Functional theory for Bose-Einstein condensates

One-particle reduced density matrix functional theory would potentially be the ideal approach for describing Bose-Einstein condensates. It namely replaces the macroscopically complex wavefunction by the simple one-particle reduced density matrix, therefore provides direct access to the degree of condensation and still recovers quantum correlations in an exact manner. We initiate and establish this novel theory by deriving the respective universal functional F for homogeneous Bose-Einstein condensates with arbitrary pair interaction. Most importantly, the successful derivation necessitates a particle-number conserving modification of Bogoliubov theory and a solution of the common phase dilemma of functional theories. We then illustrate this novel approach in several bosonic systems such as homogeneous Bose gases and the Bose-Hubbard model. Remarkably, the general form of F reveals the existence of a universal Bose-Einstein condensation force which provides an alternative and more fundamental explanation for quantum depletion.