Radiative Corrections to SIDIS

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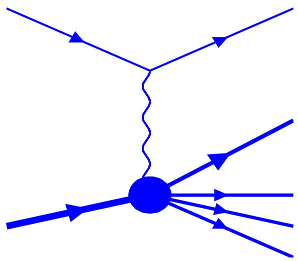
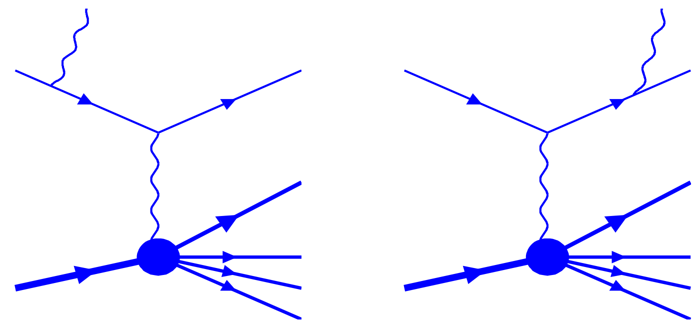
Contribution to RC in SIDIS

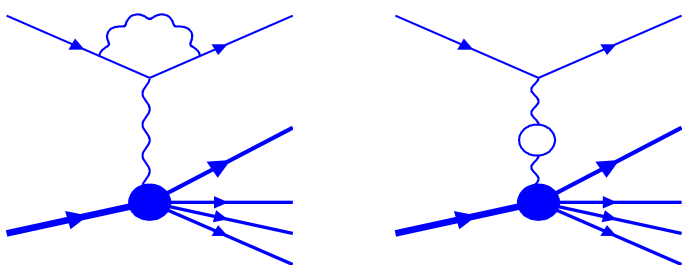
The Born cross section



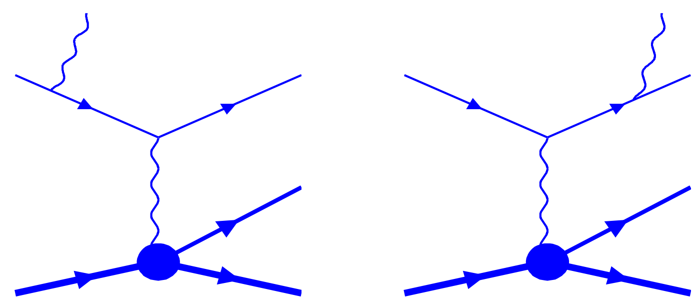
Emission of a radiated photon (semi-inclusive processes)



Loop diagrams



Emission of a radiated photon (exclusive processes)



Current Status of Our Calculations

- All analytic calculations were performed without any approximations and presented in our paper:
  - Akushevich, Igor, and Alexander Ilyichev. “Lowest order QED radiative effects in polarized SIDIS” Physical Review D 100, no. 3 (2019): 033005.
- Fortran code was created by Alexander Ilyichev
  - SIDIS SF were implemented as in (Wandzura-Wilczek-type approximation):
    - *Bastami, Saman, Harut Avakian, A. V. Efremov, Aram Kotzinian, B. U. Musch, B. Parsamyan, Alexei Prokudin et al. “Semi-inclusive deep-inelastic scattering in Wandzura-Wilczek-type approximation.” Journal of High Energy Physics 2019, no. 6 (2019): 7.*
    - *However, this model is not completely satisfactory: there are kinematical regions with unrealistic estimates of SFs, applicability of the model in entirely region necessary for RC has to be reviewed.*
  - Exclusive SFs were not implemented. Possible approaches are still reviewed.
- Extraction of the leading log contributions and analysis of shifted kinematics is completed.
  - By “exactly” calculated RC we understand the estimation of the lowest order RC contribution with any predetermined accuracy. The structure of the dependence on the electron mass in RC cross section: $\sigma_{RC} = A \log \frac{Q^2}{m^2} + B + O(m^2/Q^2)$, where A and B do not depend on the electron mass m.

Exact Formulae for RC in SIDIS

The total RC is calculated exactly in our paper (PR D100 (2019) 033005):

$$\sigma_{RC} = \frac{\alpha}{\pi} \exp(\delta_{inf}) \left(\delta_{VR} + \delta^l_{vac} + \delta^h_{vac} \right) \sigma_B + \sigma^F_R + \sigma_{AMM} + \sigma^{ex}_R$$

Here δ_{VR} and $\delta^{l,h}_{vac}$ come from the radiation of soft photons and the effects of vacuum polarization, the correction δ_{VR} is infrared-free sum of factorized parts of real and virtual photon radiation, and σ^F_R and σ^{ex}_R are contributions from the process of emission of an additional real photon:

$$\begin{aligned}
\sigma_B &= \frac{\alpha^2 S_x^2}{4Q^4\sqrt{\lambda_Y} S} \sum_{i=1}^{9} \theta_i^B \mathcal{H}_i \\
\sigma^F_R &= -\frac{\alpha^3 S S_x^2}{32\pi^2 \lambda_S \lambda_Y} \int\limits_{\tau_{\min}}^{\tau_{\max}} d\tau \int\limits_0^{2\pi} d\phi_k \int\limits_0^{R_{\max}} dR \sum_{i=1}^{9} \left[\sum_{j=1}^{k_i} \tilde{\mathcal{H}}_i \theta_{ij} \frac{R^{j-2}}{\tilde{Q}^4} - \frac{\theta_{i1}\mathcal{H}_i}{RQ^4} \right], \\
\sigma^{ex}_R &= -\frac{\alpha^3 S S_x^2}{2^8 \pi^5 \lambda_S \lambda_Y} \int\limits_{\tau_{\min}}^{\tau_{\max}} d\tau \int\limits_0^{2\pi} d\phi_k \sum_{i=1}^{9} \sum_{j=1}^{k_i} \frac{\tilde{\mathcal{H}}_i^{ex} \theta_{ij} R_{ex}^{j-2}}{(1+\tau-\mu)\tilde{Q}^4}.
\end{aligned}$$

Explicit expressions for the functions θ_{ij} are given in Appendix B of our paper.

The Cross Section in the Leading Log

The cross section of RC in the leading log approximation is:

$$\sigma_{RC}(x,Q^2,z,t,\phi_h) = \left[1+\frac{\alpha}{\pi}\delta_{\rm vac}\right]\sigma_B(x,Q^2,z,t,\phi_h)$$

$$+\frac{\alpha}{2\pi}l_m\int\limits_{z_1^m}^{1} dz_1\frac{1+z_1^2}{1-z_1}\left[\sqrt{\frac{\lambda_Y^s}{\lambda_Y}}\frac{S_x^2}{S_x^{s\,2}}\sigma_B(\tilde{x}_1,\tilde{Q}_1^2,\tilde{z}_1,\tilde{t}_1,\tilde{\phi}_{h1})-\sigma_B(x,Q^2,z,t,\phi_h)\right]$$

$$+\frac{\alpha}{2\pi}l_m\int\limits_{z_2^m}^{1} dz_2\frac{1+z_2^2}{1-z_2}\left[\frac{1}{z_2^2}\sqrt{\frac{\lambda_Y^p}{\lambda_Y}}\frac{S_x^2}{S_x^{p\,2}}\sigma_B(\tilde{x}_2,\tilde{Q}_2^2,\tilde{z}_2,\tilde{t}_2,\tilde{\phi}_{h2})-\sigma_B(x,Q^2,z,t,\phi_h)\right]$$

where quantities with tilde are calculated in the shifted kinematics.

Leading Log: Exclusive Radiative Tail

Exact contribution calculated inour 2019 paper is:

$$\sigma_R^{ex} \quad = \quad -\frac{\alpha^3 S S_x^2}{2^8 \pi^5 \lambda_S \lambda_Y} \int\limits_{\tau_{\min}}^{\tau_{\max}} d\tau \int\limits_0^{2\pi} d\phi_k \sum_{i=1}^{9} \sum_{j=1}^{k_i} \frac{\tilde{\mathcal{H}}_i^{ex} \theta_{ij} R_{ex}^{j-2}}{(1+\tau-\mu)\tilde{Q}^4}.$$

where $R_{ex} = \frac{p_x^2 - m_u^2}{1+\tau-\mu}$. The Born cross section of exclusive process is:

$$\sigma_{ex}^B = \frac{\alpha^2 S_x}{32\pi^3 Q^4 S \sqrt{\lambda_Y}} \sum_{i=1}^{9} \mathcal{H}_i^{ex} \theta_i^B$$

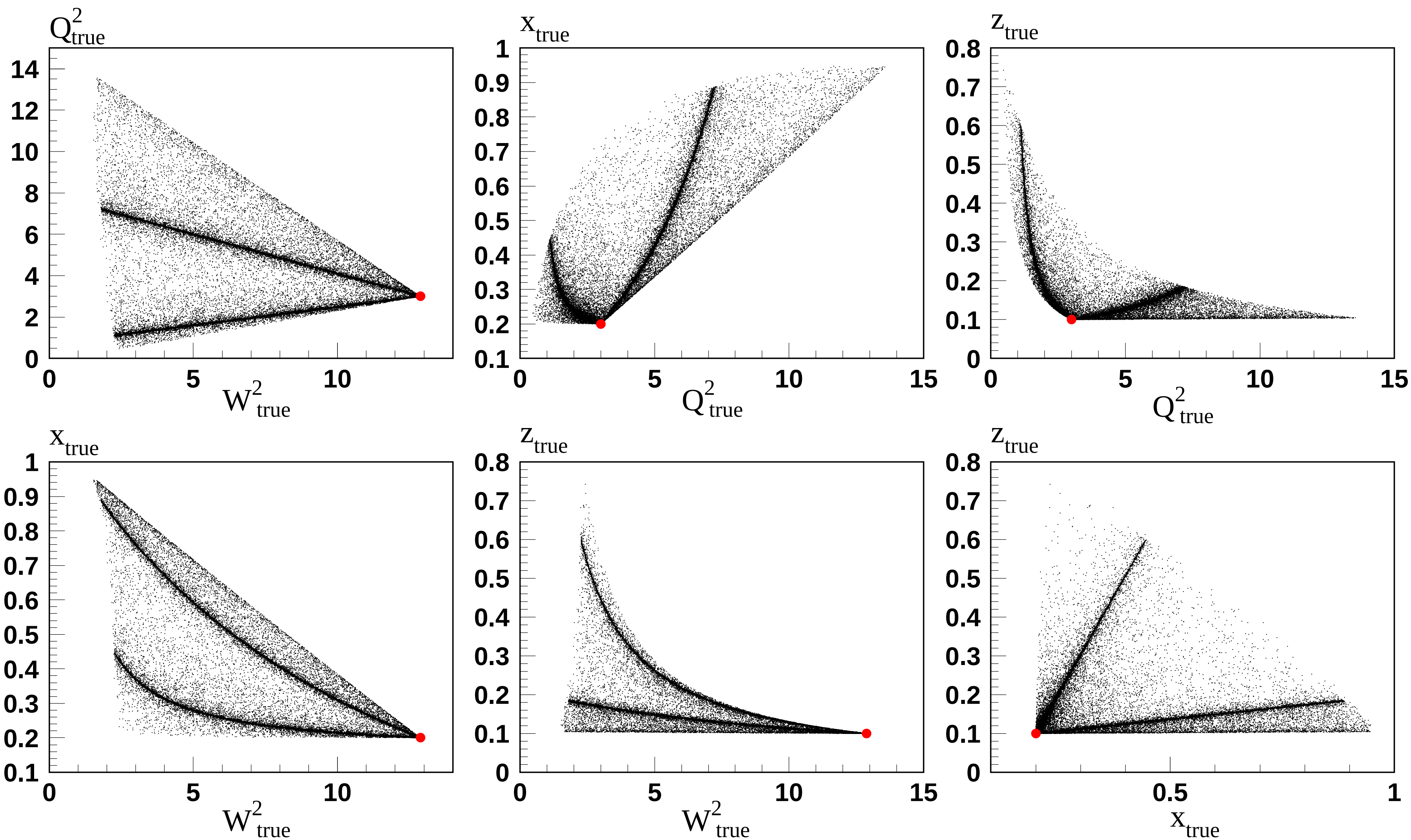
Therefore we have

$$\sigma_{sex}^1 = \frac{\alpha}{2\pi} l_m \sqrt{\frac{\lambda_Y^{se}}{\lambda_Y}} \frac{S_x^2}{S' S_x^{se}} \frac{1+z_{1ex}^2}{1-z_{1ex}} \sigma_{sex}^B,$$

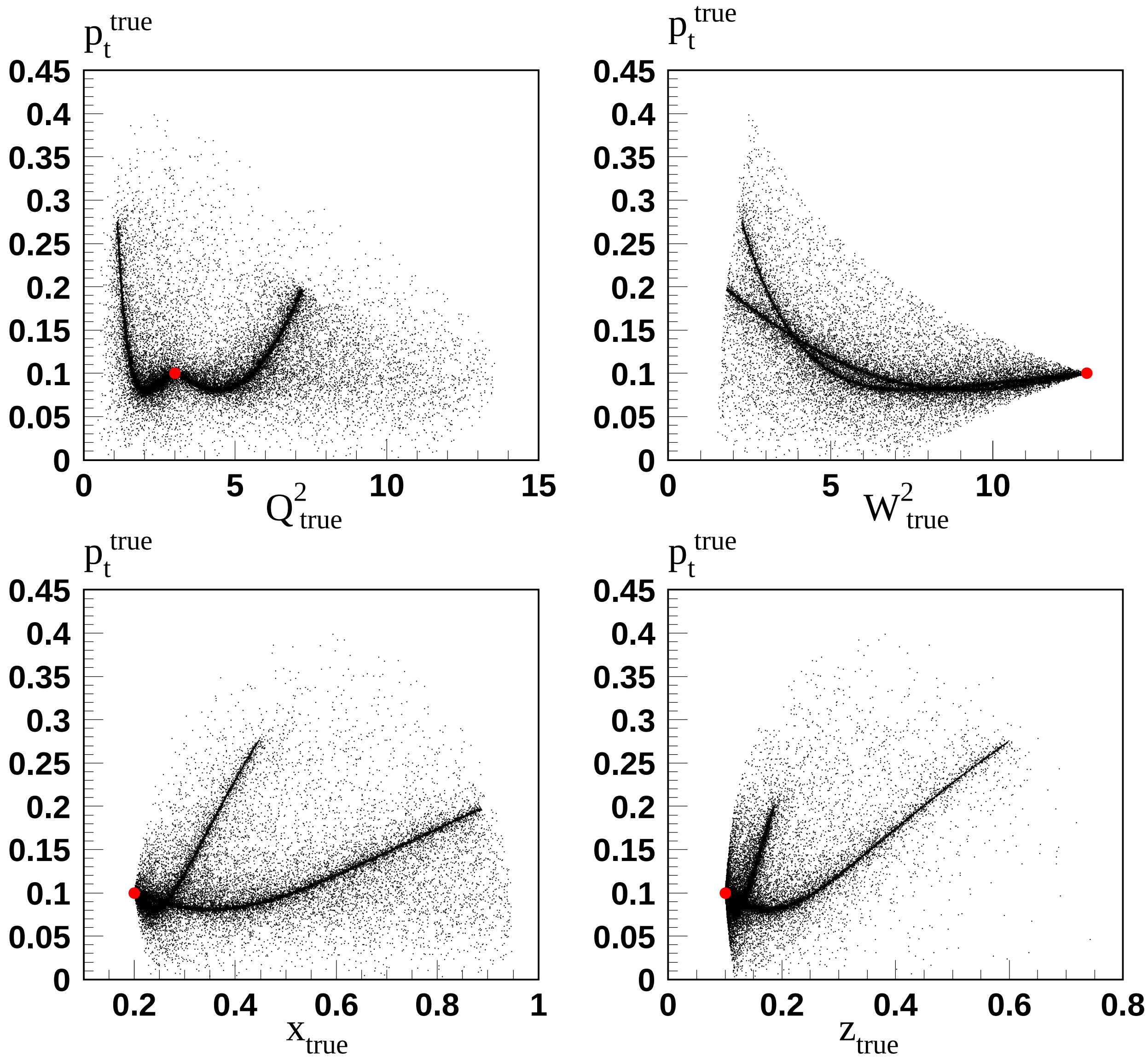
$$\sigma_{pex}^1 = \frac{\alpha}{2\pi} l_m \sqrt{\frac{\lambda_Y^{pe}}{\lambda_Y}} \frac{S_x^2}{X' S_x^{pe}} \frac{1+z_{2ex}^2}{1-z_{2ex}} \sigma_{pex}^B,$$

where $z_{1ex} = 1 - (p_x^2 - m_u^2)/S'$ and $z_{2ex} = (1 + (p_x^2 - m_u^2)/X')^{-1}$.

Kinematic Regions for SIDIS SFs (x,Q2,W2,z)



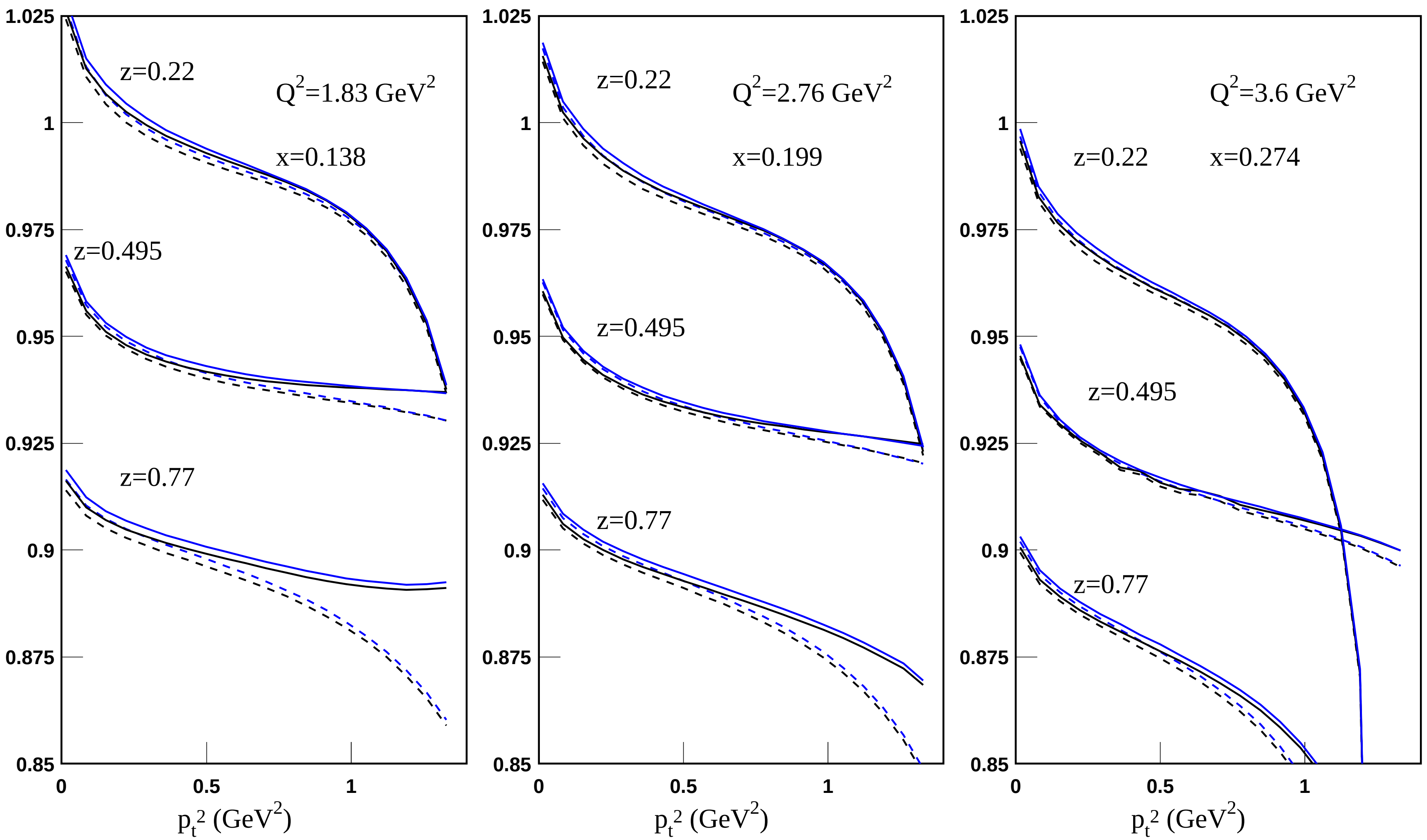
Kinematic Region for SIDIS SFs (pt)



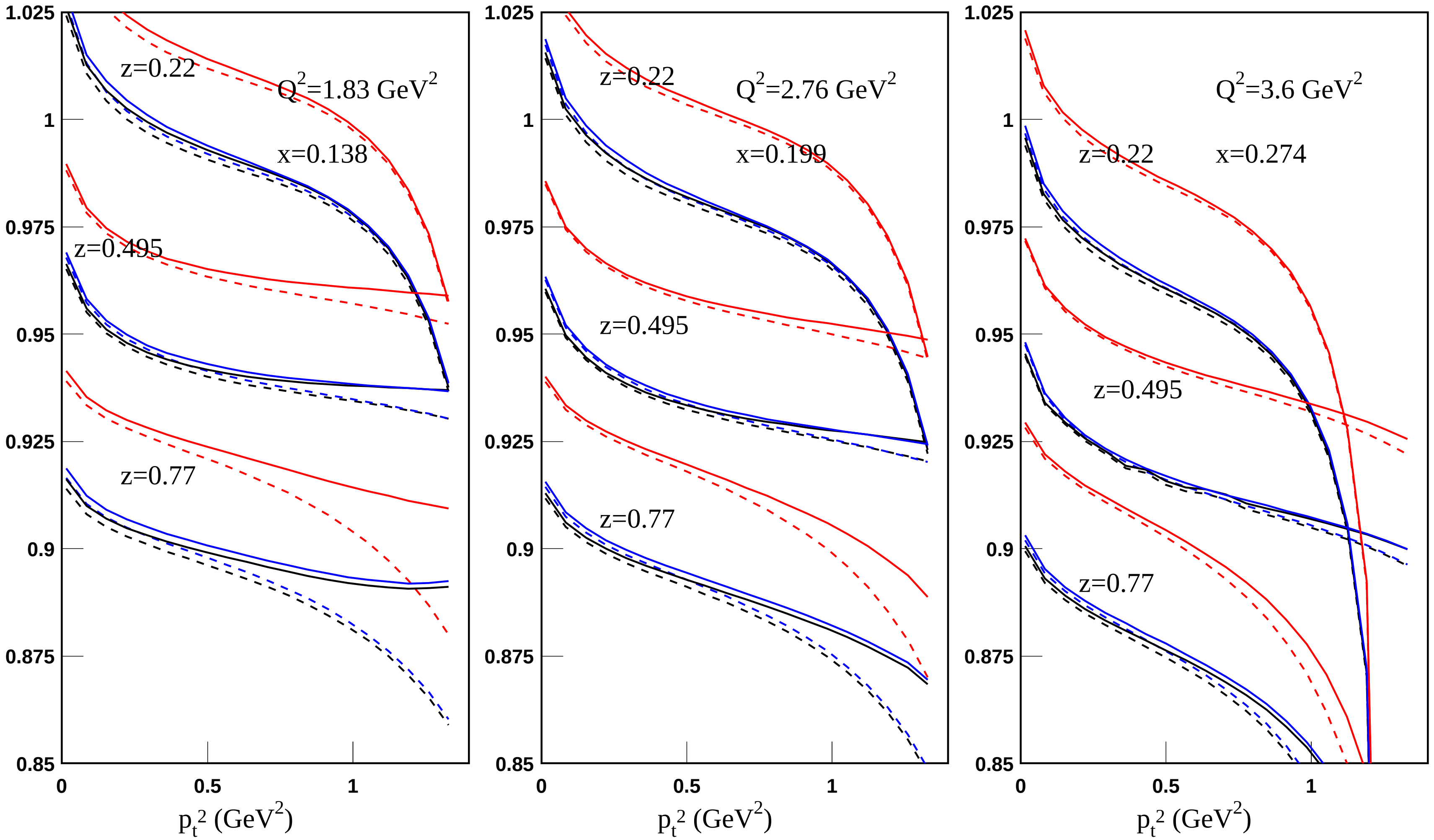
Conclusions from prior analyses

- RC to azimuthal asymmetry can reach 10-20%. We note that different input of SFs produce different corrections (even by sign).
- Exclusive radiative tail is important contribution to RC. there are kinematical regions (e.g., small M_X^2 where it is a dominant contribution.
- The RC may be very significant at large PT.
- There exist effects not observed at the level of the Born cross section (e.g., $< cos(3\phi) >$).
- RC to Sivers and Collins asymmetries can reach several dozens of persents and is expected to be sensitive to SF input.

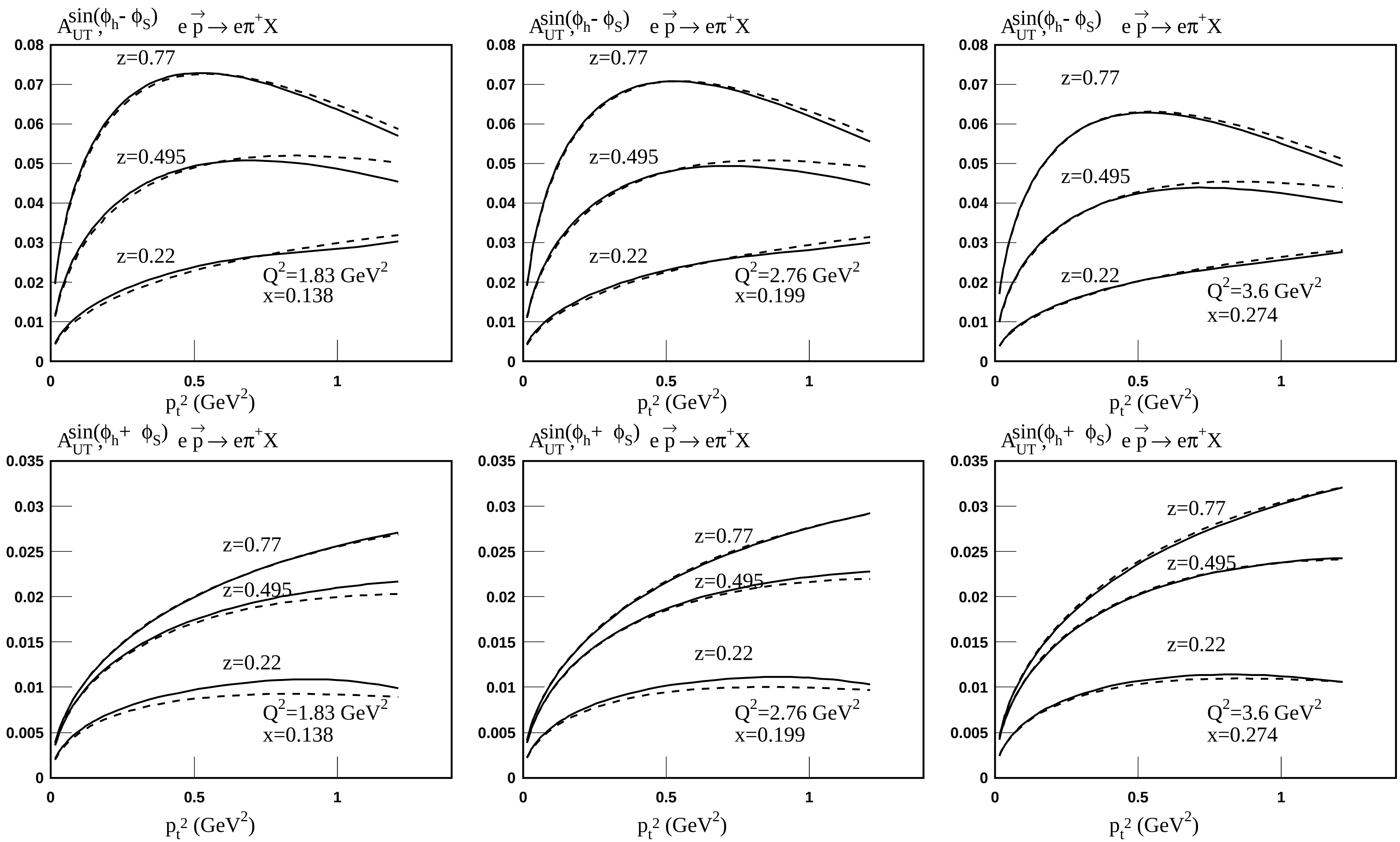
RC to Unpolarized Cross Section



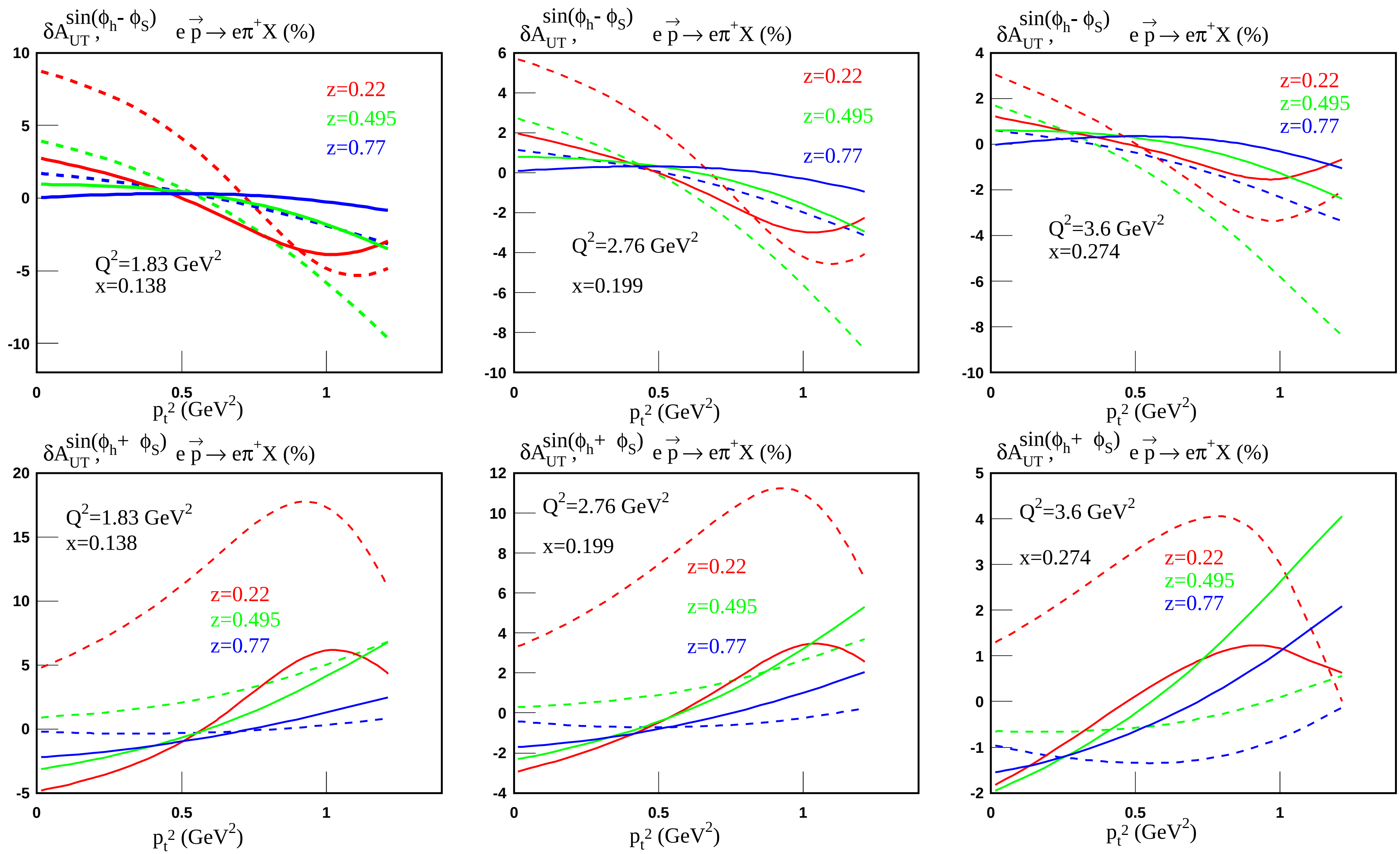
RC to Unpolarized Cross Section



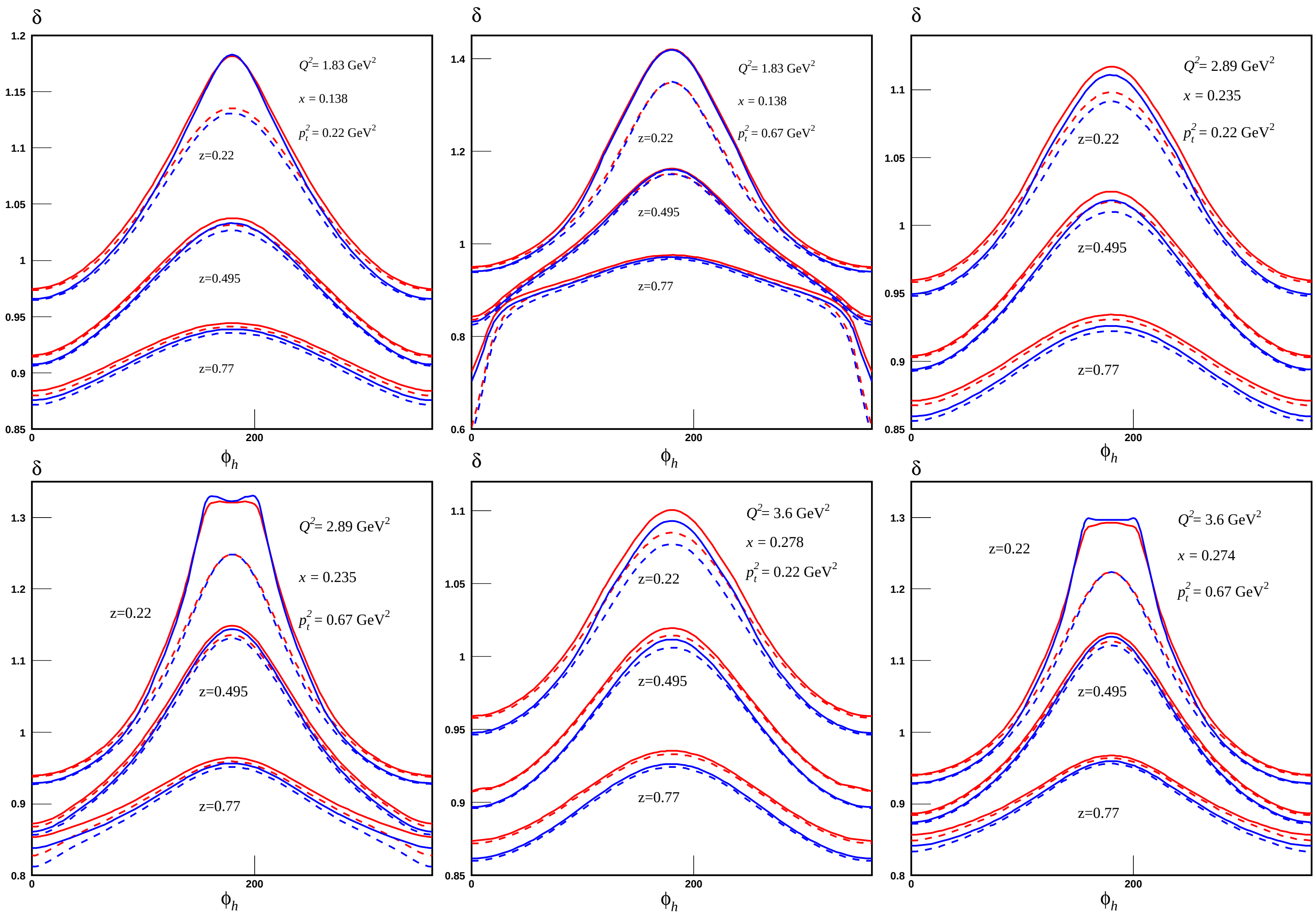
RC to Collins and Sivers Asymmetries



Effects to the Asymmetries Induced by Other SFs



RC to ϕ-dependence



Open Points

Exlusive SFs for polarized case *We are working on it, but we will need several models to test model-dependence. Any help will be appreciated.*

SIDIS SFs *We currently implemented WW approach, however, additional cross checks are required for specific regions. Bounds of applicability of WW are needed to be defined and discussed. Additional models would be helpful as well.*

Iteration procedure of RC *We need to decide whether iteration procedure of RC will be used in data analyses. We strongly recommend this (especially for asymmetry measurements) to avoid an additional bias due to RC procedure. The bias is proportional to the difference between the values of Born asymmetries in the given bin: i) finally extracted and ii) used in RC codes. These values coincide by definition when the iteration procedure is used.*

Comparison and agreement between teoretical calculations *We need to complete analyic and numeric comparison to leptonic RC calculated by Tianbo Liu, W. Melnitchouk, Jian-Wei Qiu, and N. Sato (JLAB-THY-21-3489).*

Hadronic corrections *Discuss and decide whether and how non-leptonic corrections (including box diagrams and emission by hadrons) will be calculated.*

Higher order corrections *We need to discuss approaches for higher order corrections, such as exponentiation, electron SFs, etc. Pay specific attention to the region with small* M_x^2.

Monte Carlo *Approaches to MC generators has to be discussed and implemented.*

SIDIS event generator on radiative corrections

A Monte Carlo event generator created by Duane Byer from Duke University, based on the SIDIS RC paper: https://github.com/duanebyer/sidis

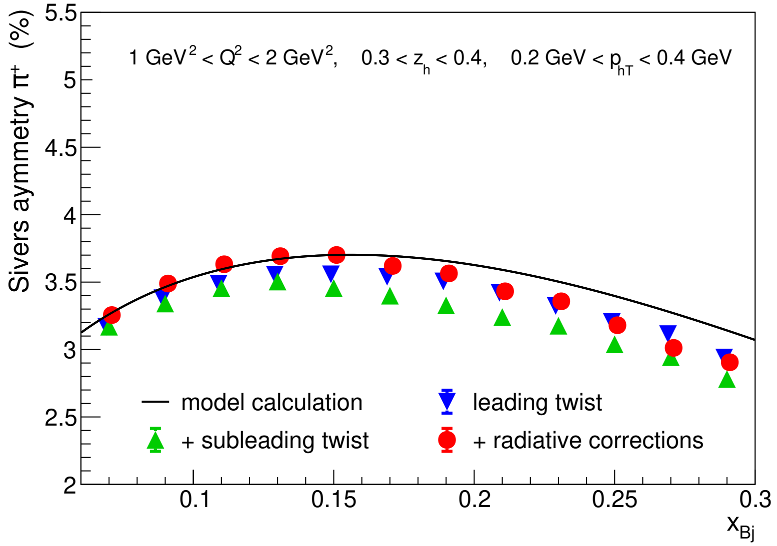
- Generates events for SIDIS six-fold cross sections computation
- All eighteen SIDIS structure functions implemented in Gaussian and Wandzura-Wilczek type approximations: S. Bastami et. al., JHEP06, 007 (2019)
- More fine tuning on the generator should be done for running it in the SoLID experiment's framework, including the neutron target
- In the meantime, the generator can be used for other experiments from medium to high energies, which also measure the SIDIS processes (ClAS12, COMPASS, etc.)

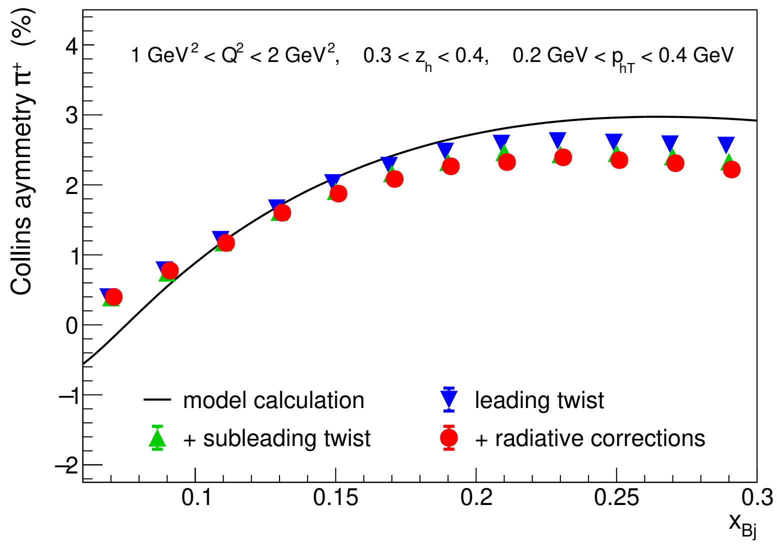
A respective paper is made by Duane Byer and Vlad Khachatryan, including also Haiyan Gao, Igor Akushevich, Alexander Ilyichev, Chao Peng, Alexei Prokudin, Stan Srednyak, and Zhiwen Zhao

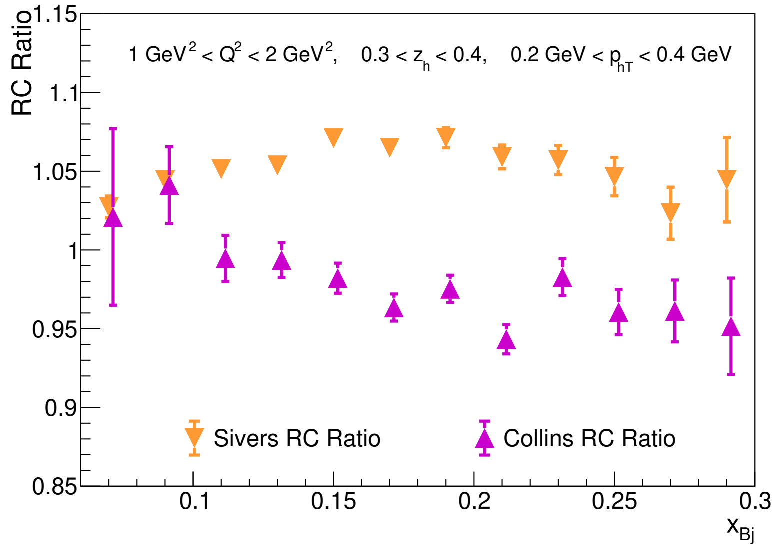
- Paper soon will be submitted to the archive and Computer Physics Communication
- Another paper is in preparation by Akushevich, Ilyichev, and Srednyak on exclusive structure functions in SIDIS
- Event generator will be updated in the future for better numerical performance and to include exclusive structure functions

SIDIS event generator on radiative corrections

Examples of extracted Collins and Sivers asymmetries without exclusive SF included







Conclusion

- Newly achieved accuracies in Jlab and new physics studied at Jlab require paying renewed attention to RC calculations and their implementation in data analysis software.
- For SIDIS RC theoretical efforts are needed both for calculation of SIDIS RC in a bin and for generation of radiated events:
  - *Hadronic tensor for the SIDIS cross sections in the covariant form is constructed and tested. Further work is required for the hadronic tensor for exclusive processes*
  - *Exact calculation of RC for the complete SIDIS cross section containing 18 SFs is completed and coding is done, however, implementation of SFs is still in progress.*
  - *We expect sensitivity of the results for RC to specific assumptions used for constructing SIDIS SFs:*
  - *Broad discussion and efforts of theoreticians and experimentalists are required to complete the evaluation of all SIDIS SFs as well as SFs in resonance region and exclusive SFs.*
  - *Iteration procedure with fitting of measured SFs and joining with models beyond SIDIS measurements at each iteration step has to be designed, implemented and involved in data analysis.*
- Tools for generation of the radiated photon in DIS cannot provide valid generation of the radiated events in SIDIS.
  - *Such generator can be constructed based on a code for RC in SIDIS in the same way of how RADGEN is constructed based on POLRAD 2.0*