

Constraining energy density functionals with dipole excitations and parity-violating electron scattering experiments

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Operativni program
**KONKURENTNOST
I KOHEZIJA**

- **nuclear energy density functional (EDF)**

unified approach to describe at the quantitative level nuclear properties across the nuclide map including exotic nuclei – astrophysically relevant properties

- ❑ static and dynamic properties of finite nuclei across the nuclide map (nuclear masses, excitations,...)
- ❑ equation of state of nuclear matter
- ❑ nuclear processes and reactions (beta decays, beta delayed neutron emission, neutron capture, neutrino-induced reactions, nuclear fission ...)

- **relativistic energy density functional (REDF)**

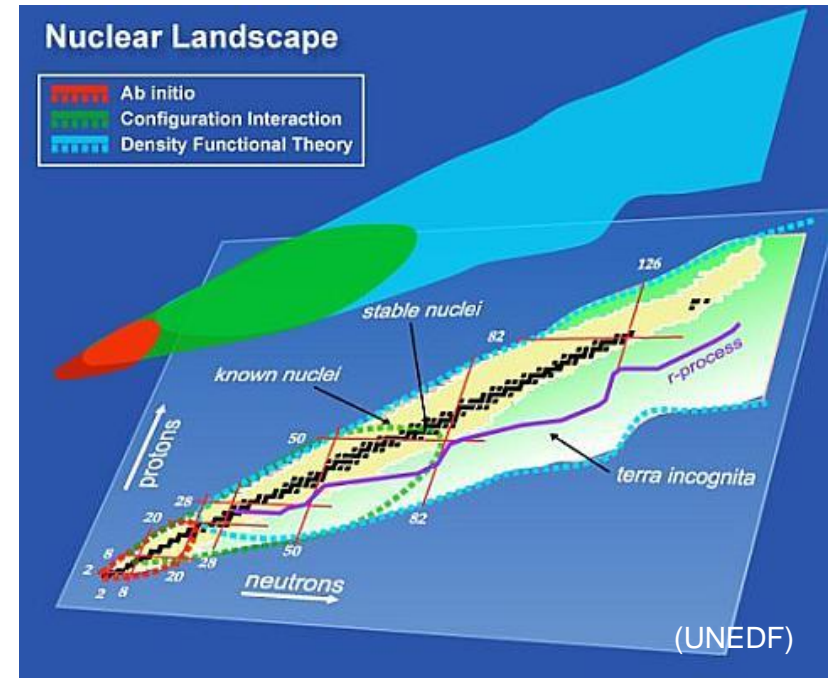


TABLE OF CONTENTS

1. Constraining the symmetry energy in the REDFs
2. Using dipole polarizability to establish REDFs with correct symmetry energy
3. Implications of parity violating electron scattering experiments for the EDFs
4. Magnetic M1 transitions in the REDF theory
5. Conclusions

RELATIVISTIC NUCLEAR ENERGY DENSITY FUNCTIONAL

- Relativistic point coupling interaction



- The basis is an effective Lagrangian with four-fermion (contact) interaction terms; isoscalar-scalar, isoscalar-vector, isovector-vector, derivative term

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\gamma \cdot \partial - m)\psi \\ & - \frac{1}{2}\alpha_S(\hat{\rho})(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_V(\hat{\rho})(\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi) \\ & - \frac{1}{2}\alpha_{TV}(\hat{\rho})(\bar{\psi}\vec{\tau}\gamma^\mu\psi)(\bar{\psi}\vec{\tau}\gamma_\mu\psi) \\ & - \frac{1}{2}\delta_S(\partial_\nu\bar{\psi}\psi)(\partial^\nu\bar{\psi}\psi) - e\bar{\psi}\gamma \cdot A \frac{(1 - \tau_3)}{2}\psi \end{aligned}$$

- many-body correlations encoded in density-dependent coupling functions that are motivated by microscopic calculations but parameterized in a phenomenological way – 10 model parameters
- Extensions: pairing correlations in finite nuclei (Gogny / separable pairing force)
 - Relativistic Hartree-Bogoliubov model [T. Niksic, et al., Comp. Phys. Comm. 185, 1808 \(2014\).](#)
- In the small amplitude limit, self-consistent quasiparticle random phase approximation (QRPA) is used to compute nuclear excitations
- supplemented by the co-variance analysis to determine statistical uncertainties of calculated quantities

CONSTRAINING THE EDFs

- The EDFs have been parametrized with the experimental data on the ground state properties of finite nuclei

$$\chi^2(\mathbf{p}) = \sum_{i=1}^m \left(\frac{\mathcal{O}_i^{\text{theo.}}(\mathbf{p}) - \mathcal{O}_i^{\text{ref.}}}{\Delta \mathcal{O}_i^{\text{ref.}}} \right)^2$$

- Nuclear ground state properties are often not enough to constrain the effective interaction completely, especially its **isovector channel** (that is especially relevant for the neutron-rich nuclei, neutron skins, symmetry energy, neutron stars, etc.).
- The protocols to determine the EDF's often included additional constraints on the *pseudo-observables* on the nuclear matter properties (often they are arbitrary).
- **The neutron skin thickness** r_{np} may be useful probe for the isovector channel of the EDFs. However, the data on r_{np} are often model dependent.
- Other observables are required to constrain the isovector channel of the EDFs.

NUCLEAR MATTER EQUATION OF STATE

- Nuclear matter equation of state (for the uniform and infinite system)

$$E(\rho, \delta) = E_{SNM}(\rho) + E_{sym}(\rho)\delta^2 + \dots$$

$$\rho = \rho_n + \rho_p \quad \delta = \frac{\rho_n - \rho_p}{\rho}$$

- Symmetry energy $S_2(\rho)$ describes the increase in the energy of the $N \neq Z$ system as protons are turned into neutrons

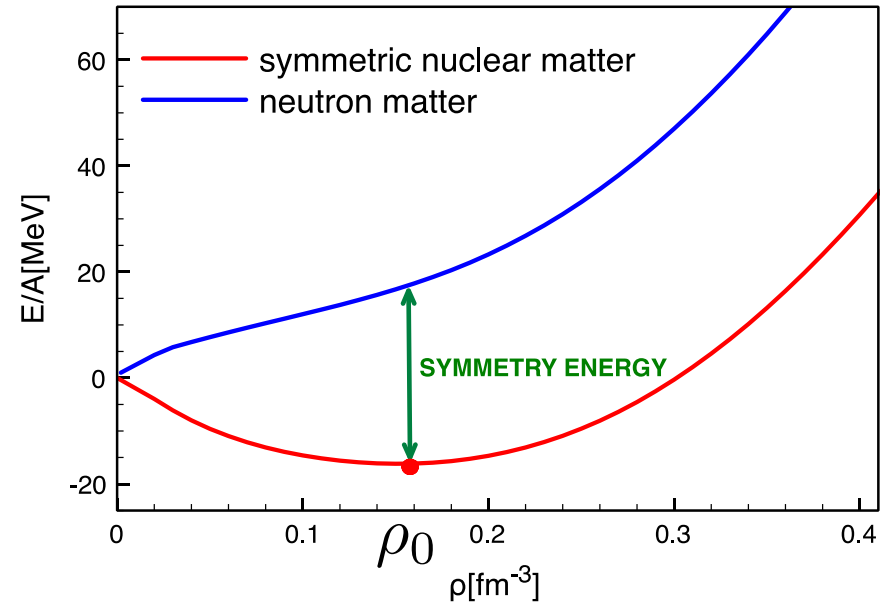
$$E_{sym}(\rho) \equiv S_2(\rho) = J - L\epsilon + \dots$$

$$\epsilon = (\rho_0 - \rho)/(3\rho_0)$$

$$L = 3\rho_0 \left. \frac{dS_2(\rho)}{d\rho} \right|_{\rho_0}$$

J – symmetry energy at saturation density

L – slope of the symmetry energy
(related to the pressure of neutron matter)



$$K_0 \equiv 9\rho_0^2 \left. \frac{\partial^2 E(\rho, 0)}{\partial \rho^2} \right|_{\rho=\rho_0}$$

K_0 - incompressibility of symmetric nuclear matter

Nuclear collective excitations provide important constraints for the EDFs

1. Isoscalar modes

- 1.1 Isoscalar giant monopole resonance
- 1.2 Isoscalar giant dipole resonance
- 1.3 Isoscalar giant quadrupole resonance

CONSTRAINTS
FOR THE ISOSCALAR
PROPERTIES

2. Isovector modes

- 2.1 Isovector giant dipole resonance
- 2.2 Dipole polarizability
- 2.3 Pygmy dipole strength
- 2.4 Isovector giant quadrupole resonance

CONSTRAINTS
FOR THE ISOVECTOR
PROPERTIES

3. Charge-exchange modes

- 3.1 Isobaric analog resonance
- 3.2 Gamow-Teller resonance
- 3.3 Spin-dipole resonance
- 3.4 Anti-analogue giant dipole resonance

Experimental and theoretical efforts in finding and measuring observables especially sensitive to the EoS properties are important, not only for low-energy nuclear physics but also for nuclear astrophysics applications.

X. Roca Maza, N. P.,

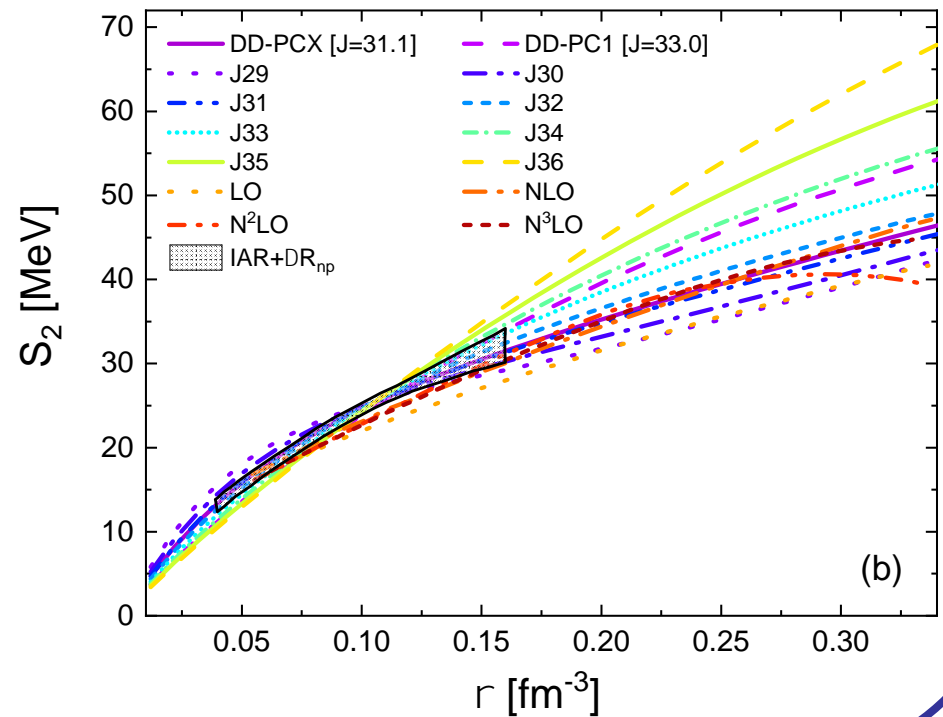
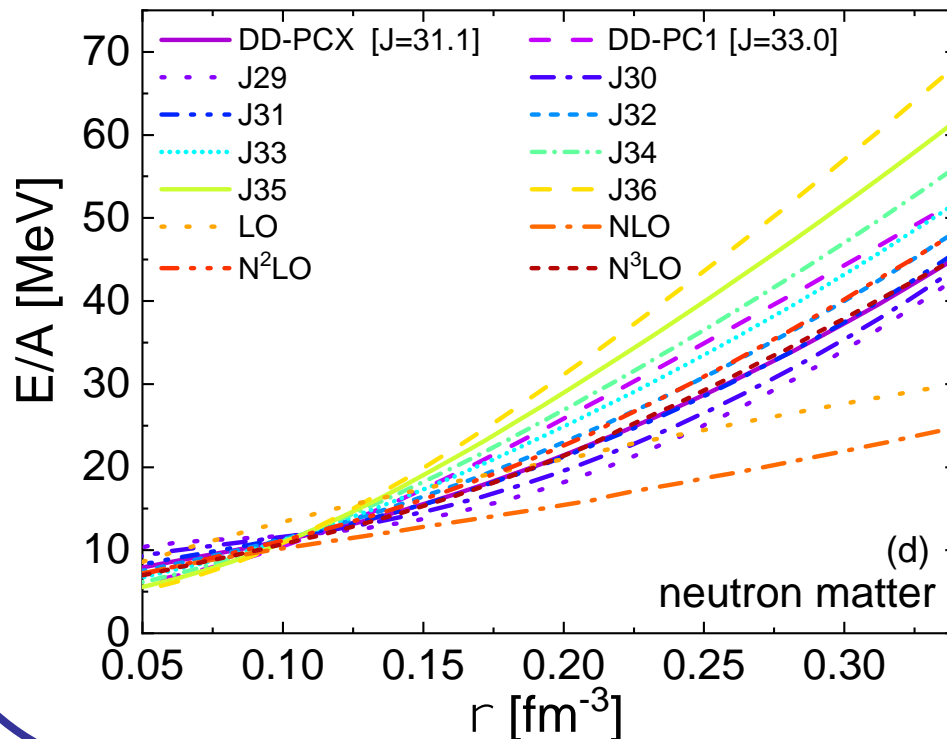
Nuclear equation of state from ground and collective excited state properties of nuclei

Progress in Particle and Nuclear Physics 101, 96-176 (2018).

VARIATION OF THE SYMMETRY ENERGY IN CONSTRAINING THE EDF

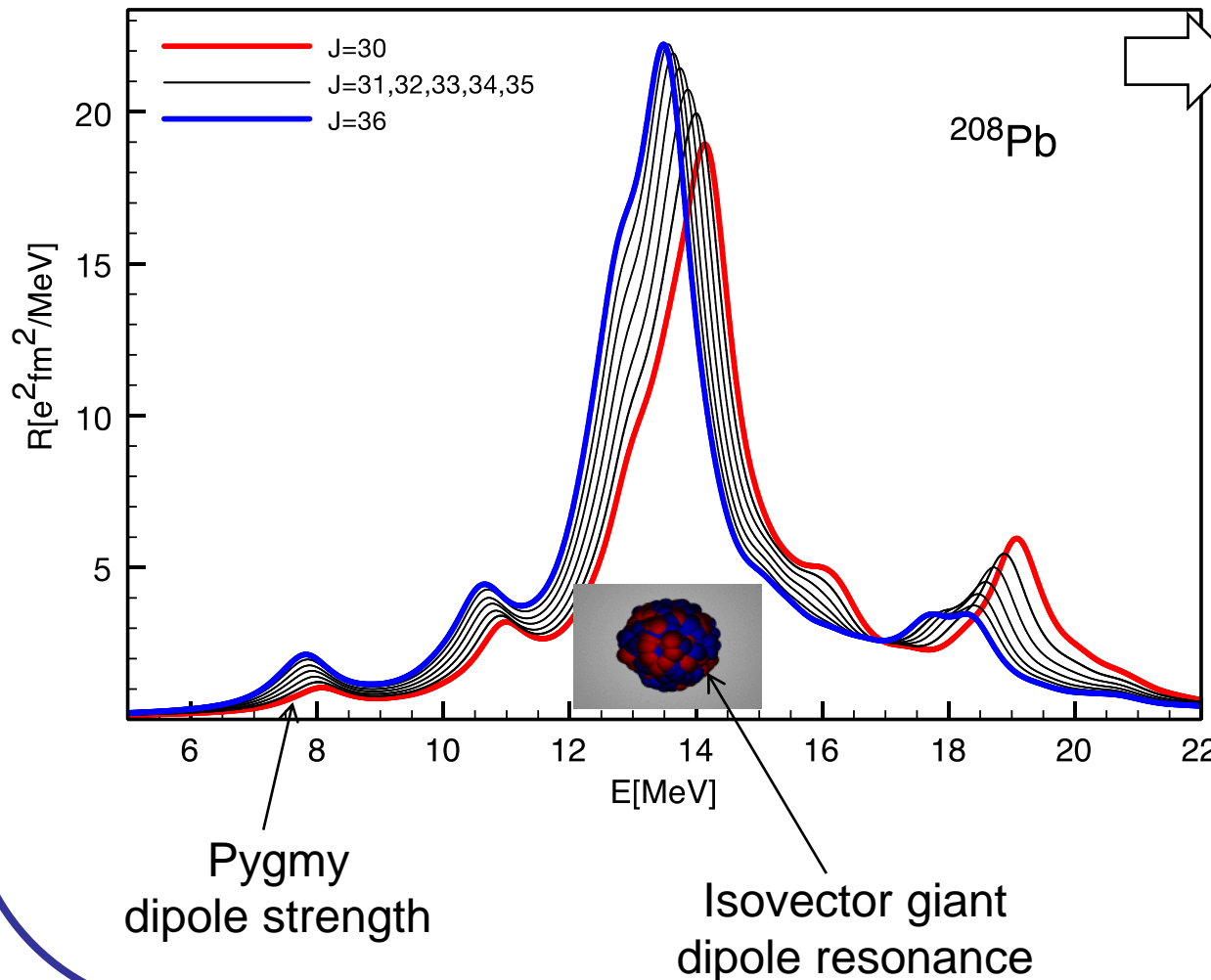
- Establish a set of 8 relativistic point coupling interactions that span the range of values of the symmetry energy at saturation density: $J=29,30,\dots,36$ MeV E. Yuksel, N.P., Universe 7, 71 (2021).
- Adjust the properties of 72 spherical nuclei to exp. data (binding energies, charge radii, diffraction radii, surface thickness, pairing gaps)
- Each interaction is determined independently using the same dataset supplemented with an additional constraint on J

NEUTRON MATTER ENERGY p.p.



CONSTRAINING THE SYMMETRY ENERGY

- Isovector dipole transition strength for ^{208}Pb using a set of relativistic point coupling interactions which vary the symmetry energy properties ($J=30,31,\dots,36$ MeV)



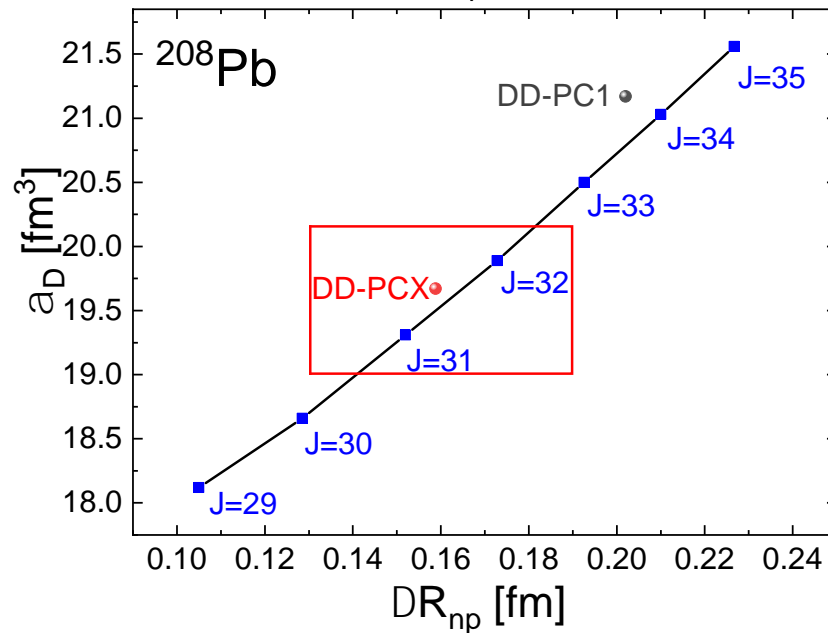
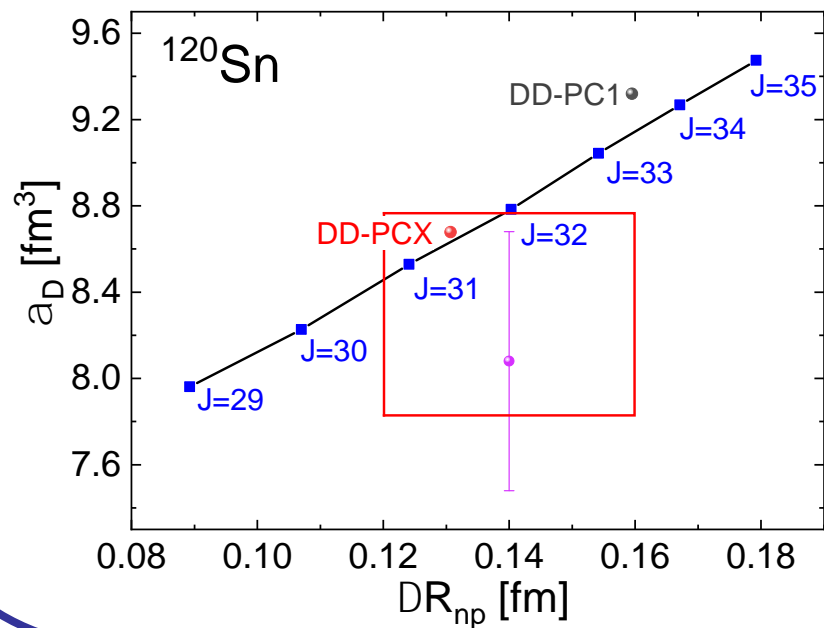
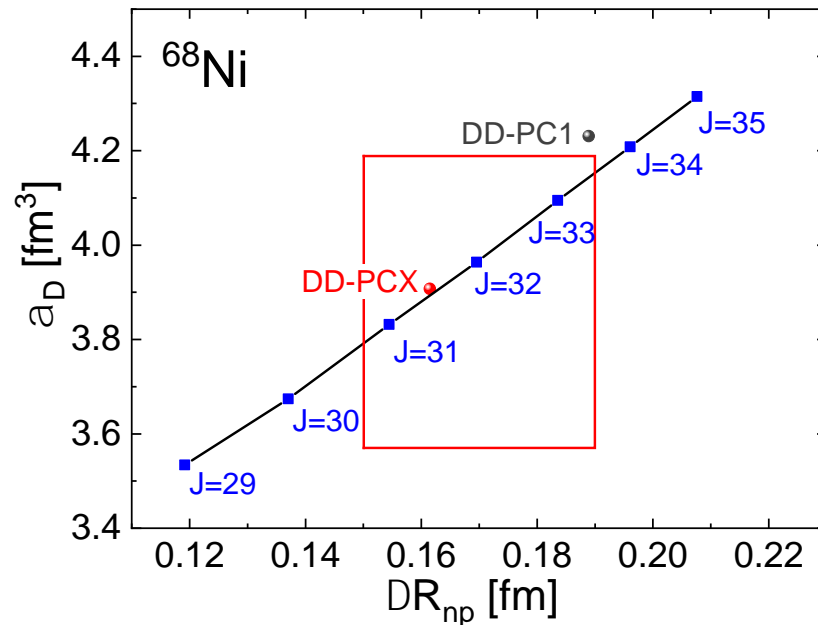
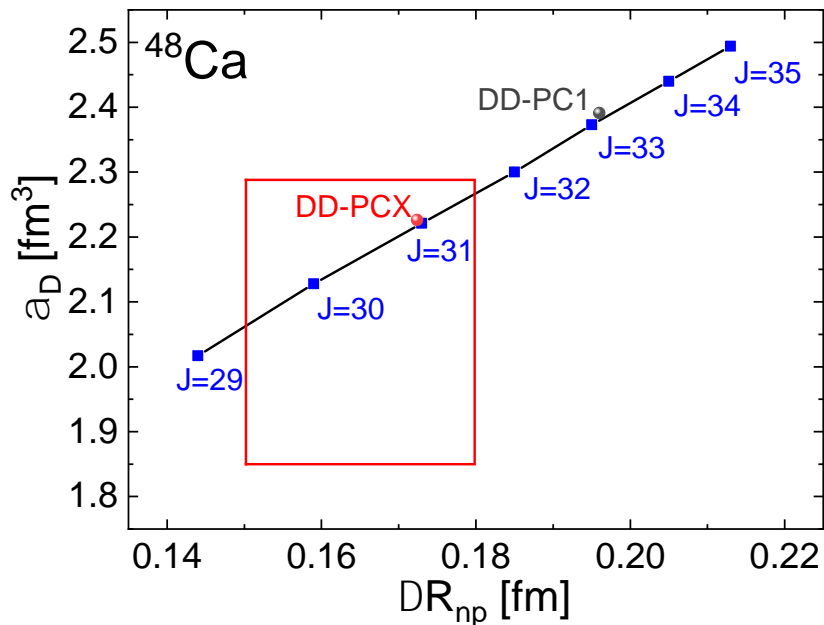
- Isovector giant dipole resonance
- Pygmy dipole strengths
- Dipole polarizability ($\alpha_D \sim m_{-1}$)

- The transition strength is sensitive on the properties of symmetry energy - (J,L)

- **Strategy to determine symmetry energy parameters:**

- build the EDF which accurately reproduces experimental data on the dipole transitions (e.g., dipole polarizability, pygmy strength, etc.)

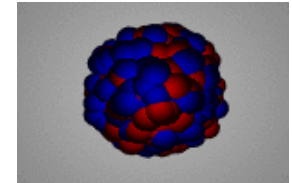
CONSTRAINING NEUTRON SKINS FROM DIPOLE POLARIZABILITY



CONSTRAINING THE NUCLEAR MATTER INCOMPRESSIBILITY

- Nuclear matter incompressibility $K_{nm} = 9\rho_0^2 \frac{d^2 E}{d\rho^2} \bigg|_{\rho=\rho_0}$
- It can be determined from the energies of compression mode in nuclei: **Isoscalar Giant Monopole Resonance (ISGMR)**
- ISGMR energies from inelastic scattering of α -particles, e.g. [D. Patel et al., PLB 726, 178 \(2013\)](#)

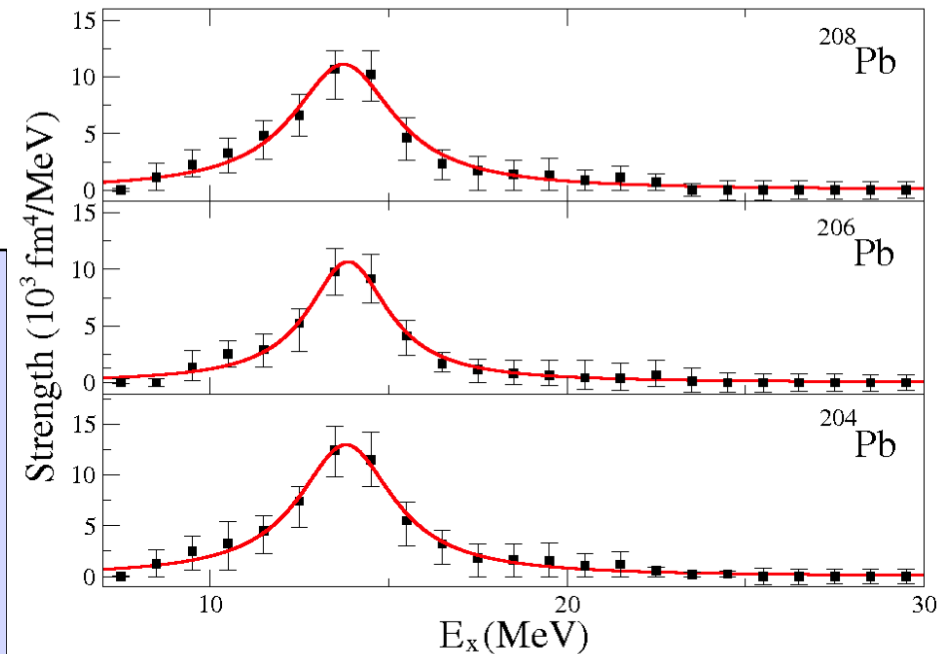
ISGMR



$$E_{ISGMR} = \hbar \sqrt{\frac{K_A}{m \langle r^2 \rangle}} \quad (\text{for nuclei})$$

Strategy to determine K_{nm} (for nuclear matter)

- ❑ Build the EDF that accurately reproduces (e.g. by using the RPA) the experimental data on the ISGMR excitation energy
- The K_{nm} value associated with the EDF that best describes the experimental ISGMR energy is considered as the “correct” one.



CONSTRAINING THE ENERGY DENSITY FUNCTIONAL

- The relativistic point coupling EDF constrained by the nuclear ground state properties and collective excitation properties in finite nuclei – **DD-PCX**
 - E. Yuksel, T. Marketin, N.P., PRC 99, 034318 (2019)

- The observables used directly in χ^2 minimization to constrain the functional:
 - **standard** – nuclear masses (34), charge radii (26), pairing gaps (15)
 - **additional** – dipole polarizability (^{208}Pb) to constrain the symmetry energy

Exp. data: $\alpha_D = 19.6 \pm 0.6 \text{ fm}^3$ A. Tamii et al. PRL 107, 062502 (2011)

A. Tamii (private communication (2015))

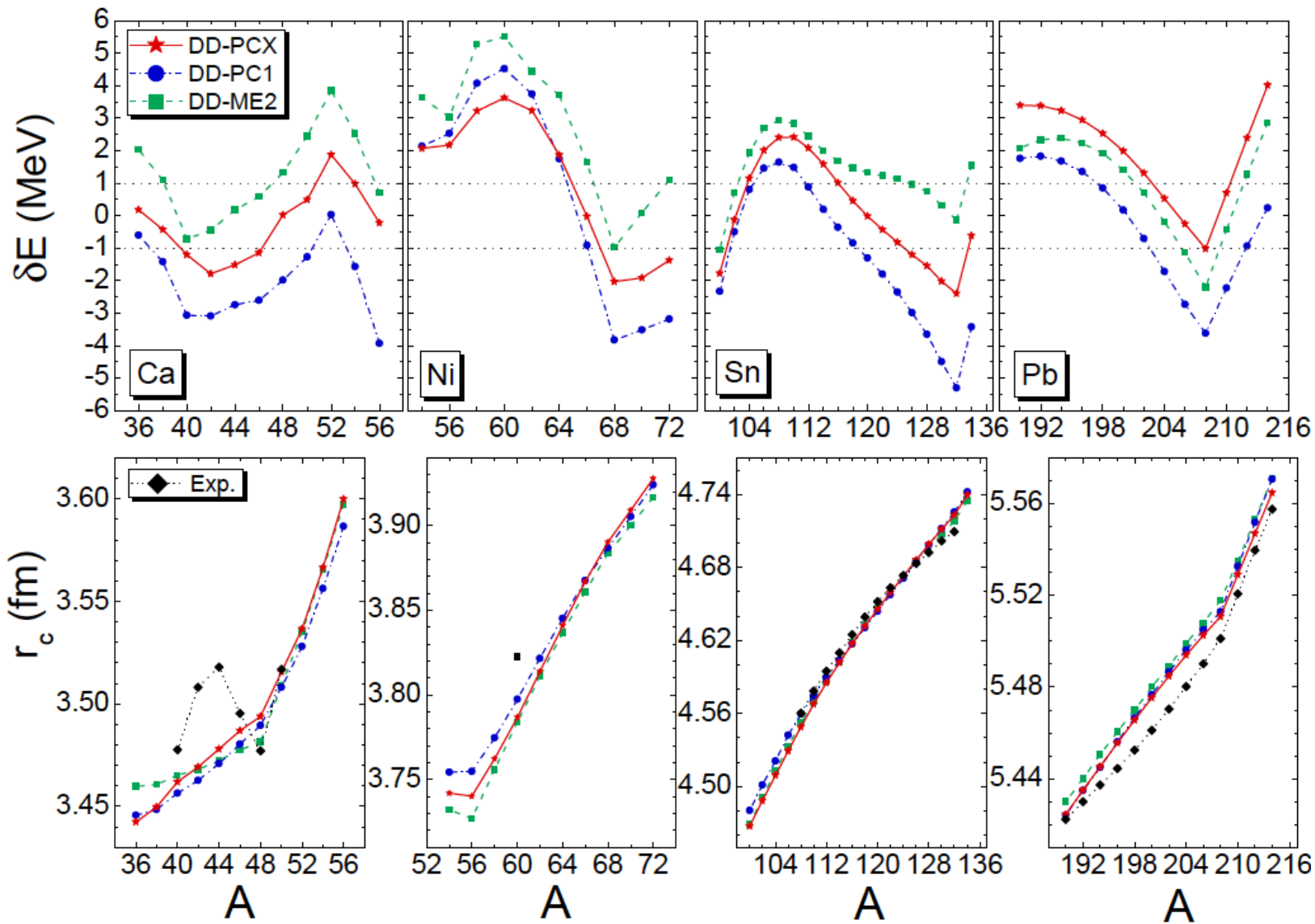
– isoscalar giant monopole resonance (^{208}Pb) to constrain the nuclear matter incompressibility

Exp. data: $E_{\text{ISGMR}} = 13.5 \pm 0.1 \text{ MeV}$ D. Patel et al., PLB 726, 178 (2013)

- ✓ **DD-PCX** → the first EDF constrained by nuclear ground state properties, dipole polarizability and ISGMR excitation energy
- The EDF constrained by this procedure will have the “*correct*” *symmetry energy* – the one that is necessary to reproduce the experimental data on dipole polarizability
- It will also have the “*correct*” *nuclear matter incompressibility* that is compatible with the measured ISGMR excitation energy

NUCLEAR BINDING ENERGIES AND CHARGE RADII

DD-PCX: new relativistic density dependent point coupling interaction



THE PROPERTIES OF FINITE NUCLEI AND NUCLEAR MATTER

DD-PCX: RMS errors for binding energies, charge radii, and mean gap values for a selection of nuclei

Interaction		B.E. (65)	r_c (46)	Mean Gap (56)
DD-PCX	Δ	1.38 MeV	0.016 fm	0.18 MeV
	δ	0.21%	0.47%	15.40%
DD-PC1	Δ	3.05 MeV	0.017 fm	0.29 MeV
	δ	0.48 %	0.49%	21.73%
DD-ME2	Δ	2.08 MeV	0.016 fm	0.35 MeV
	δ	0.27 %	0.44 %	26.13%

$$\Delta = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_i^{exp} - y_i^{th})^2}$$

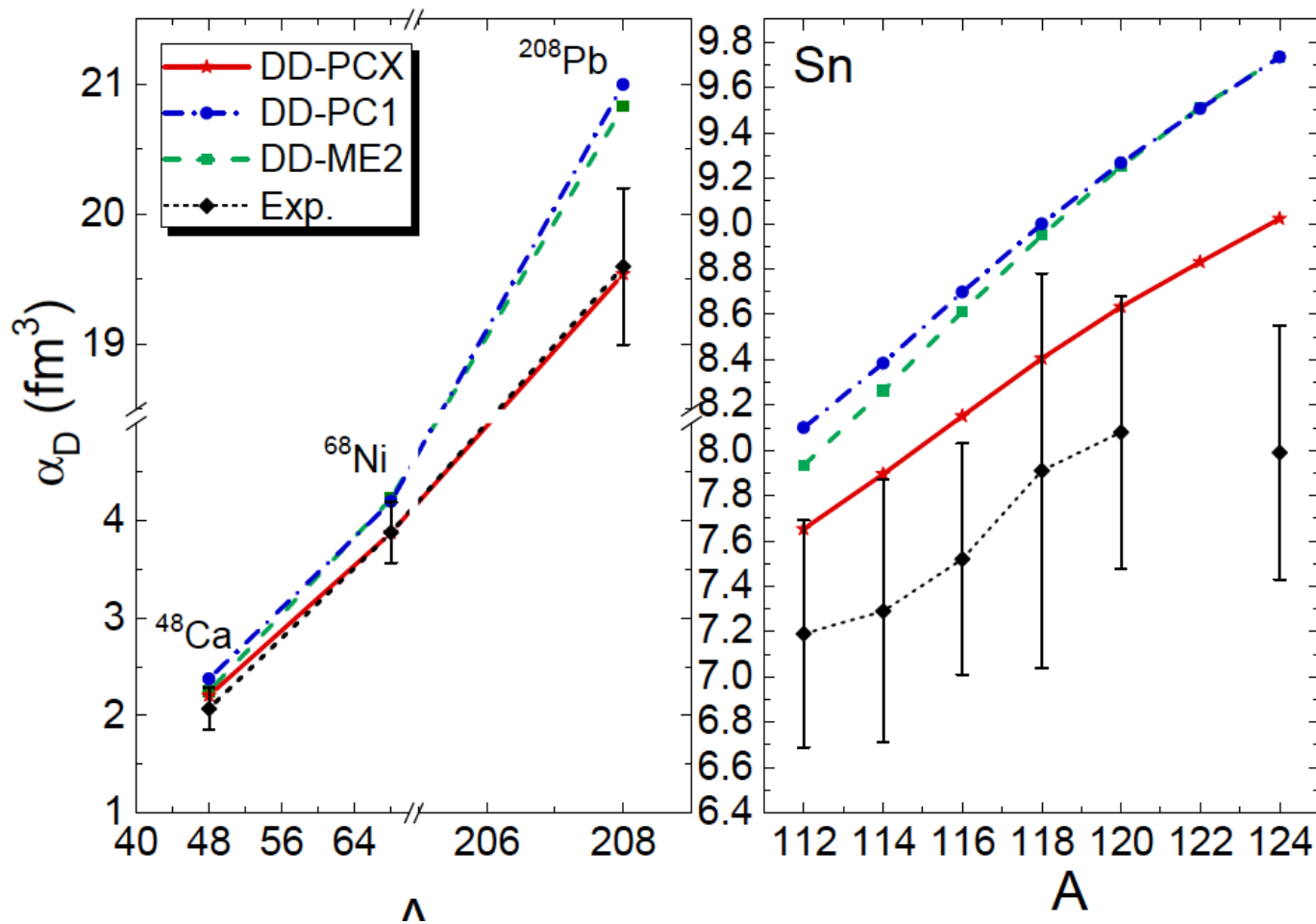
$$\delta = \sqrt{\frac{1}{N} \sum_{i=1}^N \frac{(y_i^{exp} - y_i^{th})^2}{(y_i^{exp})^2}}$$

saturation properties
of nuclear matter

	DD-PCX	DD-PC1	DD-ME2
E/A (MeV)	-16.026 ± 0.018	-16.06	-16.14
m_D^*/m	0.5598 ± 0.0008	0.580	0.572
K_0 (MeV)	213.03 ± 3.54	230.0	250.89
J (MeV)	31.12 ± 0.32	33.0	32.30
L (MeV)	46.32 ± 1.68	70.0	51.26

DIPOLE POLARIZABILITY

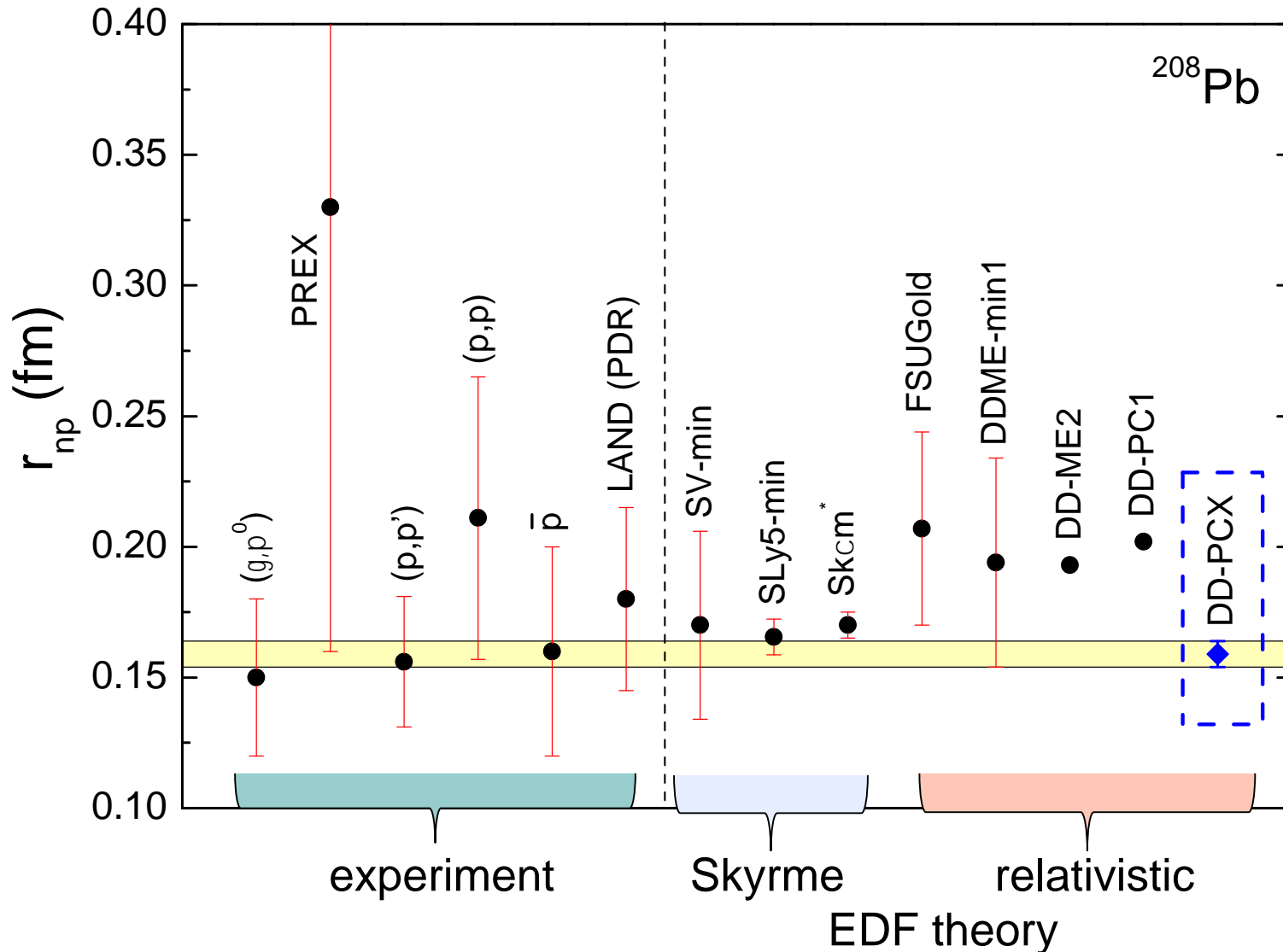
- dipole polarizability (α_D) in several nuclei – **DD-PCX** interaction improves description of the exp. data E. Yuksel, T. Marketin, N.P., PRC 99, 034318 (2019).



- Exp. data on α_D :
 - ^{48}Ca - J. Birkhan et al., PRL 118, 252501 (2017).
 - ^{68}Ni - D. M. Rossi et al., PRL 111, 242503 (2013).
 - $^{112-124}\text{Sn}$ - S. Bassauer et al., PLB 810, 135804 (2020).
 - ^{208}Pb - A. Tamii et al., PRL 107, 062502 (2011); A. Tamii (priv. comm.) (2015).

NEUTRON SKIN THICKNESS

- neutron skin thickness in neutron-rich nucleus ^{208}Pb
- **DD-PCX**: $r_{\text{np}}(^{208}\text{Pb}) = 0.159 \pm 0.005 \text{ fm}$



PREX AND CREX EXPERIMENTS

Measurement of neutral weak form factor F_W in elastic scattering of electrons on nuclei

- ❖ significantly larger coupling of the Z_0 boson to neutrons compared to protons
- ▶ □ Parity violating (PV) asymmetry

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

→ nuclear weak form factor F_W

^{48}Ca (CREX)

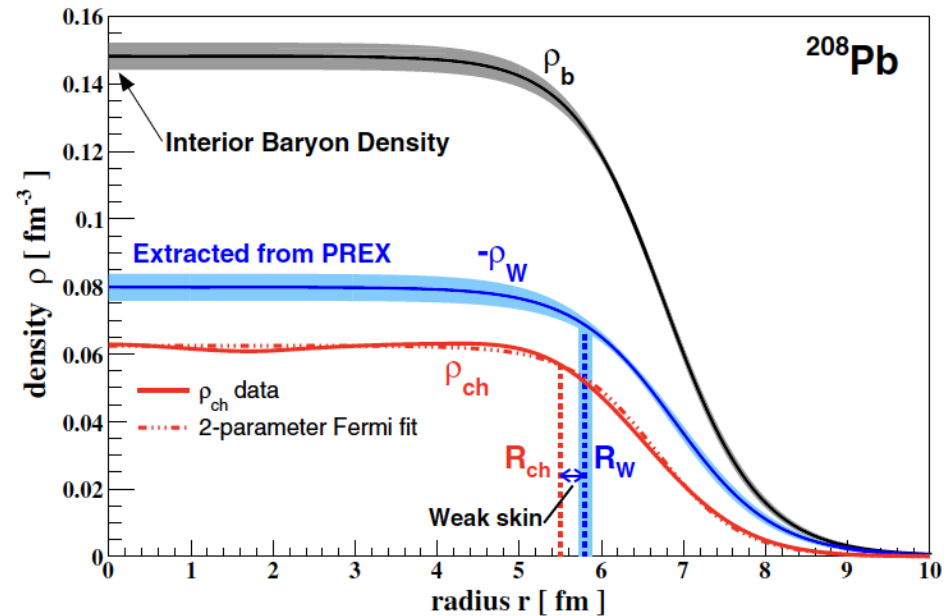
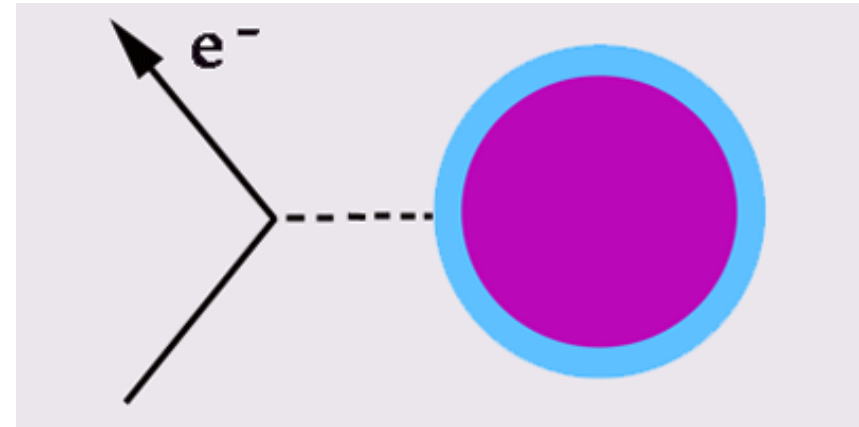
$$F_W(q=0.8733 \text{ fm}^{-1}) = 0.1304 \pm 0.0052(\text{stat}) \pm 0.0020(\text{syst.})$$

D. Adhikari et al., PRL 129, 042501 (2022).

^{208}Pb (PREX II)

$$F_W(q=0.3978 \text{ fm}^{-1}) = 0.368 \pm 0.013$$

D. Adhikari et al., PRL 126, 172502 (2021).
S. Abrahamyan et al., PRL 108, 112502 (2012).



D. Adhikari et al., PRL 126, 172502 (2021).

What are implications of new CREX and PREX experiments for the nuclear matter symmetry energy and isovector properties of finite nuclei: neutron skin thickness and dipole polarizability?

We introduce EDFs directly constrained by F_W :

- The CREX and PREX-II weak form factors F_W are employed directly in χ^2 minimization of the model parameters of the relativistic density-dependent point coupling EDFs, alongside with the usual properties of nuclei (E_B , r_c , pairing gaps)

E. Yuksel, N.P., arXiv:2206.06527 (2022).

→ 3 new relativistic point coupling interactions:

- **DDPC-CREX** – constrained by $F_W(^{48}\text{Ca})$
- **DDPC-PREX** – constrained by $F_W(^{208}\text{Pb})$
- **DDPC-REX** – constrained by $F_W(^{48}\text{Ca})$ & $F_W(^{208}\text{Pb})$

Nuclear matter properties for relativistic point coupling EDFs

	E/A (MeV)	m_D^*/m	K_0 (MeV)	J (MeV)	L (MeV)
DDPC-CREX	-15.989(15)	0.5672(13)	225.48(4.69)	27.01(16)	19.60(64)
DDPC-PREX	-16.108(17)	0.5680(15)	235.41(5.20)	36.18(47)	101.78(4.87)
DDPC-REX	-16.019(15)	0.5696(7)	242.95(2.04)	28.86(15)	30.03(63)
DD-PC1	-16.061	0.580	230.0	33.0	70.1
DD-PCX	-16.026(18)	0.5598(8)	213.03(3.54)	31.12(32)	46.32(1.68)

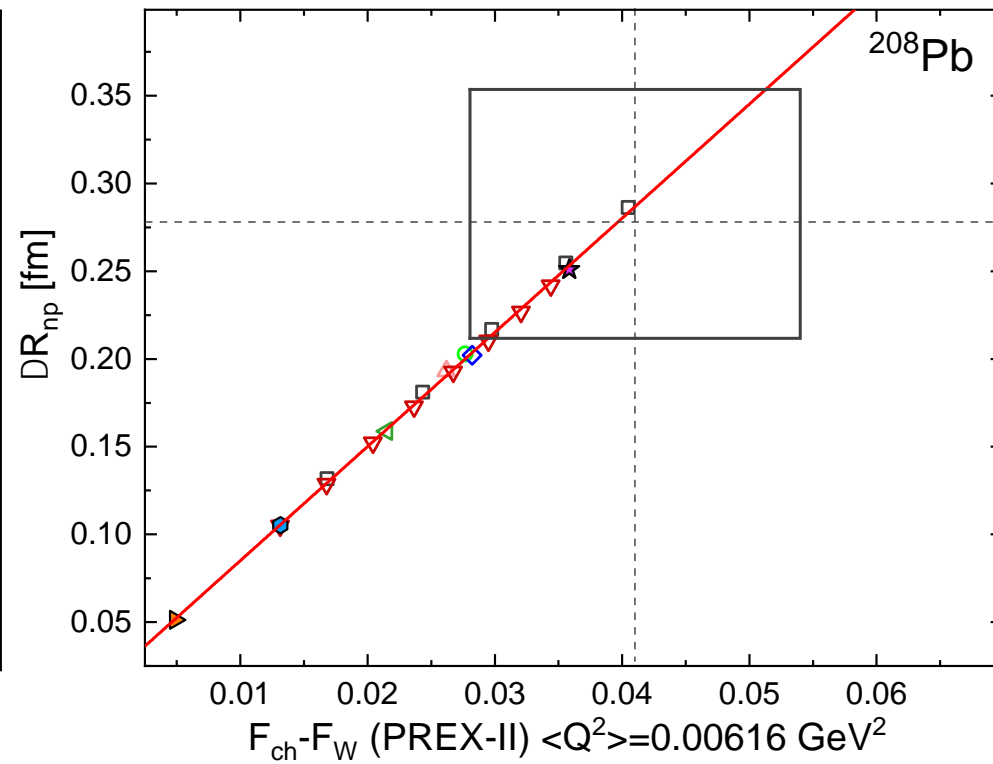
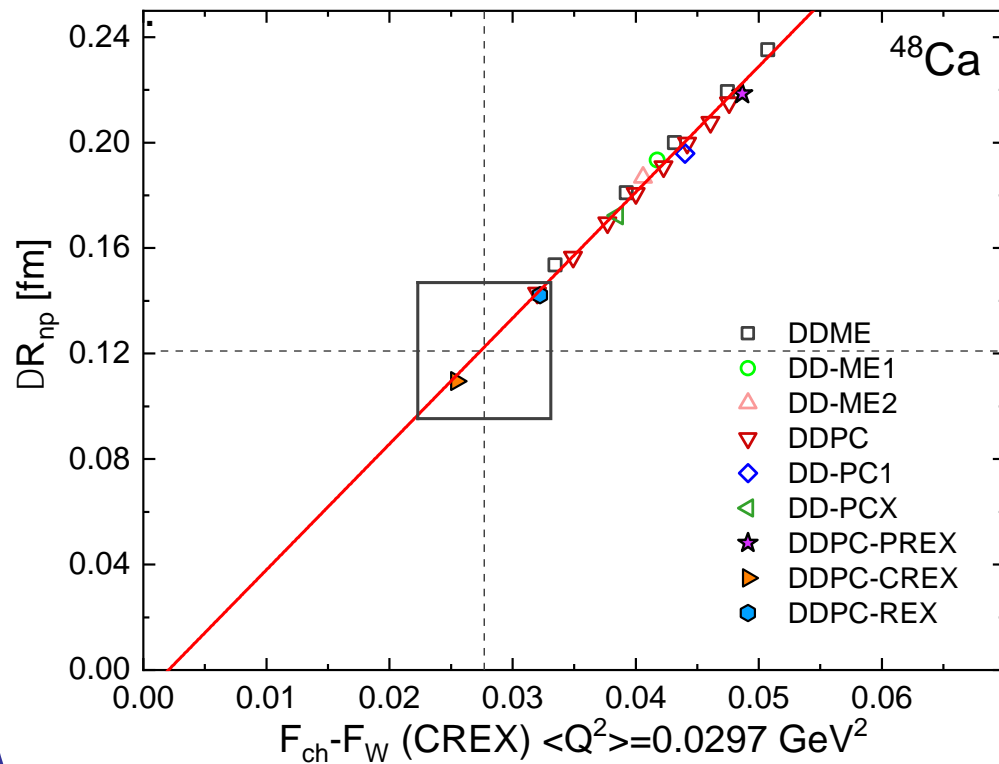
EDFs constrained by CREX and PREX data result in rather different values of the symmetry energy (J) and its slope parameter (L)

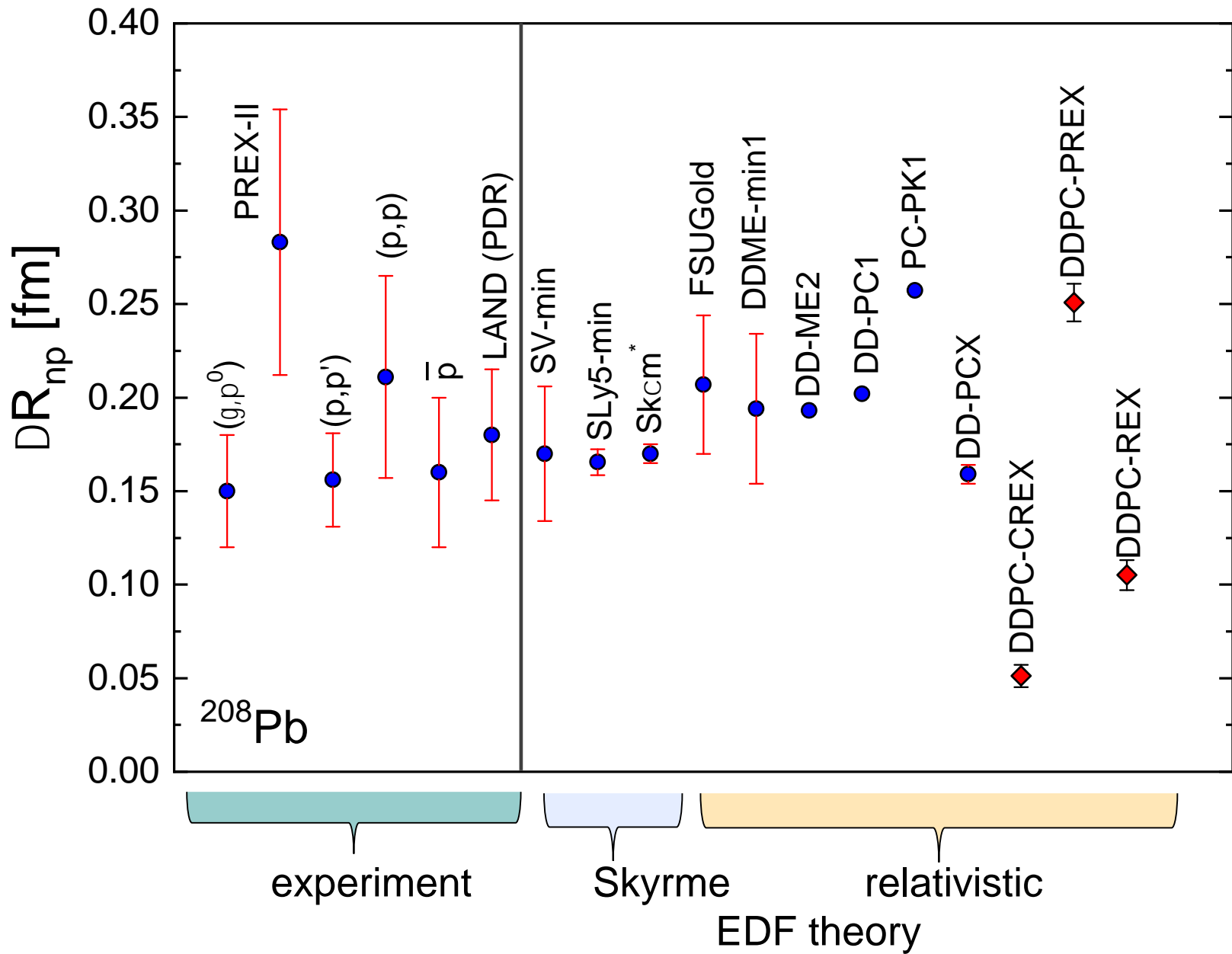
PROBING THE NEUTRON SKIN THICKNESS

The neutron skin thickness ΔR_{np} of ^{48}Ca and ^{208}Pb as a function of the form factor difference $F_{ch} - F_W$ using relativistic energy density functionals.

The limits from PREX-II and CREX are denoted with vertical and horizontal bands

(D. Adhikari et al., PRL 126, 172502 (2021); PRL 129, 042501 (2022).)



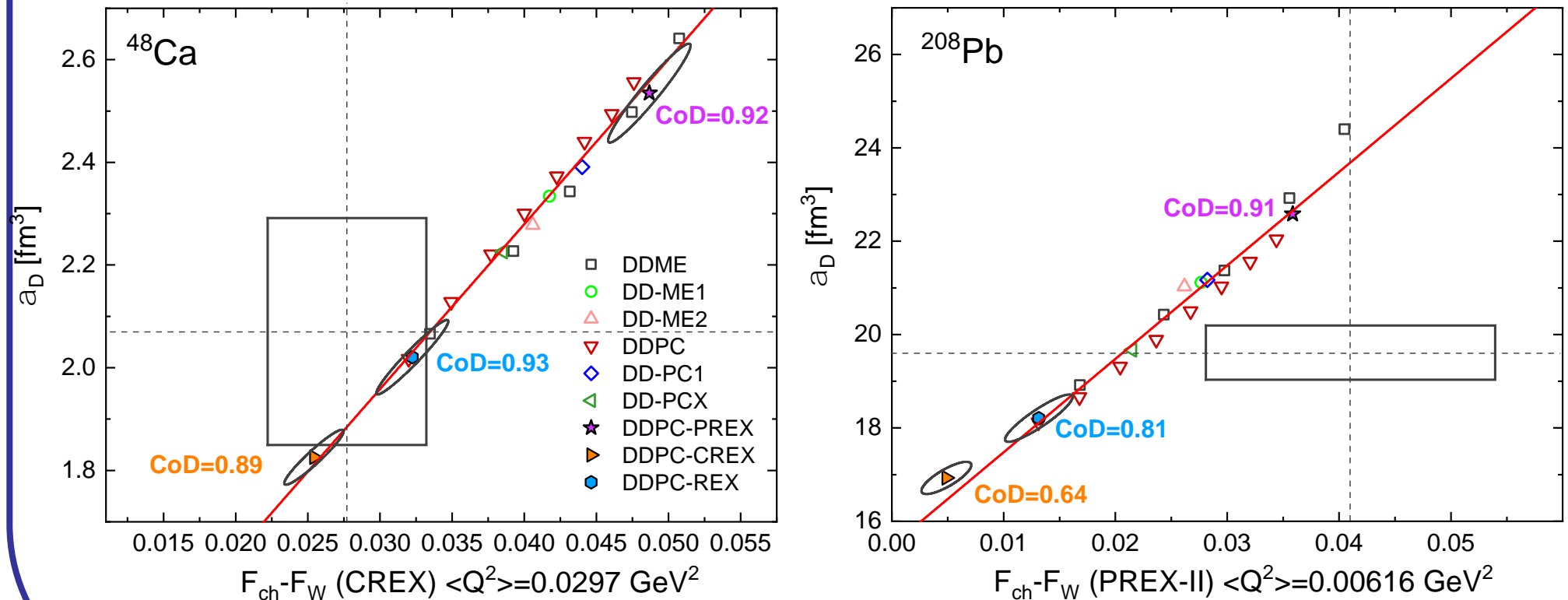


PROBING THE DIPOLE POLARIZABILITY

The dipole polarizability α_D of ^{48}Ca and ^{208}Pb as a function of the form factor difference $F_{\text{ch}} - F_{\text{W}}$ using relativistic EDFs.

Vertical bands denote $F_{\text{ch}} - F_{\text{W}}$ range of values from the CREX and PREX-II (D. Adhikari et al., PRL 126, 172502 (2021); PRL 129, 042501 (2022).)

Horizontal bands correspond to the experimental data on α_D (J. Birkhan et al., PRL 118, 252501 (2017); A. Tamii et al., PRL 107, 062502 (2011); X. Roca-Maza et al., PRC 92, 064304 (2015).)



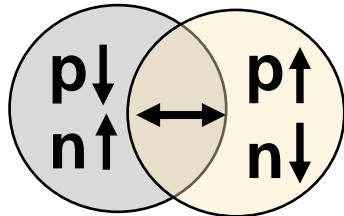
MAGNETIC DIPOLE TRANSITIONS

Isvector M1 spin-flip transitions

- induced by $\sigma\tau_3$ operator

$$\Delta J=1, \Delta L=0, \Delta\pi=0, \Delta S=1, \Delta T=1$$

$$J^\pi=0^+(\text{GS}) \rightarrow J^\pi=1^+$$

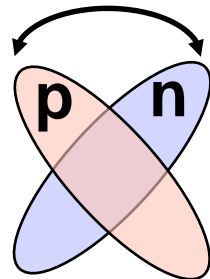


- Transitions between spin-orbit partner states allowed

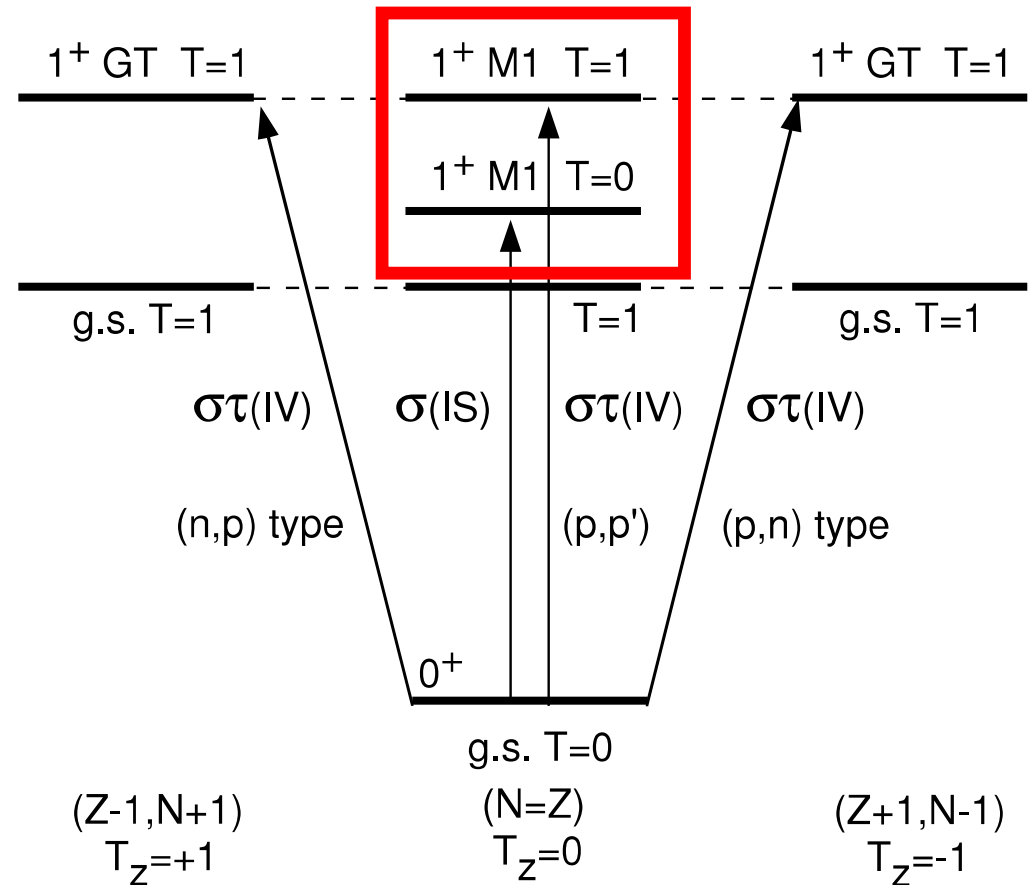
M1 transitions can also be induced by the orbital angular momentum operator \vec{l}

Scissors mode (orbital M1, $\Delta T=1$)

D. Bohle, A. Richter, et al.,
PLB 137, 27 (1984).
K. Heyde et al.,
RMP 82, 2374 (2010).



Y. Fujita et al., PNP 66, 549–606 (2011).



Isvector M1 spin-flip excited states are analogous to Gamow-Teller states in neighboring nuclei

MAGNETIC DIPOLE (M1) TRANSITION OPERATORS

➤ M1 transition operator – spin and orbital angular momentum operators

$$\hat{\mu}_{1\nu} = \sum_{k=1}^A \begin{pmatrix} \hat{\mu}_{1\nu}^{(IS)}(11)_k & 0 \\ 0 & \hat{\mu}_{1\nu}^{(IS)}(22)_k \end{pmatrix} \otimes 1_\tau - \sum_{k=1}^A \begin{pmatrix} \hat{\mu}_{1\nu}^{(IV)}(11)_k & 0 \\ 0 & \hat{\mu}_{1\nu}^{(IV)}(22)_k \end{pmatrix} \otimes \hat{\tau}_3$$

$$\hat{\mu}_{1\nu}^{(IS,IV)}(11)_k = \hat{\mu}_{1\nu}^{(IS,IV)}(22)_k = \frac{\mu_N}{\hbar} (g_\ell^{IS,IV} \hat{\ell}_k + g_s^{IS,IV} \hat{s}_k) \cdot \nabla (r Y_{1\nu}(\Omega_k))$$

➤ Gyromagnetic factors

- orbital $g_\ell^\pi = 1$ $g_\ell^\nu = 0$
- spin $g_s^{\pi(\nu)} = 5.586$ (-3.826)

- isoscalar $g_\ell^{IS} = \frac{g_\ell^\pi + g_\ell^\nu}{2} = 0.5$

$$g_s^{IS} = \frac{g_s^\pi + g_s^\nu}{2} = 0.880$$

- isovector $g_\ell^{IV} = \frac{g_\ell^\pi - g_\ell^\nu}{2} = 0.5$

$$g_s^{IV} = \frac{g_s^\pi - g_s^\nu}{2} = 4.706$$

Largest
factor

M1 TRANSITION STRENGTH DISTRIBUTIONS

- Relativistic Quasiparticle Random Phase Approximation (RQRPA) equations

$$\begin{pmatrix} A^J & B^J \\ B^{*J} & A^{*J} \end{pmatrix} \begin{pmatrix} X^{\nu, JM} \\ Y^{\nu, JM} \end{pmatrix} = \hbar\omega_\nu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X^{\nu, JM} \\ Y^{\nu, JM} \end{pmatrix}$$

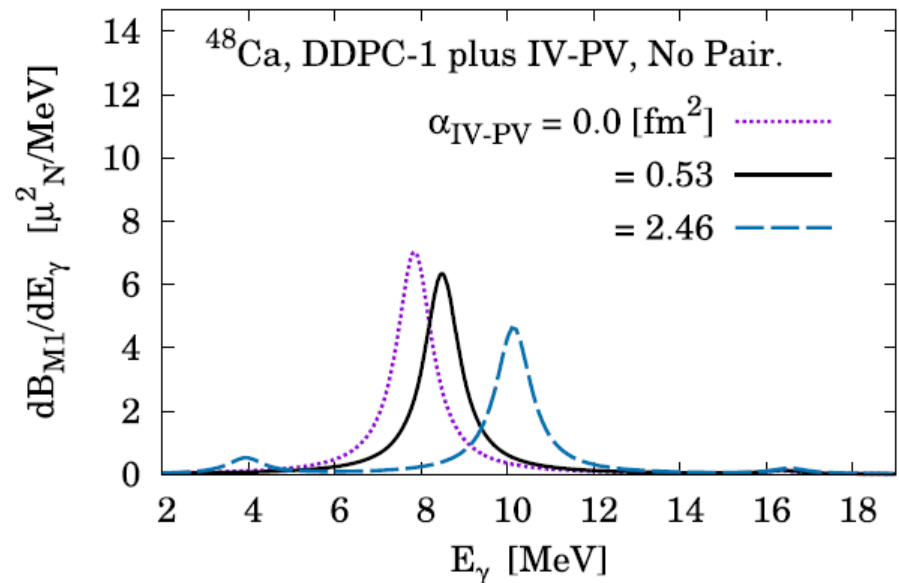
- Includes also contribution from the pseudovector interaction term to describe unnatural parity transitions $J^\pi=1^+$

$$\mathcal{L}_{IV-PV} = -\frac{1}{2} \alpha_{IV-PV} [\bar{\Psi}_N \gamma^5 \gamma^\mu \vec{\tau} \Psi_N] \cdot [\bar{\Psi}_N \gamma^5 \gamma_\mu \vec{\tau} \Psi_N]$$

- The PV coupling strength is constrained by exp. M1 energies in ^{48}Ca and ^{208}Pb

Transition strength

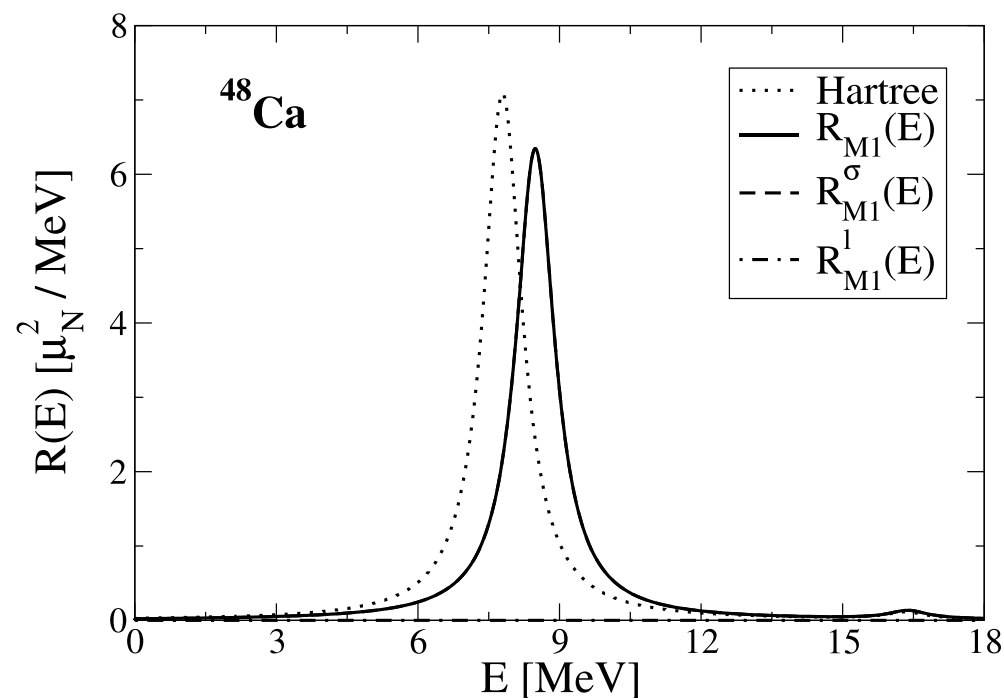
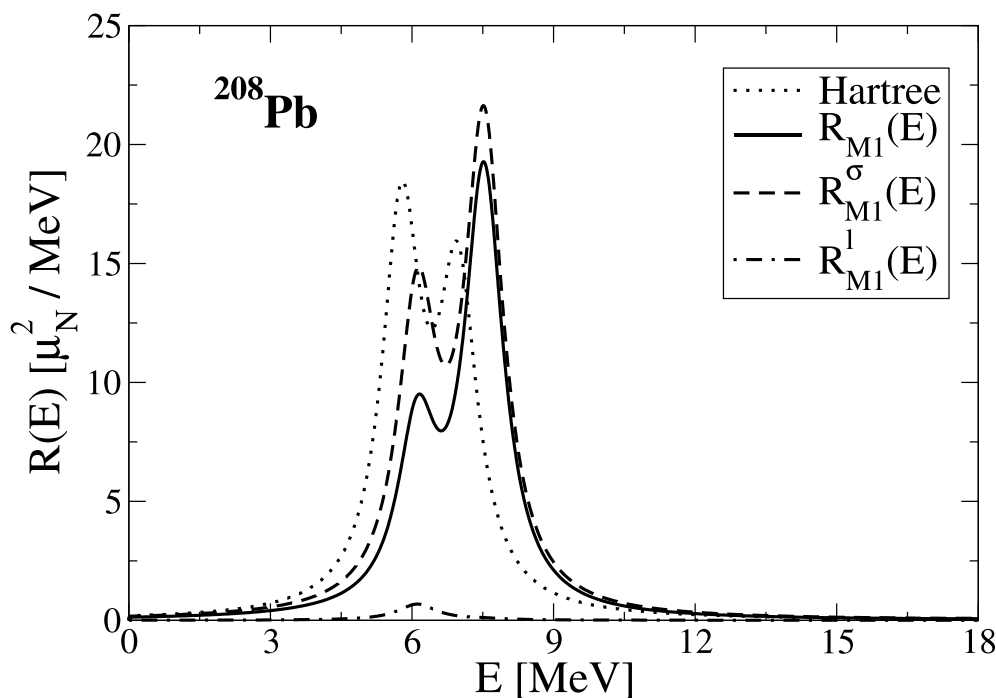
$$B(MJ, \omega_\nu) = \left| \sum_{\kappa\kappa'} \left(X_{\kappa\kappa'}^{\nu, J0} \langle \kappa || \hat{\mu}_J || \kappa' \rangle + (-1)^{j_\kappa - j_{\kappa'} + J} Y_{\kappa\kappa'}^{\nu, J0} \langle \kappa' || \hat{\mu}_J || \kappa \rangle \right) \times \left(u_\kappa v_{\kappa'} + (-1)^J v_\kappa u_{\kappa'} \right) \right|^2$$



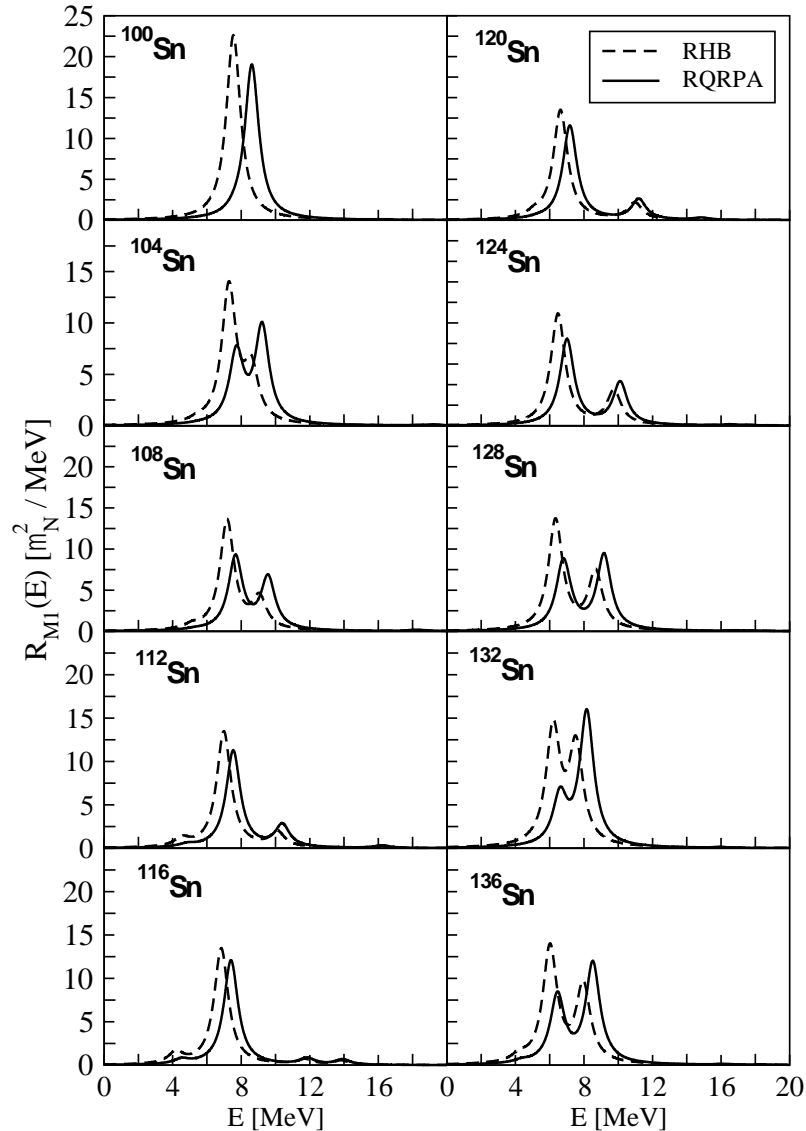
M1 TRANSITION STRENGTH DISTRIBUTIONS

M1 transition strength for ^{48}Ca and ^{208}Pb :

- unperturbed Hartree response
- spin response (large)
- orbital response (small)
- total response (dominated by the spin response)

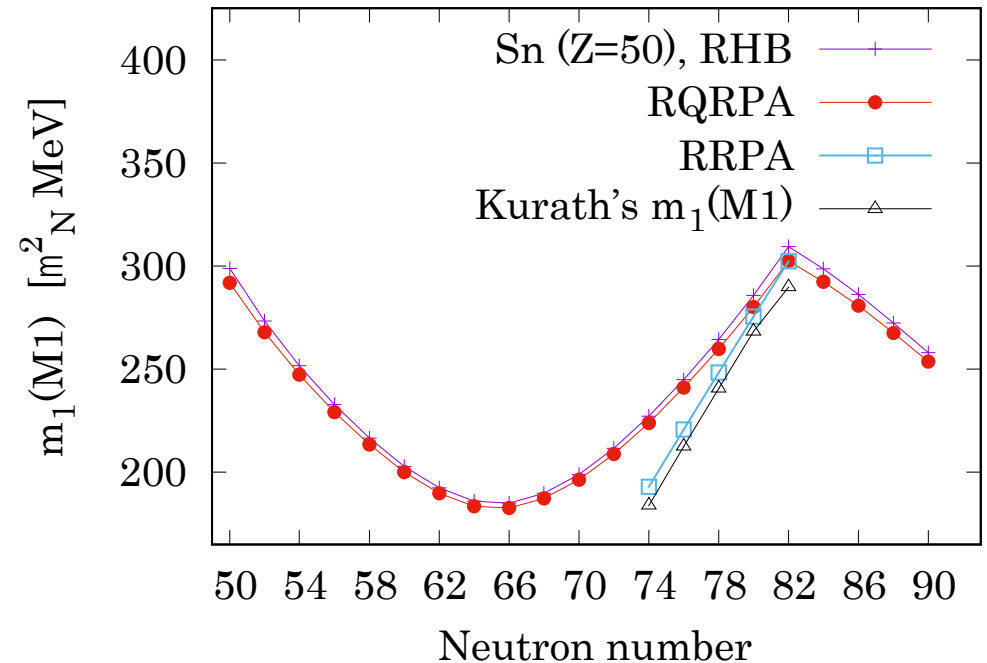


M1 transitions in Sn isotope chain – evolution toward neutron rich nuclei



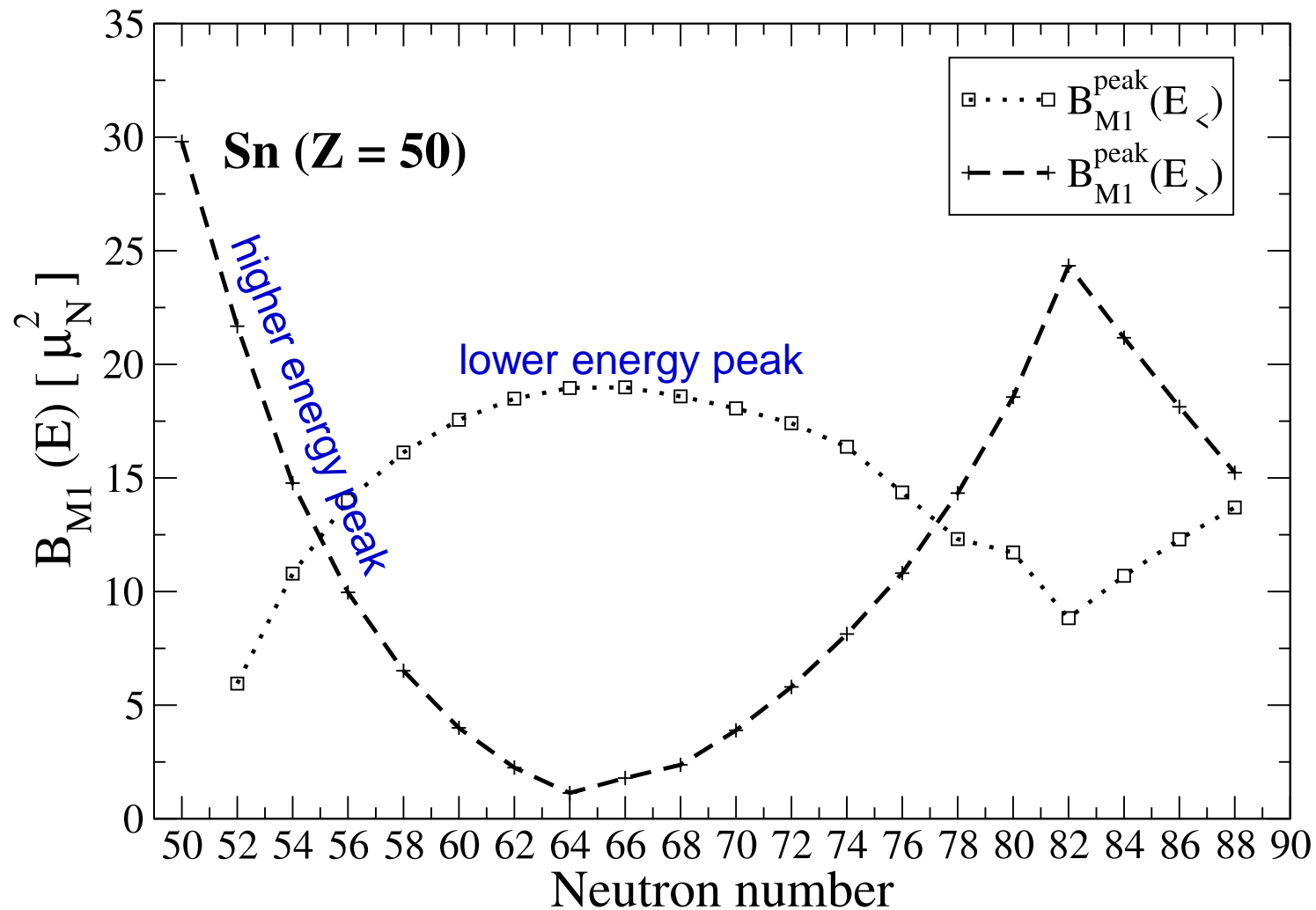
G. Kružić, T. Oishi, N. P., PRC, 103, 054306 (2021).

- Energy weighted M1 transition strength, compared with the Kurath's sum rule values



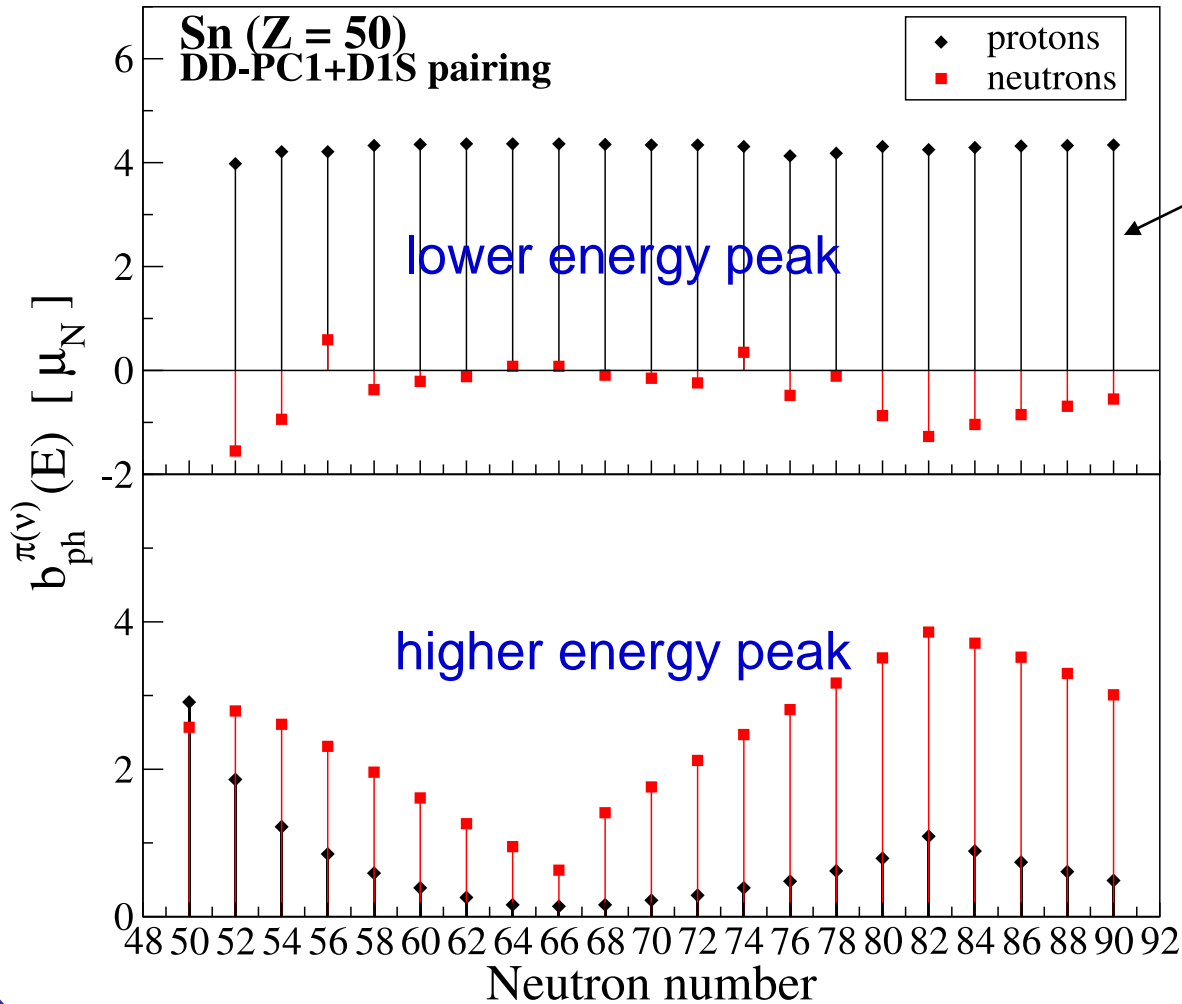
M1 TRANSITIONS IN SN ISOTOPES

The interplay of the transition strength of the two main M1 peaks along Sn isotope chain



M1 TRANSITIONS IN SN ISOTOPES

Partial M1 transition strengths for relevant 2qp configurations



relevant contributions come from proton transitions
 $(\pi 1g_{9/2} \rightarrow \pi 1g_{7/2})$

Neutron transitions

$^{104-116}\text{Sn}$:
 $(\nu 1g_{9/2} \rightarrow \nu 1g_{7/2})$
 $(\nu 2d_{5/2} \rightarrow \nu 2d_{3/2})$

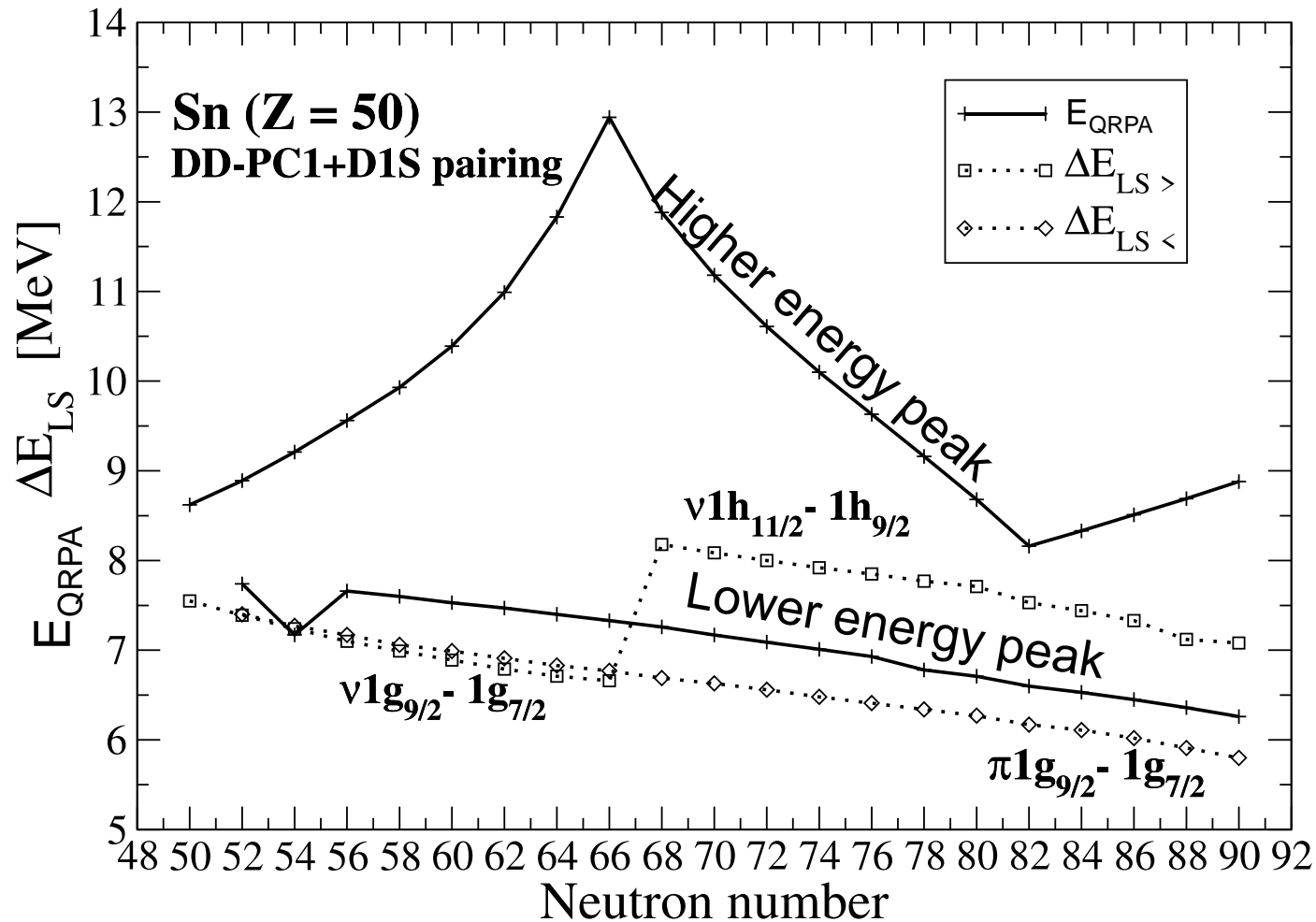
^{116}Sn - neutron transition is suppressed because $\nu 1g_{7/2}$ becomes occupied

$^{118-140}\text{Sn}$

$(\nu 1h_{11/2} \rightarrow \nu 1h_{9/2})$

M1 TRANSITIONS IN SN ISOTOPES

Comparison of the M1 excitation energies with relevant spin-orbit splittings
 - Demonstrate important role of the QRPA residual interaction for M1 transitions



THE QUENCHING OF g -FACTORS FROM M1 TRANSITIONS

- In general, model calculations provide more M1 transition strength than the experimental data. Often the quenching in g -factors (0.6-0.75) is introduced in models to resolve this discrepancy.
- The quenching of g -factors remains an open question to date.

The total RQRPA (DD-PC1) transition strength in comparison to the new experimental data from inelastic proton scattering - [S. Bassauer et al., PRC 102, 034327 \(2020\)](#).

	$\sum B_{M1}^{th.}(\mu_N^2)$	$\sum B_{M1}^{exp.}(\mu_N^2)$	g_{eff}/g_{free}
^{112}Sn	22.81	14.7(1.4)	0.80
^{114}Sn	22.61	19.6(1.9)	0.93
^{116}Sn	22.56	15.6(1.3)	0.83
^{118}Sn	22.76	18.4(2.4)	0.89
^{120}Sn	23.34	15.4(1.4)	0.81
^{124}Sn	25.55	19.1(1.7)	0.86

quenching of the g factors of the free nucleons, needed to reproduce the exp. data on M1 transition strengths.

Less quenching (**0.8-0.93**) is required in comparison to previous studies (0.6-0.75). Some M1 exp. data above neutron threshold may be missing because of limited accuracy \Rightarrow weaker quenching?

CONCLUDING REMARKS

- **DD-PCX relativistic density dependent point coupling interaction**
 - the first EDF constrained by nuclear ground state properties ($E_B, r_c, \Delta_{n,p}$) and collective excitation properties (dipole polarizability and the ISGMR excitation energy in ^{208}Pb)
- **DDPC-CREX, DDPC-PREX, DDPC-REX** functionals established using the weak form factors from parity violating electron scattering.
 - The CREX and PREX-II experiments could not provide consistent constraints on the isovector sector of the EDF.
- **The synergy of the EDF theory with the experiments** on dipole transitions (E1, M1) open perspectives to improve the EDFs
 - E1: isovector properties – neutron-rich nuclei, neutron-skin thickness, symmetry energy of the EOS
 - M1: spin-orbit splittings, constraints on the g factors, residual interactions

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