Isovector Giant Dipole Resonance: sum rules

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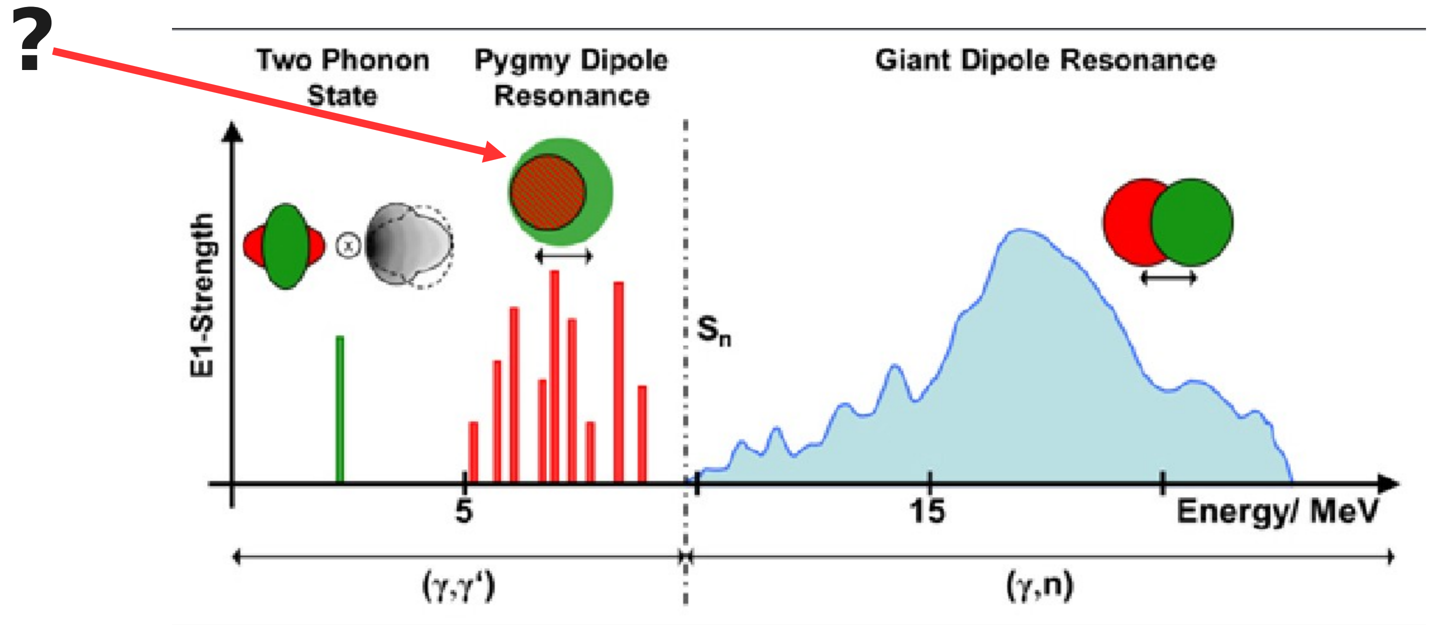


Figure from (http://www.nupecc.org/pub/np_light_2015.pdf):

This brochure **"Light to Reveal the Heart of Matter"** is one of the contributions being made by NuPECC, the Nuclear Physics European Collaboration Committee *(www.NuPECC.org)*, to the International Year of Light 2015 *(www.light2015.org)*.

Dielectric theorem:

Inverse Energy Weighted Sum Rule m_{-1} (polarizability)

Ground state |0> **perturbed** by an **external field** λF (λ → 0) so that perturbation theory holds → The **expectation value** of the **Hamiltonian** <H> and of the **operator** <F> can be written:

$$\delta\langle\mathcal{H}\rangle = \lambda^2 \sum_{\nu\neq 0} \frac{|\langle\nu|F|0\rangle|^2}{E_\nu - E_0} + \mathcal{O}(\lambda^3) = \lambda^2 m_{-1} + \mathcal{O}(\lambda^3)$$

$$\delta\langle F\rangle = -2\lambda \sum_{\nu\neq 0} \frac{|\langle\nu|F|0\rangle|^2}{E_\nu - E_0} + \mathcal{O}(\lambda^2) = -2\lambda m_{-1} + \mathcal{O}(\lambda^2)$$

$$m_{-1} = \frac{1}{2}\frac{\partial^2\langle\mathcal{H}\rangle}{\partial\lambda^2}\bigg|_{\lambda=0} = -\frac{1}{2}\frac{\partial\langle F\rangle}{\partial\lambda}\bigg|_{\lambda=0} \rightarrow \frac{1}{m_{-1}} = 2\frac{\partial^2\langle\mathcal{H}\rangle}{\partial\langle F\rangle^2}$$

Dipole polarizability, J and $\Delta r_{np} := r_n - r_p$

The dipole **polarizability** measures the **tendency** of the nuclear **charge** distribution to be **distorted.**

From a macroscopic point of view **α ~ (electric dipole moment)/($E_{external}$)**

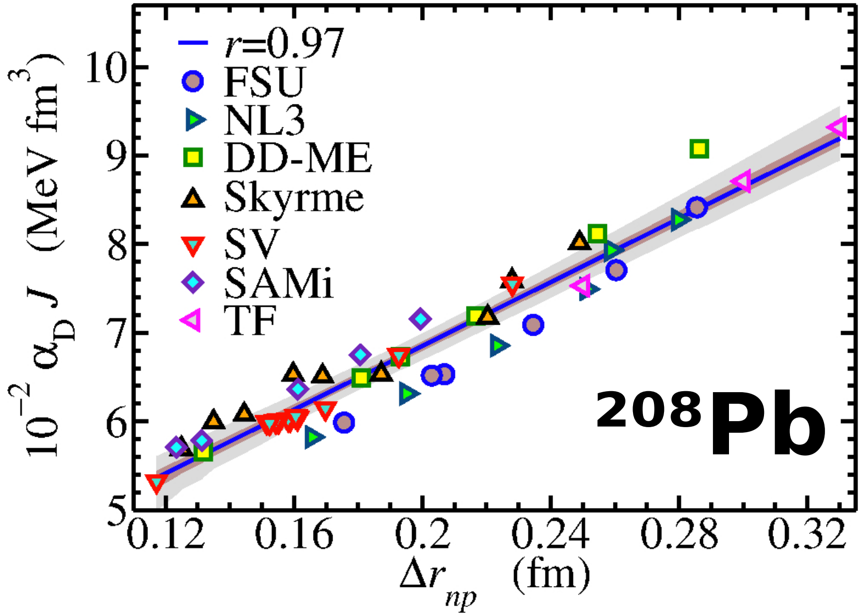
→ For guidance, using the **dielectric theorem**, the polarizability can be calculated assuming the **Droplet model**:

$$\alpha_D = \frac{8\pi e^2}{9} m_{-1}(E1)$$

Meyer et al. NPA385 (1982) 269-284

$$\alpha_D \approx \frac{\pi e^2}{54} \frac{\langle r^2 \rangle}{J} A \left(1 + \frac{5}{2} \frac{\Delta r_{np} - \Delta r_{np}^{\rm surf} - \Delta r_{np}^{\rm Coul}}{\langle r^2 \rangle^{1/2} (I - I_{\rm Coul})} \right)$$

Electric dipole polarizability in 208Pb: Insights from the droplet model - X. Roca-Maza, M. Brenna, G. Colò, M. Centelles, X. Viñas, B. K. Agrawal, N. Paar, D. Vretenar, and J. Piekarewicz
Phys. Rev. C 88, 024316 (2013)



Polarizability increases with the mass (for the dipole $A^{5/3}$, for the quadrupole $A^{7/3}$ and so on …) and it **sets a relation between the EoS parameters J and L**

$$e(\rho, \delta) = e(\rho, 0) + S(\rho)\delta^2$$

$$S(\rho) = J + Lx + \frac{1}{2} K_{\rm sym} x^2$$

Neutron skin thickness ($\Delta r_{np} := r_n - r_p$) and neutron pressure

For a fixed **(N-Z)/A**, one must **expect** that the **larger the pressure felt by nucleons**, **the larger the skin**

$$P = -\frac{\partial E}{\partial V}\Big|_A = \rho^2 \frac{\partial e(\rho,\delta)}{\partial \rho}\Big|_\delta =$$

$$\rho^2 \frac{\partial}{\partial \rho}\left[e(\rho,0) + S(\rho)\delta^2\right] =$$

$$\rho^2\delta^2 \frac{\partial S(\rho)}{\partial \rho} = \frac{1}{3}\rho\delta^2 L$$

→ From the Droplet Model:

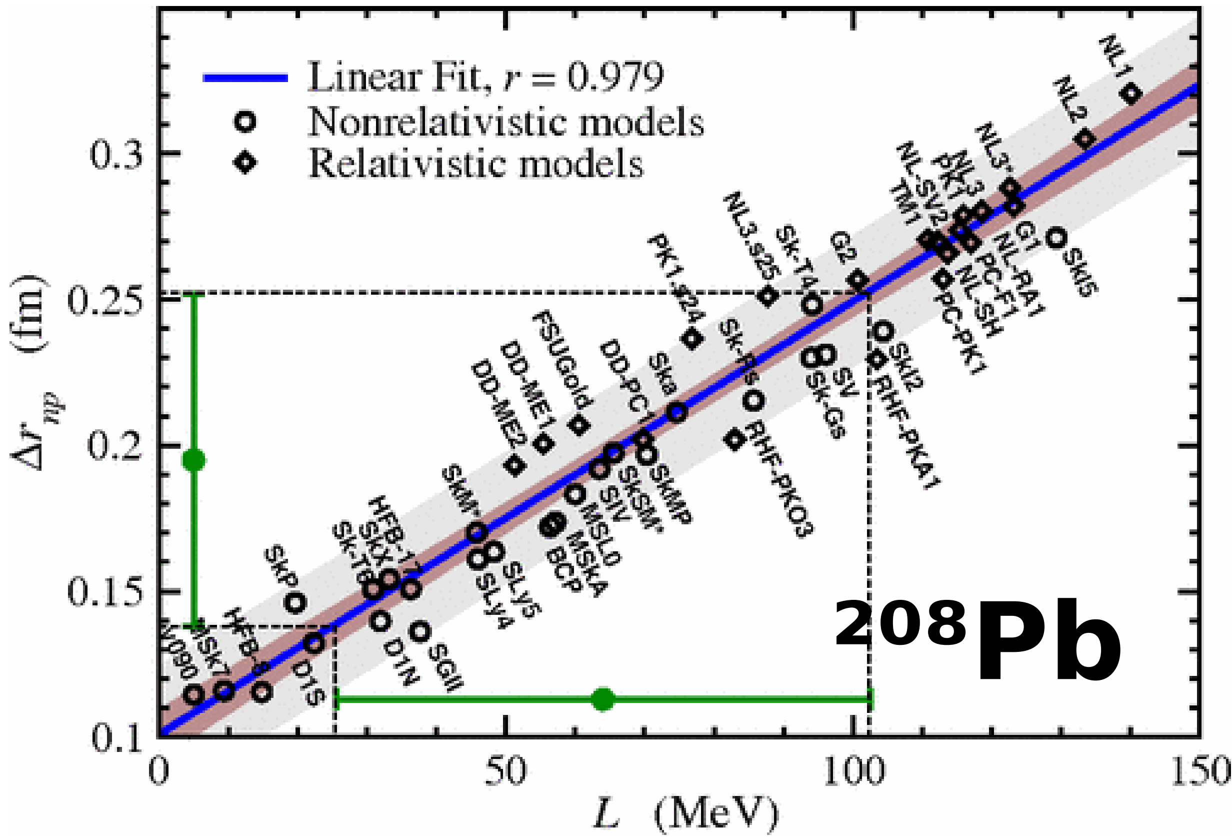
$$\Delta r_{np} \approx \frac{1}{12}\frac{N-Z}{A}\frac{R}{J}L$$

The nuclear droplet model for arbitrary shapes

W.D Myers, W.J Swiate

Annals of Physics

Volume 84, Issues 1–2, 15 May 1974, Pages 186-210



Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)

Dipole polarizability:
How good it is the Droplet Model for α_D?

Droplet model + LDA

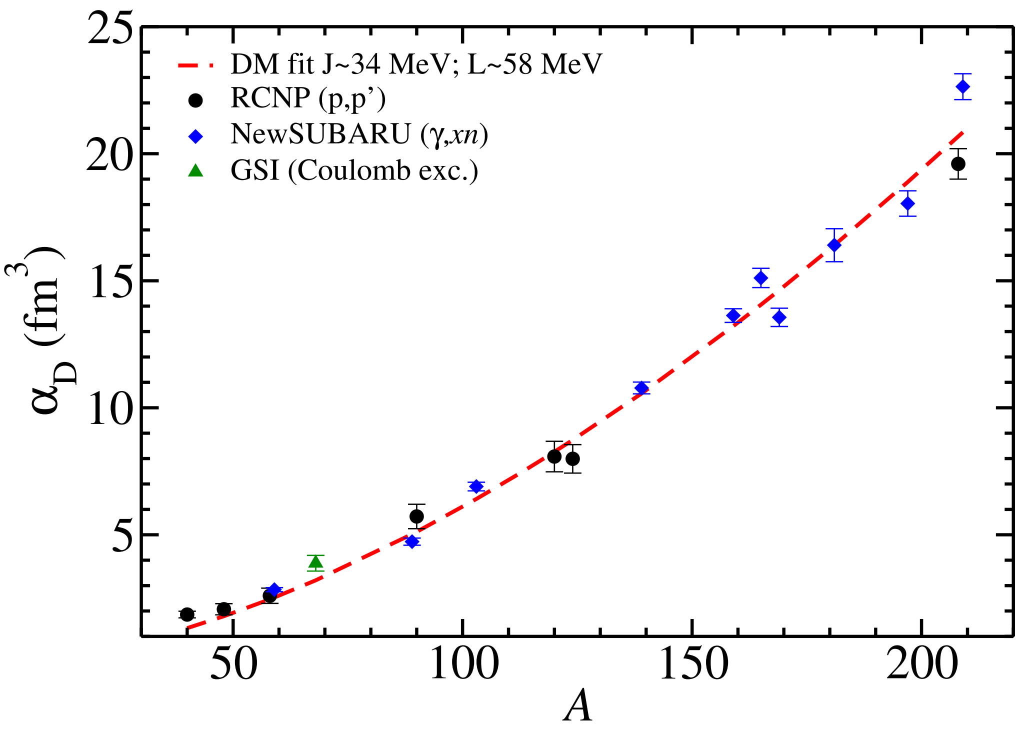
[$a_{sym}^{DM}(A) \approx S(\rho_A)$]

$$\alpha_D^{DM} \approx \frac{\pi e^2}{54}\frac{A\langle r^2\rangle}{J}\left[1+\frac{5}{3}\frac{L}{J}\epsilon_A\right]$$

Assuming:

$$\epsilon_A \equiv \frac{\rho_0-\rho_A}{3\rho_0} \approx \frac{0.16-0.08}{3\cdot 0.16}$$

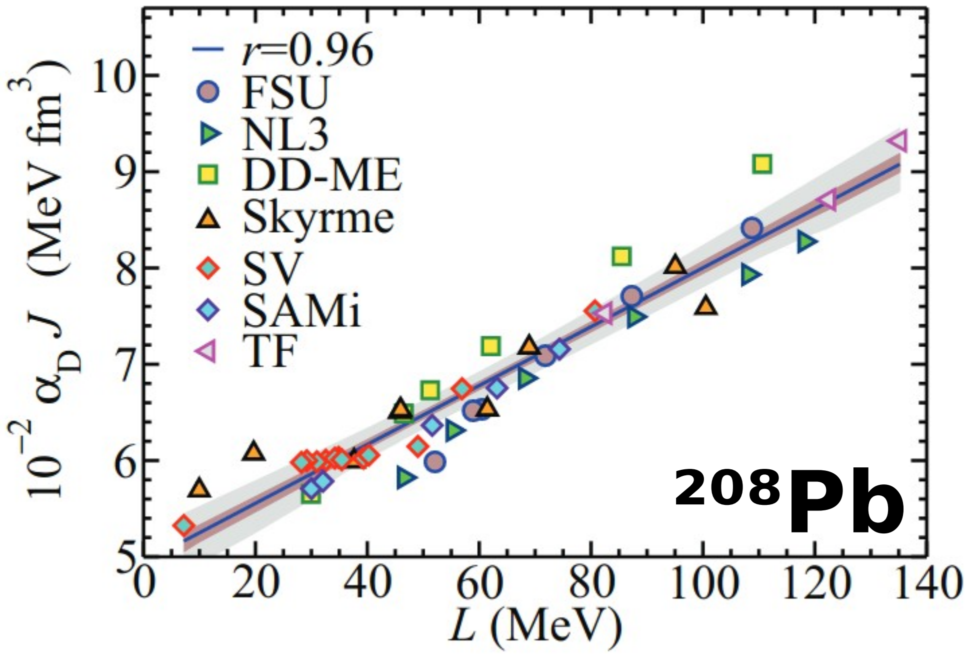
$$\langle r^2\rangle \approx \frac{3}{5}r_0^2 A^{2/3}$$



Fit to experimental data provides **J~34 MeV** and **L~58 MeV.**

Dipole polarizability, J and L

Determination of the J vs L relation from experimental data according to EDFs



$$J = 25.0(2) + 0.19(2)L \quad \text{for } ^{68}\text{Ni},$$
$$J = 25.4(1.1) + 0.17(1)L \quad \text{for } ^{120}\text{Sn},$$
$$J = 24.5(8) + 0.17(1)L \quad \text{for } ^{208}\text{Pb}.$$

Assuming Taylor expansion around ρ_0:

$$S(\langle\rho\rangle) \approx J - L\frac{\rho_0 - \langle\rho\rangle}{3\rho_0} \rightarrow J \approx S(\langle\rho\rangle) + \frac{\rho_0 - \langle\rho\rangle}{3\rho_0} L$$

one can qualitatively understand the result!!

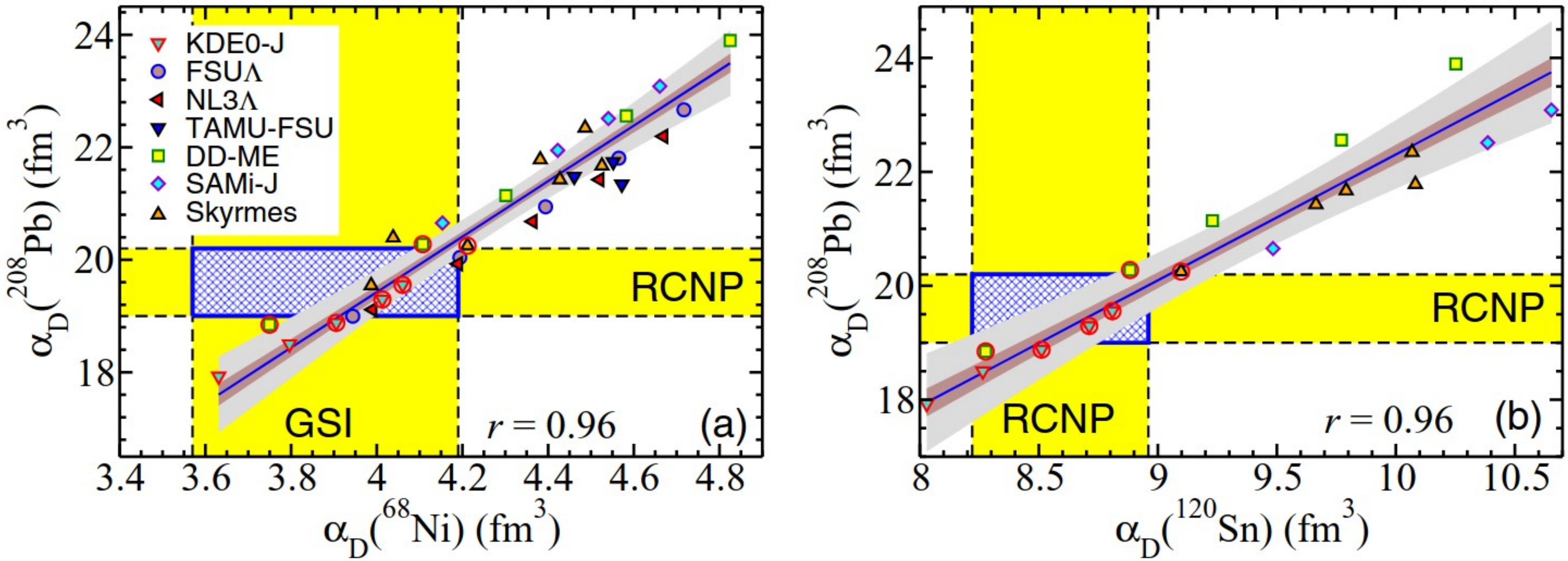
$$S(\langle\rho\rangle \approx 0.08\ \text{fm}^{-3}) \approx 25\ \text{MeV}$$

X. Roca-Maza, M. Brenna, G. Colò, M. Centelles, X. Viñas, B. K. Agrawal, N. Paar, D. Vretenar, and J. Piekarewicz
Phys. Rev. C **88**, 024316 – Published 20 August 2013

X. Roca-Maza, X. Viñas, M. Centelles, B. K. Agrawal, G. Colò, N. Paar, J. Piekarewicz, and D. Vretenar
Phys. Rev. C **92**, 064304 – Published 8 December 2015

Dipole polarizability, J and L

Alternatively: **Selection** of EDFs **(red circles) compatible** with **experimental** data



X. Roca-Maza, M. Brenna, G. Colò, M. Centelles, X. Viñas, B. K. Agrawal, N. Paar, D. Vretenar, and J. Piekarewicz
Phys. Rev. C **88**, 024316 – Published 20 August 2013

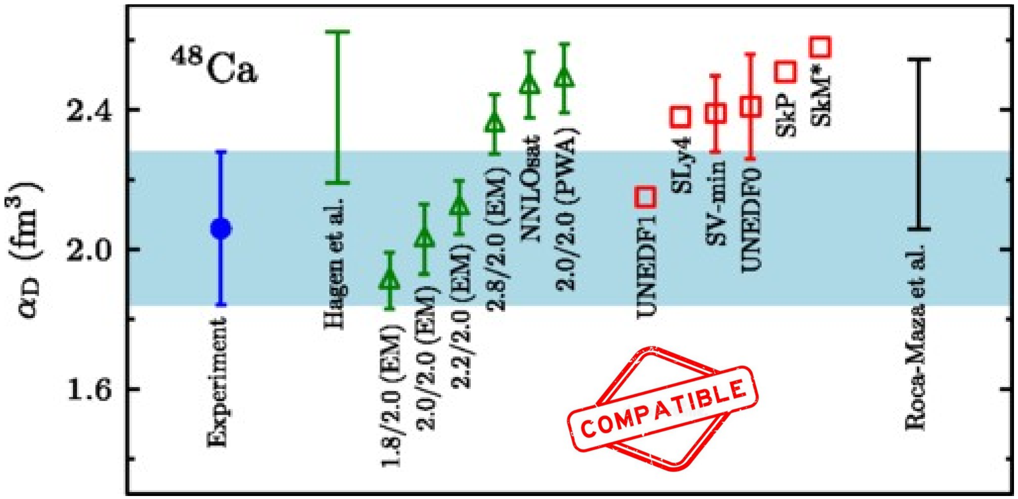
X. Roca-Maza, X. Viñas, M. Centelles, B. K. Agrawal, G. Colò, N. Paar, J. Piekarewicz, and D. Vretenar
Phys. Rev. C **92**, 064304 – Published 8 December 2015

Selection compatible EDFs and **correlation analysis** (previous slides) provides **comparable estimates** for the **neutron skin thickness**.

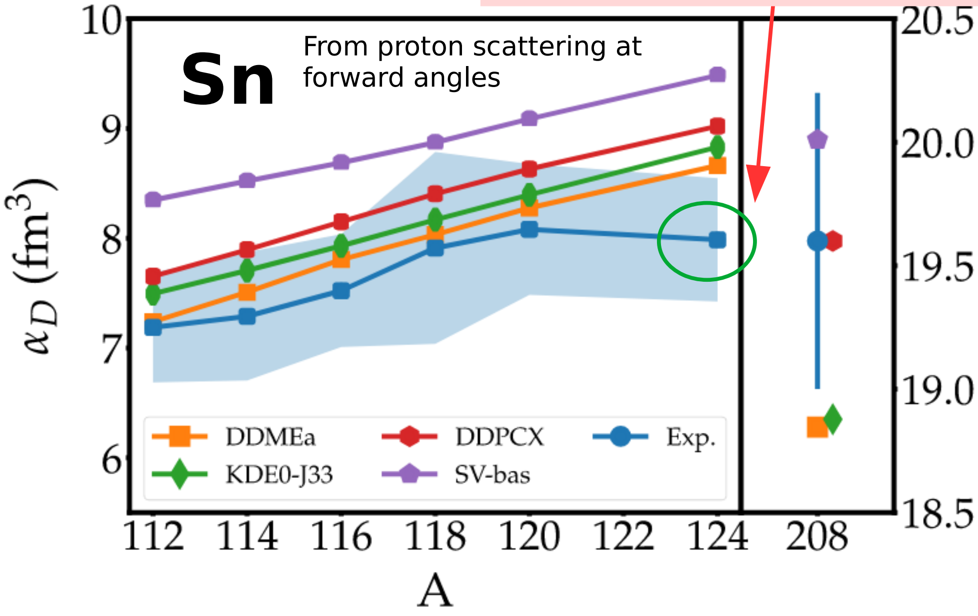
	From selected models		From α_D J vs Δr_{np}
Nucleus	Δr_{np} (a)	Δr_{np} (b)	Δr_{np} (c)
^{68}Ni	0.15–0.19	0.18 ± 0.01	0.16 ± 0.04
^{120}Sn	0.12–0.16	0.14 ± 0.02	0.12 ± 0.04
^{208}Pb	0.13–0.19	0.16 ± 0.02	0.16 ± 0.03

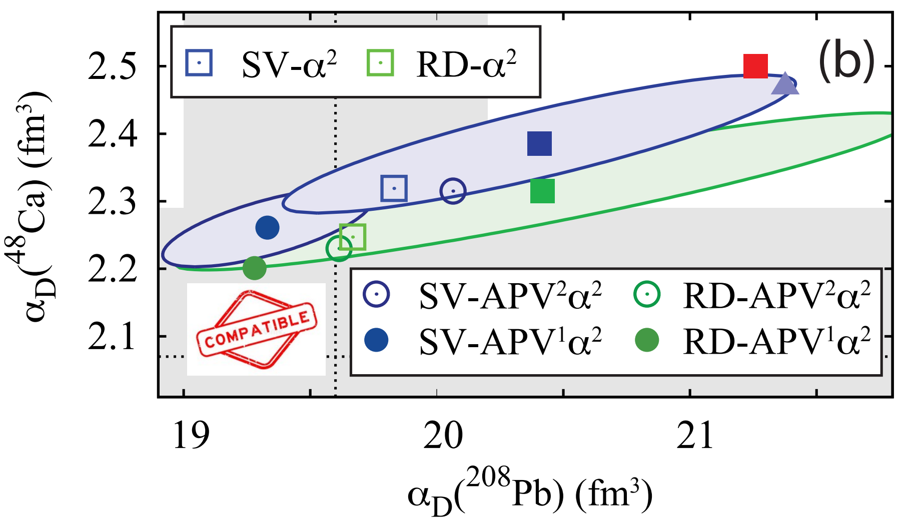
Dipole polarizability:
do we understand it?

J. Birkhan, M. Miorelli, S. Bacca, S. Bassauer, C. A. Bertulani, G. Hagen, H. Matsubara, P. von Neumann-Cosel, T. Papenbrock, N. Pietralla, V. Yu. Ponomarev, A. Richter, A. Schwenk, and A. Tamii
Phys. Rev. Lett. **118**, 252501 – Published 23 June 2017



Trend with N?





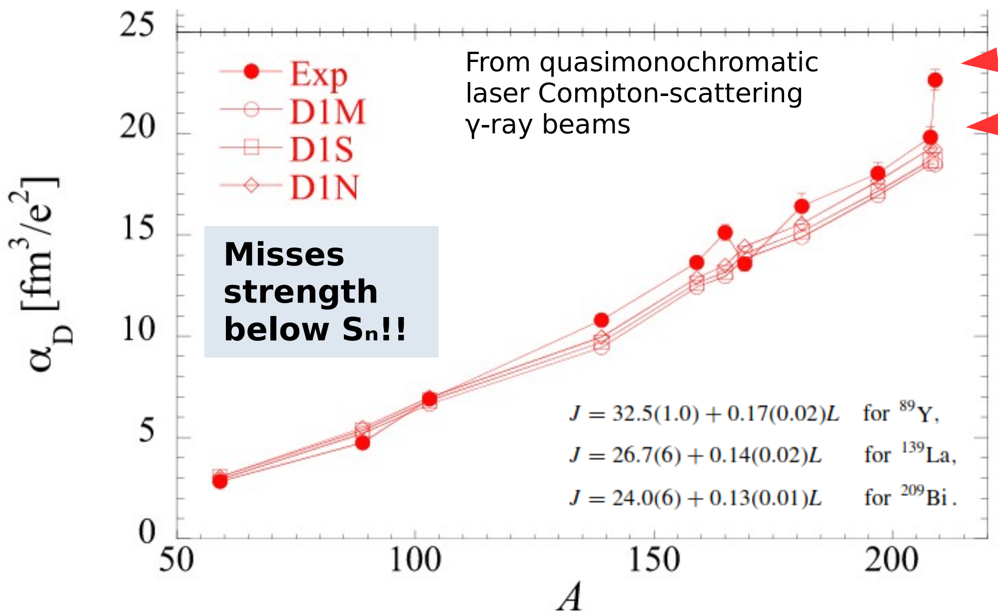
S. Bassauer, P. von Neumann-Cosel, P.-G. Reinhard et al.

Physics Letters B 810 (2020) 135804

Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz
arXiv:2206.03134 (2022)

Dipole polarizability: systematics with A
do we understand it?

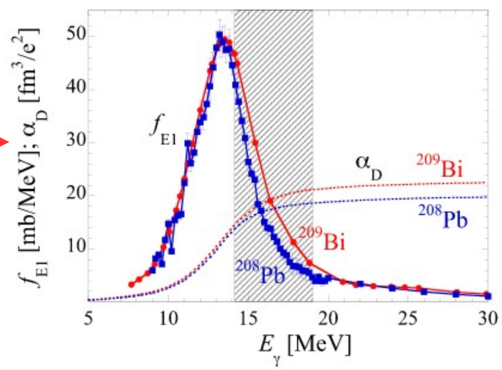
S. Goriely, S. Péru, G. Colò, X. Roca-Maza, I. Gheorghe, D. Filipescu, and H. Utsunomiya
Phys. Rev. C **102**, 064309 – Published 9 December 2020



Misses strength below S_n!!

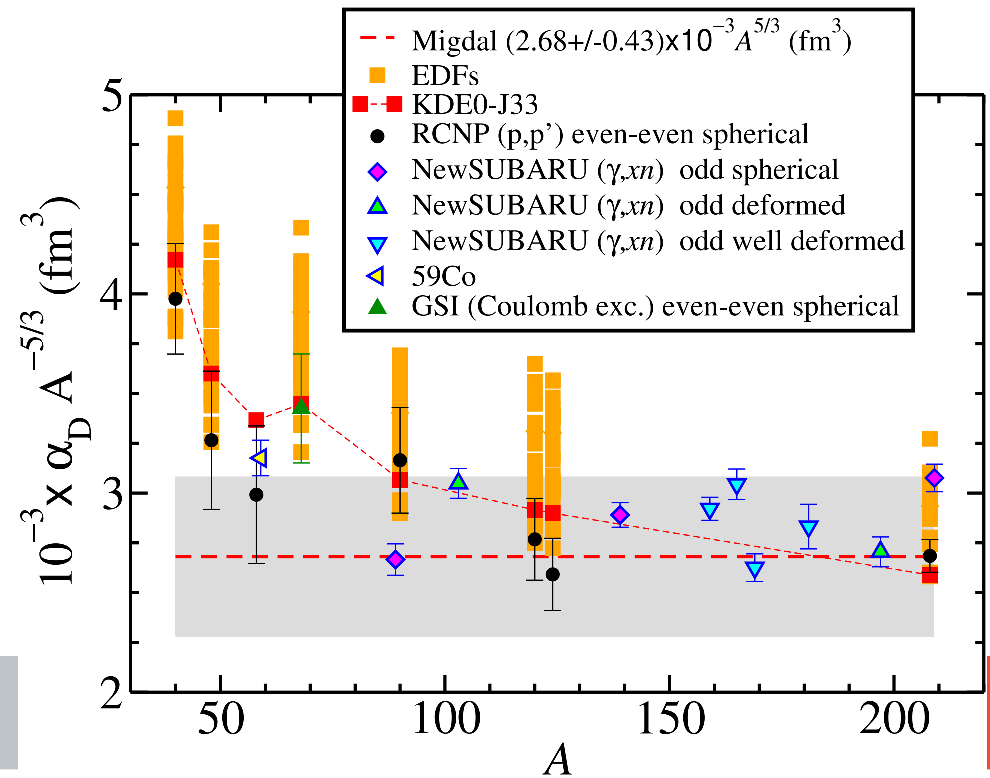
^{209}**Bi**

^{208}**Pb**



Adding one more nucleon?

All together: $A^{5/3}$ trend



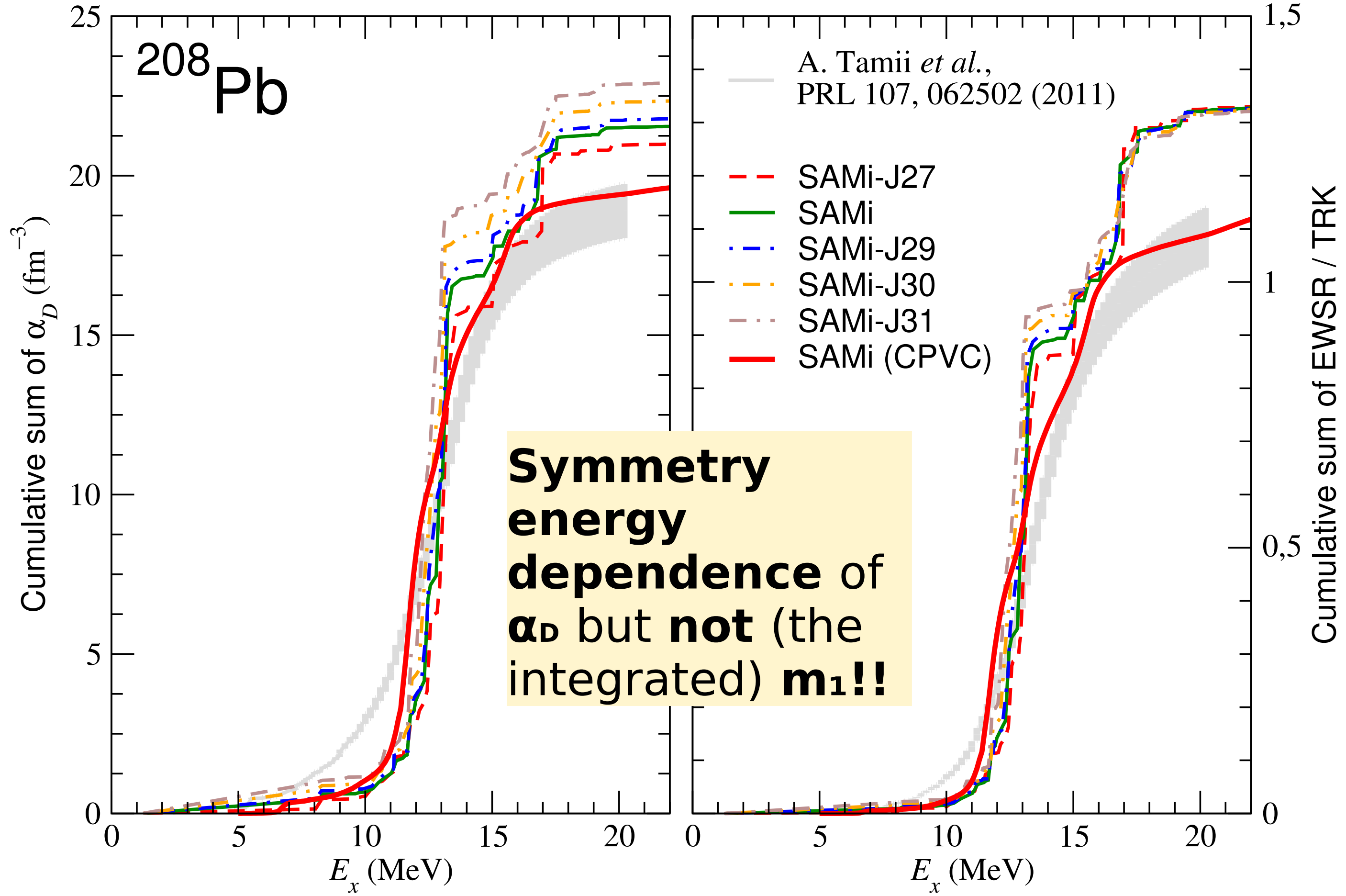
→ Overall, **EDFs agree** with experimental **systematics**

→ **Macroscopic law** (Migdal) $A^{5/3}$ reasonable for A>90

Energy weighted sum rule and αD:

$$m_1 = \sum_\nu (E_\nu - E_0)|\langle\nu|F|0\rangle|^2 = \langle 0|F^\dagger[\mathcal{H},F]|0\rangle$$

$$m_1 = \frac{9}{4\pi}\frac{\hbar^2}{2m}\frac{NZ}{A}e^2(1+\kappa)$$



Symmetry energy dependence of **αD** but **not** (the integrated) **m1!!**

If **not all the strength** is measured, **need** to include **correlations beyond RPA**

Effect of the **width Γ** on **αD** is of a **few %**

$$\Delta\alpha_D \sim -\alpha_D \frac{\Gamma^2}{4E_x^2},$$

Energy weighted sum rule:
Is the dipole operator suitable to excite a skin mode?

The m_1:

$$m_1 = \sum_\nu (E_\nu - E_0)|\langle \nu|F|0\rangle|^2 = \langle 0|F^\dagger[\mathcal{H}, F]|0\rangle$$

The dipole operator

$$\hat{F}_{1M} = \frac{eN}{A}\sum_{i=1}^{Z} r_i Y_{1M}(\hat{r}_i) - \frac{eZ}{A}\sum_{i=1}^{N} r_i Y_{1M}(\hat{r}_i);$$

Skyrme EDF

$$m_1 = \frac{9}{4\pi}\frac{\hbar^2}{2m}\frac{NZ}{A}e^2(1+\kappa)$$

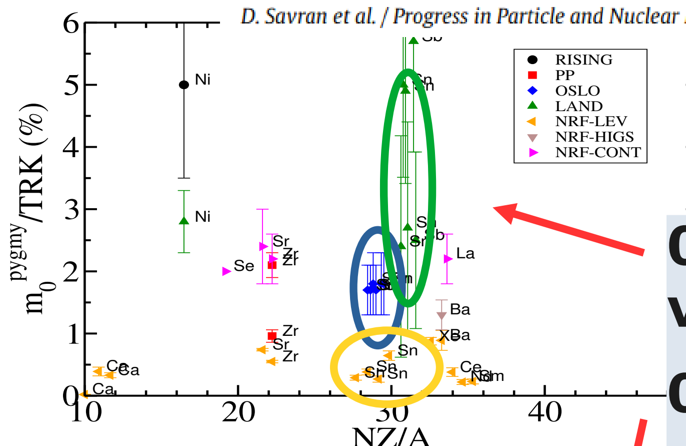
$$\kappa \equiv \frac{2m}{\hbar^2}\frac{A}{16NZ}\left[t_1\left(1+\frac{x_1}{2}\right)+t_2\left(1+\frac{x_2}{2}\right)\right] \times \int d^3r\, \rho^2\left(1-\beta^2\right)$$

Main contribution (~99%) does not depend on the neutron to proton density differences!!

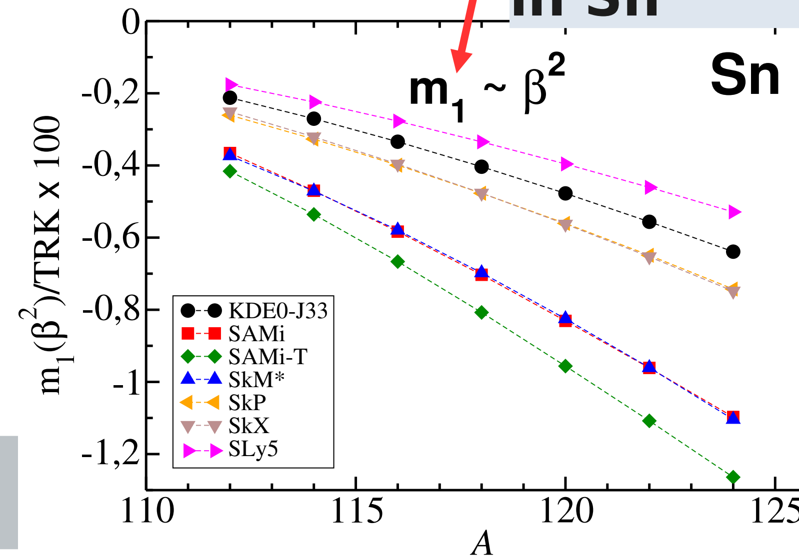
ρ_n and ρ_p differences:

→ decrease m_1

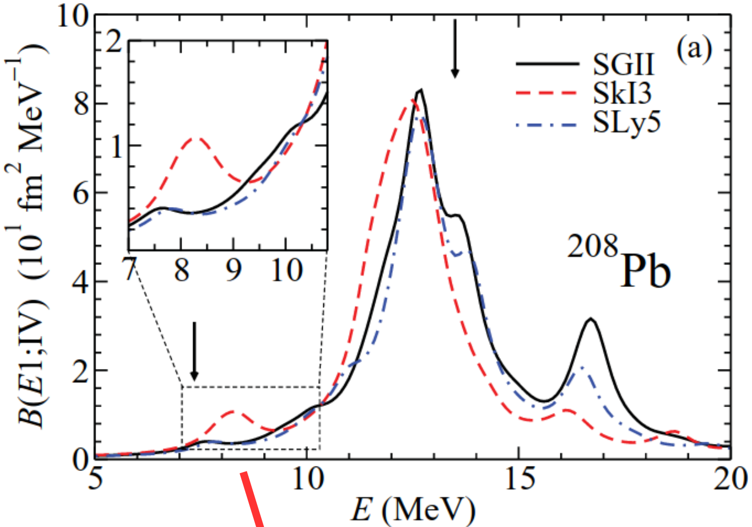
→ skin mode does not provide an explanation for the experimental pygmy EWSR from OSLO and LAND while it would be compatible with NRF-LEV



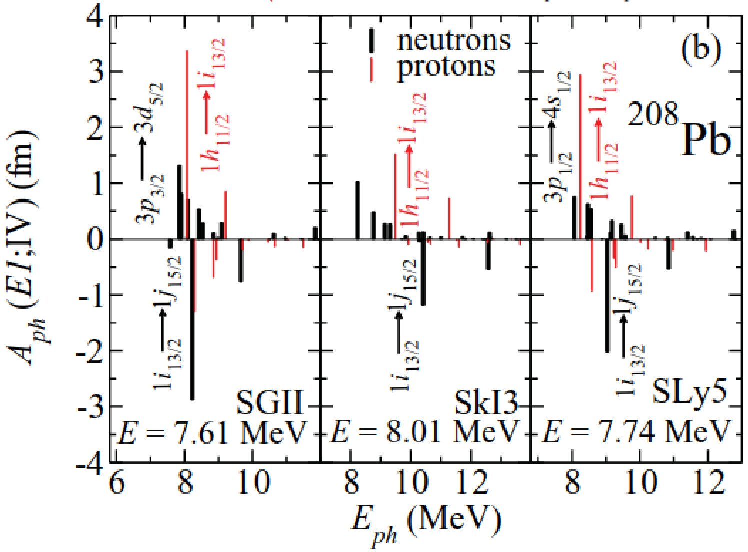
0.5-6% versus 0.2-1.2% in Sn



Low-energy isovector dipole response: RPA



$$A_{\rm ph}(EJ, 0 \to \nu) = \left(X_{\rm ph}^{(\nu)} + Y_{\rm ph}^{(\nu)}\right)\langle p||F_J||h\rangle.$$

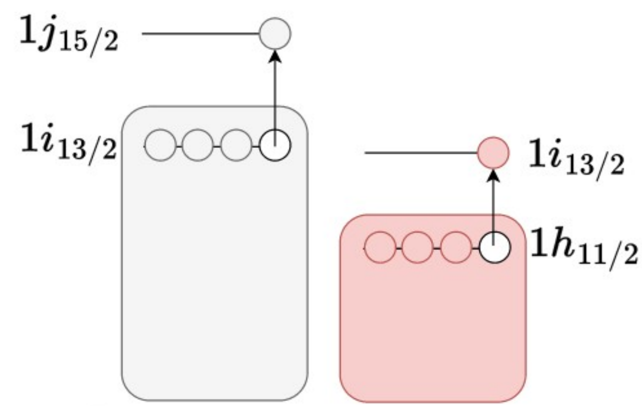


Pygmy state: not coherent and **not** involve **all neutrons** in **excess**

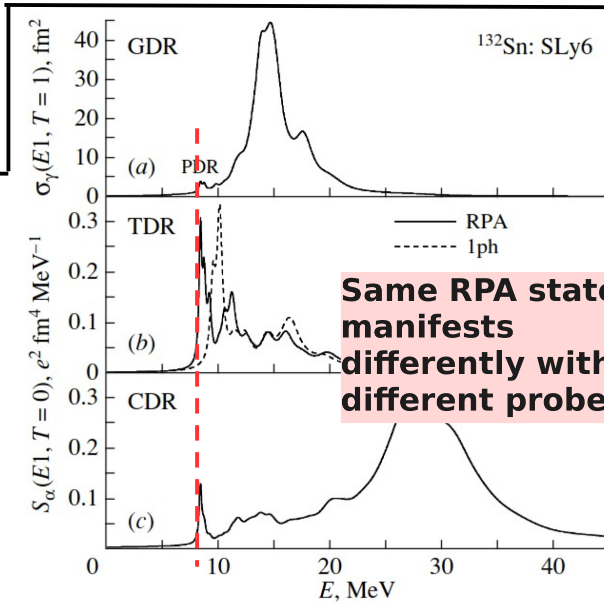
→ **Main *ph*** transitions **outermost neutrons** (~14) and **outermost protons** (~12) in opposition of phase.

→ **Other** relevant ***ph* neutron** transitions in **opposite phase** with respect to the main neutron *ph* transition.

→ **All neutrons in excess** (44) contibute to the **neutron skin**



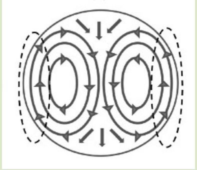
$$\Delta r_{np} \propto N - Z = 44 > 14$$



Same RPA state manifests differently with different probes

Dipole strength in the **pygmy region** a vortical **toroidal mode?**

Toroidal mode can occur also in N=Z nuclei



X. Roca-Maza, G. Pozzi, M. Brenna, K. Mizuyama, and G. Colò
Phys. Rev. C **85**, 024601 – Published 3 February 2012

V. O. Nesterenko, J. Kvasil, A. Repko, W. Kleinig & P.-G. Reinhard
Physics of Atomic Nuclei **79**, 842–850 (2016) | Cite this article

Non-energy weighted sum rule

work in progress

Model dependence of m₀

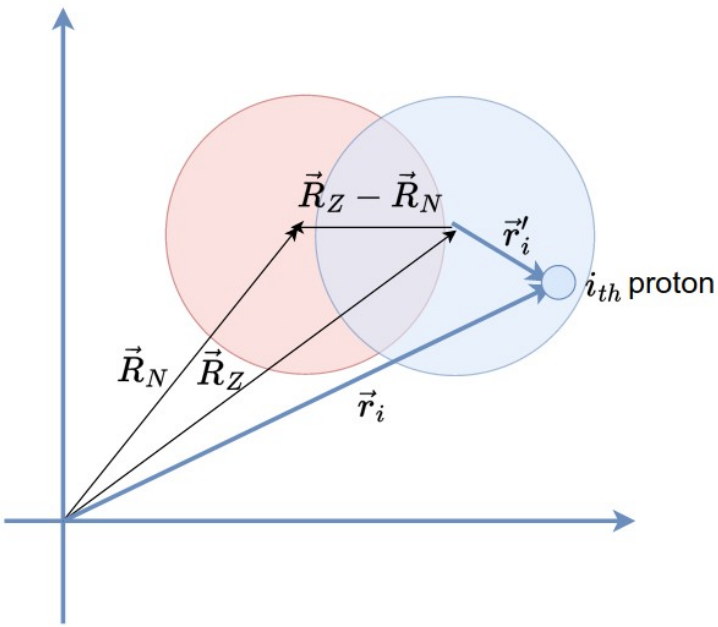
$$m_0 = \sum_\nu |\langle \nu | F | 0 \rangle|^2 = \langle 0 | F^\dagger F | 0 \rangle$$

$$F = \frac{N}{A} \sum_{i=1}^{Z} r_i Y_{10}(\hat{r}_i) - \frac{Z}{A} \sum_{i=1}^{N} r_i Y_{10}(\hat{r}_i)$$

$$m_0 = \frac{1}{4\pi} \left(\frac{NZ}{A} \right)^2 \boxed{\langle 0 | (R_Z - R_N)^2 | 0 \rangle}$$

$$\langle {r'_p}^2 \rangle = \frac{1}{Z} \langle 0 | \sum_{i=1}^{Z} (r_i - R)^2 | 0 \rangle$$

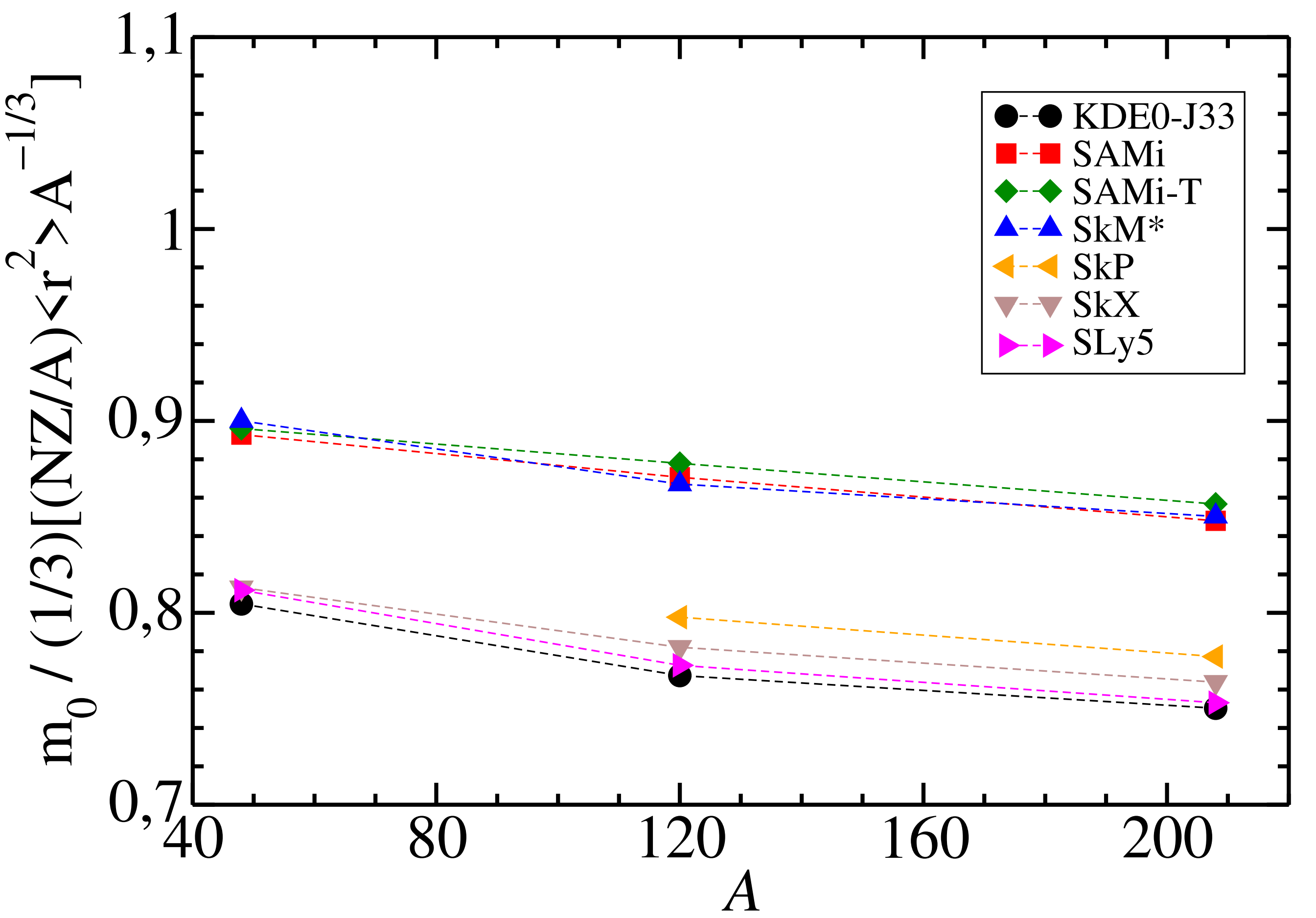
$$\langle {r'_p}^2 \rangle = \frac{N^2}{A^2} \boxed{\langle 0 | (R_Z - R_N)^2 | 0 \rangle} + \frac{1}{Z} \langle 0 | \sum_{i=1}^{Z} r_i^2 | 0 \rangle - \left(\frac{1}{Z} \langle 0 | \sum_{i=1}^{Z} r_i | 0 \rangle \right)^2$$



How to relate m₀ with ground state observables? Can be used for a better understanding of the isovector dipole response?

Non-energy weighted sum rule
work in progress

Suggested model dependence of m_0 (?)



J. S. Levinger and D. C. Kent
Phys. Rev. **95**, 418 – Published 15 July 1954

Conclusions

→ The **inverse energy weighted sum rule** or polarizability inform us about the **isovector properties** in EDFs

→ The **energy weighted sum rule** does **not** provide clear **information** on the **isovector properties** of the Skyrme EDFs. Same for other models?

→ The **non-energy weighted sum rule** depends on the **neutron to proton CM difference**. Can we gain information on the isovector channel from this sum rule?

→ … and from the last two: what about the centroid energy (m_1/m_0)?

Collaborators

→ Gianluca **Colò** (University of Milan)
→ Hiroyuki **Sagawa** (University of Aizu & RIKEN)
→ Shihang **Shen** (Forschungszentrum Jülich)
→ Xavier **Vinyes** & Mario **Centelles** (University of Barcelona)
→ Jorge **Piekarewicz** (Florida State University)
→ Nils **Paar** & Dario **Vretenar** (University of Zagreb)
→ Bijay K. **Agrawal** (Saha Institute of Nuclear Physics)
→ P.-G. **Reinhard** (University of Erlangen-Nürnberg)
→ Yifei **Niu** (Lanzhou University)
→ Witold **Nazarewicz** (FRIB and Michigan State University)
→ Stephane **Goriely** (Université Libre de Bruxelles)
→ Sophie **Péru** (Université Paris-Saclay, CEA)

Experimental techniques: PYGMY

D. Savran et al. / Progress in Particle and Nuclear Physics 70 (2013) 210–245

Table 2
Main strengths and weaknesses of the different experimental tools. The compared methods are: (a) Discrete (γ, γ') level analysis; (b) Continuous (γ, γ') analysis; (c) Quasi-mono-energetic photons; (d) Coulomb excitation of stable targets; (e) Coulomb excitation in inverse kinematics; (f) $(\alpha, \alpha'\gamma)$ and $(p, p'\gamma)$ experiments; (g) Oslo method. A "+" means a peculiar strength of the method, a "−" a weakness and a "o" stands for an average rating. See detailed discussion in the text.

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Typical energy resolution [keV]	5–10	100	30–200[a]	30	500	5–10	50
Model independence	+	−	+	o	o	o	−
Completeness of summed strength	−	+	o	+	+	o	+
Fine structure	+	−	+	o	−	+	−
Versatility	−	−	−	−	+	−	o

[a] Concerning the beam energy resolution. The γ-decay spectroscopy is equivalent to (a).

NFR-LEV **LAND** **OSLO**

Less model dependence ←

Completeness summed strength →

Summary from Progress in Particle and Nuclear Physics 101 (2018) 96–176

EoS par.	Observable	Range	Comments
ρ_0	$\langle r_{ch}^2 \rangle^{1/2}$	0.154–0.159	Most accurate EDFs on $M(N,Z)$ and $\langle r_{ch}^2 \rangle^{1/2}$ (see Section 5)
e_0	$M(N,Z)$	−16.2 to −15.6	Most accurate EDFs on $M(N,Z)$ and $\langle r_{ch}^2 \rangle^{1/2}$ (see Section 5)
K_0	$M(N,Z)$	220–245	Most accurate EDFs on $M(N,Z)$ and $\langle r_{ch}^2 \rangle^{1/2}$ (see Section 5)
	ISGMR	220–260	From EDFs in closed shell nuclei [116]
	ISGMR	250–315	Blaizot's formula [Eq. (32)] [51]
	ISGMR	~200	EDF describing also open shell nuclei [118]
J	$M(N,Z)$	29-35.6	Most accurate EDFs on $M(N,Z)$ and $\langle r_{ch}^2 \rangle^{1/2}$ (see Section 5)
	IVGDR	~24.1(8) + L/8	From EDF analysis [$S(\rho = 0.1\ \text{fm}^{-3}) = 24.1(8)$ MeV] [273]
	PDS	30.2–33.8	From EDF analysis [370]
	PDS	31.0–33.6	From EDF analysis [371]
	α_D	24.5(8) + 0.168(7)L	From EDF analysis ^{208}Pb [96]
	α_D	30–35	From EDF analysis [179]
	IAS and Δr_{np}	30.2–33.7	From EDF analysis [325]
	AGDR	31.2–35.4	From EDF analysis [401]
	PDS, α_D, IVGQR, AGDR	32–33	From EDF analysis [508]
	compilation	29.0–32.7	[106]
	compilation	30.7–32.5	[107]
	compilation	28.5–34.9	[3]
L	$M(N,Z)$	27–113	Most accurate EDFs on $M(N,Z)$ $\langle r_{ch}^2 \rangle^{1/2}$ (see Section 5)
	ρ_n	40–110	proton-^{208}Pb scattering [24]
	ρ_n	0–60	π photoproduction (^{208}Pb) [181]
	ρ_n	30–80	antiprotonic at. (EDF analysis) [102,509]
	ρ_{weak}	>20	Parity violating scattering [27]
	PDS	32–54	From EDF analysis [370]
	PDS	49.1–80.5	From EDF analysis [371]
	α_D	20–66	From EDF analysis [179]
	IVGQR and ISGQR	19–55	From EDF analysis [101]
	IAS and Δr_{np}	35–75	From EDF analysis [325]
	AGDR	75.2–122.4	From EDF analysis [401]
	PDS, α_D, IVGQR, AGDR	45.2–54.6	From EDF analysis [508]
	compilation	40.5–61.9	[106]
	compilation	42.4–75.4	[107]
	compilation	30.6–86.8	[3]

Pygmy used to get information about the nuclear EoS?

Some alternative compilations:

M. Oertel, M. Hempel, T. Klähn, S. Typel, Rev. Modern Phys. 89 (2017) 015007.

M.B. Tsang, et al., Phys. Rev. C 86 (2012) 015803.

Bao-An Li, Xiao Han, Phys. Lett. B 727 (1) (2013) 276–281.

Summary

with qualitative indication of accuracy needed to describe experiment
(note that absolute values might be subject to systematics)

→ $\rho_0 \in$ [0.154,0.159] fm^{-3} → relative accuracy 2%

→ needed to describe experiment (Rch) ≤0.1%

→ $e_0 \in$ [15.6,16.2] MeV→ relative accuracy 4%

→ needed to describe experiment (B) ≤0.0001%

→ $K_0 \in$ [200,260] MeV→ relative accuracy 25%

→ needed to describe experiment (E_x^{GMR}) ≤7%

→ J ∈ [30,35] MeV → relative accuracy 15%

→ needed to describe experiment (α) ≤15%

→ L ∈ [20,120] MeV → relative accuracy 150%

→ needed to describe experiment (α) ≤50%

→ ...