ΛNN content of Λ-nucleus potential

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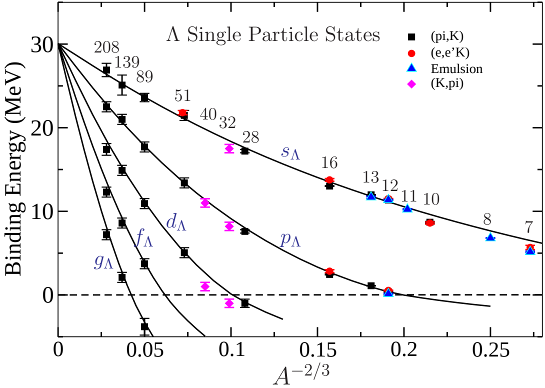
ECT* workshop: **EXOTICO**

EXOTIC ATOMS MEET NUCLEAR COLLISIONS FOR A NEW FRONTIER PRECISION ERA IN LOW-ENERGY

STRANGENESS NUCLEAR PHYSICS

Trento, October 2022

Update: Millener, Dover, Gal PRC 38, 2700 (1988)



Woods-Saxon $V = 30.05$ MeV, $r = 1.165$ fm, $a = 0.6$ fm

Λ hypernuclei: binding energies and Woods-Saxon fits, 2016

OUTLINE

- Experimental binding energies of Λ hypernuclei up to Pb
- Density-dependent potentials indicated a repulsive ρ^2 term in addition to an attractive dominant one
- At ρ_0=0.17 fm^{-3} most ΛN potential models overbind
- 'Statement of mission'
- The optical potential
- Results
- Discussion

Statement of mission

The optical potential employed in this work,
$V_\Lambda^{\rm opt}(\rho) = V_\Lambda^{(2)}(\rho) + V_\Lambda^{(3)}(\rho)$,
consists of terms representing two-body ΛN and three-body ΛNN interactions, respectively.

Our aim in the present phenomenological study is to check to what extent properly chosen Λ hypernuclear binding energy data, with minimal extra assumptions, imply repulsive $V_\Lambda^{(3)}(\rho)$, and how large it is.

The optical potential employed is

$$V_\Lambda^{(2)}(\rho) = -\frac{4\pi}{2\mu_\Lambda} f_A\, C_{\rm Pauli}(\rho)\, b_0 \rho,\ V_\Lambda^{(3)}(\rho) = +\frac{4\pi}{2\mu_\Lambda}\, f_A\, B_0\, \frac{\rho^2}{\rho_0},$$

with b_0 and B_0 in units of fm ($\hbar = c = 1$). A is the mass number of the nuclear core, $\rho_0 = 0.17$ fm^{-3} is nuclear matter density, μ_Λ is the Λ-nucleus reduced mass, f_A is a kinematical factor transforming b_0 from ΛN c.m. to Λ-nucleus c.m., and $C_{\rm Pauli}(\rho)$ for $\alpha_P = 1$ is a Pauli correlations factor:

$$f_A = 1 + \frac{A-1}{A}\frac{\mu_\Lambda}{m_N}, \qquad C_{\rm Pauli}(\rho) = [1 + \alpha_P \frac{3k_F}{2\pi}(1 + \frac{m_\Lambda}{m_N}) b_0]^{-1},$$

with Fermi momentum $k_F = (3\pi^2\rho/2)^{1/3}$.

In another version we replaced $(1 + \frac{m_\Lambda}{m_N})$ by f_A.

Densities are constrained by experimental charge r.m.s.radii:

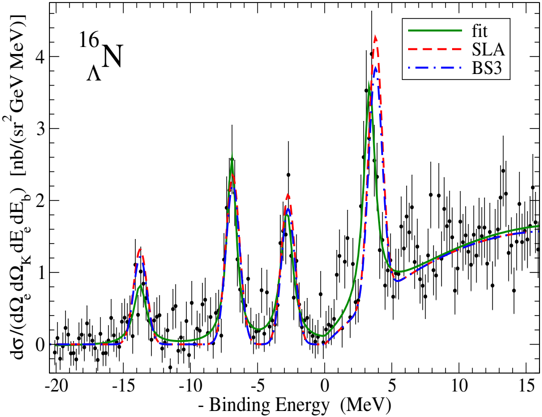
$$V_\Lambda^{(2)}(\rho) = -\frac{4\pi}{2\mu_\Lambda} f_A\, C_{\rm Pauli}(\rho)\, b_0 \rho, \;\; V_\Lambda^{(3)}(\rho) = +\frac{4\pi}{2\mu_\Lambda}\, f_A\, B_0\, \frac{\rho^2}{\rho_0}.$$

Use charge densities for ρ_p throughout. For ρ_n use the same radial parameter as for ρ_p in light and medium-weight nuclei, and slightly different parameters for ρ_n in heavy species,
$r_n - r_p = 1.1\frac{N-Z}{A} - 0.04$ fm, for r.m.s radii.

What can be expected?
When $B_0 = 0$, b_0 is expected to be the ΛN scattering length.

Experimental ΛN spin-averaged scattering length=1.7±0.1 fm

Use high-quality data for a single species for calibration.
${}^{16}_{\Lambda}$N is not too light, single proton hole in the 1p shell.
1st and 3rd peaks from left are 1s and 1p Λ-nucleus states



^{16}O$(e, e'K^{+})$, F. Garibaldi *et. al.* PRC99 (2019) 054309

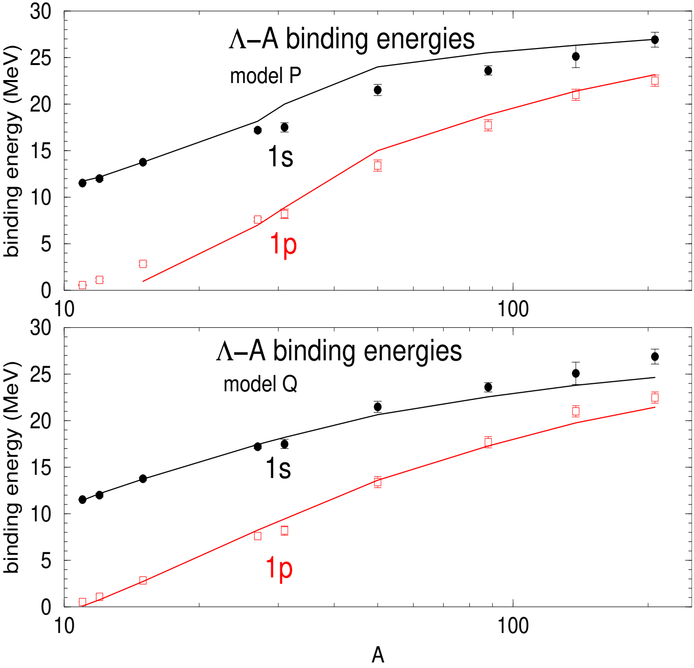
Binding energies in MeV, including uncertainties

hypernucleus	$1s_\Lambda$	$\pm$	$1p_\Lambda$	$\pm$
${}^{12}_{\Lambda}$B	11.52	0.02	0.54	0.04
${}^{13}_{\Lambda}$C	12.0	0.2	1.1	0.2
${}^{16}_{\Lambda}$N	13.76	0.16	2.84	0.18
${}^{28}_{\Lambda}$Si	17.2	0.2	7.6	0.2
${}^{32}_{\Lambda}$S	17.5	0.5	8.2	0.5
${}^{51}_{\Lambda}$V	21.5	0.6	13.4	0.6
${}^{89}_{\Lambda}$Y	23.6	0.5	17.7	0.6
${}^{139}_{\Lambda}$La	25.1	1.2	21.0	0.6
${}^{208}_{\Lambda}$Pb	26.9	0.8	22.5	0.6

We fit two parameters to the ${}^{16}_{\Lambda}$N data. Then we compare calculations with the rest of the data.

Method

- potential P: fit only b_0 to $B_{1s}({}^{16}_{\Lambda}\mathrm{N}) = 13.76 \pm 0.16$ MeV
- potential P': as P but with inevitable Pauli correlations (not shown)
- potential Q: fit b_0 and B_0 to $B_{1s}({}^{16}_{\Lambda}\mathrm{N}) = 13.76 \pm 0.16$ MeV and $B_{1p}({}^{16}_{\Lambda}\mathrm{N}) = 2.84 \pm 0.18$ MeV. No Pauli correlations.
- potential X: as Q, including Pauli correlations.
- potential Y; as X, including 'core-excess' correction.


Λ–A binding energies
model P
1s
1p
binding energy (MeV)
0
5
10
15
20
25
30
10
100
Λ–A binding energies
model Q
1s
1p
binding energy (MeV)
A

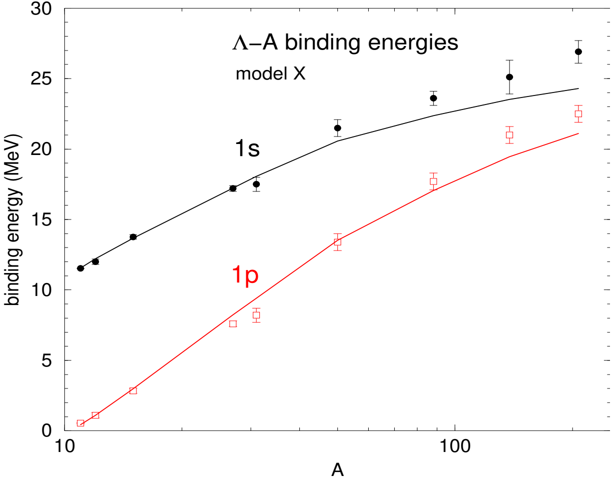
Strength parameters b_0, B_0 (fm) in models P,P',Q plus their respective potential depths $D_\Lambda^{(2)}$, $D_\Lambda^{(3)}$ and sum D_Λ (MeV) at nuclear matter density $\rho_0 = 0.17$ fm^{-3}. Pauli correlations are switched off (on) using $\alpha_P = 0\,(1)$.

Model	α_P	b_0	B_0	$D_\Lambda^{(2)}$	$D_\Lambda^{(3)}$	D_Λ
P	0	0.418	–	−34.1	–	−34.1
P'	1	0.908	–	−32.3	–	−32.3
Q	0	0.706	0.370	−57.6	30.2	−27.4

b_0 is attractive; B_0 is repulsive.

Experimental ΛN spin-averaged scattering length=1.7±0.1 fm

Model X is model Q with Pauli correlations
$b_0 = 1.85$ fm, $B_0 = 0.170 fm$.



Experimental ΛN spin-averaged scattering length=1.7±0.1 fm

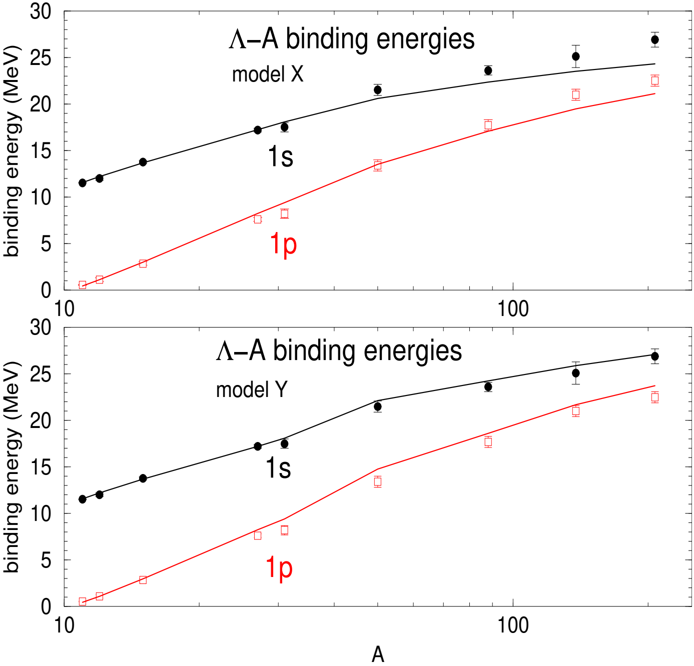
The observed underbinding while b_0 = the free ΛN scattering length suggests that the ΛNN interaction is oversimplified and too repulsive for heavy species.

For N >Z medium weight and heavy nuclei one may write $\rho = \rho_c + \rho_{ex}$ where ρ_c refers to a 'core' of Z protons and Z neutrons in parallel shell-model orbits, and ρ_{ex} represent the N-Z excess neutrons.
Expecting that direct three-body ΛNN contributions involving one 'core' nucleon and one 'excess' nucleon vanish upon summing on the T=0 'core' closed-shell nucleons, we modify ρ^2 by discarding the bilinear term $\rho_c\, \rho_{ex}$, replacing ρ^2 by

$$\rho_c^2 + \rho_{ex}^2 \quad = \quad (2\rho_p)^2 + (\rho_n - \rho_p)^2$$

in terms of the input densities ρ_p and ρ_n.

Model Y is model X with the above expression for ρ^2.
In model Y0 f_A is used in the Pauli correction.


Λ–A binding energies
model X
1s
1p
binding energy (MeV)
30
25
20
15
10
5
0
10
100
Λ–A binding energies
model Y
1s
1p
binding energy (MeV)
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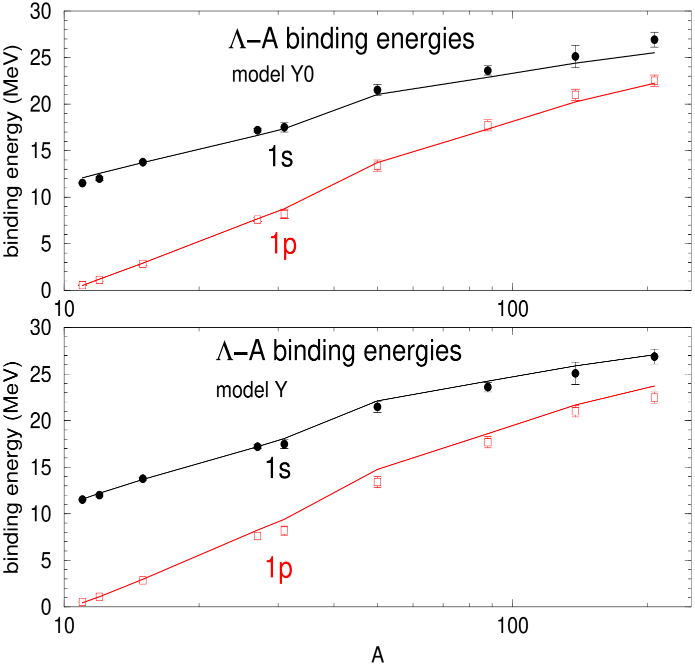
Λ–A binding energies
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1p
binding energy (MeV)
30
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20
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0
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100
A

A simple model conserving volume integrals and making small approximations on r.m.s. radii is $\rho_c = 2\rho_p$, $\rho_{ex} = \rho_n - \rho_p$.

It is easy to show that the volume integral of $(\rho_c)^2 + (\rho_{ex})^2$ is smaller than the volume integral of ρ^2 by approximately a suppression factor

$$F = [4Z^2 + (N\text{-}Z)^2]/[N + Z]^2.$$

Applying this factor to ρ^2 in the potential leads to results almost identical to results when the form $(\rho_c)^2 + (\rho_{ex})^2$ is used.

Strength parameters b_0, B_0 (fm) in models P,P',Q,X,Y plus their respective potential depths $D_\Lambda^{(2)}$, $D_\Lambda^{(3)}$ and sum D_Λ (MeV) at nuclear matter density $\rho_0 = 0.17$ fm^{-3}. Pauli correlations are switched off (on) using $\alpha_P = 0\,(1)$.

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Q	0	0.706	0.370	-57.6	30.2	-27.4
X,Y	1	1.85	0.170	-41.6	13.9	-27.7

b_0 is attractive; B_0 is repulsive.

Experimental ΛN spin-averaged scattering length$=1.7\pm0.1$ fm

Final values for $\rho_0 = 0.17\mathrm{fm}^{-3}$
$D_\Lambda^{(2)} = -(40.6 \pm 1.0)\mathrm{MeV}$, $\quad D_\Lambda^{(3)} = (13.9 \pm 1.4)\mathrm{MeV}$, and
$D_\Lambda = -26.7 \pm 1.7$ MeV.

Discussion

Other calculations do NOT apply the Pauli correction. Consequently their $D_\Lambda^{(2)}$ and $D_\Lambda^{(3)}$ come out larger than ours, but the sum D_Λ agrees with our result, within uncertainties.

Quantum Monte Carlo calculations used nuclear r.m.s. radii that are some 20% smaller than the corresponding charge radii. Estimating the very large corrections required, their results would agree with ours.

Summary

A simple phenomenological optical potential based on nuclear densities constrained by charge radii and containing ρ and ρ^2 terms, where a Pauli correlations effect is applied, is capable of describing experimental $1s$ and $1p$ Λ-nuclear binding energies with two parameters.

The ρ^2 term turns out to be repulsive and larger by a few MeV than the one leading to the Λ chemical potential to be larger than the chemical potential for neutrons in pure neutron matter. (Gerstung, Kaiser and Weise, Eur. Phys. J. **A 56**,175 (2020)).

Thanks for your attention!

Suppressing ρ^2 in medium-weight and heavy species

$\rho = \rho_c + \rho_{ex}$.

By definition:

$\int \rho^2 d\vec{r} = A \int \rho \frac{\rho}{A} d\vec{r} = A\bar{\rho}$

$\int \rho_c^2 d\vec{r} = 2Z \int \rho_c \frac{\rho_c}{2Z} d\vec{r} = 2Z\bar{\rho}_c$

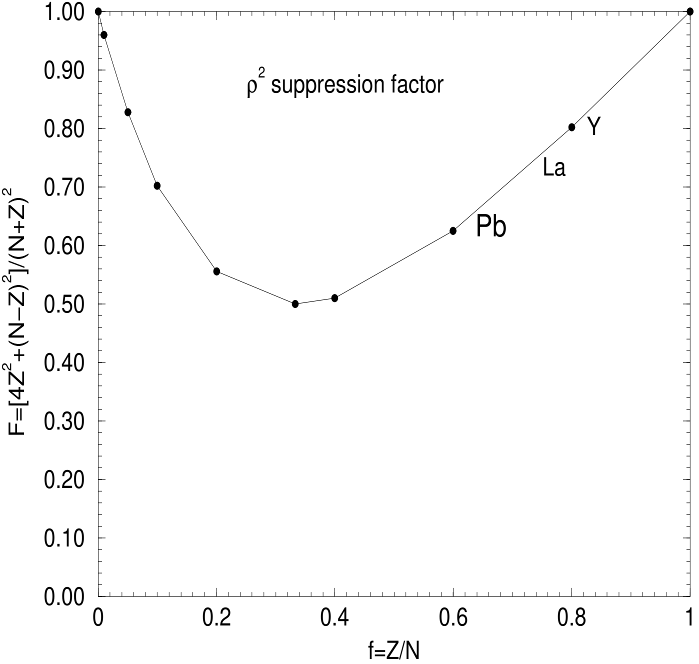
$\int \rho_{ex}^2 d\vec{r} = (N-Z) \int \rho_{ex} \frac{\rho_{ex}}{N-Z} d\vec{r} = (N-Z)\bar{\rho}_{ex}$

$\rho^2 = (\rho_c + \rho_{ex})^2 = \rho_c^2 + \rho_{ex}^2 + 2\rho_c\rho_{ex}$.

Ignoring the crossed term $2\rho_c\rho_{ex}$ and approximating
$\bar{\rho}_c = \frac{2Z}{A}\bar{\rho}$, $\bar{\rho}_{ex} = \frac{N-Z}{A}\bar{\rho}$, we get

$\int \rho^2 d\vec{r} \to \int (\rho_c + \rho_{ex})^2 d\vec{r} = \frac{(2Z)^2 + (N-Z)^2}{A^2} \int \rho^2 d\vec{r}$.

Hence we apply a suppression factor $F = \frac{(2Z)^2 + (N-Z)^2}{A^2}$ to the ρ^2 term in the potential.



1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
ρ² suppression factor
Y
La
Pb
F=[4Z²+(N−Z)²]/(N+Z)²
0
0.2
0.4
0.6
0.8
1
f=Z/N