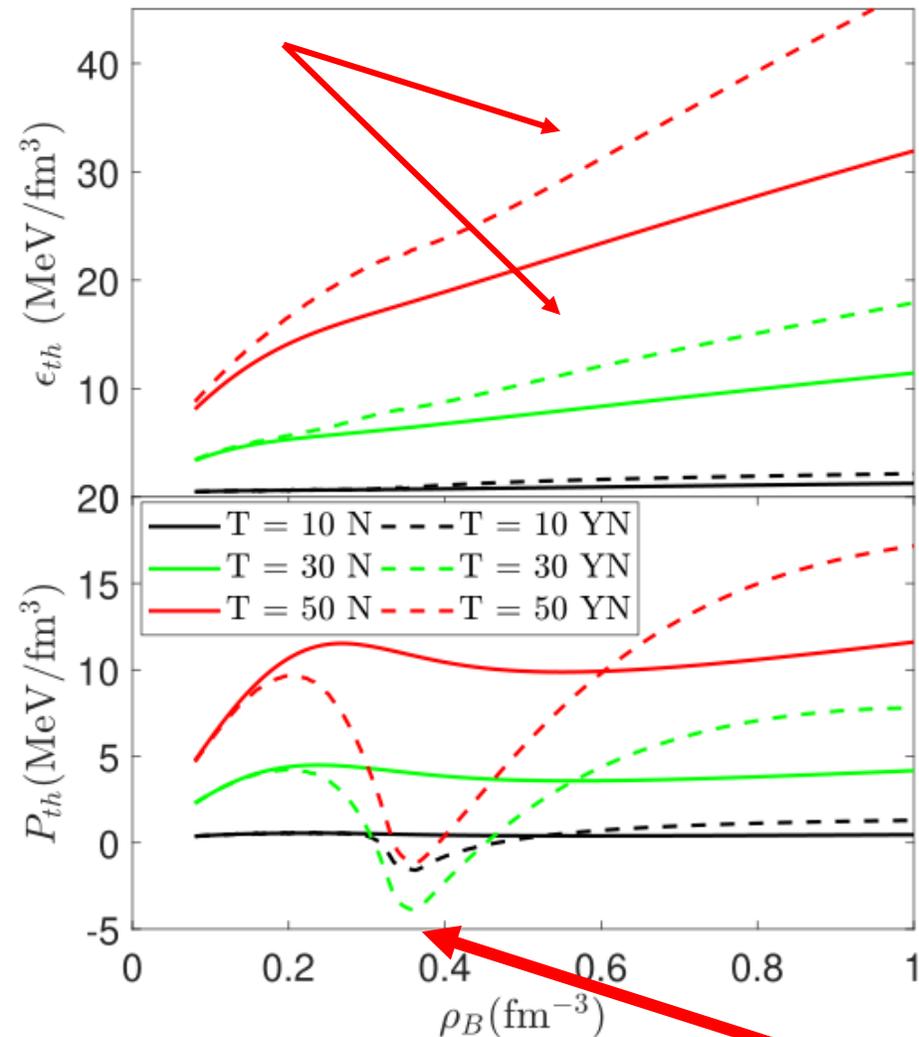
Thermal index – β - stable ν free matter in FSU2H* I



- Thermal effects are more emphasized when hyperons are included
- Significant effect on the thermal pressure – **can be lower than 0!**



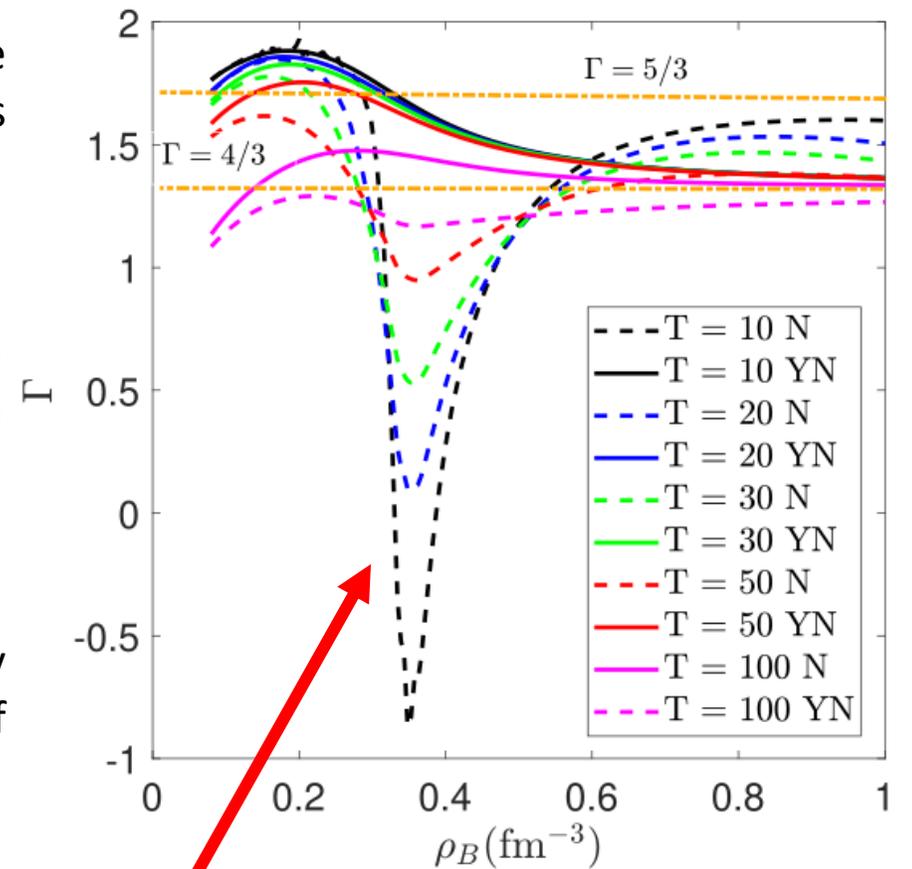
Thermal index – β - stable ν free matter in FSU2H* I



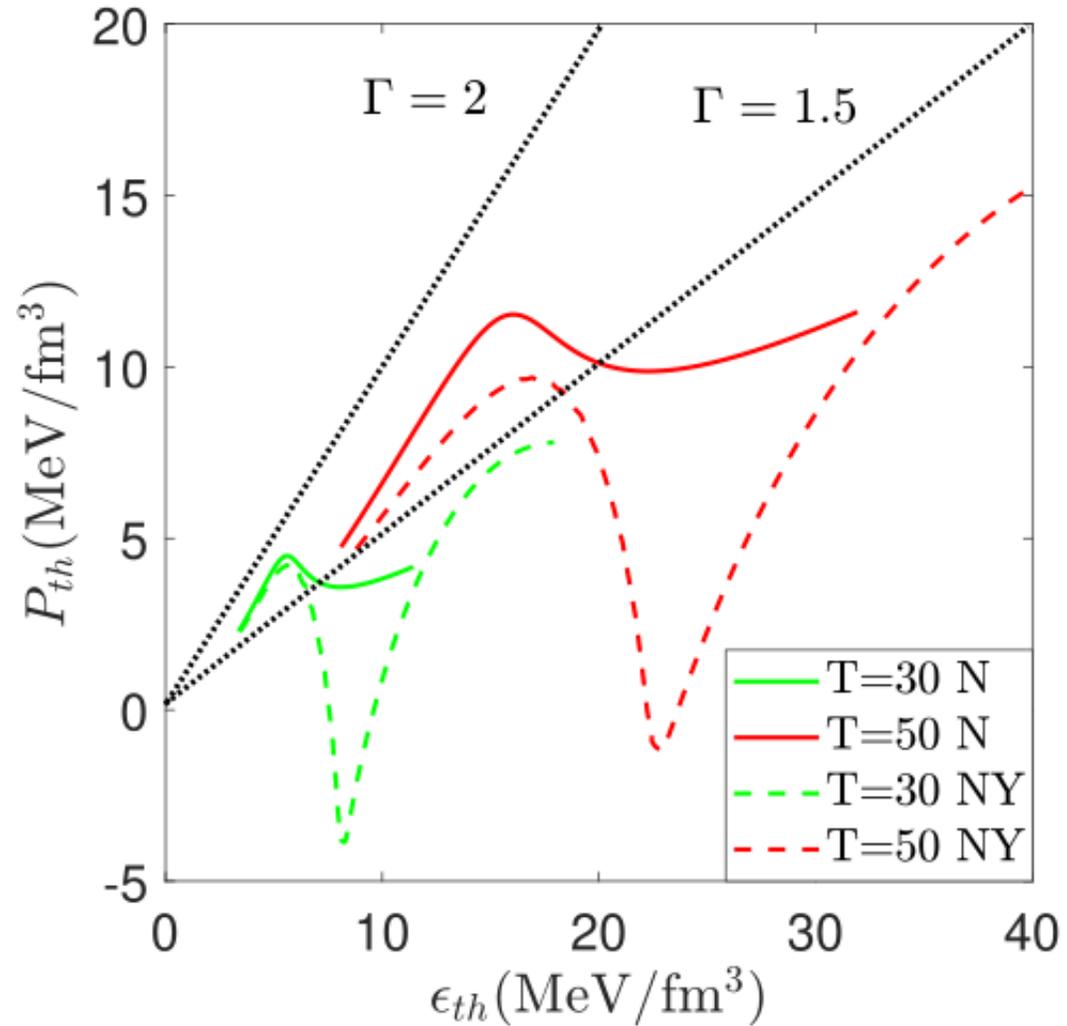
- Thermal effects are more emphasized when hyperons are included

- Significant effect on the thermal pressure – **can be lower than 0!**

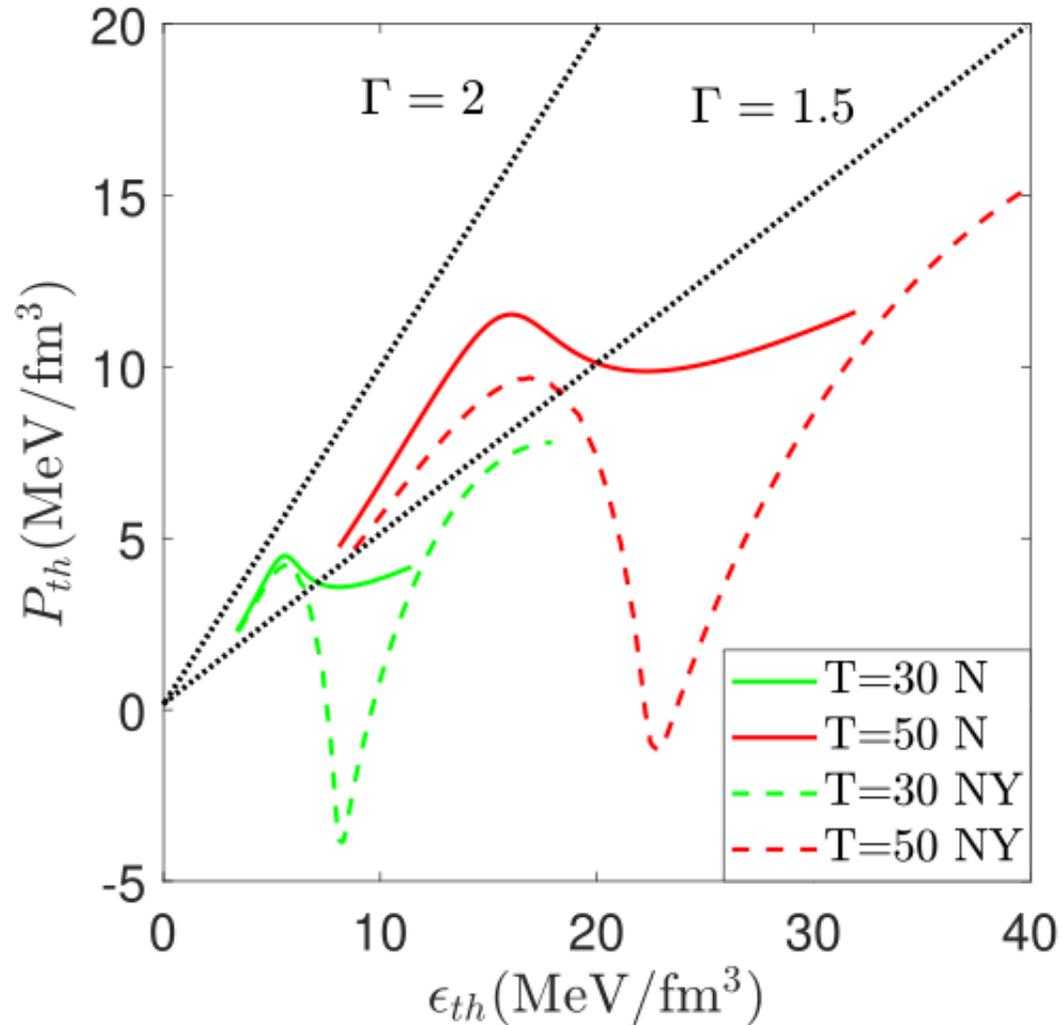
- Thermal index is directly affected by the behavior of p_{th}



Thermal index – β - stable ν free matter in FSU2H* II

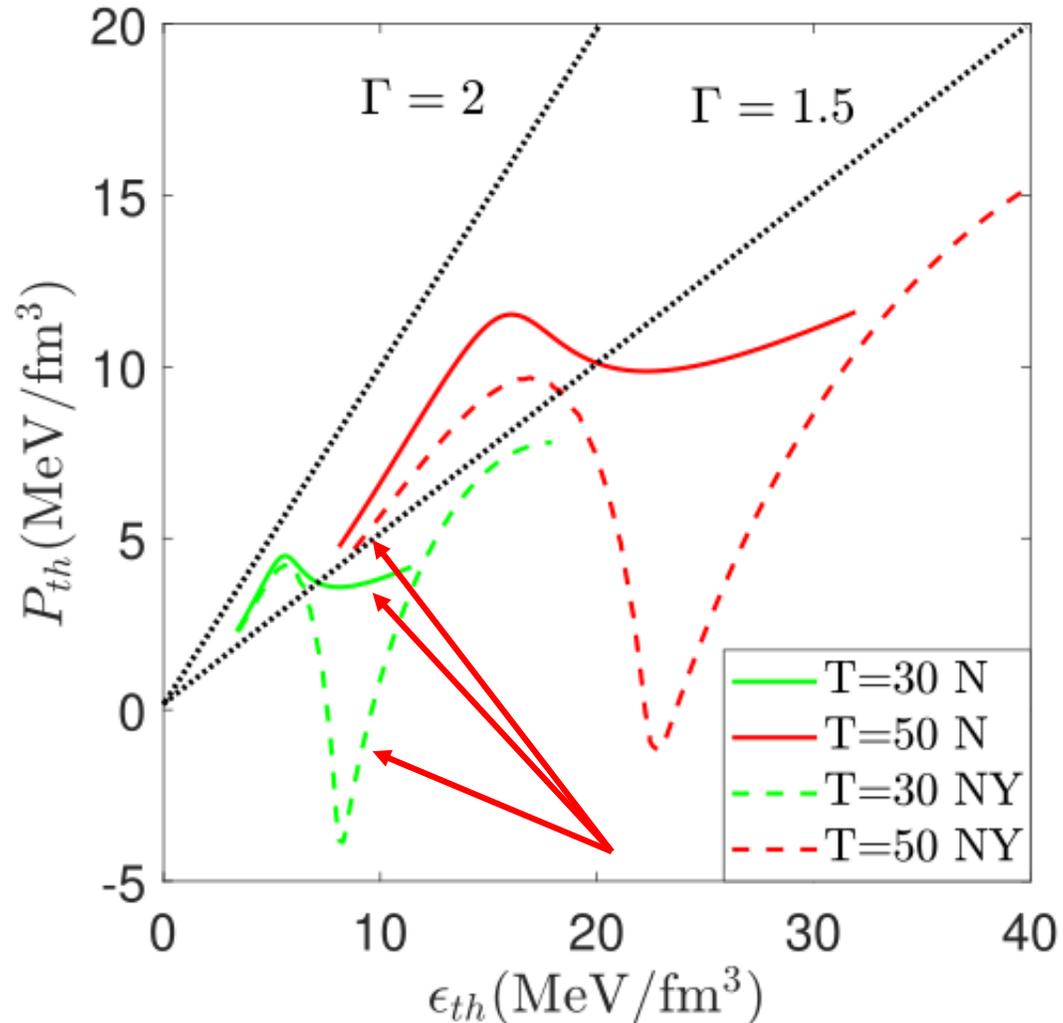


Thermal index – β - stable ν free matter in FSU2H* II



$$P_{th} = (\Gamma - 1)\epsilon_{th}$$

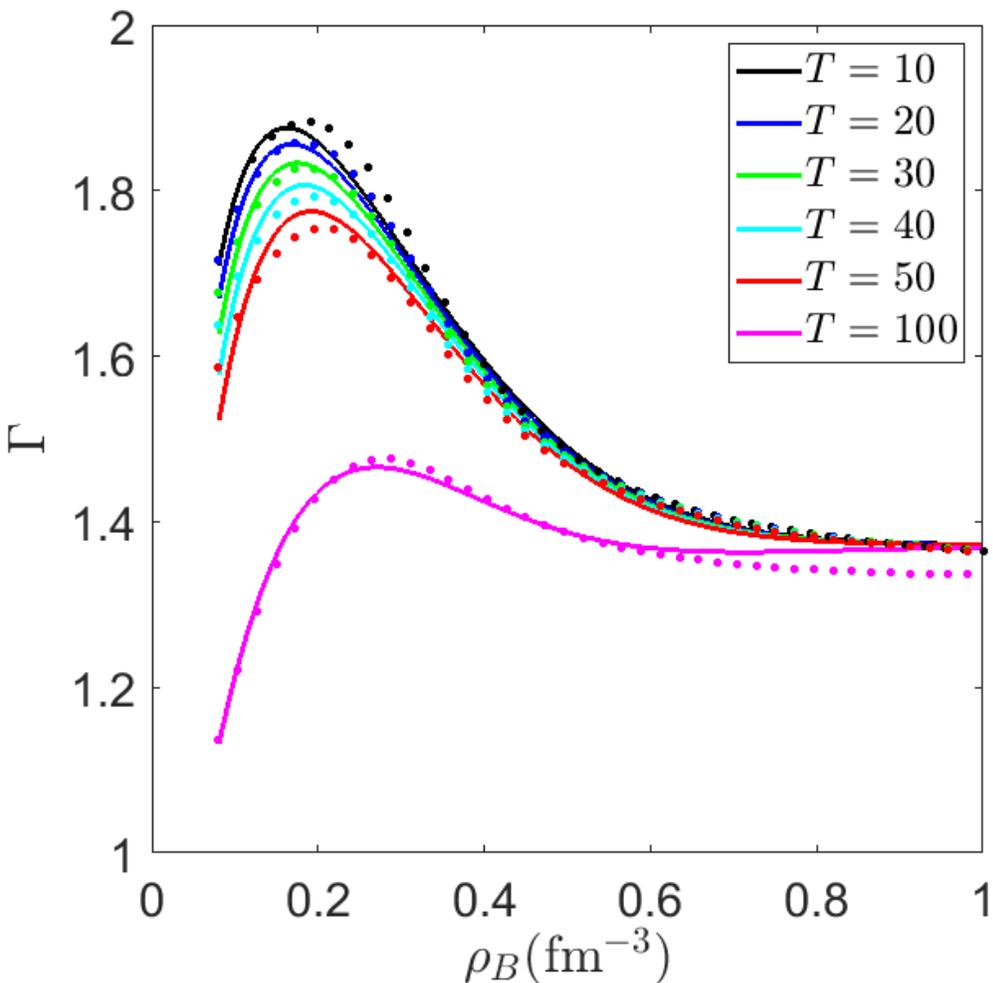
Thermal index – β - stable ν free matter in FSU2H* II



$$P_{th} = (\Gamma - 1)\epsilon_{th}$$

**Thermal effects calculated
with constant Γ index are
not accurate!**

Thermal index – parametrization of the thermal index I



NEUTRINOLESS NUCLEONS ONLY CASE

- We assume that the thermal index has the following functional dependence:

$$\Gamma(\rho_B, T) = a_1 + b_1 e^{-c(T)\rho_B^2} + d_1 e^{-e(T)\rho_B}$$

$$c(T) = c_1 T + c_2$$

$$e(T) = e_1 T + e_2$$

- The values of the parameters:

i	a_i	b_i	c_i	d_i	e_i
1	1.37	6.89×10^{-1}	1.69×10^{-3}	1.47	-1.41×10^{-1}
2	/	/	6.59	/	20.29

Thermal index – parametrization of the thermal index I

NEUTRINOLESS HYPERONIC CASE

- We assume that the thermal index has the following functional dependence:

$$\Gamma(\rho_B, T) = \frac{a(T)\rho_B + b(T)}{(\rho_B - \rho_{Bc})^2 + c(T)\rho_B + d(T) + e(T)\rho_B + f(T)\sqrt{\rho_B} + g(T)}$$

$$a(T) = a_1T + a_2$$

$$b(T) = b_1T + b_2$$

$$c(T) = c_1T^2 + c_2T + c_3$$

$$d(T) = d_1T^2 + d_2T + d_3 \quad \rho_{Bc} = 0.3715 \text{ fm}^{-3}$$

$$e(T) = e_1T^2 + e_2T + e_3$$

$$f(T) = f_1T + f_2$$

$$g(T) = g_1T^2 + g_2T + g_3$$

Thermal index – parametrization of the thermal index I

NEUTRINOLESS HYPERONIC CASE

- We assume that the thermal index has the following functional dependence:

$$\Gamma(\rho_B, T) = \frac{a(T)\rho_B + b(T)}{(\rho_B - \rho_{B_c})^2 + c(T)\rho_B + d(T)} + e(T)\rho_B + f(T)\sqrt{\rho_B} + g(T)$$

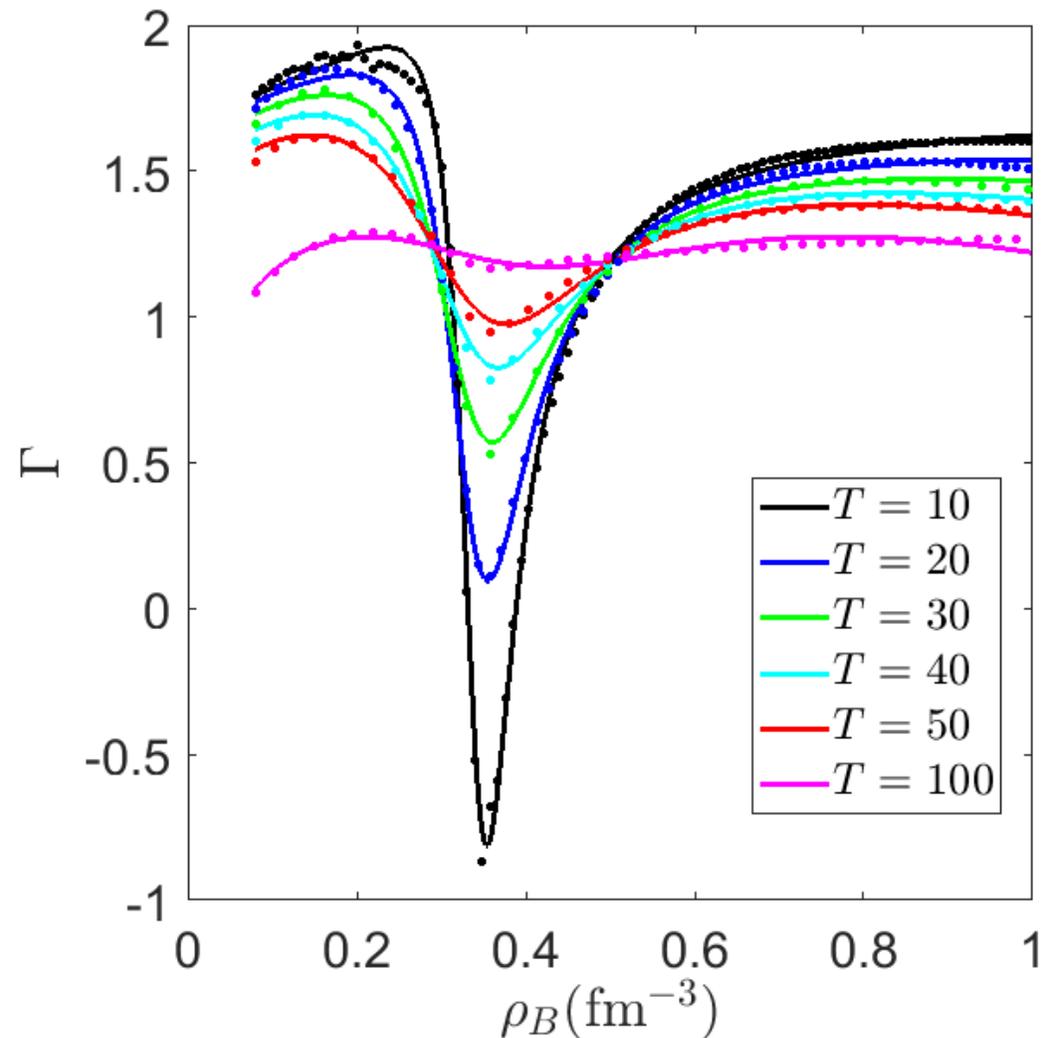
$$a(T) = a_1T + a_2$$

$$b(T) = b_1T + b_2$$

$$c(T) = c_1T^2 + c_2T + c_3$$

i	a_i	b_i	c_i	d_i	e_i	f_i	g_i
1	-2.02×10^{-4}	1.98×10^{-2}	-1.71×10^{-5}	1.24×10^{-5}	1.11×10^{-4}	3.21×10^{-2}	-6.64×10^{-5}
2	-6.14×10^{-2}	-1.26×10^{-4}	1.83×10^{-3}	-6.22×10^{-4}	-3.63×10^{-2}	2.60×10^{-1}	-5.13×10^{-3}
3	/	/	1.56×10^{-2}	-4.83×10^{-3}	7.15×10^{-2}	/	1.48

Thermal index – parametrization of the thermal index I



NEUTRINOLESS HYPERONIC CASE

- We assume that the thermal index has the following functional dependence:

$$\Gamma(\rho_B, T) = \frac{a(T)\rho_B + b(T)}{(\rho_B - \rho_{B_c})^2 + c(T)\rho_B + d(T)} + e(T)\rho_B + f(T)\sqrt{\rho_B} + g(T)$$

$$a(T) = a_1 T + a_2$$

$$b(T) = b_1 T + b_2$$

$$c(T) = c_1 T^2 + c_2 T + c_3$$

	d_i	e_i	f_i	g_i
-5	1.24×10^{-5}	1.11×10^{-4}	3.21×10^{-2}	-6.64×10^{-5}
3	-6.22×10^{-4}	-3.63×10^{-2}	2.60×10^{-1}	-5.13×10^{-3}
2	-4.83×10^{-3}	7.15×10^{-2}	/	1.48

Summary

- We extended the **FSU2H** hyper-nucleonic model to **finite temperature**, constructing the new so-called **FSU2H*** model, in order to be used in early stages of NS evolution and in NS mergers.
- The model satisfies the major constraints that come from nuclear experiments and astrophysical measurements.
- In order to get the properties of the matter computed within the framework of our model, we performed calculations for β - stable matter at different temperatures and different lepton fractions.
- The composition patterns and the EoS are strongly affected by the introduction of hyperons.
- Thermal effects in the star cannot be reproduced with high accuracy with the so-called Γ -law, and that is especially important when hyperons are considered in the core.
- A simple parametrization is constructed to account for the strong dependence of the thermal index in ν -less beta stable matter.

ECT*

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IN NUCLEAR PHYSICS AND RELATED AREAS



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Thank you for your attention

UNIVERSITAT DE
BARCELONA



Hristijan Kochankovski
Laura Tolos
Angels Ramos

EXOTICO workshop 2022
Trento, Italy

17-21 October 2022



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Backup slides



RMF model extended I

$$(i\gamma_\mu \partial^\mu - m_b^* - g_{\omega b} \gamma_0 \omega^0 - g_{\phi b} \gamma_0 \phi^0 - g_{\rho b} I_{3b} \gamma_0 \rho_3^0) \Psi_b = 0,$$

$$(i\gamma_\mu \partial^\mu - q_l \gamma_\mu A^\mu - m_l) \psi_l = 0,$$

Baryon's and lepton's equations of motions

$$m_\sigma^2 \bar{\sigma} + \frac{\kappa}{2} g_{\sigma b}^3 \bar{\sigma}^2 + \frac{\lambda}{3!} g_{\sigma b}^4 \bar{\sigma}^3 = \sum_b g_{\sigma b} \rho_b^s,$$

$$m_{\sigma^*}^2 \bar{\sigma}^* = \sum_{b^*} g_{\sigma b^*} \rho_b^s,$$

$$m_\omega^2 \bar{\omega} + \frac{\zeta}{3!} g_{\omega b}^4 \bar{\omega}^3 + 2\Lambda_\omega g_{\rho,b}^2 g_{\omega,b}^2 \bar{\omega} \bar{\rho}^2 = \sum_b g_{\omega b} \rho_b,$$

$$m_\rho^2 \bar{\rho} + 2\Lambda_\omega g_{\rho,b}^2 g_{\omega,b}^2 \bar{\omega}^2 \bar{\rho} = \sum_b g_{\rho b} I_{3b} \rho_b,$$

$$m_\phi^2 \bar{\phi} = \sum_b g_{\phi b} \rho_b,$$

Meson's equation of motion in RMF approximation

$$\rho_b = \langle \bar{\Psi}_b \gamma^0 \Psi_b \rangle = \frac{\gamma_b}{2\pi^2} \int_0^\infty dk k^2 f_b(k, T),$$

$$\rho_b^s = \langle \bar{\Psi}_b \Psi_b \rangle = \frac{\gamma_b}{2\pi^2} \int_0^\infty dk k^2 \frac{m_b^*}{\sqrt{k^2 + m_b^{*2}}} f_b(T, k)$$

Scalar and baryonic density

$$f_b(k, T) = \left[1 + \exp \left(\frac{\sqrt{k^2 + m_b^{*2}} - \mu_b^*}{T} \right) \right]^{-1}$$

Fermi – dirac distribution

$$\mu_b^* = \mu_b - g_{b\omega} \bar{\omega} - g_{b\rho} \bar{\rho} - g_{b\phi} \bar{\phi}.$$

$$m_b^* = m_b - g_{\sigma b} \sigma - g_{\sigma^* b} \sigma^*,$$

Effective chemical potential and charge neutrality

RMF model extended II

$$\mu_{b^0} = \mu_n,$$

$$\mu_{b^-} = 2\mu_n - \mu_p,$$

$$\mu_{b^+} = \mu_p,$$

$$\mu_n - \mu_p = \mu_e - \mu_{\nu_e},$$

$$\mu_e = \mu_\mu + \mu_{\nu_e} - \mu_{\bar{\nu}_\mu},$$

β equilibrium

$$\rho_B = \sum_b \rho_b,$$

$$Y_l \cdot \rho_B = \rho_l + \rho_{\nu_l}$$

Conservation of baryon and lepton numbers

$$T_{\mu\nu} = \frac{\partial \mathcal{L}}{\partial (\partial_\mu \Phi_\alpha)} \partial^\mu \Phi_\alpha - \eta_{\mu\nu} \mathcal{L},$$

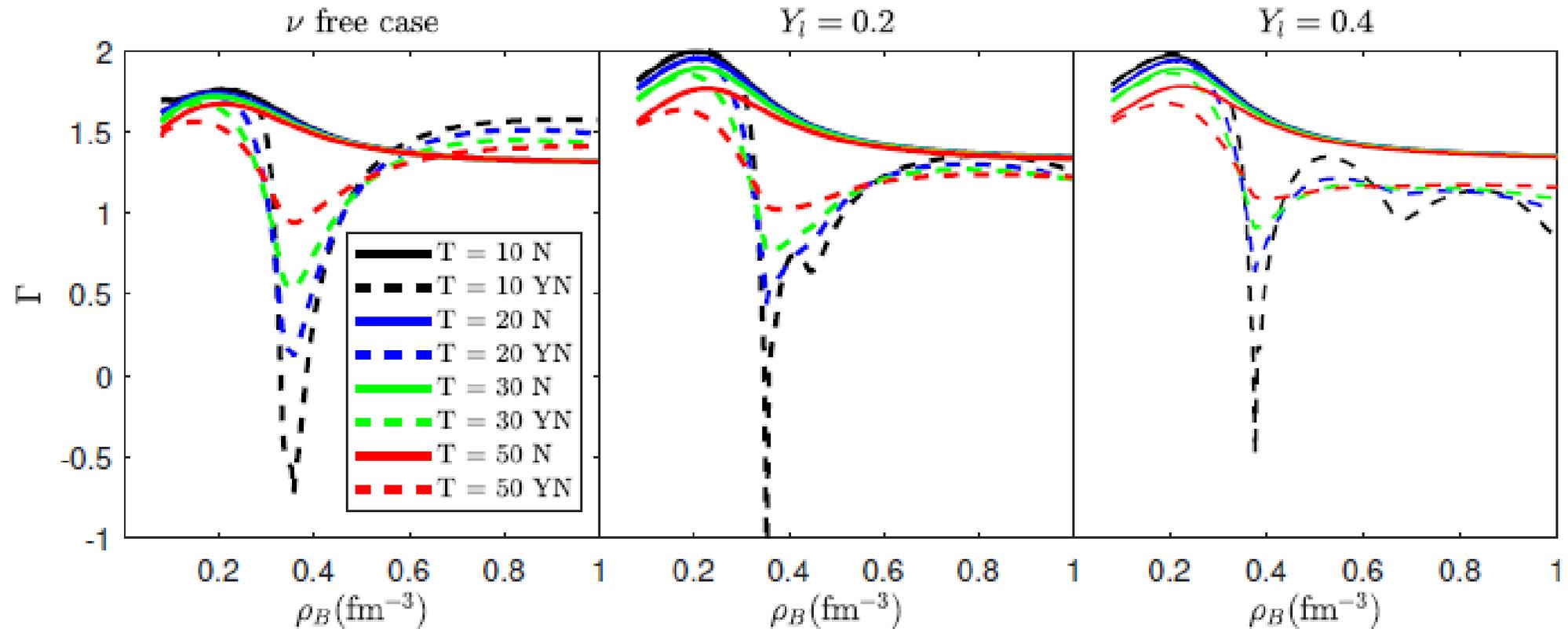
Energy-momentum tensor

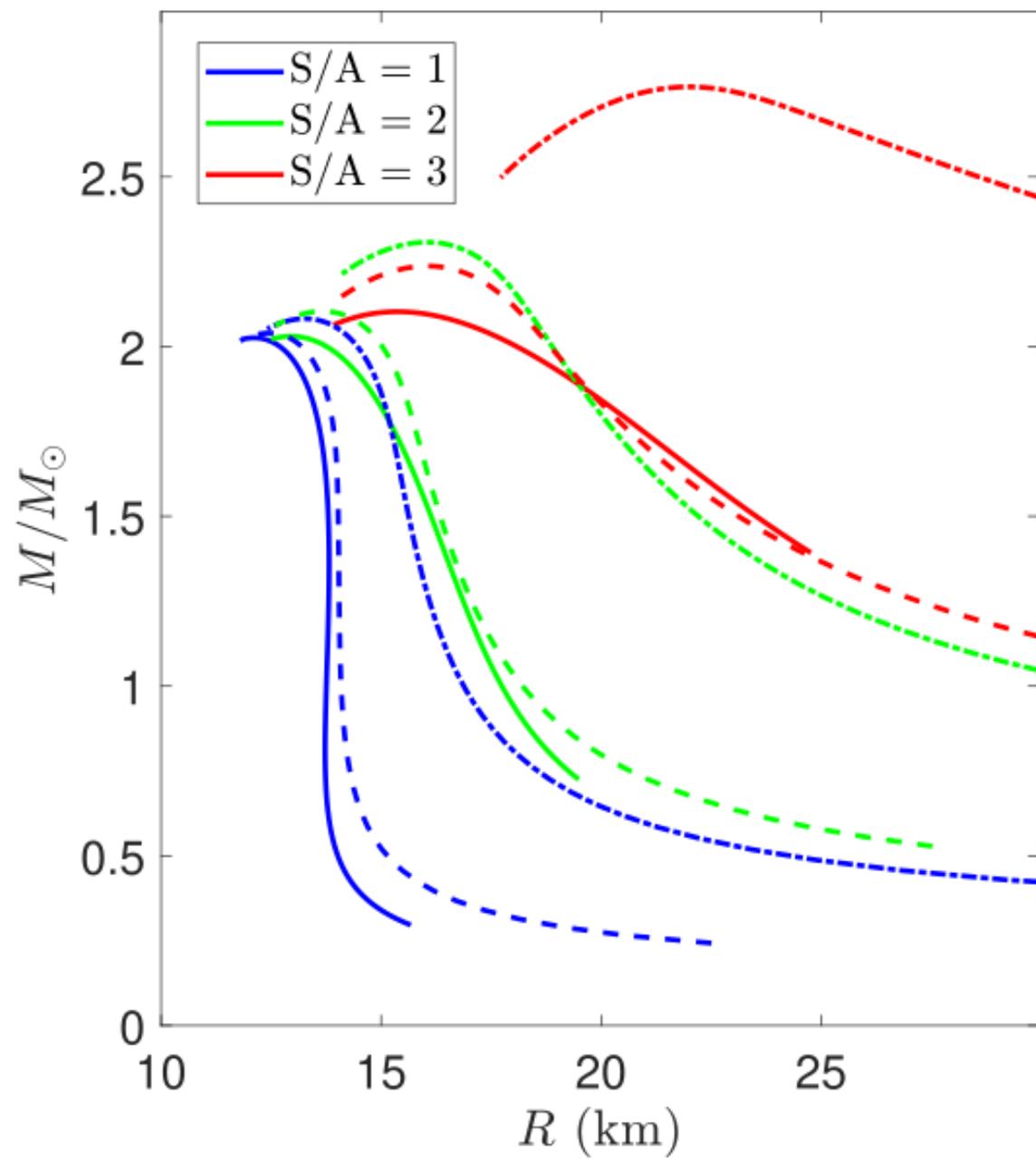
$$\begin{aligned} \epsilon &= \langle T_{00} \rangle \\ &= \frac{1}{2\pi^2} \sum_b \gamma_b \int_0^\infty dk k^2 \sqrt{k^2 + m_b^{*2}} f_b(k, T) \\ &\quad + \frac{1}{2\pi^2} \sum_l \gamma_l \int_0^\infty dk k^2 \sqrt{k^2 + m_l^2} f_l(k, T) \\ &\quad + \frac{1}{2} (m_\omega^2 \bar{\omega}^2 + m_\rho^2 \bar{\rho}^2 + m_\sigma^2 \bar{\phi}^2 + m_\sigma^2 \bar{\sigma}^2 + m_\sigma^2 \cdot \bar{\sigma}^{*2}) \\ &\quad + \frac{\kappa}{3!} (g_\sigma \bar{\sigma})^3 + \frac{\lambda}{4!} (g_\sigma \bar{\sigma})^4 + \frac{\zeta}{8} (g_\omega \bar{\omega})^4 + 3\Lambda_\omega (g_\rho g_\omega \bar{\rho} \bar{\omega})^2, \\ P &= \frac{1}{3} \langle T_{jj} \rangle \\ &= \frac{1}{6\pi^2} \sum_b \gamma_b \int_0^\infty dk \frac{k^4}{\sqrt{k^2 + m_b^{*2}}} f_b(k, T) \\ &\quad + \frac{1}{6\pi^2} \sum_l \gamma_l \int_0^\infty dk \frac{k^4}{\sqrt{k^2 + m_l^2}} f_l(k, T) \\ &\quad + \frac{1}{2} (m_\omega^2 \bar{\omega}^2 + m_\rho^2 \bar{\rho}^2 + m_\sigma^2 \bar{\phi}^2 - m_\sigma^2 \bar{\sigma}^2 - m_\sigma^2 \cdot \bar{\sigma}^{*2}) \\ &\quad - \frac{\kappa}{3!} (g_\sigma \bar{\sigma})^3 - \frac{\lambda}{4!} (g_\sigma \bar{\sigma})^4 + \frac{1}{24} \zeta (g_\omega \bar{\omega})^4 + \Lambda_\omega (g_\rho g_\omega \bar{\rho} \bar{\omega})^2, \end{aligned}$$

$$\begin{aligned} s &= \frac{1}{T} \left(\epsilon + P - \sum_i \mu_i \rho_i \right) \\ f &= \sum_i \mu_i \rho_i - P. \end{aligned}$$

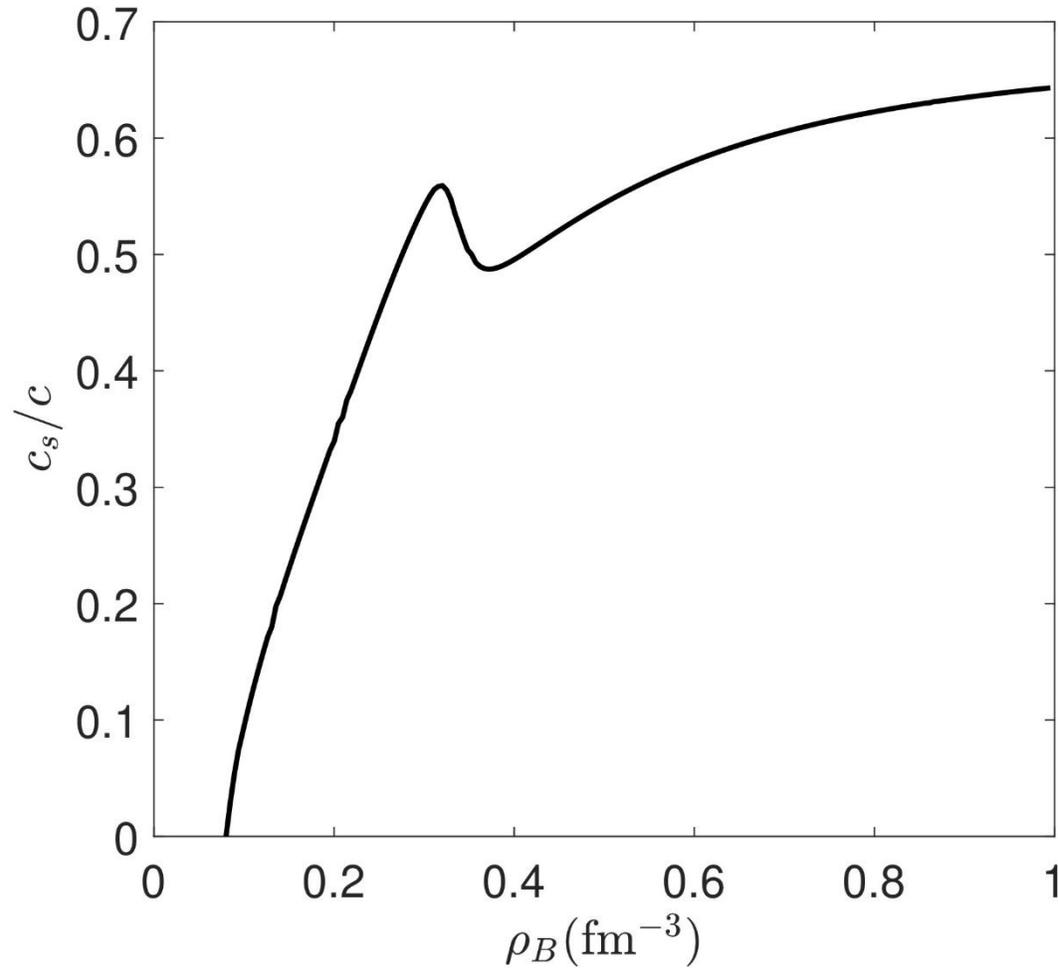
Thermodynamic quantities

Thermal index for different lepton fractions





Speed of sound ($T = 0$)



$$\frac{c_s}{c} = \sqrt{\frac{dp}{d\epsilon}}$$