I will present a reformulation of the Hohenberg-Kohn theorem and the Kohn-Sham equations in terms of a particular many-body density, the hyperradial density, which is translational and Galileian invariant and therefore is relevant for self-bound systems. A unique relation between the hyperradial density and a definite many-body potential is proved. The energy is then a functional of this density and its minimization leads to the ground-state energy of the system. As a proof of principle, I will present results where the analogous of the Kohn-Sham equation is solved in the specific case of 4He atomic clusters, to put in evidence the advantages of this new formulation in terms of physical insights. [see Phys. Rev. A 104, 030801 (2021) ]