Prospects on GPDs from lattice QCD

Martha Constantinou

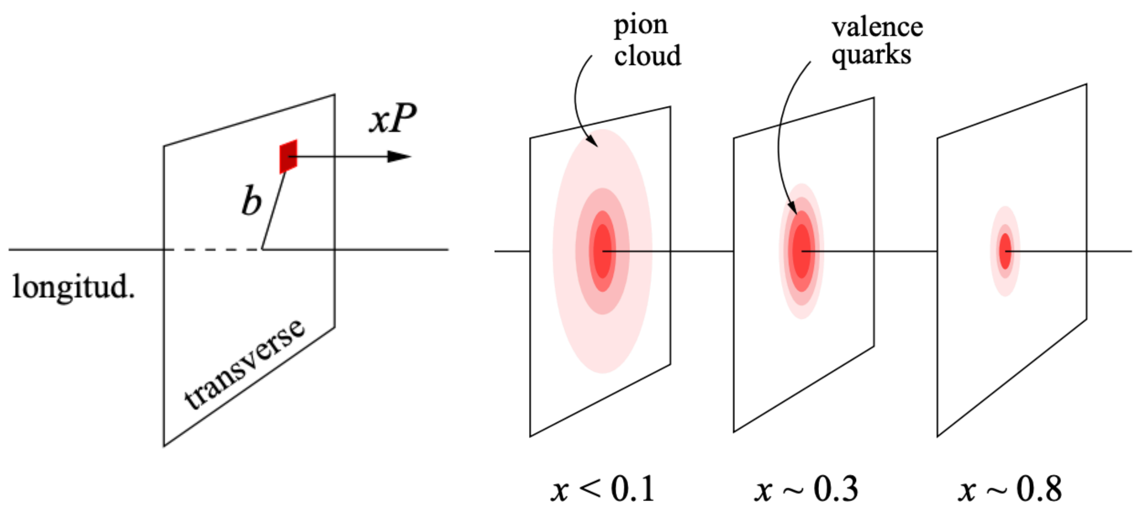
Temple University

Opportunities with JLab Energy and Luminosity Upgrade

September 28, 2022

Generalized Parton Distributions

★ Crucial in understanding hadron tomography



[H. Abramowicz et al., whitepaper for NSAC LRP, 2007]

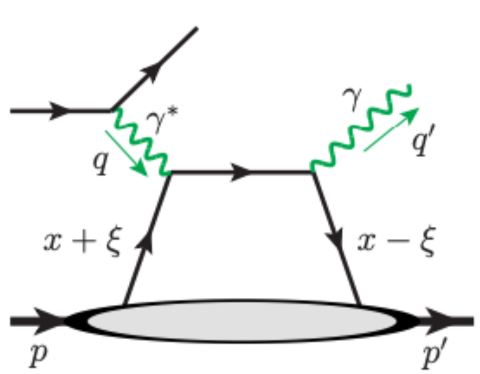
$1_{\text{mom}} + 2_{\text{coord}}$ tomographic images of quark distribution in nucleon at fixed longitudinal momentum

3-D image from FT with respect to longitudinal momentum transfer

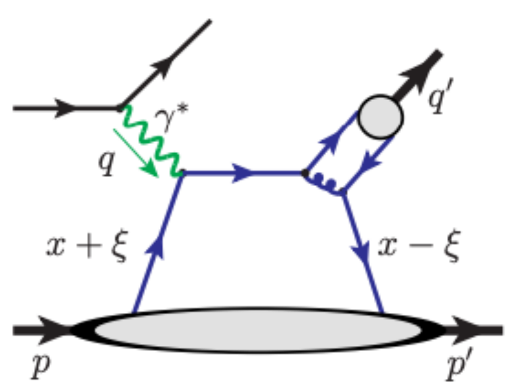
★ GPDs may be accessed via exclusive reactions (DVCS, DVMP)

★ exclusive pion-nucleon diffractive production of a γ pair of high $p_\perp$

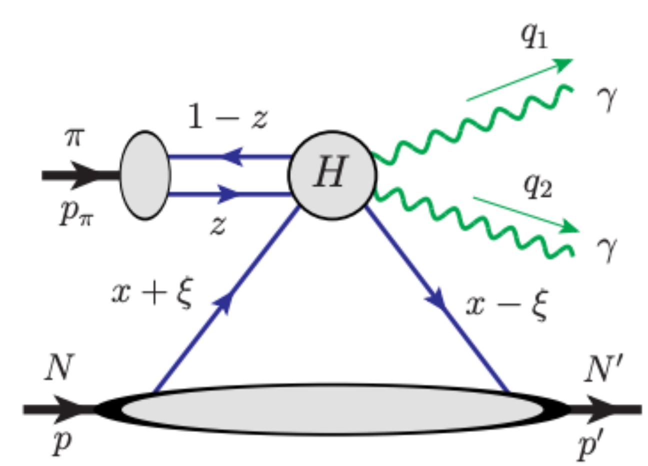
DVCS



DVMP



[X.-D. Ji, PRD 55, 7114 (1997)]



[J. Qiu et al, arXiv:2205.07846]

Generalized Parton Distributions

★ GPDs are not well-constrained experimentally:

- x-dependence extraction is not direct. DVCS amplitude: $\mathscr{H} = \int_{-1}^{+1} \frac{H(x,\xi,t)}{x-\xi+i\epsilon} dx$

(SDHEP [J. Qiu et al, arXiv:2205.07846] gives access to x)

- independent measurements to disentangle GPDs

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- and more challenges …

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★ Essential to complement the knowledge on GPD from lattice QCD

★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

Accessing information on GPDs

★ **Mellin moments**
(local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z,\tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\alpha_1}\ldots z_{\alpha_n}\left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n} q\right]$$

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1}\left\{\gamma^{\{\mu}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}A_{n,i}(t) - i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}B_{n,i}(t)\right\} + \frac{\Delta^{\mu}\Delta^{\mu_1}\ldots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\text{ even}}\Big)\Big\}$$

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(quasi-GPDs, pseudo-GPDs, …)

$$\langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

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- Frame independence
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- x dependence is integrated out
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- Geometrical twist classification (coincides with dynamical twist of scattering processes only at leading order)
- Signal-to-noise ratio decays with the addition of covariant derivatives
- Power-divergent mixing for high Mellin moments (derivatives > 3)
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Form Factors & Generalizations

★ Ultra-local operators (FFS)

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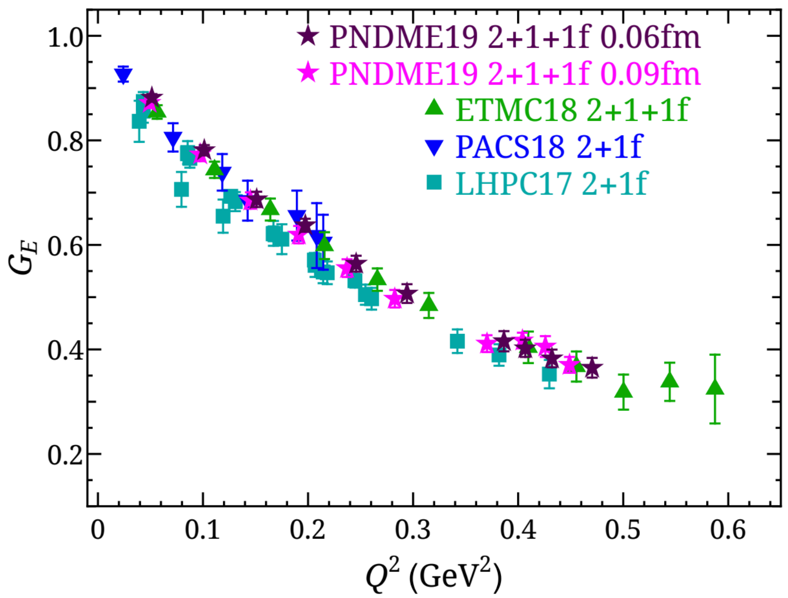

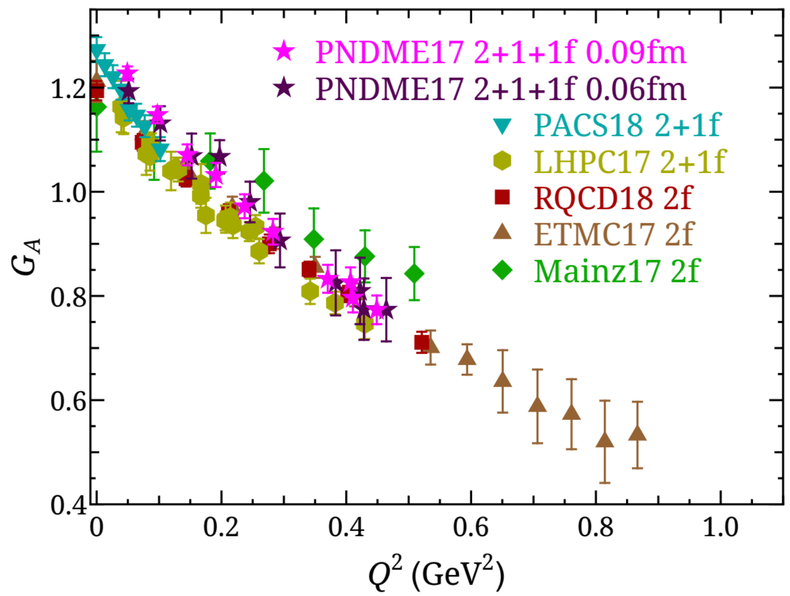

[M. Constantinou et al. (2020 PDFLattice Report), Prog.Part.Nucl.Phys. 121 (2021) 103908]

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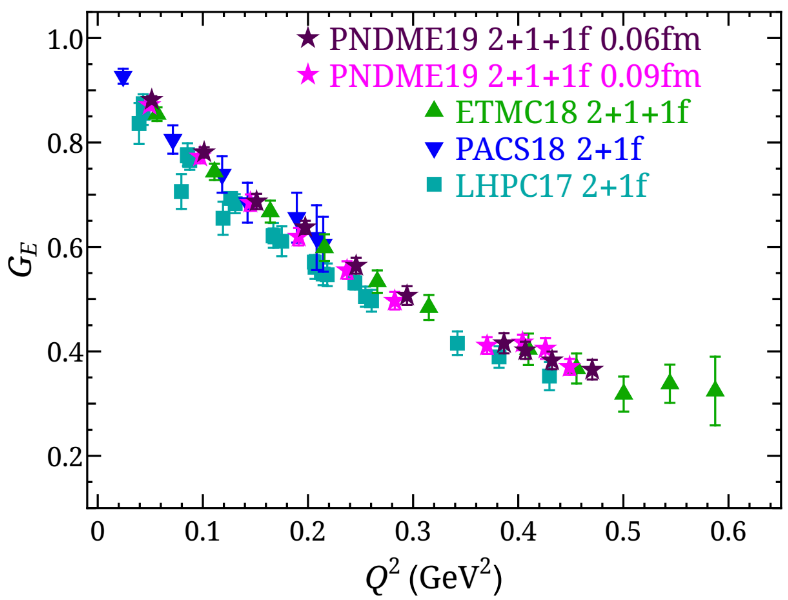

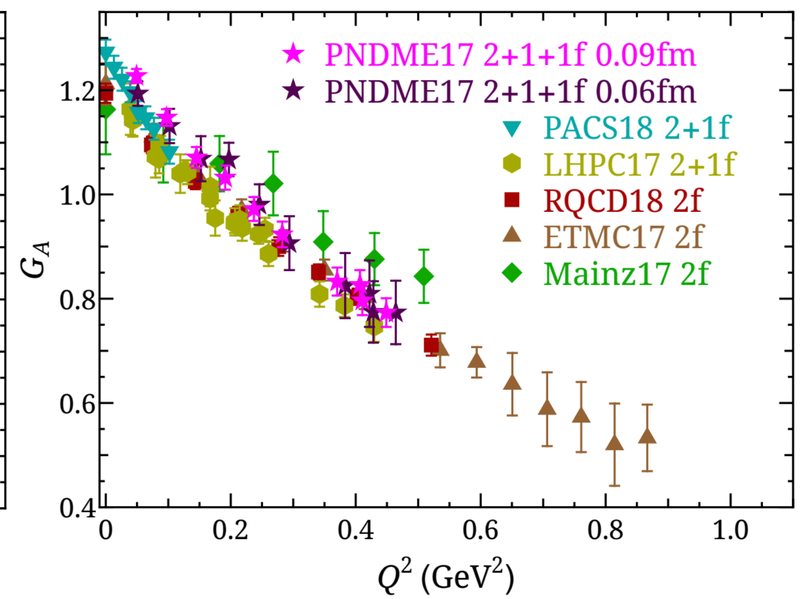

- Simulations at physical point available by multiple groups
- Precision data era
- Towards control of systematic uncertainties

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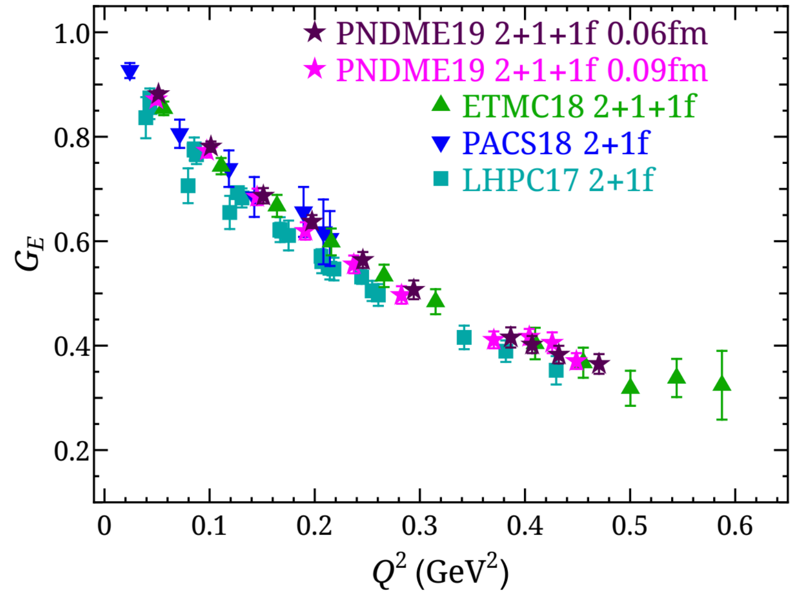

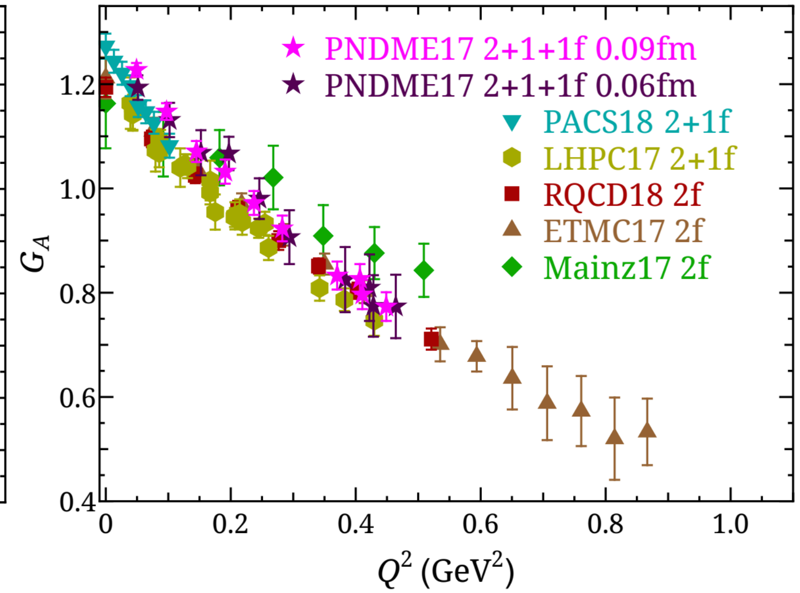

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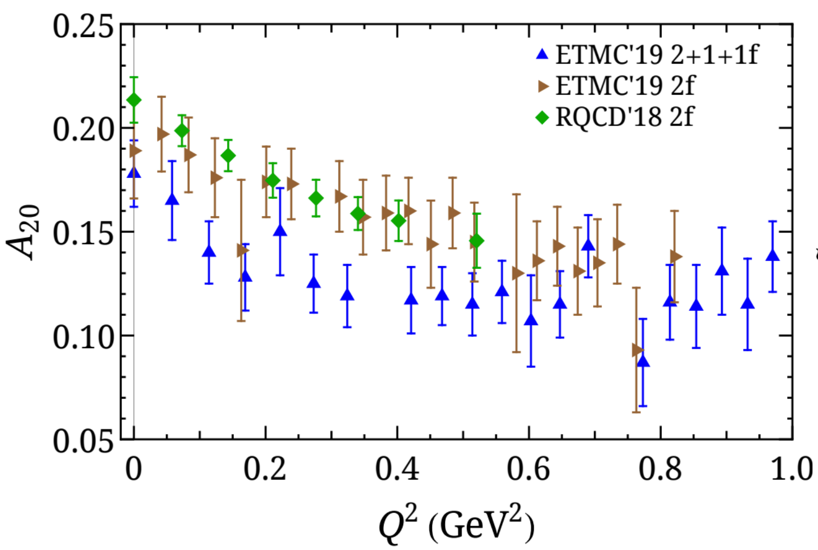

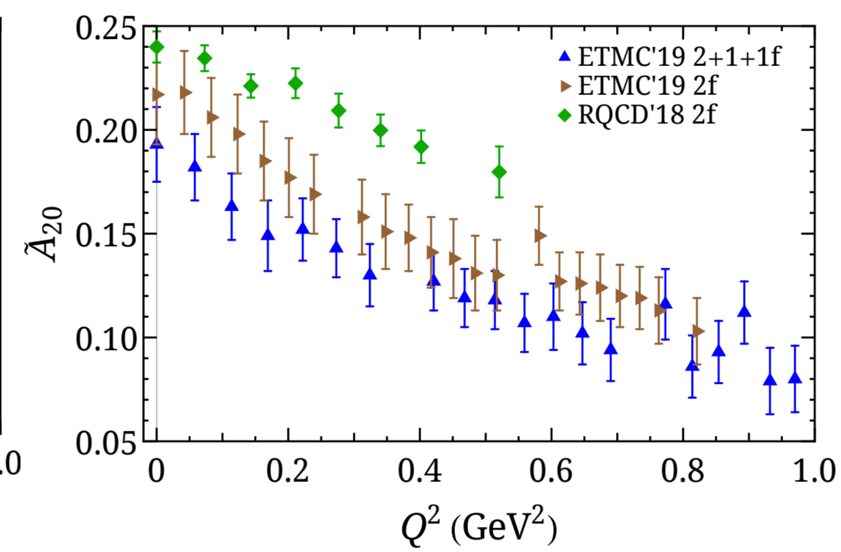

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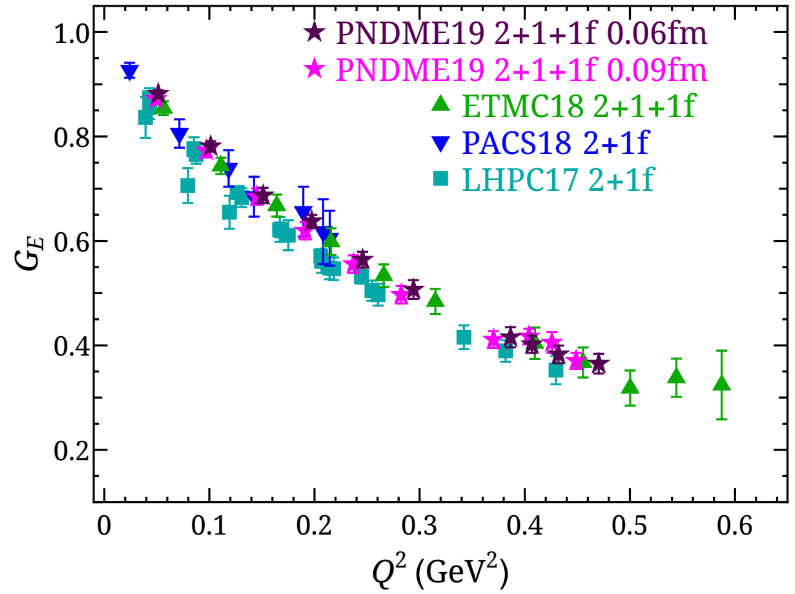

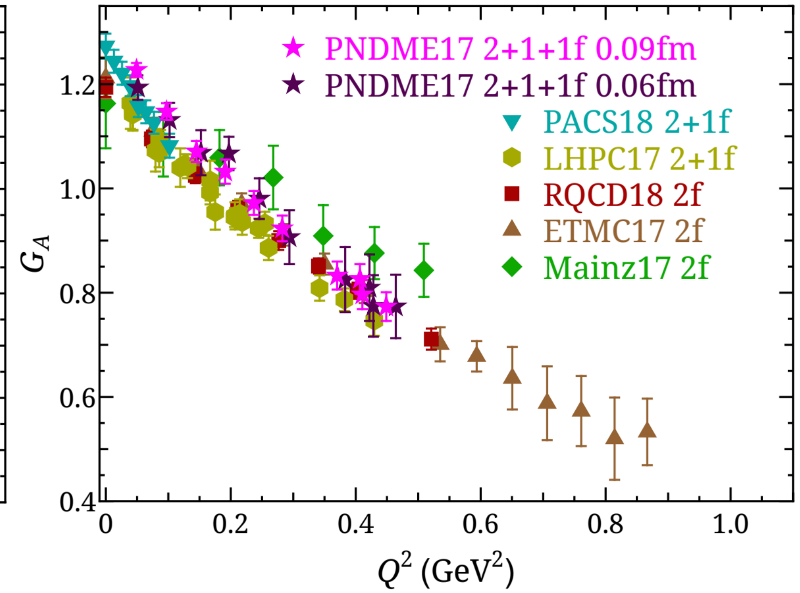

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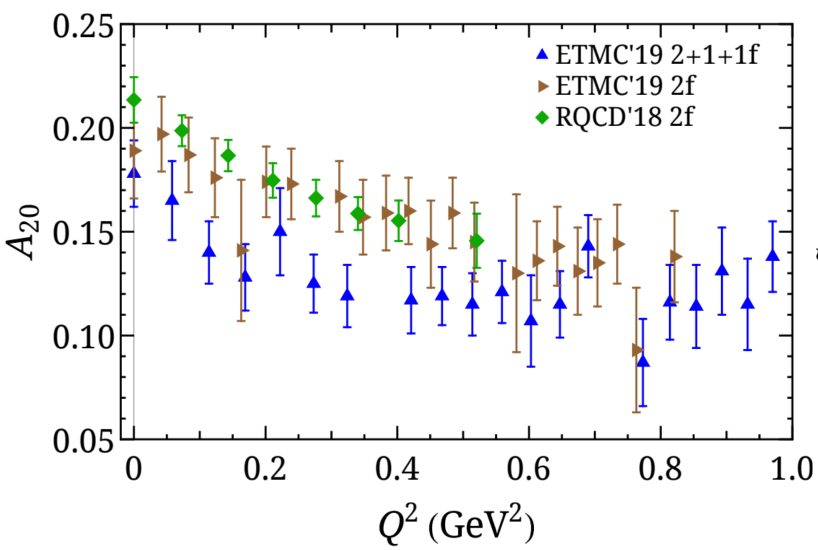

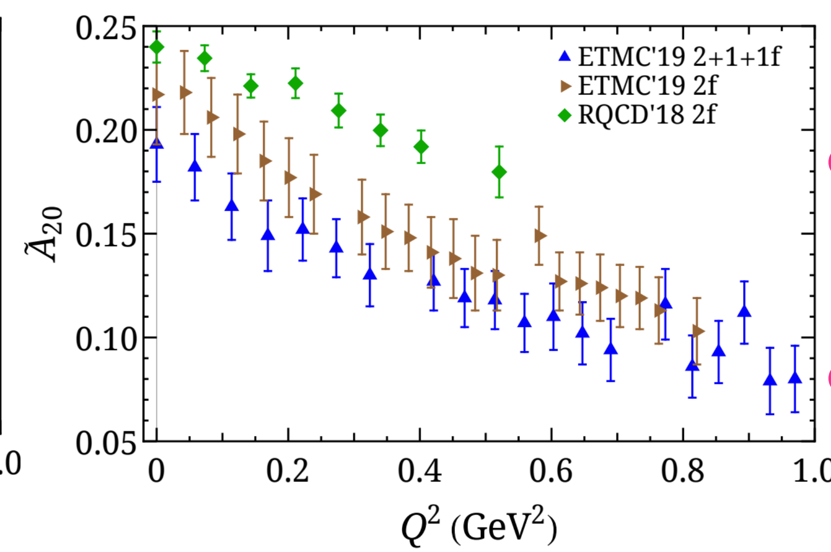

- Lesser studied compared to FFs at physical point
- Decay of signal-to-noise ratio

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GPDs

Through non-local matrix elements

of fast-moving hadrons

Access of GPDs on a Euclidean Lattice

[X. Ji, Phys. Rev. Lett. 110 (2013) 262002]

Matrix elements of nonlocal (equal-time) operators with fast moving hadrons

$$\tilde{q}_{\Gamma}^{\mathrm{GPD}}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \quad \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathcal{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_{\mu}$$

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$

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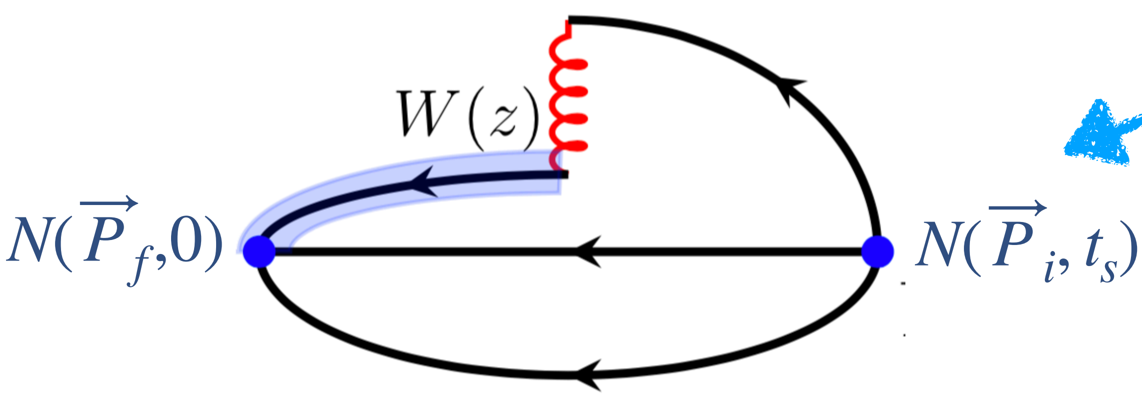

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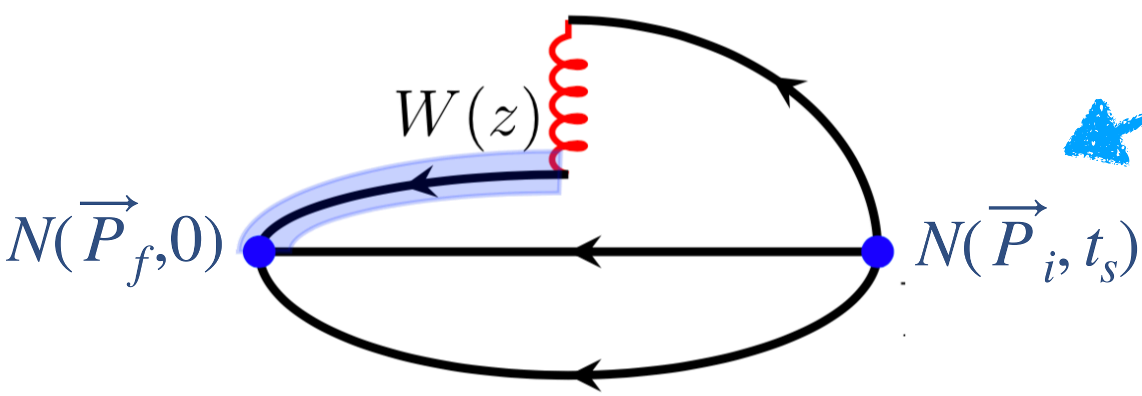

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Variables of the calculation:

- length of the Wilson line (z)
- nucleon momentum boost (P_3)
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- skewness (ξ)

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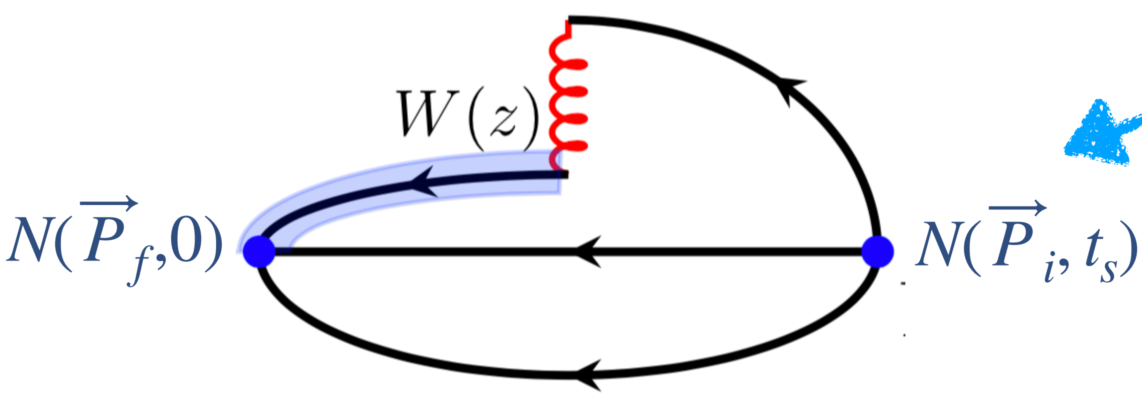

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Such matrix elements may be analyzed through LaMET formalism (quasi—GPDs) or coordinate space factorization (pseudo-ITD)

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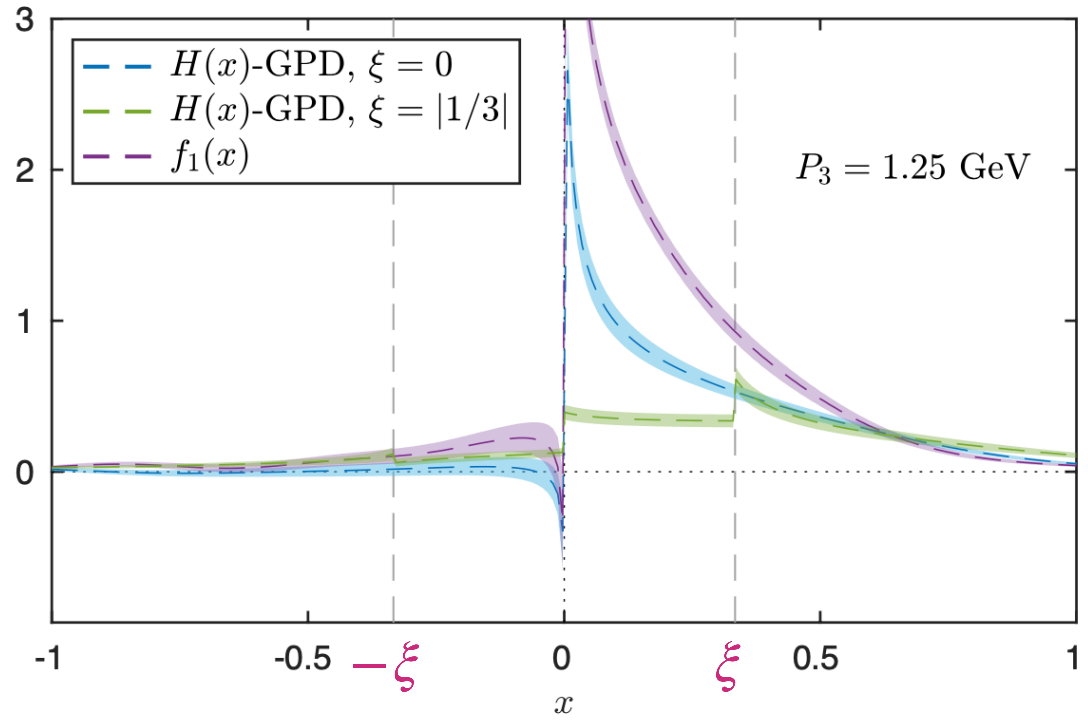

[C. Alexandrou et al., PRL 125, 262001 (2020)]

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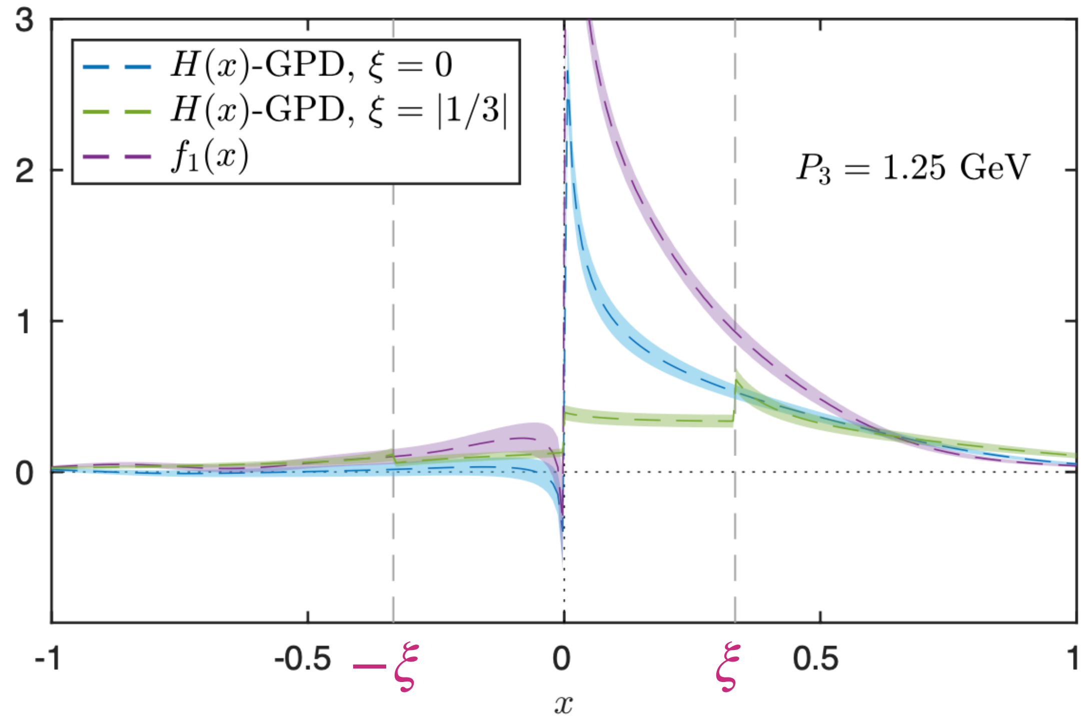

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- ★ ERBL/DGLAP: Qualitative differences
- ★ $\xi = \pm\, x$ inaccessible (formalism breaks down)
- ★ $x \rightarrow 1$ region: qualitatively comparison with power counting analysis [F. Yuan, PRD69 (2004) 051501, hep-ph/0311288]

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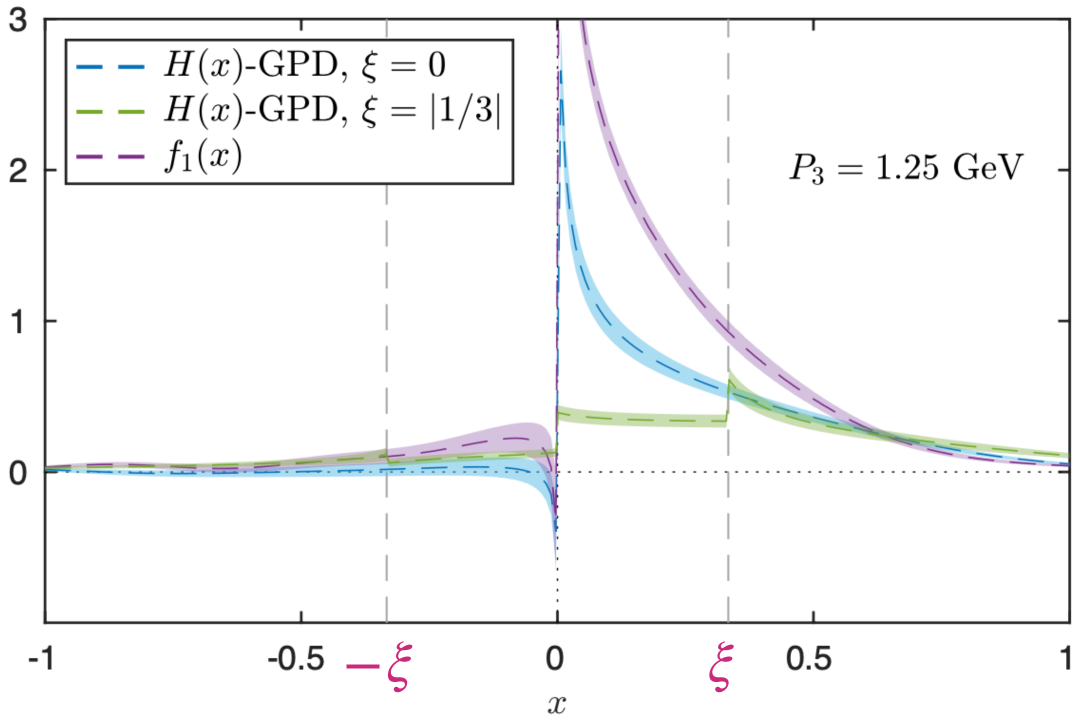

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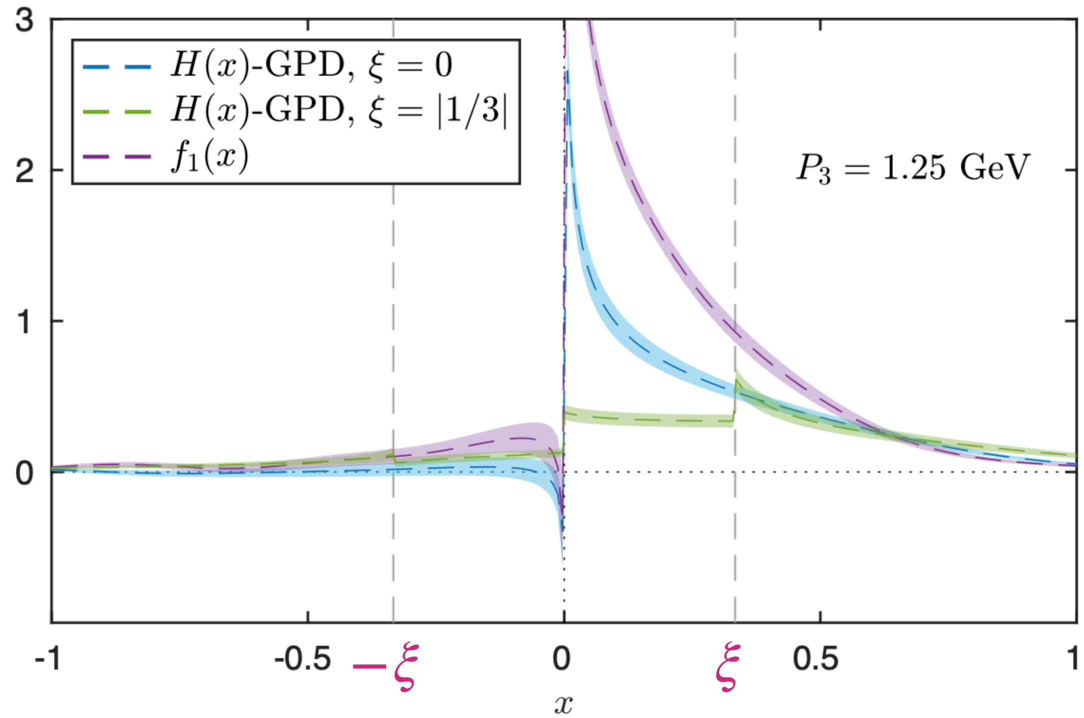

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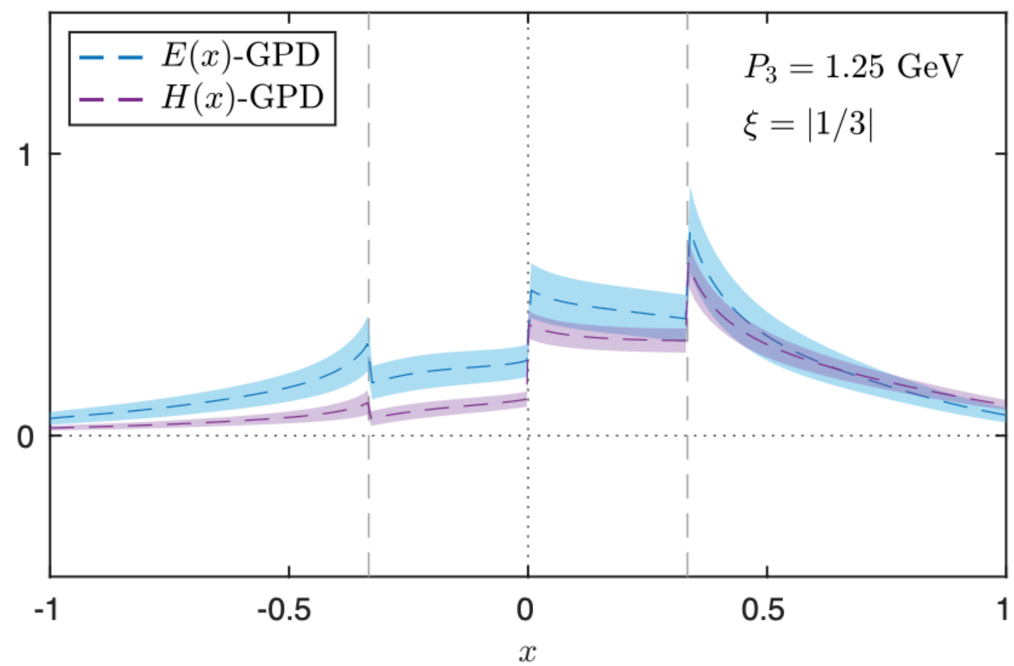

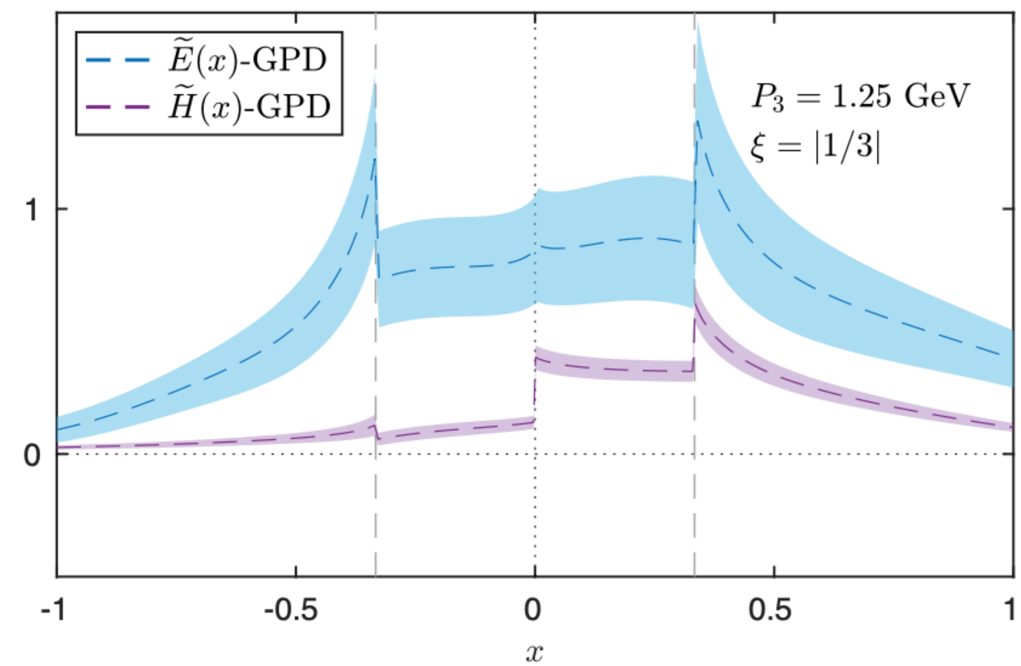

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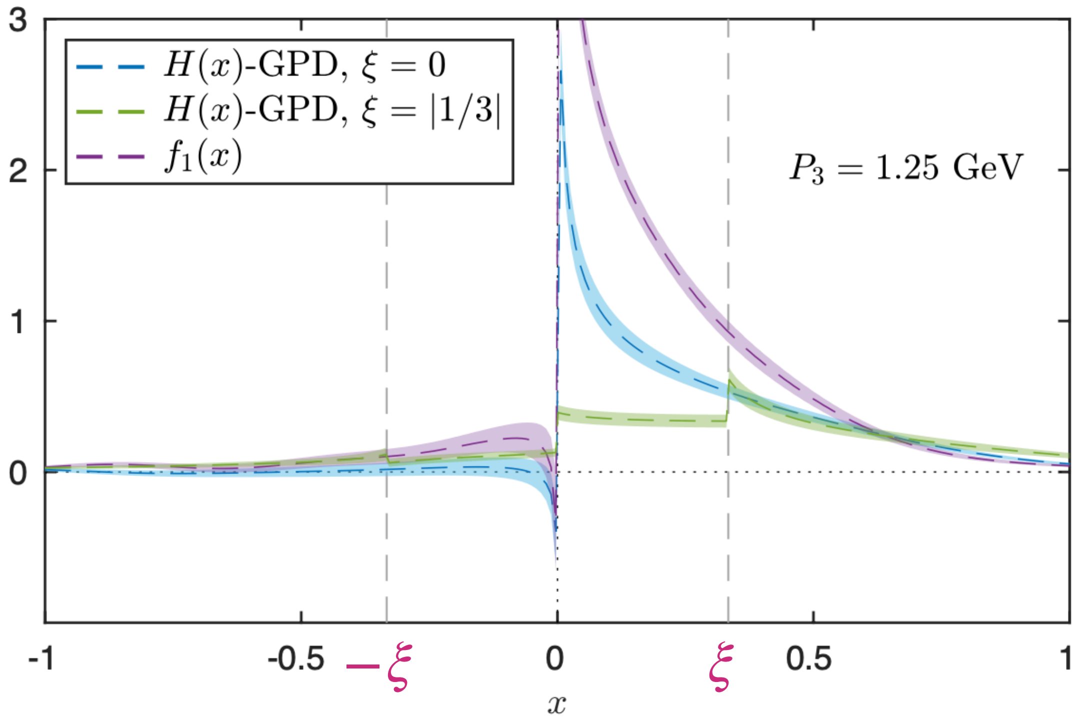

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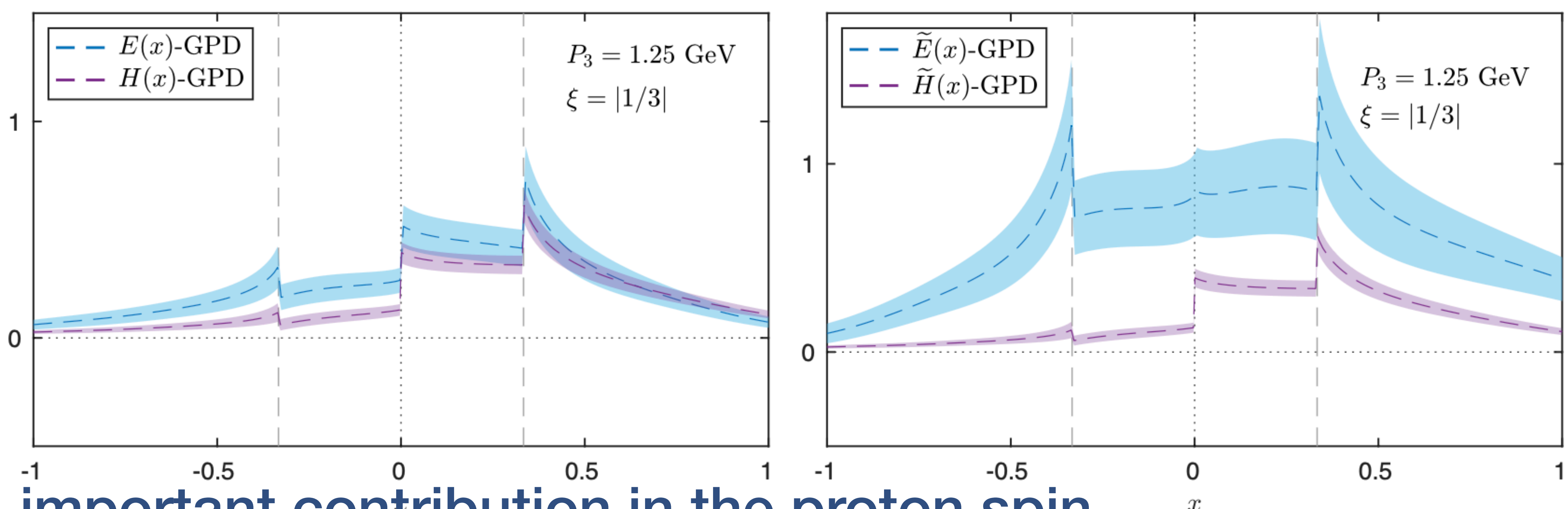

- ★ important contribution in the proton spin

$$\int_{-1}^{+1} dx x^2 H^q(x,\xi,t) = A_{20}^q(t) + 4\xi^2 C_{20}^q(t)\,, \qquad \int_{-1}^{+1} dx x^2 E^q(x,\xi,t) = B_{20}^q(t) - 4\xi^2 C_{20}^q(t)$$

What can we currently learn from lattice results?

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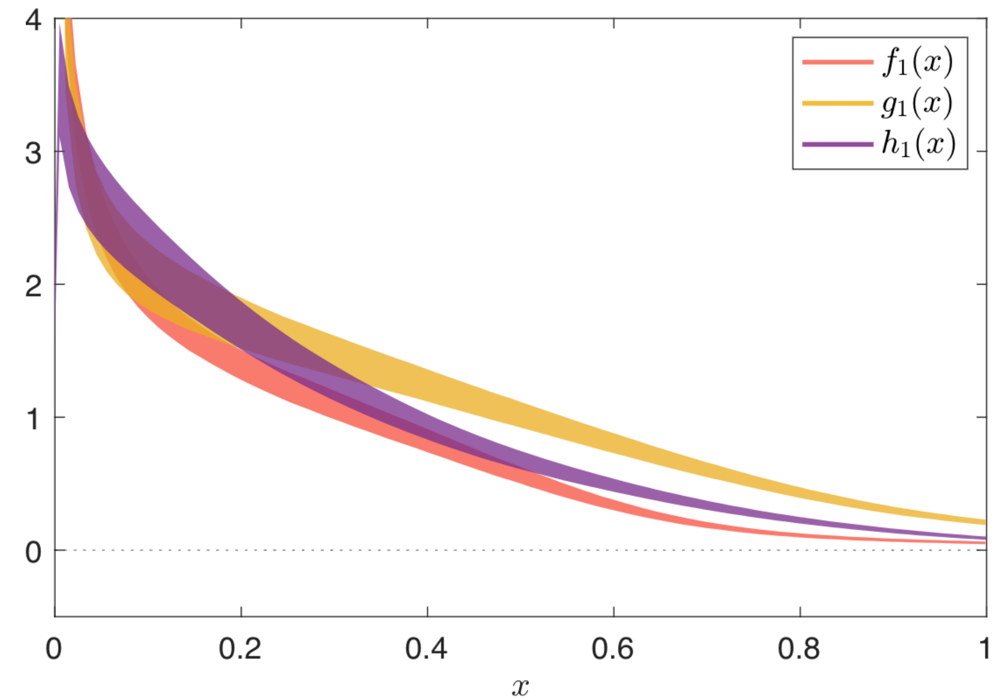
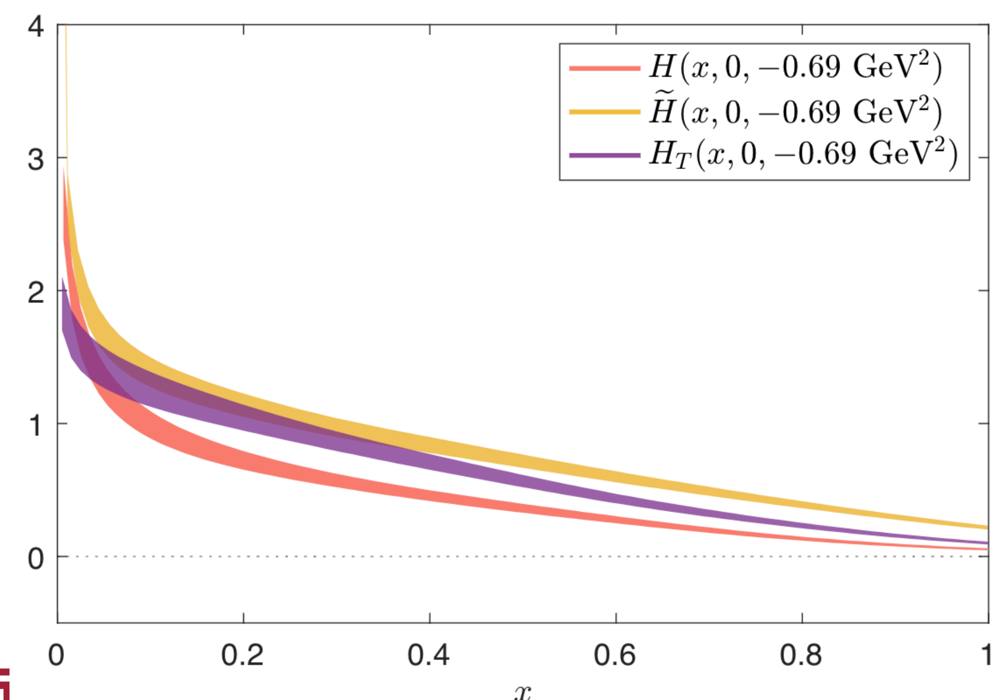
★ Qualitative understanding of GPDs and their relations

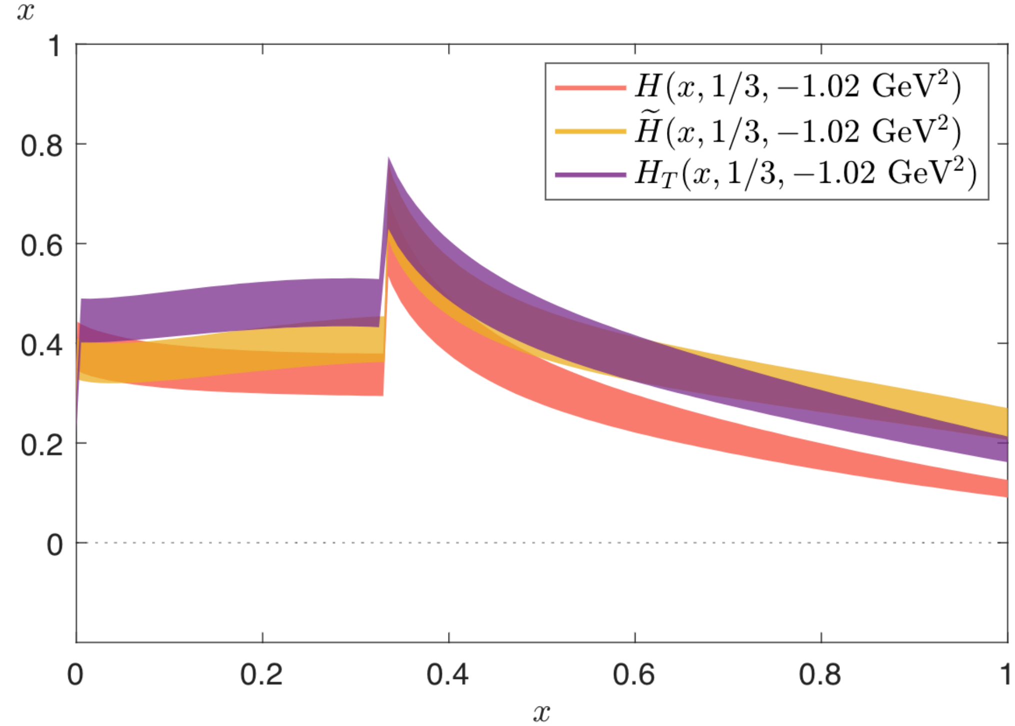
★ Qualitative understanding of ERBL and DGLAP regions



★ Relations can be identified for the t-dependence of GPDs

[C. Alexandrou et al., PRD 105, 034501 (2022)]





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★ Lattice data on transversity GPDs

$$\int_{-2}^{2} dx H_{Tq}(x,0,-0.69\,\mathrm{GeV}^2,P_3)=\{0.65(4),\,0.64(6),\,0.81(10)\}\,,\quad \int_{-2}^{2} dx H_{Tq}(x,\tfrac{1}{3},-1.02\,\mathrm{GeV}^2,1.25\,\mathrm{GeV})=0.49(5)\,,$$

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- lowest moments the same between quasi-GPDs and GPDs
- Values of moments decrease as t increases
- Higher moments suppressed compared to the lowest

What possible extensions can we achieve?

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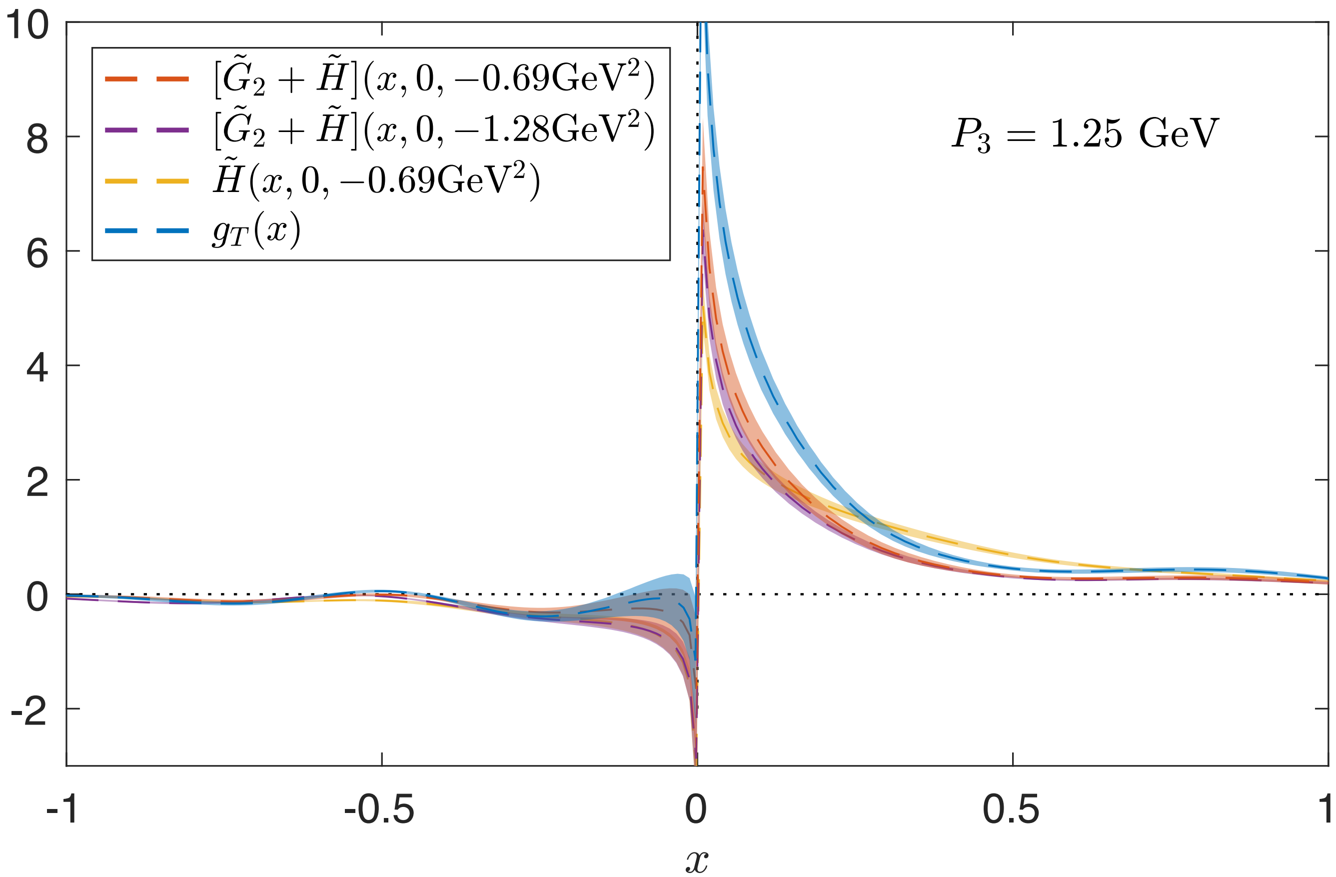
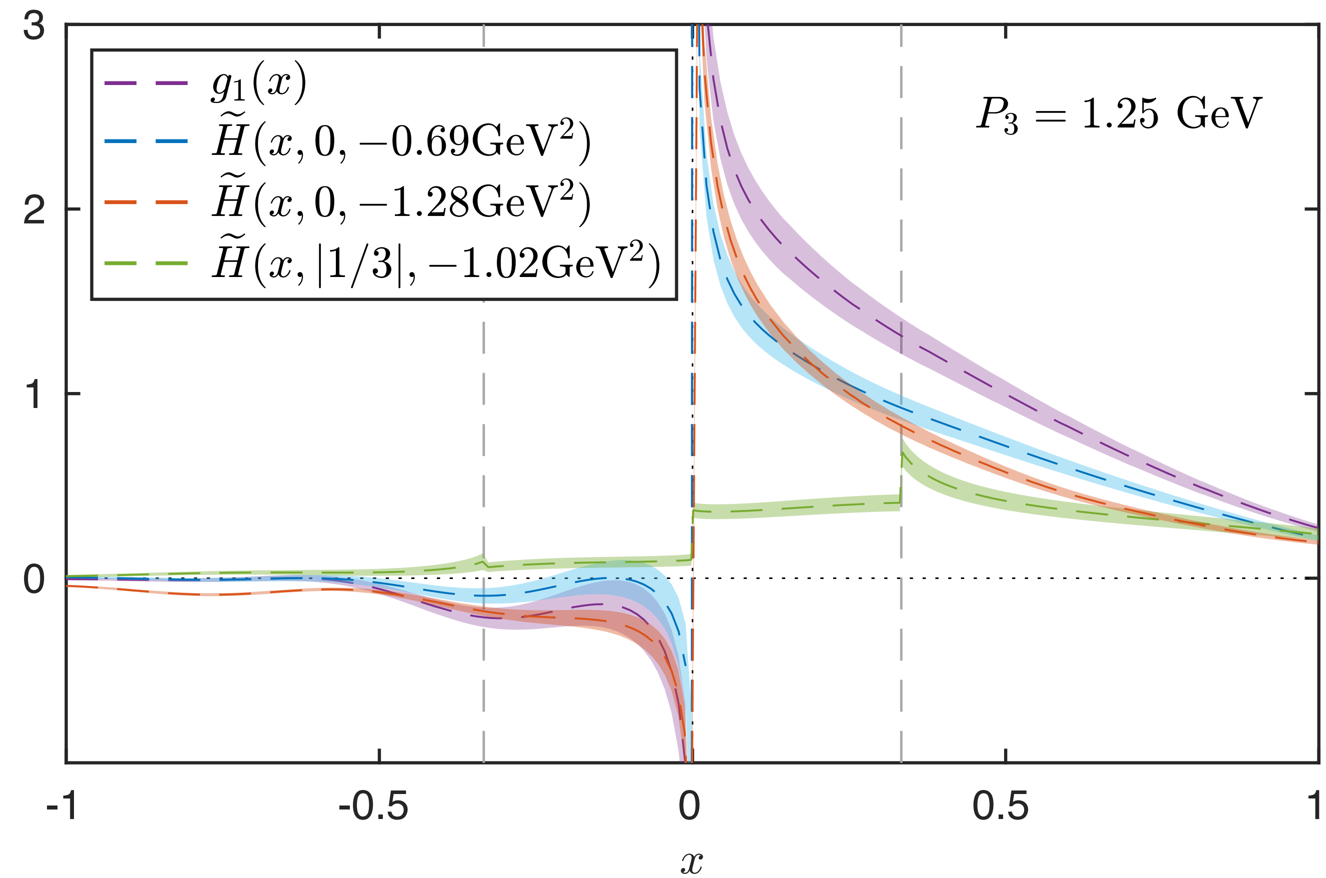
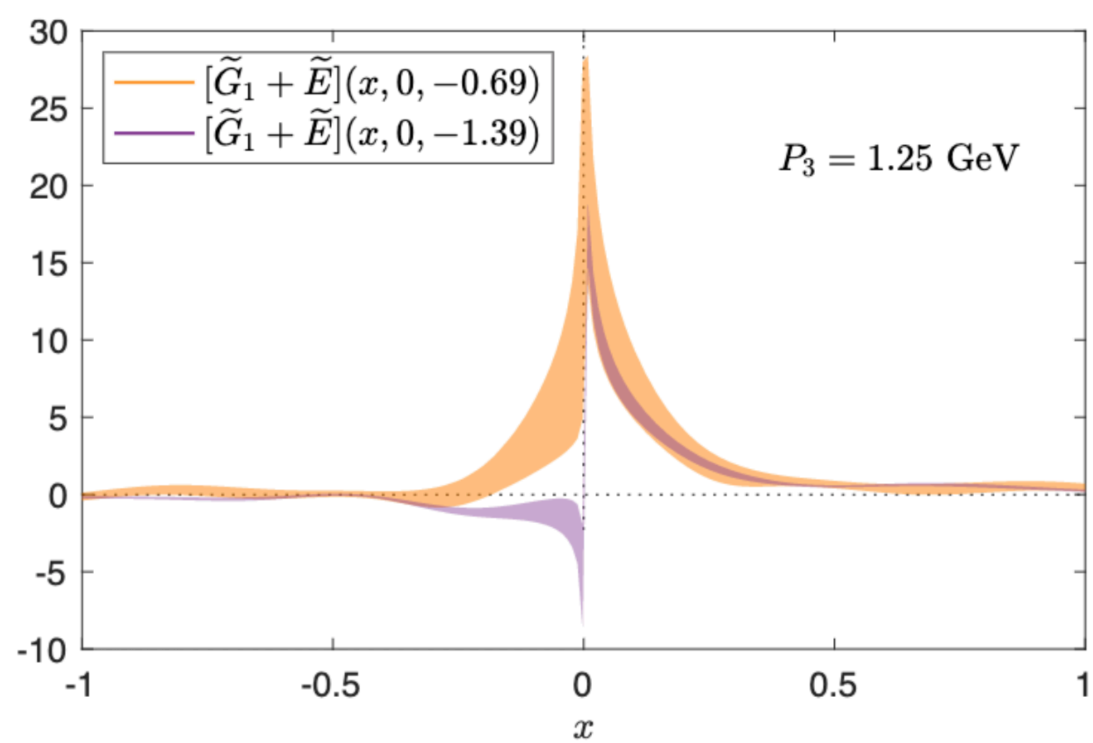
★ Twist-3 GPDs

PRELIMINARY





[S. Bhattacharya et al., PoS LATTICE2021 (2022) 054 arXiv:2112.05538]



★ $g_T(x)$: dominant distribution

★ $\widetilde{H} + \widetilde{G}_2$ similar in magnitude to $\widetilde{H}$

★ $\widetilde{G}_2$ is expected to be small

Definition of GPDs in Euclidean lattice

- ★ Calculation expected to be performed in symmetric frame to extract the "standard" GPDs
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Extraction of GPDs in the symmetric frame using lattice correlators calculated in non-symmetric frames

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1st goal:

Extraction of GPDs in the symmetric frame using lattice correlators calculated in non-symmetric frames

2nd goal:

New definition of Lorentz covariant quasi-GPDs that may have faster convergence to light-cone GPDs (elimination of kinematic corrections)

Theoretical setup

[S. Bhattacharya et al., arXiv:2209.05373]

★ Parametrization of matrix elements in Lorentz invariant amplitudes

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^{\mu}}{M}A_1 + z^{\mu}\,MA_2 + \frac{\Delta^{\mu}}{M}A_3 + i\sigma^{\mu z}\,MA_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^{\mu}i\sigma^{z\Delta}}{M}A_6 + \frac{z^{\mu}i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

Advantages

- Applicable to any kinematic frame and have definite symmetries
- Lorentz invariant amplitudes A_i can be related to the standard H, E GPDs
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★ Proof-of-concept calculation (zero quasi-skewness):

- symmetric frame: $\vec{p}^{\,s}_f = \vec{P} + \frac{\vec{Q}}{2}$, $\quad \vec{p}^{\,s}_i = \vec{P} - \frac{\vec{Q}}{2}$ $\quad t^s = -\vec{Q}^2$

- asymmetric frame: $\vec{p}^{\,a}_f = \vec{P}$, $\quad \vec{p}^{\,a}_i = \vec{P} - \vec{Q}$ $\quad t^a = -\vec{Q}^2 + (E_f - E_i)^2$

Matrix element decomposition

Symmetric

$$C_s = \frac{2m^2}{E(E+m)}$$

$$\Gamma_0 = \frac{1}{2}(1+\gamma^0)$$

$$\Gamma_j = \frac{i}{4}(1+\gamma^0)\gamma^5\gamma^j \quad (j = 1,2,3)$$

$$\Pi_s^0(\Gamma_0) = C_s\left(\frac{E\left(E(E+m)-P_3^2\right)}{2m^3}A_1 + \frac{(E+m)\left(-E^2+m^2+P_3^2\right)}{m^3}A_5 + \frac{EP_3\left(-E^2+m^2+P_3^2\right)z}{m^3}A_6\right)$$

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$$\Pi_s^0(\Gamma_2) = i\,C_s\left(-\frac{EP_3Q_1}{4m^3}A_1 + \frac{(E+m)P_3Q_1}{2m^3}A_5 + \frac{E\left(P_3^2+m(E+m)\right)zQ_1}{2m^3}A_6\right)$$

Asymmetric

$$C_a = \frac{2m^2}{\sqrt{E_iE_f(E_i+m)(E_f+m)}}$$

$$\begin{aligned}\Pi_0^a(\Gamma_0) = C_a\Bigg(&-\frac{(E_f+E_i)(E_f-E_i-2m)(E_f+m)}{8m^3}A_1 - \frac{(E_f-E_i-2m)(E_f+m)(E_f-E_i)}{4m^3}A_3\\ &+\frac{(E_i-E_f)P_3z}{4m}A_4 + \frac{(E_f+E_i)(E_f+m)(E_f-E_i)}{4m^3}A_5 + \frac{E_f(E_f+E_i)P_3(E_f-E_i)z}{4m^3}A_6\\ &+\frac{E_fP_3(E_f-E_i)^2z}{2m^3}A_8\Bigg)\end{aligned}$$

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Novel feature: z-dependence

Asymmetric

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$$\Pi^a_0(\Gamma_0) = C_a\left(-\frac{(E_f+E_i)(E_f-E_i-2m)(E_f+m)}{8m^3}A_1 - \frac{(E_f-E_i-2m)(E_f+m)(E_f-E_i)}{4m^3}A_3 + \frac{(E_i-E_f)P_3z}{4m}A_4 + \frac{(E_f+E_i)(E_f+m)(E_f-E_i)}{4m^3}A_5 + \frac{E_f(E_f+E_i)P_3(E_f-E_i)z}{4m^3}A_6 + \frac{E_fP_3(E_f-E_i)^2z}{2m^3}A_8\right)$$

$$\Pi^a_0(\Gamma_1) = i\,C_a\left(\frac{(E_f+E_i)P_3Q_2}{8m^3}A_1 + \frac{(E_f-E_i)P_3Q_2}{4m^3}A_3 + \frac{(E_f+m)Q_2z}{4m}A_4 - \frac{(E_f+E_i+2m)P_3Q_2}{4m^3}A_5 - \frac{E_f(E_f+E_i)(E_f+m)Q_2z}{4m^3}A_6 - \frac{E_f(E_f-E_i)(E_f+m)Q_2z}{2m^3}A_8\right)$$

$$\Pi^a_0(\Gamma_2) = i\,C_a\left(-\frac{(E_f+E_i)P_3Q_1}{8m^3}A_1 - \frac{(E_f-E_i)P_3Q_1}{4m^3}A_3 - \frac{(E_f+m)Q_1z}{4m}A_4 + \frac{(E_f+E_i+2m)P_3Q_1}{4m^3}A_5 + \frac{E_f(E_f+E_i)(E_f+m)Q_1z}{4m^3}A_6 + \frac{E_f(E_f-E_i)(E_f+m)Q_1z}{2m^3}A_8\right)$$

Matrix element decomposition

Symmetric

$$C_s = \frac{2m^2}{E(E+m)}$$

$$\Gamma_0 = \frac{1}{2}(1+\gamma^0)$$

$$\Gamma_j = \frac{i}{4}(1+\gamma^0)\gamma^5\gamma^j \quad (j = 1,2,3)$$

$$\Pi_s^0(\Gamma_0) = C_s\left(\frac{E\left(E(E+m)-P_3^2\right)}{2m^3}A_1 + \frac{(E+m)\left(-E^2+m^2+P_3^2\right)}{m^3}A_5 + \frac{EP_3\left(-E^2+m^2+P_3^2\right)z}{m^3}A_6\right)$$

$$\Pi_s^0(\Gamma_1) = i\,C_s\left(\frac{EP_3Q_2}{4m^3}A_1 - \frac{(E+m)P_3Q_2}{2m^3}A_5 - \frac{E\left(P_3^2+m(E+m)\right)zQ_2}{2m^3}A_6\right)$$

$$\Pi_s^0(\Gamma_2) = i\,C_s\left(-\frac{EP_3Q_1}{4m^3}A_1 + \frac{(E+m)P_3Q_1}{2m^3}A_5 + \frac{E\left(P_3^2+m(E+m)\right)zQ_1}{2m^3}A_6\right)$$

Novel feature: z-dependence

Asymmetric

$$C_a = \frac{2m^2}{\sqrt{E_iE_f(E_i+m)(E_f+m)}}$$

$$\Pi_0^a(\Gamma_0) = C_a\Bigg(-\frac{(E_f+E_i)(E_f-E_i-2m)(E_f+m)}{8m^3}A_1 - \frac{(E_f-E_i-2m)(E_f+m)(E_f-E_i)}{4m^3}A_3$$
$$+\frac{(E_i-E_f)P_3z}{4m}A_4 + \frac{(E_f+E_i)(E_f+m)(E_f-E_i)}{4m^3}A_5 + \frac{E_f(E_f+E_i)P_3(E_f-E_i)z}{4m^3}A_6$$
$$+\frac{E_fP_3(E_f-E_i)^2z}{2m^3}A_8\Bigg)$$

$$\Pi_0^a(\Gamma_1) = i\,C_a\Bigg(\frac{(E_f+E_i)P_3Q_2}{8m^3}A_1 + \frac{(E_f-E_i)P_3Q_2}{4m^3}A_3 + \frac{(E_f+m)Q_2z}{4m}A_4 - \frac{(E_f+E_i+2m)P_3Q_2}{4m^3}A_5$$
$$-\frac{E_f(E_f+E_i)(E_f+m)Q_2z}{4m^3}A_6 - \frac{E_f(E_f-E_i)(E_f+m)Q_2z}{2m^3}A_8\Bigg)$$

No definite symmetries for Π_μ^a

$$\Pi_0^a(\Gamma_2) = i\,C_a\Bigg(-\frac{(E_f+E_i)P_3Q_1}{8m^3}A_1 - \frac{(E_f-E_i)P_3Q_1}{4m^3}A_3 - \frac{(E_f+m)Q_1z}{4m}A_4 + \frac{(E_f+E_i+2m)P_3Q_1}{4m^3}A_5$$
$$+\frac{E_f(E_f+E_i)(E_f+m)Q_1z}{4m^3}A_6 + \frac{E_f(E_f-E_i)(E_f+m)Q_1z}{2m^3}A_8\Bigg)$$

Lorentz-Invariant amplitudes

Symmetric

$$A_1 = \frac{(m(E+m)+P_3^2)}{E(E+m)}\Pi_0^s(\Gamma_0) - i\,\frac{P_3 Q_1}{2E(E+m)}\Pi_0^s(\Gamma_2) - \frac{Q_1}{2E}\Pi_2^s(\Gamma_3)$$

$$A_5 = -\frac{E}{Q_1}\Pi_2^s(\Gamma_3)$$

$$A_6 = \frac{P_3}{2Ez(E+m)}\Pi_0^s(\Gamma_0) + i\,\frac{(P_3^2 - E(E+m))}{EQ_1 z(E+m)}\Pi_0^s(\Gamma_2) + \frac{P_3}{EQ_1 z}\Pi_2^s(\Gamma_3)$$

Asymmetric

$$A_1 = \frac{2m^2}{E_f(E_i+m)}\frac{\Pi_0^a(\Gamma_0)}{C_a} + i\,\frac{2(E_f-E_i)P_3 m^2}{E_f(E_f+m)(E_i+m)Q_1}\frac{\Pi_0^a(\Gamma_2)}{C_a} + \frac{2(E_i-E_f)P_3 m^2}{E_f(E_f+E_i)(E_f+m)(E_i+m)}\frac{\Pi_1^a(\Gamma_2)}{C_a}$$
$$+i\,\frac{2(E_i-E_f)m^2}{E_f(E_i+m)Q_1}\frac{\Pi_1^a(\Gamma_0)}{C_a} + \frac{2(E_i-E_f)P_3 m^2}{E_f(E_f+E_i)(E_f+m)(E_i+m)}\frac{\Pi_2^a(\Gamma_1)}{C_a} + \frac{2(E_f-E_i)m^2}{E_f(E_i+m)Q_1}\frac{\Pi_2^a(\Gamma_3)}{C_a}$$

$$A_5 = \frac{m^2 P_3}{E_f(E_f+m)(E_i+m)}\frac{\Pi_2^a(\Gamma_1)}{C_a} - \frac{(E_f+E_i)m^2}{E_f(E_i+m)Q_1}\frac{\Pi_2^a(\Gamma_3)}{C_a}$$

$$A_6 = \frac{P_3 m^2}{E_f^2(E_f+m)(E_i+m)z}\frac{\Pi_0^a(\Gamma_0)}{C_a} + i\,\frac{(E_f-E_i-2m)m^2}{E_f^2(E_i+m)Q_1 z}\frac{\Pi_0^a(\Gamma_2)}{C_a} + i\,\frac{(E_i-E_f)P_3 m^2}{E_f^2(E_f+m)(E_i+m)Q_1 z}\frac{\Pi_1^a(\Gamma_0)}{C_a}$$
$$+\frac{(-E_f+E_i+2m)m^2}{E_f^2(E_f+E_i)(E_i+m)z}\frac{\Pi_1^a(\Gamma_2)}{C_a} + \frac{2(m-E_f)m^2}{E_f^2(E_f+E_i)(E_i+m)z}\frac{\Pi_2^a(\Gamma_1)}{C_a} + \frac{2P_3 m^2}{E_f^2(E_i+m)Q_1 z}\frac{\Pi_2^a(\Gamma_3)}{C_a}$$

- ★ Asymmetric frame equations more complex
- ★ A_i have definite symmetries
- ★ System of 8 independent matrix elements to disentangle the A_i

Lorentz-Invariant amplitudes

Symmetric

$$A_1 = \frac{(m(E+m)+P_3^2)}{E(E+m)}\Pi_0^s(\Gamma_0) - i\,\frac{P_3 Q_1}{2E(E+m)}\Pi_0^s(\Gamma_2) - \frac{Q_1}{2E}\Pi_2^s(\Gamma_3)$$

$$A_5 = -\frac{E}{Q_1}\Pi_2^s(\Gamma_3)$$

$$A_6 = \frac{P_3}{2Ez(E+m)}\Pi_0^s(\Gamma_0) + i\,\frac{(P_3^2 - E(E+m))}{EQ_1 z(E+m)}\Pi_0^s(\Gamma_2) + \frac{P_3}{EQ_1 z}\Pi_2^s(\Gamma_3)$$

$=$ $=$ $=$

$$A_1 = \frac{2m^2}{E_f(E_i+m)}\frac{\Pi_0^a(\Gamma_0)}{C_a} + i\,\frac{2(E_f-E_i)P_3m^2}{E_f(E_f+m)(E_i+m)Q_1}\frac{\Pi_0^a(\Gamma_2)}{C_a} + \frac{2(E_i-E_f)P_3m^2}{E_f(E_f+E_i)(E_f+m)(E_i+m)}\frac{\Pi_1^a(\Gamma_2)}{C_a}$$
$$+i\,\frac{2(E_i-E_f)m^2}{E_f(E_i+m)Q_1}\frac{\Pi_1^a(\Gamma_0)}{C_a} + \frac{2(E_i-E_f)P_3m^2}{E_f(E_f+E_i)(E_f+m)(E_i+m)}\frac{\Pi_2^a(\Gamma_1)}{C_a} + \frac{2(E_f-E_i)m^2}{E_f(E_i+m)Q_1}\frac{\Pi_2^a(\Gamma_3)}{C_a}$$

$$A_5 = \frac{m^2 P_3}{E_f(E_f+m)(E_i+m)}\frac{\Pi_2^a(\Gamma_1)}{C_a} - \frac{(E_f+E_i)m^2}{E_f(E_i+m)Q_1}\frac{\Pi_2^a(\Gamma_3)}{C_a}$$

$$A_6 = \frac{P_3m^2}{E_f^2(E_f+m)(E_i+m)z}\frac{\Pi_0^a(\Gamma_0)}{C_a} + i\,\frac{(E_f-E_i-2m)m^2}{E_f^2(E_i+m)Q_1 z}\frac{\Pi_0^a(\Gamma_2)}{C_a} + i\,\frac{(E_i-E_f)P_3m^2}{E_f^2(E_f+m)(E_i+m)Q_1 z}\frac{\Pi_1^a(\Gamma_0)}{C_a}$$
$$+\frac{(-E_f+E_i+2m)m^2}{E_f^2(E_f+E_i)(E_i+m)z}\frac{\Pi_1^a(\Gamma_2)}{C_a} + \frac{2(m-E_f)m^2}{E_f^2(E_f+E_i)(E_i+m)z}\frac{\Pi_2^a(\Gamma_1)}{C_a} + \frac{2P_3m^2}{E_f^2(E_i+m)Q_1 z}\frac{\Pi_2^a(\Gamma_3)}{C_a}$$

- ★ Asymmetric frame equations more complex
- ★ A_i have definite symmetries
- ★ System of 8 independent matrix elements to disentangle the A_i

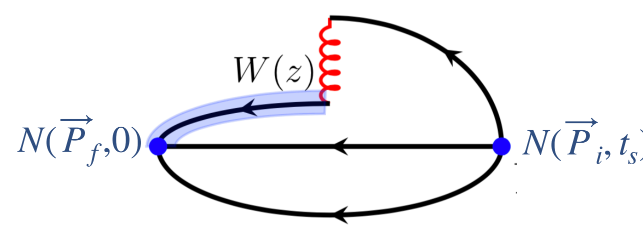
Parameters of calculation

★ Nf=2+1+1 twisted mass (TM) fermions & clover improvement

Pion mass:	260 MeV
Lattice spacing:	0.093 fm
Volume:	32^3 x 64
Spatial extent:	3 fm

★ Calculation:

- isovector combination
- zero skewness
- T_{sink}=1 fm



frame	P_3 [GeV]	$\mathbf{Q}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
symm	1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
non-symm	1.25	(±2,0,0), (0,±2,0)	0.64	0	8	269	8	17216

★ Computational cost:

- symmetric frame 4 times more expensive than asymmetric frame for same set of $\vec{Q}$ (requires separate calculations at each t)

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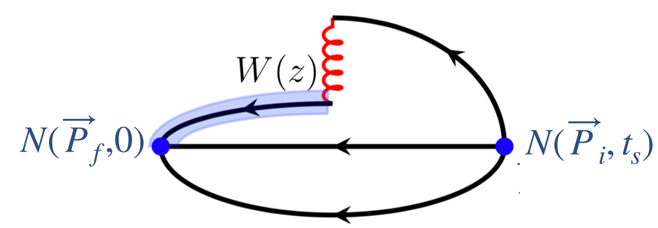

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Small difference: $t^s = -\vec{Q}^2$ $\quad t^a = -\vec{Q}^2 + (E_f - E_i)^2$

$$A(-0.64\text{GeV}^2) \sim A(-0.69\text{GeV}^2)$$

★ Computational cost:

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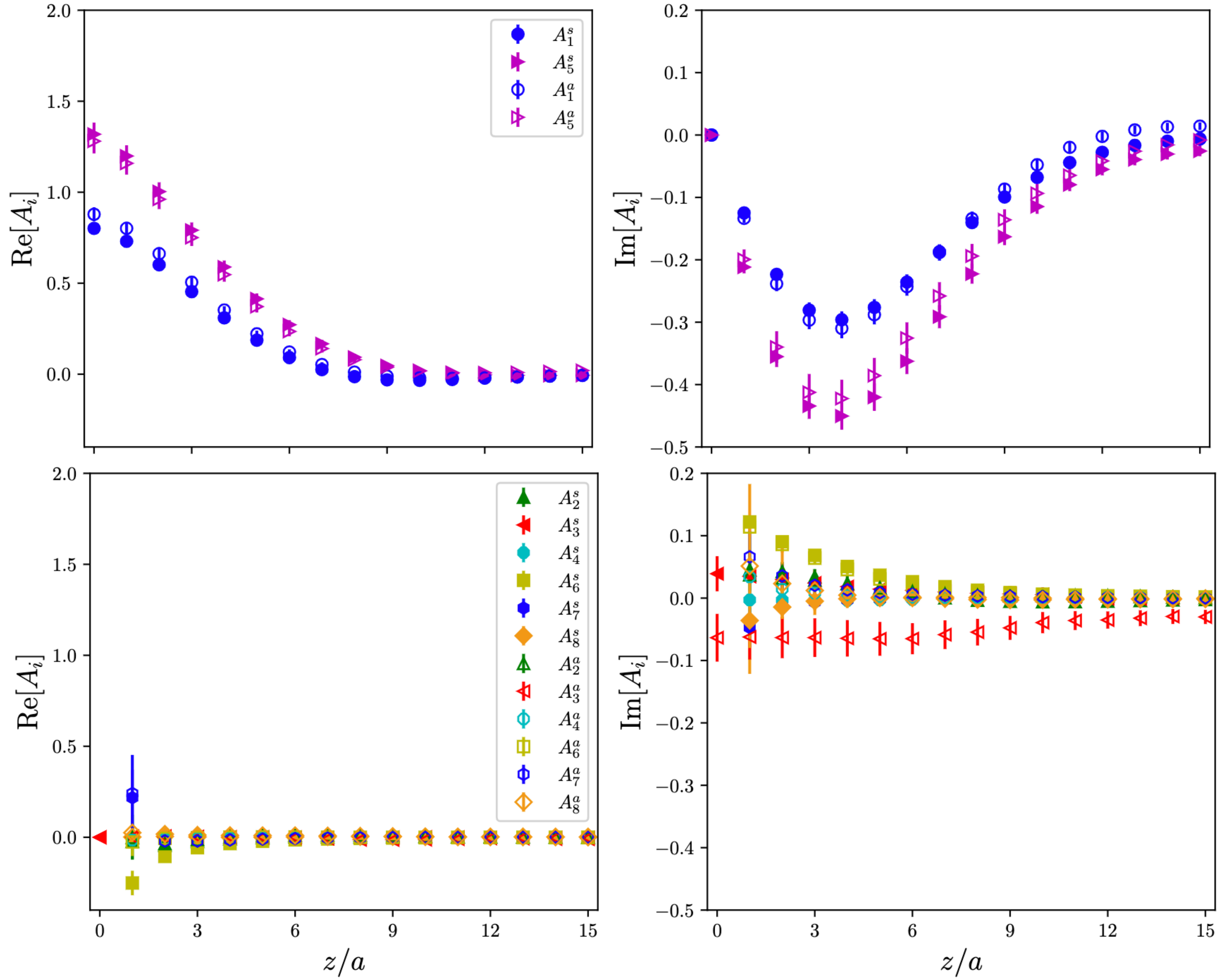
Results: A_i



- ★ A_1, A_5 dominant contributions
- ★ Full agreement in two frames for both Re and Im parts of A_1, A_5
- ★ Remaining A_i suppressed (at least for this kinematic setup and $\xi = 0$)

Π_H, Π_E in terms of A_i

★ Mapping of $\{\Pi_H, \Pi_E\}$ to A_i using $F^{[\gamma^0]} \sim \left[\gamma^0 H_{\mathrm{Q(0)}}(x,\xi,t;P^3) + \frac{i\sigma^{0\mu}\Delta_\mu}{2M} E_{\mathrm{Q(0)}}(x,\xi,t;P^3)\right]$ in each frame leading to frame dependent relations:

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$(\xi = 0)$

$$\Pi^s_H = A_1 + \frac{zQ_1^2}{2P_3} A_6$$

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$$\Pi^a_H = A_1 + \frac{Q_0}{P_0} A_3 + \frac{m^2 z Q_0}{2P_0 P_3} A_4 + \frac{z(Q_0^2 + Q_\perp^2)}{2P_3} A_6 + \frac{z(Q_0^3 + Q_0 Q_\perp^2)}{2P_0 P_3} A_8$$

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★ **Definition of Lorentz invariant Π_H & Π_E**

$(\xi = 0)$

$$\Pi_H^{\text{impr}} = A_1$$

$$\Pi_E^{\text{impr}} = -A_1 + 2A_5 + 2zP_3 A_6$$

Π_H, Π_E in terms of A_i

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1st approach: extraction of $\{\Pi_H^s, \Pi_E^s\}$ using A_i from any frame (universal)

$$\Pi_H^a = A_1 + \frac{Q_0}{P_0} A_3 + \frac{m^2 z Q_0}{2P_0P_3} A_4 + \frac{z(Q_0^2 + Q_\perp^2)}{2P_3} A_6 + \frac{z(Q_0^3 + Q_0 Q_\perp^2)}{2P_0P_3} A_8$$

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2nd approach: extraction of $\{\Pi_H, \Pi_E\}$ from a purely asymmetric frame; GPDs differ in functional form from $\{\Pi_H^s, \Pi_E^s\}$

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1st approach: extraction of $\{\Pi_H^s, \Pi_E^s\}$ using A_i from any frame (universal)

$$\Pi_H^a = A_1 + \frac{Q_0}{P_0} A_3 + \frac{m^2 z Q_0}{2P_0P_3} A_4 + \frac{z(Q_0^2 + Q_\perp^2)}{2P_3} A_6 + \frac{z(Q_0^3 + Q_0 Q_\perp^2)}{2P_0P_3} A_8$$

$$\Pi_E^a = -A_1 - \frac{Q_0}{P_0} A_3 - \frac{m^2 z (Q_0 + 2P_0)}{2P_0P_3} A_4 + 2A_5 - \frac{z\left(Q_0^2 + 2P_0Q_0 + 4P_0^2 + Q_\perp^2\right)}{2P_3} A_6 - \frac{zQ_0\left(Q_0^2 + 2Q_0P_0 + 4P_0^2 + Q_\perp^2\right)}{2P_0P_3} A_8$$

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$$\Pi_H^{\text{impr}} = A_1$$

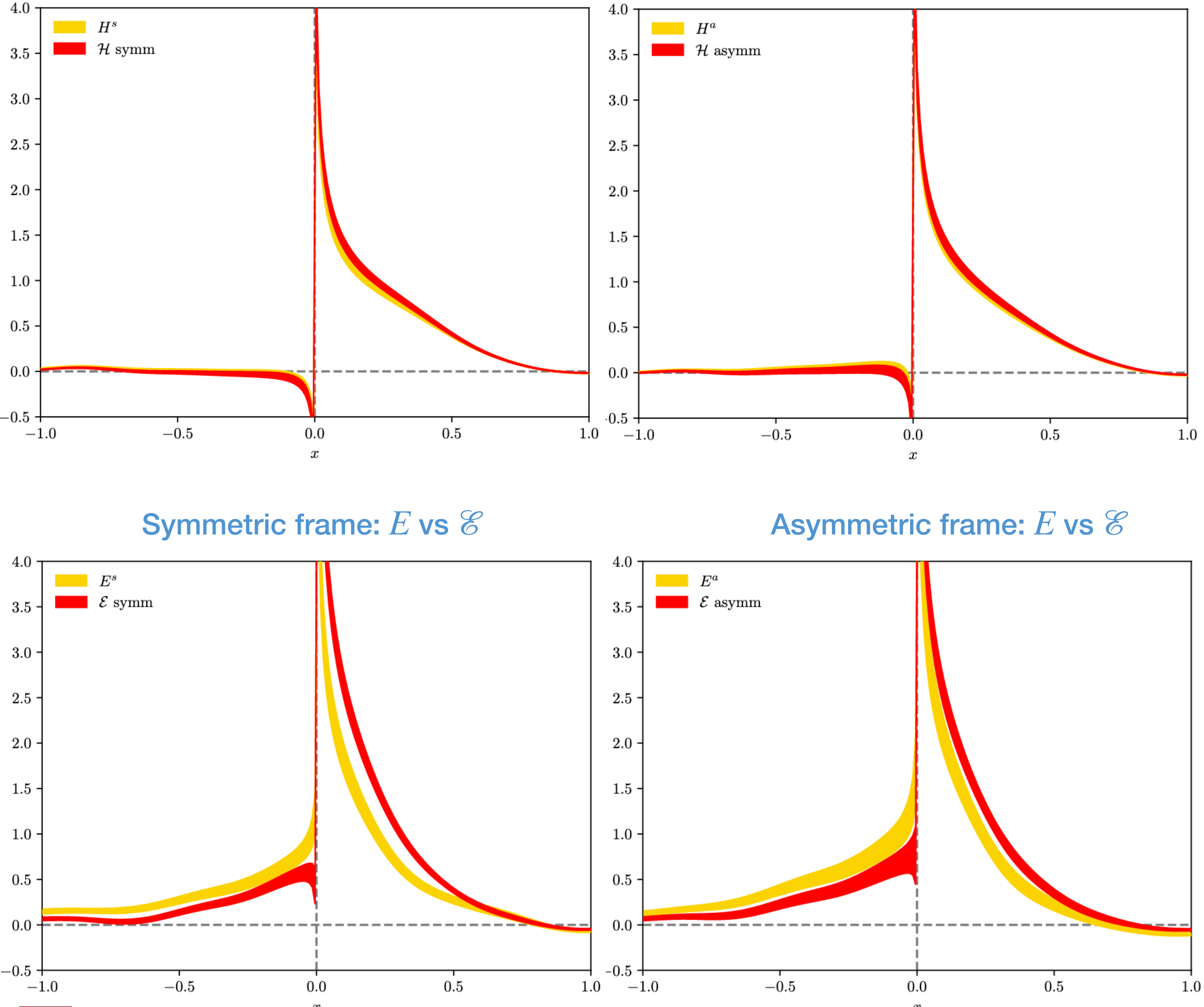
$$\Pi_E^{\text{impr}} = -A_1 + 2A_5 + 2zP_3A_6$$

3rd approach: use redefined Lorentz covariant $\{\Pi_H, \Pi_E\}$ in desired frame

Results: H − GPD

Symmetric frame: H vs $\mathscr{H}$ | Asymmetric frame: H vs $\mathscr{H}$

Symmetric frame: E vs $\mathscr{E}$ | Asymmetric frame: E vs $\mathscr{E}$

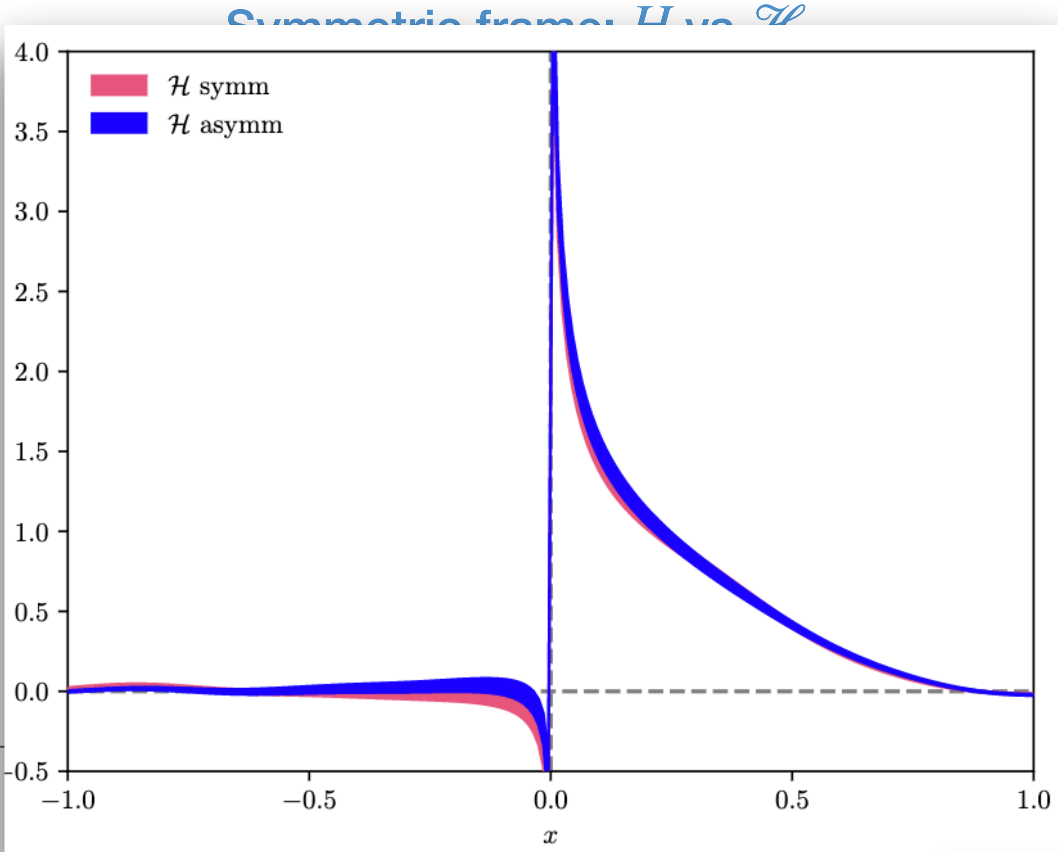


Similar results for H and $\mathscr{H}$ for both frames (agreement not by construction)

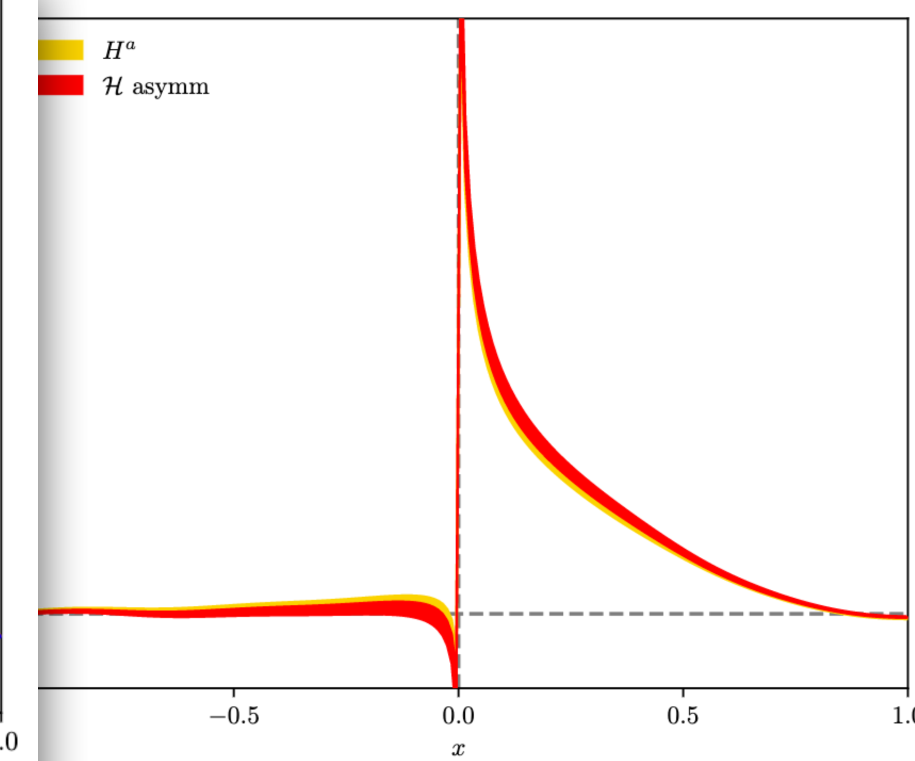
Differences between E and $\mathscr{E}$ for both frames (agreement not by construction)

Results: H – GPD

Symmetric frame: H vs $\mathscr{H}$



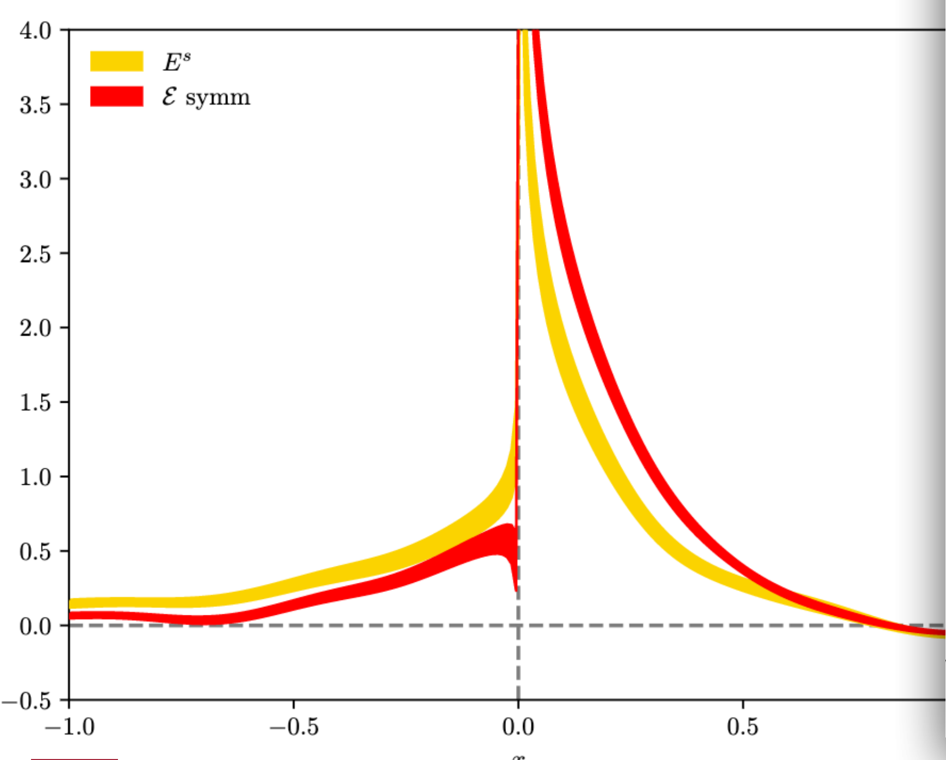
Asymmetric frame: H vs $\mathscr{H}$

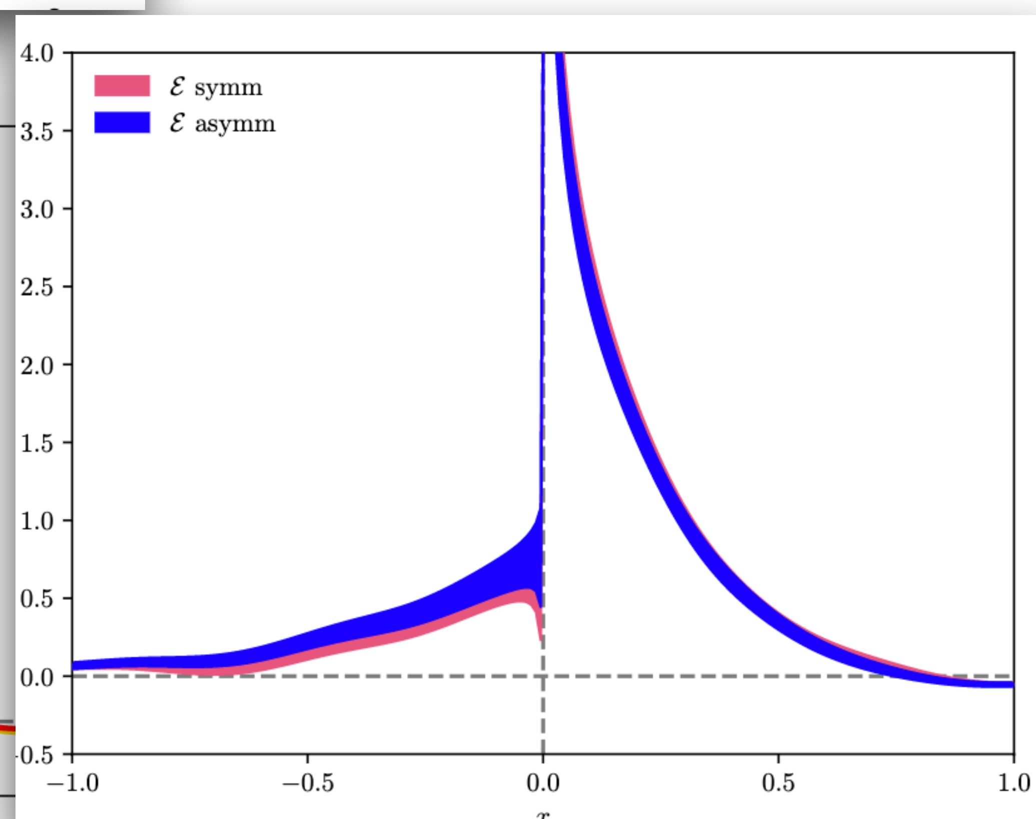


Similar results for H and $\mathscr{H}$ for both frames (agreement not by construction)

Differences between E and $\mathscr{E}$ for both frames (agreement not by construction)

Symmetric frame: E vs $\mathscr{E}$





Agreement between frames for $\mathscr{H}$ and $\mathscr{E}$ (agreement by construction)

Summary

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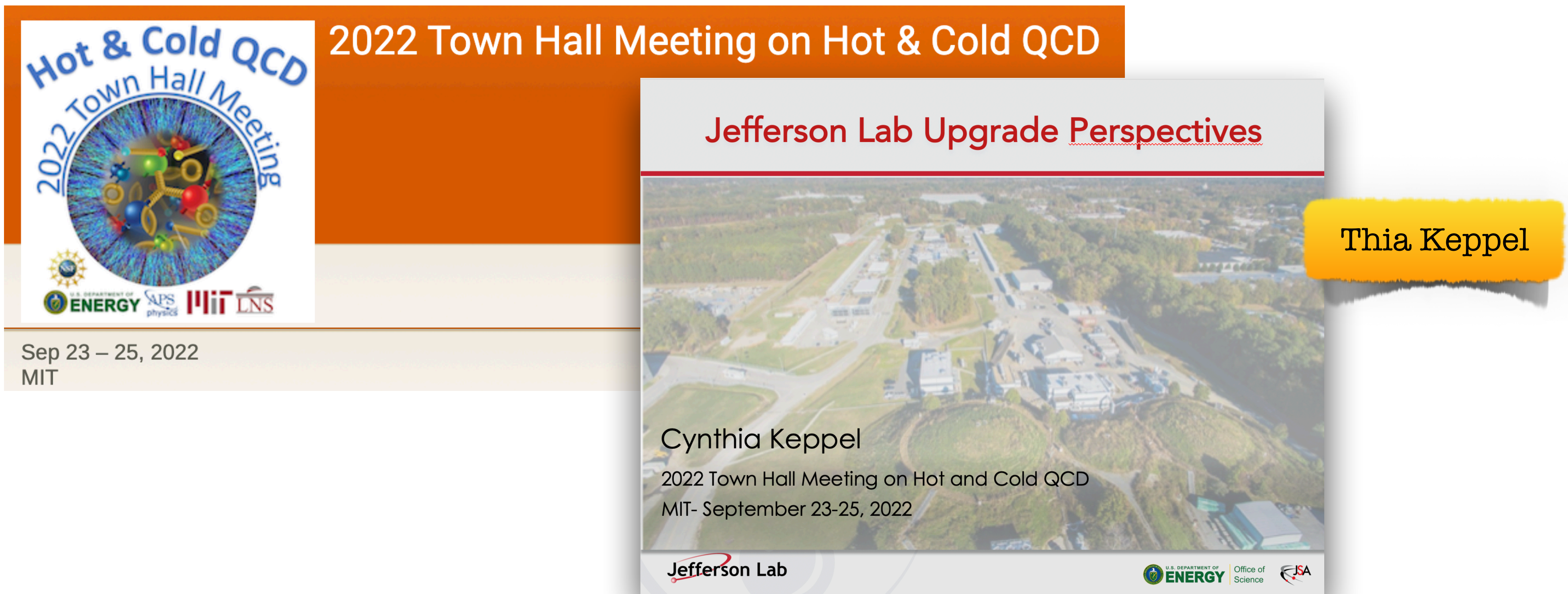

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★ JLab Upgrade included in the survey for the Town Hall Meeting recommendations

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★ Lattice QCD data on GPDs will play an important role in the pre-EIC era and can complement experimental efforts of JLab@12GeV

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- access to a broad range of t and ξ

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Thank you

DOE Early Career Award (NP)
Grant No. DE-SC0020405

BACKUP

Challenges of lattice calculation

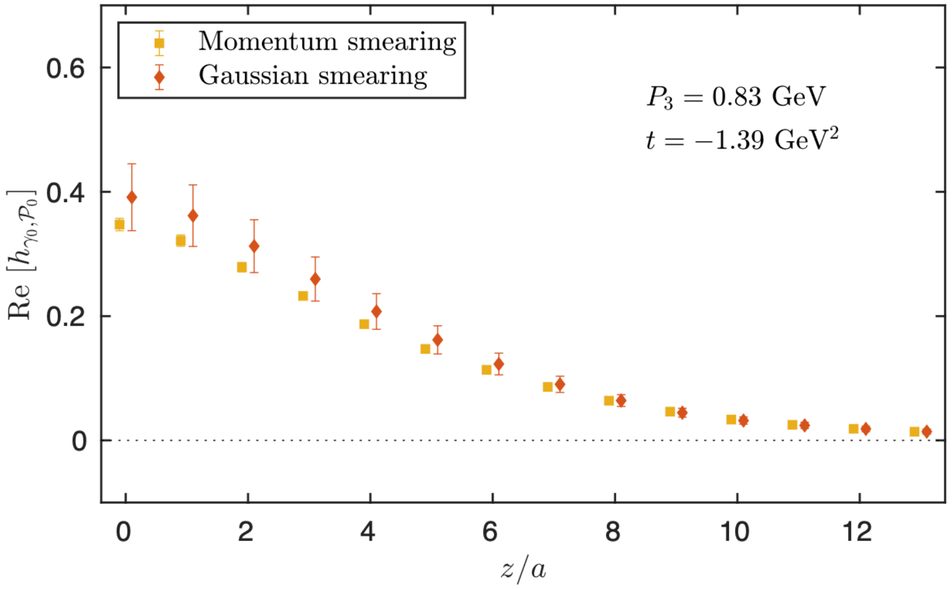
★ Statistical noise increases with P_3, t

use of momentum smearing method

Challenges of lattice calculation

★ **Statistical noise increases with P_3, t**

use of momentum smearing method



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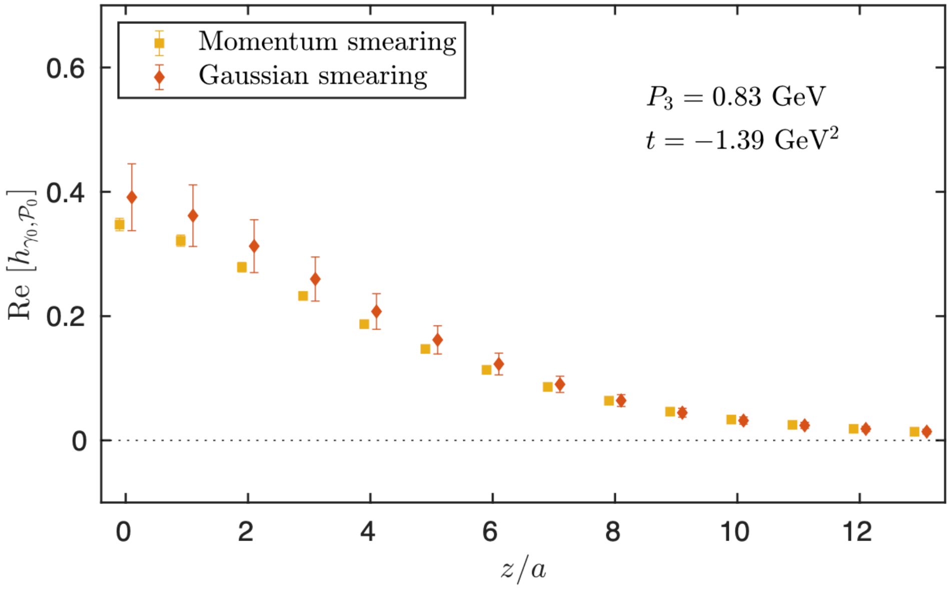

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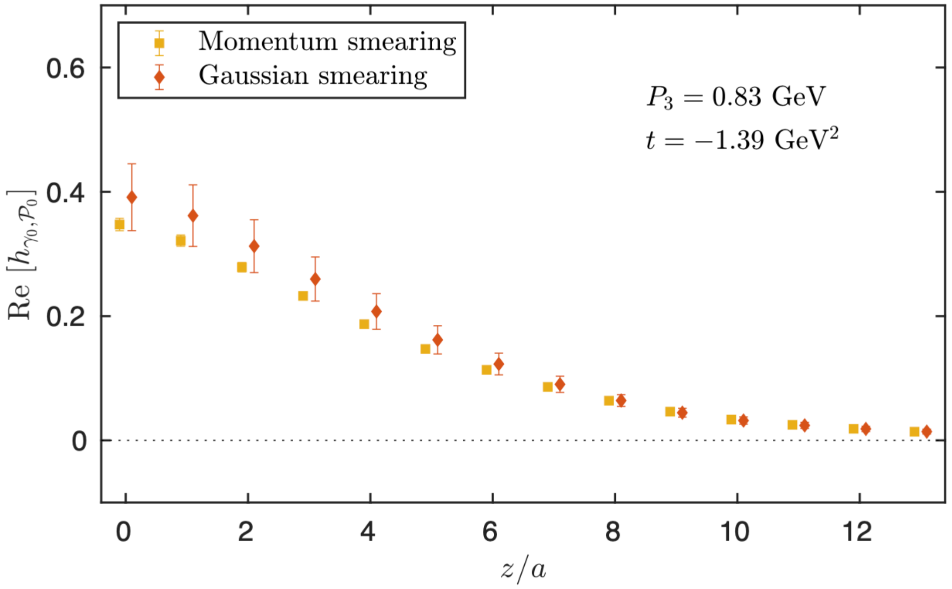

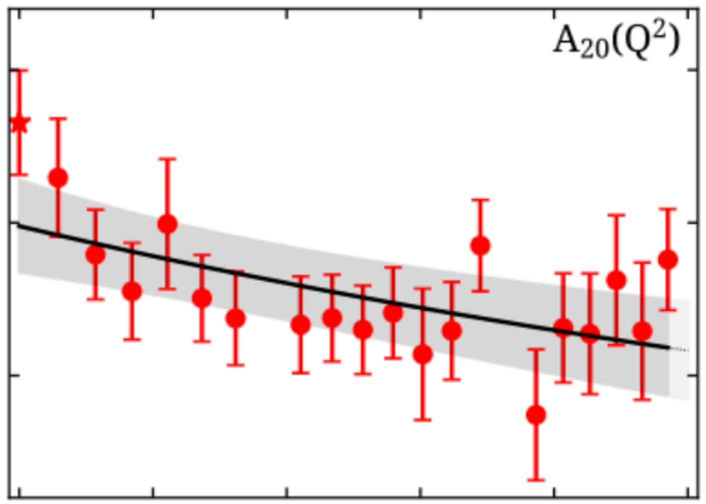

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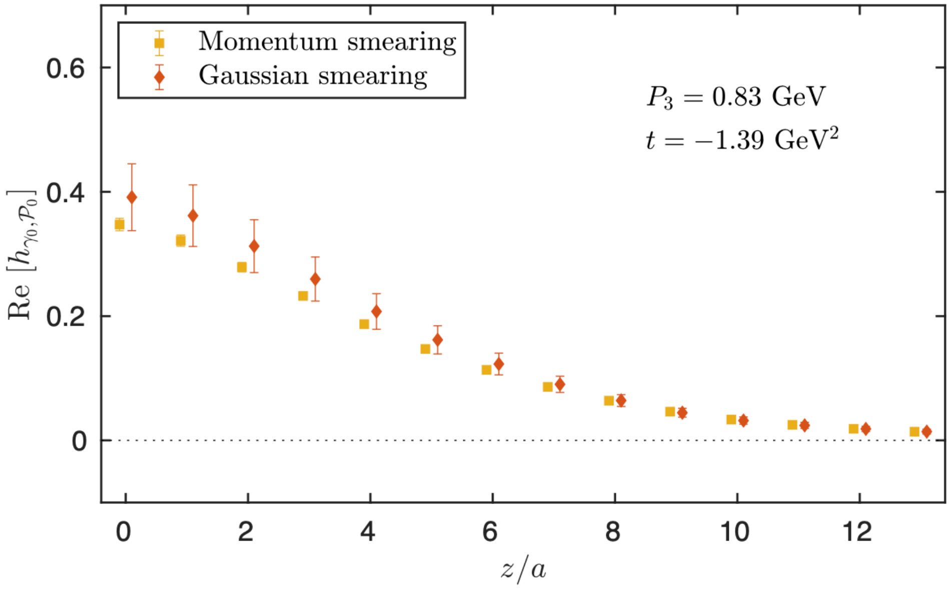

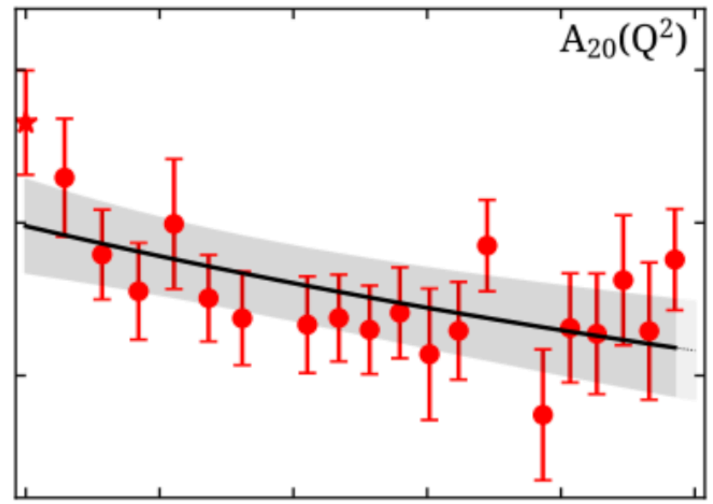

Ref.	m_π(MeV)	P_3(GeV)	$\frac{n}{s}\Big\vert_{z=0}$
quasi/pseudo [59,95]	130	1.38	6%
pseudo [92]	172	2.10	8%
current-current [98]	278	1.65	19% ⋆
quasi [72]	300	1.72	6% †
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† At $T_{\rm sink} < 1$ fm.
⋆ At smallest z value used, $z = 2$.
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[M. Constantinou, EPJA 57 (2021) 77]

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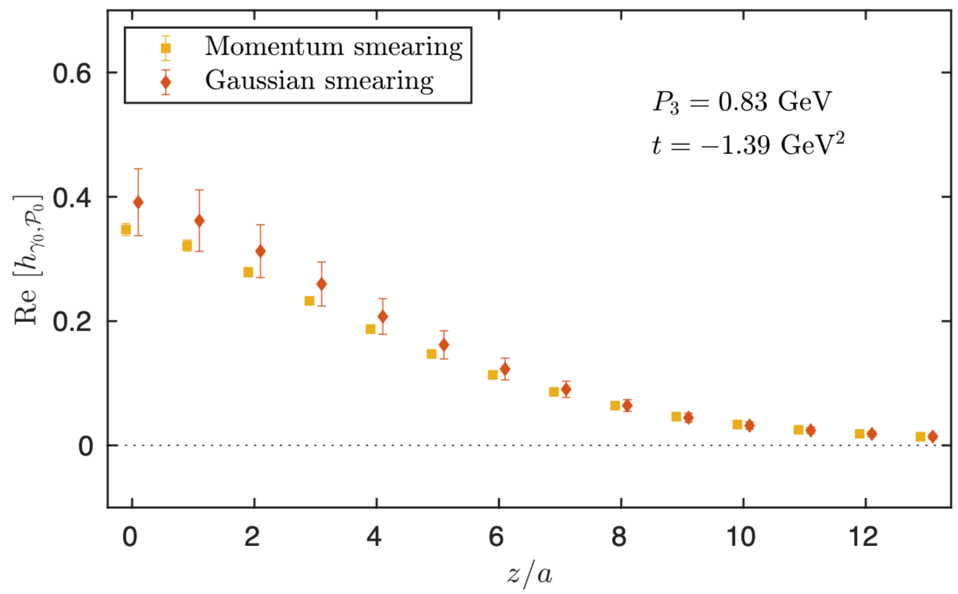

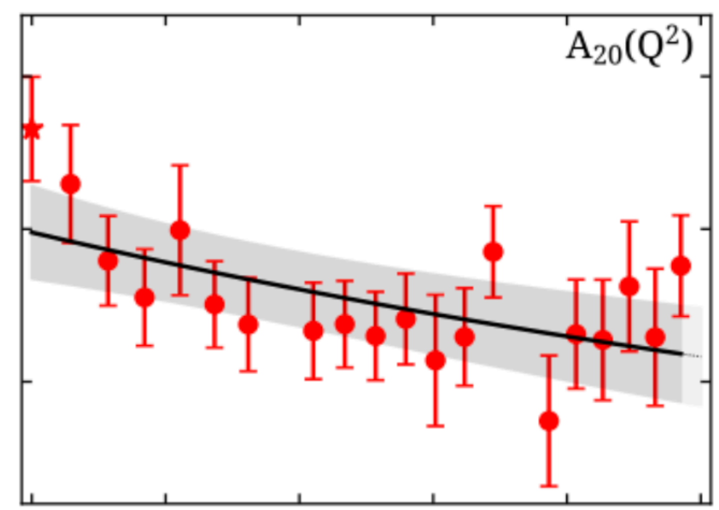

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Further increase of momentum at the cost of credibility

Twist-classification of GPDs

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

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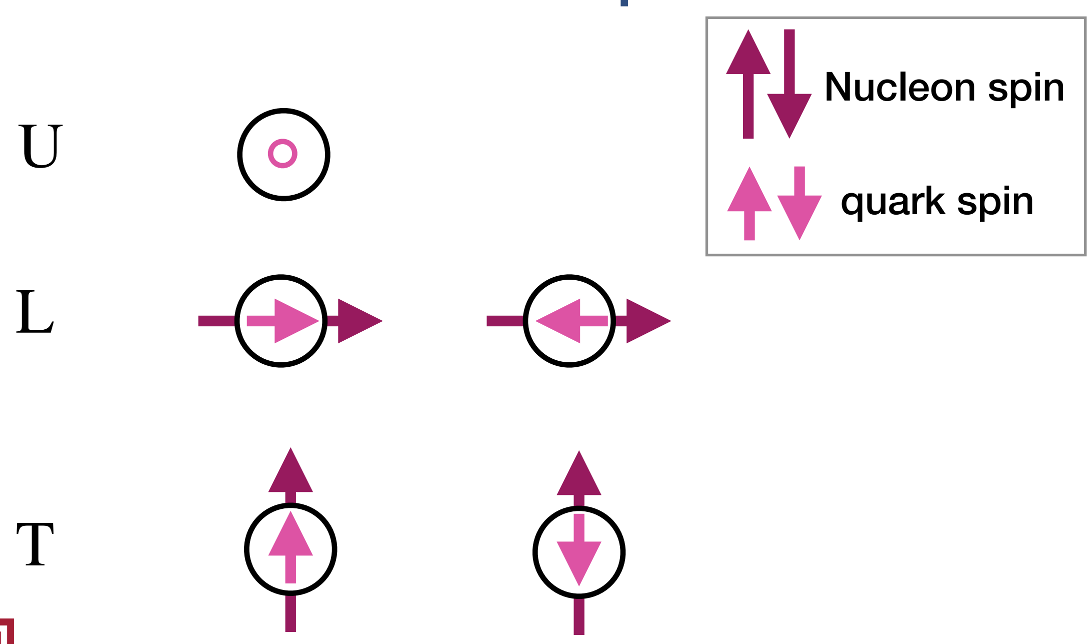
Twist-2 $(f_i^{(0)})$

Quark / Nucleon	U (γ^+)	L $(\gamma^+\gamma^5)$	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity

Probabilistic interpretation



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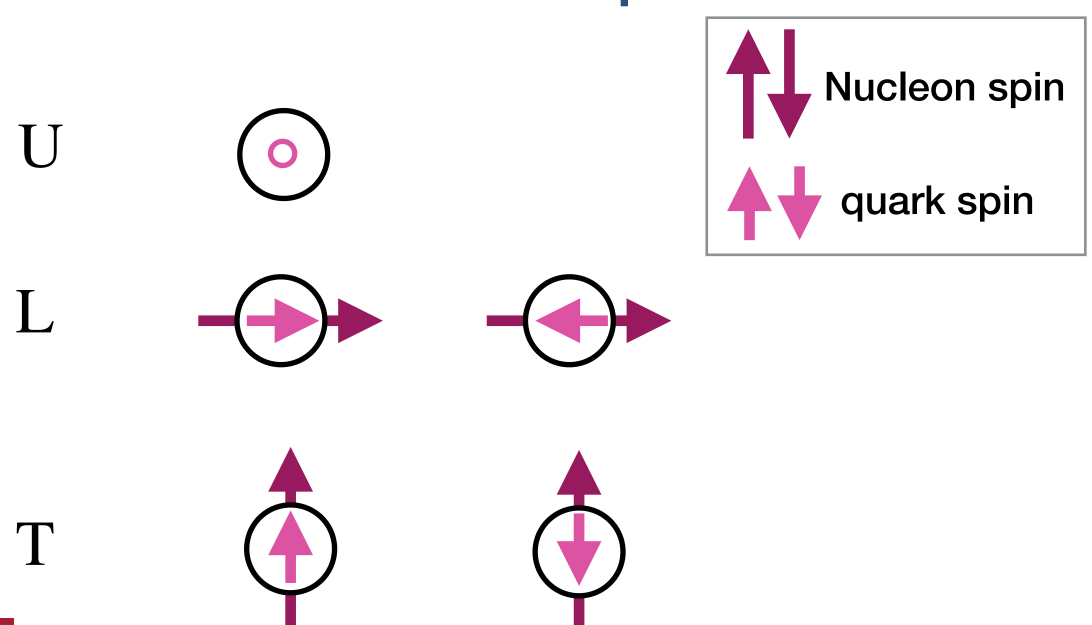
Twist-3 $(f_i^{(1)})$

$\mathcal{O}$ / Nucleon	γ^j	$\gamma^j\gamma^5$	σ^{jk}
U	G_1, G_2 G_3, G_4		
L		$\widetilde{G}_1, \widetilde{G}_2$ $\widetilde{G}_3, \widetilde{G}_4$	
T			$H_2'(x,\xi,t)$ $E_2'(x,\xi,t)$

(Selected)

Probabilistic interpretation

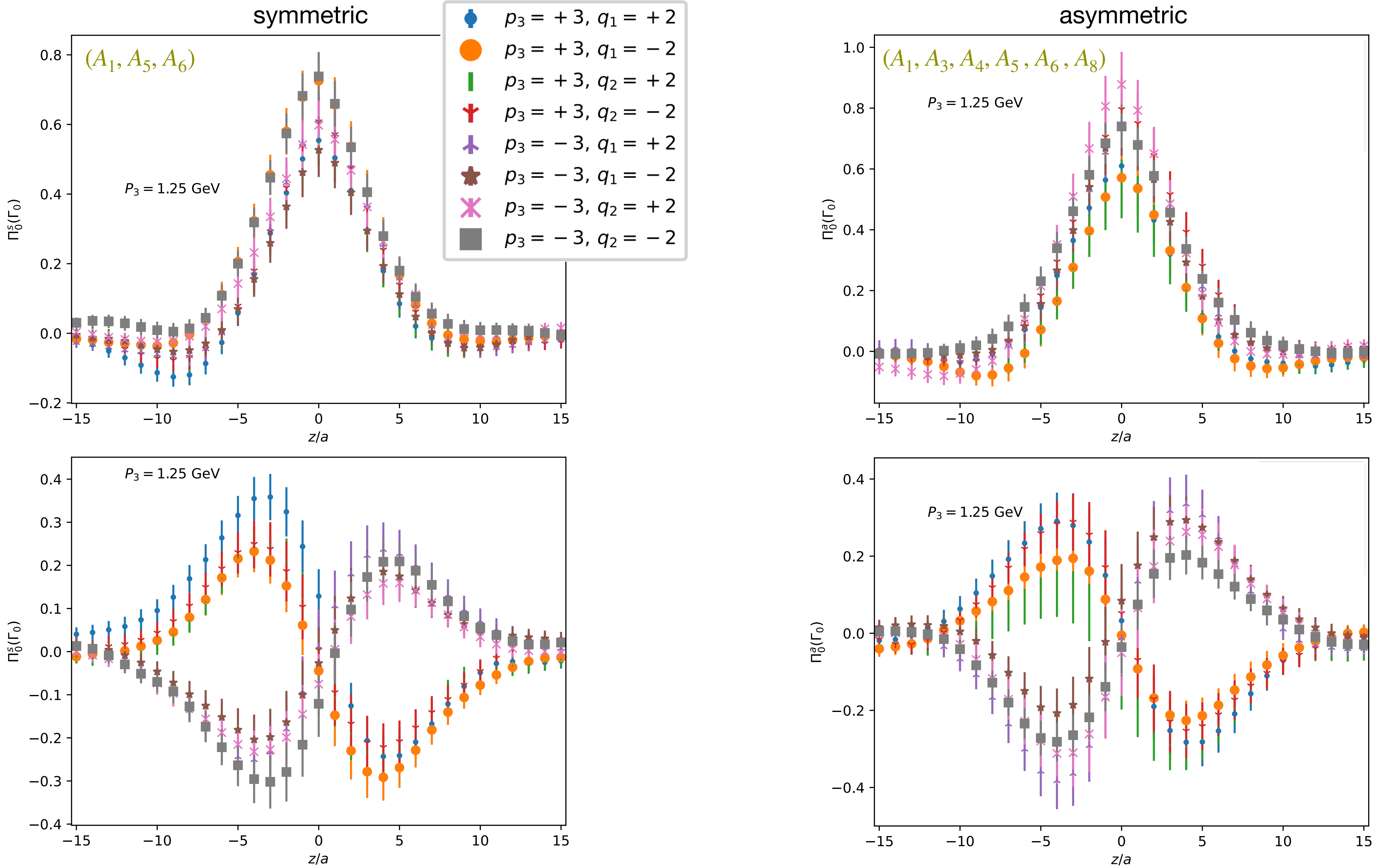


- ★ Lack density interpretation, but not-negligible
- ★ Contain info on quark-gluon-quark correlators
- ★ Physical interpretation, e.g., transverse force
- ★ Kinematically suppressed
  Difficult to isolate experimentally
- ★ Theoretically: contain $\delta(x)$ singularities

Results: matrix elements

Real

Imag

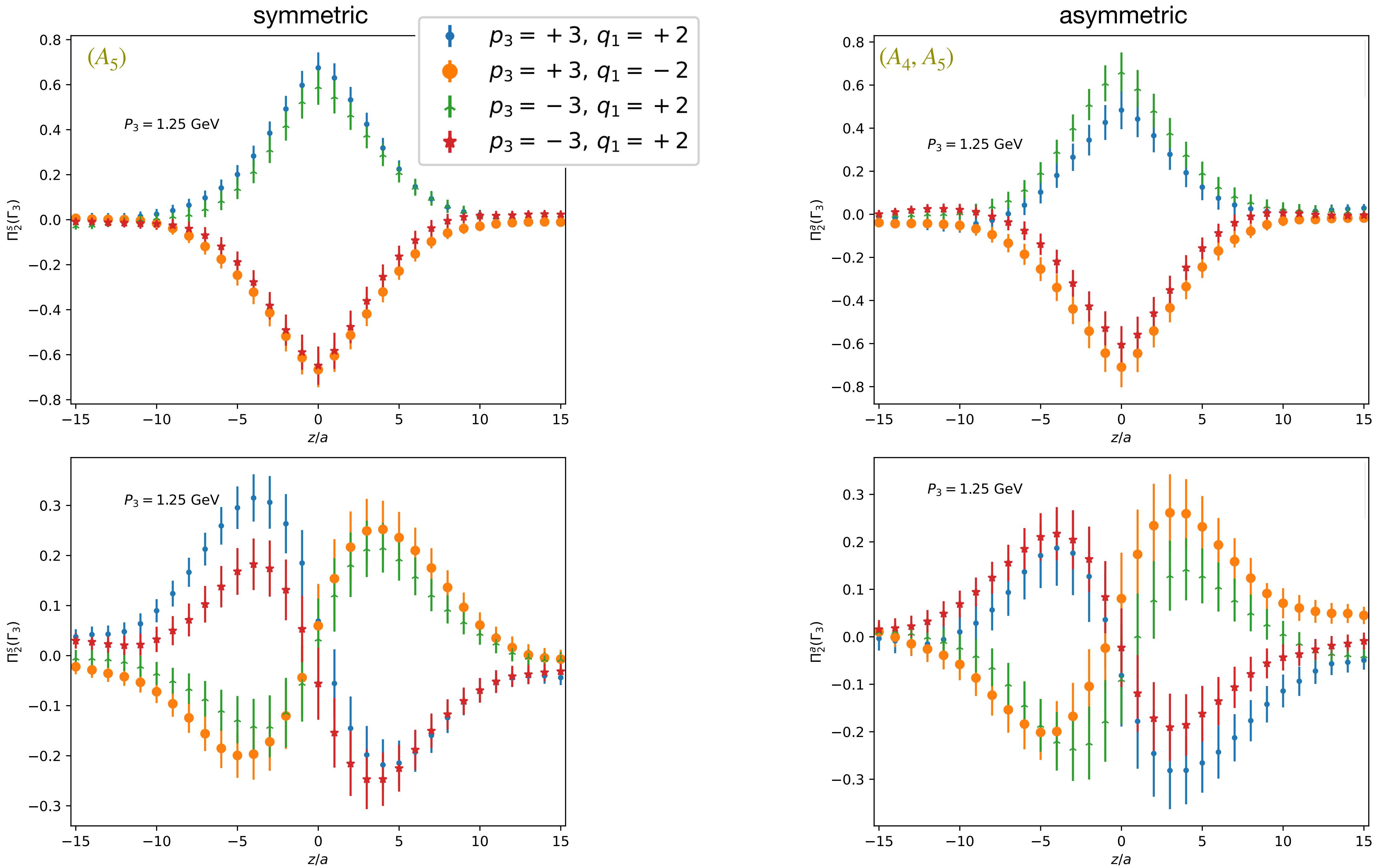


- ★ Lattice data confirm symmetries where applicable **(e.g.,** $\Pi^s_0(\Gamma_0)$ in $\pm P_3, \pm Q, \pm z$**)**
- ★ ME decompose to different A_i
- ★ Multiple ME contribute to the same quantity

Results: matrix elements

Real

Imag



★ Matrix elements depend on frame (comparison pedagogical)

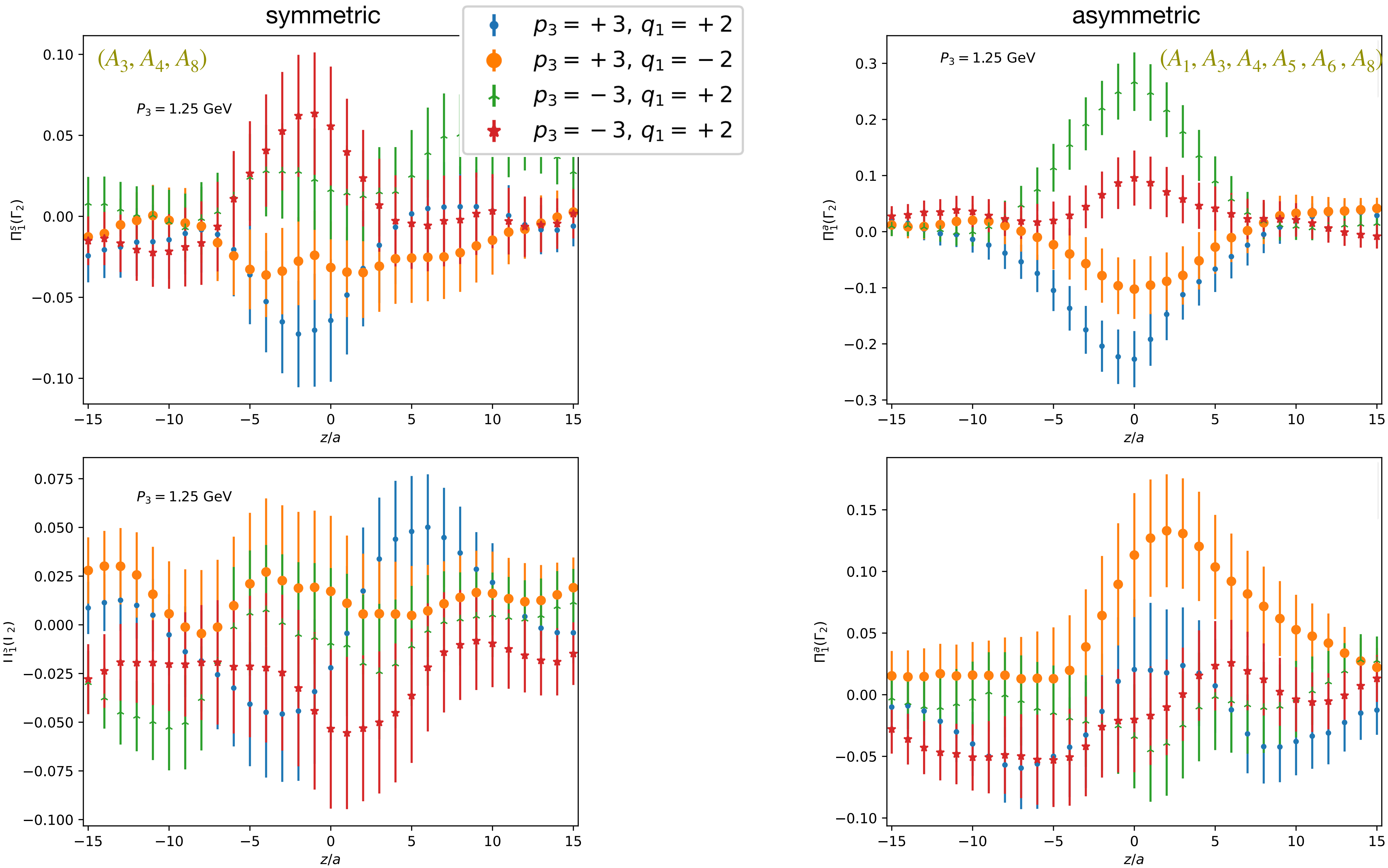
★ ME in asymmetric frame do not have definite symmetries in $\pm P_3, \pm Q, \pm z$

Frame comparison and symmetries applied on Lorentz-invariant amplitudes

Results: matrix elements

Real

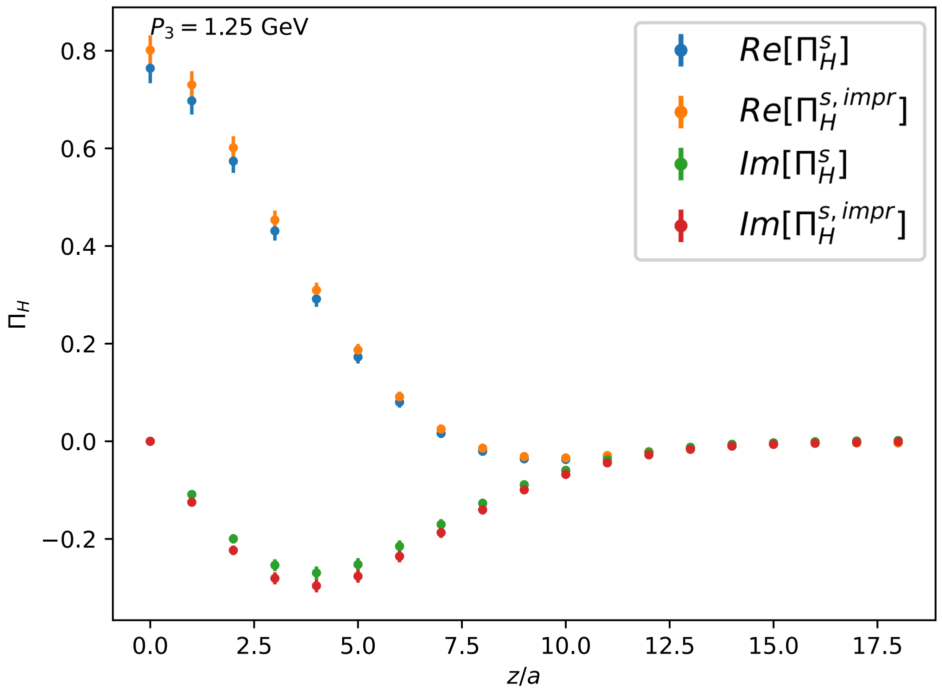
Imag



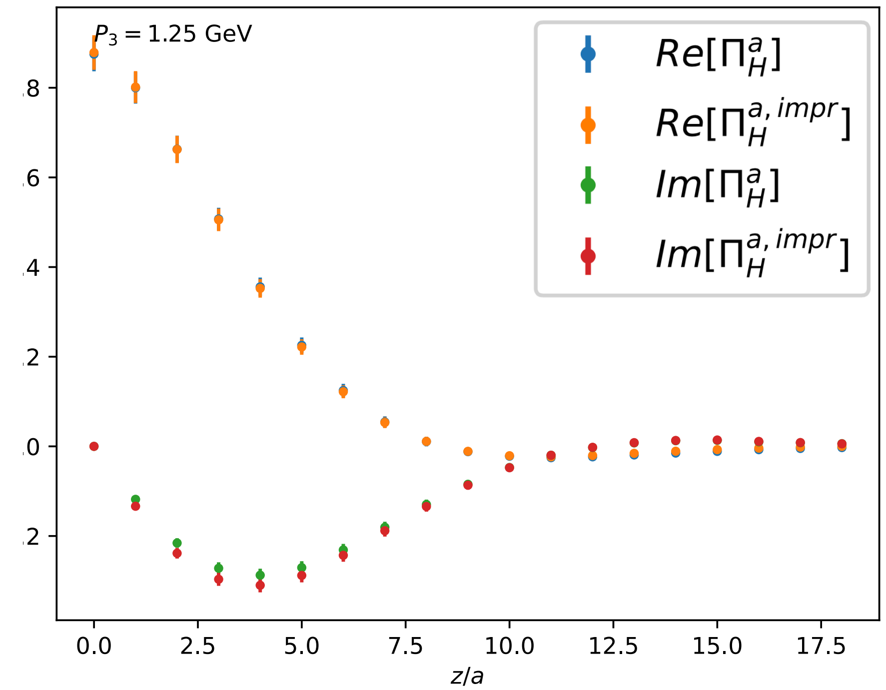
- ★ $\Pi_1(\Gamma_2)$ theoretically nonzero
- ★ Noisy contributions lead to challenges in extracting A_i of sub-leading magnitude

Results: H – GPD

Π^s_H vs $\Pi^{s,impr}_H$

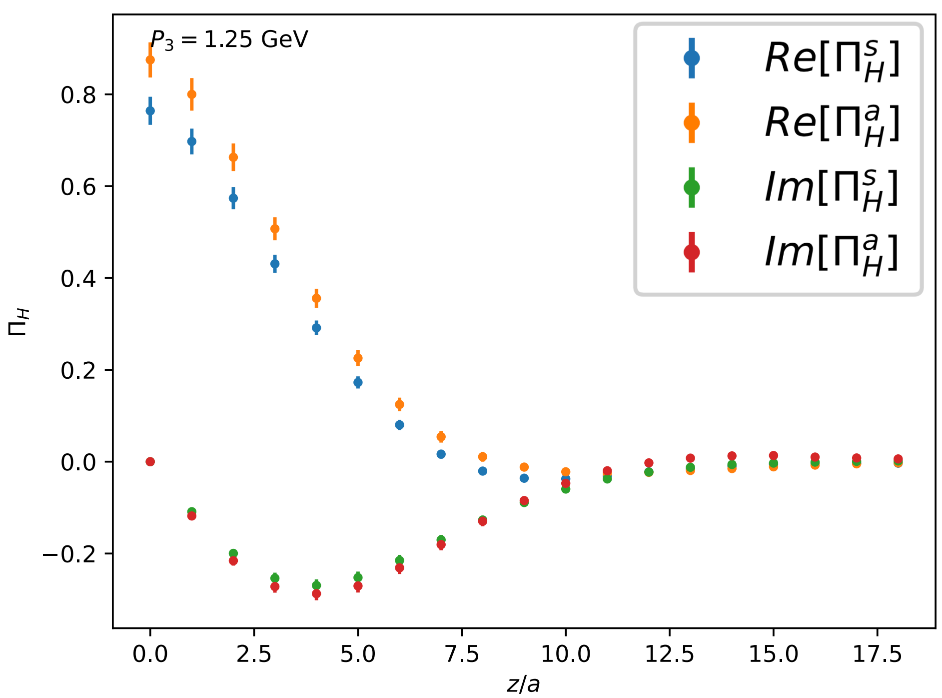


Π^a_H vs $\Pi^{a,impr}_H$

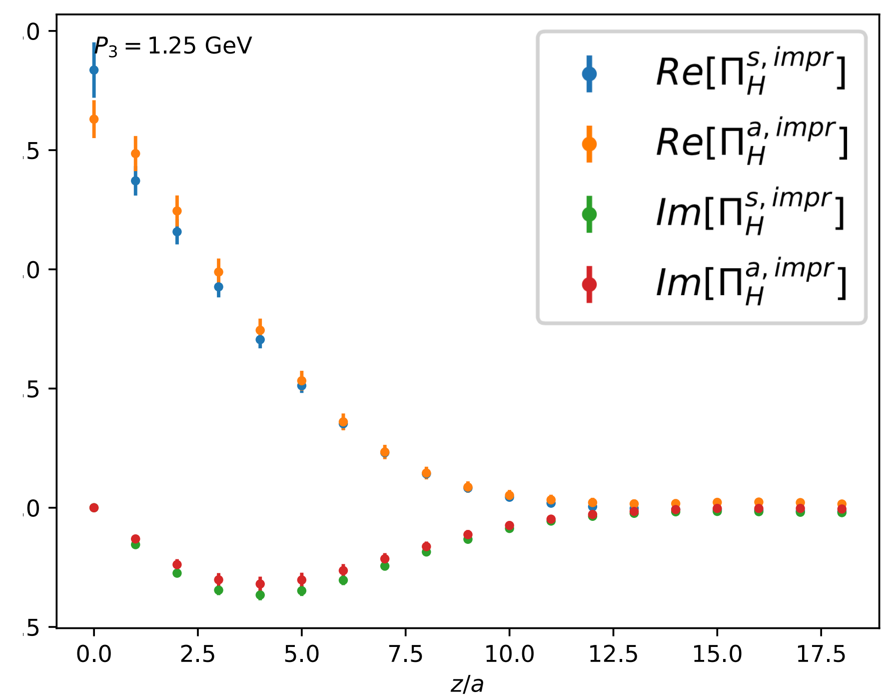


Π_H agree with Π^{impr}_H for both frames despite different definitions (agreement not by construction)

Π^s_H vs Π^a_H



$\Pi^{s,\,impr}_H$ vs $\Pi^{a,\,impr}_H$

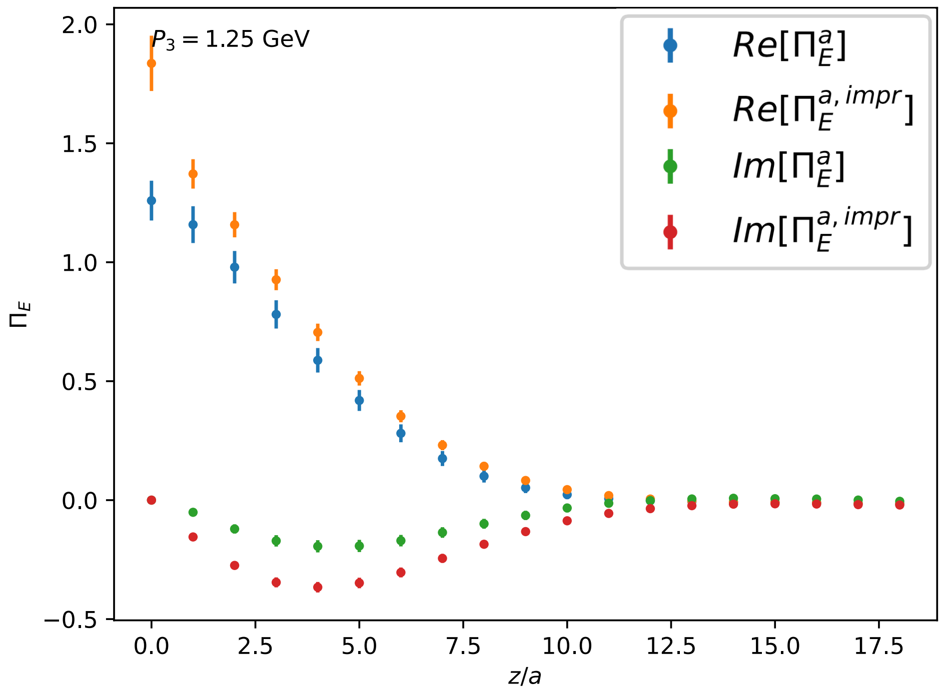


Agreement between Π^s_H and Π^a_H also not required theoretically

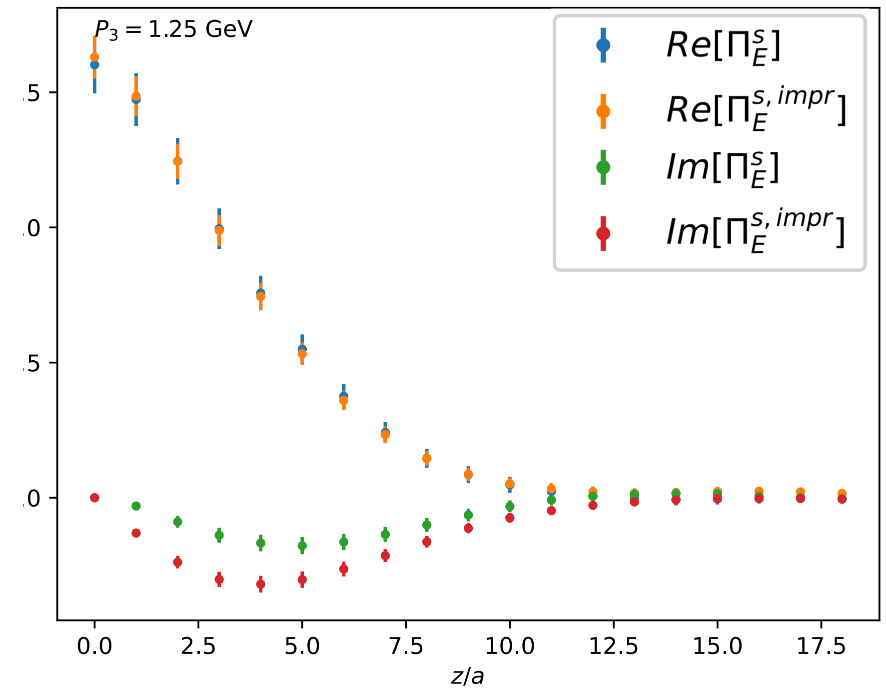
Π^s_H & Π^a_H agreement achieved for improved definition, as expected from Lorentz invariance

Results: Π_E – GPD

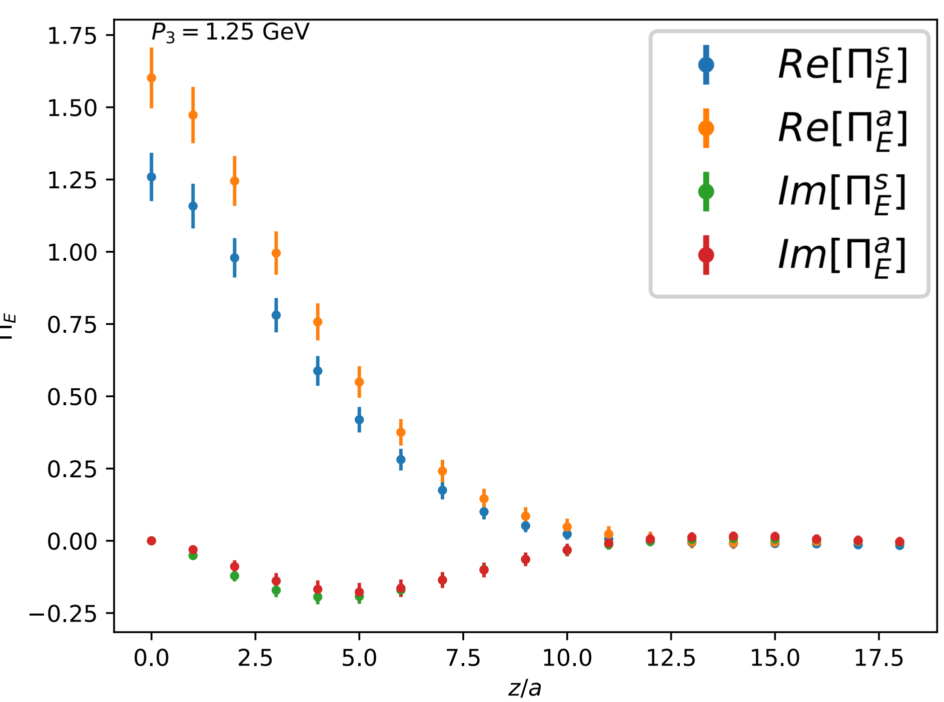
Π_E^s vs $\Pi_E^{s,impr}$



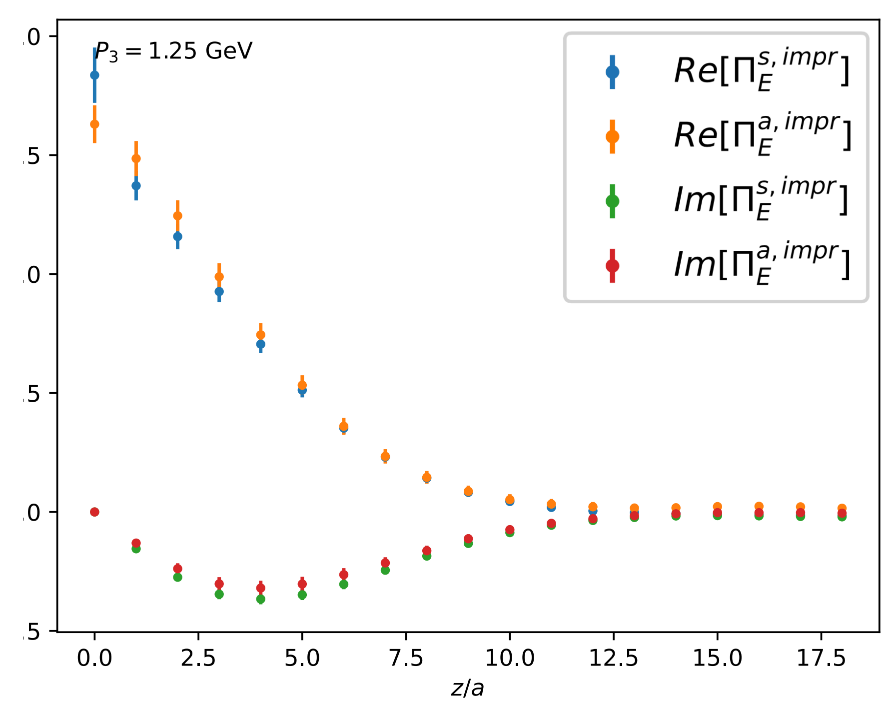
Π_E^a vs $\Pi_E^{a,impr}$



Π_E^s vs Π_E^a



$\Pi_E^{s,\, impr}$ vs $\Pi_E^{a,\, impr}$



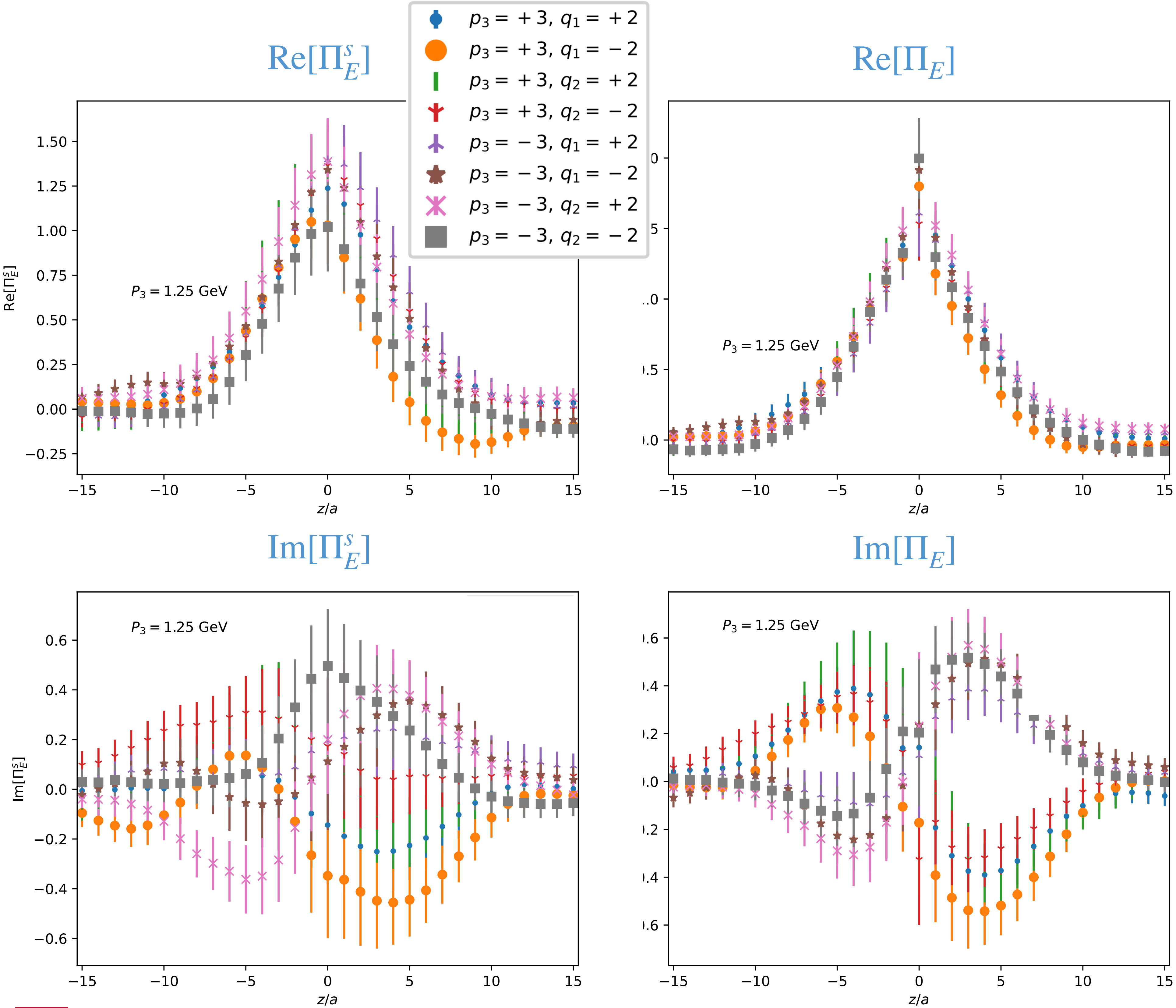
Both frames: $\mathrm{Im}[\Pi_E^{impr}]$ **enhanced compared to** $\mathrm{Im}[\Pi_E]$.

$\mathrm{Re}[\Pi_E^{s,impr}]$ **larger than other** $\mathrm{Re}[\Pi_E^s]$, $\mathrm{Re}[\Pi_E^a]$ **and** $\mathrm{Re}[\Pi_E^{a,impr}]$

Agreement reached between frames for improved definition (expected theoretically)

A comment on Lorentz covariant definitions

Example: symmetric frame



Lorentz covariant definition leads to more precise results for Π_E

Same effect of improvement also for asymmetric frame

Numerical indications that using Π_E leads to better converge to light-cone GPDs with respect to P_3

Signal quality in Π_H same across all cases (not shown)