Constraints on nucleon 3D structure from relativity and quantum mechanics

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Outline

- Rest vs. infinite momentum frame
- Frame dependence of the nucleon wave function
- Additional comments
  - GPD vs. TMD
  - TMD lattice calculations
  - GPD and spin/mass structure
- Conclusion

Rest vs. infinite momentum frame

3D structure

- 3D structure is generally a classical concept.
- One studies a system with

$$\vec{R}_{cm} = \vec{P}_{cm} = 0$$

- For a quantum system, however, there is the uncertainty principle

$$\Delta R_{cm}^{i} \cdot \Delta P_{cm}^{j} \sim \hbar \delta^{ij}$$

One cannot simultaneously have a system at rest ($\vec{P}_{cm} = 0$) and with a fixed position ($\vec{R}_{cm} = 0$)

Two relevant scales

- For a quantum system, two scales are important
  - Mass, M
  - Radius, r
- One cannot localize a system better than the Compton wavelength $r_c \sim 1/M$, thus one in principle cannot make better spatial resolution than r_c, thus

$$r \gg r_c$$

(not the case for pion)

Wave packet & scale hierarchy

- On the other hand, to have a static system, one can construct a wave-packet with momentum spread ΔP_{cm} with the condition

$$\Delta P_{cm} \ll M \qquad \text{such that} \qquad \mathrm{v}_{cm} \ll 1$$

- However, ΔP_{cm} muse large enough so that

$$\Delta R_{cm} \sim \frac{\hbar}{\Delta P_{cm}} \ll r$$

- One arrives at the following hierarchy

$$\mathrm{r_c} \ll \Delta R_{cm} \ll r$$

necessary condition to have a classic-ish 3D picture

Test cases

- For hydrogen atom

  $r = 5000 \text{ fm}, \quad r_C = 0.2 \text{ fm}$

  one can choose $\Delta \mathrm{R}_{\mathrm{cm}} = 100 \text{ fm}, \ (\text{3D resolution})$

  $\Delta P_{cm} = 2 \text{ MeV}$ (Wave Packet)

  to describe the 3D structure
- For the nucleon

  r = 0.8 fm, $r_C = 0.2 \text{ fm}$

  the hierarchy cannot be established.

  The problem has no THE solution.

The traditional method

- Consider the electric form factor in the Breit frame, where $\vec{P}_{in} = -\vec{P}_{out}$
- There is no energy transfer to the system, only 3-momentum transfer (analogous to non-relativistic situation), $\vec{q} = 2\vec{P}$, and $Q^2 = \vec{q}^2$
- The transition matrix element of the charge operator (the Sachs electric form factor) DEFINES a 3D charge distribution

$$\langle P_{out}|\hat{\rho}|P_{in}\rangle \sim G_E(q^2) \equiv F.T.\,\rho(r)$$

This in turn leads to the famous “charge radius.”

Infinite momentum frame

- Consider a proton moving at the speed of light. Its effective mass

$$M_{eff} = \gamma M \to \infty$$

effective Compton wavelength r_c^{eff} = 0

- In the transverse space, one goes back to non-relativistic case:

$$(r_c^{eff} = 0) \ll (\Delta R_{cm\perp}^{eff} \sim 0) \ll r_\perp$$

one has an infinite resolution in the transverse space!

However, one gives up 1D!!

A general Breit frame

- Boost the Breit frame in direction z.

$$\vec{P}_{in} = \left(-\frac{\vec{q}}{2}, P^z\right); \vec{P}_{out} = \left(\frac{\vec{q}}{2}, P^z\right)$$

$$with \;\; \vec{q}^{\,2} = Q^2$$

with $\vec{q}$ now only in the transverse direction.

$$G_E\left(q_\perp^2\right) \equiv F.T.\, \rho(r_\perp)$$

One ends up with a 2D distribution.

There are a lot of attempts to reconstruct 3D from 2D, but it depends on one's opinion.

Frame dependence of the nucleon wave function

Plane-wave nucleon states

- We generally consider plane-wave nucleon states with different momentum $|P\rangle$
- Different momentum states are connected to each other by Lorentz boost

$$|P'\rangle = U(\Lambda)|P\rangle$$

$U(\Lambda)$ depends on the boost operator K which, just like Hamiltoinian, depends on interactions

Properties of boost

- Properties of time evolution
  - For a Hamiltonian system, the state evolves according to $|\psi(t)\rangle = \exp(-iHt)|\psi(0)\rangle$
  - Thus t=0, and other t states are quantum mechanically totally different
  - But, certain properties are conserved in the evolution.
- Thus the nucleons with different momentum are completely different quantum mechanical states!
- 3D structure in the different Lorentz frame can be very different, which are nontrivially related.

Transverse structure: rapidity evolution

- In the non-interacting or non-relativistic case, the transverse structure are independent of boost.
- It is generally expected the transverse structure shall be independent nucleon momentum.
- However, this may not be the case.
- At small Pz, it is completely unknown how to study the evolution of transverse structure.
- However, at large Pz, one can derive rapidity evolution equation using pQCD.

Wave function amplitudes

- Euclidean WF amplitudes (Az=0 gauge)

$$\widetilde{\psi}_N^{\pm}(x_i, \vec{b}_{i\perp}, \mu, \zeta_z) = \lim_{L\to\infty} \int d\lambda_i e^{-i\lambda_i x_i - i\lambda_0 x_0} \frac{\langle 0|\mathcal{P}_N \prod_{i=1}^{N} \Phi_i^{\pm}(\lambda_i n_z + \vec{b}_{i\perp}; L)\Phi_0^{\pm}(\lambda_0 n_z; L)|P\rangle}{\sqrt{Z_E(2L, \vec{b}_{i\perp}, \mu)}}$$

- Gauge-invariant fields

$$\Phi_i^{\pm}(\xi; L) = \mathcal{P}\exp\left[ig\int_0^{\mp L \pm \xi^z} d\lambda A^z(\xi + \lambda n_z)\right]\phi(\xi)$$

Regularizing rapidity divergence (0-mode) through off-light-cone soft function

- Define two off-light-cone vectors

$$p \to p_Y = p - e^{-2Y}(P^+)^2 \bar{n},\ n \to n_{Y'} = n - e^{-2Y'} \frac{\bar{p}}{(P^+)^2}$$

- Soft functions

$$S_N^{\pm}(\vec{b}_{i\perp}, \mu, Y, Y') = \frac{\langle 0|\mathcal{P}_N \mathcal{T} \prod_{i=0}^{N} \mathcal{C}^{\pm}(\vec{b}_{i\perp}, Y, Y')|0\rangle}{\sqrt{Z_E(Y)}\sqrt{Z_E(Y')}}$$

$$\mathcal{C}^{\pm}(\vec{b}_\perp, Y, Y') = W^{\pm}_{n_{Y'}}(\vec{b}_\perp) W^{\dagger}_{p_Y}(\vec{b}_\perp) \ ,$$

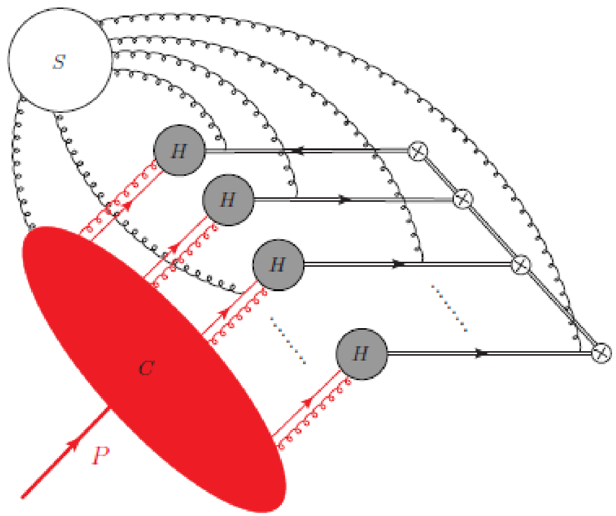
where the off-light-cone gauge-links W_{p_Y} and $W_{n'_Y}$ defined as

$$W_{p_Y}(\vec{b}_\perp) = \mathcal{P}\mathrm{exp}\left[-ig\int_0^{-\infty} d\lambda' p_Y \cdot A(\lambda' p_Y + \vec{b}_\perp)\right]$$

and

$$W^{\pm}_{n_{Y'}}(\vec{b}_\perp) = \mathcal{P}\mathrm{exp}\left[-ig\int_0^{\pm\infty} d\lambda n_{Y'} \cdot A(\lambda n_{Y'} + \vec{b}_\perp)\right]$$

Factorization the momentum dependence

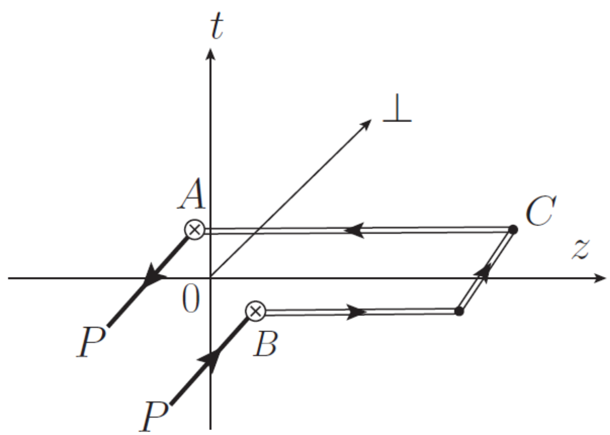


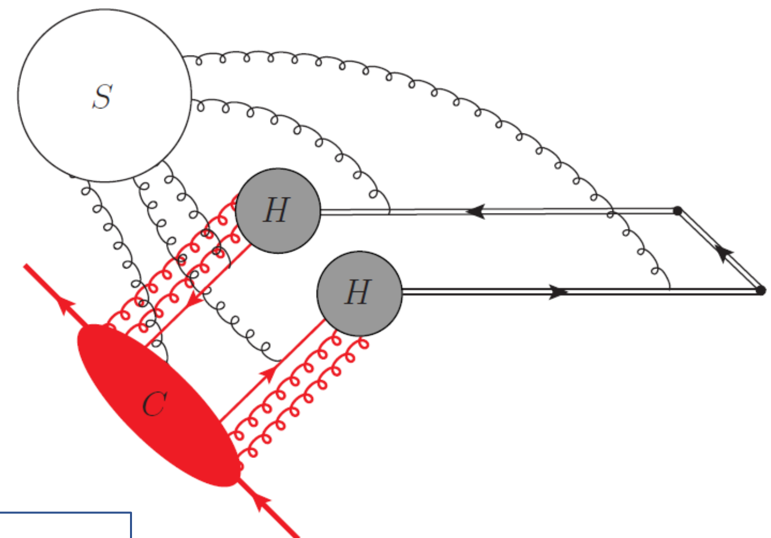
$$\widetilde{\psi}^{\pm}_{N}(x_i, \vec{b}_{i\perp}, \mu, \zeta_z)\sqrt{S_{rN}(\vec{b}_{i\perp}, \mu)} = e^{\ln \frac{\mp\zeta_z - i0}{\zeta} K_N(\vec{b}_{i\perp}, \mu)}$$

(4(

$$\times H^{\pm}_{N}\left(\zeta_{z,i}/\mu^2\right) \psi^{\pm}_{N}(x_i, \vec{b}_{i\perp}, \mu, \zeta) + \ldots ,$$

Similar things happen for TMD distribution (Ji, Liu, Liu, 2020, Ebert et al 2022)

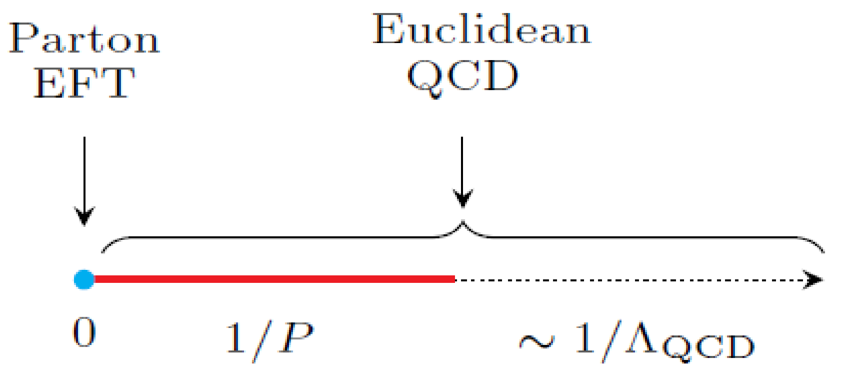




$$\tilde{f}(x, b_\perp, \mu, \zeta_z)\sqrt{S_r(b_\perp, \mu)}$$
$$= H\left(\frac{\zeta_z}{\mu^2}\right) e^{K(b_\perp,\mu)\ln(\frac{\zeta_z}{\zeta})} f^{\mathrm{TMD}}(x, b_\perp, \mu, \zeta) + \ldots$$

$$\mu \frac{d}{d\mu} \ln H\left(\frac{\zeta_z}{\mu^2}\right) = \Gamma_{\mathrm{cusp}} \ln \frac{\zeta_z}{\mu^2} + \gamma_C$$

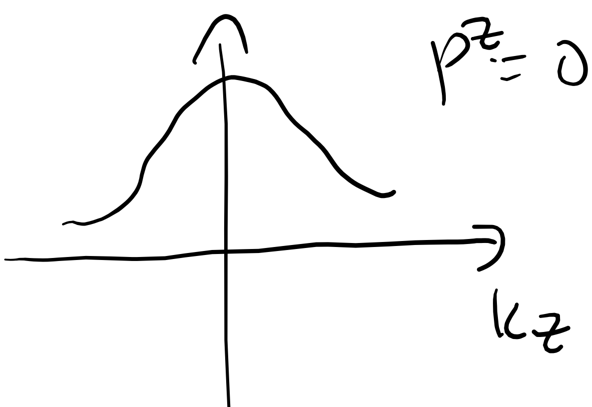
Frame dependence of the nucleon structure

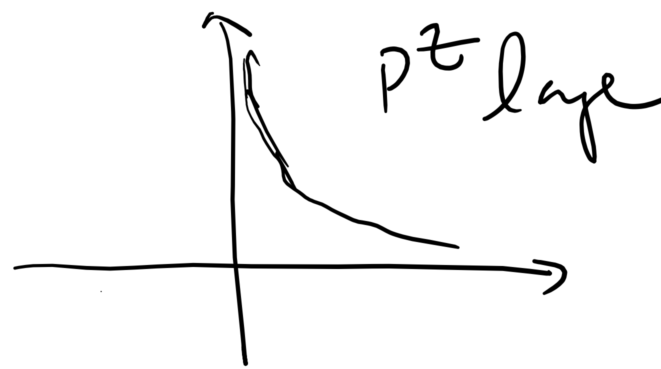


- Only the connection among the nucleon structure at large momenta can be established.
- The nucleon structure in the rest frame can be very different, or maybe not that different?
- High-energy scattering only study the structure of a fast-traveling nucleon. However, lattice can do the calculation well at low momentum.

Longitudinal structure

- Longitudinal structure, or kz dependence is similar.
- However, it can be said definitely that the longitudinal structure at large momentum is TOTALLY different because of the boost mixing.
- Thus longitudinal PDF (momentum dis. in infinite momentum frame) is totally different from the rest frame momentum density.





- Therefore, 3D structure at large momentum shall really be considered as 2D+1D structure, as the z direction is mixed with boost.
- Unless one studies the structure of $\vec{P} = 0$ state on lattice, which is unfortunately out of experimental reach.

Additional comments

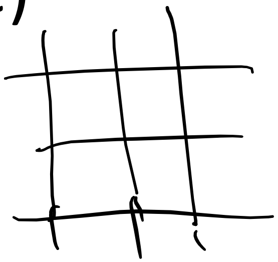
TMD& GPD: similarities

- Both TMD and GPD are single particle Green's function.
- For quarks in the nucleon, there are 16 amplitudes

  $A(\Lambda'\lambda'; \Lambda'\lambda')$ 2x2x2x2=16

- Parity symmetry constraint yields 8 independent amplitudes, thus there are 8 independent leading twist GPDs & TMDs (1 to 1 correspondence)

  2 for unpolarized nucleon
  2 for longitudinally polarized nucleon
  4 for transversely polarized nucleon



- If not forbidden, they exist! (all effects are there)

TMD& GPD: Differences

- GPD give $(x, k_\perp)$ distribution.

Since experiments measure momentum of particles, they easily appear in various processes (nice tool for HEP)

however, their connection with bound state physical properties is not straightforward.

- TMD give $(x, r_\perp)$ distribution.

Since they involves coordinates, only diffractive scattering (DVEP) can measure them.

however, their connection with bound state physics is simpler.

Relation in simple systems

- They are related for simple systems, such as for the ground state of hydrogen atom.
- In general, there is no connection other than they are parts of the Wigner distribution.

TMD lattice calculations

- TMD Moments: Busch, Hagler, Engelhardt, Schaefer, Negele et al.
- TMD Distribution from LaMET
  - X. Ji, F. Yuan, Y. Zhao, J. Zhang, Y. S. Liu, Y. Z. Liu …
    2013, 2015, 2019-2022
  - MIT group, Y. Zhao, Ebert, I. Stewart, P. Shanahan, M. Wagman et al. 2018-2022
- Talk by Y. Zhao

LPC collaboration ($\mu = \sqrt{\zeta} = 2$ GeV)

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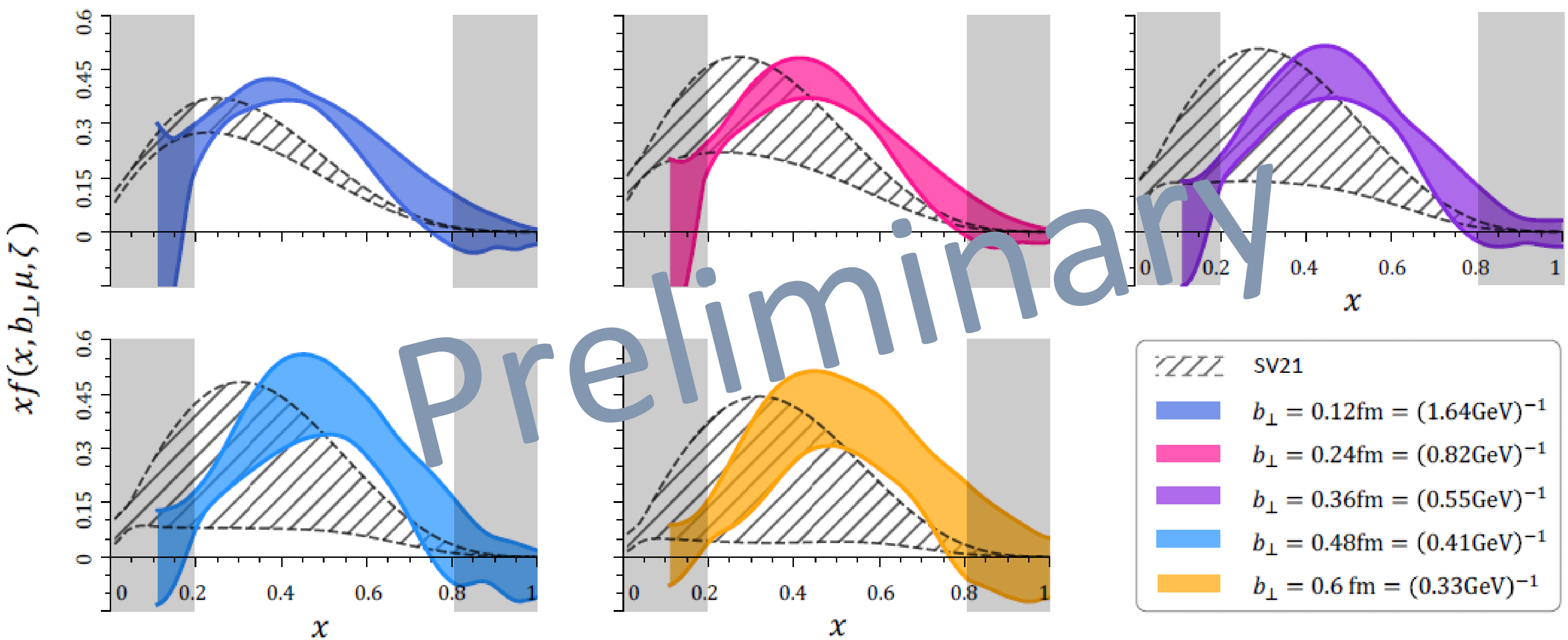


FIG. 13. Comparison with SV21.

GPD from lattice QCD

- Lattice calculations
  - ETMC, Matha’s talk
  - MSU group
  - Other groups
- Experimental program
  - DVCS, DVMP
  - Only 2D information, except DDVCS
- Global fitting
  - GUMP, lattice & exp data, Y. Guo’s talk

GPD & Gravitations form factors

- Form factors of EMT for quarks and gluons

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \overline{U}(P')[A_{q,g}(\Delta^2)\gamma^{(\mu}\overline{P}^{\nu)} + B_{q,g}(\Delta^2)\overline{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M + C_{q,g}(\Delta^2)(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)/M + \overline{C}_{q,g}(\Delta^2)g^{\mu\nu}M]U(P),$$

- Form factors for the total EMT

$$\langle P'|T^{\mu\nu}|P\rangle = \bar{u}(P')\Big[A(Q^2)\gamma^{(\mu}\bar{P}^{\nu)} + B(Q^2)\bar{P}^{(\mu}i\sigma^{\nu)\alpha}q_\alpha/2M + C(Q^2)(q^\mu q^\nu - g^{\mu\nu}q^2)/M\Big]u(P),$$

$A = A_q + A_g,\ B\ \&\ C\ etc.,\ \ \bar{C}_q + \bar{C}_g = 0$

Physics of EMT form factors

- Spin structure
- Mass radius
- Momentum current form factor C & Tensor monopole moment
- Scalar fields and radius
- Anomalous contribution to proton mass

C(q): momentum current FF

- The physics of this form factor is best seen in the Breit frame in which it is the form factor for momentum current.

$$\langle T^{ij}\rangle \sim \left(q^i q^j - \delta^{ij} q^2\right) C(q)$$

which generates gravitational field according to Einstein's equation.

- Tensor-monopole moment,

$$\tau \sim \int d^3r \, (Y_2 \times T)\wedge 0 \sim C(0)/M$$

which generates a particular type of gravitationl potential (Ji & Liu, e-Print: 2110.14781)

Tensor monopole moment of hydrogen atom

- Zero-th order

$$\tau_0 = \hbar^2/4m_e$$

positive!

for proton, it might be negative, $\tau_p = D(0)\hbar^2/4M_p$

- Radiative correction

$$\tau = \tau_0\left(1 + \frac{4\alpha}{3\pi}ln\alpha\right)$$

X. ji & Y. Liu, hep-ph/2208.05029

Trace anomaly, mass scale, and scalar form factor

- Form factor of the scalar density

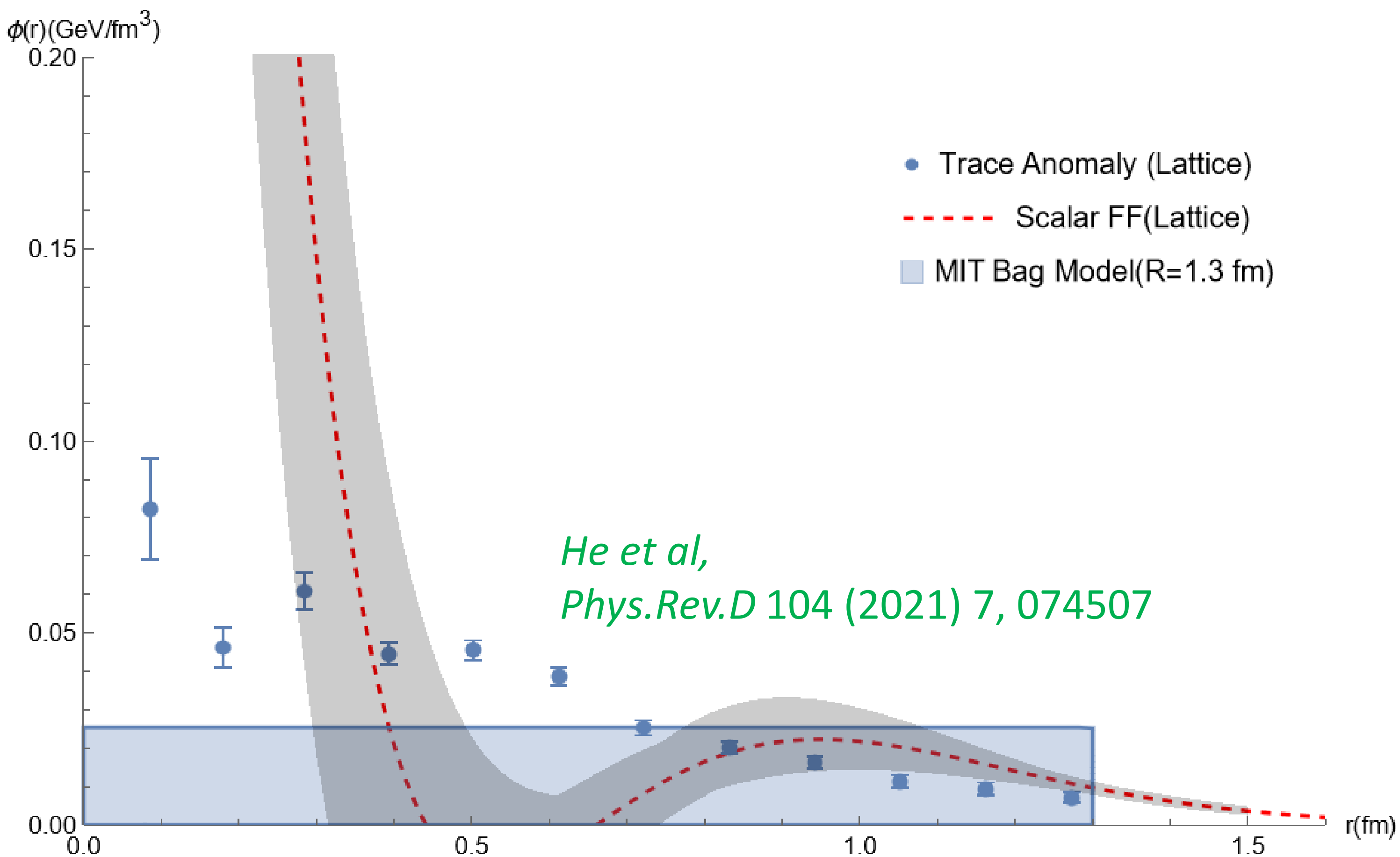
$$\langle P' | T^{\mu}_{\mu} | P \rangle = \bar{u}(P') u(P) G_s(Q^2) \ ,$$

where,

$$G_s(Q^2) = \left[M A(Q^2) - B(Q^2) \frac{Q^2}{4M} + C(Q^2) \frac{3Q^2}{M} \right]$$

- Fourier transformation of Gs gives us the scalar field distribution inside the Nucleon
- Dynamical MIT "bag constant".
- One can determine the mass scale without directly measuring F^2 matrix element! (EMT conservation)

Scalar field (QAE) distribution inside the proton



He et al,
Phys.Rev.D 104 (2021) 7, 074507

Scalar (confinement) and mass radii

- Scalar radius

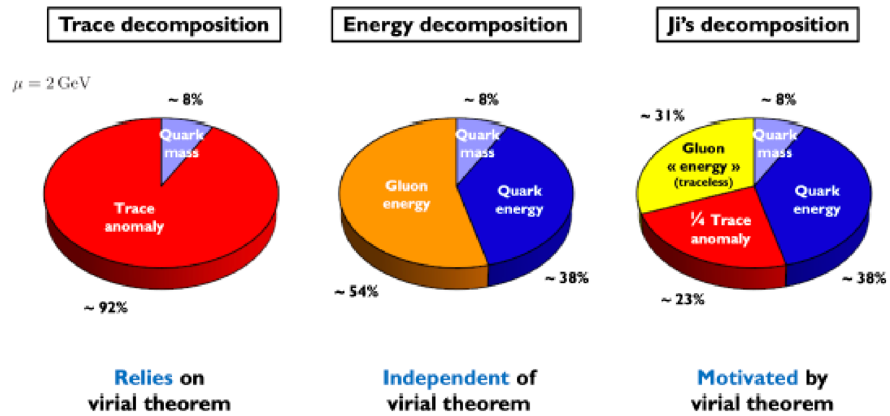
$$\langle r^2 \rangle_s = -6\frac{dA(Q^2)}{dQ^2} - 18\frac{C(0)}{M^2}$$

- The difference

$$\langle r^2 \rangle_s - \langle r^2 \rangle_m = -12\frac{C(0)}{M^2}$$

- Conjecture $\langle r^2 \rangle_s > \langle r^2 \rangle_m$ or C(0)<0

Mass sum rule



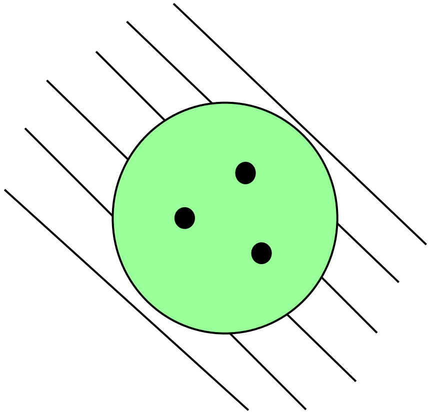
Proton mass budget decompositions, C. Lorce (from 2022 INT workshop)

- Trace decomposition is not a proper mass sum rule.
- The so-called "energy decomposition" fails to recognize there are two types of the gluon energy contributions (although it is obtained by explicit sum)
  - Gluon energy as seen in Higgs production at LHC
  - Gluon scalar field response to the valence quarks (quantum anomalous energy!)

Quantum anomalous energy (QAE) contribution to the proton mass:

- The scalar field has a VEV: $\langle 0|F^2|0\rangle$
- QAE comes from the scalar response to the presence of the quarks.

$$\phi = F^2 - \langle 0|F^2|0\rangle$$



- QCD Higgs mechanism, with gluon scalar as a dynamical Higgs field.
- This contribution is like bag constant in MIT bag model.
- Instanton susceptibility (I. Zahed, *Phys.Rev.D* 104 (2021) 5, 054031)