# Beyond-RPA approaches in the equation-of-motion framework


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**Collaborators:** *Peter Schuck, Peter Ring, Caroline Robin, Yinu Zhang, Manqoba Hlatshwayo, Herlik Wibowo, …*

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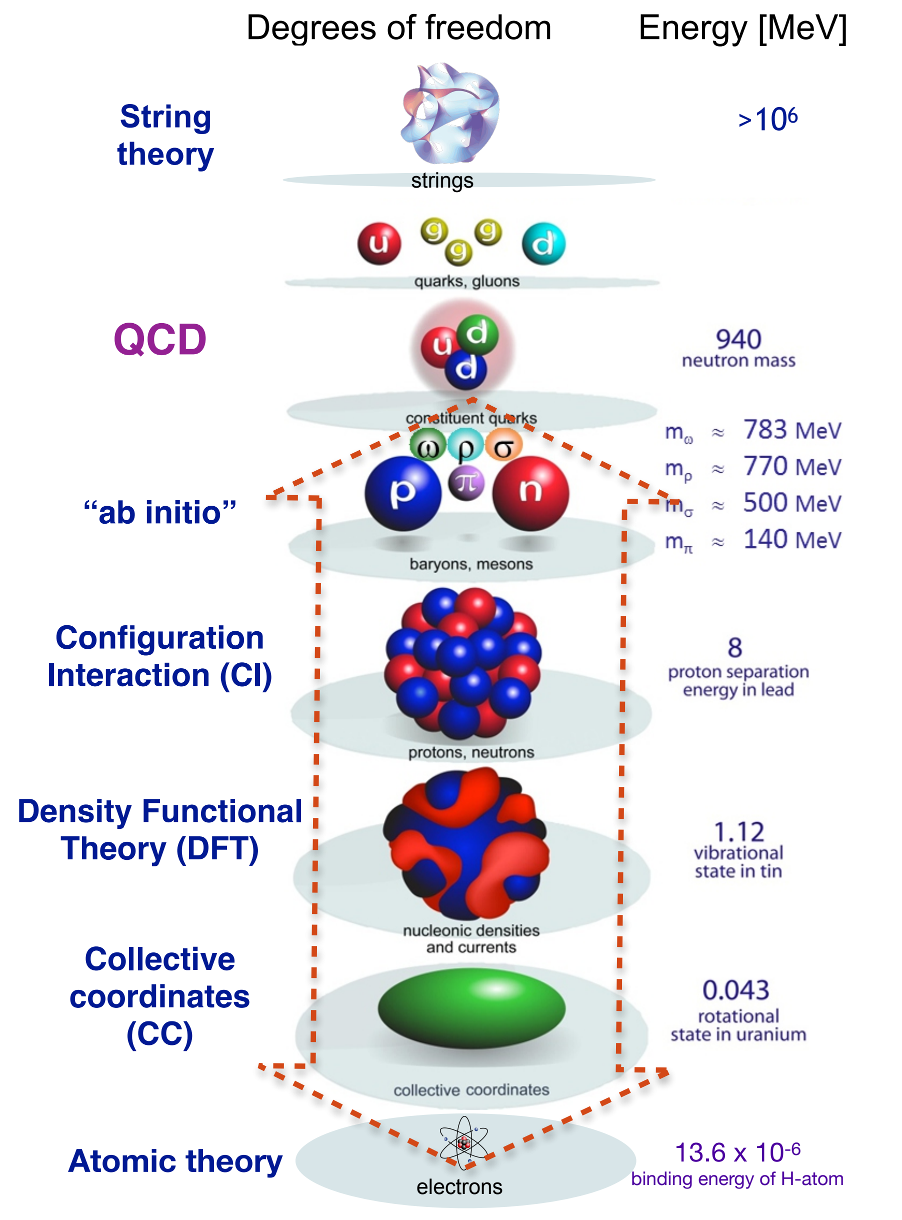
*ECT* Workshop "Giant and Soft Modes of Excitation in Nuclear Structure and Astrophysics"*

*Trento, Italy, October 24-28*

# Hierarchy of energy scales and nuclear many-body problem




- ***The major conflict: reductionism vs "emergentism"***

*Separation of energy scales => effective field theories*

*vs*

*The physics on a certain scale is governed by the next higher-energy scale*

***Hamiltonian:***

$$H = K + V$$

*center of mass* (K)

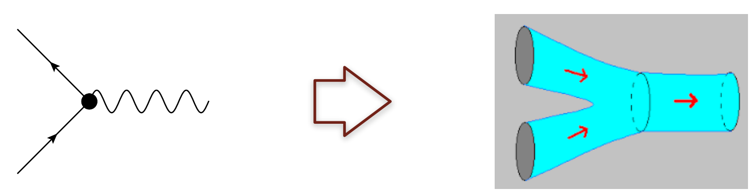
*internal degrees of freedom: next energy scale* (V)

*Standard Model:*

*free propagation and interaction, singularities & renormalizations*

*String theory:*

*merging strings NO "Interaction"*



- ***Possible solution:***

*Keep/establish connections between the scales via emergent phenomena*

# The frontiers of fundamental physics

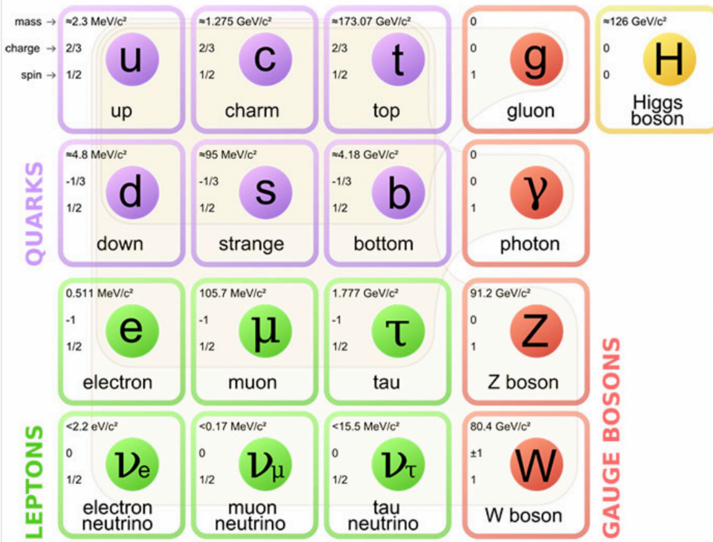
- *The Standard Model: explains (almost) all experimental data, provides a consistent and powerful framework based on the gauge origin of EM, weak and strong interactions.*

- *Some experimental facts point out that the SM is incomplete or is an effective theory with respect to some more fundamental theory:*
  - *Non-zero neutrino mass;*
  - *Dark matter & dark energy;*
  - *Gravity can not be quantized and included in the SM .*

- *Other open Big questions:*
  - *Why does the universe exist?*
  - *Why is it so large?*
  - *Are there extra dimensions?*
  - *Why time is one-way?*

- *The frontiers: the big, the small, and the complex.*
  *Making very large and very small compatible; emergence at various scales:*
  - *Big Bang*
  - *Black Holes*
  - *Stars: Interface of subatomic physics and astrophysics.*

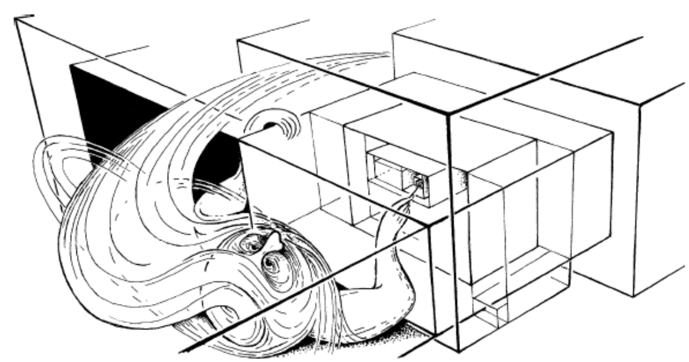
## *The Standard Model*




The Standard Model of physics describes the known particles and forces that operate at the tiny quantum scale. *(Wikimedia Commons: Miss J)*

## *God setting up the Universe*

*Fig. from R. Penrose: The Road to Reality*



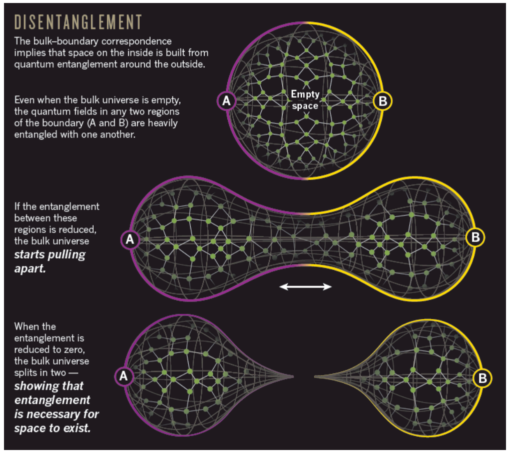
# Emergent phenomena in strongly-correlated quantum systems

- *Emergence in interacting many-body systems: drastically different behavior of the entire system originating from collective behavior and interactions of the constituent local degrees of freedom.*

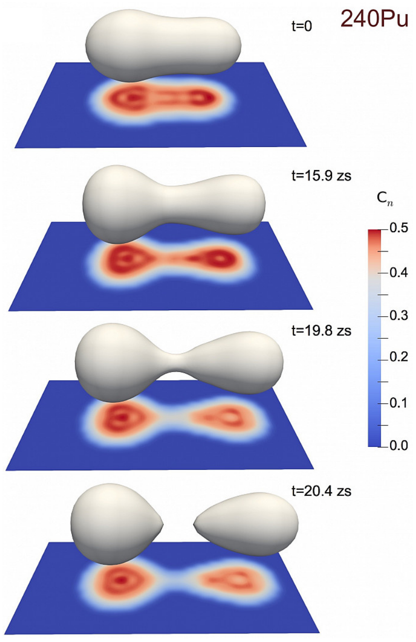
- *The emergence is associated with resolving conceptual difficulties in various contexts, such as quark confinement, scattering amplitudes, topological phases of materials, superfluidity and superconductivity. Therefore, understanding emergence has a broad impact.*

- *Answering Big Questions at fundamental physics frontiers by extensions of the Standard Model to include gravity, connecting explicitly UV and IR physics: emergent character of space(-time). Fundamentality of emergence.*

***Bulk-boundary correspondence:***

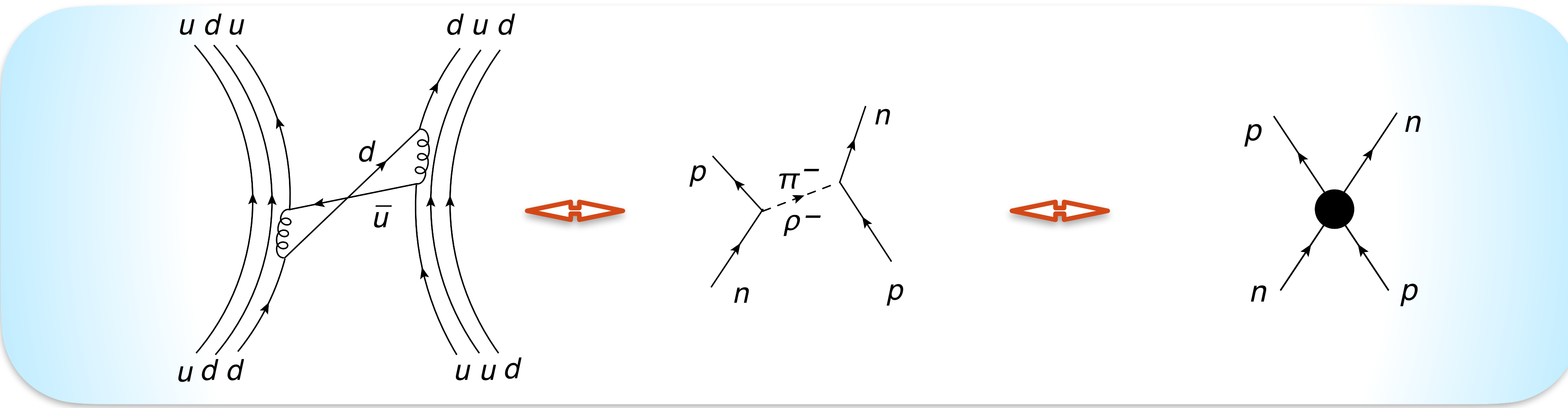



***Nuclear fission:***



# *The underlying mechanism of NN-interaction :*

| *Quantum Chromodynamics (QCD, high energy)* | *Quantum Hadrodynamics (QHD, intermediate energy)* | *Nuclear Structure (NS, low energy)* |
|---|---|---|



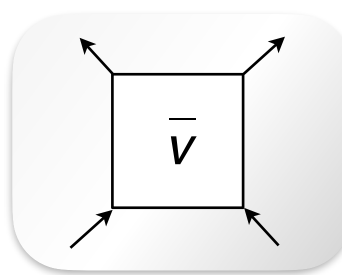

*Relay of EFTs*

QCD → QHD → NS

***Nuclear applications:***

*Generic "bare interaction": model-independent, ALL channels included:*

$$H = \sum_{12} t_{12} \psi^{\dagger}{}_1 \psi_2 + \frac{1}{4} \sum_{1234} \bar{v}_{1234} \psi^{\dagger}{}_1 \psi^{\dagger}{}_2 \psi_4 \psi_3$$




***In ideal model:***

- *Non-local √*
- *Time-dependent ?*
- *Covariant √*

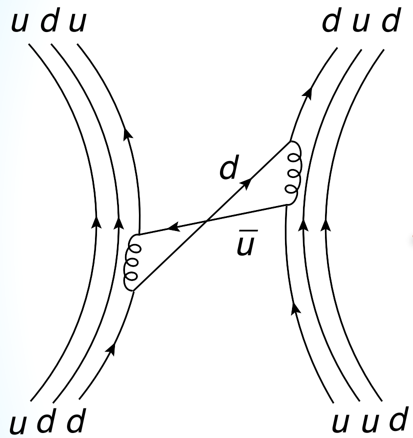
***In ideal theory:***

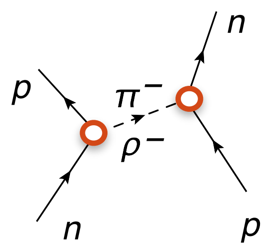
*Derivable from the underlying DoFs*

# The underlying mechanism of NN-interaction :

Quantum Chromodynamics (QCD, high energy)

Quantum Hadrodynamics (QHD, intermediate energy)



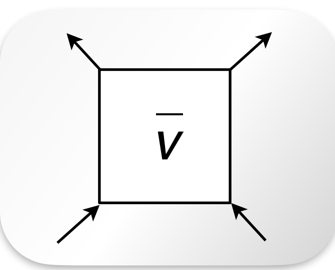

- *The full many-body scheme* has not been (yet) executed neither for the "bare" meson-exchange (ME) interaction nor for any other bare interaction.
- A good starting point - the use of *effective ME interactions* adjusted to nuclear bulk properties on the mean-field level (J. Walecka, M. Serot, …, **P. Ring**) and to supplement the *many-body correlation theory with proper subtraction techniques* (**V. Tselyaev**), in the covariant framework.

*Relay of EFTs*

QCD → QHD → NS

**Nuclear applications:**

Generic "bare interaction": model-independent, ALL channels included:




$$H = \sum_{12} t_{12}\psi^\dagger{}_1\psi_2 + \frac{1}{4}\sum_{1234}\bar{v}_{1234}\psi^\dagger{}_1\psi^\dagger{}_2\psi_4\psi_3$$

**In ideal model:**

- Non-local √
- Time-dependent ?
- Covariant √

**In ideal theory:**

Derivable from the underlying DoFs

# Exact equations of motion (EOM) for binary interactions: one-body problem

$$G_{11'}(t-t') = -i\langle T\psi_1(t)\psi^\dagger_{1'}(t')\rangle$$

**EOM**: *Dyson Equation*

$$G(\omega) = G^{(0)}(\omega) + G^{(0)}(\omega)\Sigma(\omega)G(\omega) \qquad (*) \qquad \Sigma(\omega) = \Sigma^{(0)} + \Sigma^{(r)}(\omega)$$

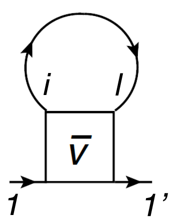
*Free propagator* → $G^{(0)}$

***Irreducible kernel (Self-energy, exact):*** → $\Sigma(\omega)$

*Instantaneous term (Hartree-Fock incl. "tadpole")*
***Short-range correlations***

$$\Sigma^{(0)}_{11'} = -\delta(t-t')\langle[[V,\psi_1],{\psi^\dagger}_{1'}]_+\rangle$$

$$= -\sum_{jl}\bar{v}_{1j1'l}\rho_{lj} =$$



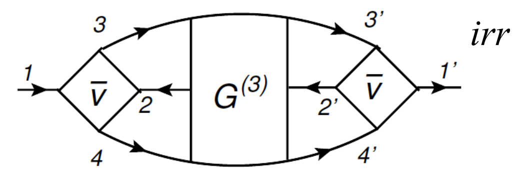

$$\rho_{ij} = -i\lim_{t=t'-0}G_{ij}(t-t')$$

*t-dependent (dynamical) term*
***Long-range correlations***

$$\Sigma^{(r)}_{11'}(t-t') = -i\langle T[\psi_1,V](t)[V,{\psi^\dagger}_{1'}](t')\rangle^{irr}$$

$$= -\frac{1}{4}\sum_{234}\sum_{2'3'4'}\bar{v}_{1234}G^{irr}(432',23'4')\bar{v}_{4'3'2'1'}$$

$$= -\frac{1}{4}$$




Koltun-Migdal-Galitsky sum rule: ***the binding energy***

$$E_0 = \frac{1}{2\pi}\int_{-\infty}^{\varepsilon_F^-} d\varepsilon\sum_{12}(T_{12}+\varepsilon\delta_{12})\mathrm{Im}G_{21}(\varepsilon)$$

***"Ab-initio DFT"***

***This self-energy is:***

- ***Ab-initio Exact & Universal***
- ***valid in relativistic regimes***
- ***including fermionic EOMs in the Standard Model***

# Equation of motion (EOM) for the particle-hole response

*Particle-hole response (correlation function):*

$$R^{(ph)}_{12,1'2'}(t-t') = -i\langle T(\psi^\dagger_1\psi_2)(t)(\psi^\dagger_{2'}\psi_{1'})(t')\rangle$$

*spectra of excitations, masses, decays, …*

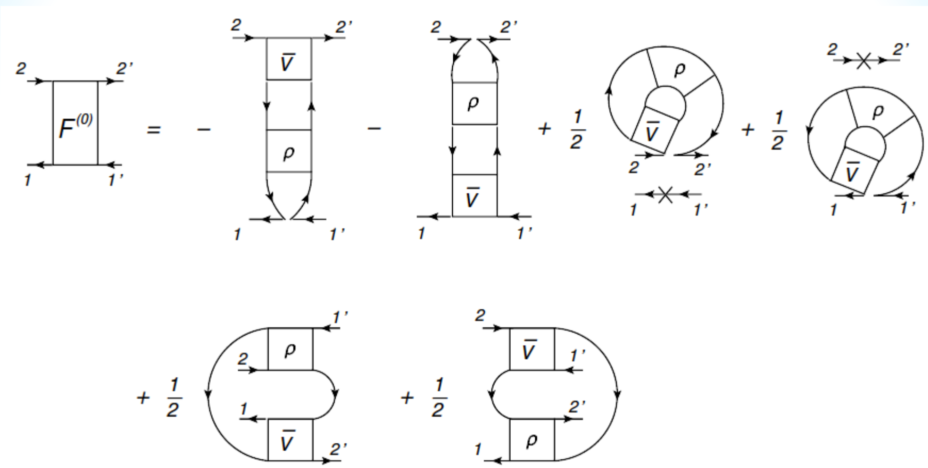
***EOM**: Bethe-Salpeter-Dyson Eq.*

$$R(\omega) = R^{(0)}(\omega) + R^{(0)}(\omega)F(\omega)R(\omega) \qquad (**) \qquad F(t-t') = F^{(0)}\delta(t-t') + F^{(r)}(t-t')$$

*Free propagator*

***Irreducible kernel (exact):***

*Instantaneous term ("bosonic" mean field):*
***Short-range correlations***



*Self-consistent mean field $F^{(0)}$, where*

$$\rho_{12,1'2'} = \delta_{22'}\rho_{11'} - i\lim_{t'\to t+0} R_{2'1,21'}(t-t')$$

*contains the full solution of (**) **including the dynamical term!***

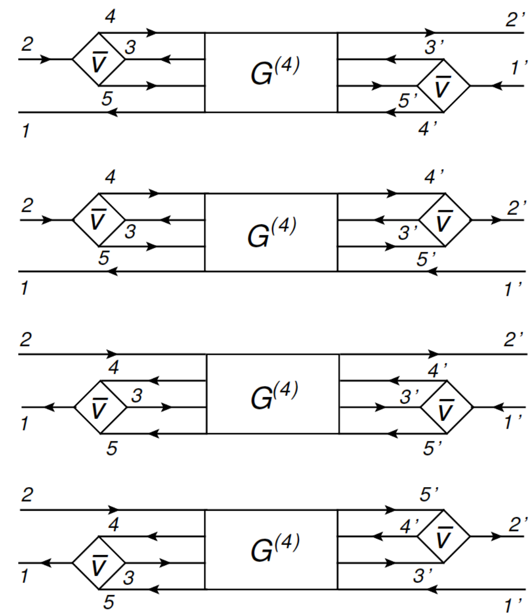
*t-dependent (dynamical) term:*
***Long-range correlations***

$F^{(r;11)}_{121'2'} =$

$F^{(r;12)}_{121'2'} =$

$F^{(r;21)}_{121'2'} =$

$F^{(r;22)}_{121'2'} =$



$$F^{(r)}_{12,1'2'}(t-t') = \sum_{ij} F^{(r;ij)}_{12,1'2'}(t-t')$$

# *Non-perturbative treatment of two-point $G^{(n)}$ in the dynamical kernels*

- ***Quantum many-body problem in a nutshell:*** *Direct EOM for $G^{(n)}$ generates $G^{(n+2)}$ in the (symmetric) dynamical kernels and further high-rank correlation functions (CFs); an equivalent of the BBGKY hierarchy. $N_{Equations} = N_{Particles}$ & Coupled 🙈 !!!*

*Truncation on two-body level*

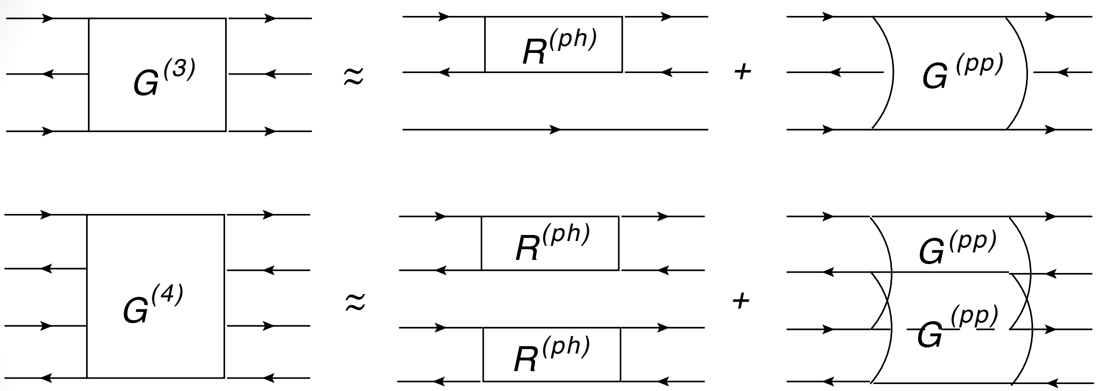
"SCGF" — This work

- ***Non-perturbative solution:***

$$\text{✦ } G^{(3)} = G^{(1)}G^{(1)}G^{(1)} + G^{(2)}G^{(1)} + \Xi^{(3)}$$

***Cluster decomposition***

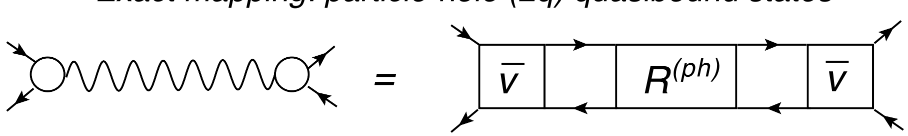
$$\text{✦ } G^{(4)} = G^{(1)}G^{(1)}G^{(1)}G^{(1)} + G^{(2)}G^{(2)} + G^{(3)}G^{(1)} + \Xi^{(4)}$$

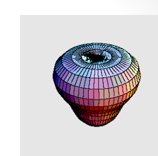



- *P. C. Martin and J. S. Schwinger, Phys. Rev.115, 1342 (1959).*
- *N. Vinh Mau, Trieste Lectures 1069, 931 (1970)*
- *P. Danielewicz and P. Schuck, Nucl. Phys. A567, 78 (1994)*
- *…*

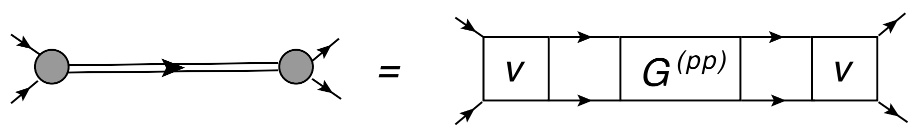
*Exact mapping: particle-hole (2q) quasibound states*

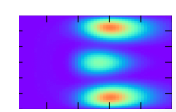
*Emergence of effective "particles" (phonons, vibrations):*



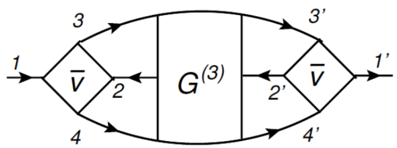
*Emergence of superfluidity:*



# Emergence of effective degrees of freedom

*Dynamical self-energy:*

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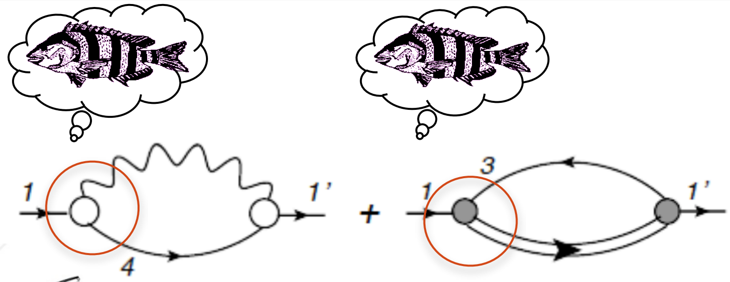
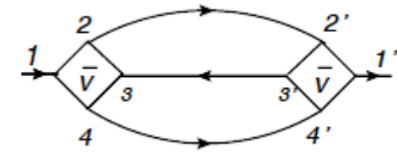



*"Fish" diagrams*

*Second-order*

*Quasiparticle-vibration coupling (QVC)*

***Emergent*** *phonon vertices and propagators: calculable from the underlying H, which does not contain phonon degrees of freedom*

*"Ab-initio"*

$$H = \sum_{12} h_{12}\psi_1^\dagger\psi_2 + \frac{1}{4}\sum_{1234}\bar{v}_{1234}\psi_1^\dagger\psi_2^\dagger\psi_4\psi_3$$

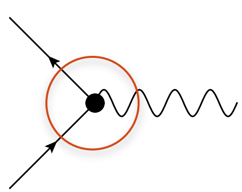
***Derivable****, unlike the NFT*

$$H = \sum_{12}\tilde{h}_{12}\psi_1^\dagger\psi_2 + \sum_{\lambda\lambda'}\mathcal{W}_{\lambda\lambda'}Q_\lambda^\dagger Q_{\lambda'} + \sum_{12\lambda}\Big[\Theta_{12}^{\lambda}\psi_1^\dagger Q_\lambda^\dagger\psi_2 + h.c.\Big]$$

*Effective*

*Cf.: The Standard Model elementary interaction vertices: boson-exchange interaction is the input:*

*Possibly derivable?*



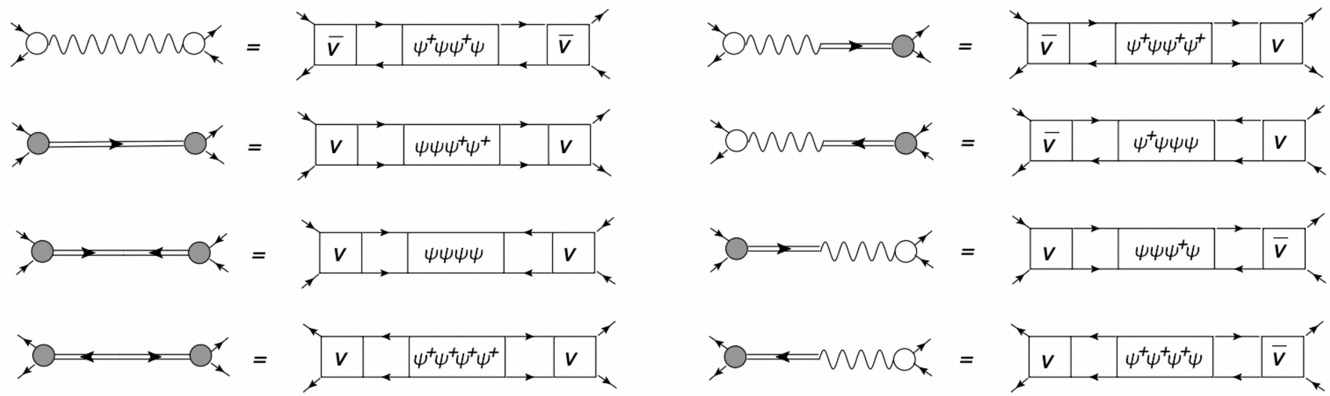
$$\gamma, g, W^\pm, Z^0$$


*E.L., P. Schuck, PRC 100, 064320 (2019)*
*E.L., Y. Zhang, PRC 104, 044303 (2021)*

# Superfluid systems

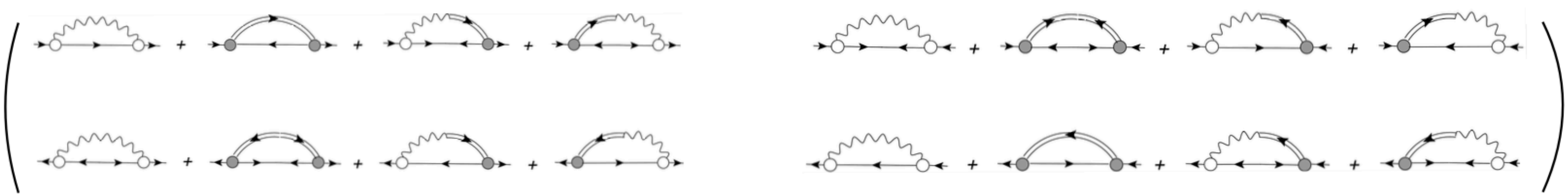
## Superfluid dynamical kernel: adding particle-number violating contributions

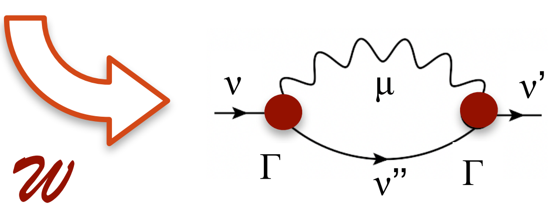
*Mapping on the QVC in the canonical basis*




*Quasiparticle dynamical self-energy (matrix):*
*normal and pairing phonons are unified*

$\hat{\Sigma}^r =$






*Cf.: Quasiparticle static self-energy (matrix) in HFB*

$$\hat{\Sigma}^0 = \begin{pmatrix} \tilde{\Sigma}_{11'} & \Delta_{11'} \\ -\Delta^*_{11'} & -\tilde{\Sigma}^T_{11'} \end{pmatrix}$$


E.L., Y. Zhang, PRC 104, 044303 (2021)
Y. Zhang et al., PRC 105, 044326 (2022)

# Transformation to quasiparticle basis

*Bogolyubov transformation:*

$$\psi_1 = \sum_\nu (U_{1\nu}\alpha_\nu + V^*_{1\nu}\alpha^\dagger_\nu), \qquad \psi^\dagger_1 = \sum_\nu (V_{1\nu}\alpha_\nu + U^*_{1\nu}\alpha^\dagger_\nu)$$

$$G^{(+)}_{\nu\nu'}(\varepsilon) = \sum_{12}\begin{pmatrix} U^\dagger_{\nu 1} & V^\dagger_{\nu 1}\end{pmatrix}\hat{G}_{12}(\varepsilon)\begin{pmatrix} U_{2\nu'} \\ V_{2\nu'}\end{pmatrix}$$

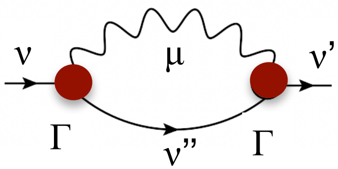
$$G^{(-)}_{\nu\nu'}(\varepsilon) = \sum_{12}\begin{pmatrix} V^T_{\nu 1} & U^T_{\nu 1}\end{pmatrix}\hat{G}_{12}(\varepsilon)\begin{pmatrix} V^*_{2\nu'} \\ U^*_{2\nu'}\end{pmatrix}$$

*Propagator becomes diagonal*

*Dyson Eqs. decouple for η=1 and η=-1: Eq. for η=-1 is redundant*

$$G^{(\eta)}_{\nu\nu'}(\varepsilon) = \tilde{G}^{(\eta)}_{\nu\nu'}(\varepsilon) + \sum_{\mu\mu'}\tilde{G}^{(\eta)}_{\nu\mu}(\varepsilon)\Sigma^{r(\eta)}_{\mu\mu'}(\varepsilon)G^{(\eta)}_{\mu'\nu'}(\varepsilon)$$

$$\Sigma^{r(+)}_{\nu\nu'}(\varepsilon) = \sum_{\nu''\mu}\Big[\frac{\Gamma^{(11)\mu}_{\nu\nu''}\Gamma^{(11)\mu*}_{\nu'\nu''}}{\varepsilon - E_{\nu''} - \omega_\mu + i\delta} + \frac{\Gamma^{(02)\mu*}_{\nu\nu''}\Gamma^{(02)\mu}_{\nu'\nu''}}{\varepsilon + E_{\nu''} + \omega_\mu - i\delta}\Big] \Longleftrightarrow$$




*HFB basis*

*Dynamical self-energy: acquires the same form as the non-superfluid one!*

*Superfluid quasiparticle-vibration coupling (QVC) vertices:*

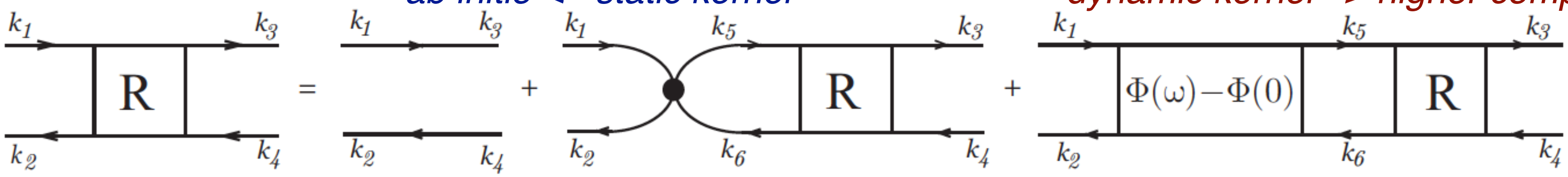
$$\Gamma^{(11)\mu}_{\nu\nu'} = \sum_{12}\Big[U^\dagger_{\nu 1}g^\mu_{12}U_{2\nu'} + U^\dagger_{\nu 1}\gamma^{\mu(+)}_{12}V_{2\nu'} - V^\dagger_{\nu 1}(g^\mu_{12})^T V_{2\nu'} - V^\dagger_{\nu 1}(\gamma^{\mu(-)}_{12})^T U_{2\nu'}\Big]$$

$$\Gamma^{(02)\mu}_{\nu\nu'} = -\sum_{12}\Big[V^T_{\nu 1}g^\mu_{12}U_{2\nu'} + V^T_{\nu 1}\gamma^{\mu(+)}_{12}V_{2\nu'} - U^T_{\nu 1}(g^\mu_{12})^T V_{2\nu'} - U^T_{\nu 1}(\gamma^{\mu(-)}_{12})^T U_{2\nu'}\Big]$$

*E.L., Y. Zhang, PRC 104, 044303 (2021)*

# Higher-order correlations: toward a complete theory

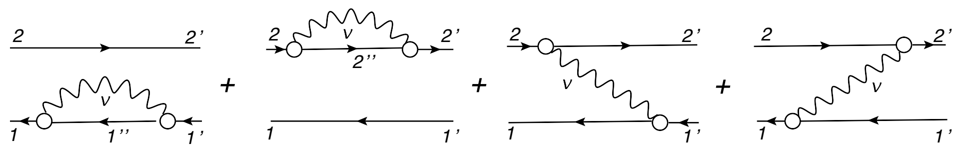
*ab initio <= static kernel* — *dynamic kernel => higher complexity*



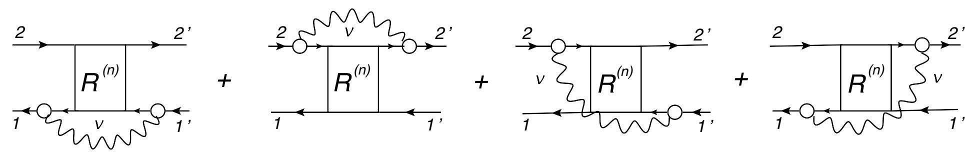

*Dyson-Bethe-Salpeter Equation:*

$$R(\omega) = R^0(\omega) + R^0(\omega)\,[V+\Phi(\omega)-\Phi(0)\,]\,R(\omega)$$

*Conventional NFT* $\Phi_{21,2'1'} =$



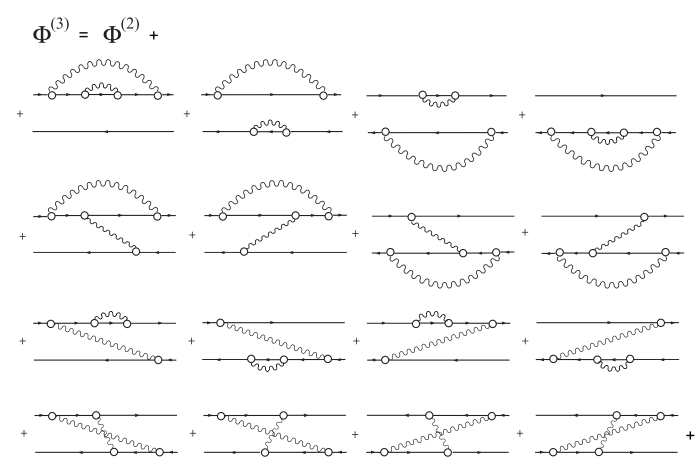

*Extended NFT:* $\Phi^{(n+1)}_{212'1'} =$




*Generalized approach for the correlated propagators*

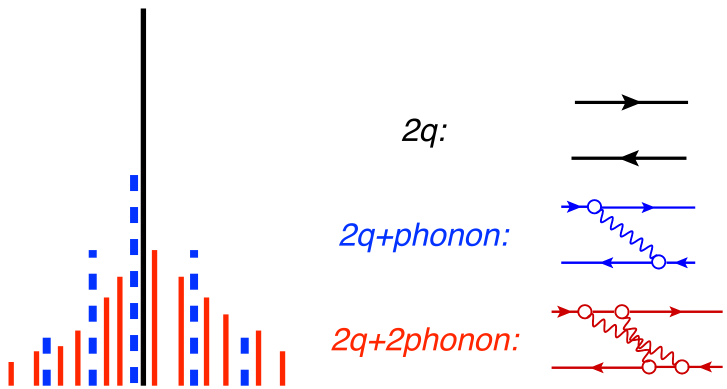
*n-th order: E.L. PRC 91, 034332 (2015)*

*Ab-initio formulation,*
*$\Phi^{(3)}$ implementation; 2q+2phonon correlations:*
*E.L., P. Schuck, PRC 100, 064320 (2019)*

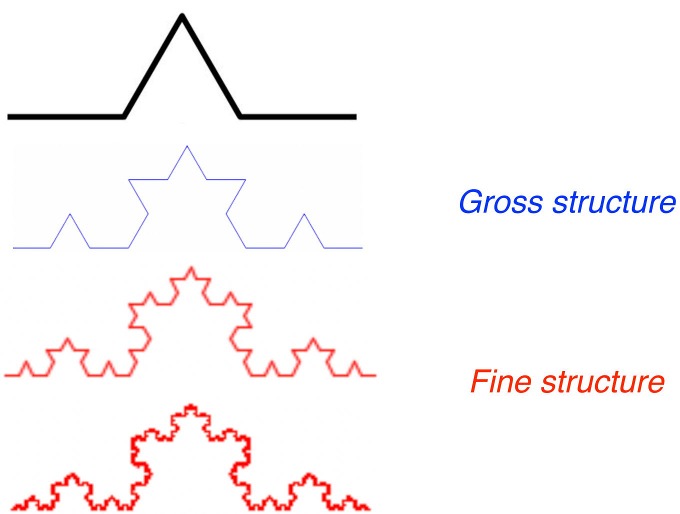


# Excitation spectrum: Hierarchy of configuration complexity

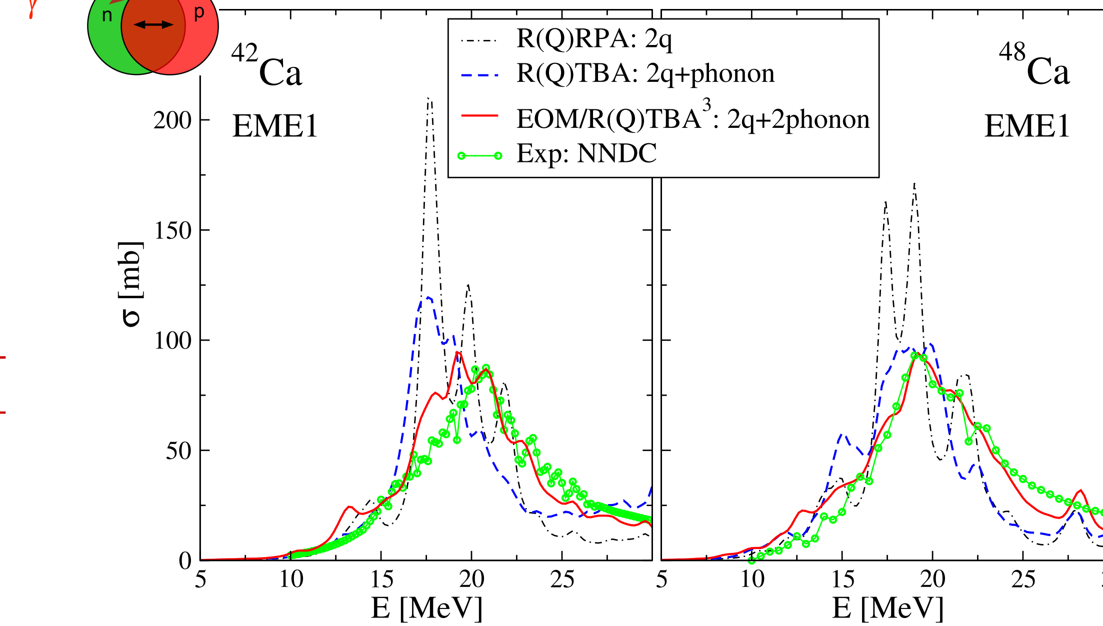
## *Fragmentation mechanism*



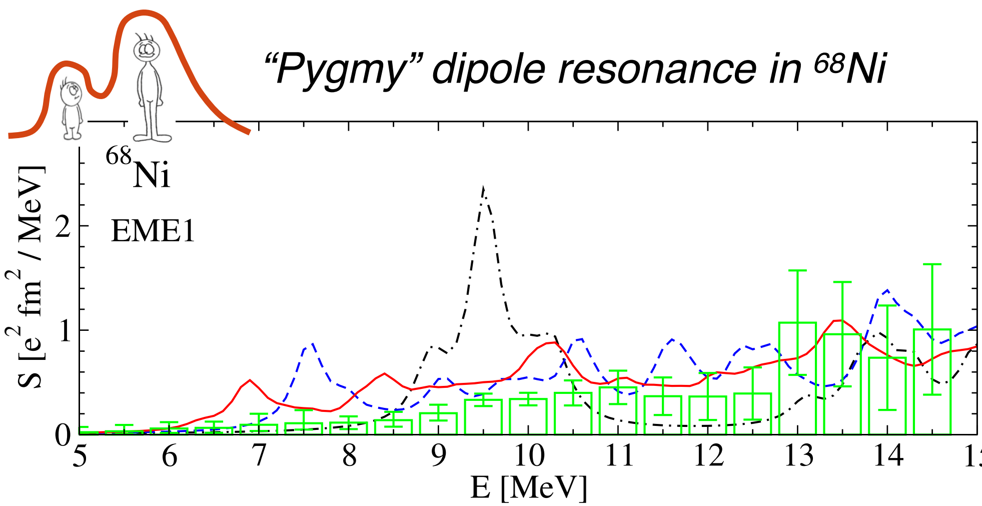

## *Fractals: Koch curve*




## *Giant dipole resonance in $^{42,48}$Ca*




## *"Pygmy" dipole resonance in $^{68}$Ni*




*Data: O. Wieland et al., Phys. Rev. C 98, 064313 (2018)*

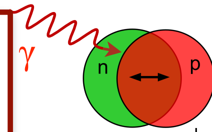
# Excitation spectrum: Hierarchy of configuration complexity

***A high-quality self-consistent (Q)RPA is the key to quantitative success:***

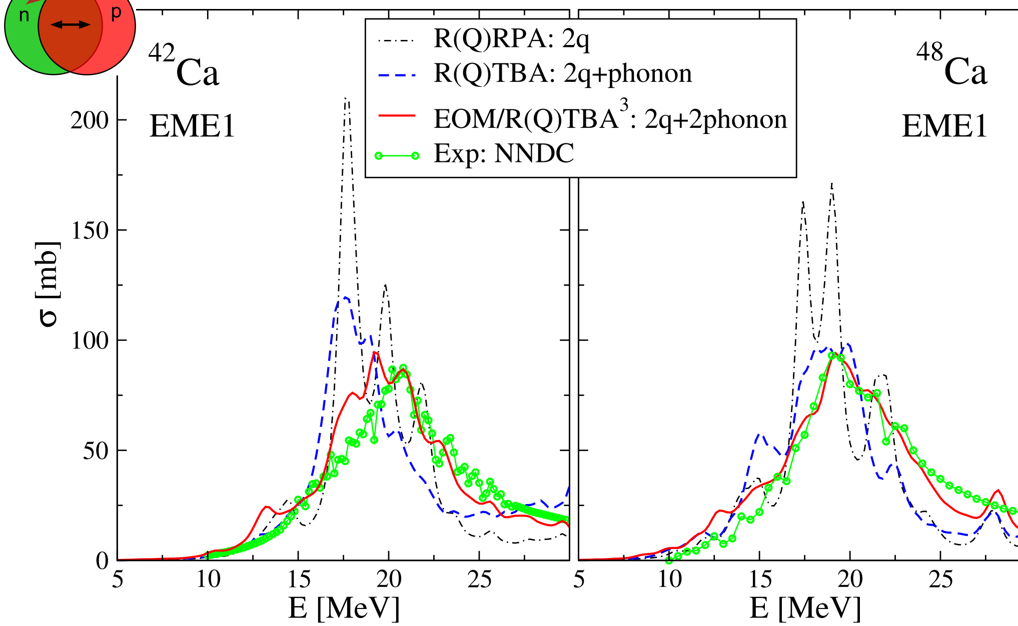
*N. Paar, T. Niksic, D. Vretenar, P. Ring et al.*
*Rep. Prog, Phys. 70, 691 (2007)*
*PRC 69, 054303 (2004)*
*PRC 67, 034312 (2003)*
*PRC 63, 047301 (2001)*

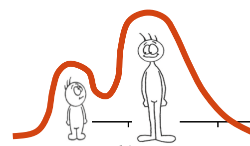
***Beyond-RQRPA: fully parameter-free***

- *The RQRPA solutions are the building blocks.*
- *Reasonable energies and transition probabilities of the RQRPA modes are extremely important for the quantitative success.*
- *Beyond-RQRPA correlations can be included based on the same computational framework.*
- *Cross-check: Momentum-space vs configuration (Dirac) space solutions.*

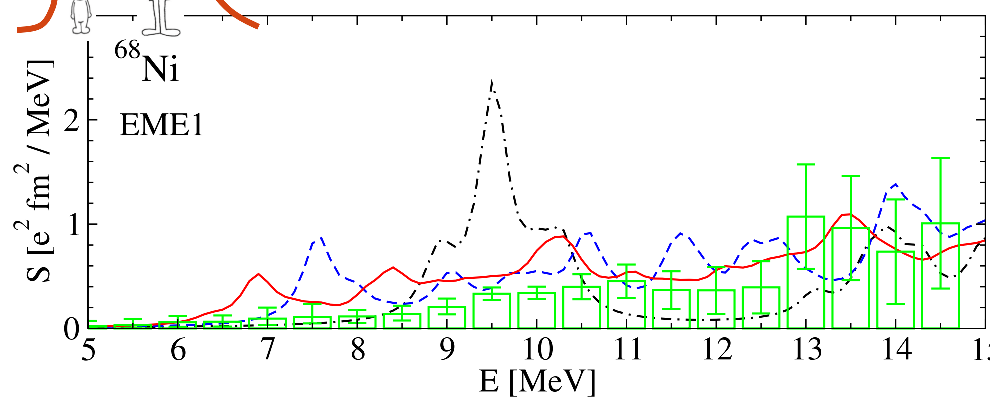



*Giant dipole resonance in $^{42,48}$Ca*



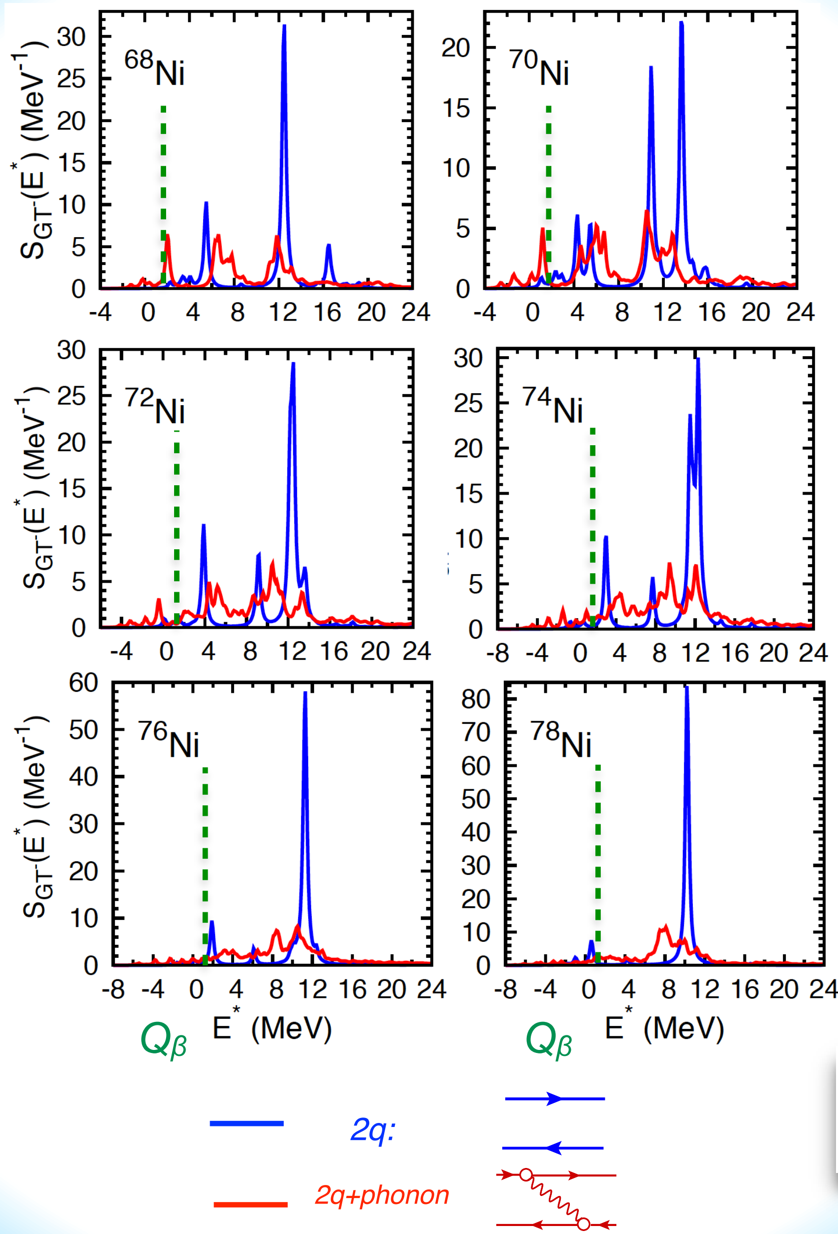



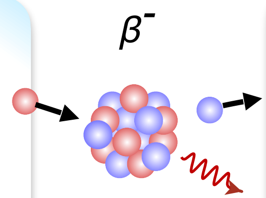
*"Pygmy" dipole resonance in $^{68}$Ni*

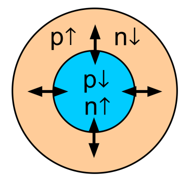



*Data: O. Wieland et al., Phys. Rev. C 98, 064313 (2018)*

# Spin-isospin excitations: Gamow-Teller resonance in neutron-rich nickel



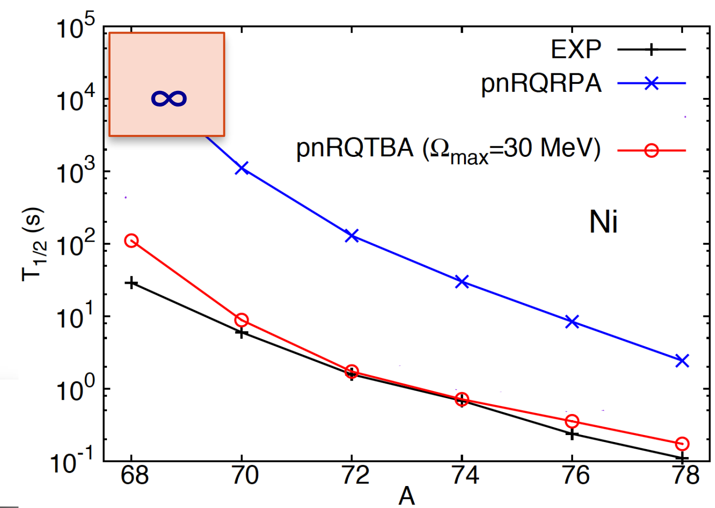




*Excitation operator:*

$$P = \sum_i \sigma^{(i)} \tau_-^{(i)}$$

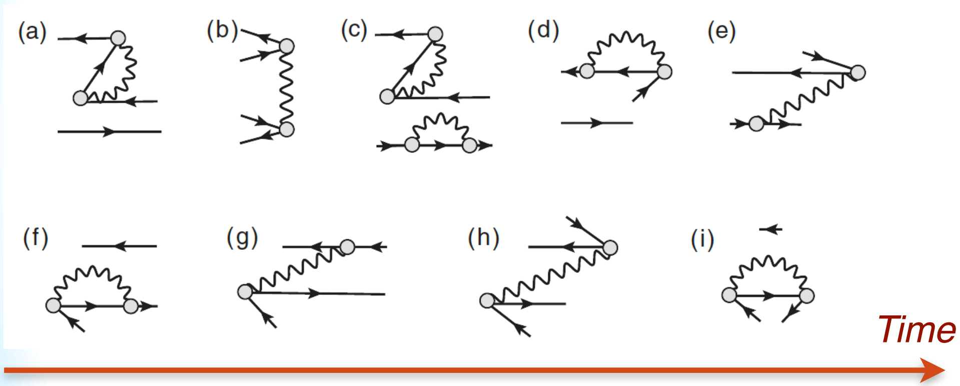
*$\beta^-$ decay half-life*

$$T_{1/2}^{-1} = \frac{g_A^2}{D} \int_{Q_\beta} f(Z, \Delta_{np} - E) S(E) dE$$

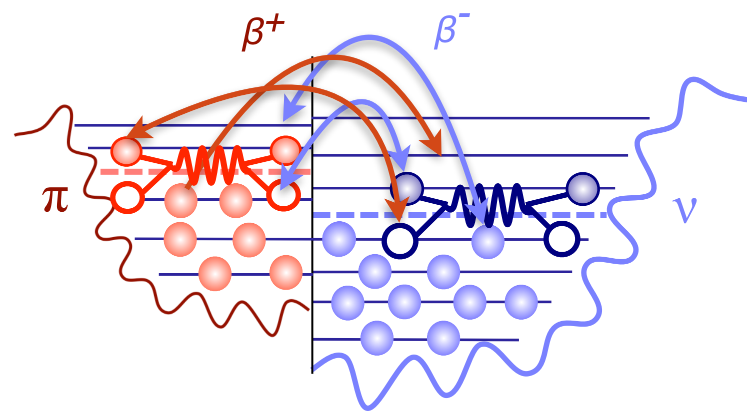



***C. Robin**, E.L.,*
*EPJA 52, 205*

*Ground state correlations induced by QVC:*
*backward-going diagrams (V. Tselyaev, 1989)*

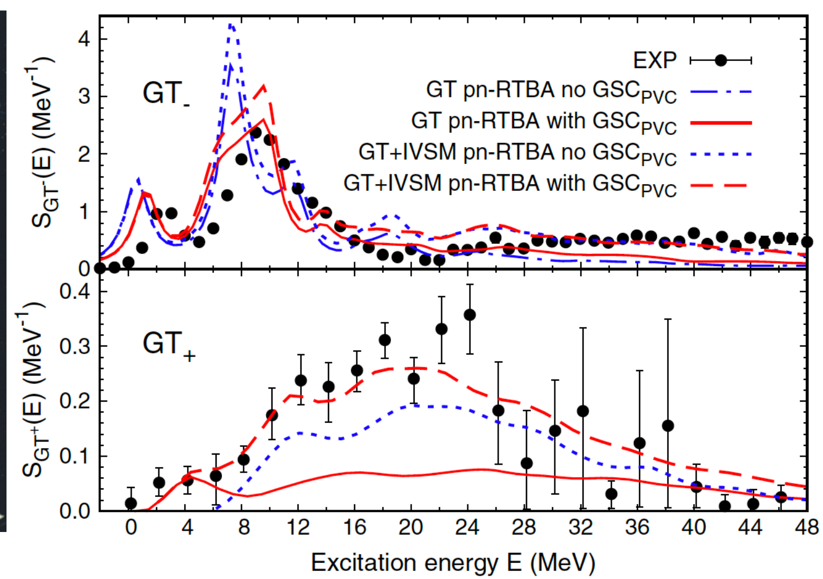



*New unblocking mechanism:*




*The backward-going diagrams are solely responsible for the β+ strength in neutron-rich nuclei*

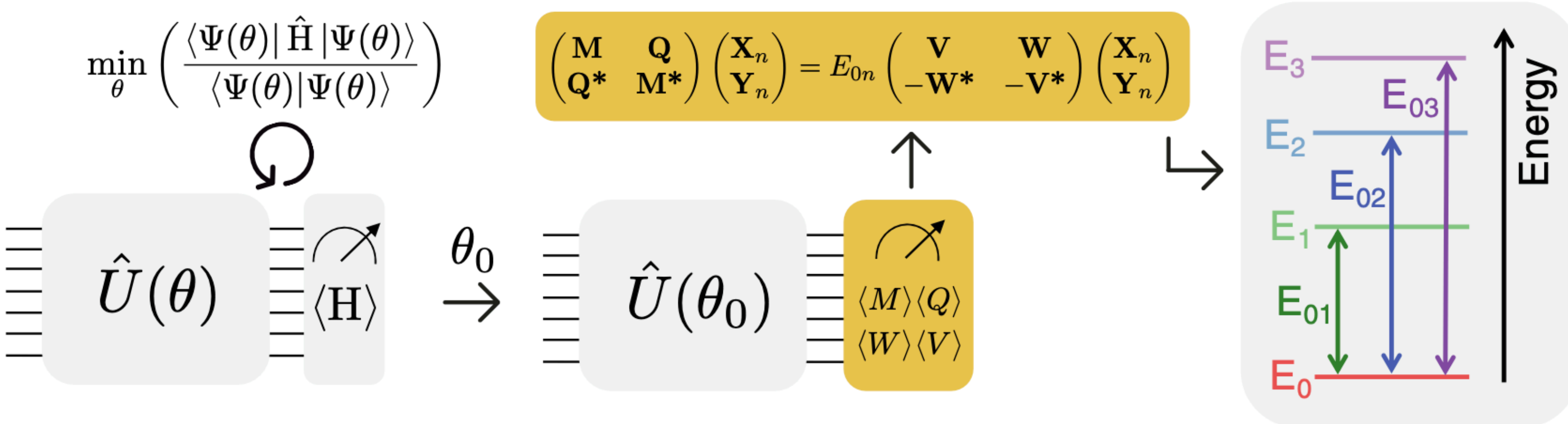
*Gamow-Teller strength in 90-Zr:*




*C. Robin, E.L., Phys. Rev. Lett. 123, 202501 (2019)*

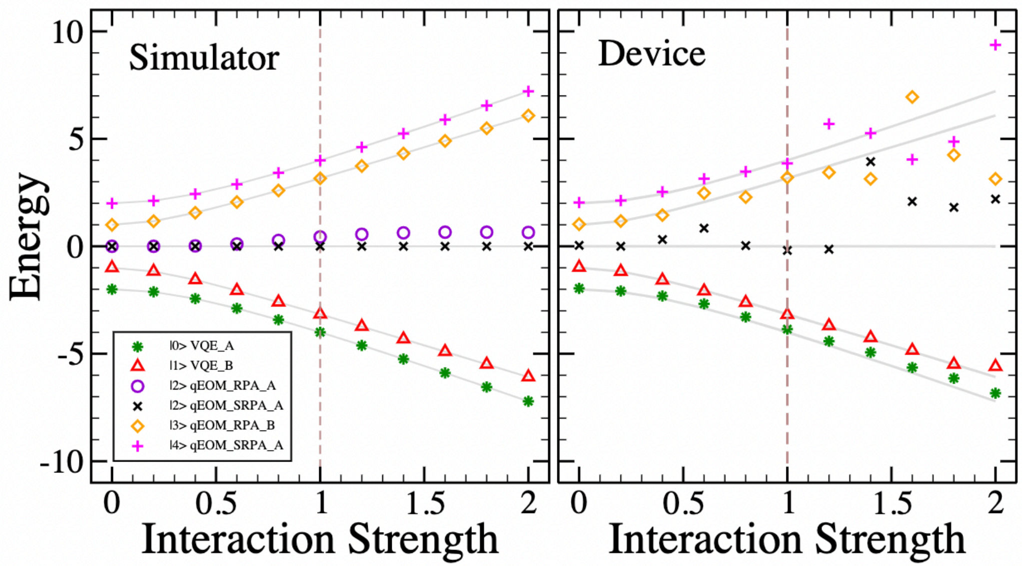
# Atomic nuclei on quantum computer: accessing emergence via entanglement

*Variational Quantum Eigensolver (VQE) + Quantum Equation of Motion (qEOM):*



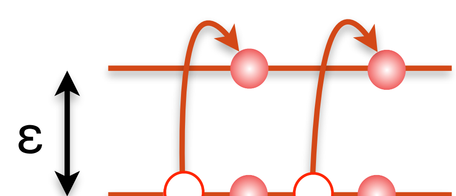

*P. Ollitrault et al., Phys. Rev. Res. 2, 043140 (2020)*

*Implementation for N = 4 (IBM-Q): RPA vs SRPA vs exact*




*Two-level*
*Lipkin Hamiltonian:*
***exactly solvable***

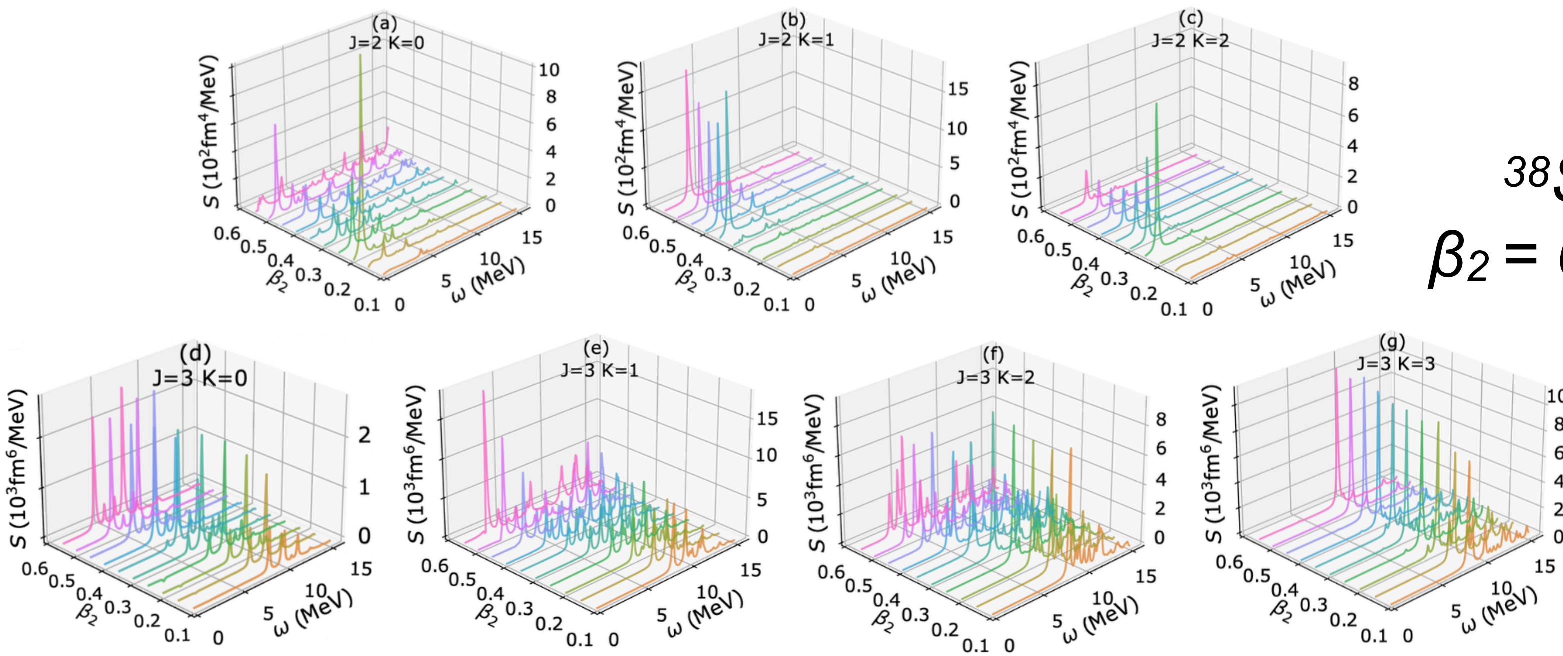
$$\hat{H} = \epsilon \hat{J}_z - \frac{v}{2}\left(\hat{J}_+^2 + \hat{J}_-^2\right)$$




*M. Hlatshwayo et al.,*
*Phys. Rev. C 106, 024319 (2022)*

# Single-(quasi)particle states. New implementation: FAM-QRPA+QVC for deformed nuclei

*(i)* Relativistic meson-nucleon Lagrangian + *(ii)* Relativistic Hartree-Bogoliubov (RHB) + *(iii)* Quasiparticle random phase approximation (QRPA): $J = 2^+ - 5^-$, $K = [0,J]$. Finite amplitude method (FAM): A. Bjelčić et al., CPC 253, 107184 (2020). Relativistic DD-PC1 interaction.




$^{38}Si$

$\beta_2 = 0.31$

*(iv)* QVC vertex extraction:

$$\Gamma^{(ij)\varkappa}_{\nu\nu'} = \lim_{\delta\to 0}\sqrt{\frac{\delta}{\pi S(\omega_\varkappa)}}\,\mathrm{Im}\Big(\delta\mathcal{H}^{(ij)}_{\nu\nu'}(\omega_\varkappa + i\delta)\Big)$$

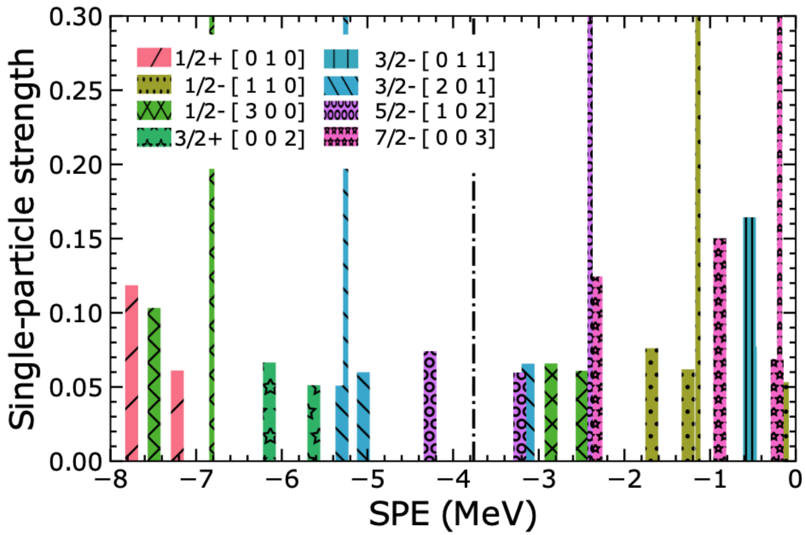
Variation of the HFB Hamiltonian at the QRPA pole

*(v)* Dyson Eq. solution

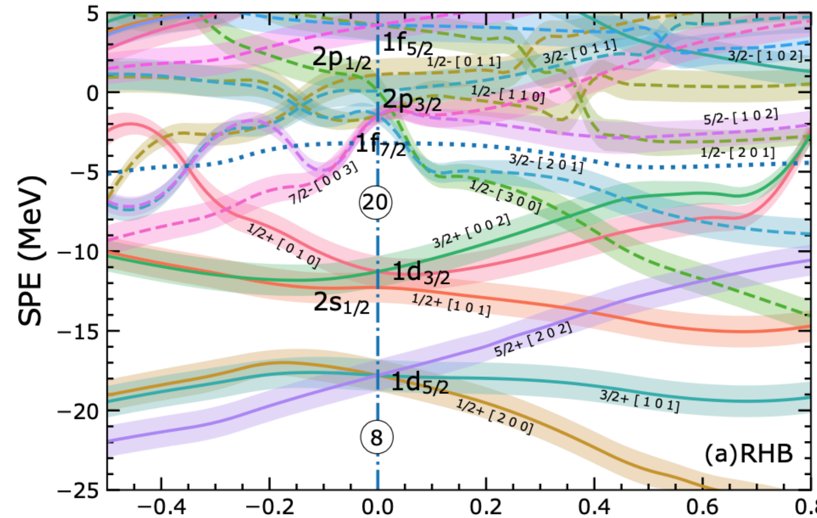
[E.L., Y. Zhang, PRC 104, 044303 (2021)]

# Single-(quasi)particle states in $^{38}Si$

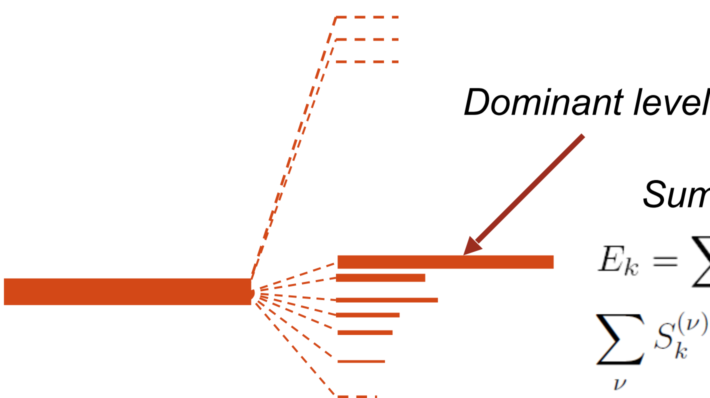
*Fragmentation of quasiparticle states: RHB vs RHB+QVC*




*Nilsson diagram: RHB*




*Fragmentation mechanism: schematic*

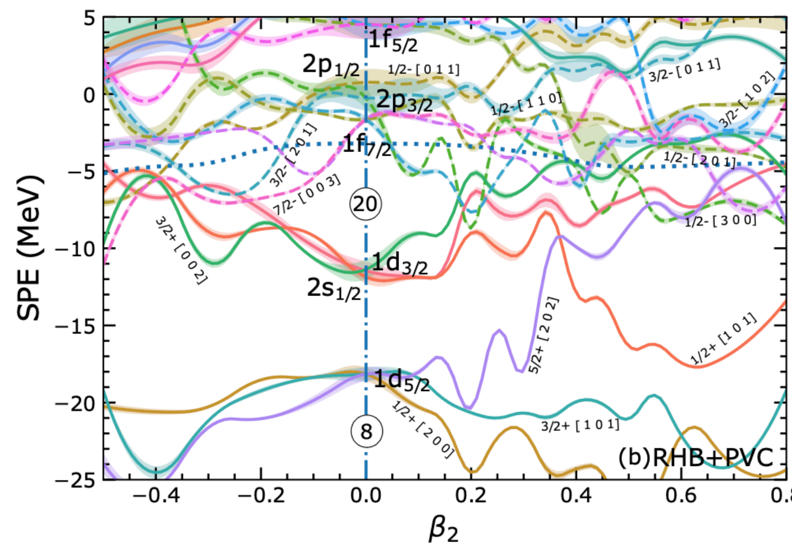



*Sum rules:*

$$E_k = \sum E_k^{(\nu)} S_k^{(\nu)}$$

$$\sum_{\nu} S_k^{(\nu)} = 1$$

*Nilsson diagram: RHB+QVC (dominant only)*



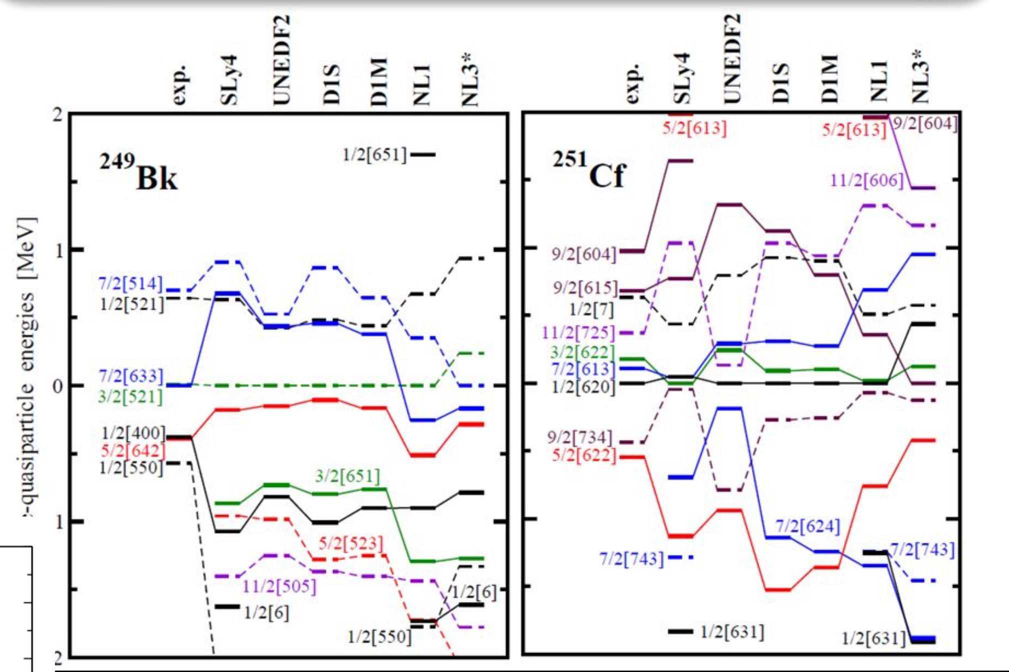

*Y. Zhang et al., PRC 105, 044326 (2022)*

# Single-(quasi)particle states in $^{249,251}Cf$

*A. Afanasjev et al.: Long-standing problem of the description of single-particle states in deformed nuclei.*

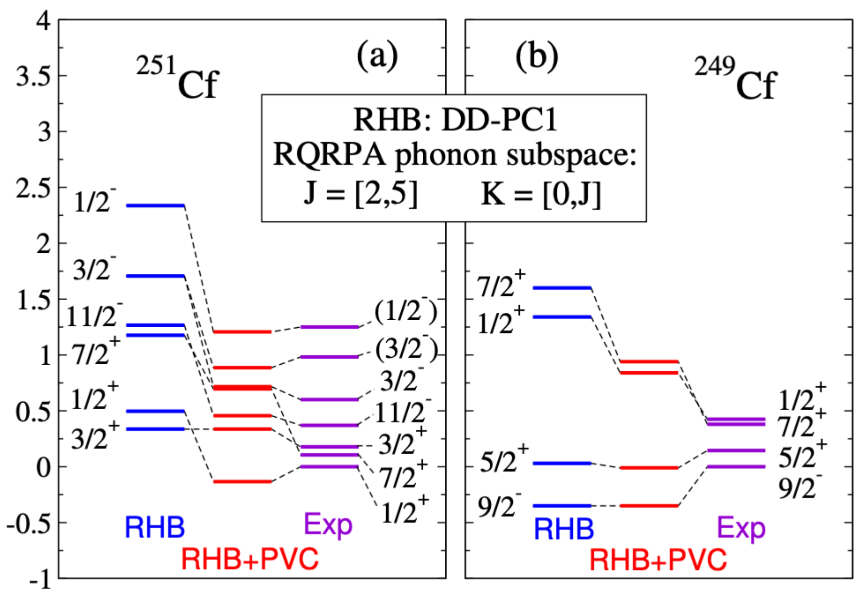
*Systematic studies for $^{249}Bk$ and $^{251}Cf$ in the mean-field approximation:*

*Deformed one-quasiparticle states: covariant and non-relativistic mean-field calculations vs experiment:*




$^{250}Cf$

$\beta_2 = 0.29$




*Beyond mean field: RHB+QVC calculations. Dominant fragments in $^{251}Cf$ and $^{249}Cf$.*

*The spectroscopic factors are quenched even stronger than in spherical nuclei. Can this be measured?*

*Y. Zhang et al., PRC 105, 044326 (2022)*

# Finite amplitude method extended beyond QRPA (preliminary):

Generalized FAM (FAM-QVC)

QVC

$$\delta\mathcal{R}^{(20)}_{\mu\nu}(\omega) = \frac{\delta\mathcal{H}^{20}_{\mu\nu}(\omega) + \sum_{\mu'\nu'} \Phi^{(+)}_{\mu\nu'\nu\mu'}(\omega)\delta\mathcal{R}^{(20)}_{\mu'\nu'}(\omega) + F^{20}_{\mu\nu}}{\omega - E_\mu - E_\nu}$$

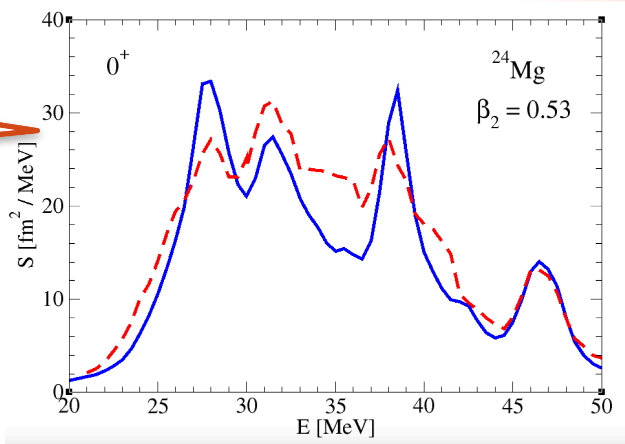
$$\delta\mathcal{R}^{(02)}_{\mu\nu}(\omega) = \frac{\delta\mathcal{H}^{02}_{\mu\nu}(\omega) + \sum_{\mu'\nu'} \Phi^{(-)}_{\mu\nu'\nu\mu'}(\omega)\delta\mathcal{R}^{(02)}_{\mu'\nu'}(\omega) + F^{02}_{\mu\nu}}{-\omega - E_\mu - E_\nu}.$$

QVC amplitude (leading approximation):

$$\Phi^{(+)}_{\mu\nu'\nu\mu'}(\omega) = \sum_n \Big[\delta_{\mu\mu'} \sum_{\nu''} \frac{\bar\Gamma^{(11)n}_{\nu''\nu}\bar\Gamma^{(11)n*}_{\nu''\nu'}}{\omega - E_\mu - E_{\nu''} - \omega_n} + \delta_{\nu\nu'} \sum_{\mu''} \frac{\Gamma^{(11)n}_{\mu\mu''}\Gamma^{(11)n*}_{\mu'\mu''}}{\omega - E_{\mu''} - E_\nu - \omega_n} - \frac{\Gamma^{(11)n}_{\mu\mu'}\bar\Gamma^{(11)n*}_{\nu\nu'}}{\omega - E_{\mu'} - E_\nu - \omega_n} - \frac{\Gamma^{(11)n*}_{\mu'\mu}\bar\Gamma^{(11)n}_{\nu'\nu}}{\omega - E_\mu - E_{\nu'} - \omega_n}\Big];$$

E.L., Y. Zhang, arXiv:2208.07843

***Proof of principle:***
*GMR in 24-Mg*
*in a restricted model space*




***Ongoing:***
- *Convergence improvement*
- *Optimization*
- *Cross-check routines*

# Finite-temperature response: the ph+phonon dynamical kernel

$$R_{12,1'2'}(t-t') = -i < \mathcal{T}(\psi^{\dagger}_1\psi_2)(t)\psi^{\dagger}_{2'}\psi_{1'})(t') > \quad \rightarrow \quad \mathcal{R}_{12,1'2'}(t-t') = -i < \mathcal{T}(\psi^{\dagger}_1\psi_2)(t)\psi^{\dagger}_{2'}\psi_{1'})(t') >_T$$

$$< ... > \equiv < 0|...|0 > \quad \rightarrow \quad < ... >_T \equiv \sum_n exp\Big(\frac{\Omega - E_n - \mu N}{T}\Big) < n|...|n >$$

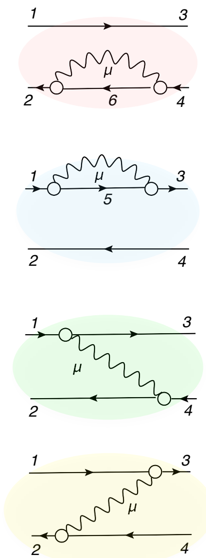
*averages* → *thermal averages*

***Method: EOM for Matsubara Green's functions***

$$\mathcal{R}_{14,23}(\omega,T) = \tilde{\mathcal{R}}^0_{14,23}(\omega,T) + $$
$$+ \sum_{1'2'3'4'} \tilde{\mathcal{R}}^0_{12',21'}(\omega,T)\big[\tilde{V}_{1'4',2'3'}(T) + \delta\Phi_{1'4',2'3'}(\omega,T)\big]\mathcal{R}_{3'4,4'3}(\omega,T)$$
$$\delta\Phi_{1'4',2'3'}(\omega,T) = \Phi_{1'4',2'3'}(\omega,T) - \Phi_{1'4',2'3'}(0,T)$$

*T > 0:*     *1p1h+phonon dynamical kernel:*     *T = 0:*

$$\Phi^{(ph)}_{14,23}(\omega,T) = \frac{1}{n_{43}(T)} \sum_{\mu;\eta_\mu=\pm1} \eta_\mu \Big[\delta_{13}\sum_6 \gamma^{\eta_\mu}_{\mu;62}\gamma^{\eta_\mu *}_{\mu;64} \times$$
$$\times \frac{\big(N(\eta_\mu\Omega_\mu) + n_6(T)\big)\big(n(\varepsilon_6 - \eta_\mu\Omega_\mu,T) - n_1(T)\big)}{\omega - \varepsilon_1 + \varepsilon_6 - \eta_\mu\Omega_\mu} +$$
$$+ \delta_{24}\sum_5 \gamma^{\eta_\mu}_{\mu;15}\gamma^{\eta_\mu *}_{\mu;35} \times$$
$$\times \frac{\big(N(\eta_\mu\Omega_\mu) + n_2(T)\big)\big(n(\varepsilon_2 - \eta_\mu\Omega_\mu,T) - n_5(T)\big)}{\omega - \varepsilon_5 + \varepsilon_2 - \eta_\mu\Omega_\mu} -$$
$$- \gamma^{\eta_\mu}_{\mu;13}\gamma^{\eta_\mu *}_{\mu;24} \times$$
$$\times \frac{\big(N(\eta_\mu\Omega_\mu) + n_2(T)\big)\big(n(\varepsilon_2 - \eta_\mu\Omega_\mu,T) - n_3(T)\big)}{\omega - \varepsilon_3 + \varepsilon_2 - \eta_\mu\Omega_\mu} -$$
$$- \gamma^{\eta_\mu *}_{\mu;31}\gamma^{\eta_\mu}_{\mu;42} \times$$
$$\times \frac{\big(N(\eta_\mu\Omega_\mu) + n_4(T)\big)\big(n(\varepsilon_4 - \eta_\mu\Omega_\mu,T) - n_1(T)\big)}{\omega - \varepsilon_1 + \varepsilon_4 - \eta_\mu\Omega_\mu}\Big],$$




$$\Phi^{(ph,ph)}_{14,23}(\omega) = \sum_\mu \times$$
$$\times \Big[\delta_{13}\sum_6 \frac{\gamma^\mu_{62}\gamma^{\mu *}_{64}}{\omega - \varepsilon_1 + \varepsilon_6 - \Omega_\mu} +$$
$$+ \delta_{24}\sum_5 \frac{\gamma^\mu_{15}\gamma^{\mu *}_{35}}{\omega - \varepsilon_5 + \varepsilon_2 - \Omega_\mu} -$$
$$- \frac{\gamma^\mu_{13}\gamma^{\mu *}_{24}}{\omega - \varepsilon_3 + \varepsilon_2 - \Omega_\mu} -$$
$$- \frac{\gamma^{\mu *}_{31}\gamma^\mu_{42}}{\omega - \varepsilon_1 + \varepsilon_4 - \Omega_\mu}\Big]$$

# Finite-temperature response: the ph+phonon dynamical kernel

$$R_{12,1'2'}(t-t') = -i < \mathcal{T}(\psi^\dagger_1\psi_2)(t)\psi^\dagger_{2'}\psi_{1'})(t') > \;\rightarrow\; \mathcal{R}_{12,1'2'}(t-t') = -i < \mathcal{T}(\psi^\dagger_1\psi_2)(t)\psi^\dagger_{2'}\psi_{1'})(t') >_T$$

$$< ... > \equiv < 0|...|0 > \;\rightarrow\; < ... >_T \equiv \sum_n exp\Big(\frac{\Omega - E_n - \mu N}{T}\Big) < n|...|n >$$

*averages* → *thermal averages*

**Method: EOM for Matsubara Green's functions**

$$\mathcal{R}_{14,23}(\omega,T) = \tilde{\mathcal{R}}^0_{14,23}(\omega,T) + + \sum_{1'2'3'4'} \tilde{\mathcal{R}}^0_{12',21'}(\omega,T)\big[\tilde{V}_{1'4',2'3'}(T) + \delta\Phi_{1'4',2'3'}(\omega,T)\big]\mathcal{R}_{3'4,4'3}(\omega,T)$$

$$\delta\Phi_{1'4',2'3'}(\omega,T) = \Phi_{1'4',2'3'}(\omega,T) - \Phi_{1'4',2'3'}(0,T)$$

**See the talk of Herlik Wibowo**

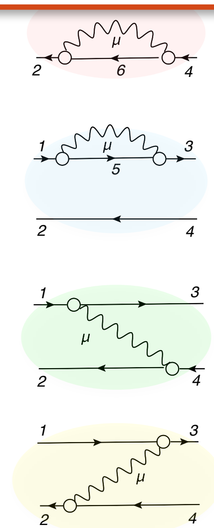
$$\ldots n_{43}(T) \sum_{\mu;\eta_\mu=\pm1} \Big[ \ldots \sum_6 \ldots$$

$$\times \frac{(N(\eta_\mu\Omega_\mu) + n_6(T))(n(\varepsilon_6 - \eta_\mu\Omega_\mu, T) - n_1(T))}{\omega - \varepsilon_1 + \varepsilon_6 - \eta_\mu\Omega_\mu} + \delta_{24}\sum_5 \gamma^{\eta_\mu}_{\mu;15}\gamma^{\eta_\mu *}_{\mu;35} \times$$

$$\times \frac{(N(\eta_\mu\Omega_\mu) + n_2(T))(n(\varepsilon_2 - \eta_\mu\Omega_\mu, T) - n_5(T))}{\omega - \varepsilon_5 + \varepsilon_2 - \eta_\mu\Omega_\mu} - \gamma^{\eta_\mu}_{\mu;13}\gamma^{\eta_\mu *}_{\mu;24} \times$$

$$\times \frac{(N(\eta_\mu\Omega_\mu) + n_2(T))(n(\varepsilon_2 - \eta_\mu\Omega_\mu, T) - n_3(T))}{\omega - \varepsilon_3 + \varepsilon_2 - \eta_\mu\Omega_\mu} - \gamma^{\eta_\mu *}_{\mu;31}\gamma^{\eta_\mu}_{\mu;42} \times$$

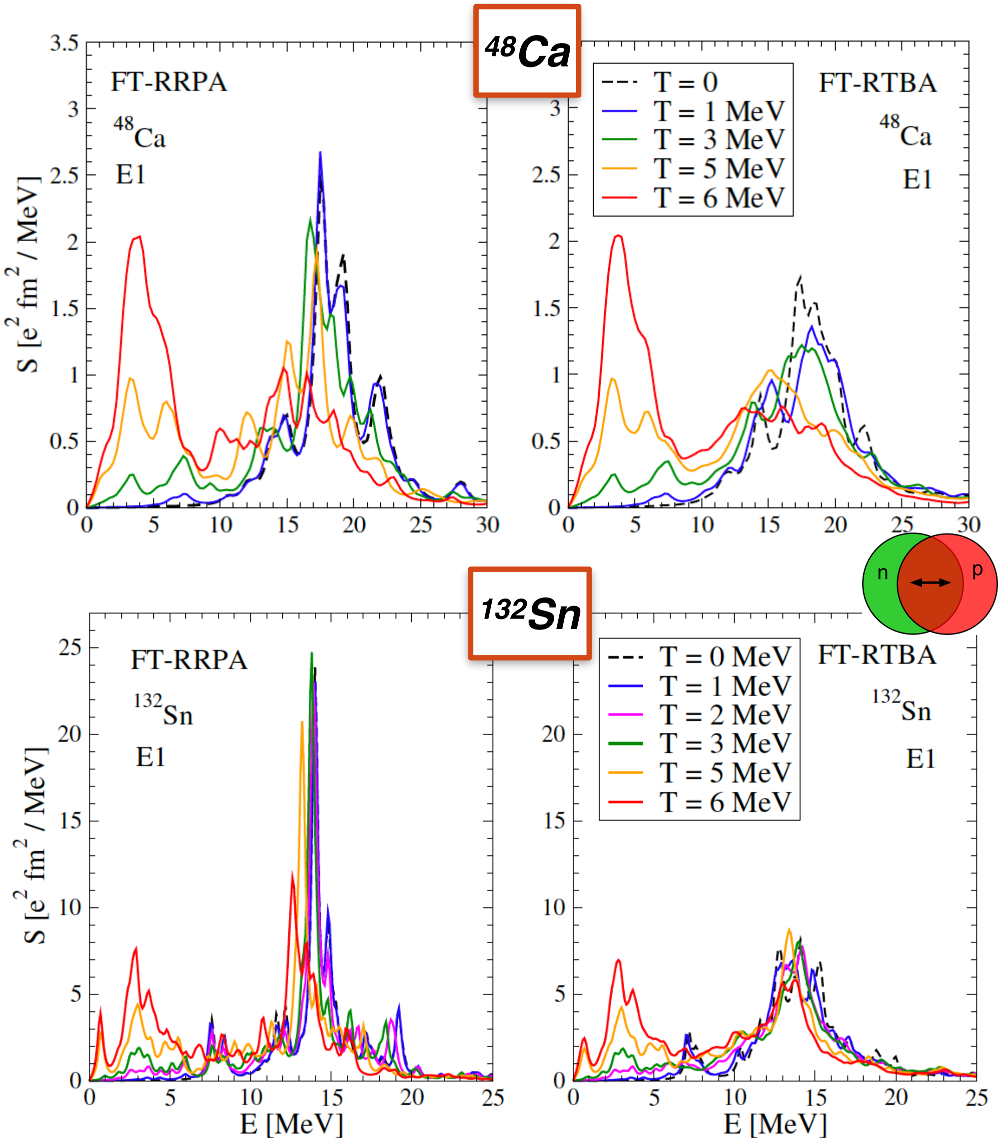
$$\times \frac{(N(\eta_\mu\Omega_\mu) + n_4(T))(n(\varepsilon_4 - \eta_\mu\Omega_\mu, T) - n_1(T))}{\omega - \varepsilon_1 + \varepsilon_4 - \eta_\mu\Omega_\mu}\Big],$$



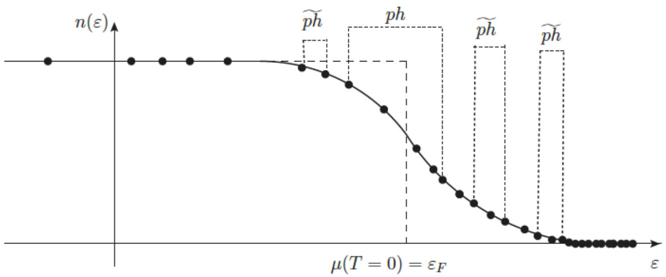
$$\Phi_{14,23}(\omega) = \sum_\mu \times \Big[\delta_{13}\sum_6 \frac{\gamma^\mu_{62}\gamma^{\mu *}_{64}}{\omega - \varepsilon_1 + \varepsilon_6 - \Omega_\mu} + \delta_{24}\sum_5 \frac{\gamma^\mu_{15}\gamma^{\mu *}_{35}}{\omega - \varepsilon_5 + \varepsilon_2 - \Omega_\mu} - \frac{\gamma^\mu_{13}\gamma^{\mu *}_{24}}{\omega - \varepsilon_3 + \varepsilon_2 - \Omega_\mu} - \frac{\gamma^{\mu *}_{31}\gamma^\mu_{42}}{\omega - \varepsilon_1 + \varepsilon_4 - \Omega_\mu}\Big]$$

# Dipole Strength at T>0: $^{48}$Ca and $^{132}$Sn

**Static only (FT-RRPA)** — **Static + dynamic (FT-RTBA)**




*Thermal unblocking:*




*0th approximation:*
*Uncorrelated propagator*

$$\tilde{R}^0_{14,23}(\omega) = \delta_{13}\delta_{24}\frac{n_2 - n_1}{\omega - \varepsilon_1 + \varepsilon_2}$$

- *New transitions due to the thermal unblocking effects*
- *More collective and non-collective modes contribute to the PVC self-energy (~400 modes at T=5-6 MeV)*
- *Broadening of the resulting GDR spectrum*
- *Development of the low-energy part => a feedback to GDR*

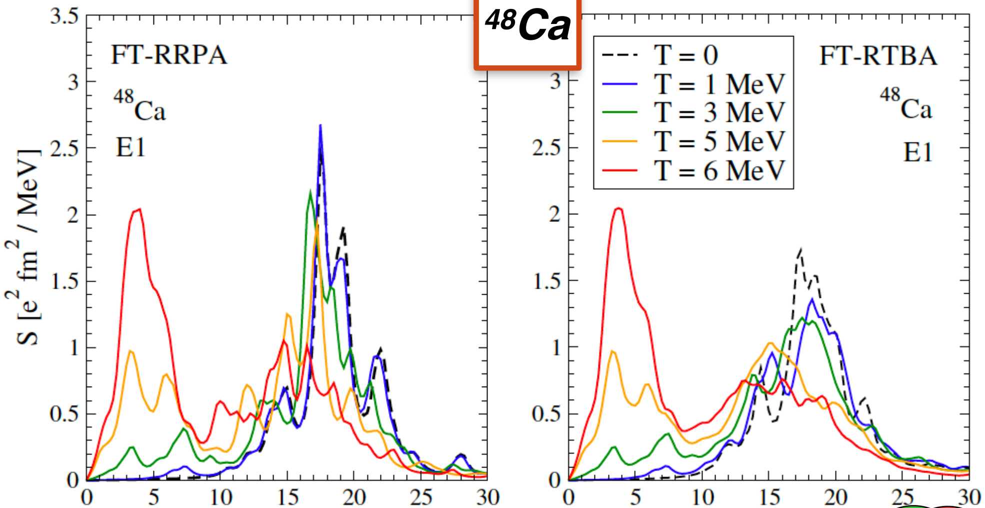
- *The spurious translation mode is properly decoupled as the mean field is modified consistently*
- *The role of the new terms in the Φ amplitude increases with temperature*
- *The role of dynamical correlations and fragmentation remain significant in the high-energy part*

*E.L., H. Wibowo, Phys. Rev. Lett. 121, 082501 (2018)*
*H. Wibowo, E.L., Phys. Rev. C 100, 024307 (2019)*

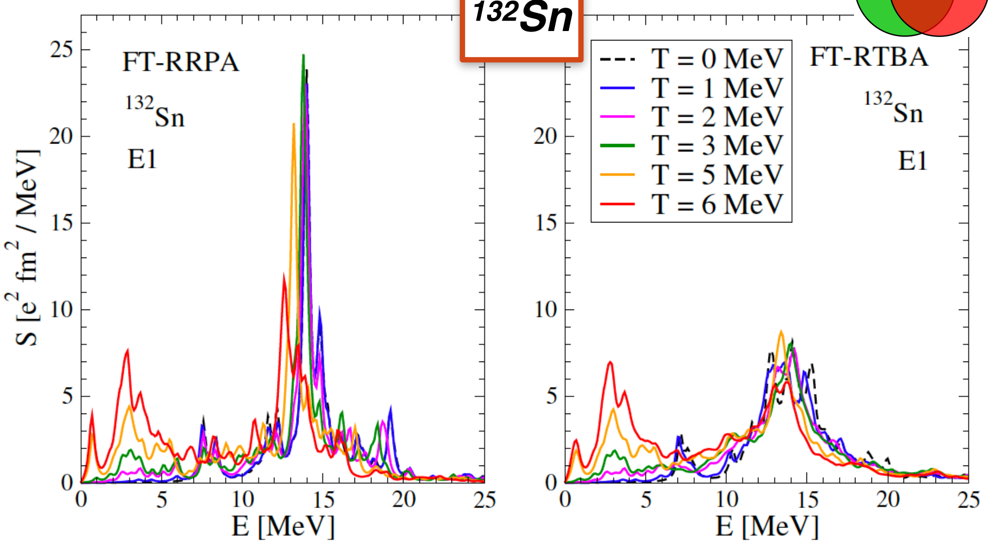
# Dipole Strength at T>0: $^{48}$Ca and $^{132}$Sn

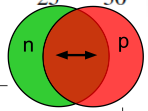
**Static only (FT-RRPA)** | **Static + dynamic (FT-RTBA)**

**$^{48}$Ca**

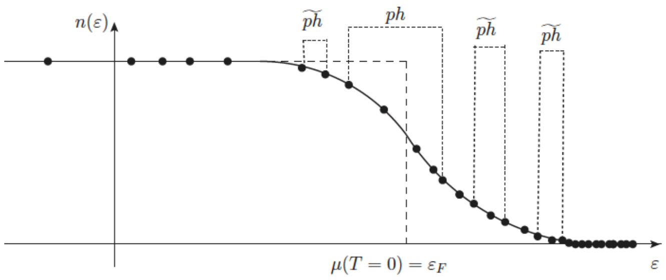



**$^{132}$Sn**



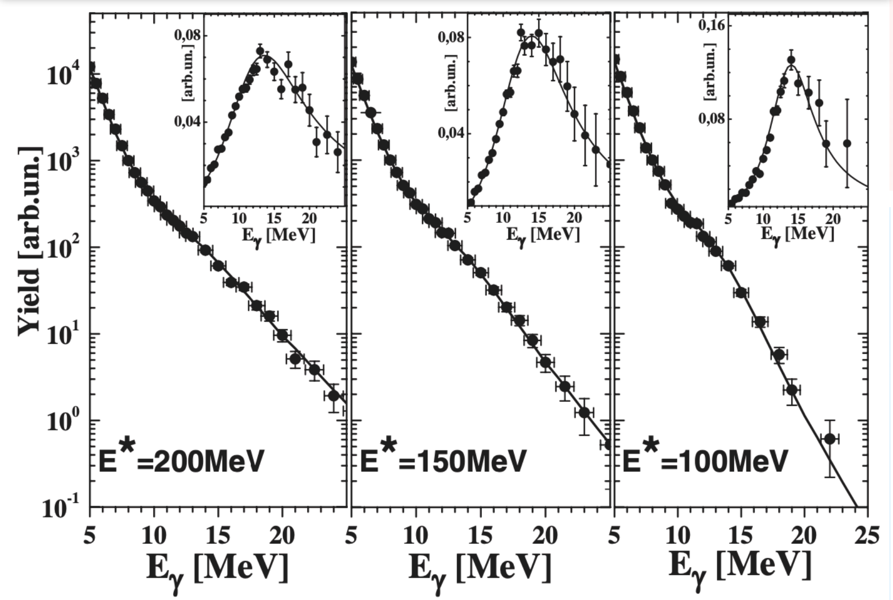

*Thermal unblocking:*



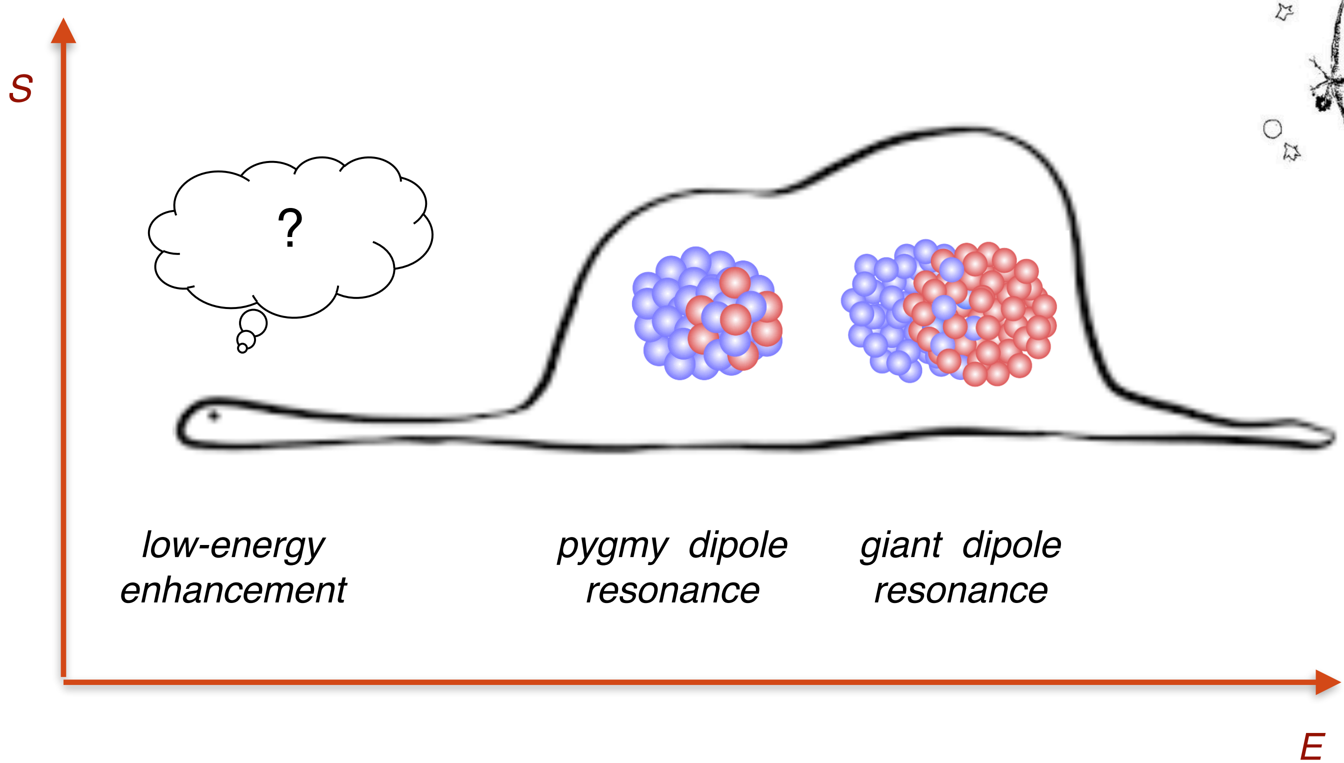

*0th approximation:*
*Uncorrelated propagator*

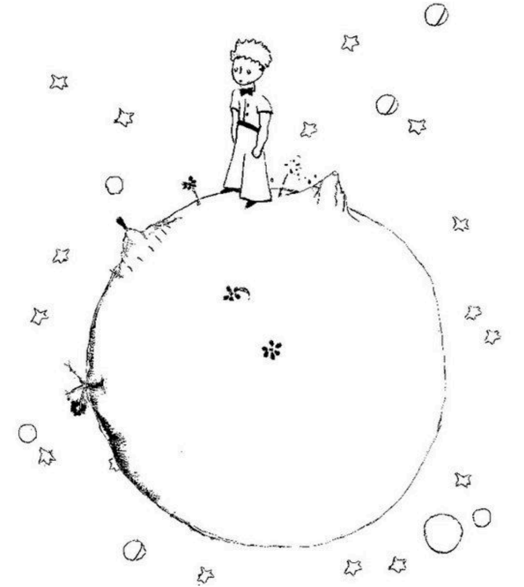
$$\tilde{R}^0_{14,23}(\omega) = \delta_{13}\delta_{24}\frac{n_2 - n_1}{\omega - \varepsilon_1 + \varepsilon_2}$$

*O. Wieland et al., PRL **97,** 012501 (2006):*
*GDR in $^{132}$Ce*

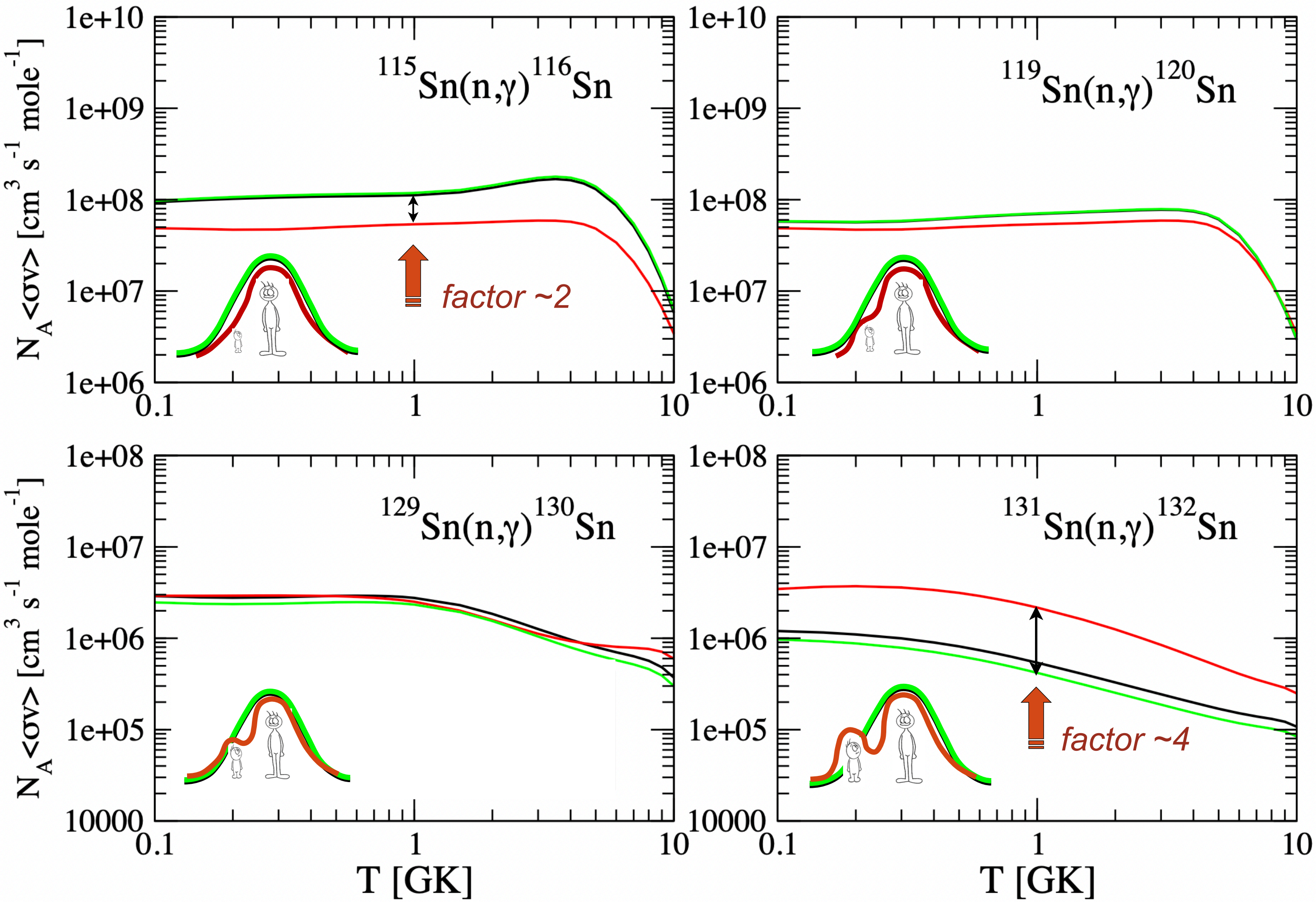


# Dipole strength in r-process nuclei



# (n,γ) stellar reaction rates





E. L., et al. Nucl. Phys. A 823, 26 (2009)

# Strength functions at finite temperature

*Response redefined:*

$$R_{12,1'2'}(t-t') = -i < \mathcal{T}(\psi_1^\dagger\psi_2)(t)\psi_{2'}^\dagger\psi_{1'})(t') > \;\;\rightarrow\;\; \mathcal{R}_{12,1'2'}(t-t') = -i < \mathcal{T}(\psi_1^\dagger\psi_2)(t)\psi_{2'}^\dagger\psi_{1'})(t') >_T$$

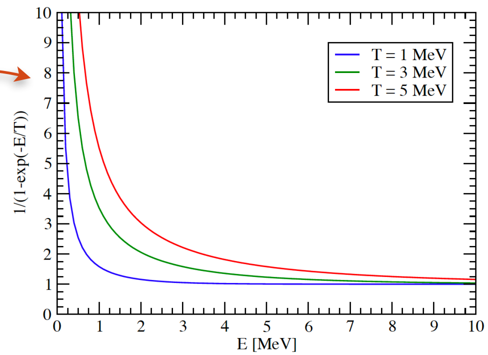
*Grand Canonical average:* $< \ldots > \equiv < 0|\ldots|0 > \;\;\rightarrow\;\; < \ldots >_T \equiv \sum_n exp\Big(\frac{\Omega - E_n - \mu N}{T}\Big) < n|\ldots|n >$

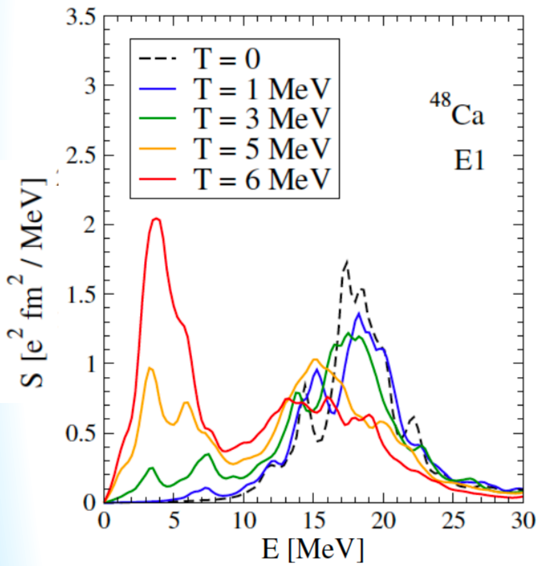
## *Dipole*

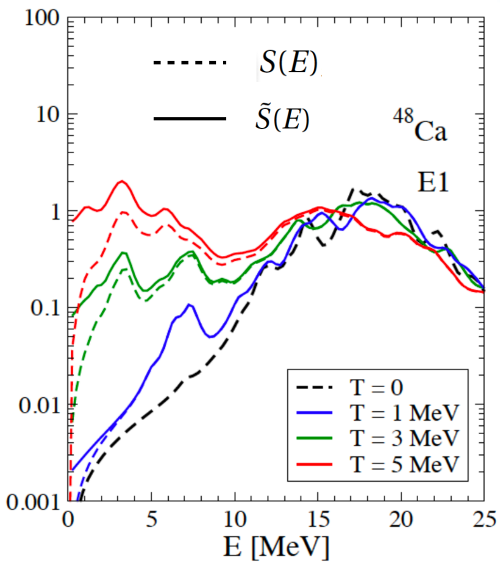
*The "upbend":*

$$\tilde{S}(E) = \frac{1}{1-e^{-E/T}} S(E)$$

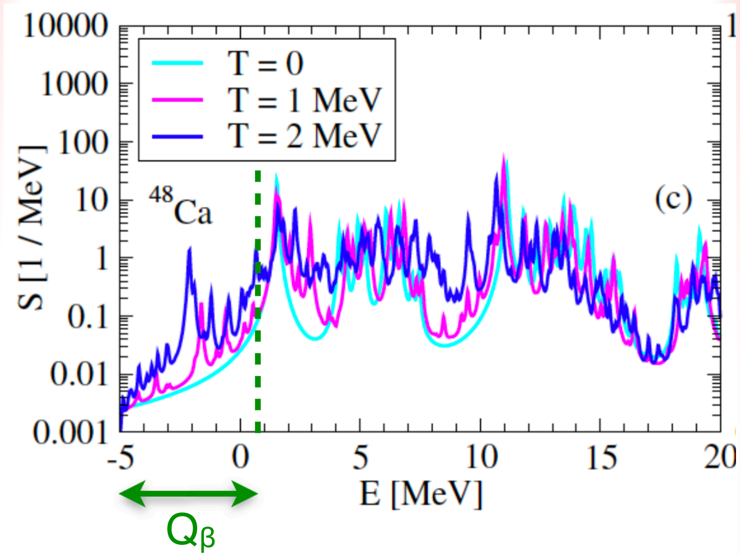
$$\lim_{E\to 0} S(E,T) = 0$$




## *Gamow-Teller*

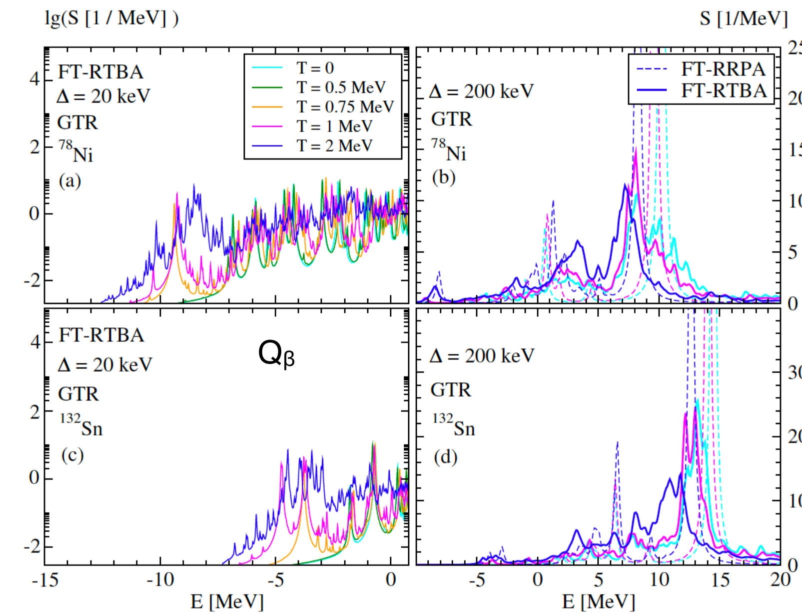




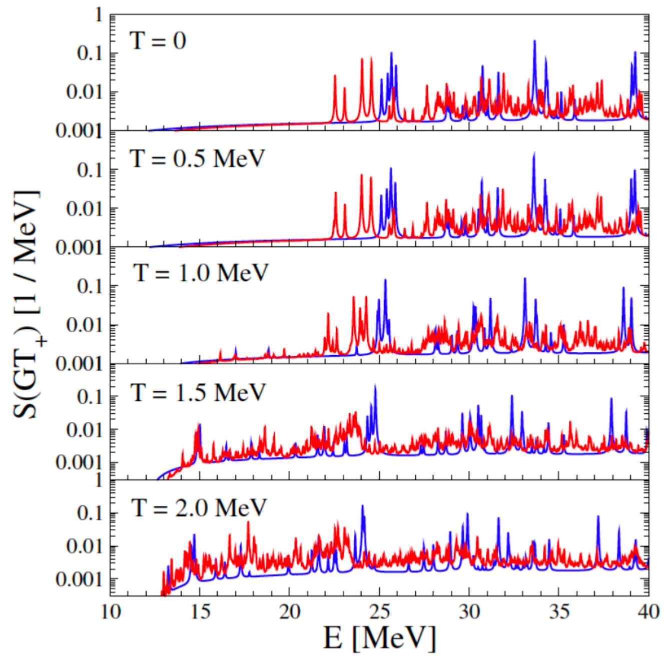
- *E. L, H. Wibowo, Phys. Rev. Lett. 121, 082501 (2018)*
- *H. Wibowo, E. L., Phys. Rev. C 100, 024307 (2019)*
- *E. L., C. Robin, H. Wibowo, Phys. Lett. B 800, 135134 (2020)*
- *E.L., C. Robin, Phys. Rev. C 103, 024326 (2021)*

# Spin-Isospin response and weak rates in hot stellar environments

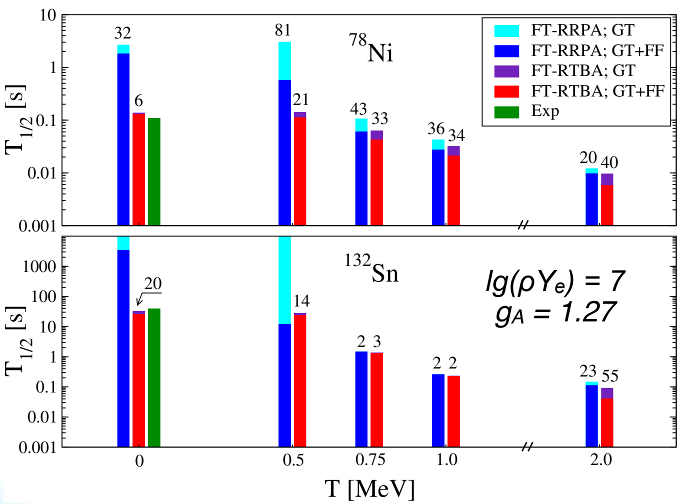
## Gamow-Teller GT_ response of $^{78}$Ni and $^{132}$Sn

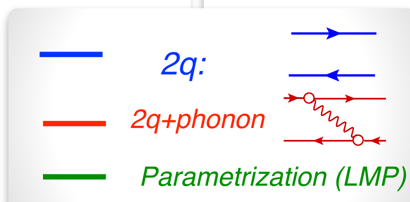



## Gamow-Teller GT+ response of $^{78}$Ni




## Beta decay half-lives

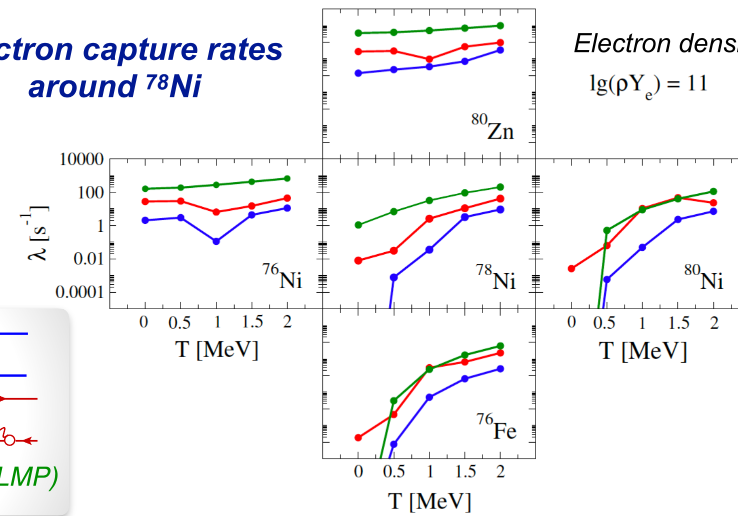


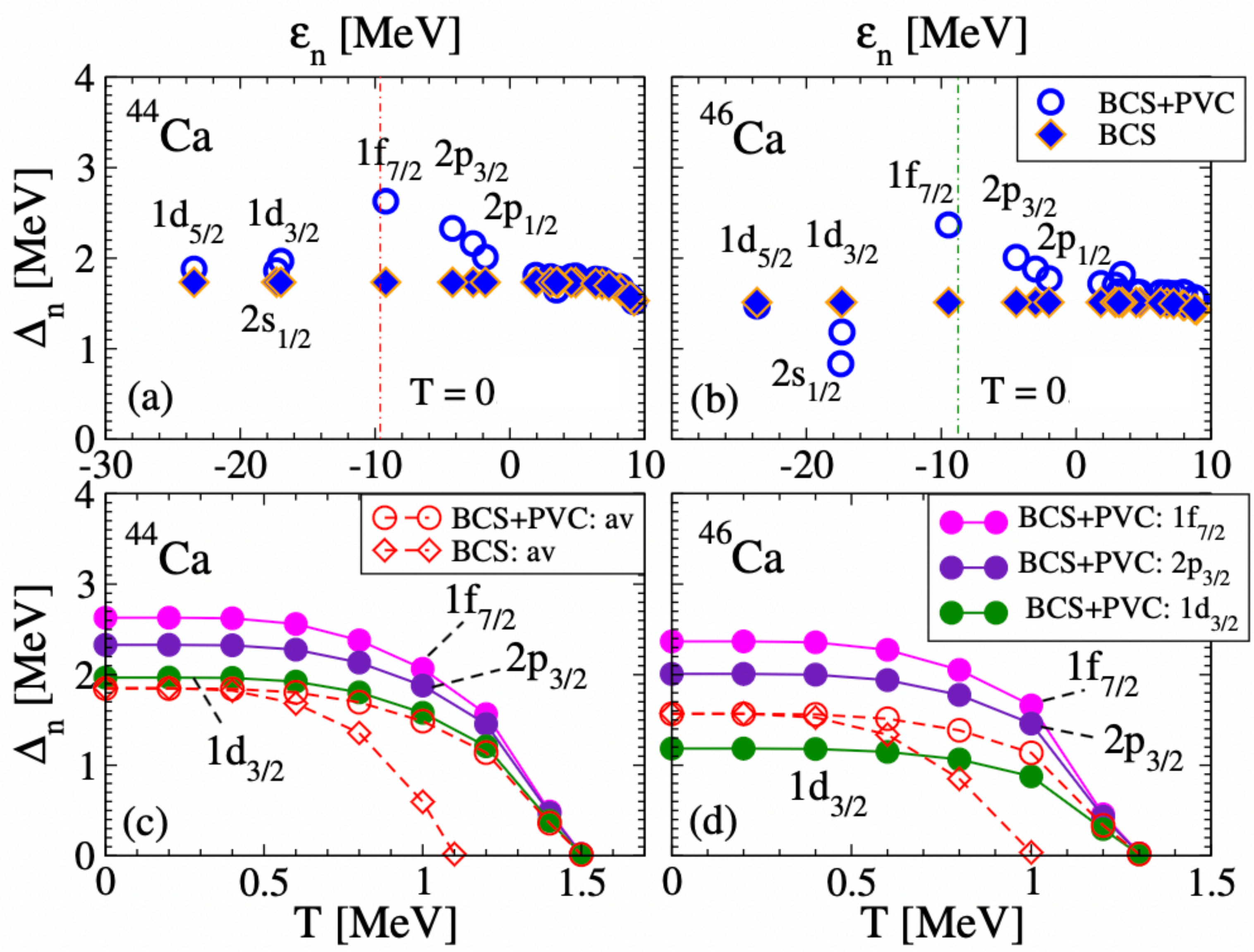

## Electron capture rates around $^{78}$Ni

*Electron density*

$\lg(\rho Y_e) = 11$



# Pairing gap at T = 0, T>0 and critical temperature




E.L., P. Schuck, Phys. Rev. C 104, 044330 (2021)

# Outlook

## Summary:

- The nuclear field theory (NFT) is formulated and advanced in the Equation of Motion (EOM) framework, with the emphasis on emergent collectivity. Not the “self-consistent Green’s functions”. Not Second RPA.
- The emergent collective effects renormalize interactions in correlated media, underly the spectral fragmentation mechanisms, affect superfluidity and weak decay rates.
- Relativistic NFT is generalized to finite temperature and applied to neutral and charge-exchange response of medium-heavy nuclei.
- Weak rates at astrophysical conditions are extracted: the correlations beyond mean field are found significant.
- Uncertainties (of the many-body theory) are quantified via building a hierarchy of approximations of growing complexity.

## Current and future developments:

- Deformed nuclei: correlations vs shapes; first results just released (Yinu Zhang et al.);
- Efficient algorithms; quantum computing (Manqoba Hlatshwayo et al.);
- Implementation of the EC rates into the core-collapse supernovae simulations;
- Toward an “ab initio” description: implementations with bare NN-interactions (in-medium beyond-the-leading order, non-perturbative);
- Superfluid pairing at $T>0$ to extend the application range (r-process);
- Relativistic EOM’s, bosonic EOM’s, beyond Standard Model, …

# *Many thanks for collaboration and support:*

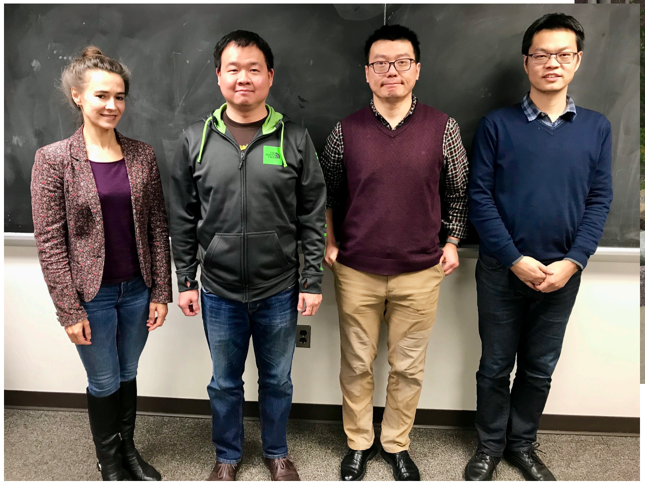
*Yinu Zhang (WMU)*
*Manqoba Hlatshwayo (WMU)*
*Herlik Wibowo (U. York)*
*Caroline Robin (U. Bielefeld & GSI)*
*Peter Schuck (IPN Orsay)*
*Peter Ring (TU München)*
*Tamara Niksic  (U Zagreb)*

*US-NSF CAREER PHY-1654379 (2017-2023)*

*US-NSF PHY-2209376 (2022-2025)*

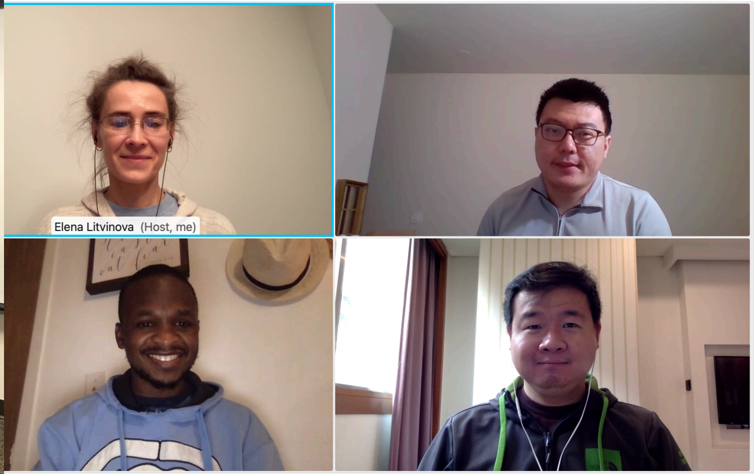
*2017:*

*2018:*



*2019-2020:*


*2020-2022:*



# *Many thanks for collaboration and support:*

*Yinu Zhang (WMU)*
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*Herlik Wibowo (AS Taipei)*
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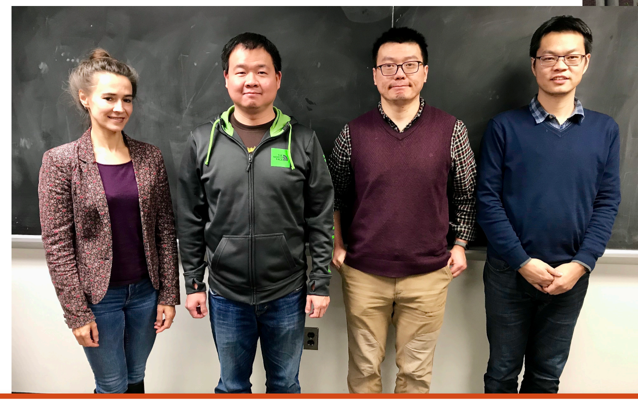
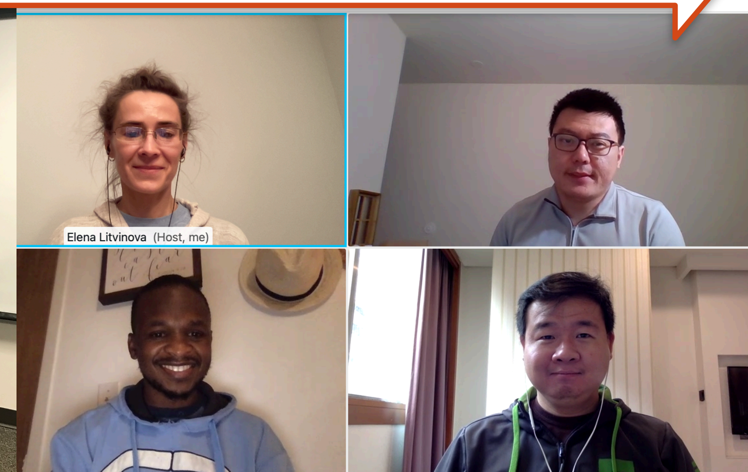
*2017:*

*2018:*



**A Postdoctoral position is currently open**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Award

# Thank you!