

**EXOTICO: EXOTIc atoms meet nuclear Collisions  
for a new frontier precision era in low-energy strangeness nuclear physics**  
ECT\*, 17 October, 2022

arXiv:2209.08254 [nucl-ex]

# $\pi\Sigma$ mass spectra measured in $d(K^-, N)\pi\Sigma$ reactions

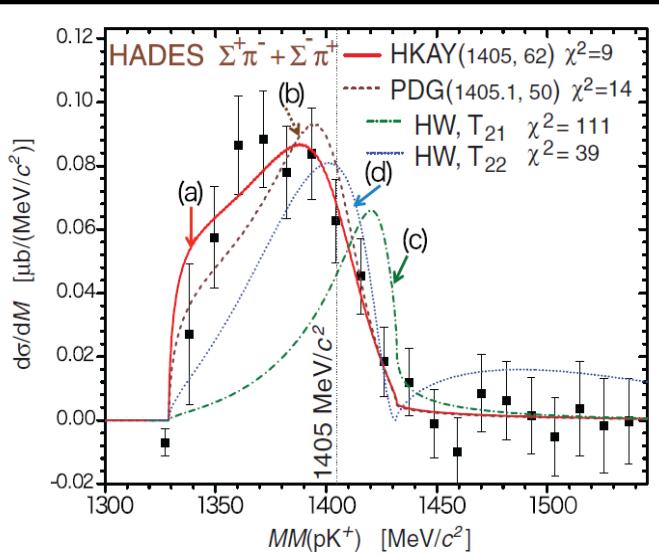
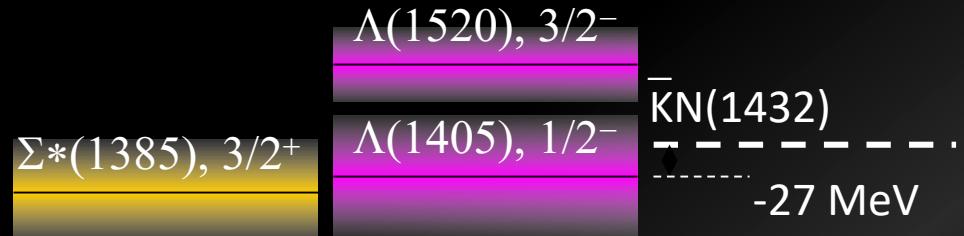
Hiroyuki Noumi<sup>\*,#</sup> for the J-PARC E31 collaboration

*\* RCNP, Osaka University*

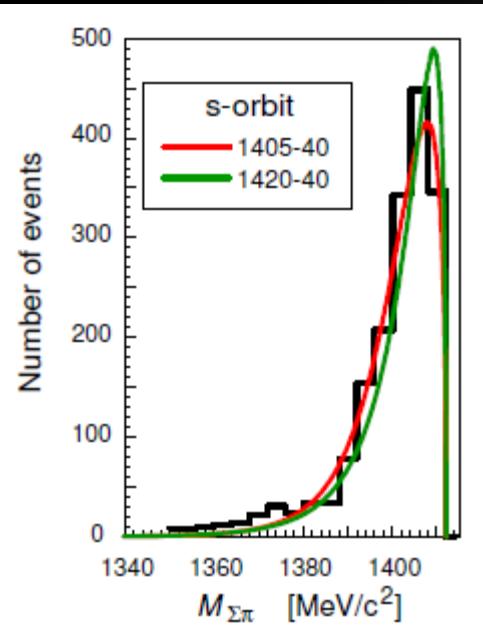
*# Institute of Particle and Nuclear Studies, KEK*

# $\Lambda(1405) : 1405.1^{+1.3}_{-0.9} \text{ MeV}$ (PDG in 2022)

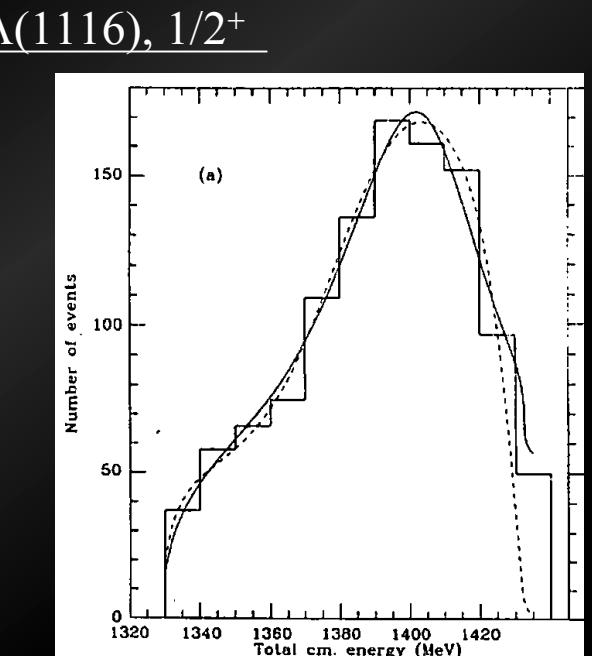
$J^P = \frac{1}{2}^-$ ,  $I = 0$ ,  $M_{\Lambda(1405)} < M_{K\bar{N}}$ , lightest in neg. parity baryons



M. Hassanvand et al:  $\pi\Sigma$  IM  
Spec. of  $p p \rightarrow K^+ \pi \Sigma$



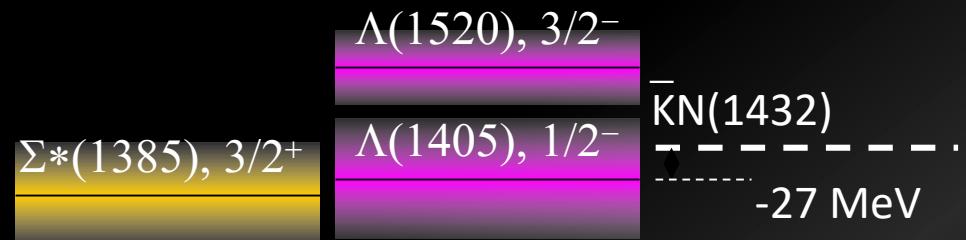
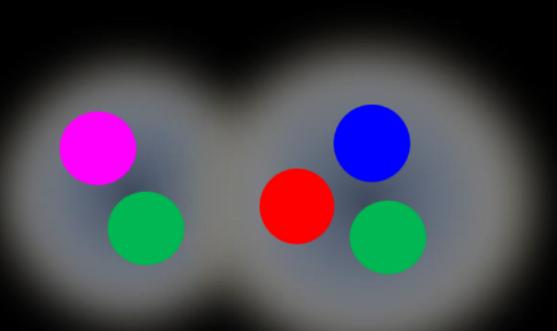
J. Esmaili et al:  $\pi\Sigma$  IM Spec. of  
Stopped  $K^-$  on  ${}^4\text{He}$



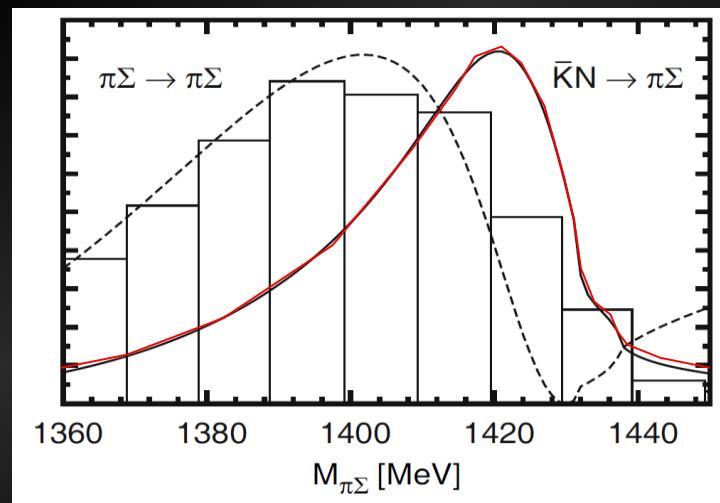
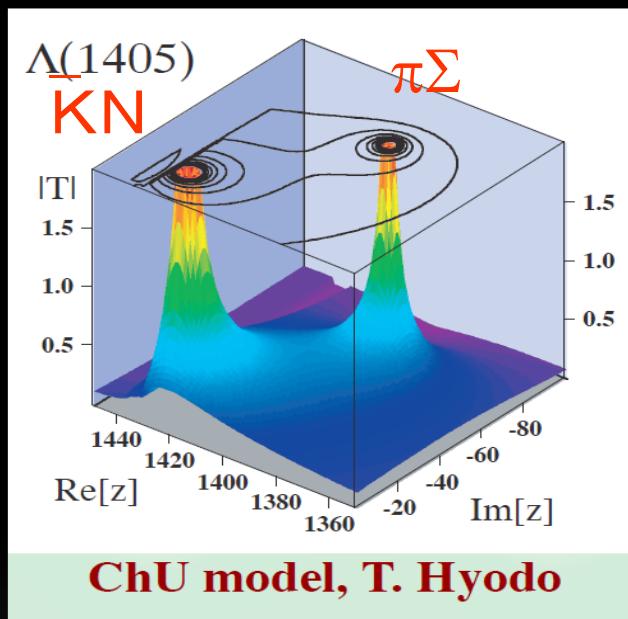
R.H. Dalitz et al:  $\pi\Sigma$  IM Spec.  
 ${}^2$   
in  $K-p \rightarrow \pi\pi\Sigma$  w/ M-matrix

# $\Lambda(1405)$ : Double pole?

$J^P = \frac{1}{2}^-$ ,  $I = 0$ ,  $M_{\Lambda(1405)} < M_{K\bar{N}}$ , lightest in neg. parity baryons



$$\frac{\Sigma(1192), 1/2^+}{\Lambda(1116), 1/2^+}$$



Chiral Unitary Model:  
D. Jido et al., NPA725(03)181

# Pole Structure of the Lambda(1405) Region

PDG Reviews: Ulf-G. Meissner and T. Hyodo (since Nov. 2015)

**Table 1:** Comparison of the pole positions of  $\Lambda(1405)$  in the complex energy plane from next-to-leading order chiral unitary coupled-channel approaches including the SIDDHARTA constraint.

approach	pole 1 [MeV]	pole 2 [MeV]
Refs. 11,12, NLO	$1424^{+7}_{-23} - i \ 26^{+3}_{-14}$	$1381^{+18}_{-6} - i \ 81^{+19}_{-8}$
Ref. 14, Fit II	$1421^{+3}_{-2} - i \ 19^{+8}_{-5}$	$1388^{+9}_{-9} - i \ 114^{+24}_{-25}$
Ref. 15, solution #2	$1434^{+2}_{-2} - i \ 10^{+2}_{-1}$	$1330^{+4}_{-5} - i \ 56^{+17}_{-11}$
Ref. 15, solution #4	$1429^{+8}_{-7} - i \ 12^{+2}_{-3}$	$1325^{+15}_{-15} - i \ 90^{+12}_{-18}$

$\Lambda(1405) : 1405.1^{+1.3}_{-1.0}$  MeV (Part. Listing in '22)

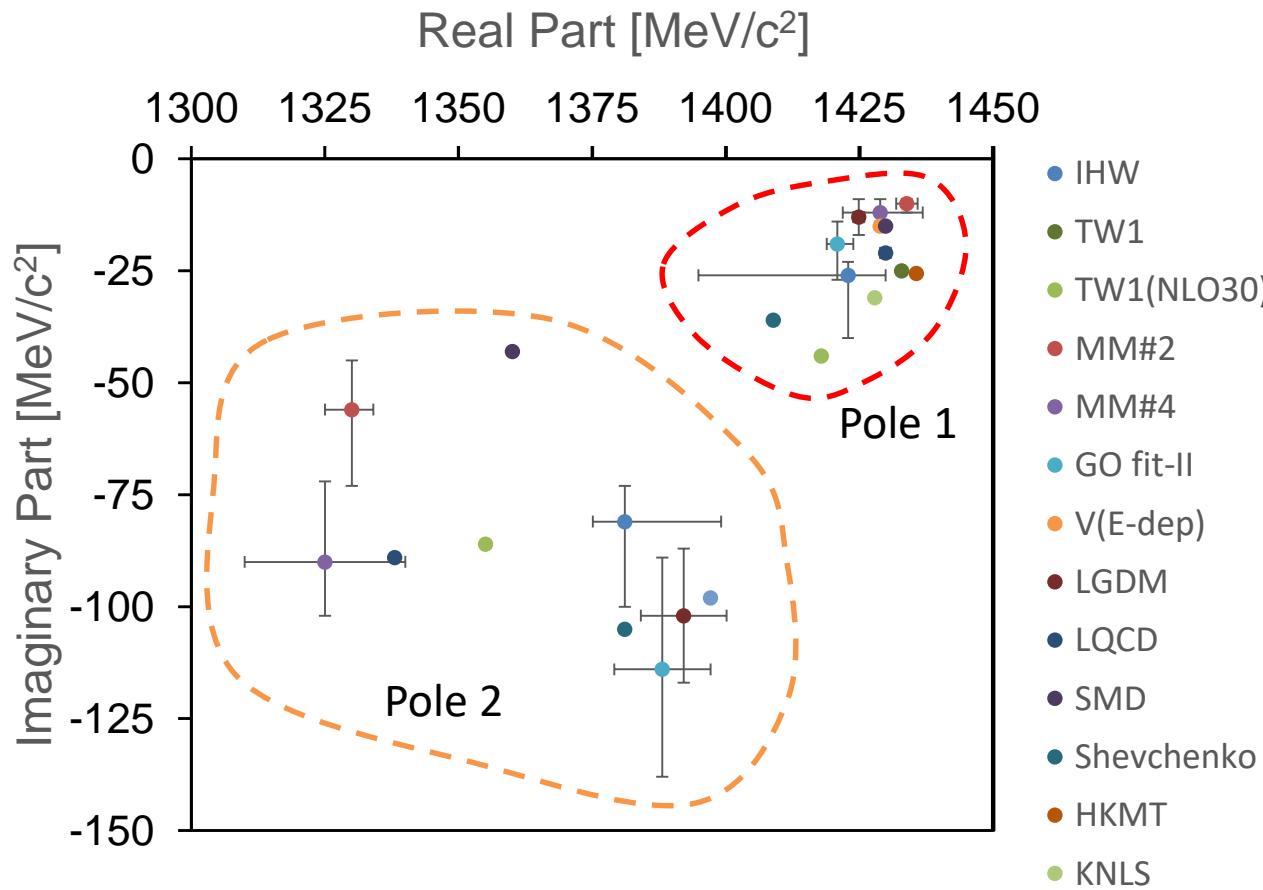
$J^P = \frac{1}{2}^-$ ,  $I = 0$ ,  $M_{\Lambda(1405)} < M_{K\bar{N}}$ , lightest in neg. parity baryons

M. Hassanvand et al:  $\pi\Sigma$  IM  
Spec. of  $p\bar{p} \rightarrow K^+\pi\Sigma$

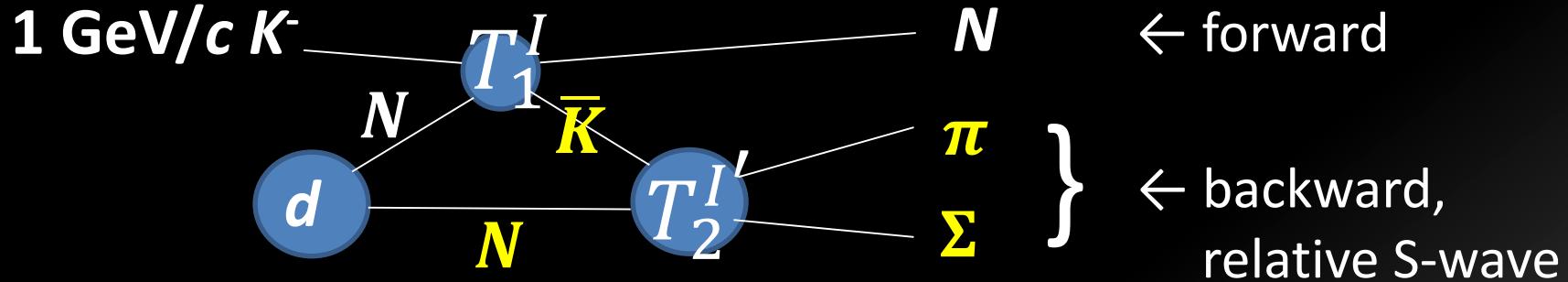
J. Esmaili et al:  $\pi\Sigma$  IM Spec. of  
Stopped  $K^-$  on  ${}^4\text{He}$

R.H. Dalitz et al:  $\pi\Sigma$  IM Spec.  
in  $K-p \rightarrow \pi\pi\Sigma$  w/ M-matrix

# Two-pole structure of Lambda(1405) in Meson-Baryon dynamics



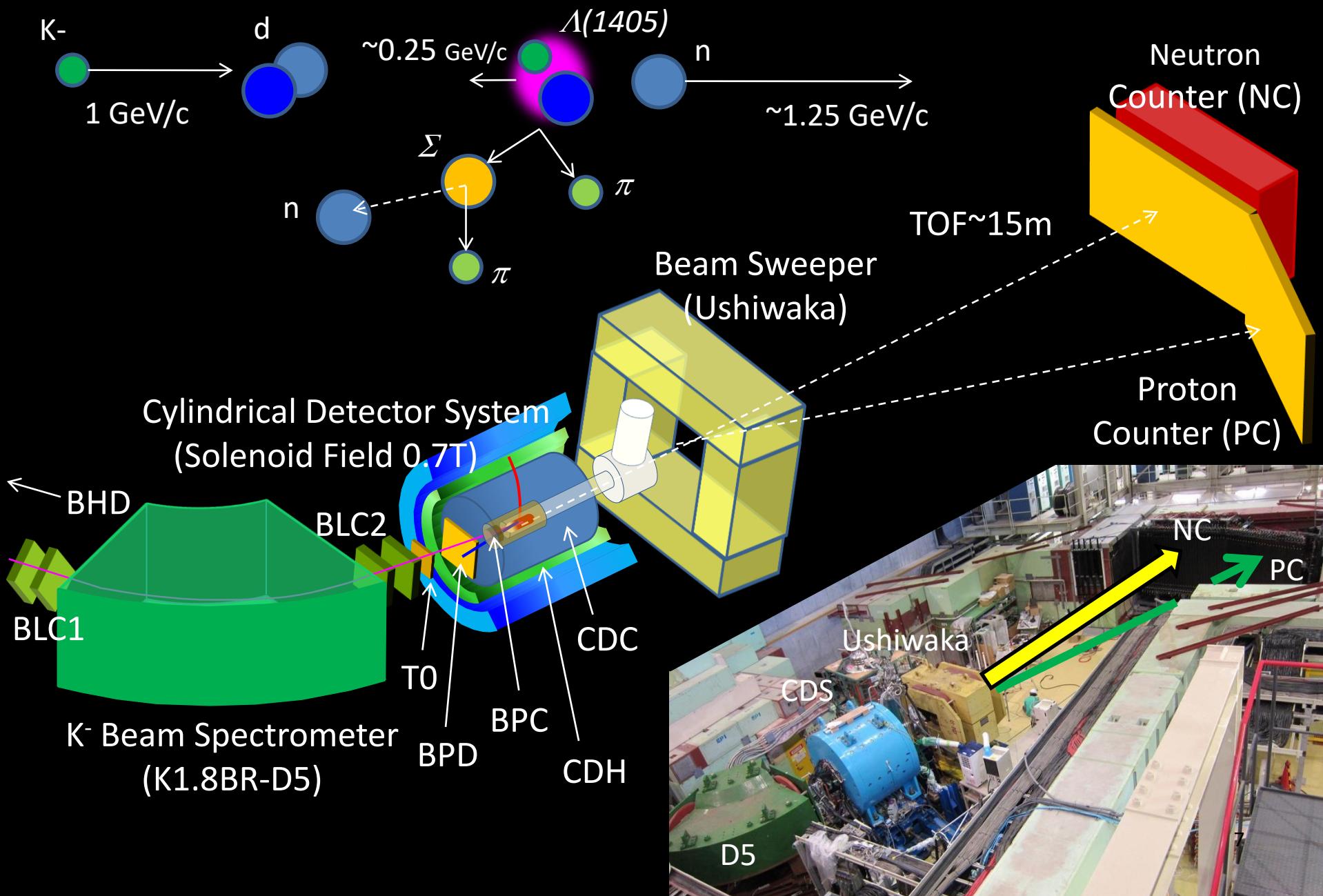
# $K^{\bar{N}}$ scattering below the $K^{\bar{N}}$ thres. (J-PARC E31)



- measuring an **S-wave**  $\bar{K}N \rightarrow \pi\Sigma$  scattering below the  $\bar{K}N$  threshold in the  $d(K^-, n)\pi\Sigma$  reactions at a forward angle of  $N$ .
- ID's all the final states to decompose the  $I=0$  and  $1$  ampl's.

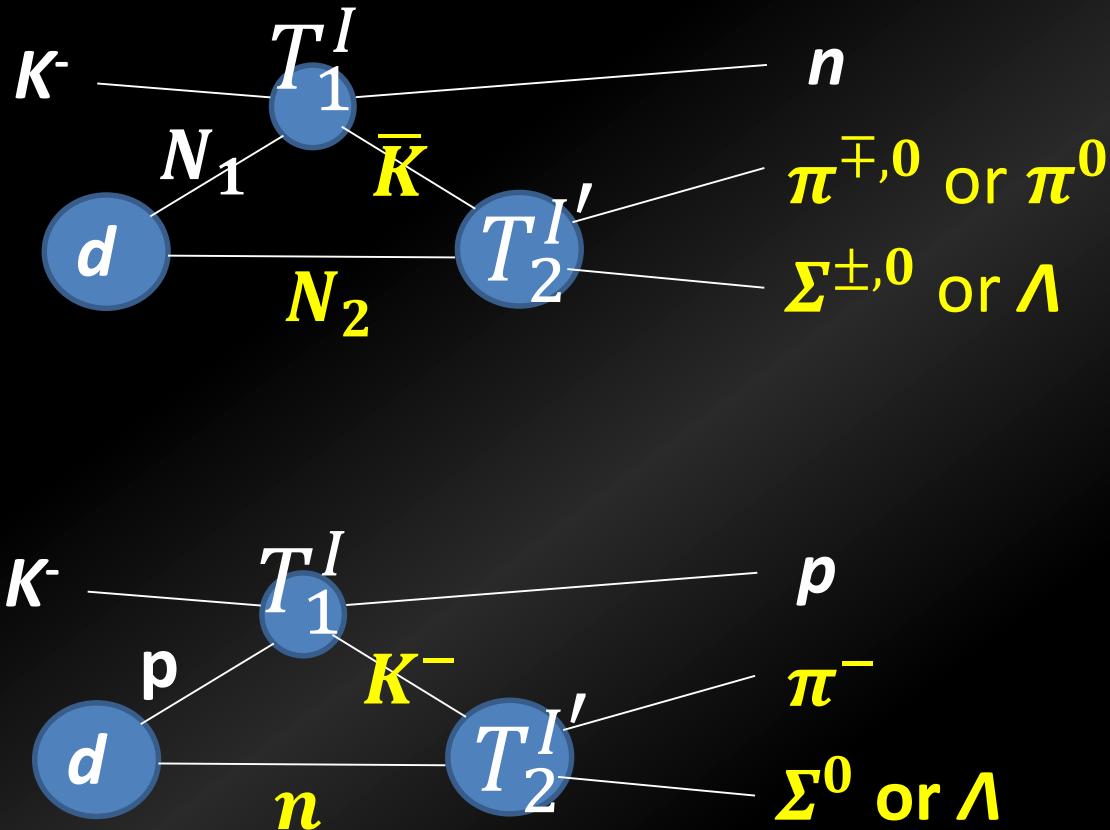
Fwd $N$	$\pi\Sigma$ mode	Isospin	Expected resonance
$n$	$\pi^\pm\Sigma^\mp$	0, 1	$\Lambda(1405)$ interference btw $I=0$ and $1$ ampl's.
$p$	$\pi^-\Sigma^0$	1	P-wave $\Sigma^*(1385)$ to be suppressed
$n$	$\pi^0\Sigma^0$	0	$\Lambda(1405)$

# Experimental Setup for E31

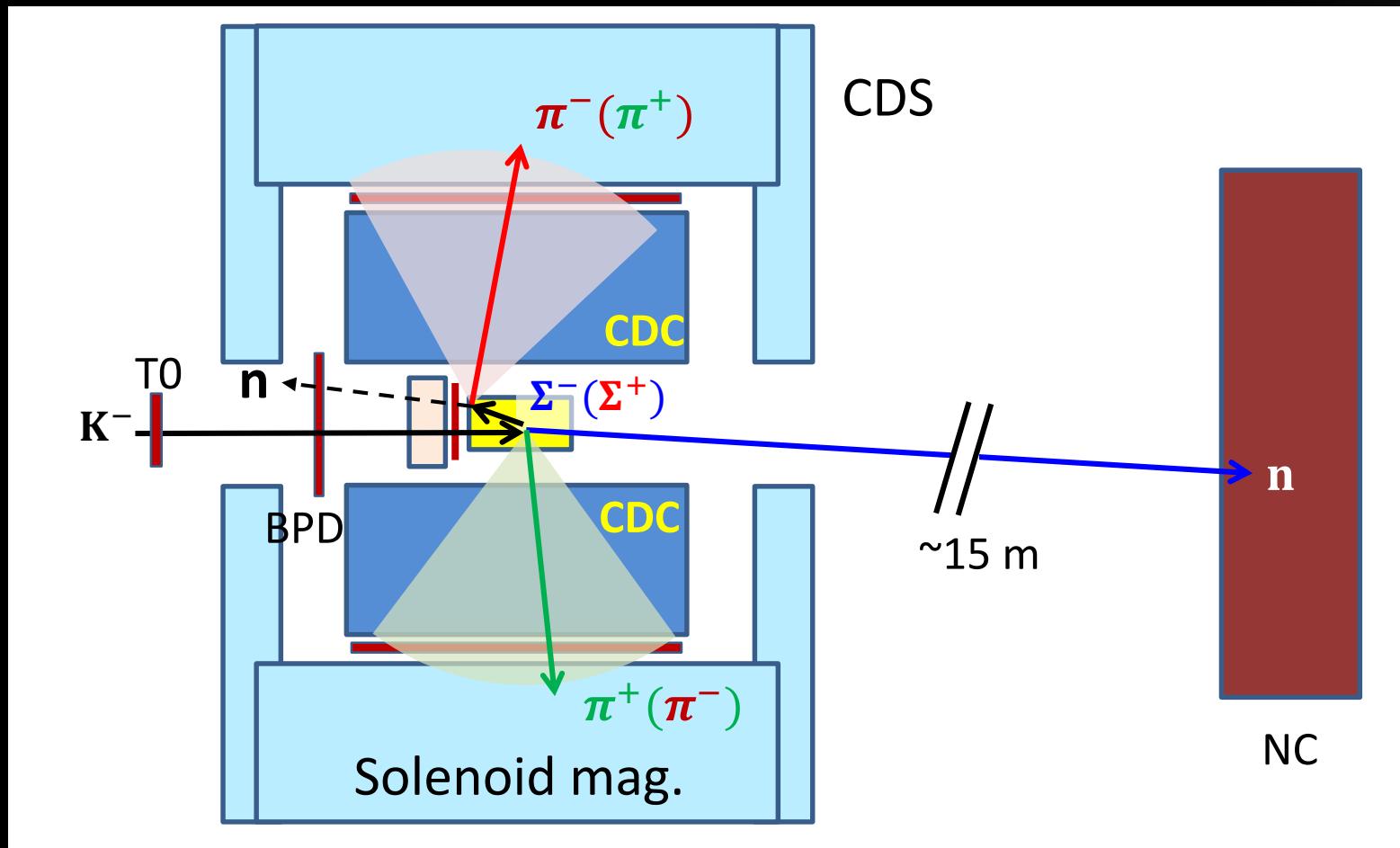


# missing $\pi\Sigma/\pi\Lambda$ mass spectra

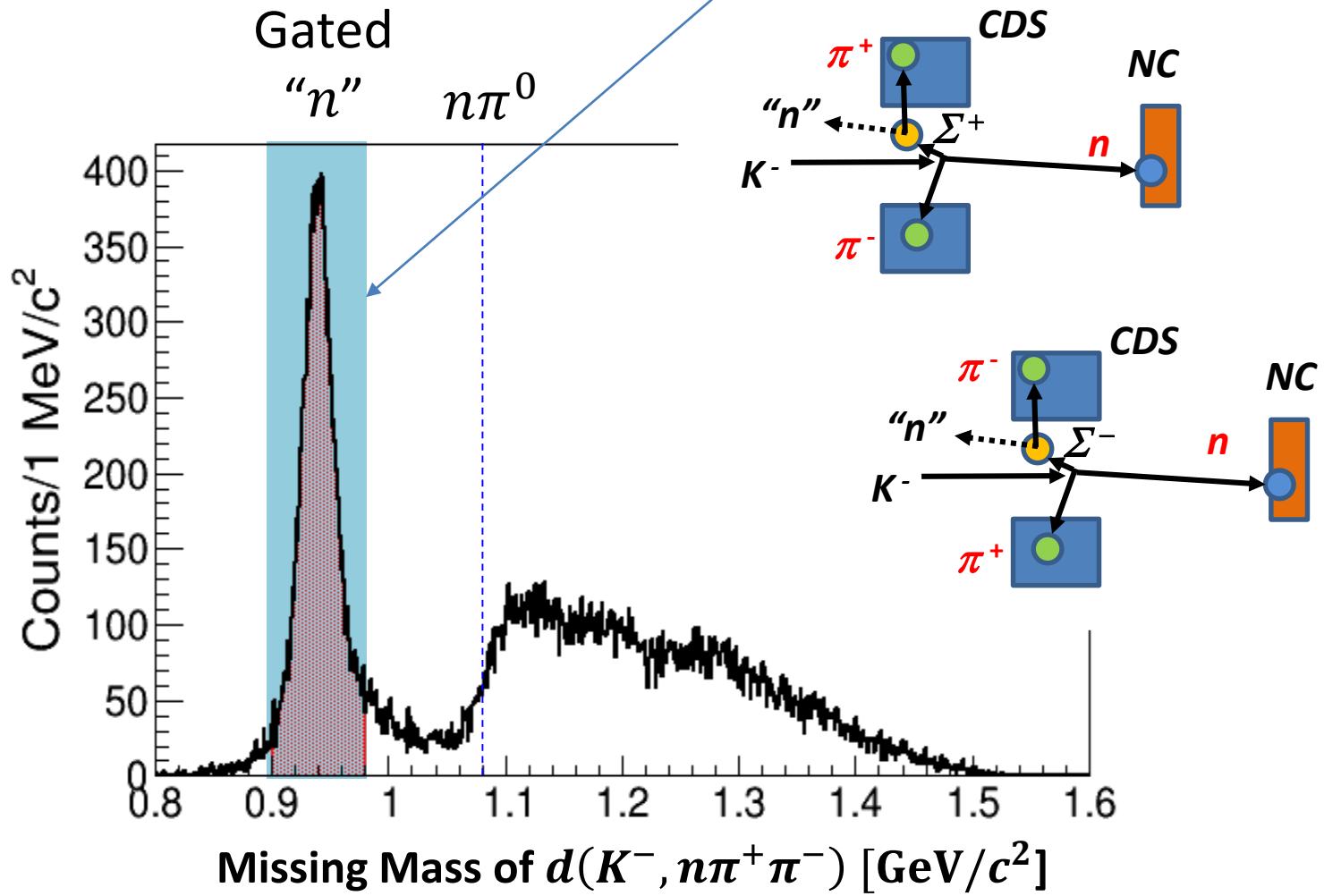
- $d(K^-, n)X_{\pi^\pm\Sigma^\mp}$
- $d(K^-, n)X_{\pi^0\Sigma^0}$
- $d(K^-, p)X_{\pi^-\Sigma^0}$
- $d(K^-, p)X_{\pi^-\Lambda}$
- $d(K^-, n)X_{\pi^0\Lambda}$



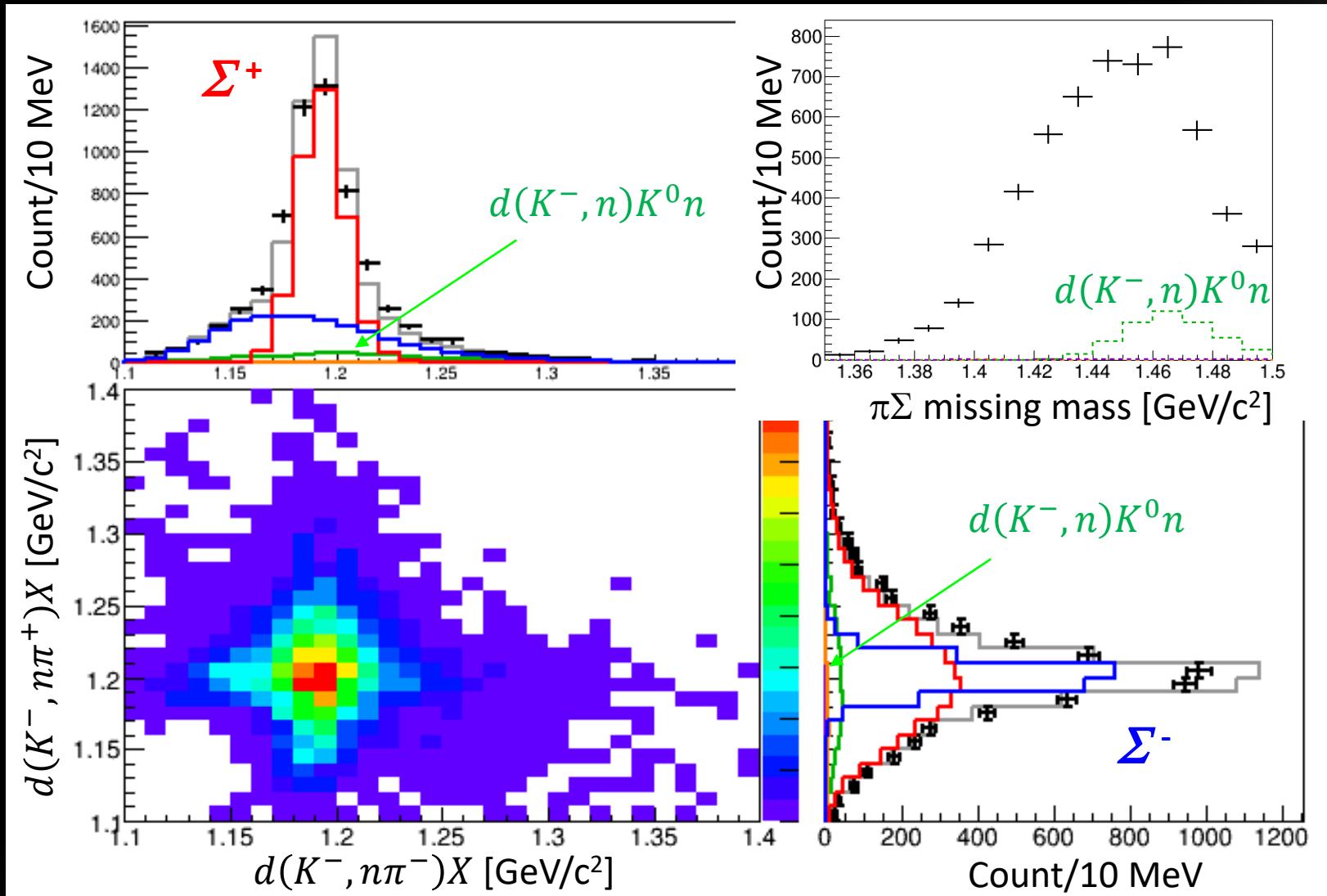
# Event topology of $d(K^-, n)X_{\pi^\pm\Sigma^\mp}$



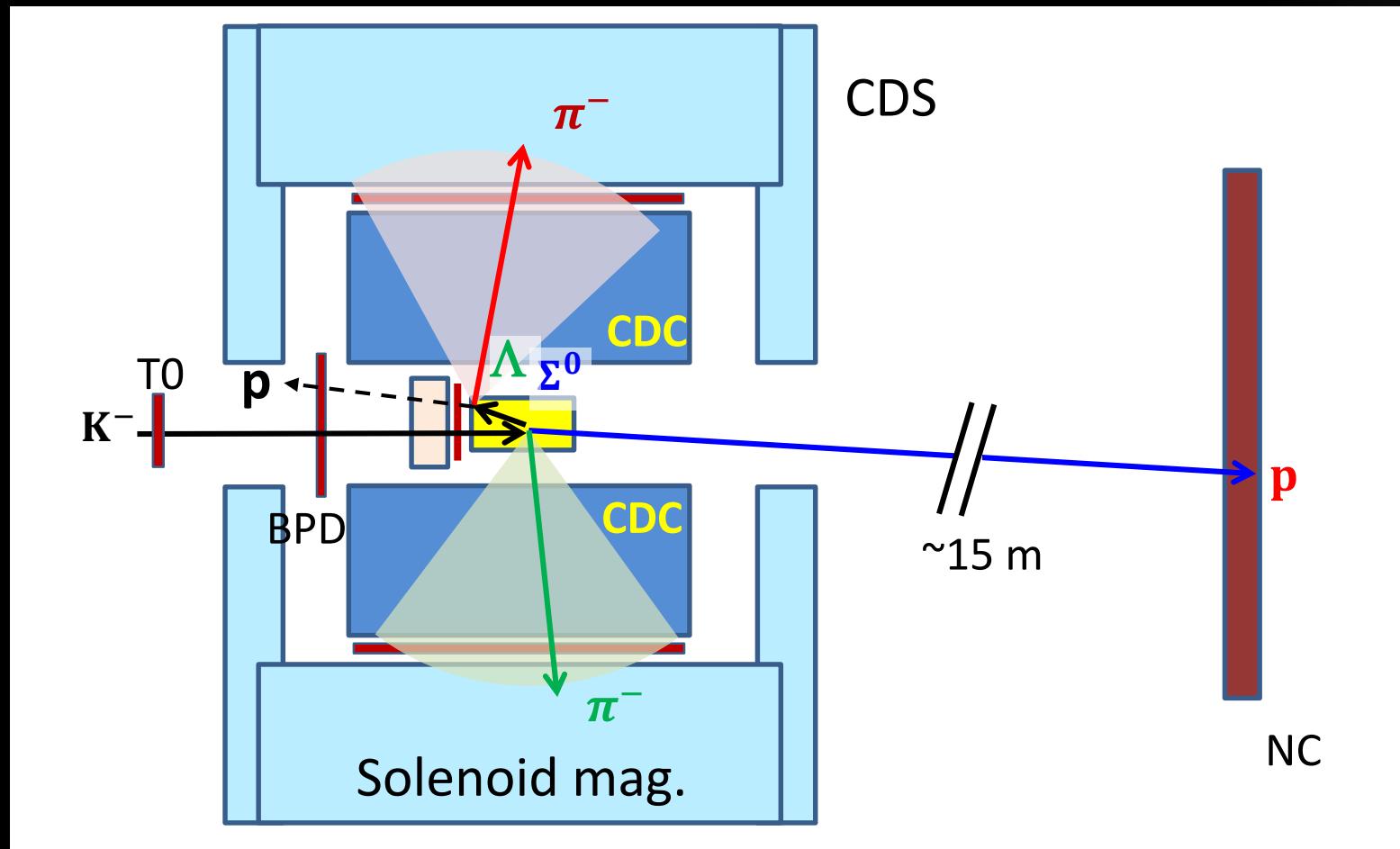
$d(K^-, n\pi^+\pi^-) \underline{n_{missing}}$



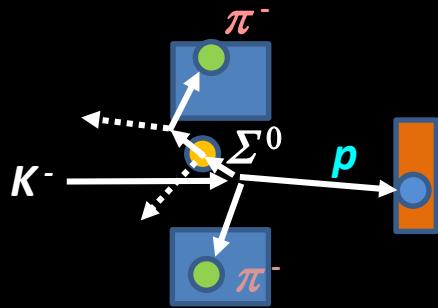
# $\pi^+\Sigma^-/\pi^-\Sigma^+$ Mode separation (template fitting, Run78)



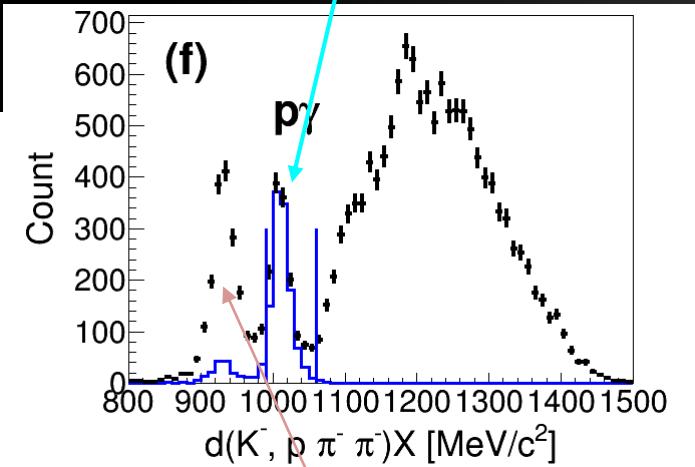
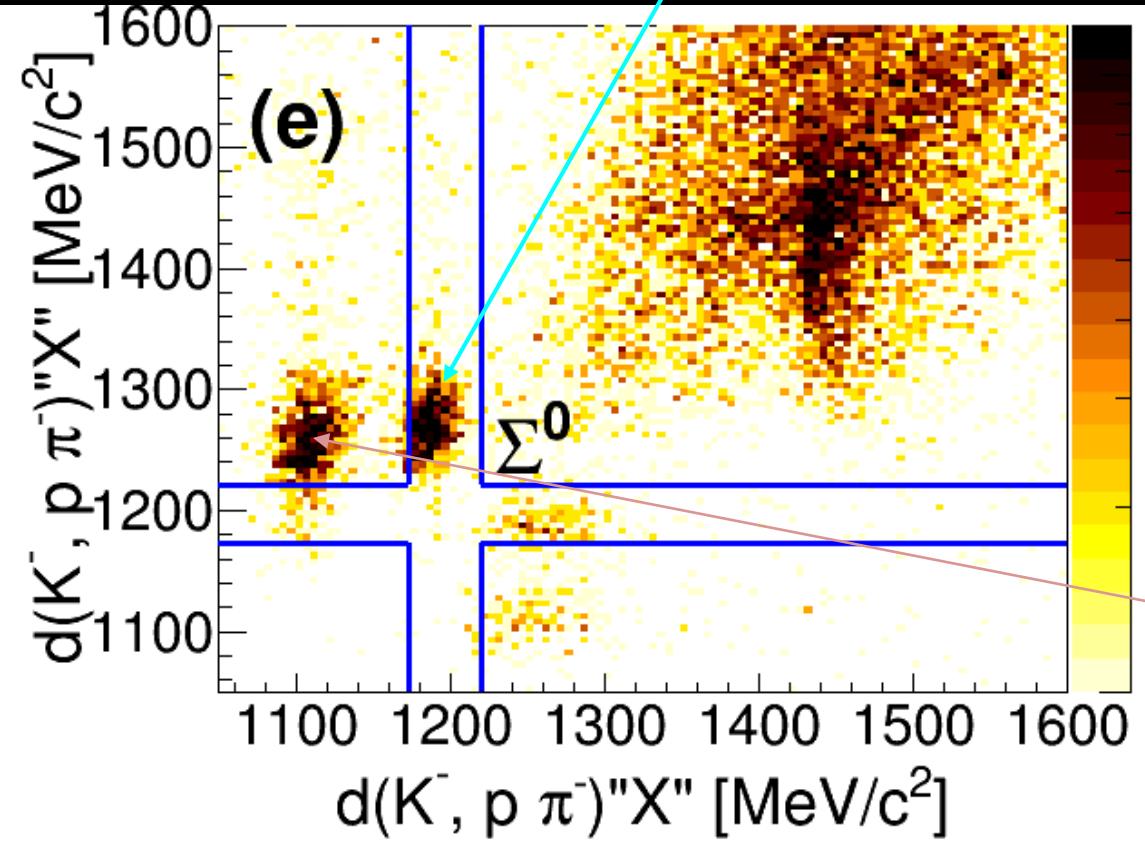
# Event topology of $d(K^-, p)X_{\pi^-\Sigma^0}$



# $d(K^-, p)X_{\pi^-\Sigma^0}$ Mode ( $I = 1$ )



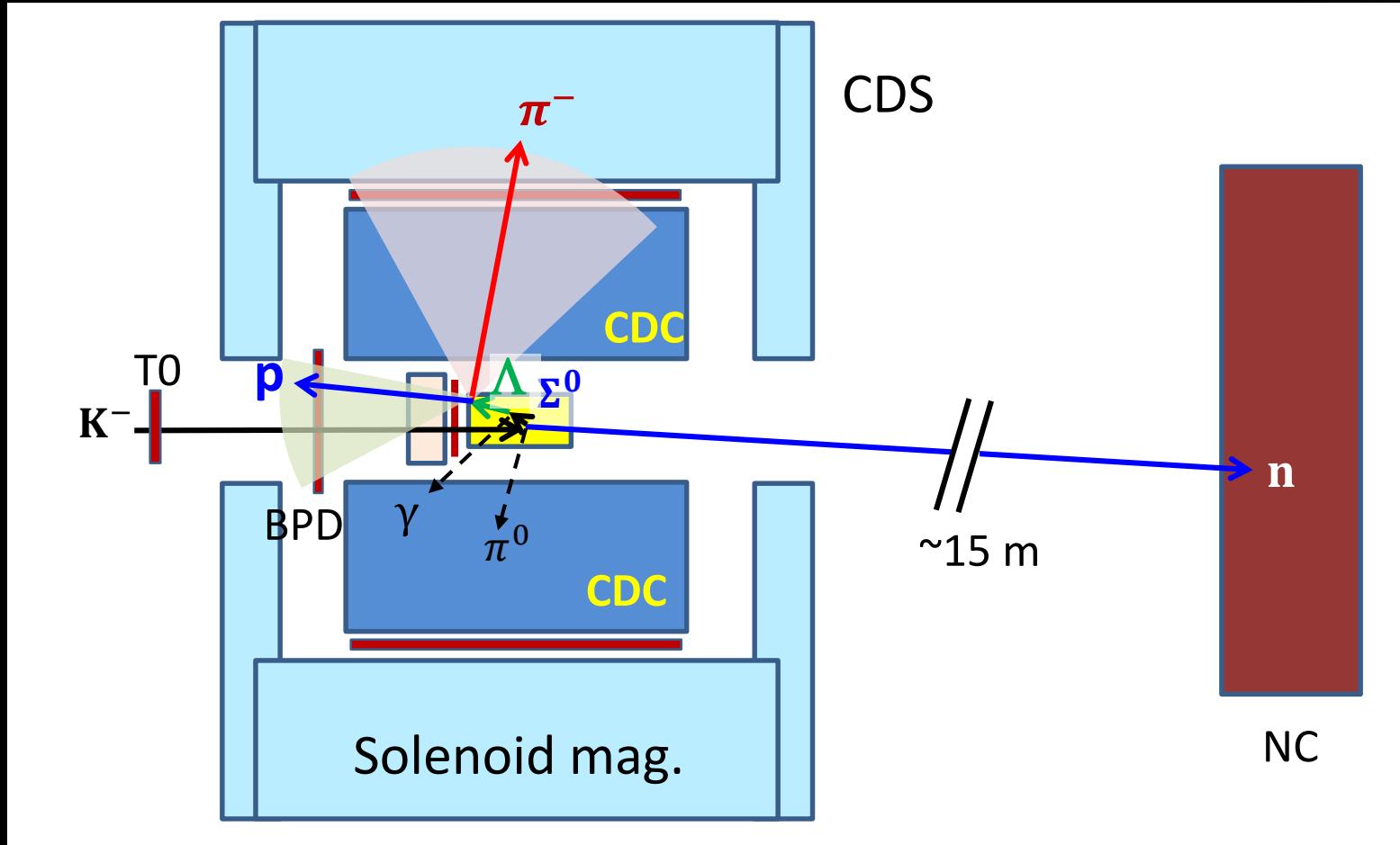
From  $d(K^-, p\pi^-\pi^-)"p\gamma"$  sample



$d(K^-, p\pi^-\pi^-)"p"$

$d(K^-, p)X_{\pi^-\Lambda}$

# Event topology of $d(K^-, n)X_{\pi^0 \Sigma^0}$

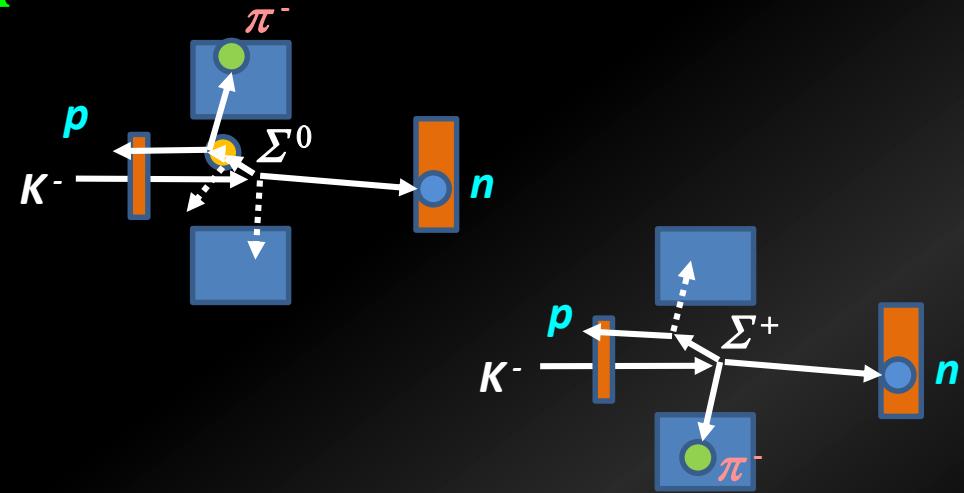
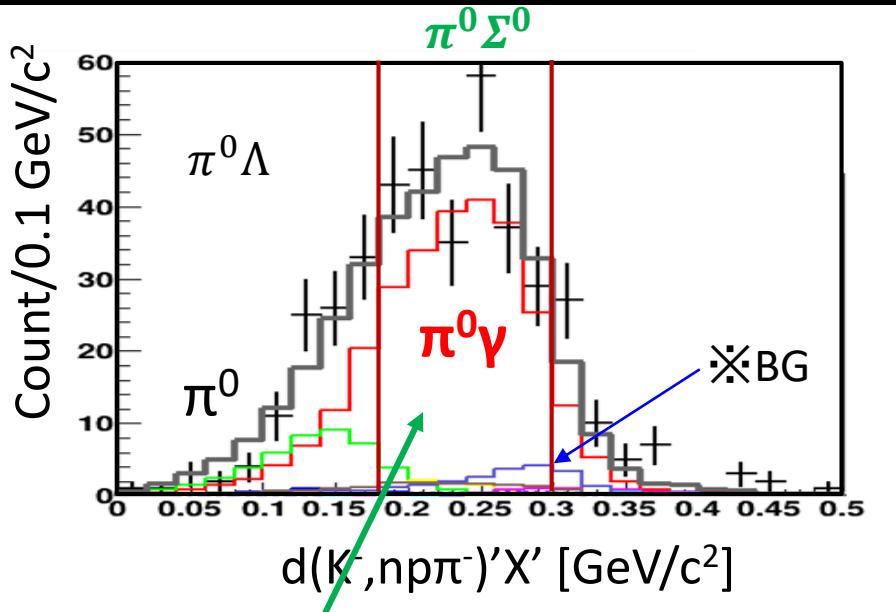


Other major process:  $d(K^-, n)X_{\pi^0 \Lambda}, d(K^-, n)X_{\pi^- \Sigma^+}$ ,  
Minor processes:  $d(K^-, n)X_{\pi^0 \pi^0 \Lambda}, d(K^-, Yp)X, \dots$

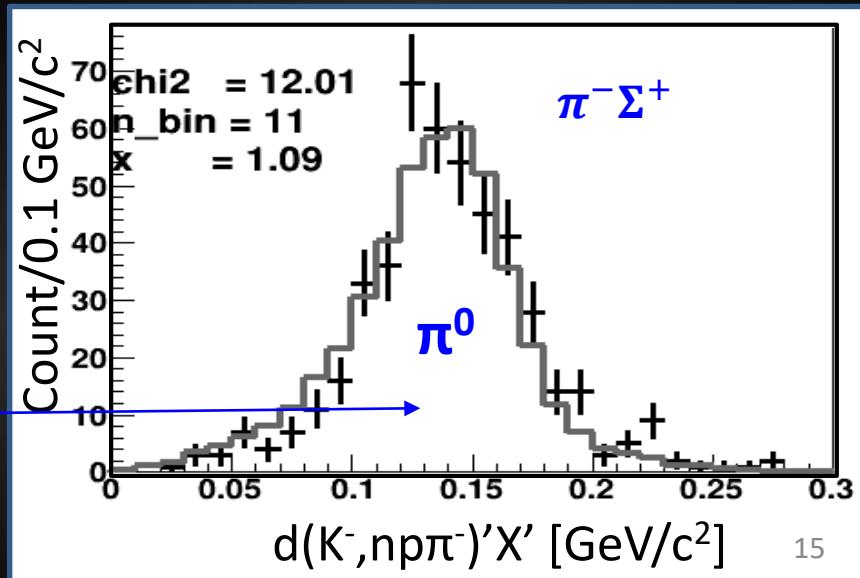
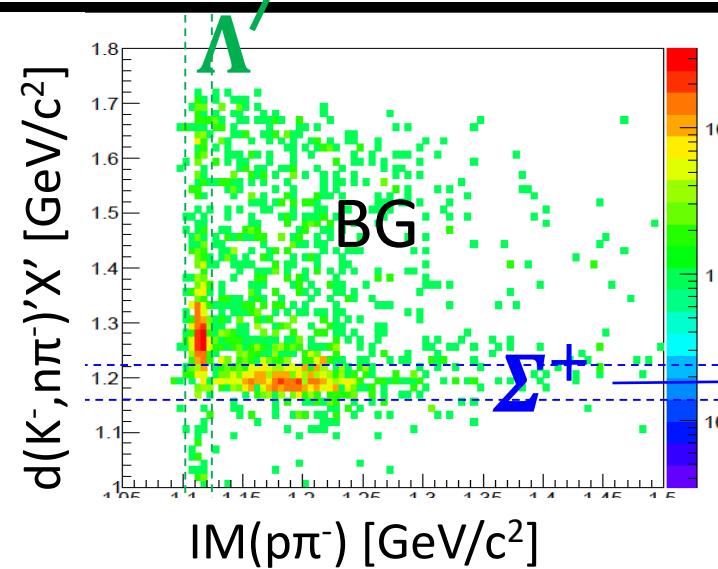
$d(K^-, n) \underline{\pi^0 \Sigma^0}$  vs  $d(K^-, n) \underline{\pi^- \Sigma^+}$

$\downarrow \pi^0 \gamma \Lambda$

$\downarrow \pi^- p \pi^0$

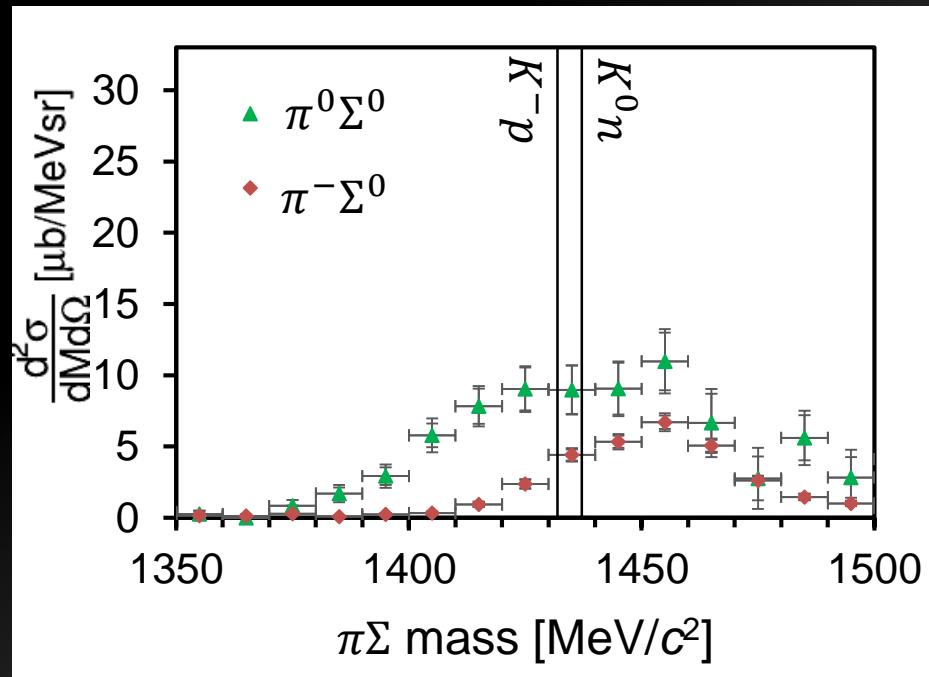
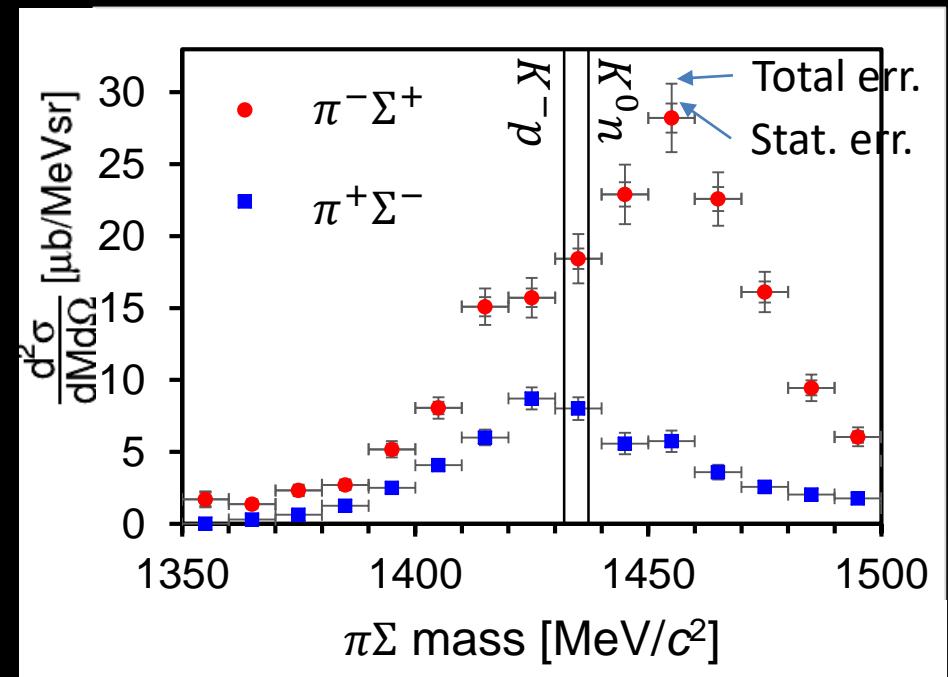


$\times$  BG: ( $K^- d \rightarrow p(\gamma\pi^-)$ , QF-K induced  $\gamma$  prod.)



# $\pi^+\Sigma^-/\pi^-\Sigma^+$ ( $I' = 0, 1$ )

# $\pi^0\Sigma^0(I' = 0)$ $\pi^-\Sigma^0(I' = 1)$

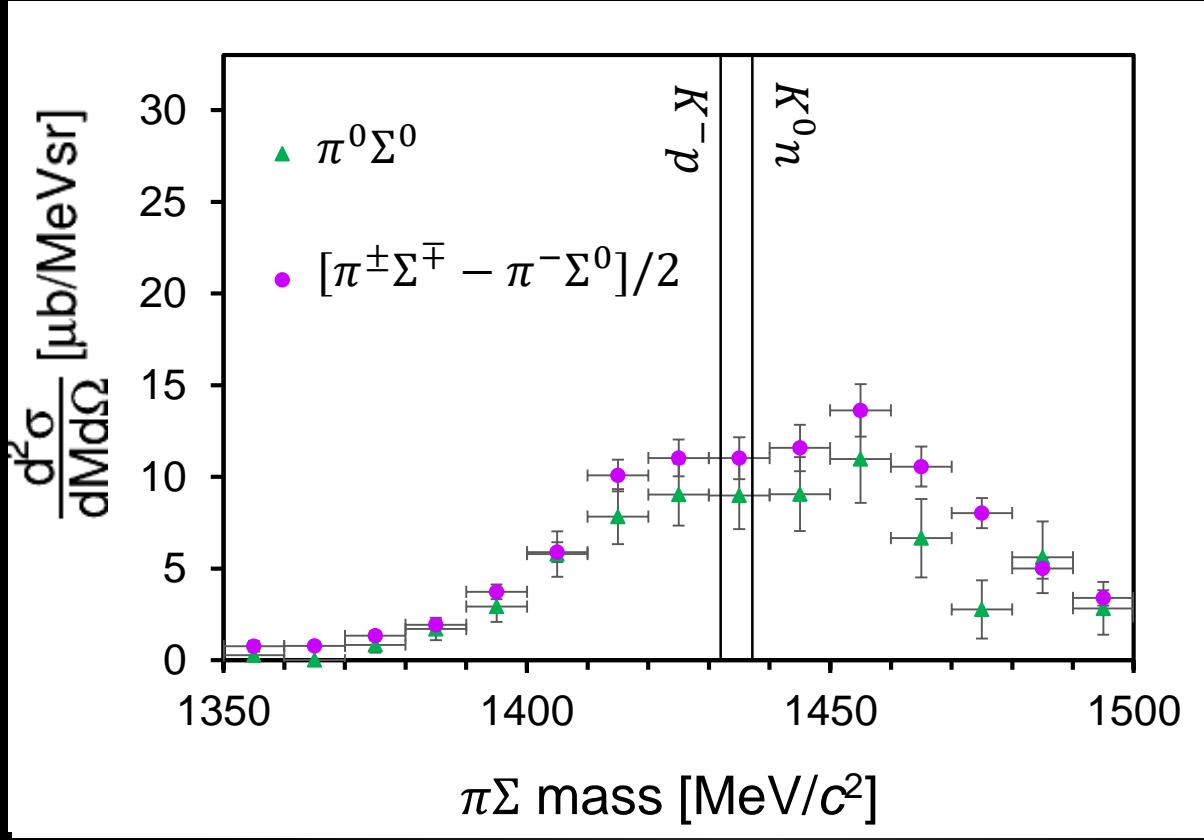


$$\frac{d\sigma}{d\Omega}(\pi^-\Sigma^+/\pi^+\Sigma^-) \propto \left| \frac{3T_1^{I=0} - T_1^{I=1}}{4\sqrt{3}} \textcolor{red}{T_2^{I'=0}} \pm \frac{T_1^{I=0} + T_1^{I=1}}{4\sqrt{2}} \textcolor{red}{T_2^{I'=1}} \right|^2$$

$$\frac{d\sigma}{d\Omega}(\pi^0\Sigma^0) \propto \left| -\frac{3T_1^{I=0} - T_1^{I=1}}{4\sqrt{3}} \textcolor{red}{T_2^{I'=0}} \right|^2$$

$$\frac{d\sigma}{d\Omega}(\pi^-\Sigma^0) \propto \left| -\frac{T_1^{I=0} + T_1^{I=1}}{4} \textcolor{red}{T_2^{I'=1}} \right|^2$$

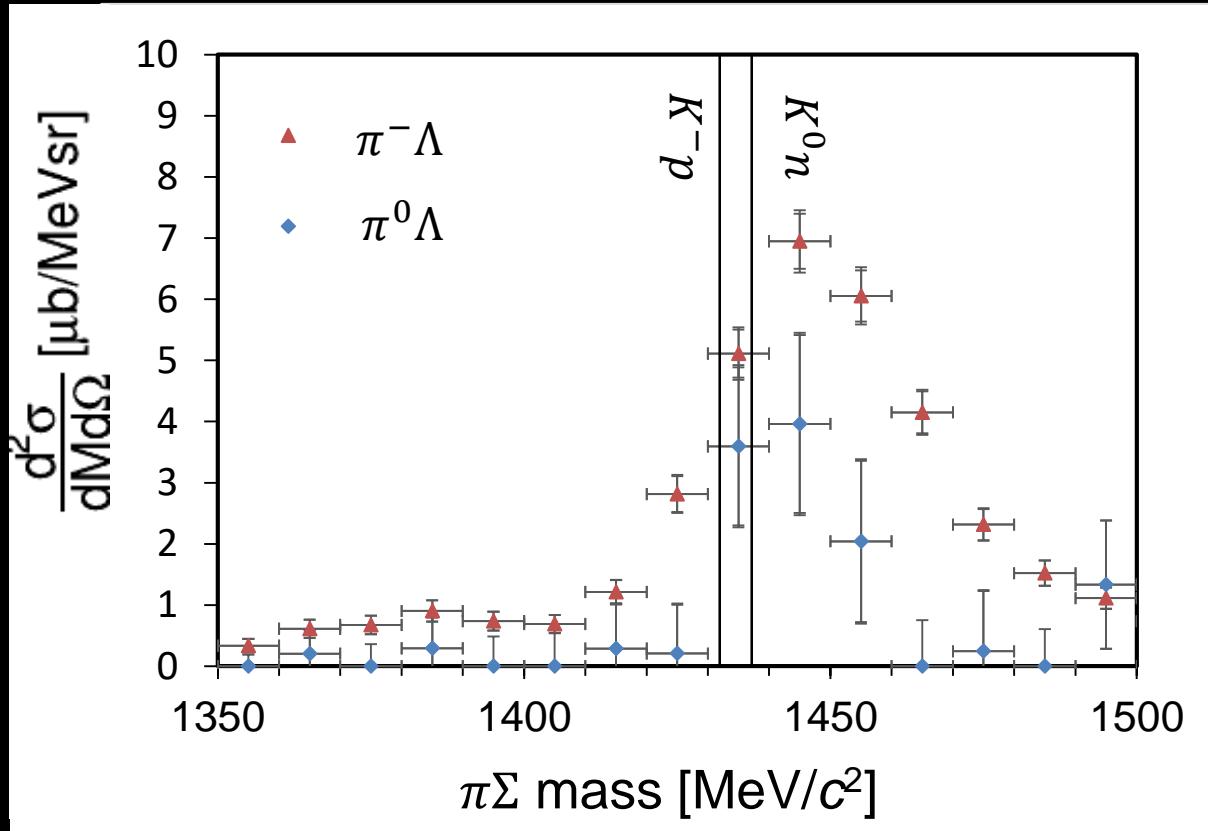
# $[\pi^\pm \Sigma^\mp - \pi^- \Sigma^0]/2$ vs $\pi^0 \Sigma^0 (I' = 0)$



$$\frac{d\sigma}{d\Omega}([\pi^\pm \Sigma^\mp - \pi^- \Sigma^0]/2) \propto \left| -\frac{3T_1^{I=0} - T_1^{I=1}}{4\sqrt{3}} \textcolor{red}{T}_2^{I'=0} \right|^2 \approx \frac{d\sigma}{d\Omega}(\pi^0 \Sigma^0) \propto \left| -\frac{3T_1^{I=0} - T_1^{I=1}}{4\sqrt{3}} \textcolor{red}{T}_2^{I'=0} \right|^2$$

*Isospin relation seems to be satisfied.*

# $\pi^- \Lambda$ vs $\pi^0 \Lambda$ ( $I' = 1$ )



$$\frac{d\sigma}{d\Omega}(\pi^- \Lambda) \propto \left| \frac{T_1^{I=0} + T_1^{I=1}}{2\sqrt{2}} \textcolor{red}{T'}_2^{I'=1} \right|^2 \approx 2 \times \frac{d\sigma}{d\Omega}(\pi^0 \Lambda) \propto \left| -\frac{T_1^{I=0} + T_1^{I=1}}{4} \textcolor{red}{T'}_2^{I'=1} \right|^2$$

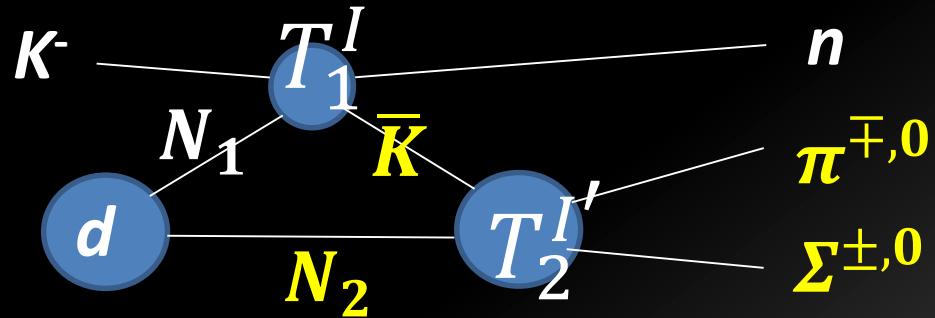
*Isospin relation seems to be satisfied.*

# Analysis of $\pi\Sigma$ spectra

- Description of the two step process in  $d(K^-, n)\pi\Sigma$
- Spectral fitting to extract scattering amplitude  $T_2^{I'=0}(\bar{K}N \rightarrow \pi\Sigma)$  and  $T_2^{I'=0}(\bar{K}N \rightarrow \bar{K}N)$

# Extracting Scattering Amplitude

- 2-step process



$$\begin{aligned} \frac{d\sigma}{dM_{\pi\Sigma}} \Big|_{\theta_n=3^\circ} &\sim | \left\langle n\pi\Sigma \left| T_2^{I'} (\bar{K}N_2 \rightarrow \pi\Sigma) G_0 T_1^I (K^-N_1 \rightarrow \bar{K}n) \right| K^- \Phi_d \right\rangle |^2 \\ &\sim \left| T_2^{I'} (\bar{K}N \rightarrow \pi\Sigma) \right|^2 F_{\text{res}}(M_{\pi\Sigma}) \end{aligned}$$

**Factorization Approximation**

$$F_{\text{res}}(M_{\pi\Sigma}) \sim \left| \int_0^\infty dq_{N_2}^3 T_1^I \frac{1}{E_{\bar{K}} - E_{\bar{K}}(q_{\bar{K}}) + i\epsilon} \Phi_d(q_{N_2}) \right|^2, q_{\bar{K}} + q_{N_2} = q_{\pi\Sigma}$$

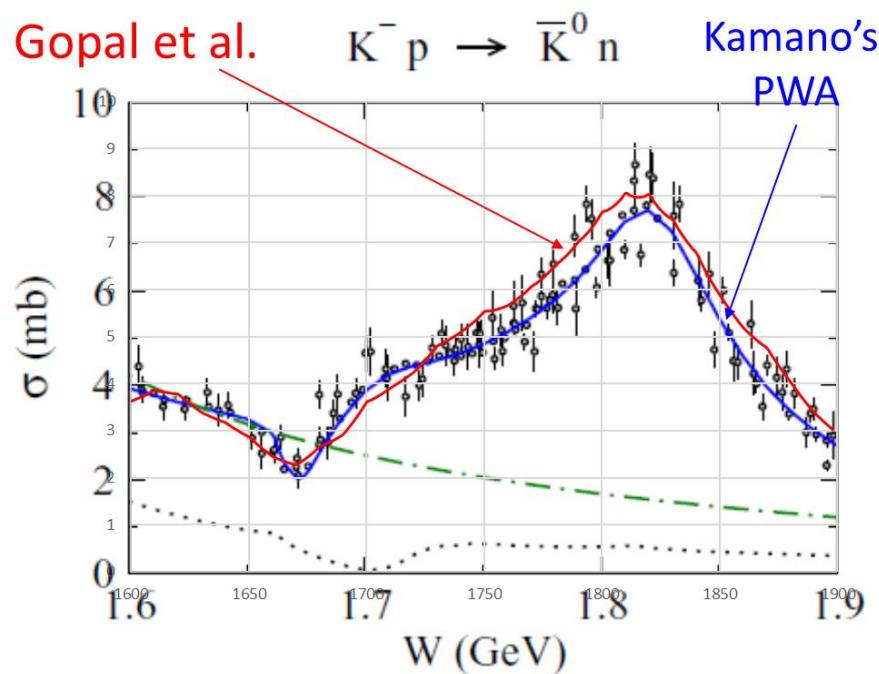
# E31: Response Function, $F_{\text{res}}(M_{\pi\Sigma})$

- $F_{\text{res}}(M_{\pi\Sigma}) = \left| \int G_0(q_2, q_1) T_1 \Phi_d(q_2) d^3 q_2 \right|^2$ 
  - $G_0(q_2, q_1) = \frac{1}{q_0^2 - q'^2 + i\varepsilon} f(q_0, q') \frac{\left( \sqrt{P_{\pi\Sigma}^2 + M_{\pi\Sigma}^2} + \sqrt{P_{\pi\Sigma}^2 + W(q')^2} \right)}{M_{\pi\Sigma} + W(q')}$ ,
  - $f(q_0, q')^{-1} = [E_1(q_0) + E_1(q')]^{-1} + [E_2(q_0) + E_2(q')]^{-1}$   
**Miyagawa and Haidenbauer, PRC85, 065201(2012)**
  - $T_1: K^- n \rightarrow K^- n$  ( $I = 1$ ),  $K^- p \rightarrow \bar{K}^0 n$  ( $I = 0, 1$ ) amplitude,  
**Gopal et al., NPB119, 362(1977)**
    - $T_1(K^- n \rightarrow K^- n) = f(I = 1)$
    - $T_1(K^- p \rightarrow \bar{K}^0 n) = [f(I = 1) - f(I = 0)]/2$
  - Off-shell treatment :See eq.(17) in PRC94, 065205
  - $\Phi_d(q_2)$ : deuteron wave function, **PRC63, 024001(2001)**

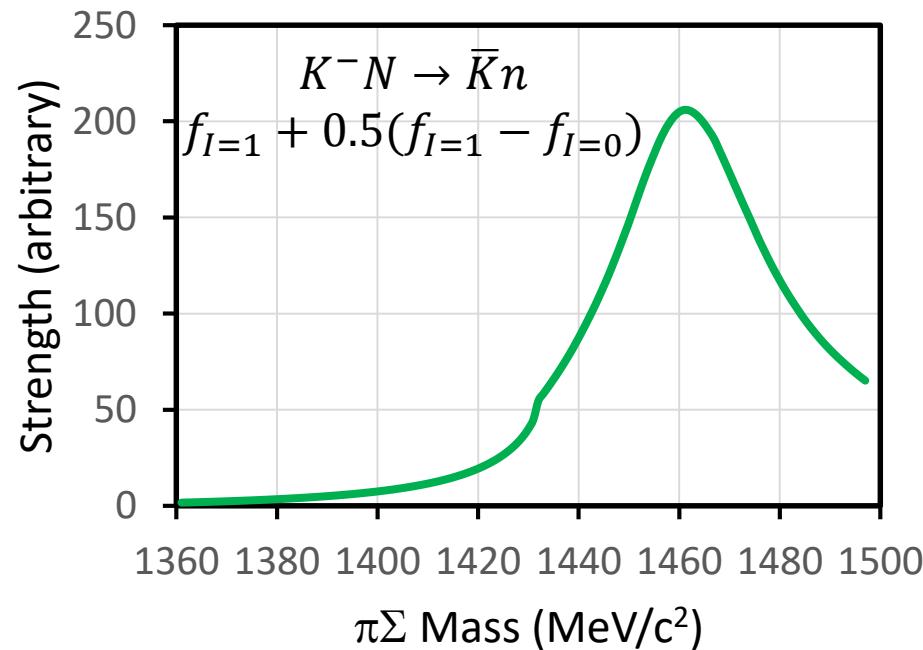
# E31: Response Function, $F_{\text{res}}(M_{\pi\Sigma})$

$$F_{\text{res}}(M_{\pi\Sigma}) \sim p_\pi^{cm} p_n^2 / |(E_{K^-} + m_d)\beta_n - p_{K^-} \cos \theta| \times \\ \int d\Omega_\pi^{cm} E_\pi E_\Sigma \left| \int q_2 T_1^I(p_{K^-}, q_N, p_n, q_{\bar{K}}, \cos \theta_{n\bar{K}}; M_{\pi\Sigma}) G_0(q_2, q_1) \Phi_d(q_2) d^3 q_2 \right|^2$$

Elementary Cross Section for  $T_1^I$



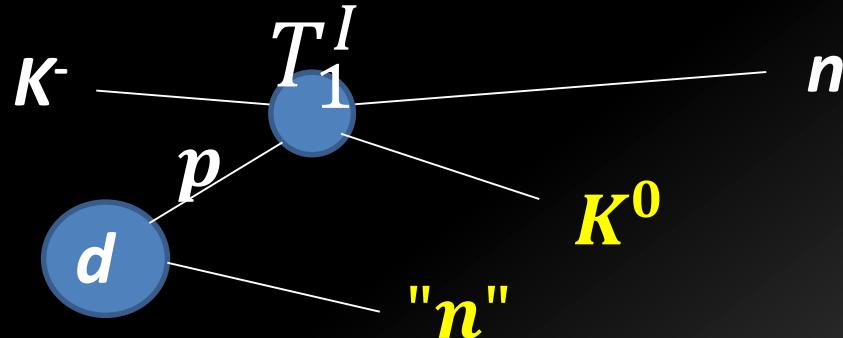
$F_{\text{res}}(M_{\pi\Sigma})$



Gopal et al., NPB119, 362(1977)

# Demonstration of the $T_1^I$ amplitude

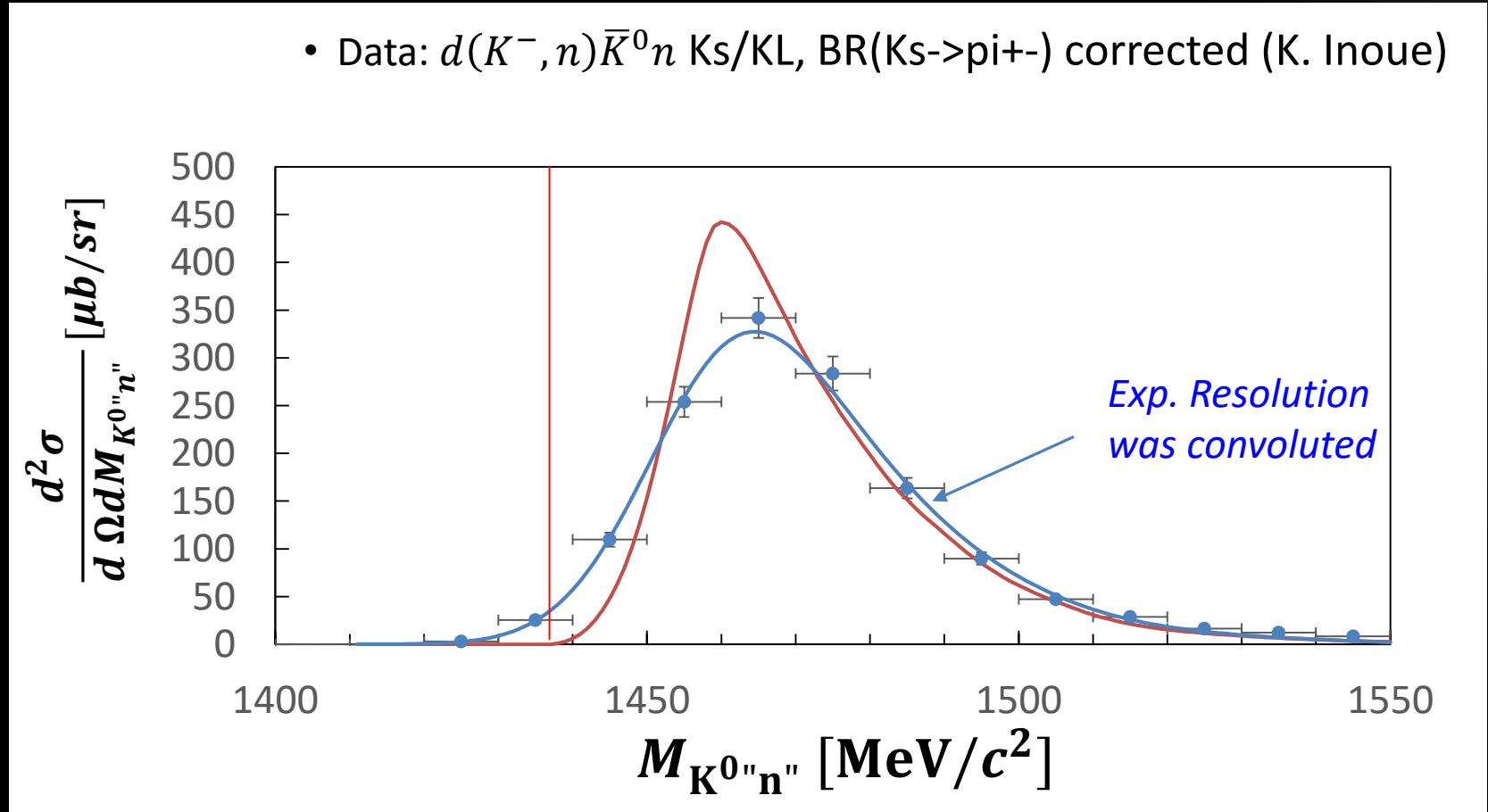
- 1-step process



$$\frac{d\sigma}{dM_{\pi\Sigma}} \Big|_{\theta_n=3^\circ} \sim |\langle n K^0 \textcolor{blue}{n} | T_1^I (K^- p \rightarrow \overline{K^0} n) | K^- \Phi_d \rangle|^2$$

$$\frac{d\sigma}{dM_{\pi\Sigma}} \sim \left| \int_0^\infty dq_{N_2}^3 T_1^I \delta(p_{K^-} + p_p - p_n - p_{K^0}) \Phi_d(q_{N_2}) \right|^2$$

# Demonstration for fitting data with the 1-step $K^- d \rightarrow n K^0 "n"$ reaction calculation

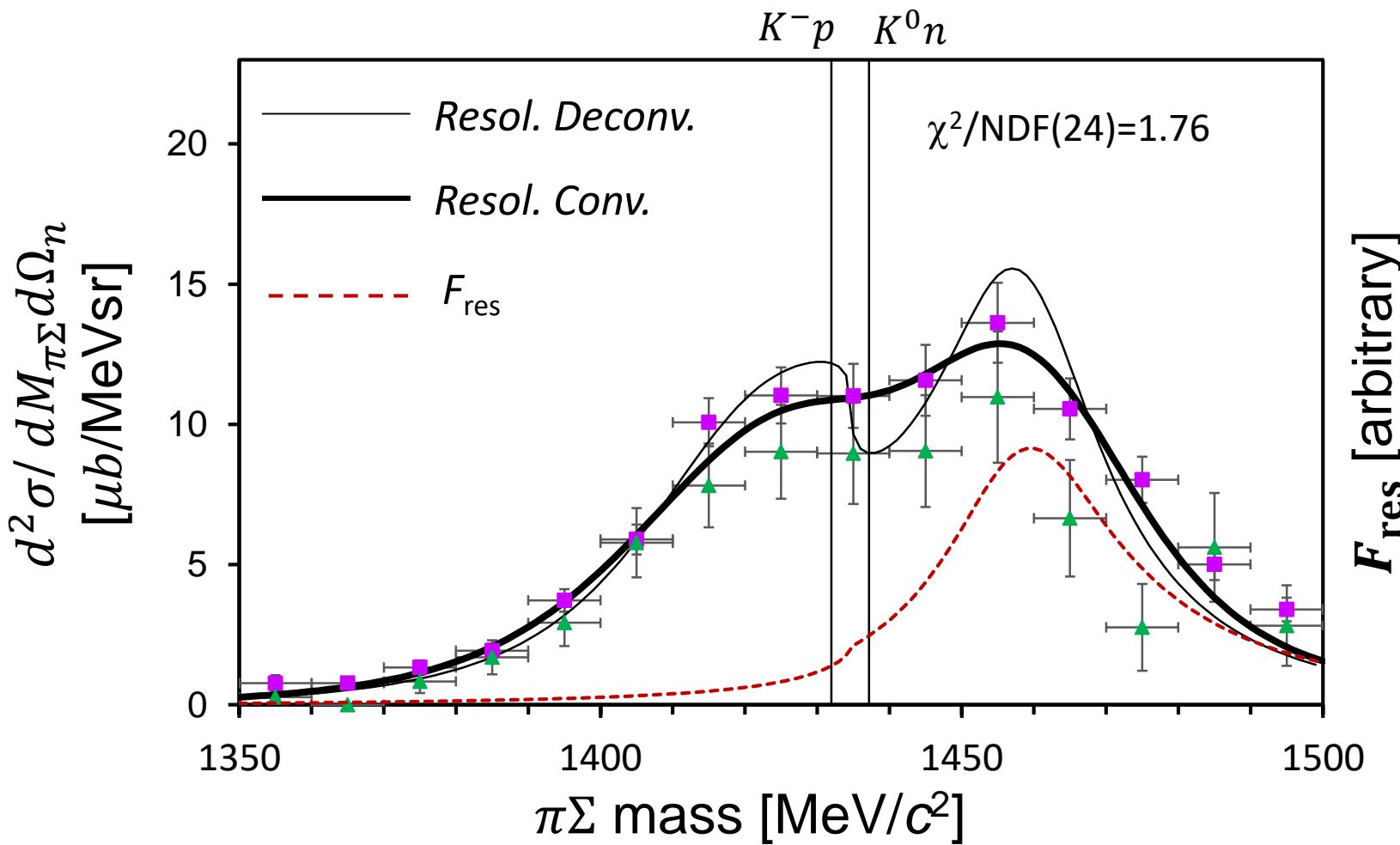


# $\bar{K}N$ Scattering Amplitude

L. Lensniak, arXiv:0804.3479v1(2008)

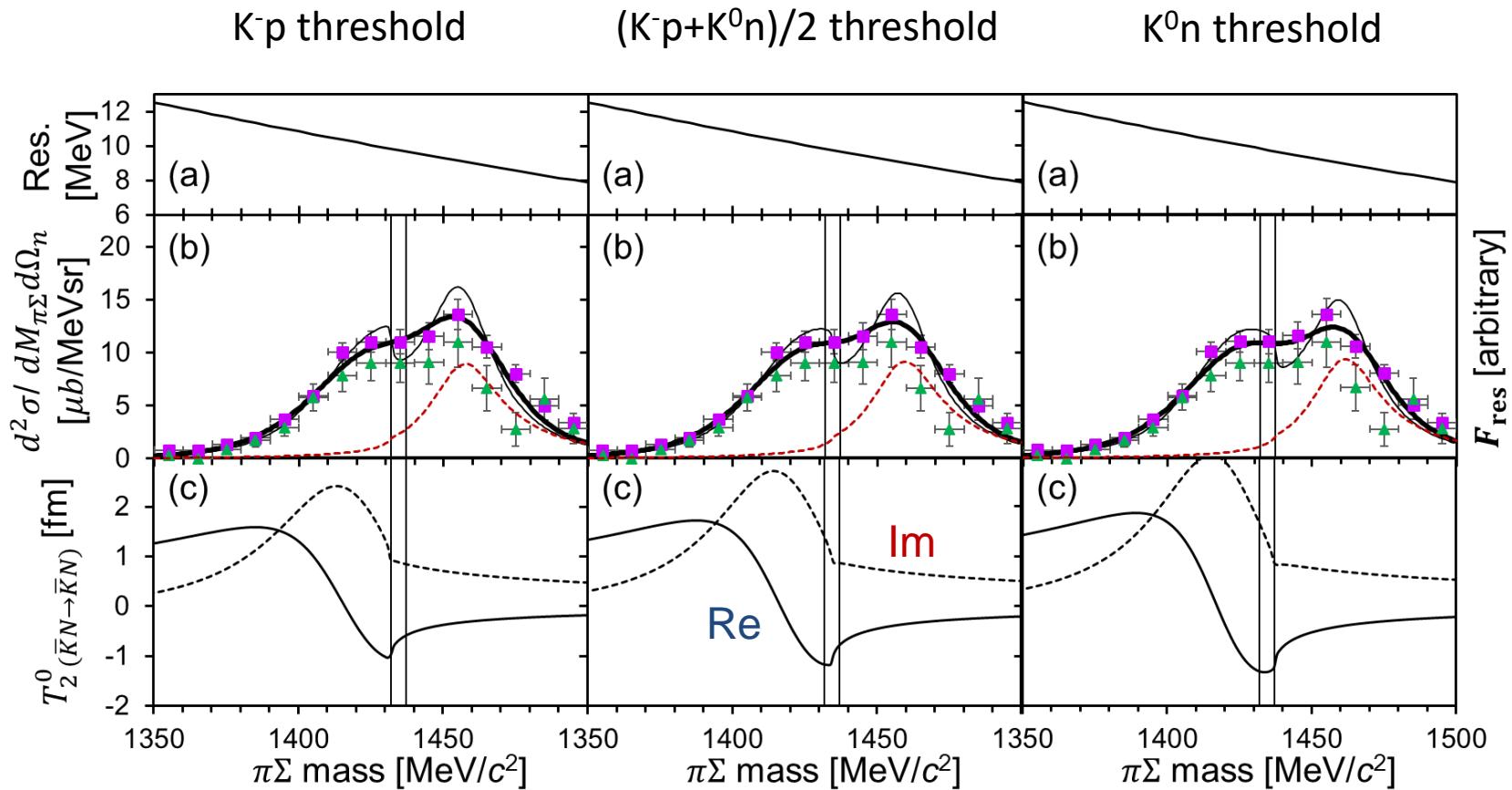
- $T_2^{I'}(\bar{K}N \rightarrow \bar{K}N) = \frac{\textcolor{red}{A}}{1-i\textcolor{red}{A}k_2+\frac{1}{2}\textcolor{red}{A}Rk_2^2}$
- $T_2^{I'}(\bar{K}N \rightarrow \pi\Sigma) = \frac{1}{\sqrt{k_1}} e^{i\delta_0} \frac{\sqrt{Im\textcolor{red}{A}-\frac{1}{2}|\textcolor{red}{A}|^2 Im\textcolor{blue}{R}k_2^2}}{1-iAk_2+\frac{1}{2}\textcolor{red}{A}Rk_2^2}$
- $T_2^{I'}(\pi\Sigma \rightarrow \pi\Sigma)$   
 $= \frac{e^{i\delta_0}}{k_1} \frac{\left(\sin \delta_0 + iIm(e^{-i\delta_0} A)k_2 - \frac{1}{2}Im(e^{-i\delta_0} AR)k_2^2\right)}{1-iAk_2+\frac{1}{2}\textcolor{red}{A}Rk_2^2}$
- 5 real number parameters (effective range expansion)
  - $A$ : scattering length,  $R$ : effective range,  $\delta_0$ : phase

# Fit the spectra to deduce $\bar{K}N$ scattering amplitude

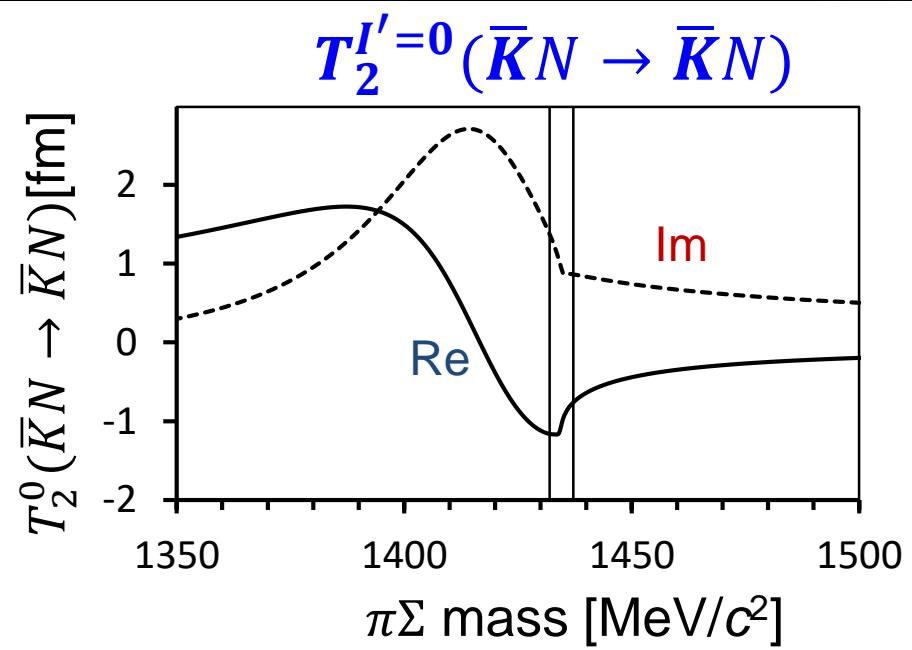
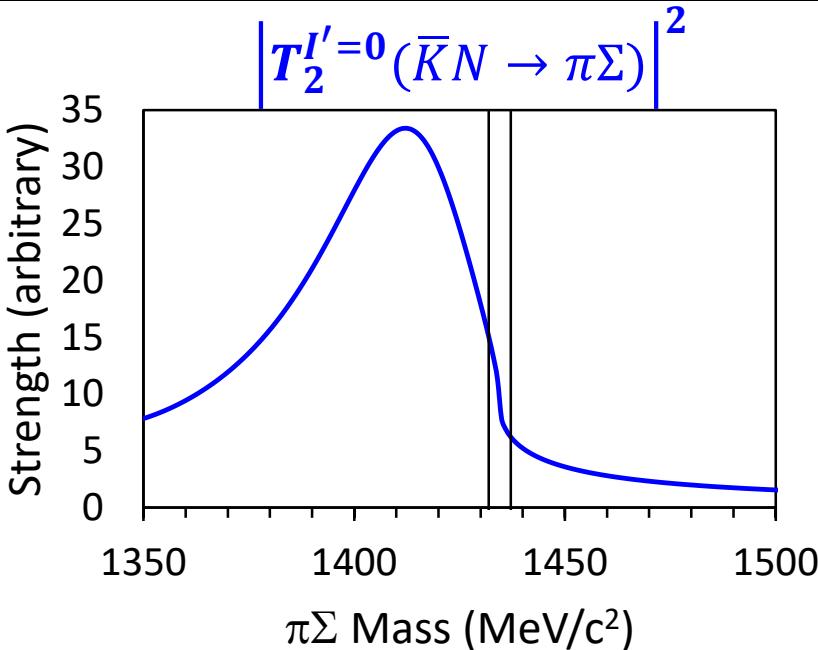


# Systematics of the fitting result by the assumed $\bar{K}N$ mass threshold

$$\left. \frac{d\sigma}{dM_{\pi\Sigma}} \right|_{\theta_n=0} \sim \left| T_2^{I'}(\bar{K}N \rightarrow \pi\Sigma) \right|^2 F_{\text{res}}(M_{\pi\Sigma})$$



# Best fit $\bar{K}N$ scattering amplitude



A pole at  $(1417.7_{-7.4-1.0}^{+6.0+1.1}) + (-26.1_{-7.9-2.0}^{+6.0+1.7})i$   $\text{MeV}/c^2$

$$\left|T_2^{I'=0}(\bar{K}N \rightarrow \bar{K}N)\right|^2 / \left|T_2^{I'=0}(\bar{K}N \rightarrow \pi\Sigma)\right|^2 = 2.2_{-0.6-0.3}^{+1.0+0.3}$$

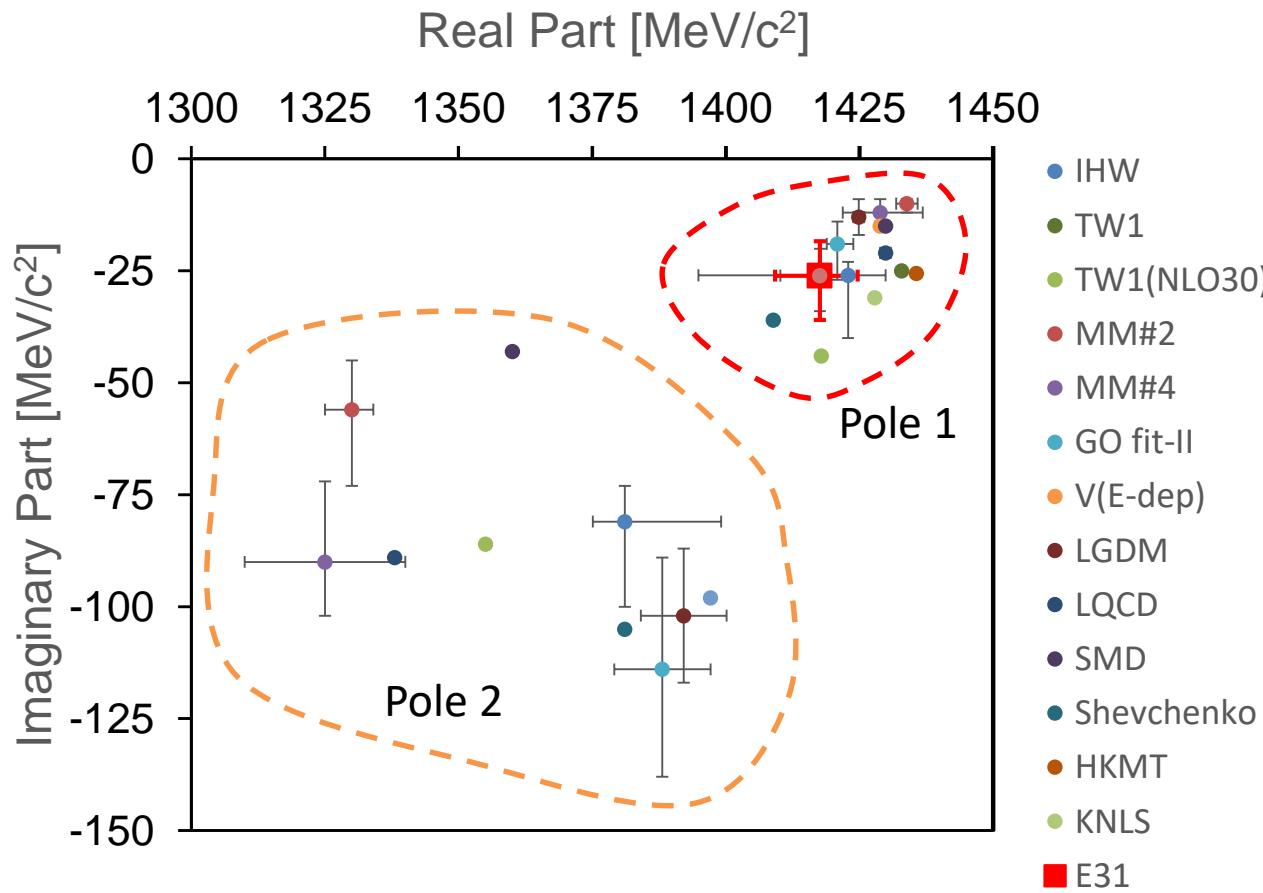
$$A^{I'=0} = (-1.12 \pm 0.11_{-0.07}^{+0.10}) + i(0.84 \pm 0.12_{-0.07}^{+0.08}) \text{ fm}$$

$$R^{I'=0} = (-0.18 \pm 0.31_{-0.06}^{+0.08}) + i(0.41 \pm 0.13_{-0.09}^{+0.09}) \text{ fm}$$

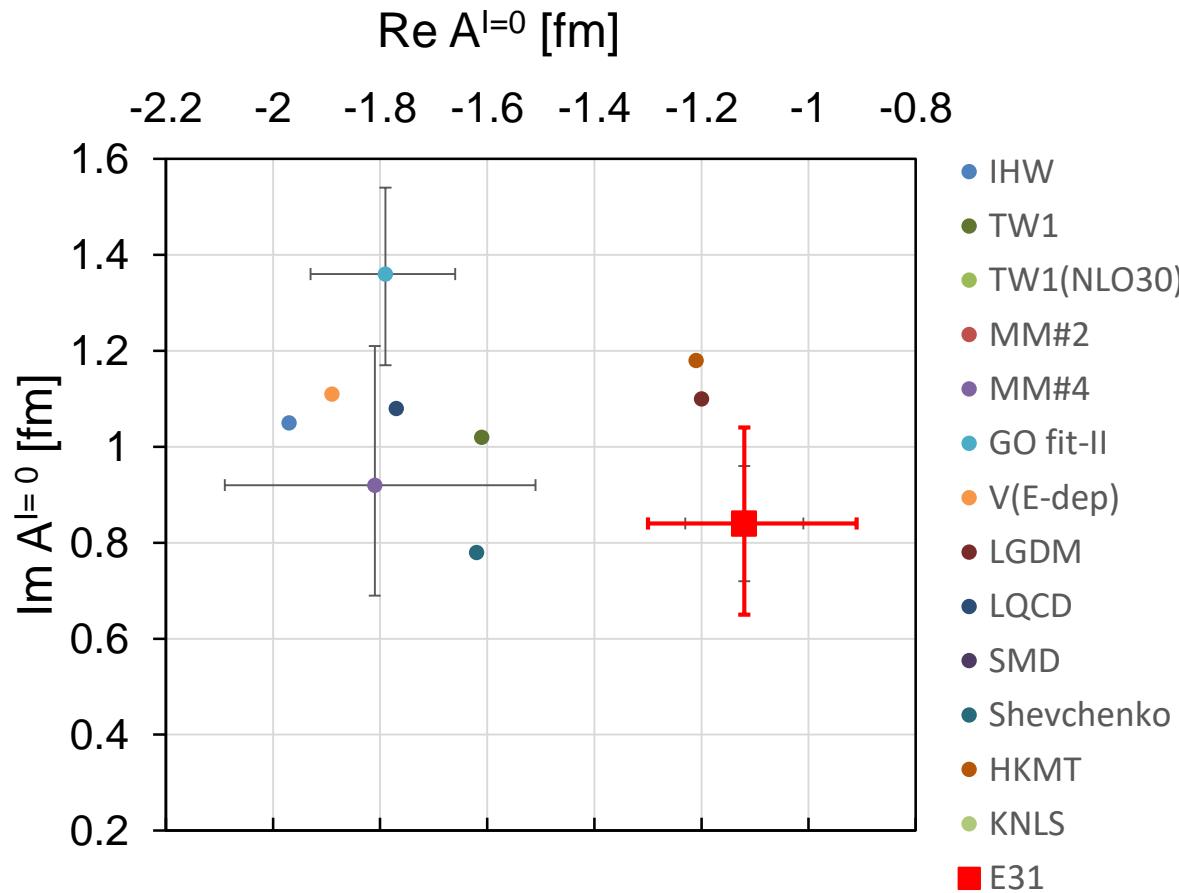
\*best fit value  $\pm$  fitting error  $\pm$  systematic error

systematic errors assuming the  $K^-p/K^0n$  mass threshold

# Two-pole structure of Lambda(1405) in Meson-Baryon dynamics



# Two-pole structure of Lambda(1405) in Meson-Baryon dynamics



# Conclusion

- We measured the  $\pi\Sigma$  mass spectra in the  $K^-d \rightarrow N\pi\Sigma$  reactions, knocked-out  $N$  measured at  $\sim 0$  degree.
  - well described with the two-step reaction process,  $K^-N_1 \rightarrow N\bar{K}$ ,  $\bar{K}N_2 \rightarrow \pi\Sigma$
  - S-wave  $\bar{K}N_2 \rightarrow \pi\Sigma$  scattering is dominant.
  - Isospin relations among the cross sections are well satisfied:
$$\frac{d\sigma}{d\Omega}([\pi^\pm\Sigma^\mp - \pi^-\Sigma^0]/2) = \frac{d\sigma}{d\Omega}(\pi^0\Sigma^0)$$
$$\frac{d\sigma}{d\Omega}(\pi^-\Lambda) = 2 \times \frac{d\sigma}{d\Omega}(\pi^0\Lambda)$$
- S-wave  $\bar{K}N$  scattering amplitude ( $I=0$ ) was deduced.
- We found a resonance pole at  $1417.7 - 26.1i$  [MeV], which seems consistent to that of the so-called higher pole of  $\Lambda(1405)$  suggested by the ChUM based calculations.
- The pole is likely to couple to the  $K^{\bar{b}ar}N$  state.