Heavy Ion Double Charge Exchange reactions and their role in gaining information on Double Beta decay Nuclear Matrix Elements

" Giant and soft modes of excitation in nuclear structure and astrophysics "

Trento, 24-28/10/22

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Heavy Ion Double Charge Exchange nuclear reactions

$$A(Z_A, N_A) + a(Z_a, N_a) \rightarrow B(Z_A \pm 2, N_A \mp 2) + b(Z_a \mp 2, N_a \pm 2)$$

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- new powerful spectroscopic tool for study of unstable neutron-rich nuclei

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H. Lenske, F. Cappuzzello, M. Cavallaro, M. Colonna,
Progress in Particle and Nuclear Physics 109 (2019) 103716.

F. Cappuzzello, H. Lenske et al.,
"Shedding light on nuclear aspects of neutrinoless double beta decay by heavy-ion double charge exchange reactions"
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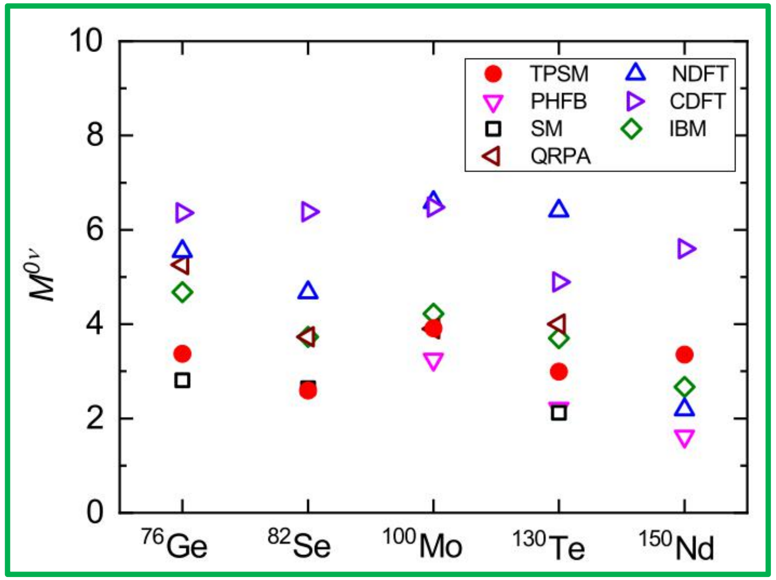
0νββ

$$[T^{0\nu}_{1/2}(\mathcal{N})]^{-1} = G^{\mathcal{N}}_{0\nu} |M^{\mathcal{N}}_{0\nu}|^2 \frac{|m_{2\beta}|^2}{m_e^2}$$

PS factor

J. Kotila, F. Iachello, Phys. Rev. C 85 (2012) 034316



Y.K. Wang et al., Phys. Rev. C **104**, 014320 (2021)

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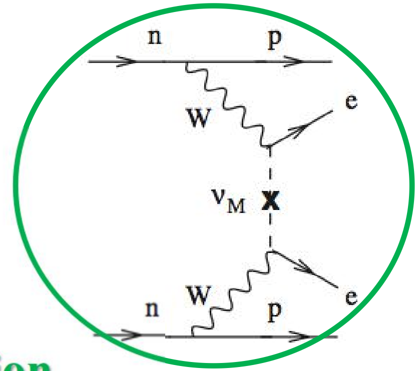
0νββ

- not yet observed
- possible only if neutrinos are massive Majorana fermions [$\nu \equiv \bar{\nu}$]



- **ΔL = ± 2 → total lepton number violation**
  - **probe physics beyond SM**
  - **give insight on matter-antimatter asymmetry**
- **provide information on the absolute neutrino mass**

Heavy Ion Double Charge Exchange nuclear reactions

$$A(Z_A, N_A) + a(Z_a, N_a) \rightarrow B(Z_A \pm 2, N_A \mp 2) + b(Z_a \mp 2, N_a \pm 2)$$

collisional Heavy Ion DCE reactions

- differences:

- DCE mediated by **strong interaction**, 0νββ by **weak interaction**
- Decay vs reaction **dynamics**
- DCE includes **sequential** transfer **mechanism**

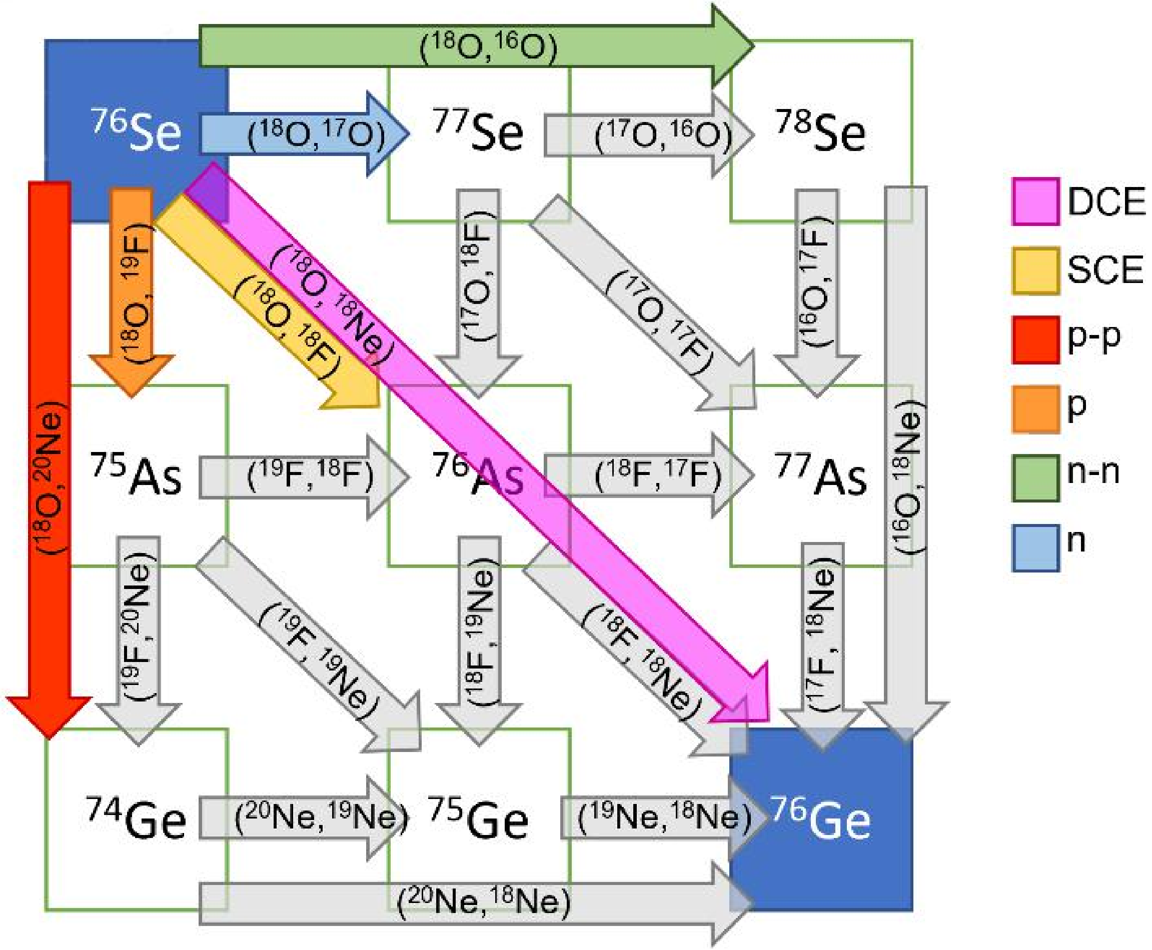
- analogies:

- **Same initial and final states:** Parent/daughter states of the $0\nu\beta\beta$ decay are the same as those of the target/residual nuclei in the DCE
- **Similar operator:** Short-range Fermi, Gamow-Teller and rank-2 tensor components are present in both the transition operators, with tunable weight in DCE
- **Large linear momentum** (~100 MeV/c) available in the virtual intermediate channel
- **Non-local** processes: characterized by two vertices localized in a pair of nucleons
- **Same nuclear medium**: Constraint on the theoretical determination of quenching phenomena on $0\nu\beta\beta$
- **Off-shell propagation** through virtual intermediate channels

F. Cappuzzello *et al.*, Eur. Phys. J. A (2018), **54**: 72

Reaction mechanisms for describing Heavy Ion Double Charge Exchange

$$A(Z_A, N_A) + a(Z_a, N_a) \rightarrow B(Z_A \pm 2, N_A \mp 2) + b(Z_a \mp 2, N_a \pm 2)$$



1) charged meson-induced (direct DCE)

2) mean field -driven
multi-nucleon transfer reactions feeding DCE
(transfer DCE)

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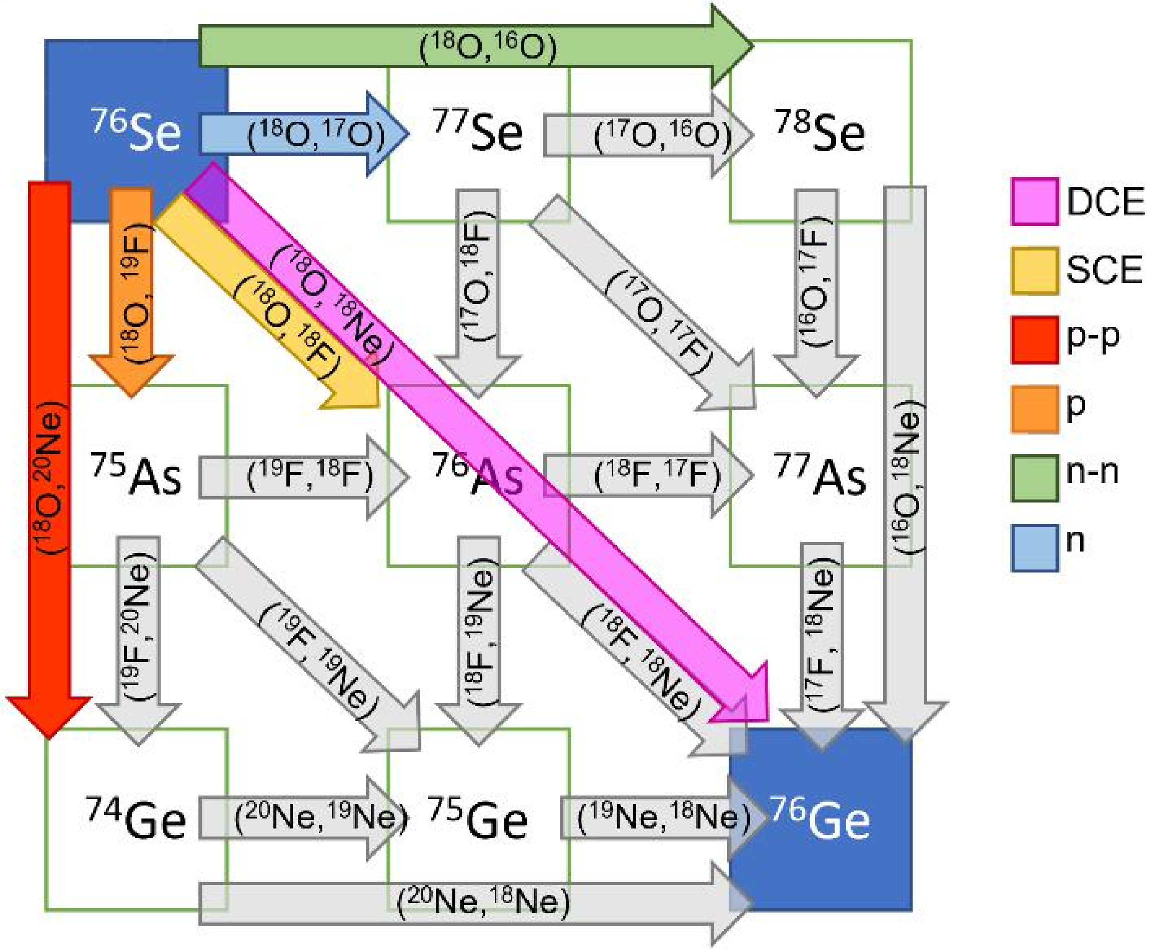

1) charged meson-induced (direct DCE)

2) mean field -driven
multi-nucleon transfer reactions feeding DCE
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safely neglegible for the nuclear systems studied

J. Ferreira et al., Phys. Rev. C 105 (2022) 014630

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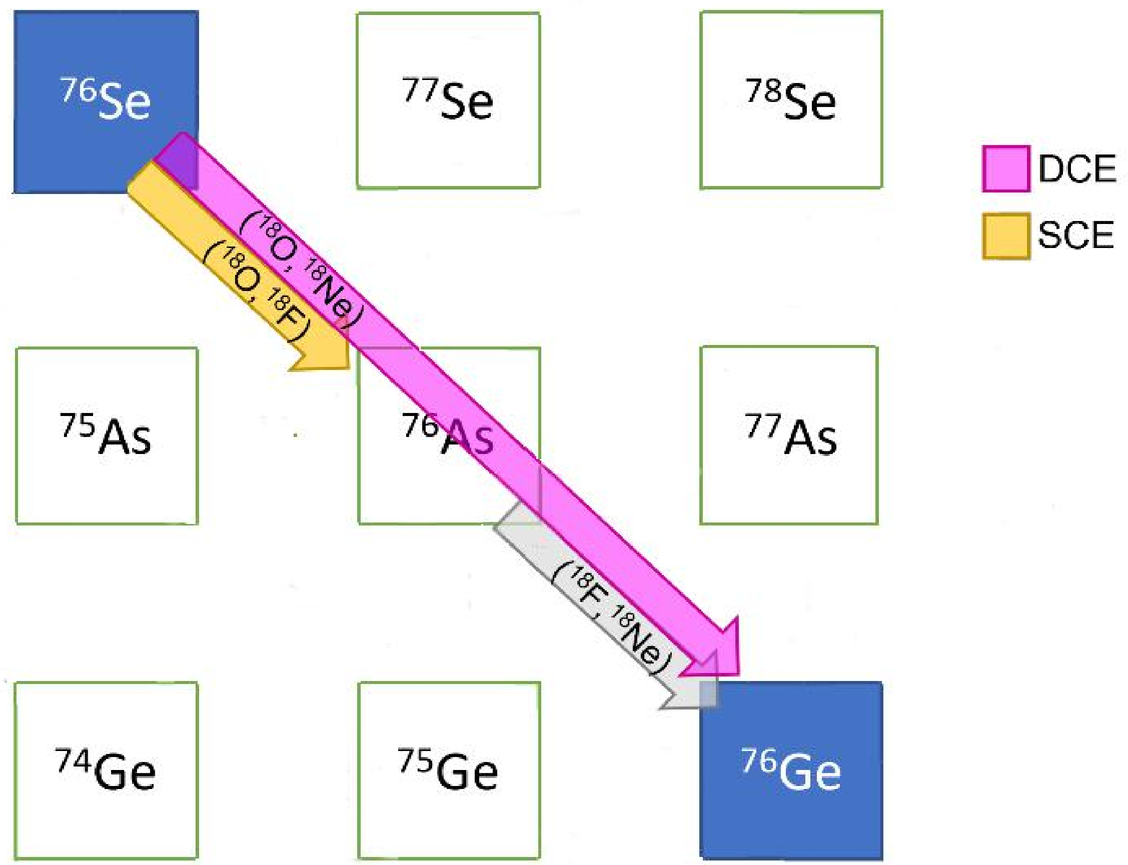
- Heavy Ion DCE reactions: analogies with $0\nu\beta\beta$

F. Cappuzzello *et al.*, Eur. Phys. J. A (2018), **54**: 72



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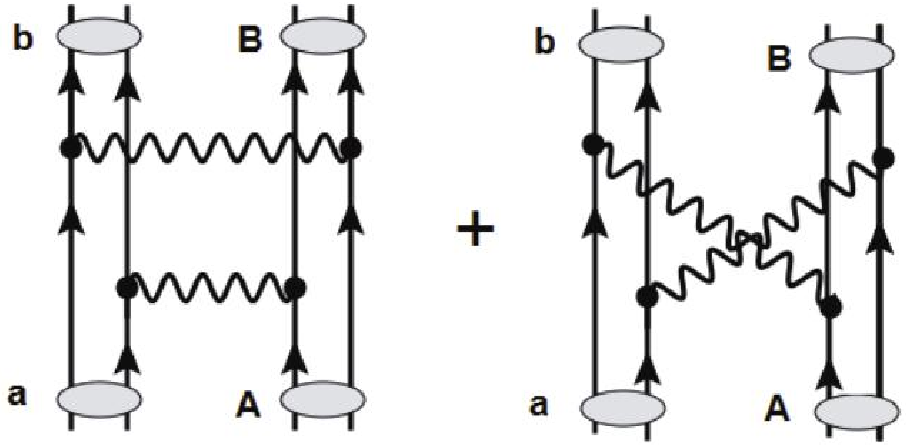
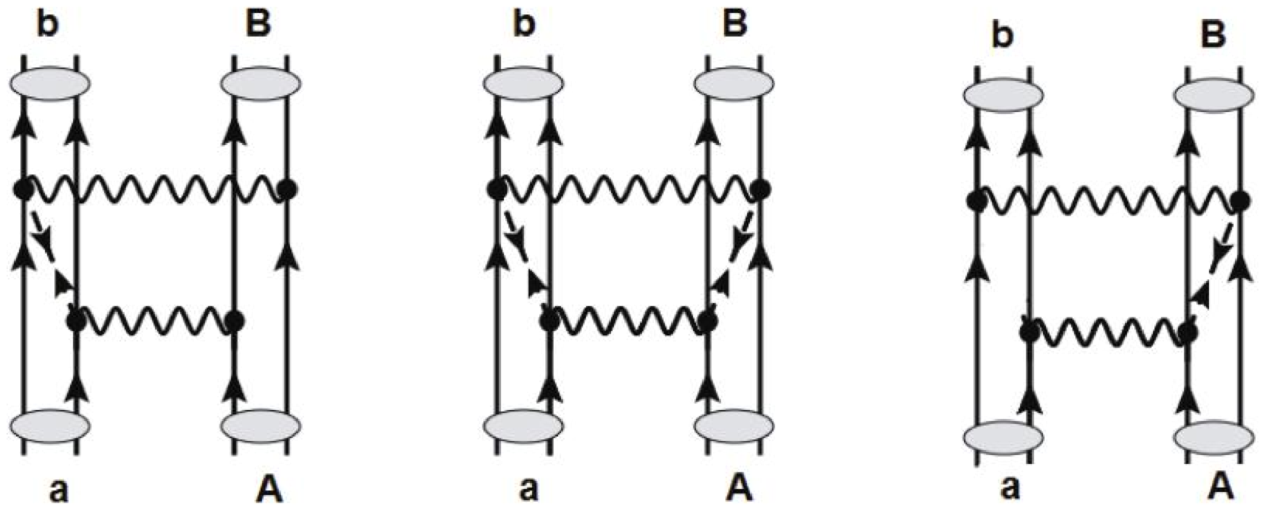
Sequential DCE (dSCE)

sequence of two ***Uncorrelated*** SCE reactions
(2νββ-like mechanism)



Majorana-like DCE (mDCE)

sequence of two ***Correlated*** SCE reactions
(0νββ-like mechanism)



J.I. B., S. Burrello, M. Colonna, J.A. Lay, H. Lenske, PLB 807 (2020) 135528
F. Cappuzzello et al., Progress in Particle and Nuclear Physics, submitted

H. Lenske, IOP Conf. Series: Journal of Physics: Conf. Series **1056 (2018) 012030**
E. Santopinto *et al.* , Phys. Rev. C 98 061601 (R) (2018)
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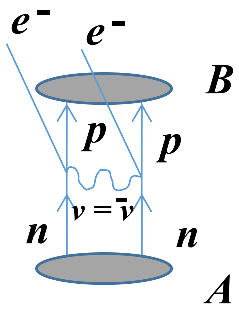
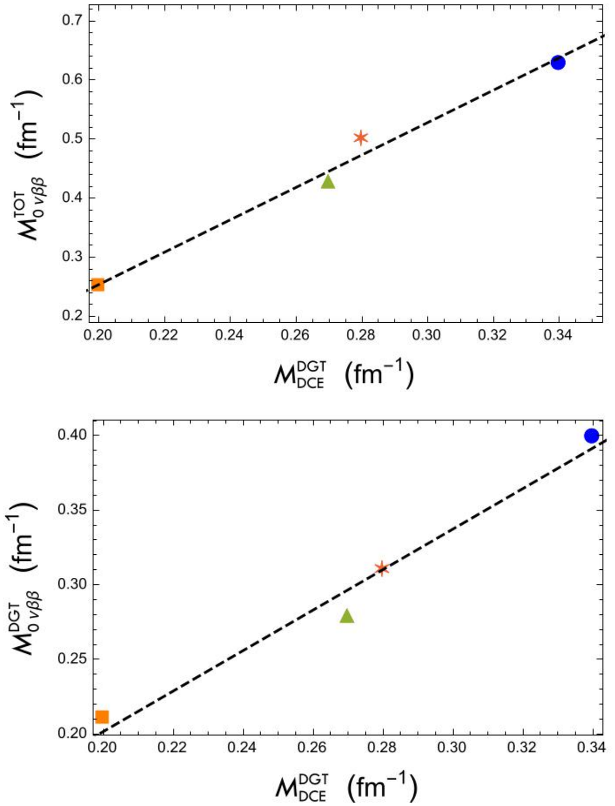
0νββ





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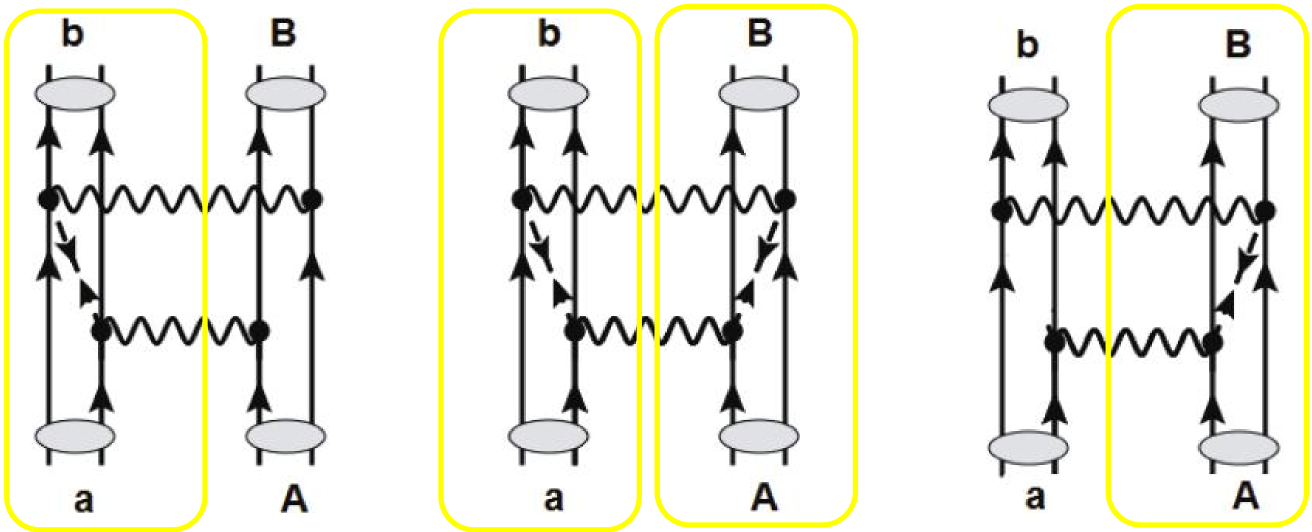

N. Shimizu et al. Phys Rev. Lett. 120, 142502 (2018)
E. Santopinto *et al.* , Phys. Rev. C 98 061601 (R) (2018)
→ actually connection for NME provided at tree level

H. Lenske, IOP Conf. Series: Journal of Physics: Conf. Series **1056 (2018) 012030**
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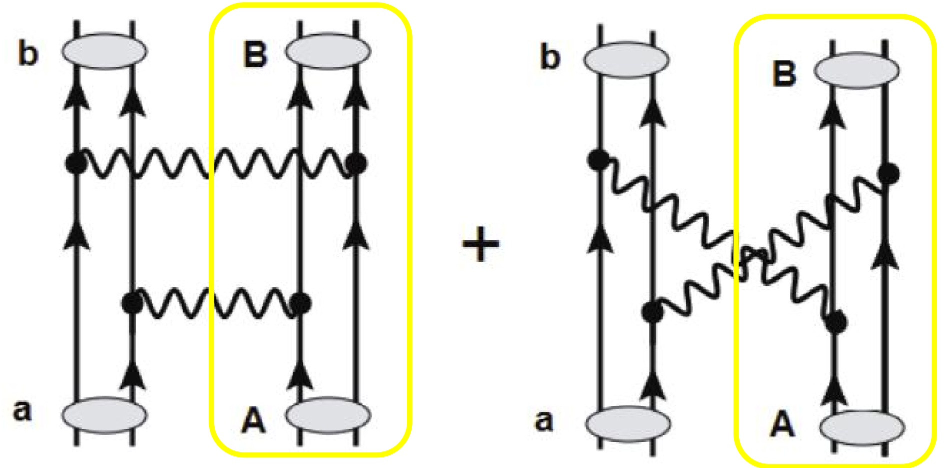
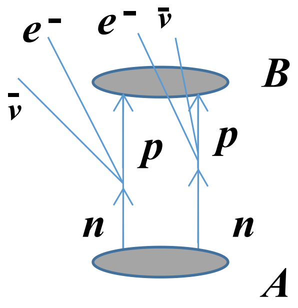
sequence of two ***Uncorrelated*** SCE reactions
(2νββ-like mechanism)



2νββ



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focusing on sequential Heavy Ion Double Charge Exchange

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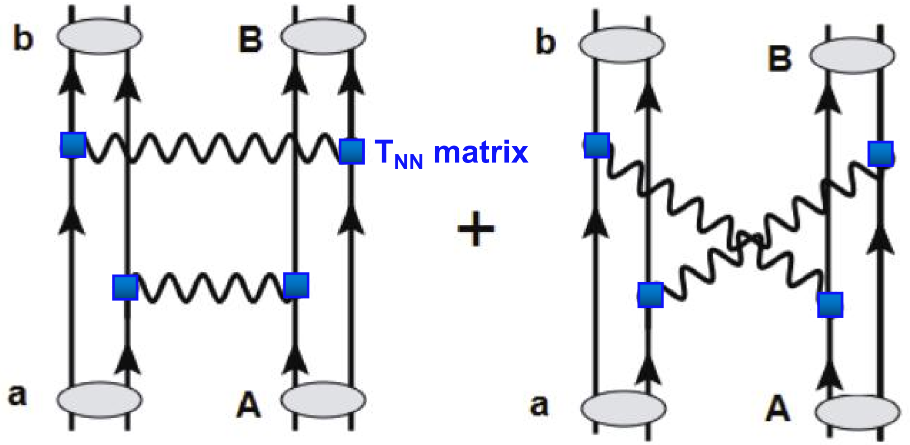

$$T_{NN}(\mathbf{q}) = \sum_{S=0,1,T=0,1} \{V^{(C)}_{ST}(q^2)[\boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_A]^S + \delta_{S1} V^{(Tn)}_T(q^2) S_{12}(\mathbf{q})\}[\boldsymbol{\tau}_a \cdot \boldsymbol{\tau}_A]^T$$

$$S_{12}(\mathbf{q}) = \frac{1}{q^2}(3\boldsymbol{\sigma}_a \cdot \mathbf{q}\,\boldsymbol{\sigma}_A \cdot \mathbf{q} - \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_A q^2)$$

(β) $B(Z_A \pm 2, N_A \mp 2) + b(Z_a \mp 2, N_a \pm 2)$

$$\uparrow$$

(γ) $C(Z_A \pm 1, N_A \mp 1) + c(Z_a \mp 1, N_a \pm 1)$

$$\uparrow$$

(α) $A(Z_A, N_A) + a(Z_a, N_a)$

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focusing on sequential Heavy Ion Double Charge Exchange

dSCE cross section

$$d\sigma_{\alpha\beta}^{(DSCE)} = \frac{m_\alpha m_\beta}{(2\pi\hbar^2)^2}\frac{k_\beta}{k_\alpha}\frac{1}{(2J_a+1)(2J_A+1)}\sum_{M_a,M_A\in\alpha;M_b,M_B\in\beta}\left|M_{\alpha\beta}^{(2)}(\mathbf{k}_\alpha,\mathbf{k}_\beta)\right|^2 d\Omega$$

$$M_{\alpha\beta}^{(2)}(\mathbf{k}_\alpha,\mathbf{k}_\beta) = \sum_{\gamma=c,C}\int\frac{d^3k_\gamma}{(2\pi)^3}\int d^3q_1\tilde{\rho}_{1P}(\mathbf{q}_1)\tilde{\rho}_{1T}(\mathbf{q}_1)\tilde{V}_{PT}(\mathbf{q}_1)N_{\alpha\gamma}(\mathbf{q}_1)G_\gamma(\omega_\alpha,\omega_\gamma)\int d^3q_2\,\tilde{\rho}_{2P}^*(\mathbf{q}_2)\tilde{\rho}_{2T}^*(\mathbf{q}_2)\tilde{V}_{PT}^*(\mathbf{q}_2)N_{\gamma\beta}(\mathbf{q}_2)$$

focusing on sequential Heavy Ion Double Charge Exchange

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I-step SCE TME
$M^{(1)}_{\alpha\gamma}(\mathbf{k}_\alpha, \mathbf{k}_\gamma)$

II-step SCE TME
$M^{(2)}_{\gamma\beta}(\mathbf{k}_\gamma, \mathbf{k}_\beta)$

focusing on sequential Heavy Ion Double Charge Exchange

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- sum and integral over the angular and linear momenta involved in the (off-shell) intermediate channel

I-step SCE TME
$M^{(1)}_{\alpha\gamma}(\mathbf{k}_\alpha\,,\,\mathbf{k}_\gamma)$

$$G_\gamma(\omega_\alpha,\omega_\gamma) = \frac{1}{\omega_\alpha-\omega_\gamma+i\eta}$$

II-step SCE TME
$M^{(2)}_{\gamma\beta}(\mathbf{k}_\gamma\,,\,\mathbf{k}_\beta)$

- Green function accounting for the propagation of the nuclear system in the (off-shell) intermediate channel

focusing on sequential Heavy Ion Double Charge Exchange

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SCE One-Body Transition density

$$\tilde{\rho}_{iX}(q_i) \equiv \tilde{\rho}^{(X)J_aJ_c}_{L_iS_iJ_i}(q_i) = \frac{1}{\hat{J}_i}\langle J_c||T_{(L_iS_i)J_i}||J_a\rangle$$

$$\hat{J} = \sqrt{2J+1}$$

$$X = P,T$$

$$i = 1,2$$

$$T_{(L_iS_i)J_iM_i}(\mathbf{r};q_i) = \sum_{m_L,m_s}\left[i^{L_i}j_{L_i}(rq_i)Y_{L_i,m_L}(\hat{r})\otimes(\boldsymbol{\sigma})^{S_i}_{m_S}\right]_{J_iM_i}\boldsymbol{\tau}$$

SCE Transition operator

focusing on sequential Heavy Ion Double Charge Exchange

dSCE cross section

$$d\sigma^{(DSCE)}_{\alpha\beta} = \frac{m_\alpha m_\beta}{(2\pi\hbar^2)^2}\frac{k_\beta}{k_\alpha}\frac{1}{(2J_a+1)(2J_A+1)}\sum_{M_a,M_A\in\alpha;M_b,M_B\in\beta}\left|M^{(2)}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta)\right|^2 d\Omega$$

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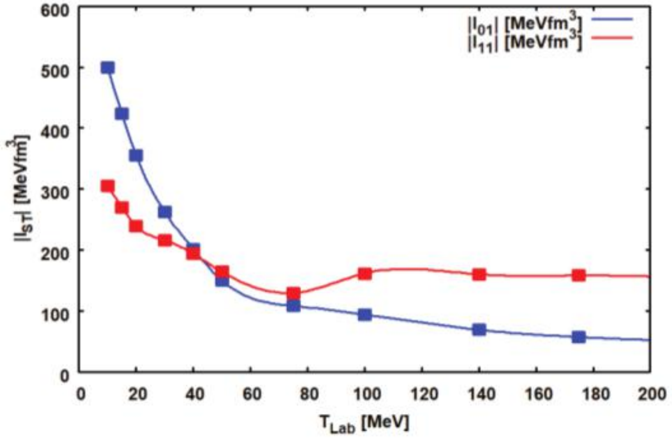

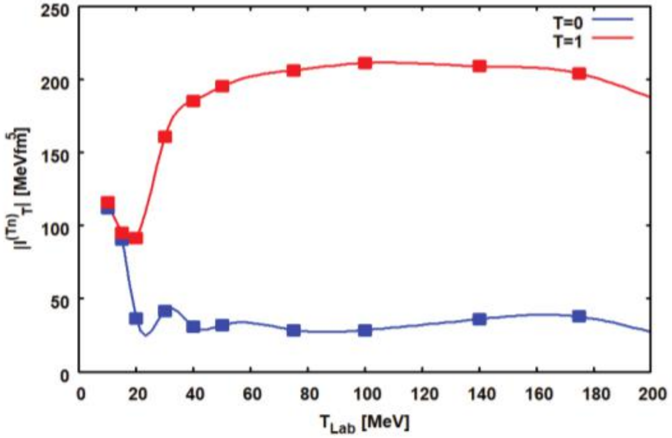

Two-Body effective local NN interaction potential

parameterized with the sum of 3 Yukawa functions according to H. Lenske, Nucl. Phys. A 482 (1988)

focusing on sequential Heavy Ion Double Charge Exchange

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SCE distortion factor

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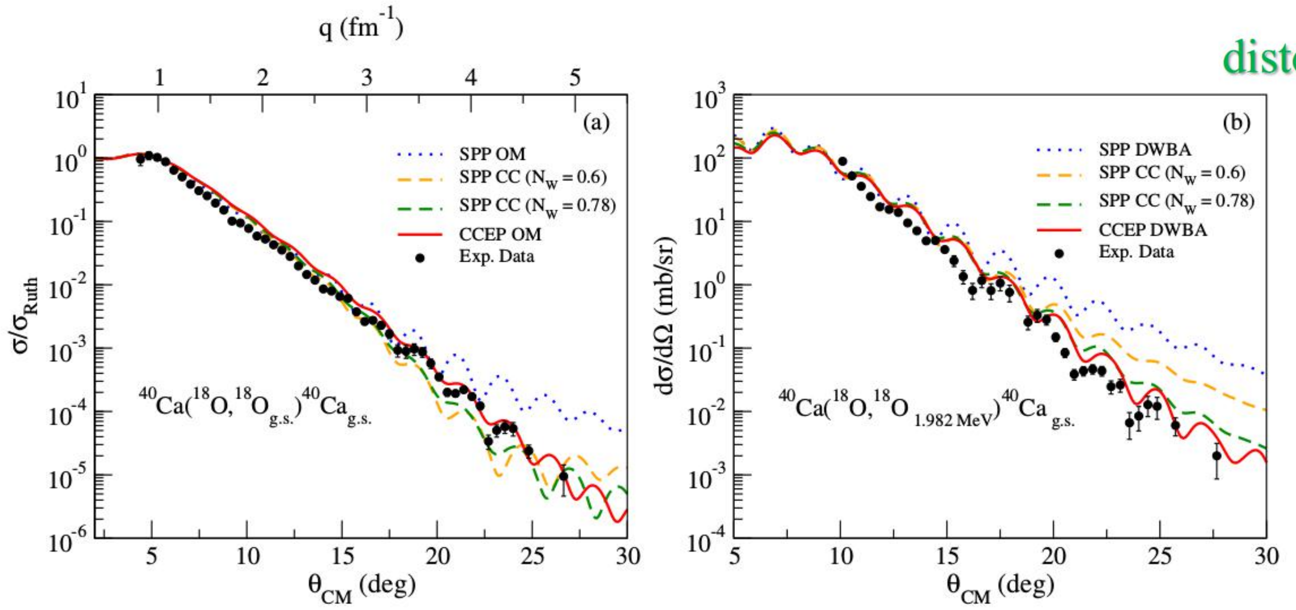

$$N_{\gamma\beta}(\mathbf{q}_2) = \int\frac{d^3r}{(2\pi)^3}\chi^*_{\mathbf{k}_\beta}(\mathbf{r})\chi_{\mathbf{k}_\gamma}(\mathbf{r})e^{-i\mathbf{r}\cdot\mathbf{q}_2}$$

← fixing Optical Potential by elastic and inelastic channel analysis

M.Cavallaro, J.I. B. et. al., Front. Astron. Space Sci. 8:659815 (2021)

focusing on sequential Heavy Ion Double Charge Exchange

dSCE cross section

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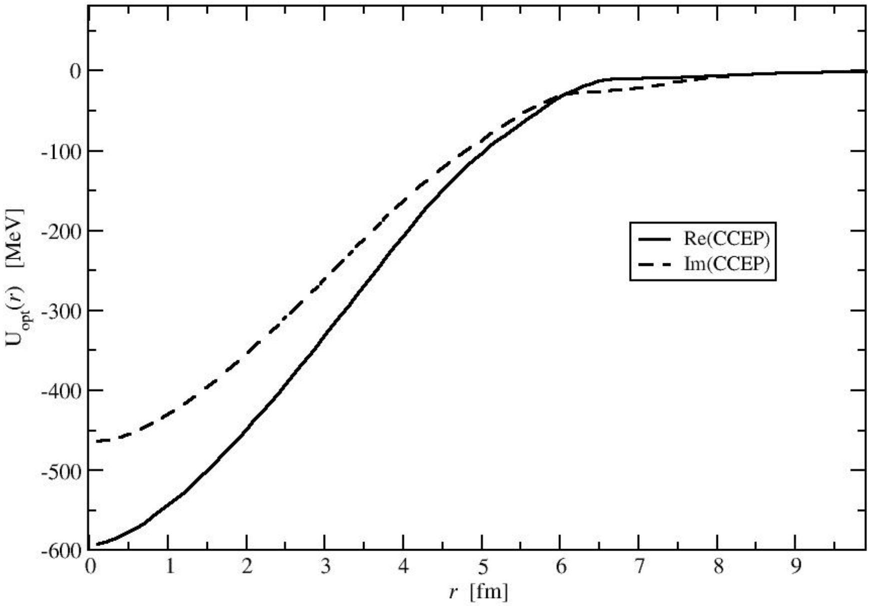
$$M^{(2)}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta) = \sum_{\gamma=c,C}\int\frac{d^3k_\gamma}{(2\pi)^3}\int d^3q_1\tilde{\rho}_{1P}(\mathbf{q}_1)\tilde{\rho}_{1T}(\mathbf{q}_1)\tilde{V}_{PT}(\mathbf{q}_1)N_{\alpha\gamma}(\mathbf{q}_1)G_\gamma(\omega_\alpha,\omega_\gamma)\int d^3q_2\,\tilde{\rho}^*_{2P}(\mathbf{q}_2)\tilde{\rho}^*_{2T}(\mathbf{q}_2)\tilde{V}^*_{PT}(\mathbf{q}_2)N_{\gamma\beta}(\mathbf{q}_2)$$

SCE
distortion factor

SCE
distortion factor



$$N_{\gamma\beta}(\mathbf{q}_2) = \int\frac{d^3r}{(2\pi)^3}\chi^*_{\mathbf{k}_\beta}(\mathbf{r})\chi_{\mathbf{k}_\gamma}(\mathbf{r})e^{-i\mathbf{r}\cdot\mathbf{q}_2}$$

← fixing Optical Potential by elastic and inelastic channel analysis
(SPPotential + CC effects effectively included = CCEP)

M.Cavallaro, J.I. B. et. al., Front. Astron. Space Sci. 8:659815 (2021)

focusing on sequential Heavy Ion Double Charge Exchange

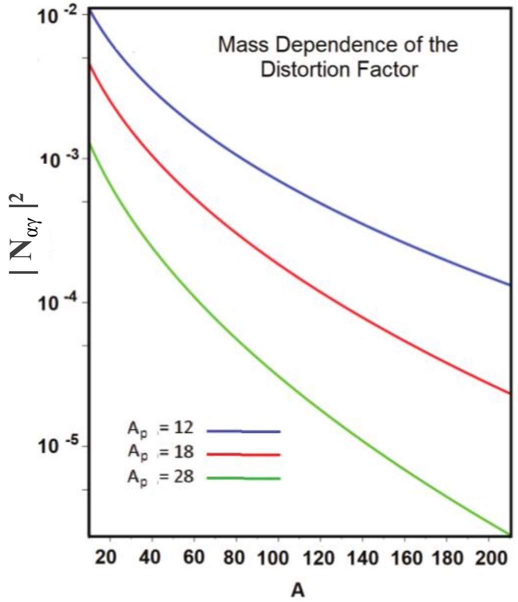
dSCE cross section

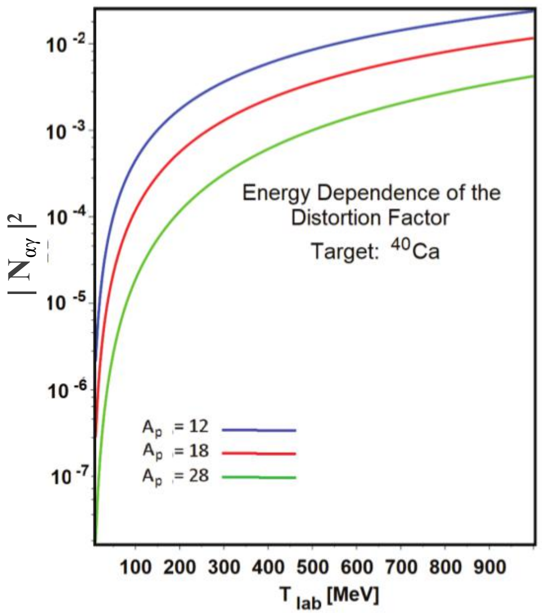
$$d\sigma^{(DSCE)}_{\alpha\beta} = \frac{m_\alpha m_\beta}{(2\pi\hbar^2)^2}\frac{k_\beta}{k_\alpha}\frac{1}{(2J_a+1)(2J_A+1)}\sum_{M_a,M_A\in\alpha;M_b,M_B\in\beta}\left|M^{(2)}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta)\right|^2 d\Omega$$

$$M^{(2)}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta) = \sum_{\gamma=c,C}\int\frac{d^3k_\gamma}{(2\pi)^3}\int d^3q_1\tilde{\rho}_{1P}(\mathbf{q}_1)\tilde{\rho}_{1T}(\mathbf{q}_1)\tilde{V}_{PT}(\mathbf{q}_1)N_{\alpha\gamma}(\mathbf{q}_1)G_\gamma(\omega_\alpha,\omega_\gamma)\int d^3q_2\,\tilde{\rho}^*_{2P}(\mathbf{q}_2)\tilde{\rho}^*_{2T}(\mathbf{q}_2)\tilde{V}^*_{PT}(\mathbf{q}_2)N_{\gamma\beta}(\mathbf{q}_2)$$

SCE distortion factor

SCE distortion factor





→ **Black Disk Approximation +**
Gaussian Eikonal Approximation =

analytical dependence of distortion factor on mass and energy of the system

H. Lenske, J. I. Bellone, M. Colonna, J. A. Lay
Phys. Rev. C (2018) 98, 044620.

Analogies between DSCE and 2νββ NMEs

DSCE cross section

$$d\sigma_{\alpha\beta}^{(DSCE)} = \frac{m_\alpha m_\beta}{(2\pi\hbar^2)^2}\frac{k_\beta}{k_\alpha}\frac{1}{(2J_a+1)(2J_A+1)}\sum_{M_a,M_A\in\alpha;M_b,M_B\in\beta}\left|M_{\alpha\beta}^{(2)}(\mathbf{k}_\alpha,\mathbf{k}_\beta)\right|^2 d\Omega$$

DSCE transition matrix element

$$M_{\beta\alpha}^{(2)}(\mathbf{k}_\beta,\mathbf{k}_\alpha) = \int d^3q_1 d^3q_2 \oint_{C+}\frac{d\nu}{2i\pi}\sum_{S_1,S_2}\Pi_{\alpha\beta}^{(S_2S_1)}(\mathbf{q}_2,\mathbf{q}_1;\nu)\int\frac{d^3k_\gamma}{(2\pi)^3}N_{\beta\gamma}(\mathbf{q}_2)V_{S_2T}(q_2^2)\frac{\tilde{S}_\gamma^\dagger}{\omega_\alpha-\nu-T_\gamma+i\eta}N_{\gamma\alpha}(\mathbf{q}_1)V_{S_1T}(q_1^2).$$

$$\Pi_{\alpha\beta}^{(S_2S_1)}(\mathbf{q}_2,\mathbf{q}_1;\nu) = \sum_{cC}\frac{F_{S_2}^{(BC)}(\mathbf{q}_2)\cdot F_{S_2}^{(bc)}(\mathbf{q}_2)F_{S_1}^{(ca)}(\mathbf{q}_1)\cdot F_{S_1}^{(CA)}(\mathbf{q}_1)}{\nu-(E_A-E_C+E_a-E_c)}$$

H. Lenske, J.I. B., M. Colonna, D. Gambacurta, Universe 7 (2021) 4, 98

$$\Pi_{(S_1S_2)SM}^{(AB)}(\mathbf{q}_2,\mathbf{q}_1;\omega) = \sum_C\frac{\left[F_{S_2}^{(BC)}(\mathbf{q}_2)\otimes F_{S_1}^{(CA)}(\mathbf{q}_1)\right]_{SM}}{\omega-(E_A-E_C)}$$

DSCE NME

J. Barea, J. Kotila, F. Iachello, Phys. Rev. C 91 034304 (2015)

$$\mathcal{M}^{(2\nu)} = g_V^2\frac{\left\langle 0_F^+\right|\sum_{n,n'}\tau_n^\dagger\tau_{n'}^\dagger\left|0_I^+\right\rangle}{\frac{1}{2}(Q_{\beta\beta}+2m_ec^2)+\langle E_{0^+,N}\rangle-E_I} - g_A^2\frac{\left\langle 0_F^+\right|\sum_{n,n'}\tau_n^\dagger\tau_{n'}^\dagger\boldsymbol{\sigma}_n\cdot\boldsymbol{\sigma}_{n'}\left|0_I^+\right\rangle}{\frac{1}{2}(Q_{\beta\beta}+2m_ec^2)+\langle E_{1^+,N}\rangle-E_I}$$

2νββ NME

How to extract information on ββ decay strengths from DCE cross section measurements?

DCE cross section factorization

- Relation between Light Ion SCE reaction cross section at intermediate and high energies provided since the 80's

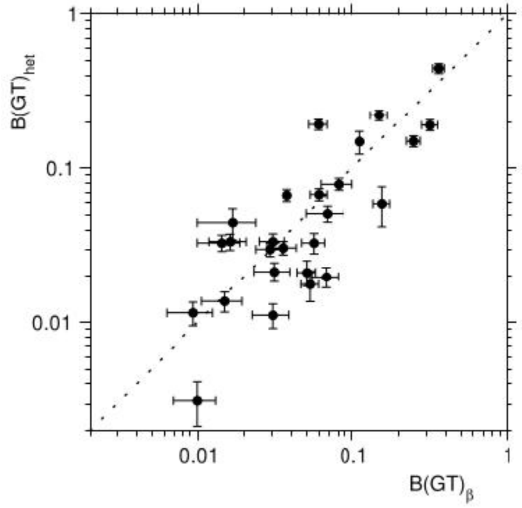
C. D. Goodman *et al.*, Phys. Rev. Lett. 44 (1980) 1755

	SCE	β decay
$(S=1)$	$\sum_{ij} V_{\sigma\tau}(\mathbf{r}_{ij})\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$	$g_A\sum_i \boldsymbol{\sigma}_i\boldsymbol{\tau}_i$
$(S=0)$	$\sum_{ij} V_{\tau}(\mathbf{r}_{ij})\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$	$g_V\sum_i \boldsymbol{\tau}_i$

$$\frac{d^2\sigma}{dEd\Omega} = \hat{\sigma}_X(E_b, A)F_X(q,\omega)B(X)$$

factorized expression working for $q_{\alpha\beta} \leqslant 0.25\, fm^{-1}$ for Light Ions

T. N. Taddeucci *et al.*, Nucl. Phys. A 469 (1987) 125 – 172



0° cross section for (^{3}He,t) reactions at RCNP

Y. Fujita et al., PPNP 66 (2011) 549-606

How to extract information on ββ decay strengths from DCE cross section measurements?

DCE cross section factorization

$$\frac{d^2\sigma}{dEd\Omega} = \hat{\sigma}_X(E_b, A)F_X(q,\omega)B(X)$$

?

$$\frac{d\sigma}{d\Omega}^{DCE}(q, E_x) = \hat{\sigma}_\alpha^{DCE}(E_p, A)F_\alpha^{DCE}(q, E_x)B_T^{DCE}(\alpha)B_P^{DCE}(\alpha)$$

factorized expression working for $q_{\alpha\beta} \leqslant 0.25\, fm^{-1}$ for Light Ions

T. N. Taddeucci *et al*., Nucl. Phys. A 469 (1987) 125 – 172

for Heavy Ions DCE

How to extract information on ββ decay strengths from DCE cross section measurements?

DCE cross section factorization

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for Light Ions

T. N. Taddeucci *et al*., Nucl. Phys. A 469 (1987) 125 – 172

?

for Heavy Ions SCE

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T. N. Taddeucci *et al*., Nucl. Phys. A 469 (1987) 125 – 172

Yes

H. Lenske, J. I. Bellone, M. Colonna, J. A. Lay
Phys. Rev. C (2018) 98, 044620.

✓ **HI SCE cross section factorization can be reached for small momentum transfer ($q_{\alpha\beta}$ < 0.25 fm^{-1})**

$$\frac{d^2\sigma}{dEd\Omega} = \hat{\sigma}(T_{lab}, A, a) F(q_{\alpha\beta}, \omega) |b_{ab}^{(0,S,S)}|^2 |b_{AB}^{(0,S,S)}|^2$$

Unit Cross Section

$$\hat{\sigma}(T_{lab}, A, a) = K_f(T_{lab}, 0) |V_{ST}^{(C)}(0)|^2 |1 - n_{BD}|^2 (2S+1)$$

Shape Factor

$$F(q_{\alpha\beta}, \omega) = \frac{K_f(T_{lab}, \omega)}{K_f(T_{lab}, 0)} e^{-\frac{1}{3} q_{\alpha\beta}^2 (\langle r^2 \rangle_a + \langle r^2 \rangle_A)} \xrightarrow{(q_{\alpha\beta}, \omega) \to (0,0)} 1$$

$\omega = E_x - Q_{reac}$ energy loss

How to extract information on ββ decay strengths from DCE cross section measurements?

DCE cross section factorization

$$\frac{d^2\sigma}{dEd\Omega} = \hat{\sigma}_X(E_b, A)F_X(q,\omega)B(X)$$

factorized expression working for $q_{\alpha\beta} \leqslant 0.25\, fm^{-1}$
for Light Ions

T. N. Taddeucci *et al.*, Nucl. Phys. A 469 (1987) 125 – 172

$$\frac{d\sigma}{d\Omega}^{DCE}(q, E_x) = \hat{\sigma}_\alpha^{DCE}(E_p, A)F_\alpha^{DCE}(q, E_x)B_T^{DCE}(\alpha)B_P^{DCE}(\alpha)$$

for Heavy Ions DCE

Yes

H. Lenske, J. I. Bellone, M. Colonna, J. A. Lay
Phys. Rev. C (2018) 98, 044620.

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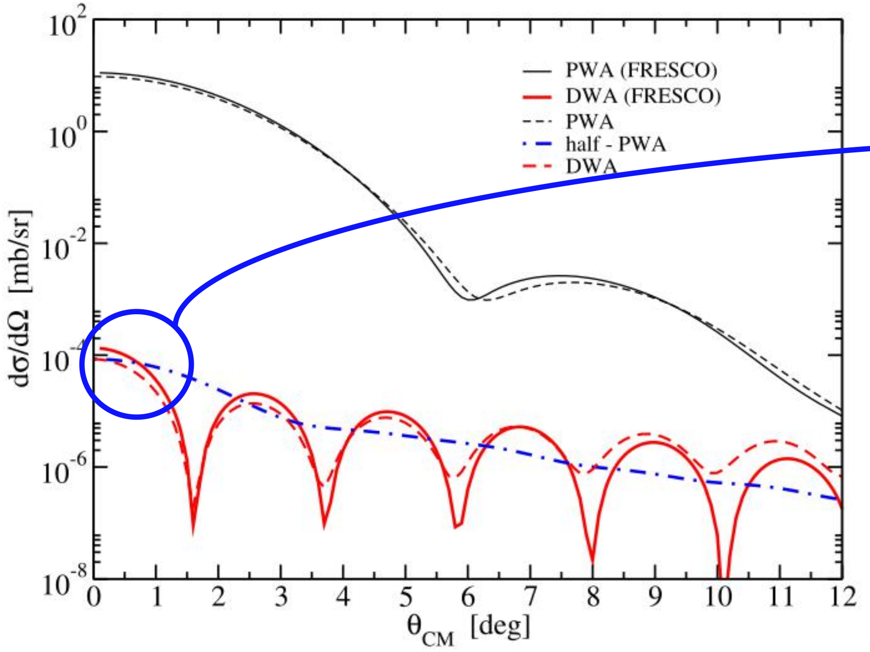
focusing on sequential Heavy Ion Double Charge Exchange

dSCE cross section

$$d\sigma_{\alpha\beta}^{(DSCE)} = \frac{m_\alpha m_\beta}{(2\pi\hbar^2)^2}\frac{k_\beta}{k_\alpha}\frac{1}{(2J_a+1)(2J_A+1)}\sum_{M_a,M_A\in\alpha;M_b,M_B\in\beta}\left|M_{\alpha\beta}^{(2)}(\mathbf{k}_\alpha,\mathbf{k}_\beta)\right|^2 d\Omega$$

$$M_{\alpha\beta}^{(2)}(\mathbf{k}_\alpha,\mathbf{k}_\beta) = \sum_{\gamma=c,C}\int\frac{d^3k_\gamma}{(2\pi)^3}\int d^3q_1\tilde{\rho}_{1P}(\mathbf{q}_1)\tilde{\rho}_{1T}(\mathbf{q}_1)\tilde{V}_{PT}(\mathbf{q}_1)N_{\alpha\gamma}(\mathbf{q}_1)G_\gamma(\omega_\alpha,\omega_\gamma)\int d^3q_2\,\tilde{\rho}^*_{2P}(\mathbf{q}_2)\tilde{\rho}^*_{2T}(\mathbf{q}_2)\tilde{V}^*_{PT}(\mathbf{q}_2)N_{\gamma\beta}(\mathbf{q}_2)$$



- neglegible net absorption effect of the intermediate channel (at forward scattering angles)

$$\mathcal{M}_{\alpha\beta}^{DSCE}(\mathbf{k}_\alpha,\mathbf{k}_\beta) \approx N_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta)\sum_{\gamma=c,C}$$
$$\times\int\frac{d^3k_\gamma}{(2\pi)^3}\mathcal{U}_{\gamma\beta}^{SCE}(\mathbf{k}_\gamma-\mathbf{k}_\beta)\frac{1}{\omega_\alpha-\omega_\gamma+i\eta}\mathcal{U}_{\alpha\gamma}^{SCE}(\mathbf{k}_\alpha-\mathbf{k}_\gamma)$$

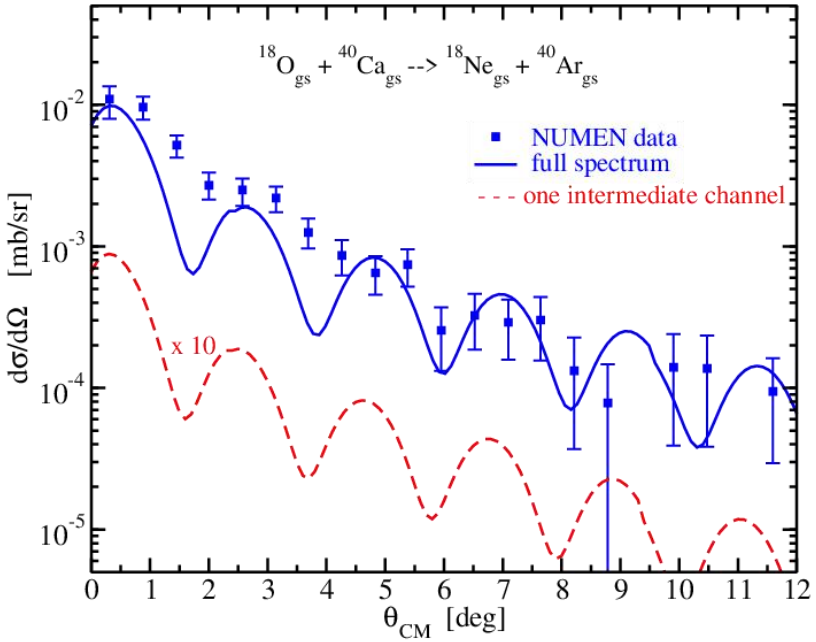
J. I. Bellone, S. Burrello, M. Colonna, J. A. Lay, H. Lenske, Phys. Lett. B 807 (2020) 135528

focusing on sequential Heavy Ion Double Charge Exchange

dSCE cross section

$$d\sigma^{(DSCE)}_{\alpha\beta} = \frac{m_\alpha m_\beta}{(2\pi\hbar^2)^2}\frac{k_\beta}{k_\alpha}\frac{1}{(2J_a+1)(2J_A+1)}\sum_{M_a,M_A\in\alpha;M_b,M_B\in\beta}\left|M^{(2)}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta)\right|^2 d\Omega$$

$$M^{(2)}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta) = \sum_{\gamma=c,C}\int\frac{d^3k_\gamma}{(2\pi)^3}\int d^3q_1\tilde{\rho}_{1P}(\mathbf{q}_1)\tilde{\rho}_{1T}(\mathbf{q}_1)\tilde{V}_{PT}(\mathbf{q}_1)N_{\alpha\gamma}(\mathbf{q}_1)G_\gamma(\omega_\alpha,\omega_\gamma)\int d^3q_2\,\tilde{\rho}^*_{2P}(\mathbf{q}_2)\tilde{\rho}^*_{2T}(\mathbf{q}_2)\tilde{V}^*_{PT}(\mathbf{q}_2)N_{\gamma\beta}(\mathbf{q}_2)$$



- order of magnitude of the data recovered only considering several nuclear states in the intermediate channel

J. I. Bellone, S. Burrello, M. Colonna, J. A. Lay, H. Lenske, Phys. Lett. B 807 (2020) 135528

focusing on sequential Heavy Ion Double Charge Exchange

dSCE cross section

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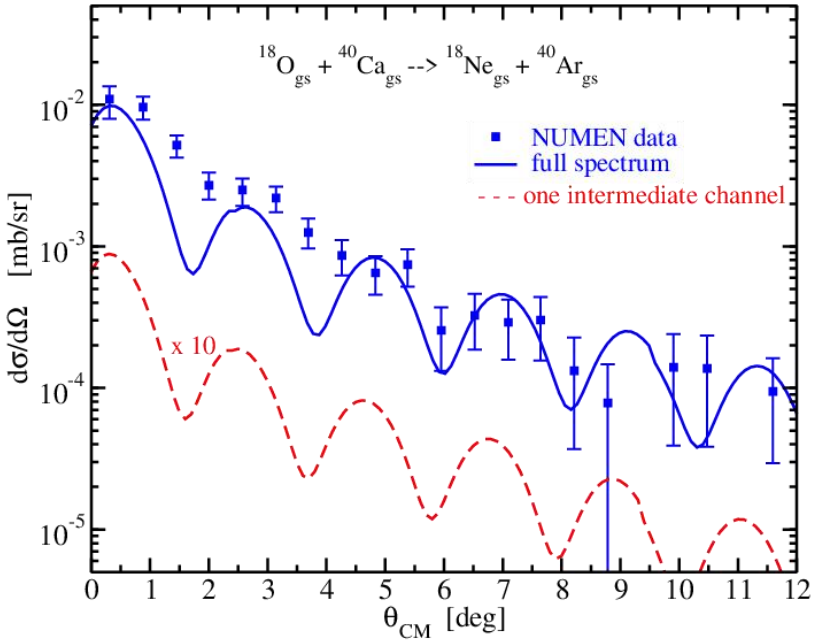
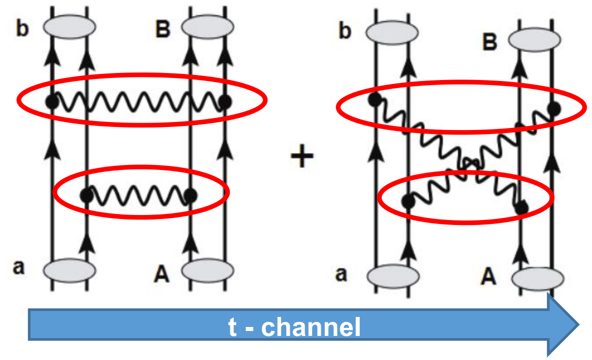
$$M^{(2)}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta) = \overline{G}_\gamma \int d^3q_1 \int d^3q_2\, \tilde{\rho}_{1P}(\mathbf{q}_1)\tilde{\rho}_{1T}(\mathbf{q}_1)\tilde{V}_{PT}(\mathbf{q}_1)\tilde{\rho}^*_{2P}(\mathbf{q}_2)\tilde{\rho}^*_{2T}(\mathbf{q}_2)\tilde{V}^*_{PT}(\mathbf{q}_2)N_{\alpha\beta}(\mathbf{q}_1+\mathbf{q}_2)$$



- projectile and target NMEs still not disentangled

J. I. Bellone, S. Burrello, M. Colonna, J. A. Lay, H. Lenske,
Phys. Lett. B 807 (2020) 135528



focusing on sequential Heavy Ion Double Charge Exchange

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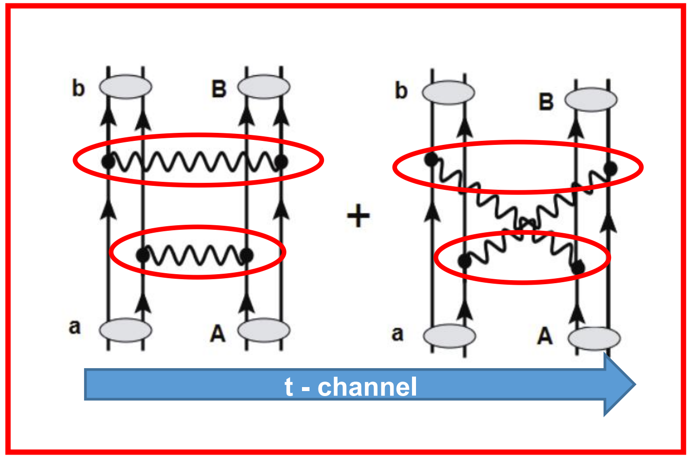
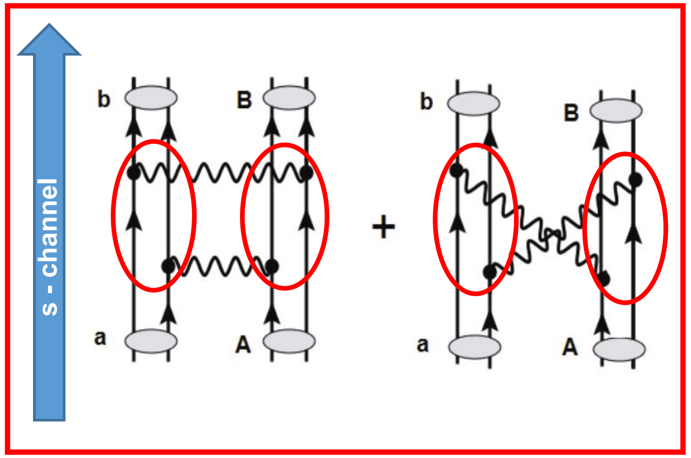
$$d\sigma_{\alpha\beta}^{(DSCE)} = \frac{m_\alpha m_\beta}{(2\pi\hbar^2)^2}\frac{k_\beta}{k_\alpha}\frac{1}{(2J_a+1)(2J_A+1)}\sum_{M_a,M_A\in\alpha;M_b,M_B\in\beta}\left|M_{\alpha\beta}^{(2)}(\mathbf{k}_\alpha,\mathbf{k}_\beta)\right|^2 d\Omega$$

$$M_{\alpha\beta}^{(2)}(\mathbf{k}_\alpha,\mathbf{k}_\beta) = \sum_{\gamma=c,C}\int\frac{d^3k_\gamma}{(2\pi)^3}\int d^3q_1\tilde{\rho}_{1P}(\mathbf{q}_1)\tilde{\rho}_{1T}(\mathbf{q}_1)\tilde{V}_{PT}(\mathbf{q}_1)N_{\alpha\gamma}(\mathbf{q}_1)G_\gamma(\omega_\alpha,\omega_\gamma)\int d^3q_2\,\tilde{\rho}^*_{2P}(\mathbf{q}_2)\tilde{\rho}^*_{2T}(\mathbf{q}_2)\tilde{V}^*_{PT}(\mathbf{q}_2)N_{\gamma\beta}(\mathbf{q}_2)$$





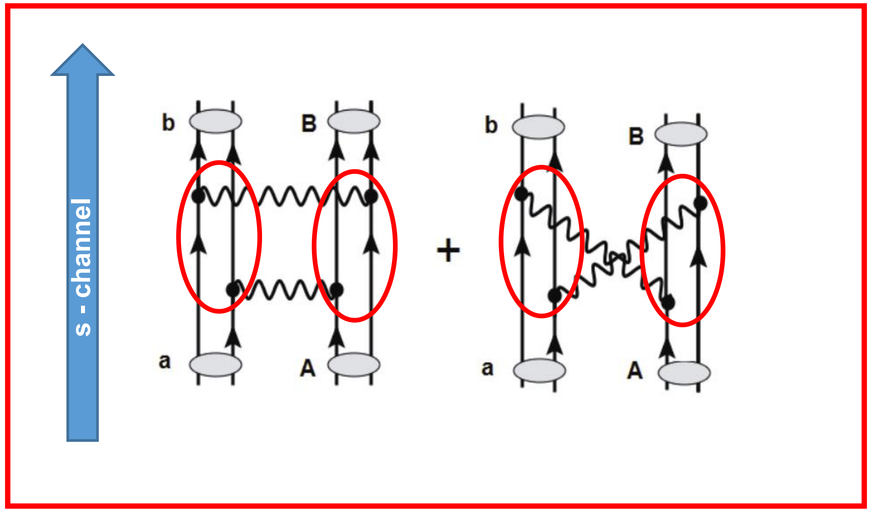
✓ **t-** and **s- channel** formulations analytically equivalent, because simply related by unitary transformation in angular momentum space

focusing on sequential Heavy Ion Double Charge Exchange

dSCE cross section

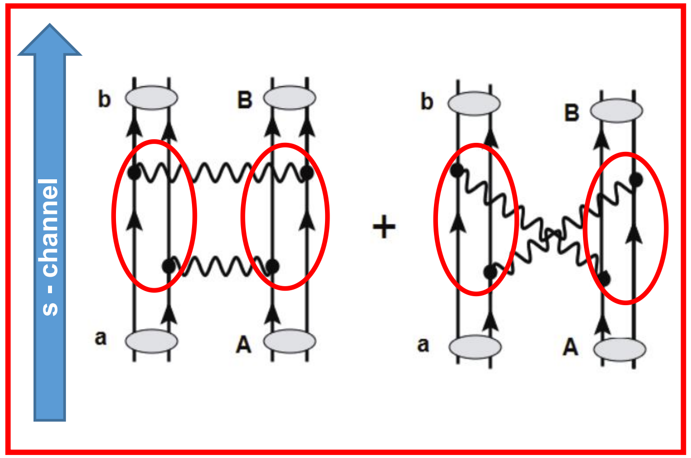
$$d\sigma^{(DSCE)}_{\alpha\beta} = \frac{m_\alpha m_\beta}{(2\pi\hbar^2)^2}\frac{k_\beta}{k_\alpha}\frac{1}{(2J_a+1)(2J_A+1)}\sum_{M_a,M_A\in\alpha;M_b,M_B\in\beta}\left|M^{(2)}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta)\right|^2 d\Omega$$

$$M^{(2)}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta) = \sum_{\gamma=c,C}\int\frac{d^3k_\gamma}{(2\pi)^3}\int d^3q_1\tilde{\rho}_{1P}(\mathbf{q}_1)\tilde{\rho}_{1T}(\mathbf{q}_1)\tilde{V}_{PT}(\mathbf{q}_1)N_{\alpha\gamma}(\mathbf{q}_1)G_\gamma(\omega_\alpha,\omega_\gamma)\int d^3q_2\,\tilde{\rho}^*_{2P}(\mathbf{q}_2)\tilde{\rho}^*_{2T}(\mathbf{q}_2)\tilde{V}^*_{PT}(\mathbf{q}_2)N_{\gamma\beta}(\mathbf{q}_2)$$



✓ **Advantage** :
get information on projectile
and target DCE NMEs separately,
from cross section measurements

Towards dSCE separation of projectile and target NMEs: the s-channel formalism (1)



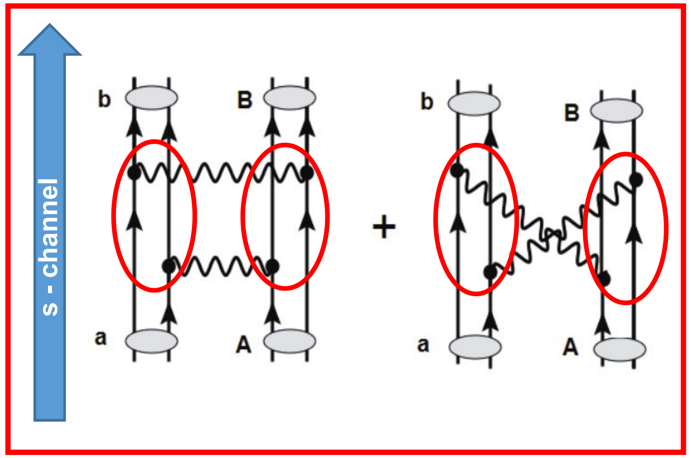
- fixing the depedence on k_γ within the reaction kernels and thus exploiting bi-orthonormality of the distorted waves describing the intermediate channel
- summing over all intermediate nuclear states
- fixing average value of intermediate channel energy within the propagator

} ~ Closure Approximation

- $\begin{cases} \mathbf{q}_1 - \mathbf{q}_2 = \xi \\ \mathbf{q}_1 + \mathbf{q}_2 = \eta \end{cases} \Leftrightarrow \begin{cases} \mathbf{q}_1 = \frac{\xi+\eta}{2} \\ \mathbf{q}_2 = \frac{\eta-\xi}{2} \end{cases}$

$$M^{(2)}_{\alpha\beta}(\mathbf{k}_\alpha, \mathbf{k}_\beta) \sim \int d^3\frac{\xi}{2} d^3\eta \, \tilde{\rho}_{1P}(\frac{\xi+\eta}{2})\tilde{\rho}_{1T}(\frac{\xi+\eta}{2})\tilde{V}_{PT}(\frac{\xi+\eta}{2})\tilde{\rho}^*_{2P}(\frac{\eta-\xi}{2})\tilde{\rho}^*_{2T}(\frac{\eta-\xi}{2})\tilde{V}^*_{PT}(\frac{\eta-\xi}{2})N_{\alpha\beta}(\eta)$$

Towards dSCE separation of projectile and target NMEs: the s-channel formalism (1)



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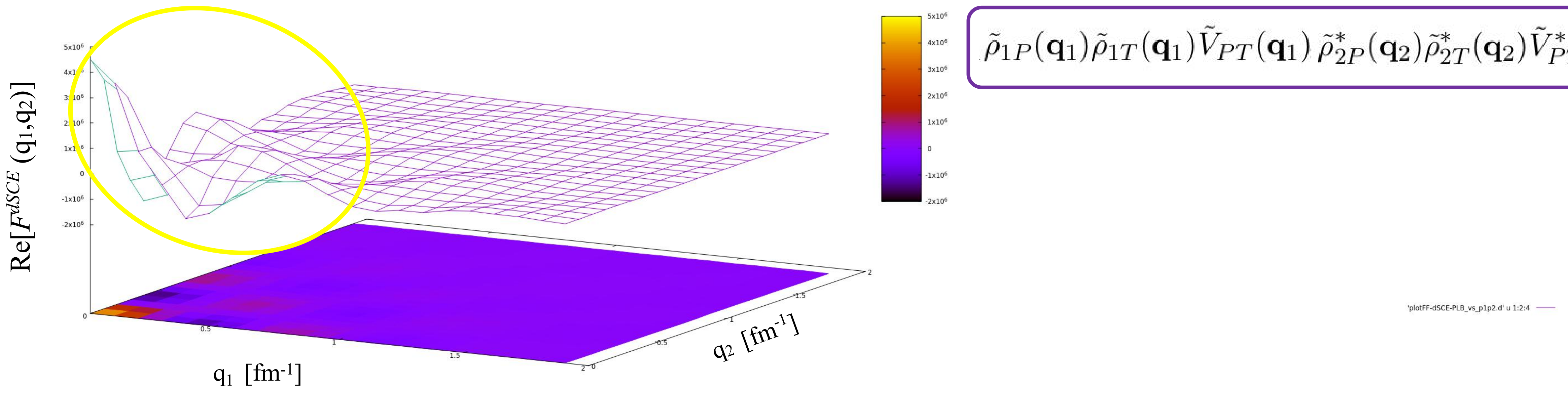
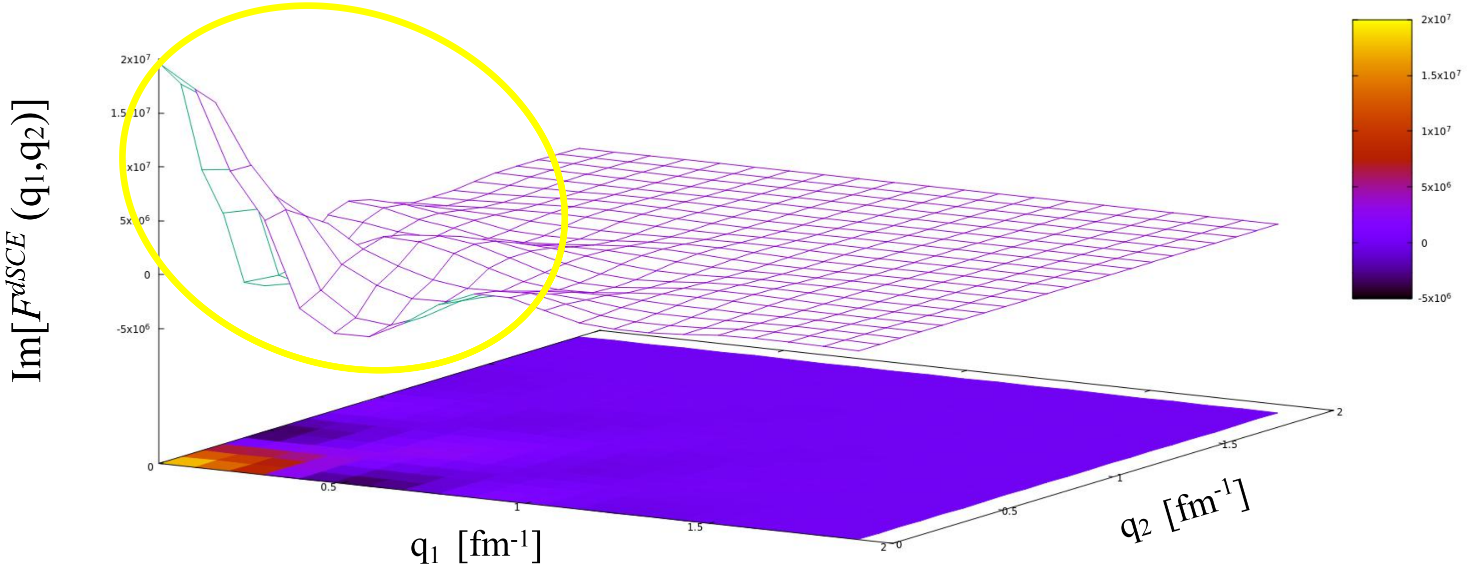
$$M^{(2)}_{\alpha\beta}(\mathbf{k}_\alpha, \mathbf{k}_\beta) \sim \int d^3\frac{\xi}{2} d^3\eta\, \tilde{\rho}_{1P}(\frac{\xi+\eta}{2})\tilde{\rho}_{1T}(\frac{\xi+\eta}{2})\tilde{V}_{PT}(\frac{\xi+\eta}{2})\tilde{\rho}^*_{2P}(\frac{\eta-\xi}{2})\tilde{\rho}^*_{2T}(\frac{\eta-\xi}{2})\tilde{V}^*_{PT}(\frac{\eta-\xi}{2})N_{\alpha\beta}(\eta)$$

- to extract dSCE NMEs proper recoupling of all angular momenta + need no ξ dependence

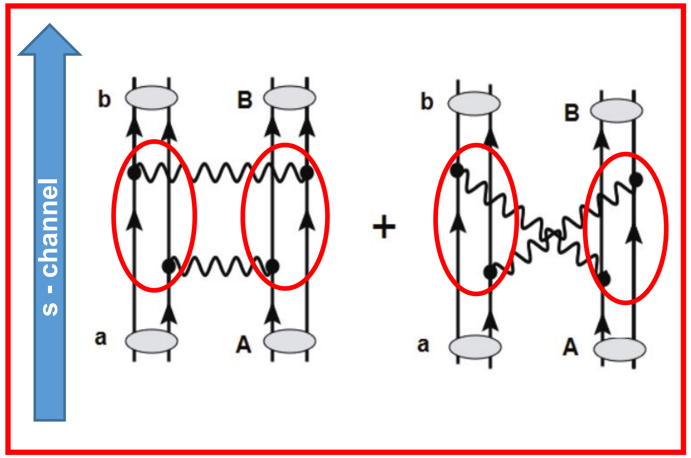
Sequential DCE Cross Section: the t-channel formalism → dSCE transition Form Factor

$$\tilde{\rho}_{1P}(\mathbf{q}_1)\tilde{\rho}_{1T}(\mathbf{q}_1)\tilde{V}_{PT}(\mathbf{q}_1)\,\tilde{\rho}^*_{2P}(\mathbf{q}_2)\tilde{\rho}^*_{2T}(\mathbf{q}_2)\tilde{V}^*_{PT}(\mathbf{q}_2)$$





Sequential DCE Cross Section: the s-channel formalism (2)



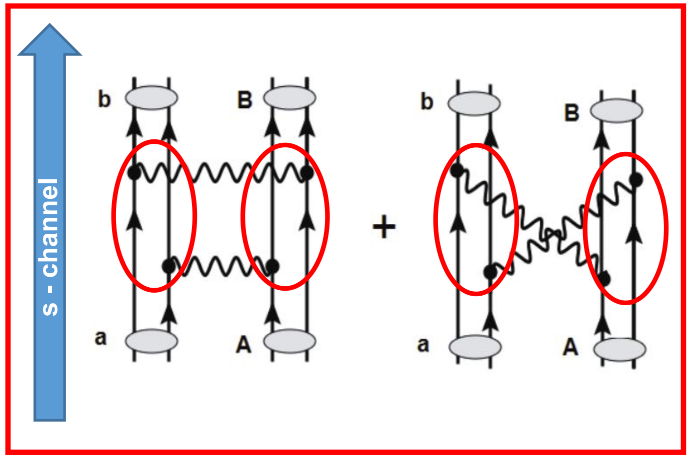
- one can take into account only the contribution for $\mathbf{q}_1 = \mathbf{q}_2 = \mathbf{0}$ in dSCE transition Form Factor ($\Leftrightarrow \xi = 0$)
  → particular case of ***collinear approximation***

- $$\tilde{\rho}_{1P}(\frac{\eta}{2})\tilde{\rho}^{*}_{2P}(\frac{\eta}{2}) \equiv \tilde{\rho}^{2BTD}_{P}(\frac{\boldsymbol{\eta}}{2})$$
  ~ 2-body dSCE transition density (2BTD)

Four-Body NN effective local interaction potential

- $$(2\pi)^3 \int d^3r\, e^{i\boldsymbol{\eta}\cdot\mathbf{r}} |V(\mathbf{r})|^2 \equiv \tilde{V}^{dSCE}_{NN}(\boldsymbol{\eta})$$

Sequential DCE Cross Section: the s-channel formalism (2)



- one can take into account only the contribution for $\mathbf{q}_1 = \mathbf{q}_2 = \mathbf{0}$ in dSCE transition Form Factor ($\Leftrightarrow \xi = 0$)
  → particular case of ***collinear approximation***

- $$\tilde{\rho}_{1P}(\frac{\eta}{2})\tilde{\rho}^*_{2P}(\frac{\eta}{2}) \equiv \tilde{\rho}^{2BTD}_P(\frac{\boldsymbol{\eta}}{2})$$ ~ **2-body dSCE transition density (2BTD)**

Four-Body NN effective local interaction potential

- $$(2\pi)^3 \int d^3r\, e^{i\boldsymbol{\eta}\cdot\mathbf{r}} |V(\mathbf{r})|^2 \equiv \tilde{V}^{dSCE}_{NN}(\boldsymbol{\eta})$$

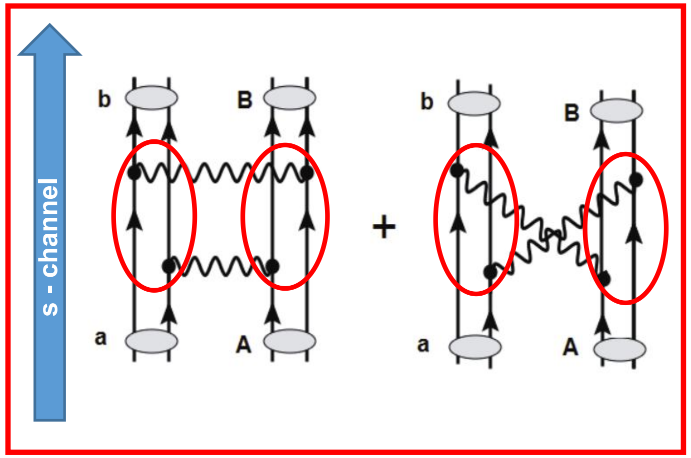
dSCE TME

$$M^{(2)}_{\alpha\beta}(\mathbf{k}_\alpha, \mathbf{k}_\beta) \simeq \int d^3\eta\, \tilde{\rho}^{2BTD}_P(\frac{\boldsymbol{\eta}}{2})\tilde{\rho}^{2BTD}_T(\frac{\boldsymbol{\eta}}{2})\tilde{V}^{dSCE}_{NN}(\boldsymbol{\eta})N_{\alpha\beta}(\boldsymbol{\eta})$$

like a single-step formalism
→ factorization at low momentum transfer

H. Lenske, J. I. B., M. Colonna, J. A. Lay
Phys. Rev. C (2018) 98, 044620.

Sequential DCE Cross Section: the s-channel formalism (2)



- one can take into account only the contribution for $\mathbf{q}_1 = \mathbf{q}_2 = \mathbf{0}$ in dSCE transition Form Factor ($\Leftrightarrow \boldsymbol{\xi} = 0$)
  $\rightarrow$ particular case of ***collinear approximation***

- $$\tilde{\rho}_{1P}(\frac{\eta}{2})\tilde{\rho}^{*}_{2P}(\frac{\eta}{2}) \equiv \tilde{\rho}^{2BTD}_{P}(\frac{\boldsymbol{\eta}}{2})$$ ~ **2-body dSCE transition density (2BTD)**

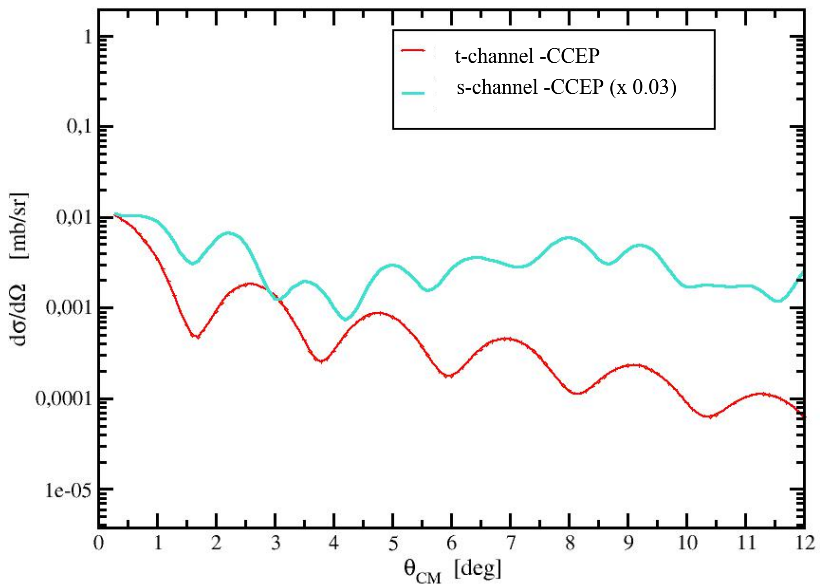
Four-Body NN effective local interaction potential

- $$(2\pi)^3 \int d^3r\, e^{i\boldsymbol{\eta}\cdot\mathbf{r}}|V(\mathbf{r})|^2 \equiv \tilde{V}^{dSCE}_{NN}(\boldsymbol{\eta})$$

dSCE TME

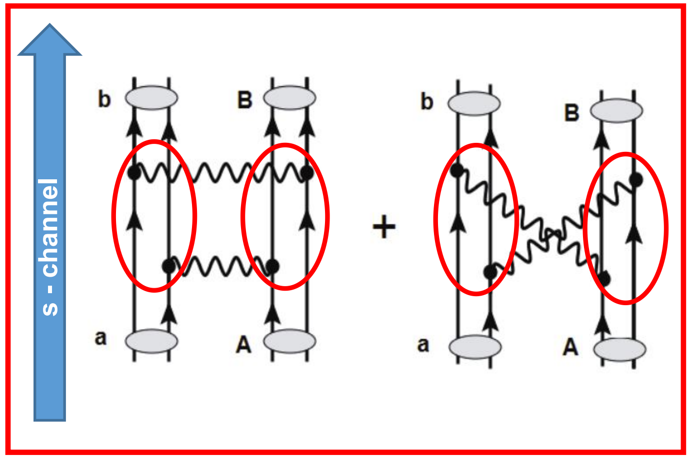
$$M^{(2)}_{\alpha\beta}(\mathbf{k}_\alpha, \mathbf{k}_\beta) \simeq \int d^3\eta\, \tilde{\rho}^{2BTD}_{P}(\frac{\boldsymbol{\eta}}{2})\tilde{\rho}^{2BTD}_{T}(\frac{\boldsymbol{\eta}}{2})\tilde{V}^{dSCE}_{NN}(\boldsymbol{\eta})N_{\alpha\beta}(\boldsymbol{\eta})$$

$^{40}Ca(^{18}O, ^{18}Ne)^{40}Ar$ at E_{lab} = 275 MeV



> ➢ but neither the diffraction pattern nor the order of magnitude of t-channel calculations are recovered
> $\rightarrow$ other values of ξ play non-neglegible role

Sequential DCE Cross Section: the s-channel formalism (3)



- averaging over the half-off-shell ξ linear relative momentum

- $$\frac{(2\pi)^3}{N}\int d^3r\, e^{i\boldsymbol{\eta}\cdot\mathbf{r}}\rho_{1P}(\mathbf{r})\rho^*_{2P}(\mathbf{r}) \equiv \tilde{\rho}_P^{2BTD}(\boldsymbol{\eta})$$

~ **average 2-body dSCE transition density (2BTD)**

Four-Body NN effective local interaction potential

- $$(2\pi)^3\int d^3r\, e^{i\boldsymbol{\eta}\cdot\mathbf{r}}|V(\mathbf{r})|^2 \equiv \tilde{V}_{NN}^{dSCE}(\boldsymbol{\eta})$$

dSCE TME

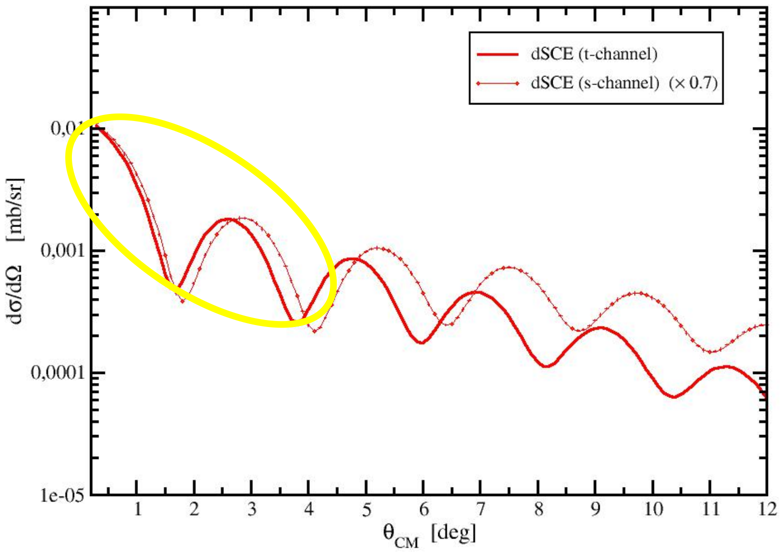
$$M^{(2)}_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta) \simeq \int d^3\eta\, \tilde{\rho}_P^{2BTD}(\boldsymbol{\eta})\tilde{\rho}_T^{2BTD}(\boldsymbol{\eta})\tilde{V}_{NN}^{dSCE}(\boldsymbol{\eta})N_{\alpha\beta}(\boldsymbol{\eta})$$

like a single-step formalism → factorization at low momentum transfer

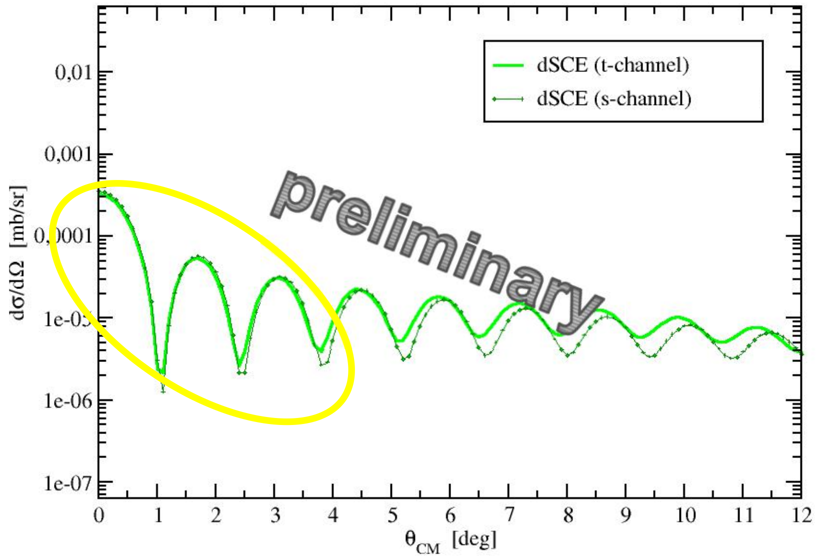
H. Lenske, J. I. Bellone, M. Colonna, J. A. Lay
Phys. Rev. C (2018) 98, 044620.

Sequential DCE Cross Section: t-channel vs s-channel formalism

^{40}Ca (^{18}O, ^{18}Ne) ^{40}Ar @ E_{lab} = 15.3 AMeV

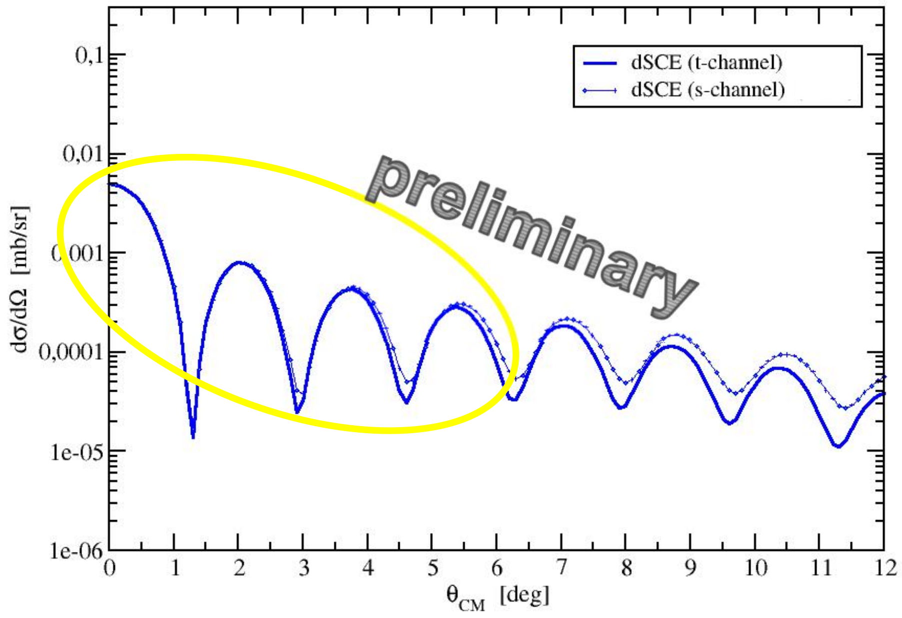


^{116}Cd (^{20}Ne, ^{20}O) ^{116}Sn @ 15.3 AMeV

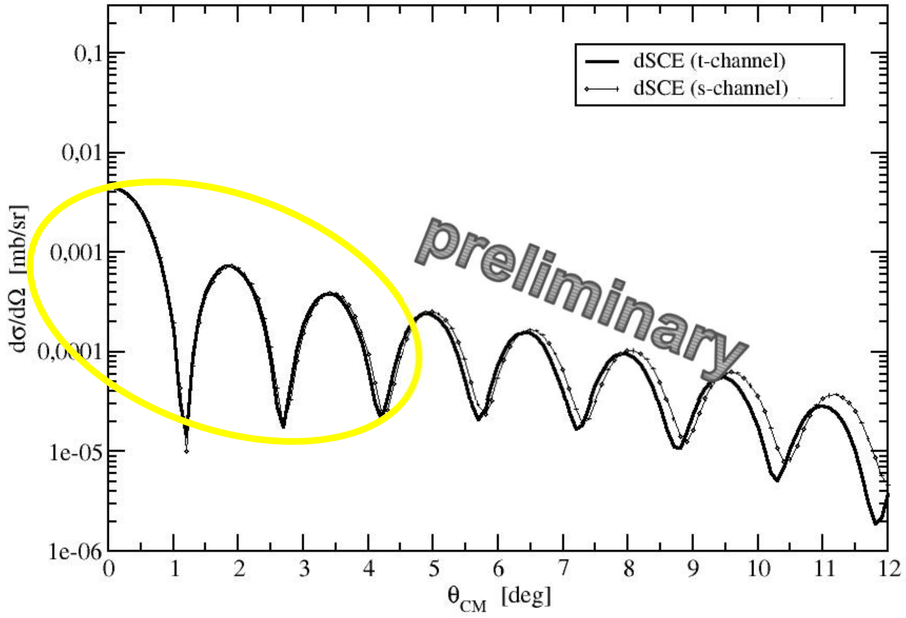


- ✓ the diffraction pattern (at very small scattering angles) recovered for all the nuclear dSCE reaction cross sections studied
- ➢ checks in progress for better reproducing the order of magnitude together with trend and diffraction pattern for larger scattering angles
- ➢ current results show that s-channel formalism represents a promising tool to get separate information on projectile and target dSCE NMEs.

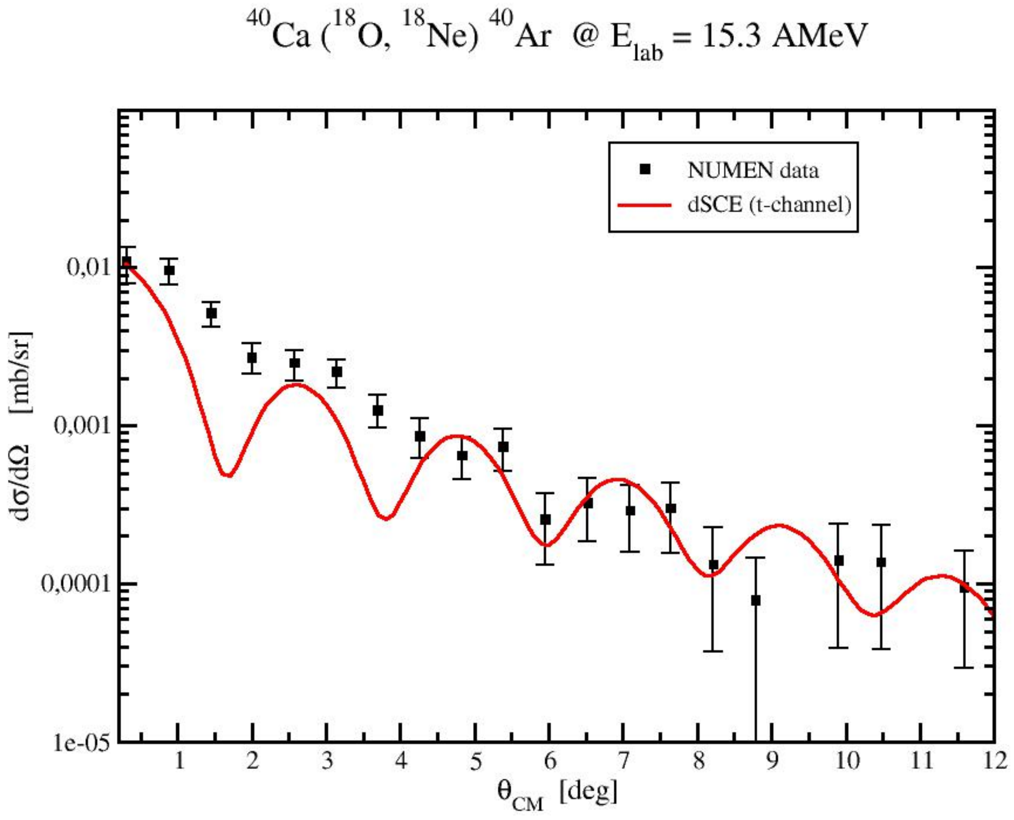
^{76}Se (^{18}O, ^{18}Ne) ^{76}Ge @ 15.3 AMeV



^{76}Ge (^{20}Ne, ^{20}O) ^{76}Se @ 15.3 AMeV



Sequential DCE Cross Section calculations compared to the data



J.I. B., S. Burrello, M. Colonna, J.A. Lay, H. Lenske, PLB 807 (2020) 135528
F. Cappuzzello et al., Progress in Particle and Nuclear Physics, submitted

Integral xsec (nb) in 3°< θ_{lab} < 13°	
$^{116}Cd(^{20}Ne,^{20}O)^{116}Sn$	
Exp.	Theo.
13 ± 2	1.4

Integrated σ [nb] in 0°< θ_{lab} < 8°			
$^{76}Se(^{18}O,^{18}Ne)^{76}Ge$		$^{76}Ge(^{20}Ne,^{20}O)^{76}Se$	
exp.	theo.	exp.	theo.
29 ± 6	14.6	30 ± 6	11.3

Discrepancies with the data could be related to:
- effect of nuclear structure deformations (affecting also SCE calculations)
- contribution of Majorana-like mechanism

..some hint on Majorana-like HIDCE reaction mechanism...

$$A(Z_A, N_A) + a(Z_a, N_a) \to B(Z_A \pm 2, N_A \mp 2) + b(Z_a \mp 2, N_a \pm 2)$$

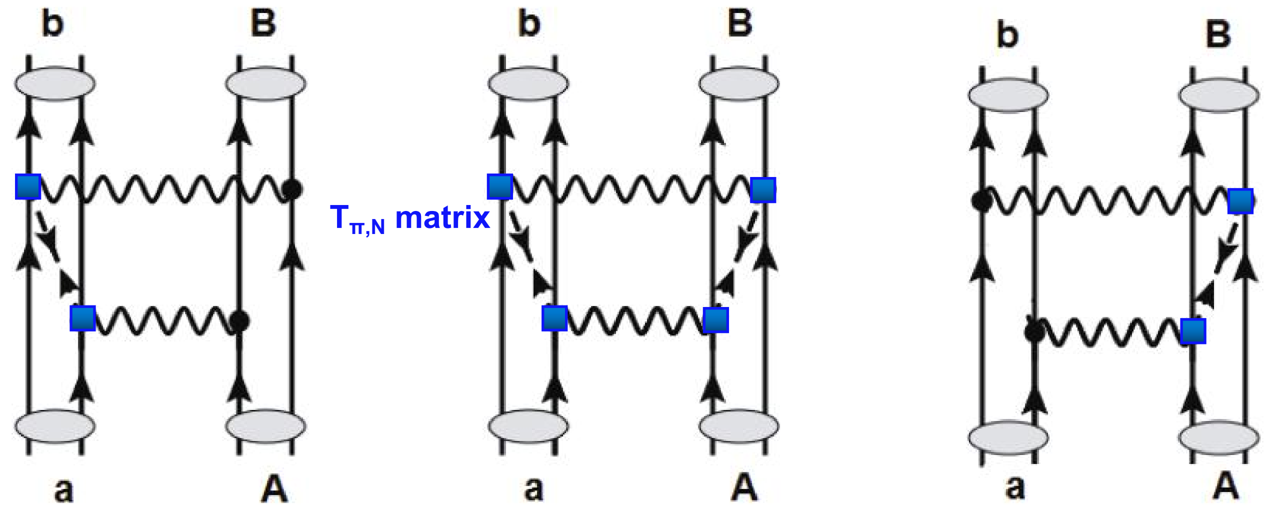
$$\mathcal{T}_{\pi N} = (T_0 + T_1 \mathbf{p}\cdot\mathbf{p}' + iT_2\sigma\cdot(\mathbf{p}\times\mathbf{p}'))\,\mathbf{T}\cdot\tau$$

(β) $B(Z_A \pm 2, N_A \mp 2) + b(Z_a \mp 2, N_a \pm 2)$

(α) $A(Z_A, N_A) + a(Z_a, N_a)$

Majorana-like DCE (mDCE)

sequence of two ***Correlated*** SCE reactions
(0νββ-like mechanism)

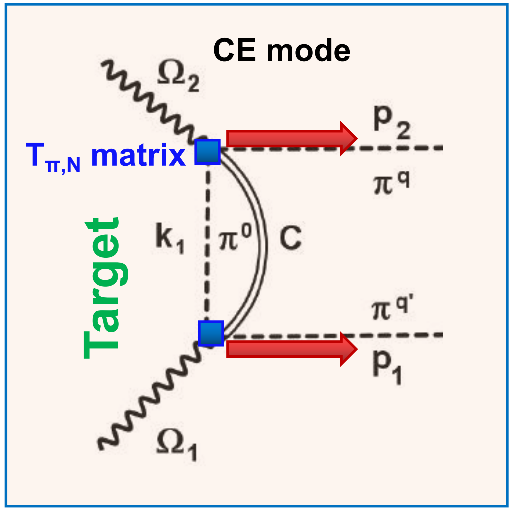


H. Lenske, IOP Conf. Series: Journal of Physics: Conf. Series **1056 (2018) 012030**
E. Santopinto *et al.* , Phys. Rev. C 98 061601 (R) (2018)
H. Lenske et al., Progress in Particle and Nuclear Physics 109 (2019) 103716
F. Cappuzzello et al., Progress in Particle and Nuclear Physics, submitted

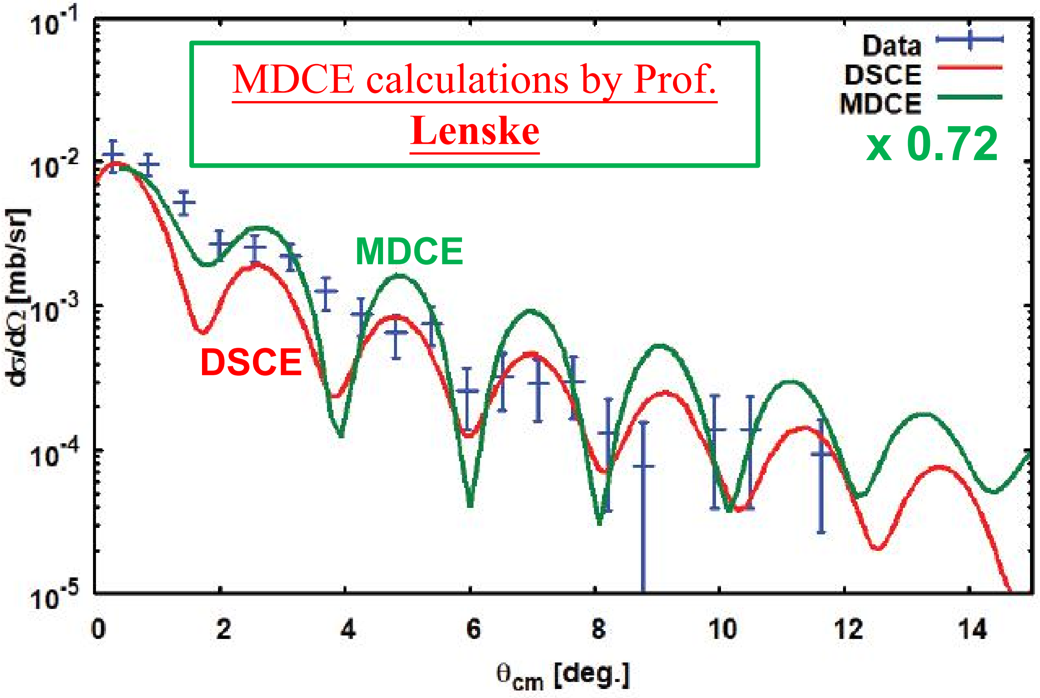
DCE reactions by double meson exchange: MDCE

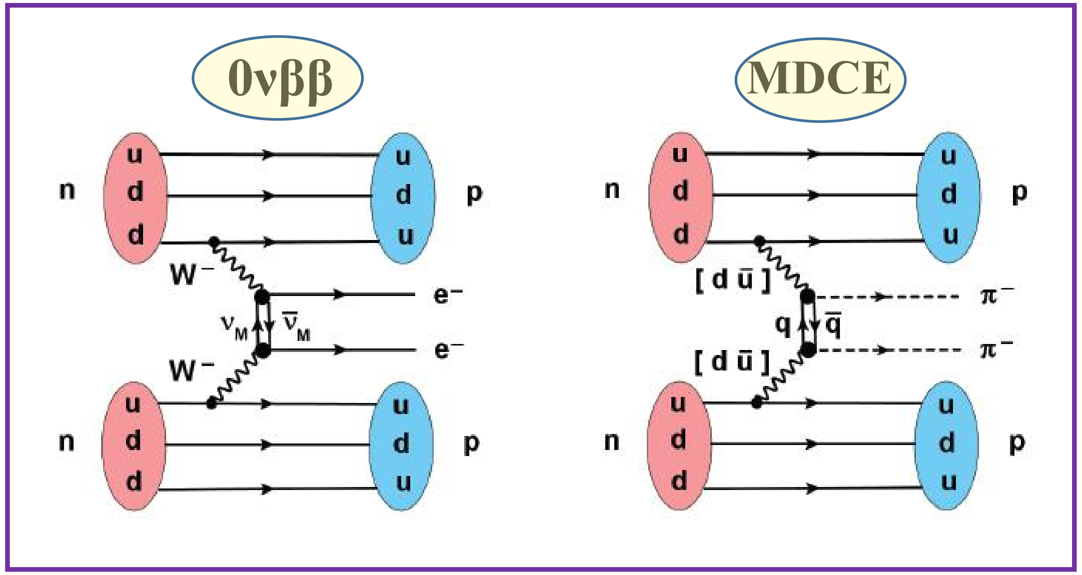
- Analogies with $0\nu\beta\beta$ matrix elements
  → Majorana-like DCE (MDCE)

H. Lenske et al., Progr. in Part. and Nucl. Phys. 109 (2019) 103716
F. Cappuzzello et al., PPNP (2022), submitted



- MDCE differential cross section
(*diagonal s-wave and p-wave* $T_{\pi,N}$ *contributions in* NMEs)





$^{40}Ca(^{18}O,^{18}Ne_{g.s.})^{40}Ar_{g.s.}$ @ 15.3 MeV/u

SUMMARY and OUTLOOK

- Sequential DCE (dSCE) cross section calculations progressively underestimate data for increasingly heavier nuclear systems:
  → contribution of Majorana-like DCE reaction mechanism, which should be coherently added to the sequential one
  → possible effect of nuclear deformation not properly treated (already affecting SCE)
  ---> further improvement of nuclear structure inputs → testing new QRPA code
  → check the effect of nuclear deformations
  → try to establish a protocol allowing to better reproduce experimental energy spectra

- to refine dSCE - s-channel fomalism in order to be able to extract separately projectile and target nuclear matrix elements from DCE cross section measurements

- calculations already performed for g.s. → g.s. DCE transitions : extension to g.s. → (excited states) DCE

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THANK YOU FOR YOUR KIND ATTENTION !