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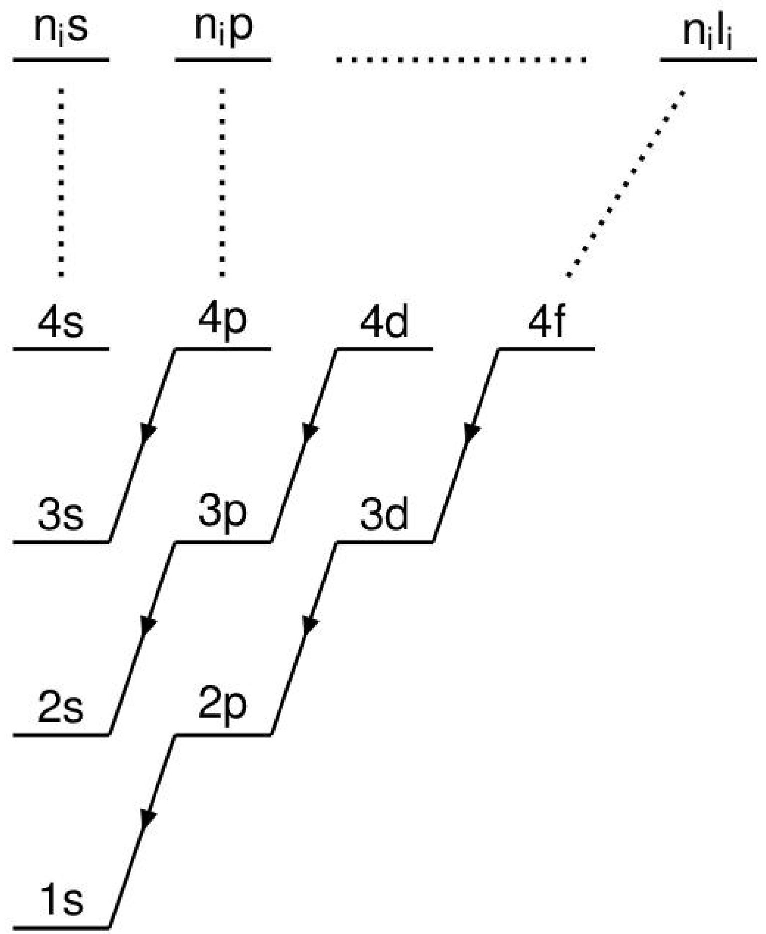
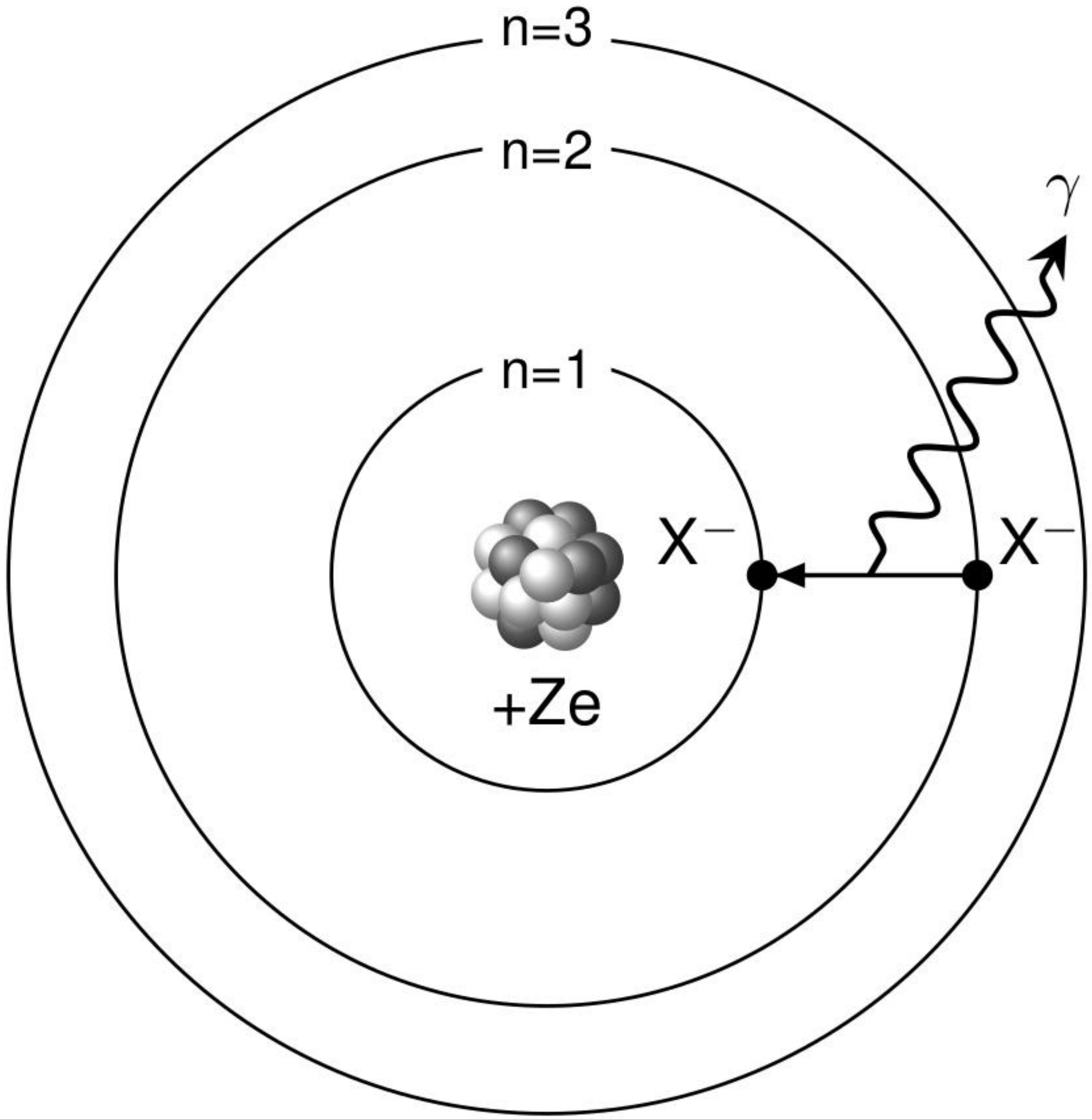
ECT*
EUROPEAN CENTRE
FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS

Cascade models for atomic transitions in kaonic atoms

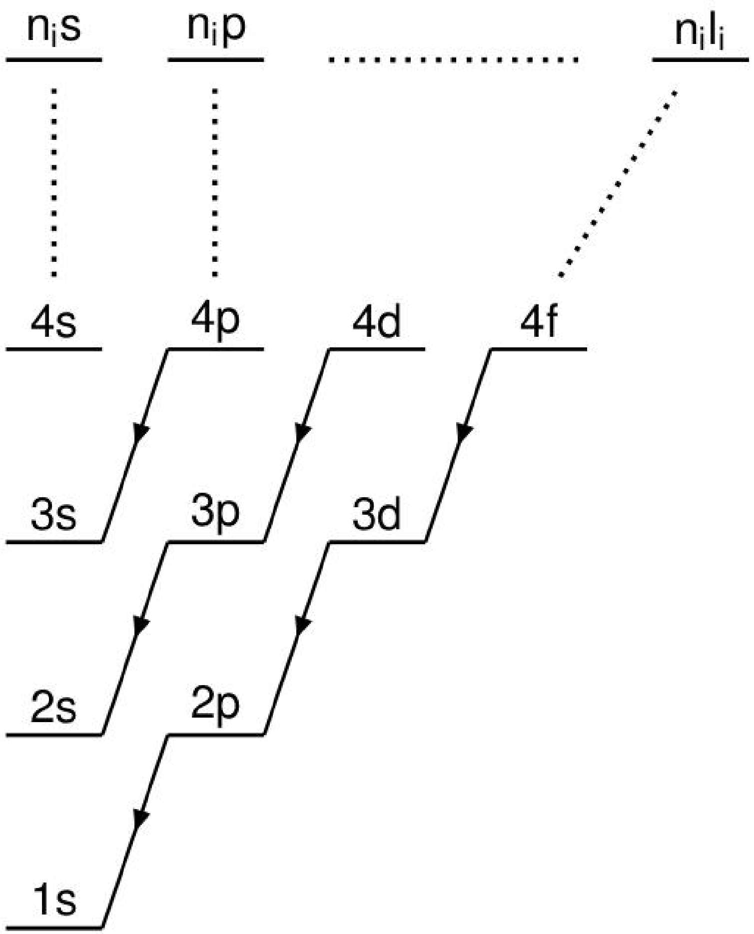
Simone Manti
17 October 2022


$n_i s$
$n_i p$
$n_i l_i$
4s
4p
4d
4f
3s
3p
3d
2s
2p
1s

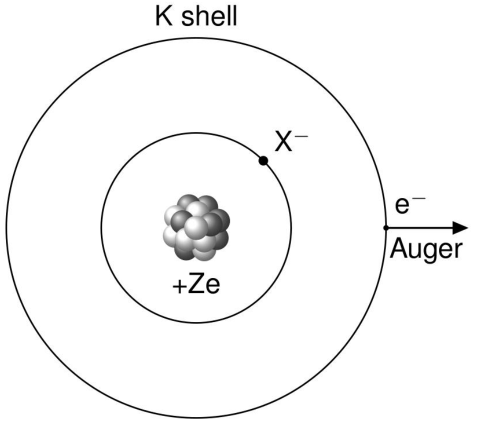

n=3
n=2
γ
n=1
X^-
X^-
+Ze

Model the cascade of the exotic atom with a Monte Carlo

- 1962 - M. Leon and H. A. Bethe, Phys. Rev. 127, 636
- 1980 - E. Borie and M. Leon, Phys. Rev. A 21, 1460
- 1989 - G. Reifenröther, E. Klempt, Nucl. Phys. A, 503, 3–4
- 1997 - T. P. Terada and R. S. Hayano, Phys. Rev. C 55, 73
- 2002 - Jensen T., Markushin V., Eur. Phys. J. D 21, 271–283



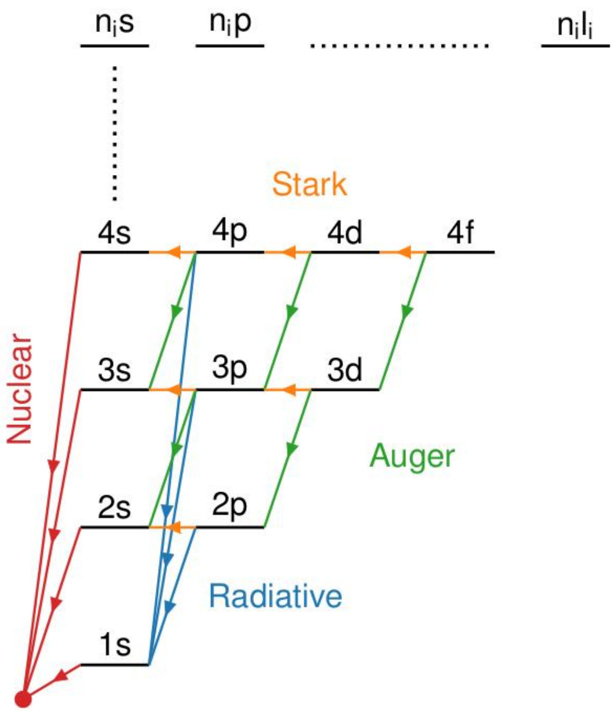
Cascade model for exotic atoms (KN)

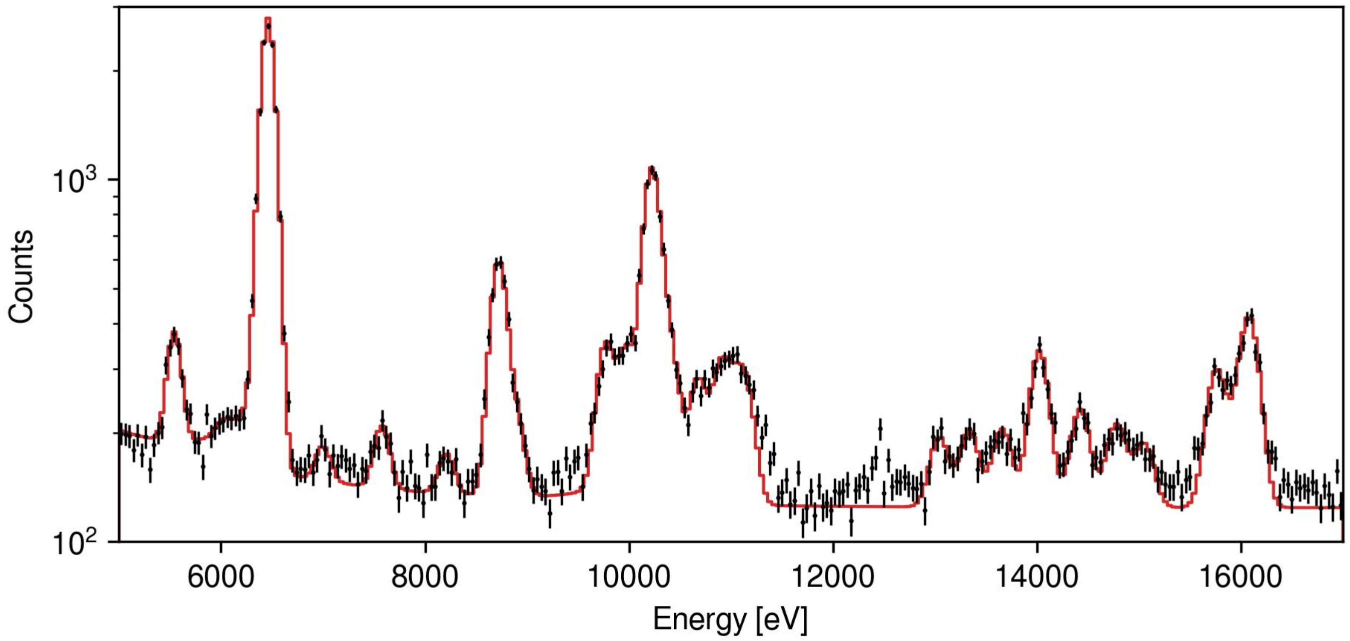


Exotic atom = Atom + X^-

$X^- = \mu^-, \pi^-, K^-,$

$$r_n = \frac{n^2}{\mu Z} \qquad E_n = -\frac{\mu Z^2}{2n^2}$$





Radiative rate from scaling the (Z,μ) hydrogen rate

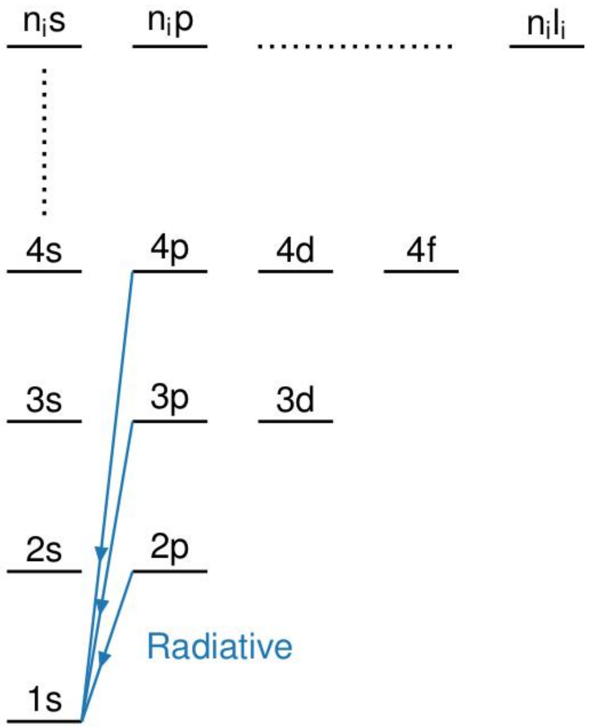
$$(\mathrm{x}^-\mathrm{X})_{n_i l_i} \rightarrow (\mathrm{x}^-\mathrm{X})_{n_f l_f} + \gamma$$

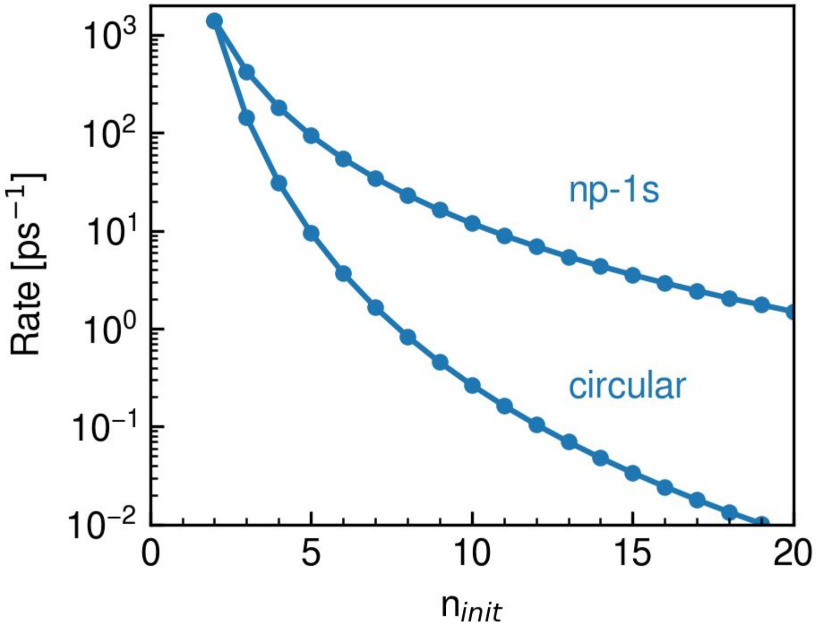
$$\Gamma^{rad}_{n_i l_i \rightarrow n_f l_f} = \mu Z^4 \Gamma^{rad}_{n_i l_i \rightarrow n_f l_f}(H)$$

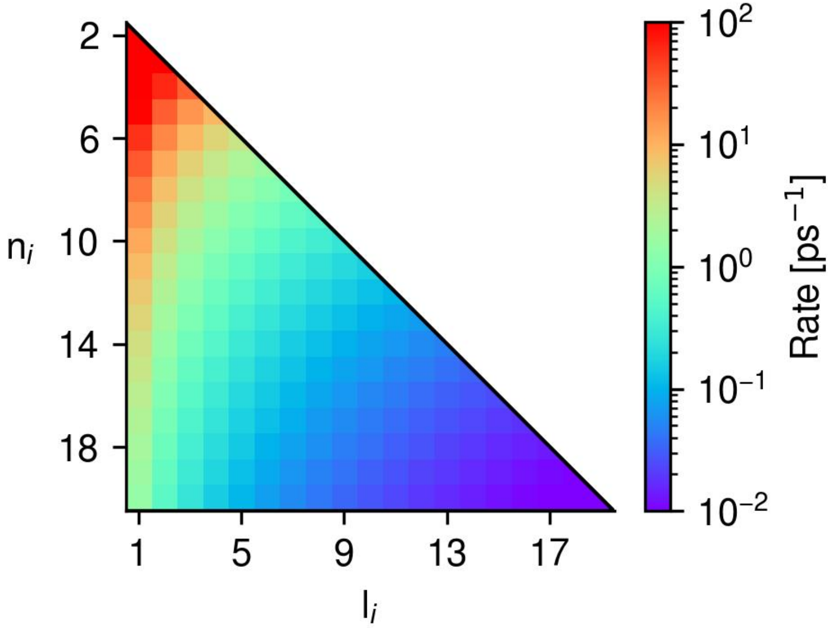
$$\Gamma^{rad}_{n_i l_i \rightarrow n_f l_f}(H) = \frac{4}{3}\alpha^3 \mathrm{R}^2_{if}\omega^3_{if}$$

$$\Gamma^{circ}_{n,n-1\rightarrow n-1,n-2} = \frac{\mu Z^4 \alpha^3}{3}\frac{2^{4n} n^{2n-4}(n-1)^{2n-2}}{(2n-1)^{4n-1}}$$

$$\Delta \mathrm{l} = 0, \pm 1$$



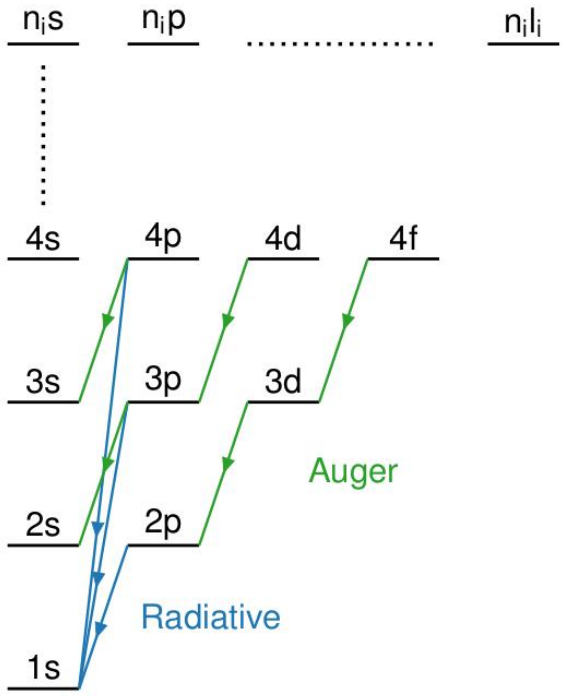


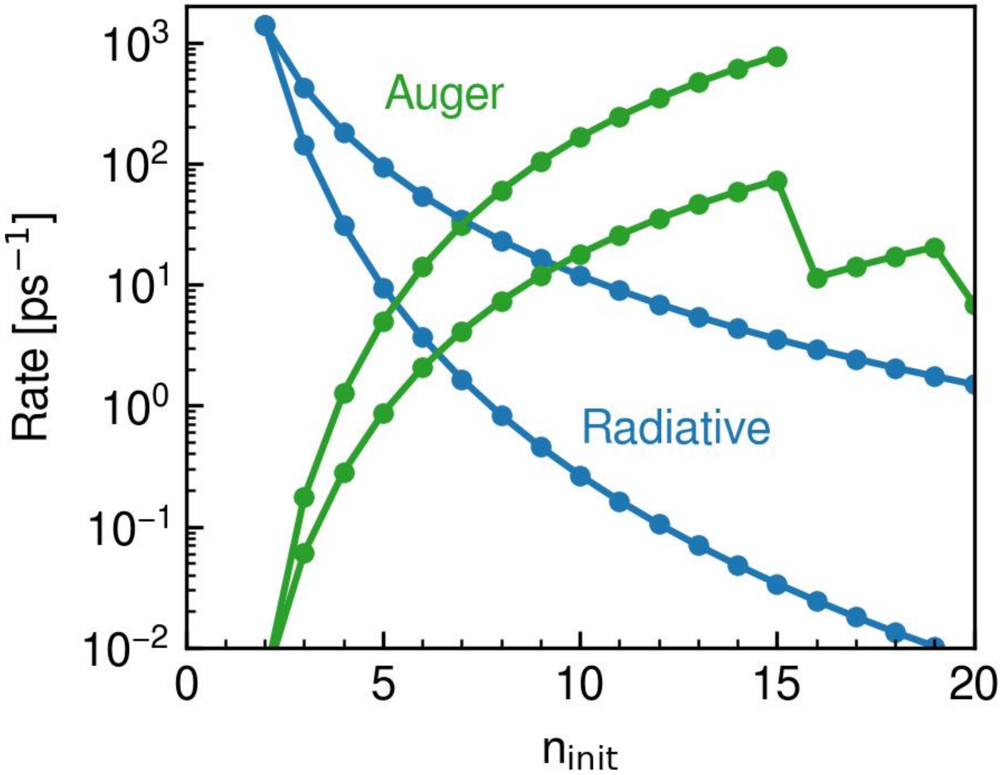


Auger rate from the e-K coulombic interaction

$$(x^-X)_{n_i l_i} 2e^- \to (x^-X)_{n_f l_f} e^- + e^-$$

$$\Gamma^{\text{Auger}} = \left| \int \int \chi_f^*(\mathbf{r}_1)\psi_f^*(\mathbf{r}_2)\frac{1}{r_{12}}\chi_i(\mathbf{r}_2)\psi_i(\mathbf{r}_1)d\mathbf{r}_1 d\mathbf{r}_2 \right|^2$$



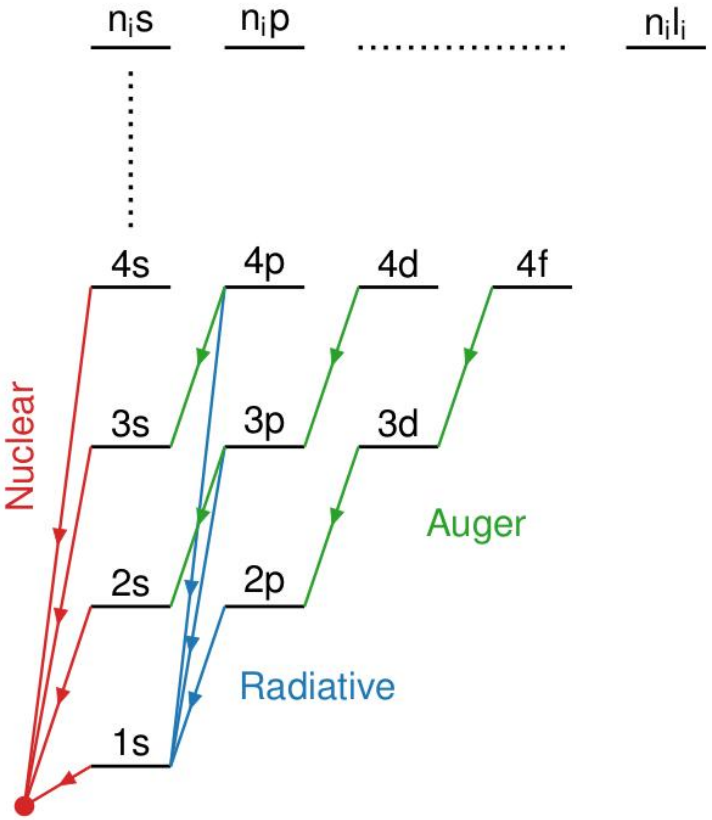


$$E = \frac{Z_x\mu}{2n_1^2} - \frac{Z_x\mu}{2n_2^2} - \frac{Ze^2}{2}$$

$$\psi_f \simeq e^{ikr}$$

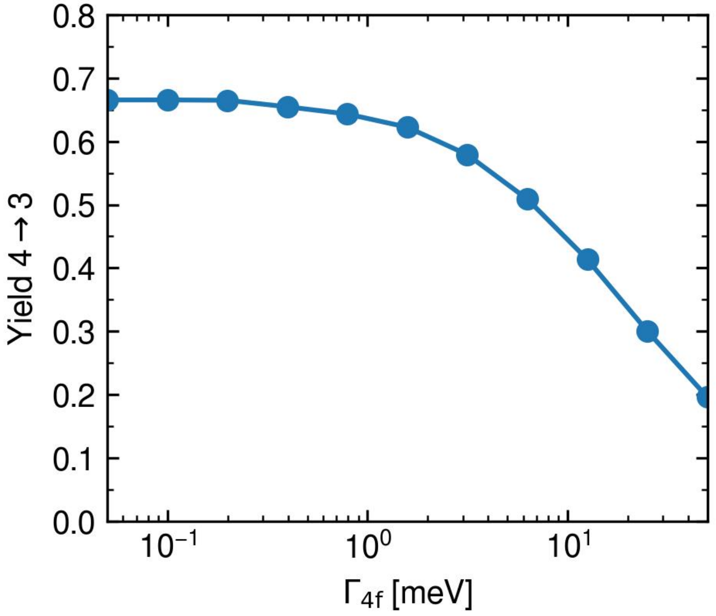
G. R. Burbidge and A. H. de Borde, Phys. Rev. 89, 189 -1953

Nuclear absorption rate from recursion relations



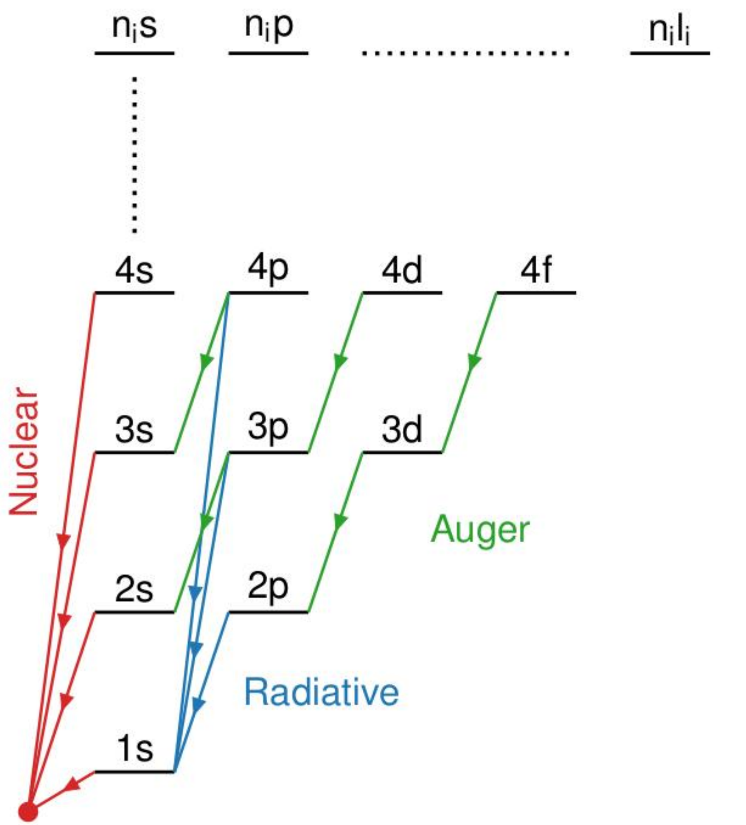
$$\frac{\Gamma^{\mathrm{nucl}}_{n+1,l}}{\Gamma^{\mathrm{nucl}}_{n,l}} = \left(\frac{n}{n+1}\right)^{2l+4}\frac{n+l+1}{n-l}$$

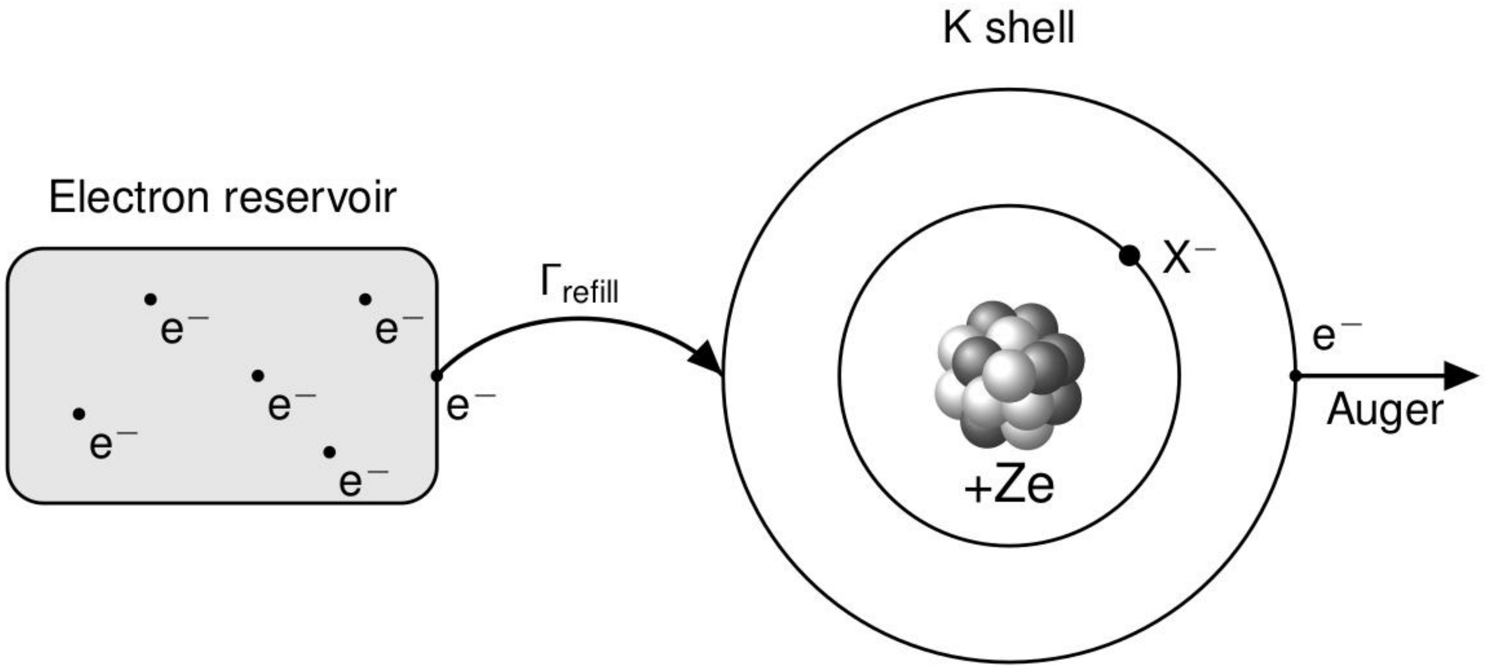
$$\Gamma^{\mathrm{nucl}}_{ns} = \frac{\Gamma^{\mathrm{nucl}}_{1s}}{n^3}$$



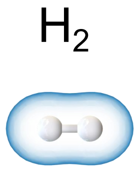
C.J. Batty, Nuclear Physics A, 372, 3, - 1981

Refilling rate to include the effect of the density

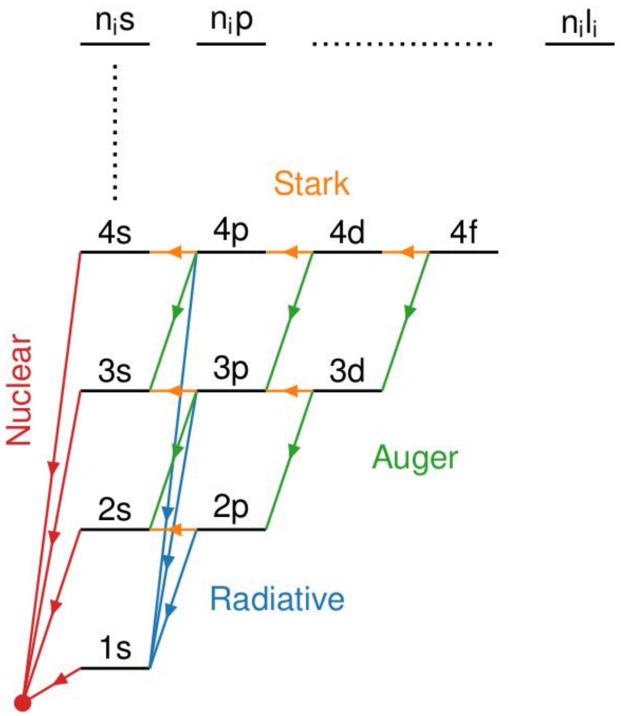


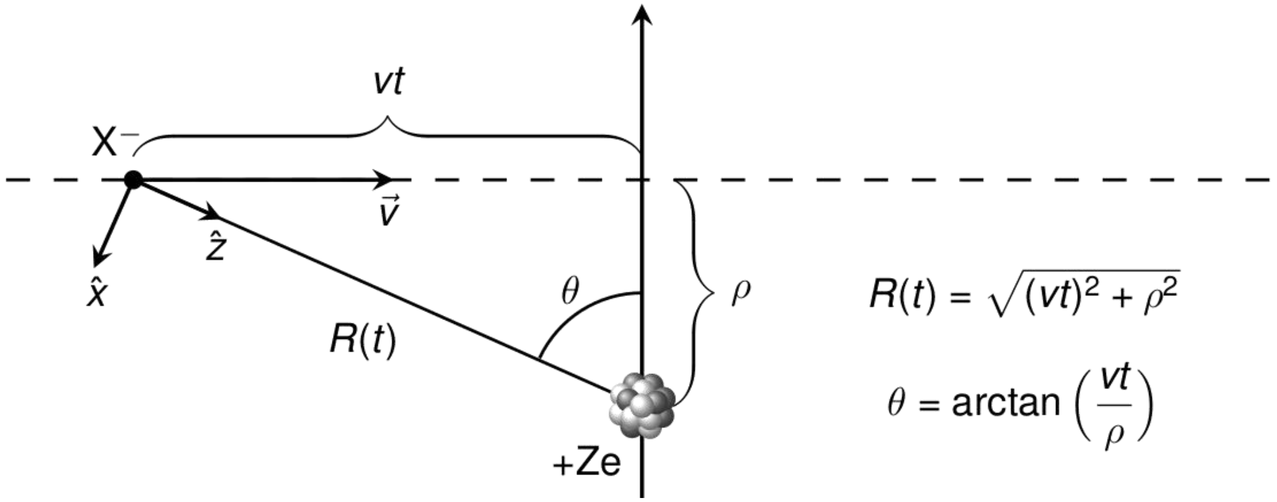


The atomic efield couples the n^2 degenerate sublevels



$$n(r) \rightarrow V(r) \rightarrow F(r)$$





$$R(t) = \sqrt{(vt)^2 + \rho^2}$$

$$\theta = \arctan\left(\frac{vt}{\rho}\right)$$

$$i\frac{d\psi}{dt} = H(t)\psi(t) \qquad \psi(t) = \sum_{l,m} c_l^m(t)\, u_l^m(t)$$

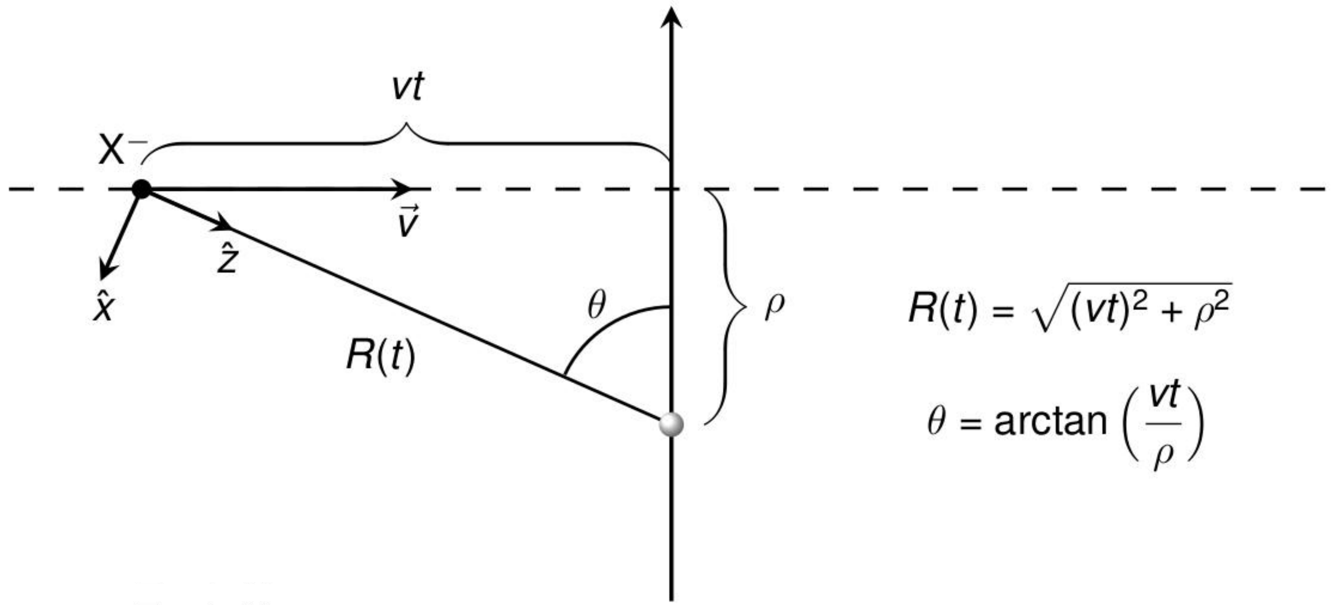
$$i\frac{dc_l^m}{dt} = \underbrace{\frac{3}{2}Fn\,[\alpha_{l-1}^m c_{l-1}^m + \alpha_{l+1}^m c_{l+1}^m]}_{\text{Fixed Field}} - \underbrace{\tfrac{1}{2}i\dot{\theta}[\beta_l^{m-1} c_l^{m-1} - \beta_l^{m+1} c_l^{m+1}]}_{\text{Rotating Field}}$$

$$\alpha_{l-1}^m = \left[\frac{(l^2 - m^2)(n^2 - l^2)}{(2l+1)(2l-1)}\right]^{\frac{1}{2}} \qquad \beta_l^{m-1} = [l(l+1) - m(m-1)]^{\frac{1}{2}}$$

$$\alpha_{l+1}^m = \left[\frac{((l+1)^2 - m^2)(n^2 - (l+1)^2)}{((2l+3)(2l+1))}\right]^{\frac{1}{2}} \qquad \beta_l^{m+1} = [l(l+1) - m(m+1)]^{\frac{1}{2}}$$

Jensen, T. and Markushin V., Eur. Phys. J. D 19, 165–181 - 2002

The Stark effect for the hydrogen atom

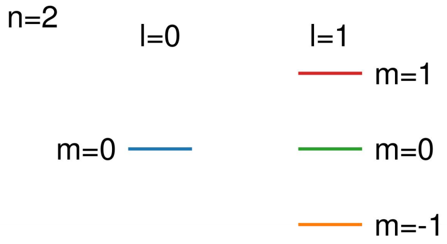


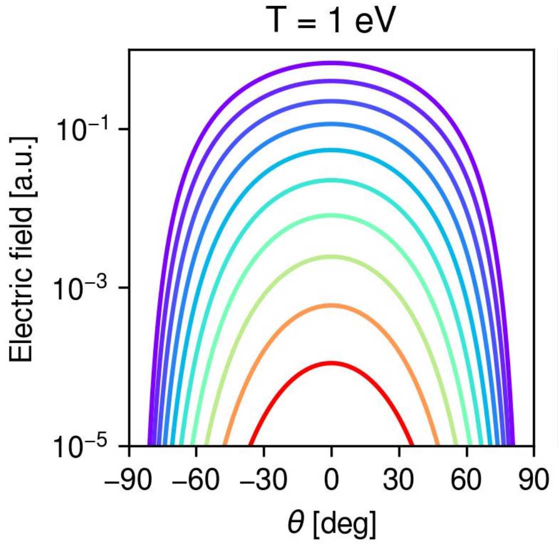
$$R(t) = \sqrt{(vt)^2 + \rho^2}$$

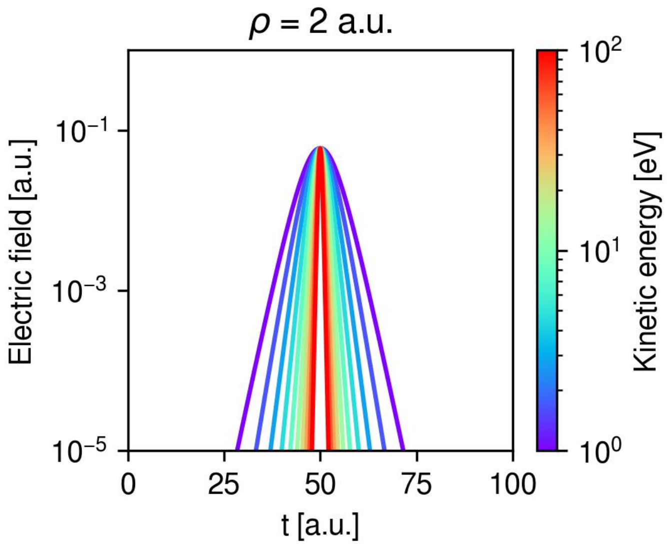
$$\theta = \arctan\left(\frac{vt}{\rho}\right)$$

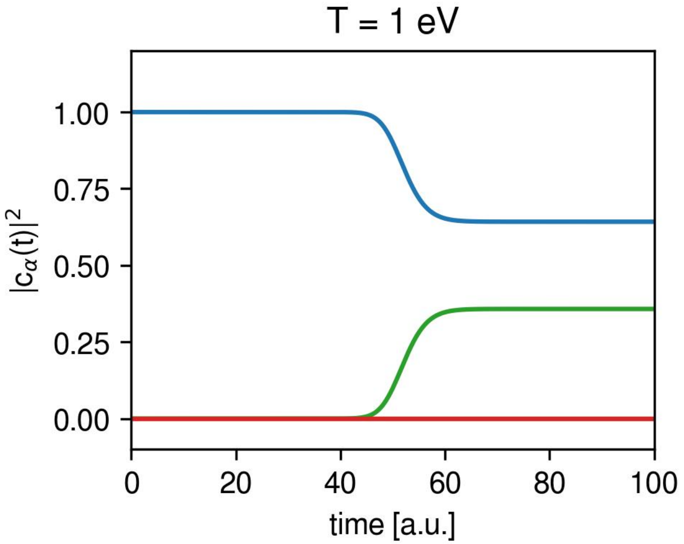
$$n(r) = \delta(r) - \frac{1}{\pi}e^{-2r}$$

$$\mathrm{F}(\mathrm{R}(t)) = \frac{e^{-2\mathrm{R}(t)}}{\mathrm{R}^2(t)}[1 + 2\mathrm{R}(t) + 2\mathrm{R}^2(t)]$$

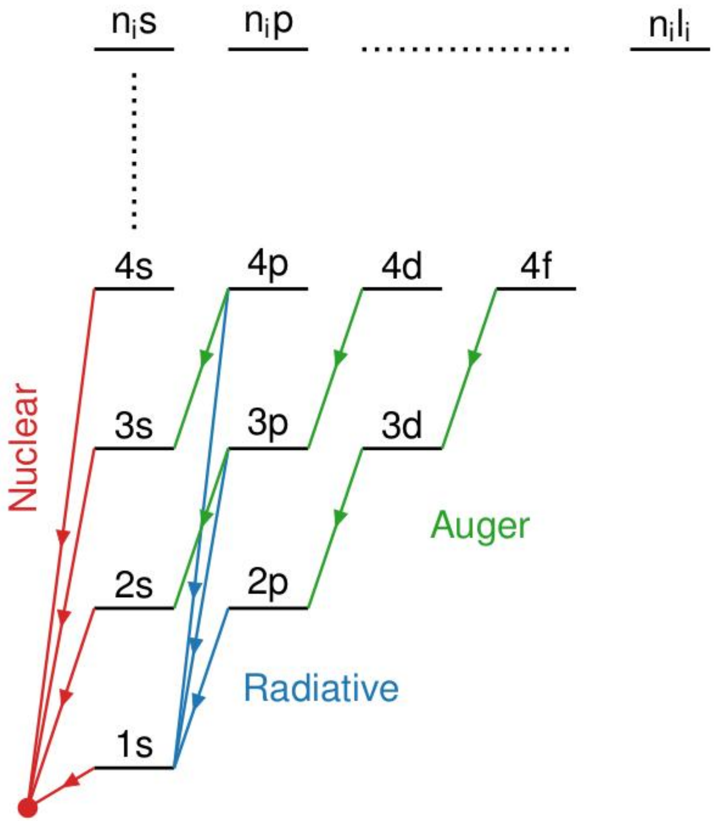


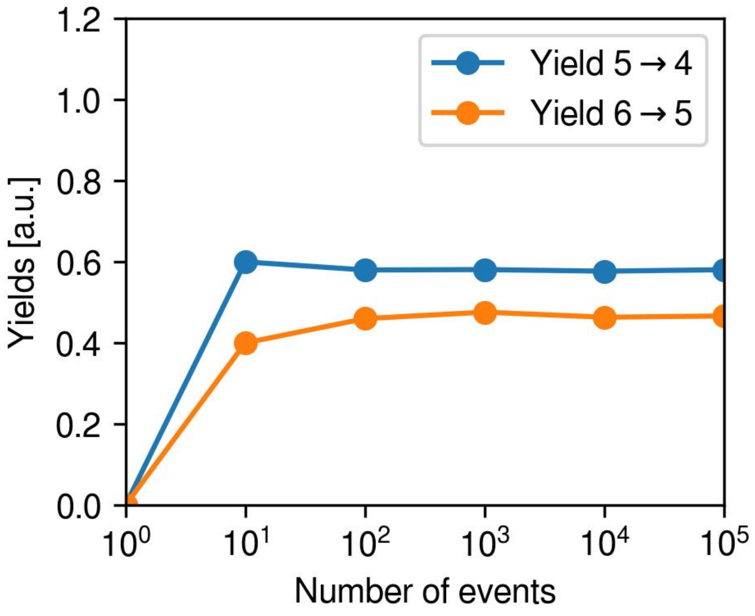


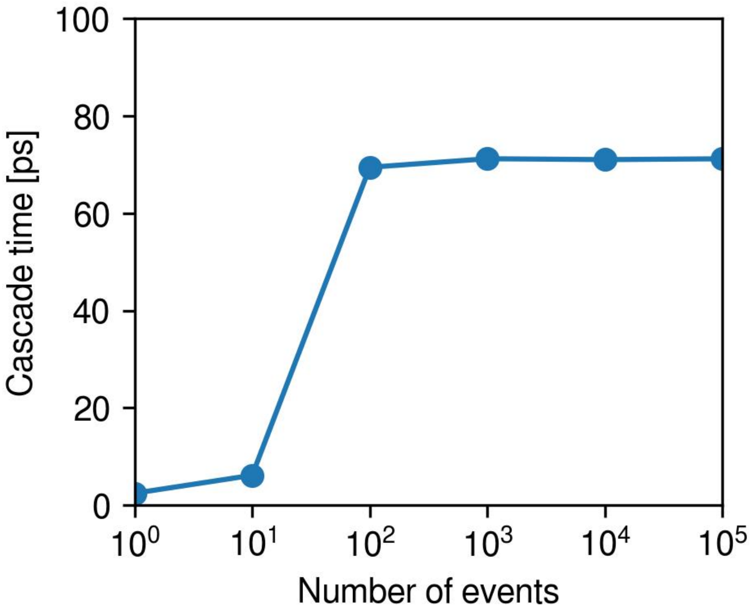




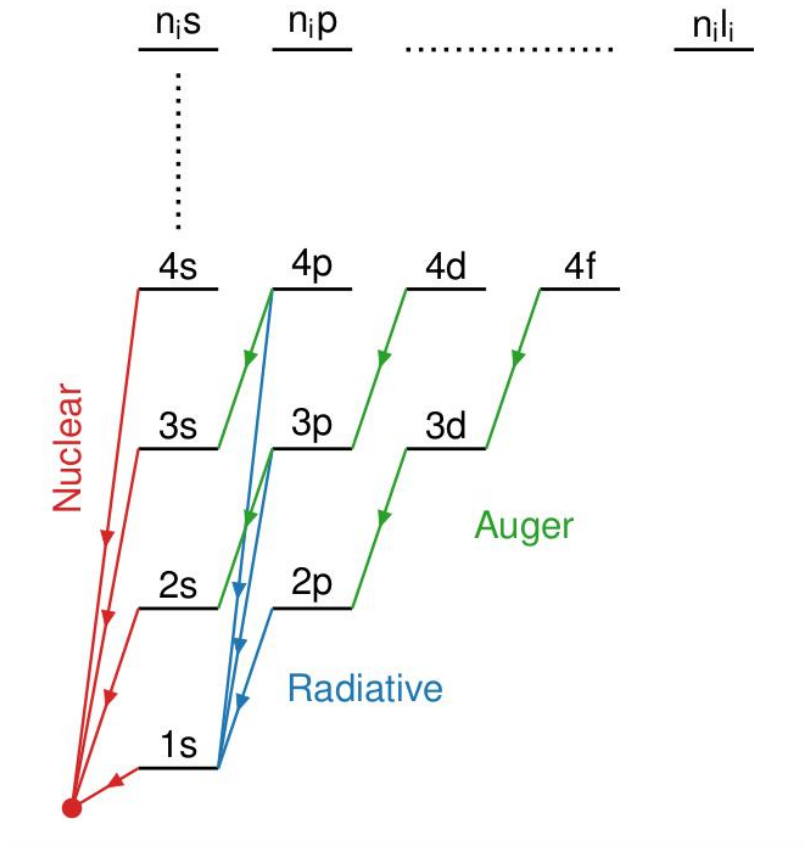
The atomic cascade is converged after 10^5 events

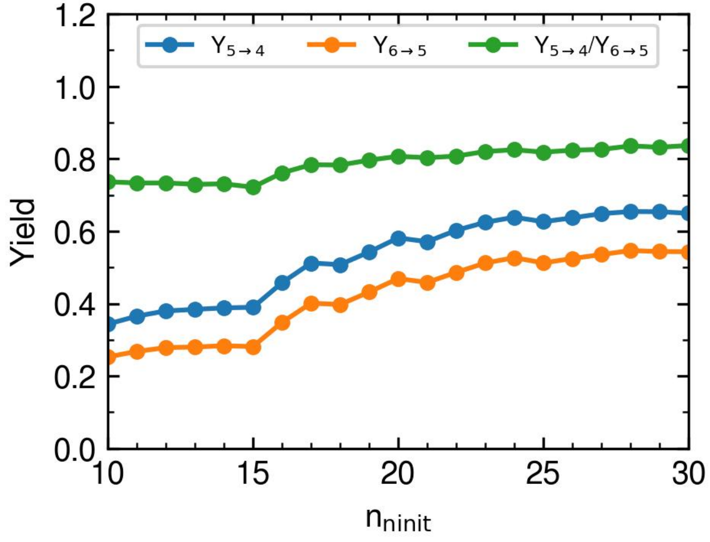


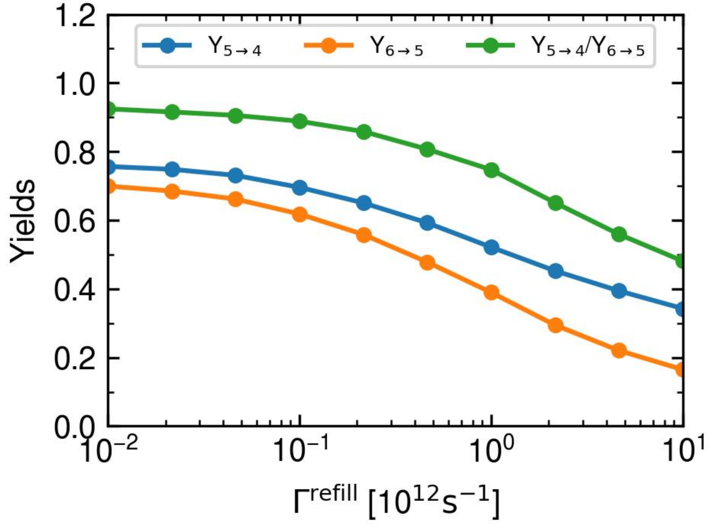


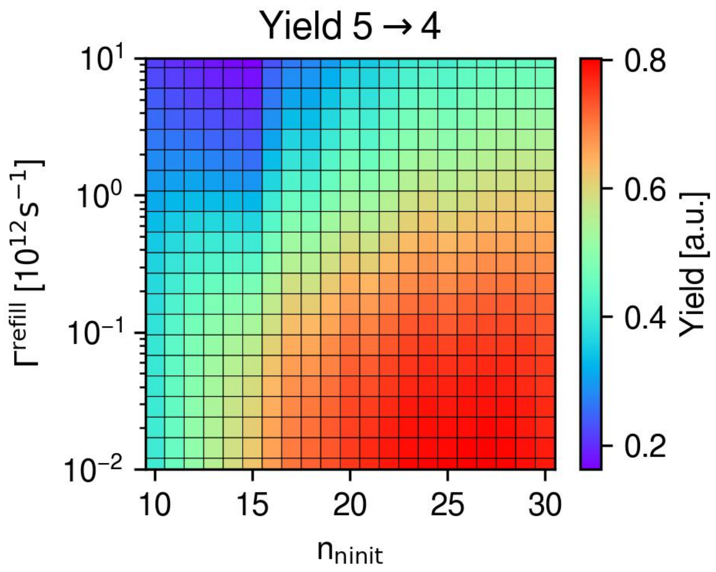


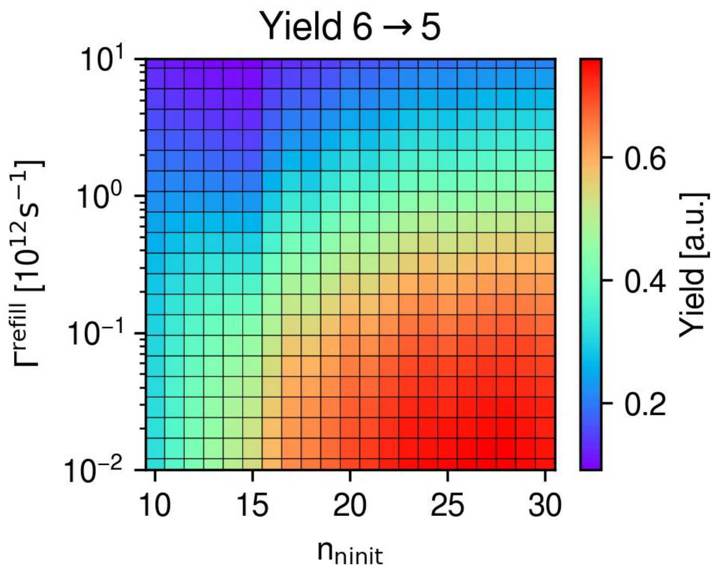
Yields of interest as function of the n_{init} and Γ^{refill}



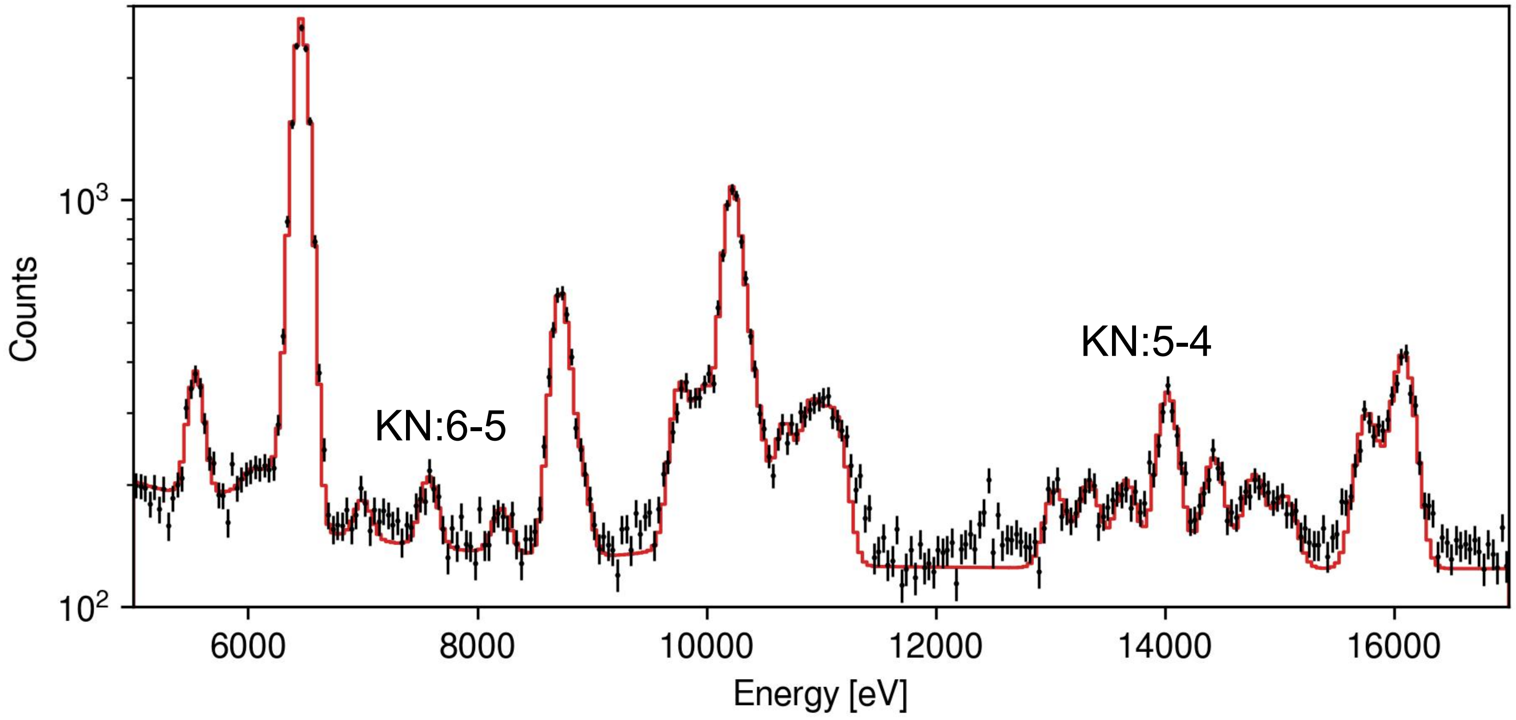




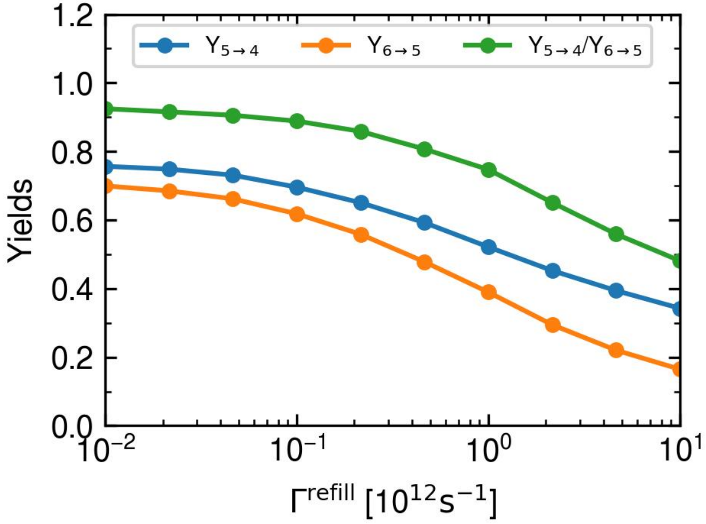


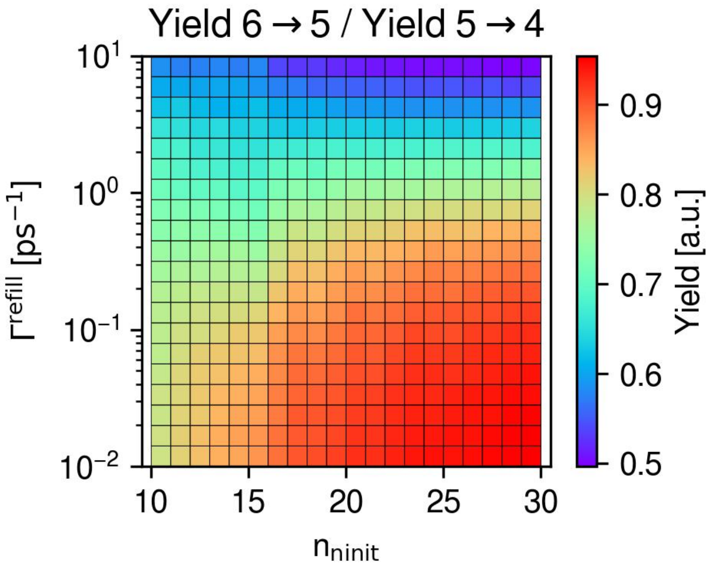


Cascade results: comparison with experiment

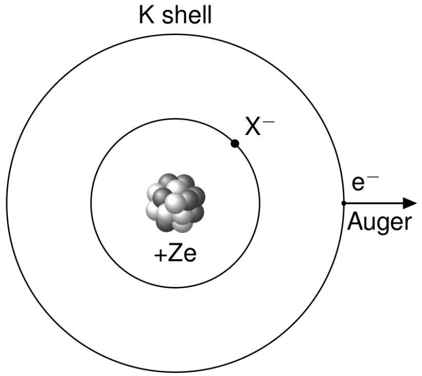


Experimental Yield 6,5 / Yields 5,4 = 0.29

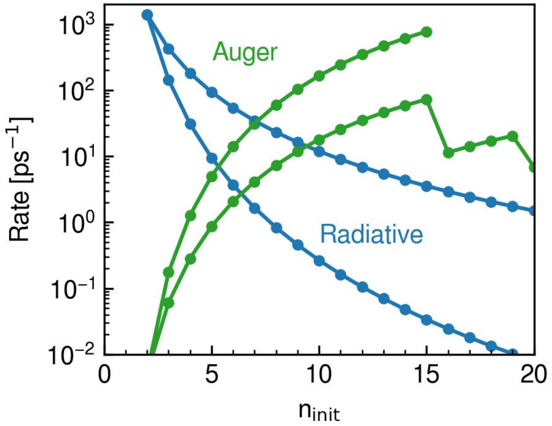




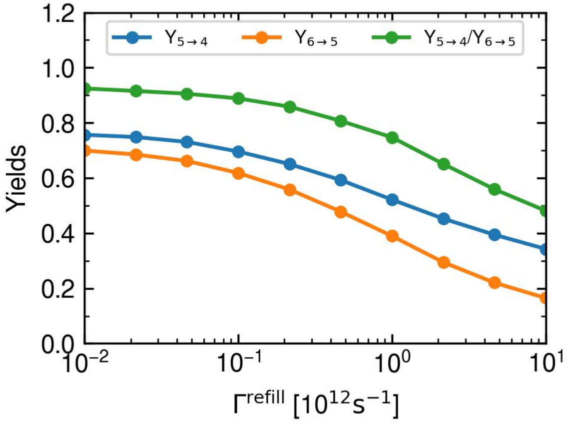
Conclusion



1. Cascade models to connect theory and experiment

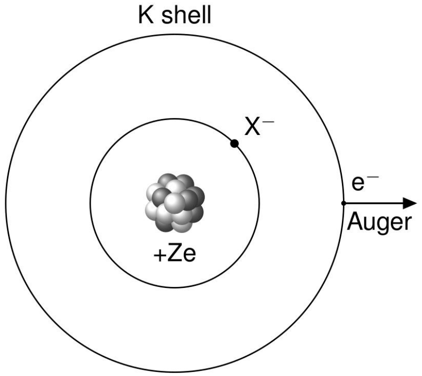


2. Different rates for different mechanisms

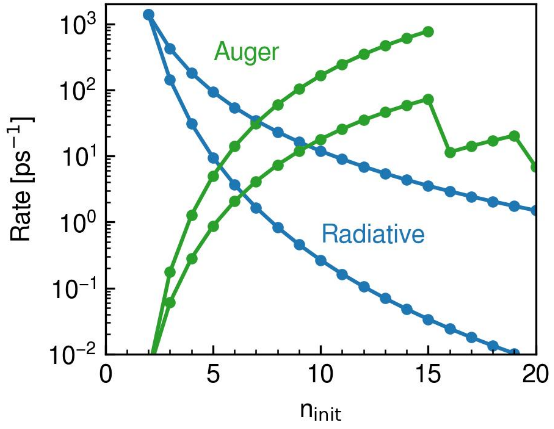


3. Comparison with the experimental yields

Conclusion



1. Cascade models to connect theory and experiment



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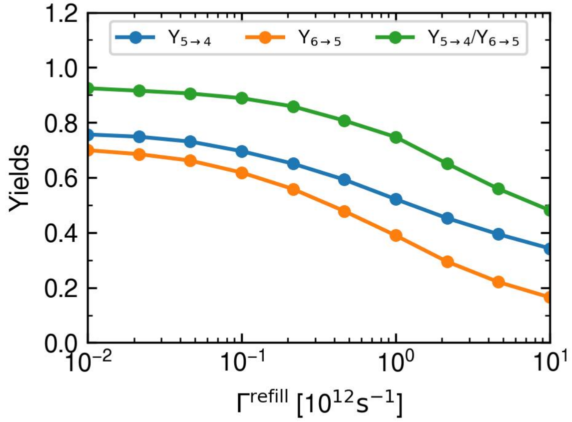

3. Comparison with the experimental yields

Thanks for the attention!