



UNIVERSITÀ DI PAVIA



Istituto Nazionale di Fisica Nucleare

TMDs extraction

Framework and Validation

Matteo Cerutti

Available Global fits

	Accuracy	SIDIS	DY	Z production	N of points	χ^2/N_{data}
Pavia 2017 arXiv:1703.10157	NLL	✓	✓	✓	8059	1.55
SV 2019 arXiv:1912.06532	N^3LL^-	✓	✓	✓	1039	1.06
MAPTMD22	N^3LL^-	✓	✓	✓	2031	1.06

Bacchetta, Bertone, Bissolotti, Bozzi, MC, Piacenza, Radici, Signori arXiv: 2206.07598

Unpolarized TMD structure

Collins, "Foundations of Perturbative QCD"

Fourier space

$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^2} e^{i \mathbf{b}_T \cdot \mathbf{k}_\perp} f_1^q(x, k_\perp^2; \mu_f, \zeta_f)$$

$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = [C \otimes f_1](x, \mu_{b_*}) e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu} (\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_f}}{\mu})} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}} \right)^{K_{\text{resum}} + g_K} f_{1NP}(x, b_T^2; \zeta_f, Q_0)$$

Scales $\mu_f = Q$ $\mu_{b_*} = \frac{2e^{-\gamma_E}}{b_*}$

$\zeta_f = Q^2$

Unpolarized TMD structure

Fourier space

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$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = \int \frac{d^2 k_\perp}{(2\pi)^2} e^{i \mathbf{b}_T \cdot \mathbf{k}_\perp} f_1^q(x, k_\perp^2; \mu_f, \zeta_f)$$

perturbative Sudakov form factor

$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = [C \otimes f_1](x, \mu_{b_*}) e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu} (\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_f}}{\mu})} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}} \right)^{K_{\text{resum}} + g_K} f_{1NP}(x, b_T^2; \zeta_f, Q_0)$$

collinear PDF

matching coefficients (perturbative)

Collins-Soper kernel (perturbative and nonperturbative)

nonperturbative part of TMD

Scales

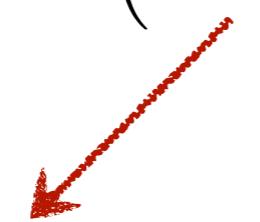
$\mu_f = Q$	$\mu_{b_*} = \frac{2e^{-\gamma_E}}{b_*}$
$\zeta_f = Q^2$	

MAPTMD22 – Parameterization of TMDs

$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{k_\perp^2}{g^{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g^{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g^{1C}}} \right)$$

MAPTMD22 – Parameterization of TMDs

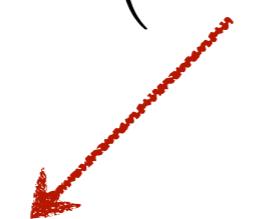
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$$g_1(x) = N_1 \frac{(1-x)^\alpha \ x^\sigma}{(1-\hat{x})^\alpha \ \hat{x}^\sigma}$$

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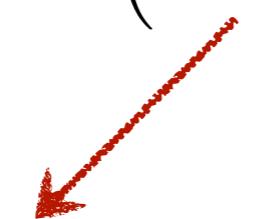


$$D_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{P_\perp^2}{g_{3A}}} + \lambda_{FB} k_\perp^2 e^{-\frac{P_\perp^2}{g_{3B}}} \right)$$

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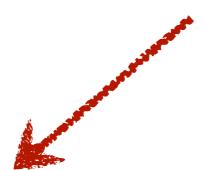
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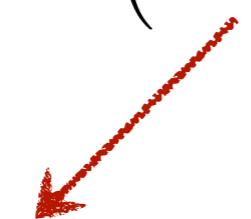
$$g_1(x) = N_1 \frac{(1-x)^\alpha \ x^\sigma}{(1-\hat{x})^\alpha \ \hat{x}^\sigma}$$



$$g_3(z) = N_3 \frac{(z^\beta + \delta)(1-z)^\gamma}{(\hat{z}^\beta + \delta)(1-\hat{z})^\gamma}$$

MAPTMD22 – Parameterization of TMDs

$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{k_\perp^2}{g^{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g^{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g^{1C}}} \right)$$



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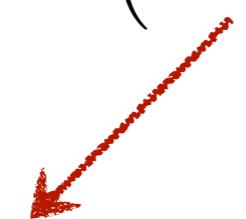


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$$g_K(b_T^2) = -g_2^2 \frac{b_T^2}{4}$$

MAPTMD22 – Parameterization of TMDs

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$$g_3(z) = N_3 \frac{(z^\beta + \delta)(1-z)^\gamma}{(\hat{z}^\beta + \delta)(1-\hat{z})^\gamma}$$

11 parameters for TMD PDF
+ 1 for NP evolution + 9 for TMD FF
= 21 free parameters

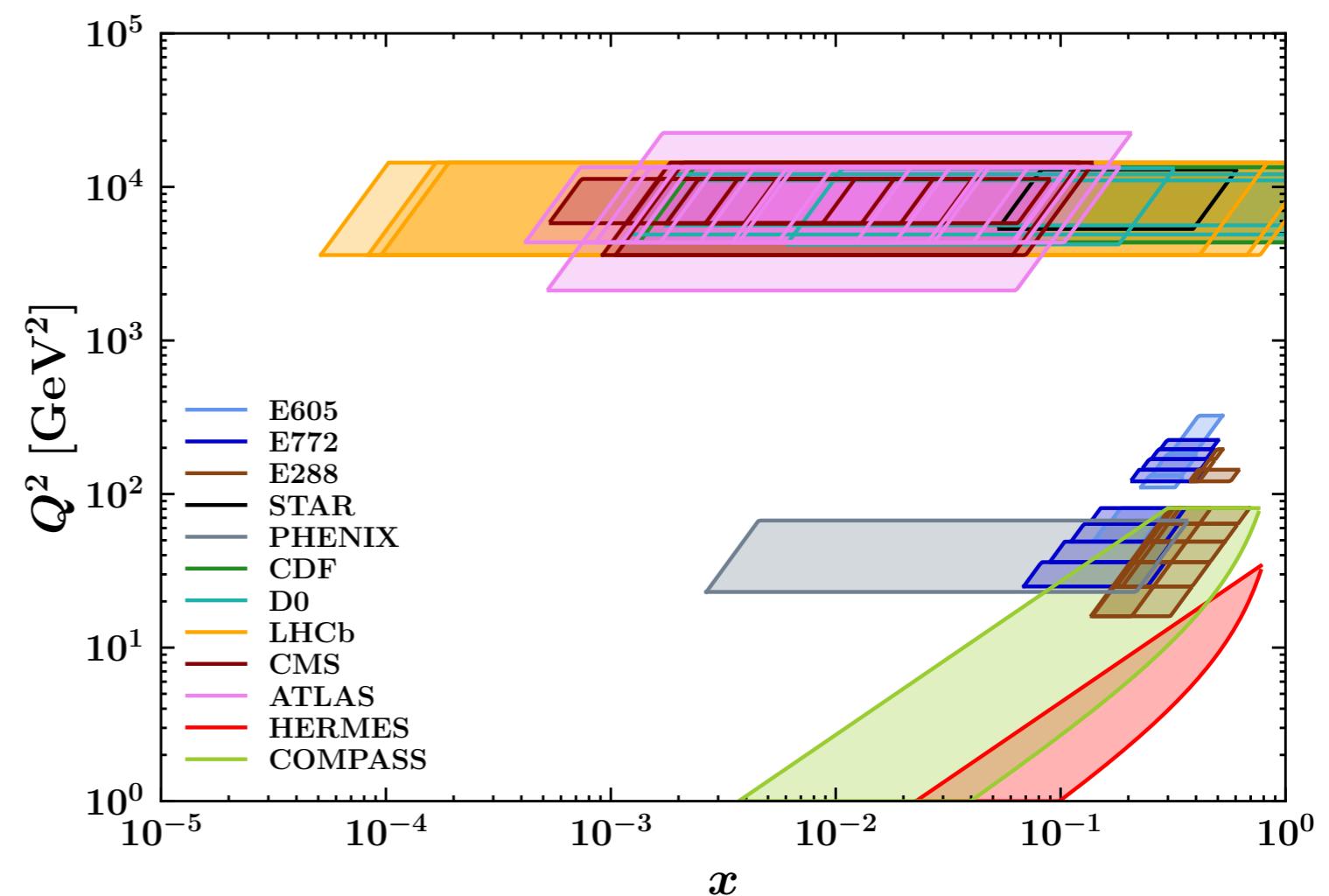
MAPTMD22 – Datasets included

Drell–Yan

484 experimental points

SIDIS

1547 experimental points



Total: 2031 fitted experimental points

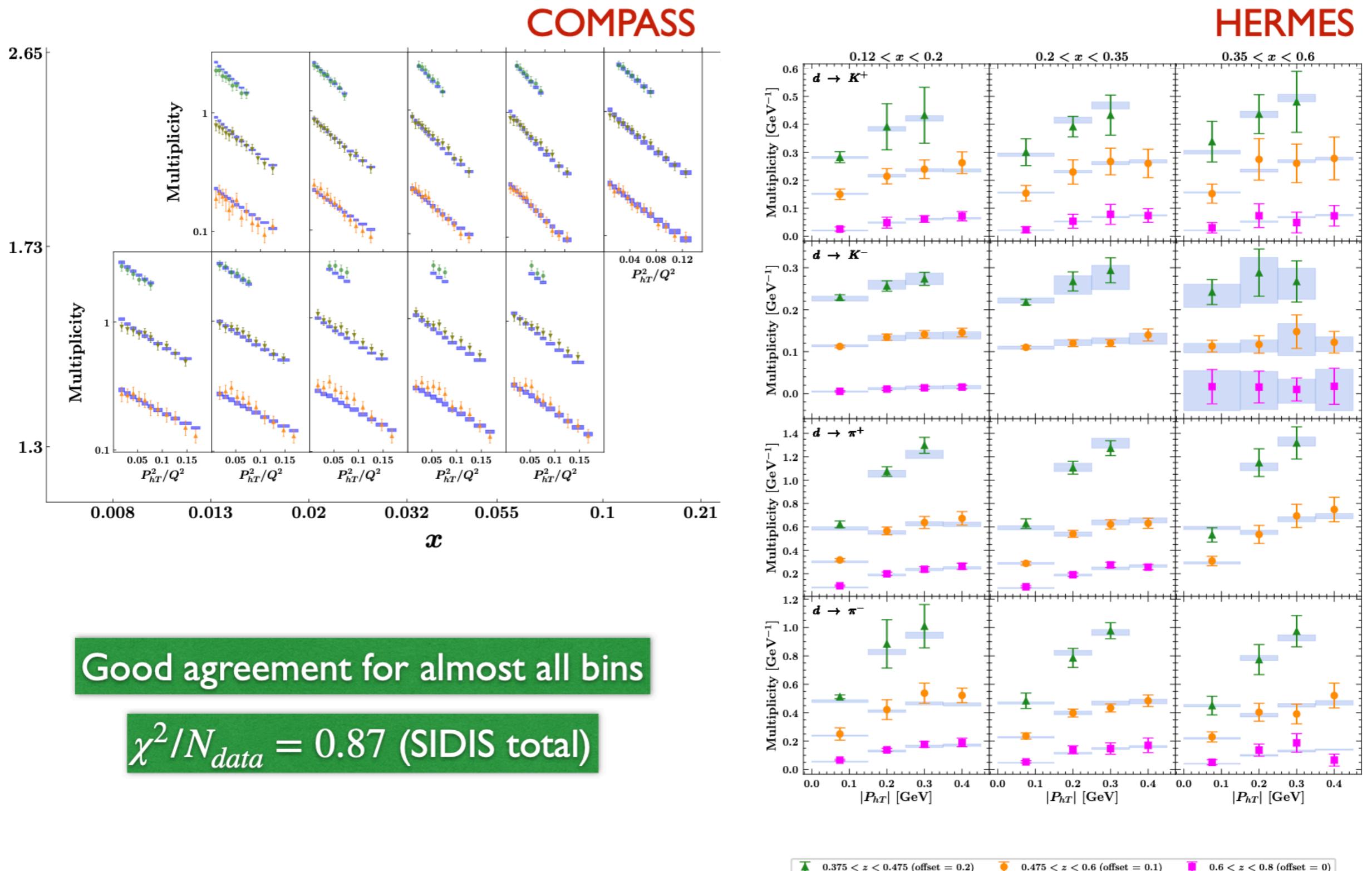


BEAST 1: NORMALIZATION

Slide by A. Bacchetta

see talk on Monday

MAPTMD22 – Quality of the fit



MAPTMD22 – Results

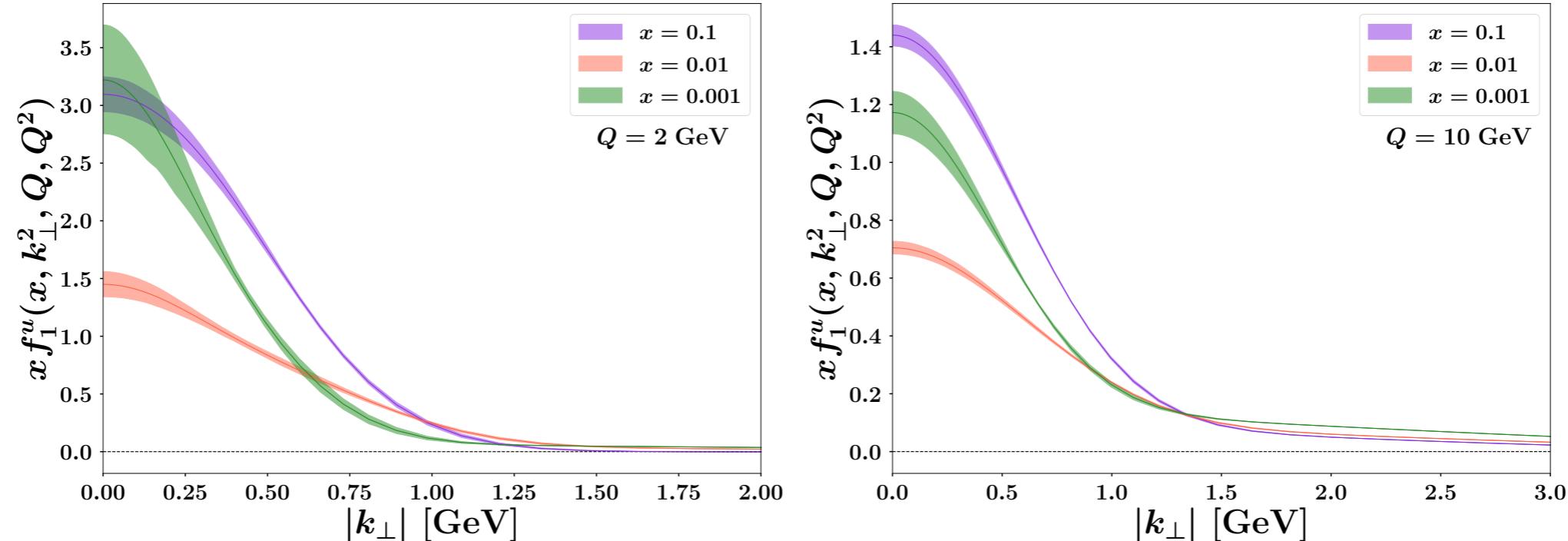


FIG. 13: The TMD PDF of the up quark in a proton at $\mu = \sqrt{\zeta} = Q = 2 \text{ GeV}$ (left panel) and 10 GeV (right panel) as a function of the partonic transverse momentum $|k_\perp|$ for $x = 0.001, 0.01$ and 0.1 . The uncertainty bands represent the 68% CL.

MAPTMD22 – Results

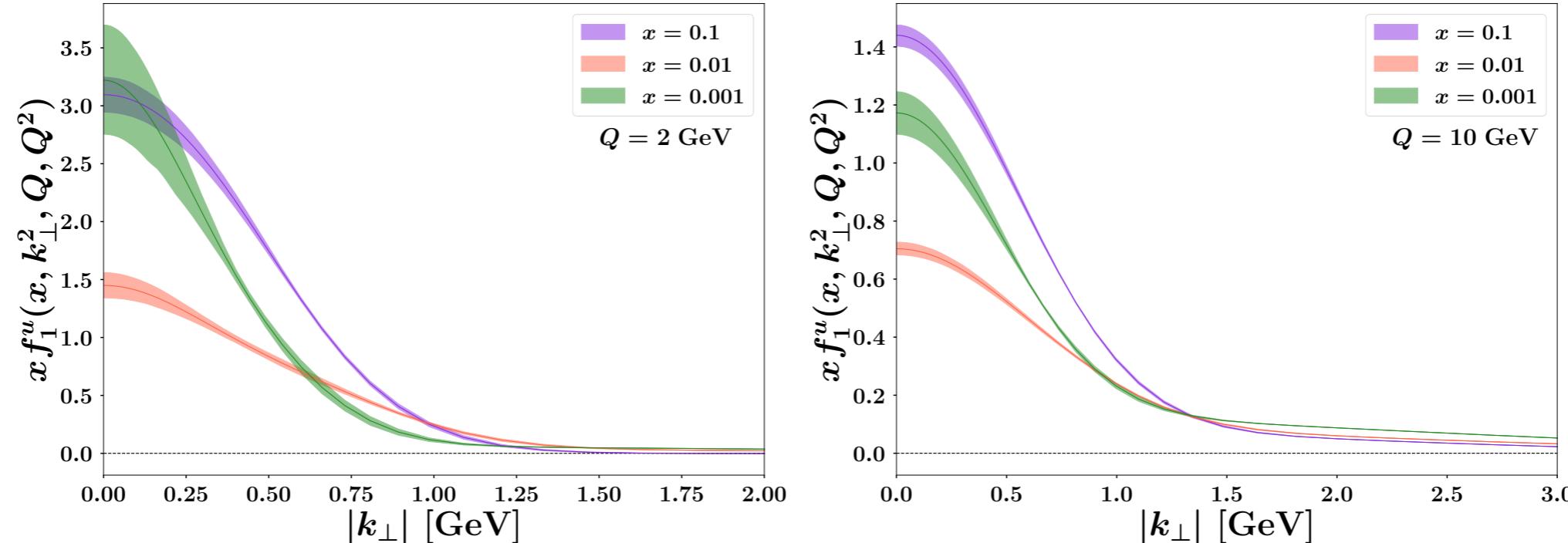


FIG. 13: The TMD PDF of the up quark in a proton at $\mu = \sqrt{\zeta} = Q = 2$ GeV (left panel) and 10 GeV (right panel) as a function of the partonic transverse momentum $|k_\perp|$ for $x = 0.001, 0.01$ and 0.1 . The uncertainty bands represent the 68% CL.

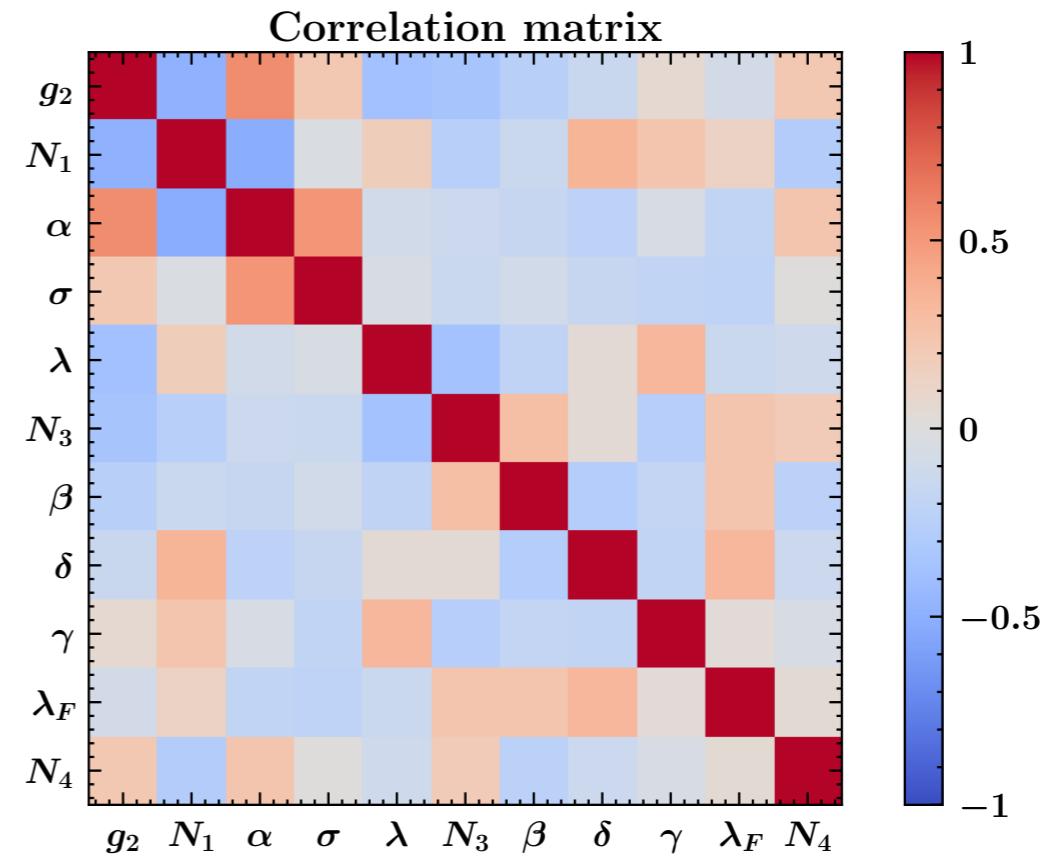
As usual, the rigidity of the functional form plays a role
and probably leads to underestimated bands

What about new data sets?



EIC impact studies with PV17

Parameter	Average over replicas
g_2	0.1171 ± 0.0145
N_1	0.283 ± 0.0368
α	2.2393 ± 1.2967
σ	-0.1416 ± 0.0959
λ	0.2548 ± 0.2549
N_3	0.2203 ± 0.0222
β	2.9304 ± 0.9978
δ	0.1175 ± 0.0506
γ	2.4736 ± 0.1649
λ_F	7.5475 ± 3.2037
N_4	0.0318 ± 0.0068

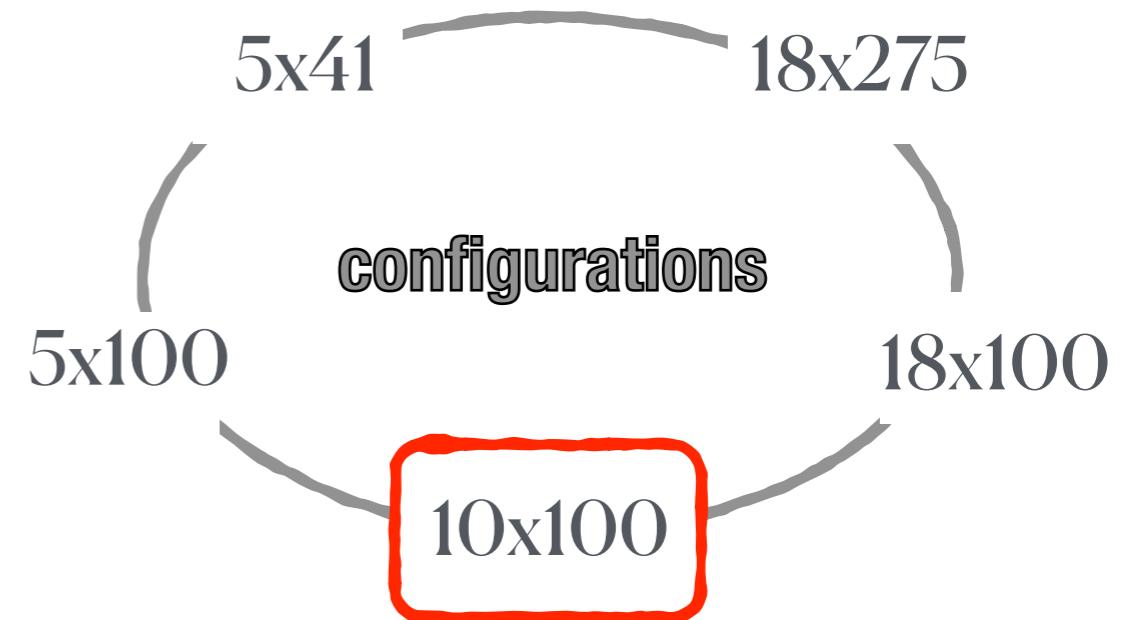
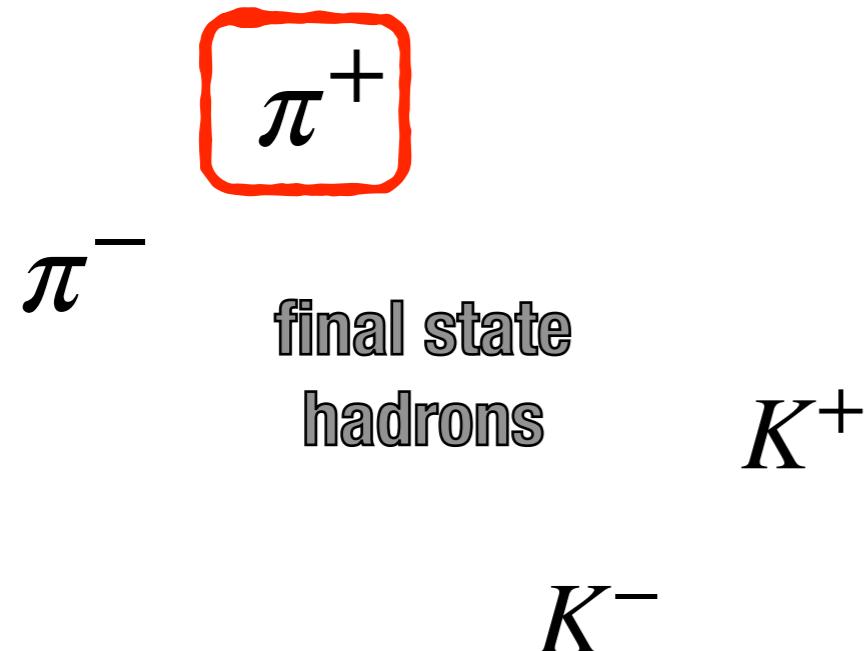


with **NangaParbat**
 a **new fit** with uncertainties
 similar to **PV17**
 $\chi^2_{\text{d.o.f.}} = 1.14 \pm 0.06$

~ 2000 fitted data

EIC impact studies with PV17

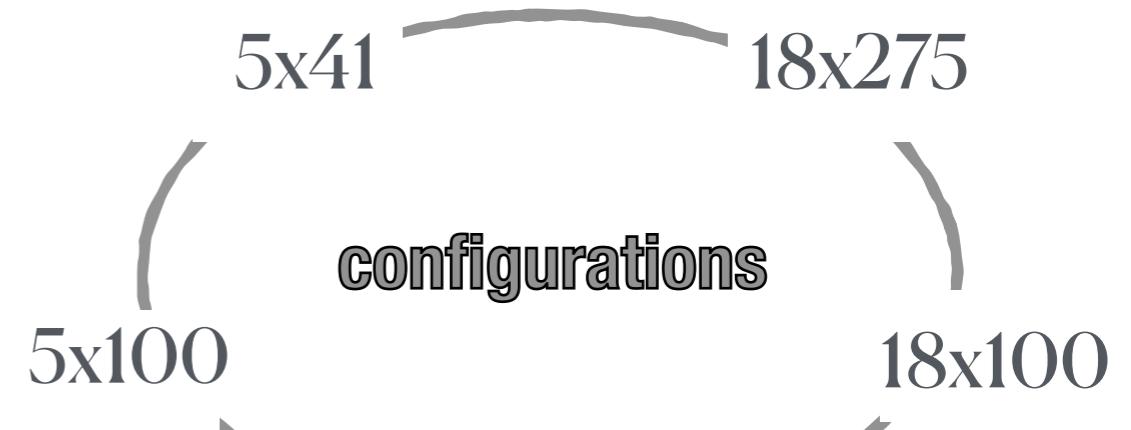
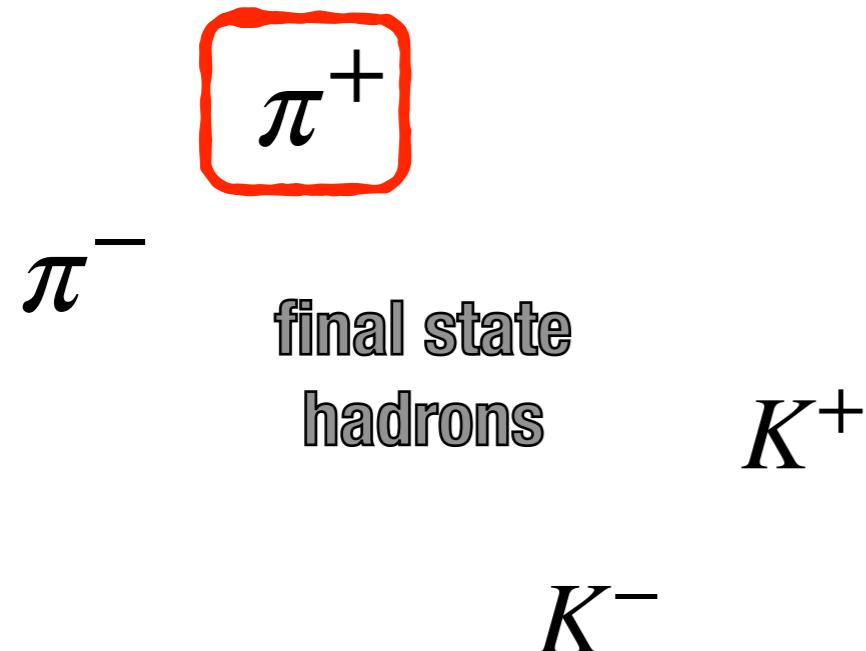
generation of pseudo data



```
Name,unpol.10x100_pip_ACC_opt8_cut
Comment,Ralf's pseudo data for EIC.
Reference,Ralf
Process type,SIDIS
Number of points,3837
Number of uncorr.errors,2
Number of corr.errors,0
Number of norm.errors,1
List of norm.errors (relative),0.03
Total cross-section nomalized,False
List of points
Point id,process
id,s[GeV^2],<Q>[GeV],Qmin[GeV],Qmax[GeV],<x>,xMin,xMax,<z>,zMin,zMax,<pT>[GeV],pTMin[GeV],pTMax[GeV],xSec,Uncorr.Err.0,Uncorr.Err.1,Th.Fac
tor,FiducialCuts,yMin,yMax,W2min[GeV^2],W2max[GeV^2],TargetMass[GeV],ProductMass[GeV]
```

EIC impact studies with PV17

generation of pseudo data



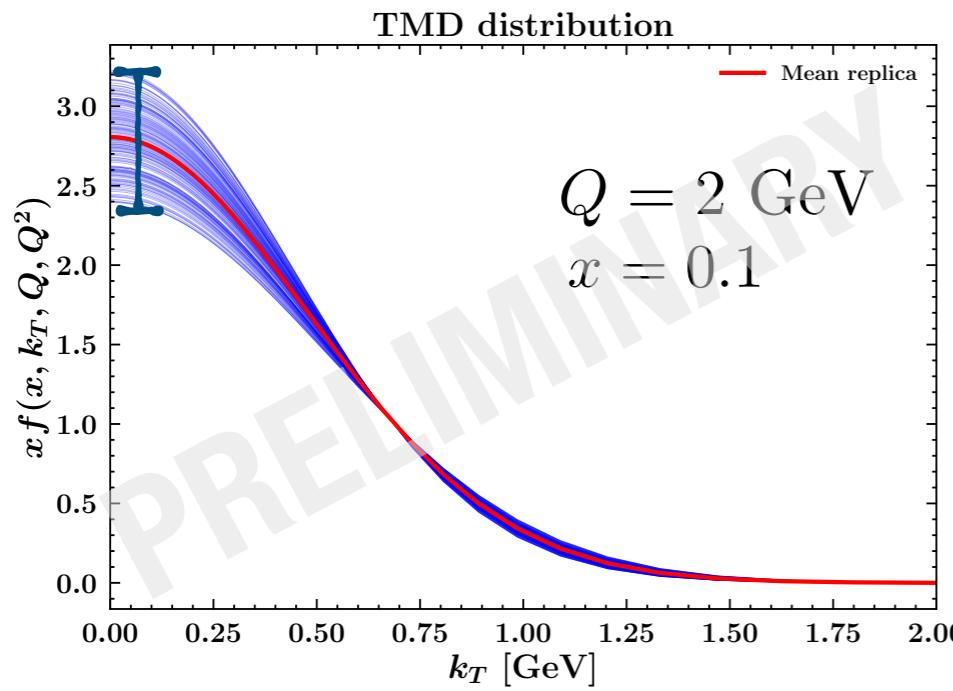
~ 2500
pseudodata points

central value of pseudo data obtained
using average parameters of the PV17 baseline fit

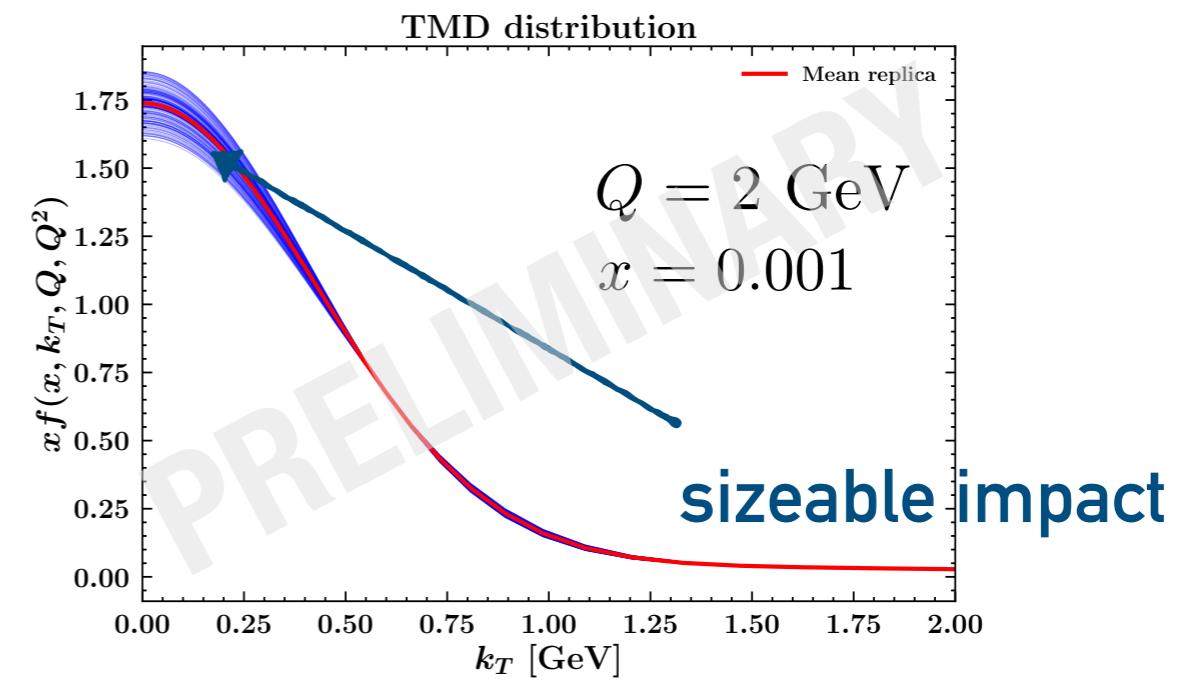
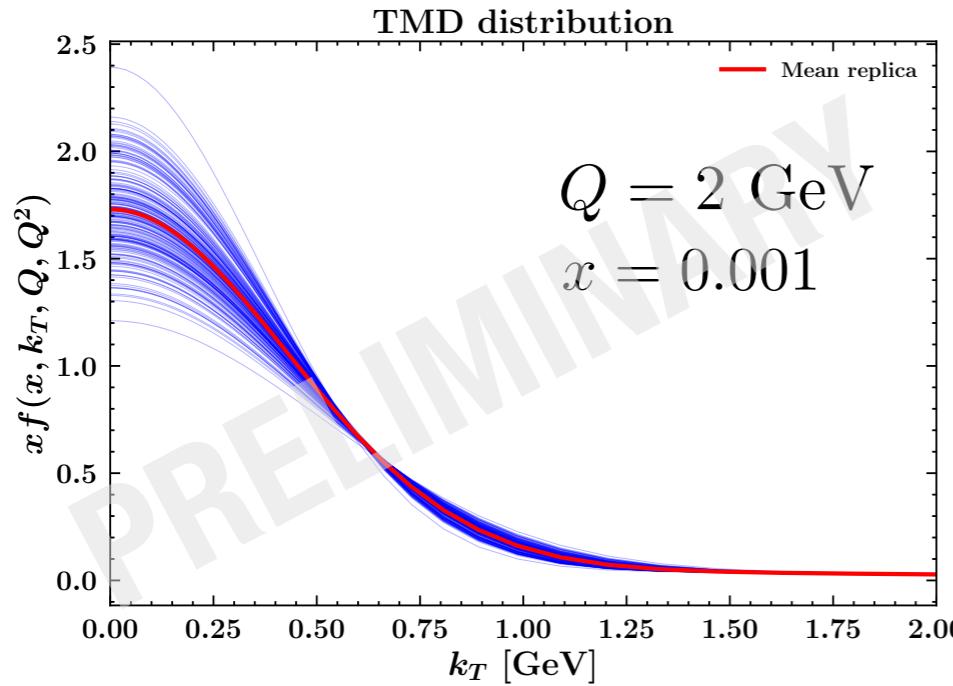
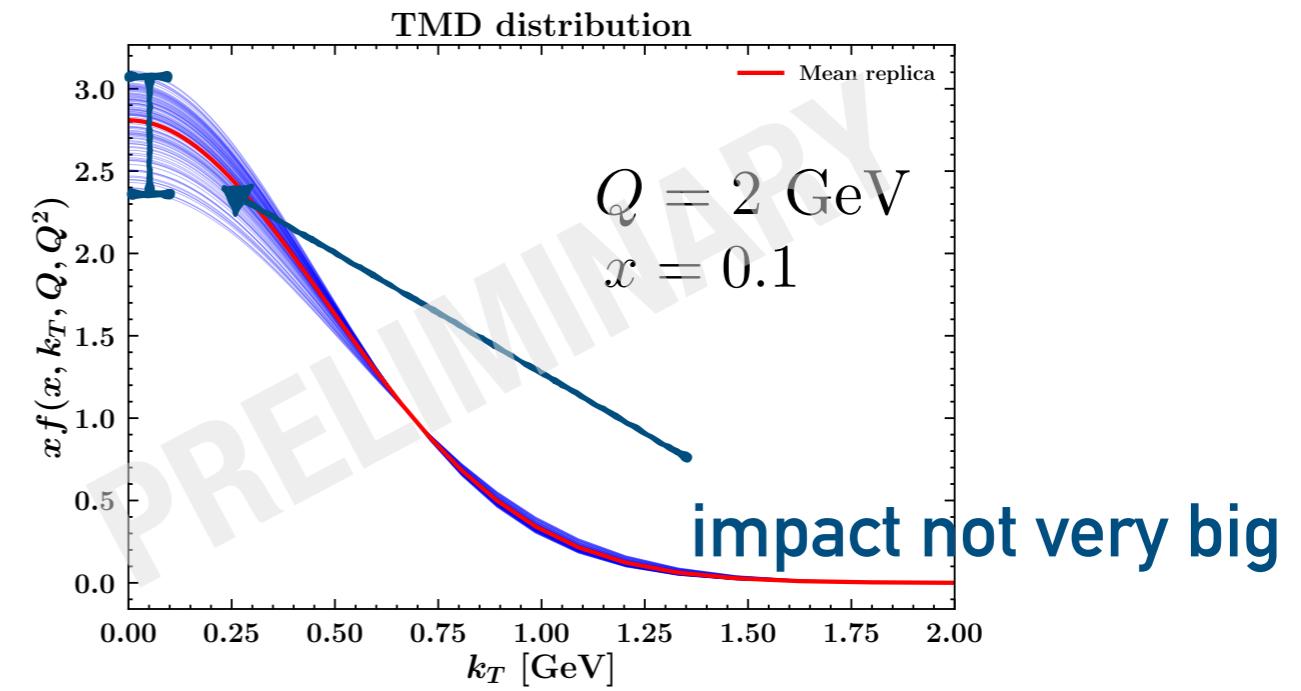
uncertainties of pseudo data
are given by simulations done
by the EIC SIDIS working group

EIC impact studies with PV17

PV17 baseline



PV17 baseline + EIC



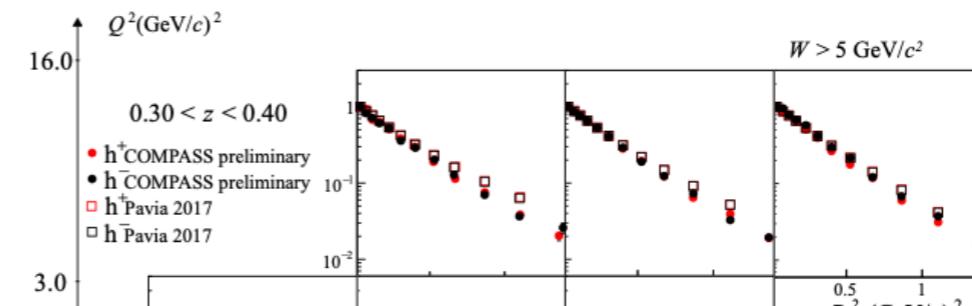
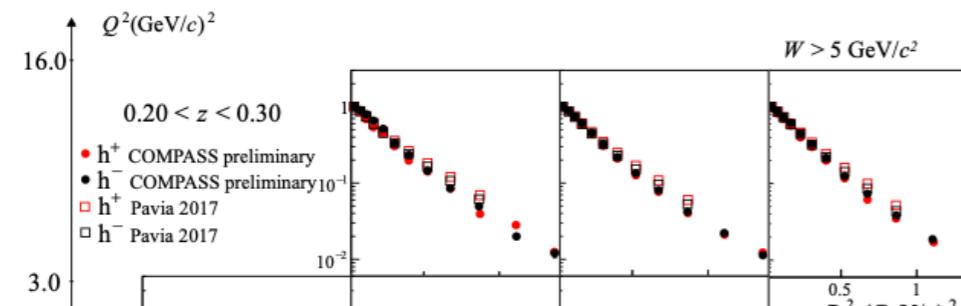
COMPASS Proton predictions with PV17

Measurement on LH₂: Results for the P_T -distributions

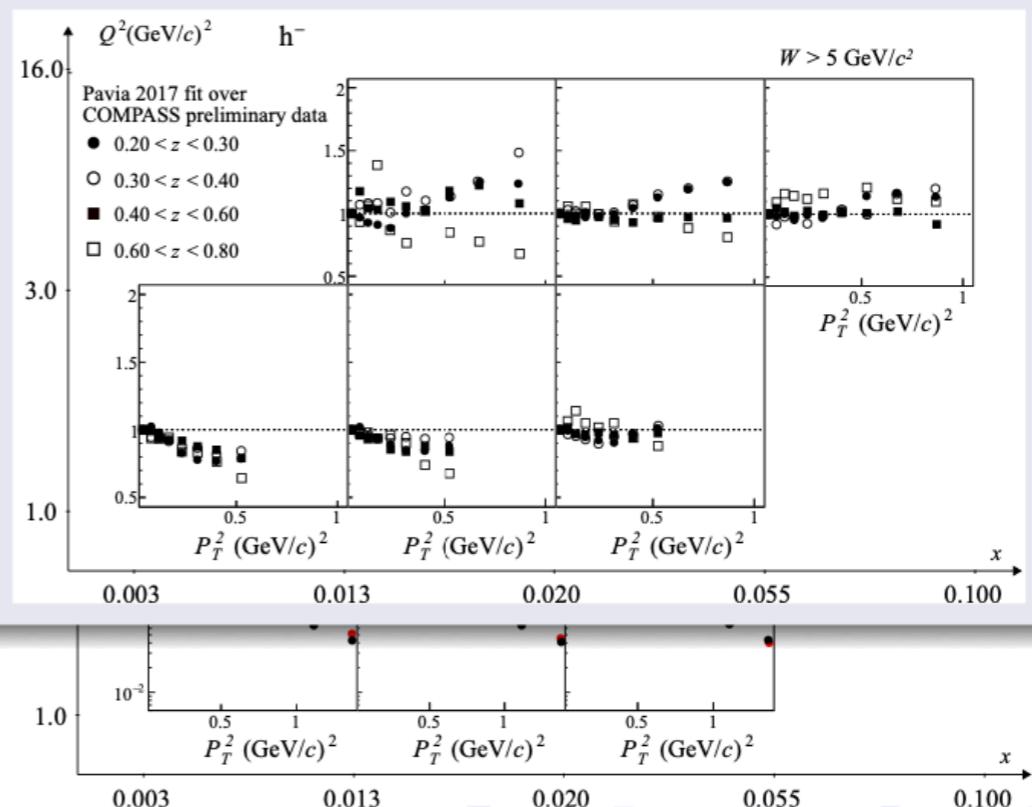
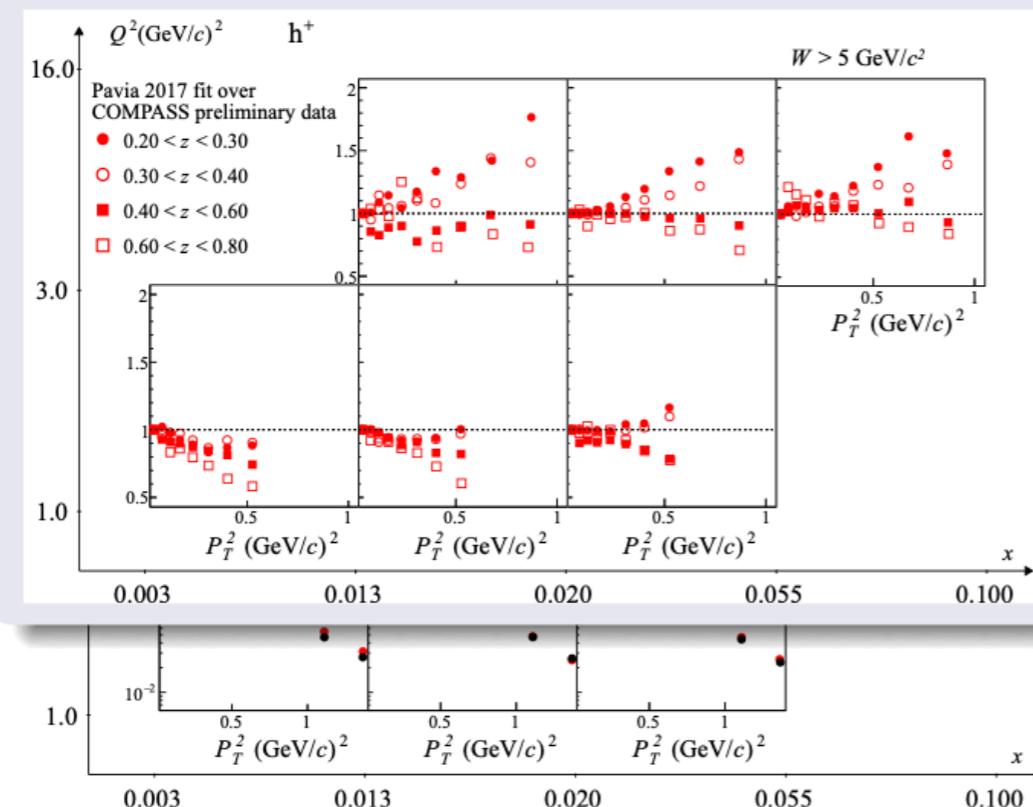


Comparison with Pavia 2017 fit [A. Bacchetta et al., JHEP 06 (2017) 081]

- SIDIS ep(D) → e π^\pm (K $^\pm$)X (HERMES)
- SIDIS μ D → μ hX (COMPASS)
- Drell-Yan (E228, E605)
- Z boson production (CDF, D0)



The ratio of the prediction from the fit over the preliminary data:





**JLab20+ impact studies
with MAPTMD22**

JLab20+ Impact Study

1000000	1	1	1	1	1	1.70	0.06	0.05	0.06	0.67	28
1000001	2	1	1	1	2	1.74	0.06	0.06	0.16	0.69	76
1000002	3	1	1	1	3	1.71	0.06	0.07	0.26	0.68	119
1000003	4	1	1	1	4	1.68	0.06	0.07	0.35	0.67	159
1000004	5	1	1	1	5	1.69	0.06	0.06	0.45	0.67	205
1000005	6	1	1	1	6	1.71	0.06	0.06	0.55	0.68	280
1000006	7	1	1	1	7	1.70	0.06	0.06	0.65	0.68	297
1000007	8	1	1	1	8	1.71	0.06	0.06	0.75	0.67	375
1000008	9	1	1	1	9	1.70	0.06	0.06	0.85	0.68	413
1000009	10	1	1	1	10	1.71	0.06	0.07	0.95	0.68	449
1000010	11	1	1	1	11	1.72	0.06	0.06	1.05	0.68	462
1000011	12	1	1	1	12	1.69	0.06	0.06	1.15	0.67	548
1000012	13	1	1	1	13	1.70	0.06	0.06	1.25	0.68	592
1000013	14	1	1	1	14	1.71	0.06	0.06	1.35	0.68	655
1000014	15	1	1	1	15	1.70	0.06	0.06	1.45	0.68	723
1000015	16	1	1	1	16	1.70	0.06	0.06	1.55	0.68	788
1000016	17	1	1	1	17	1.70	0.06	0.06	1.65	0.67	816
1000017	18	1	1	1	18	1.69	0.06	0.06	1.75	0.67	883
1000018	19	1	1	1	19	1.70	0.06	0.06	1.85	0.68	968
1000019	20	1	1	1	20	1.70	0.06	0.06	1.95	0.68	929

Kinematics

19 bins in Q^2 : [1, 20] GeV 2 ($\Delta Q^2 = 1$ GeV 2)

20 bins in x : [0.04, 0.84] ($\Delta x = 0.04$)

20 bins in z : [0.05, 0.95] ($\Delta z = 0.05$)

80 bins in q_T : [0.05, 7.95] GeV ($\Delta q_T = 0.1$ GeV)

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1000019	20	1	1	1	20	1.70	0.06	0.06	1.95	0.68	929

Pseudodata generation

Central value obtained using **average parameters** of MAPTMD22 baseline fit

Uncertainties of pseudodata

Stat $1/\sqrt{N}$

Sys ? (1 – 5%)

JLab20+ Impact Study

Included dataset

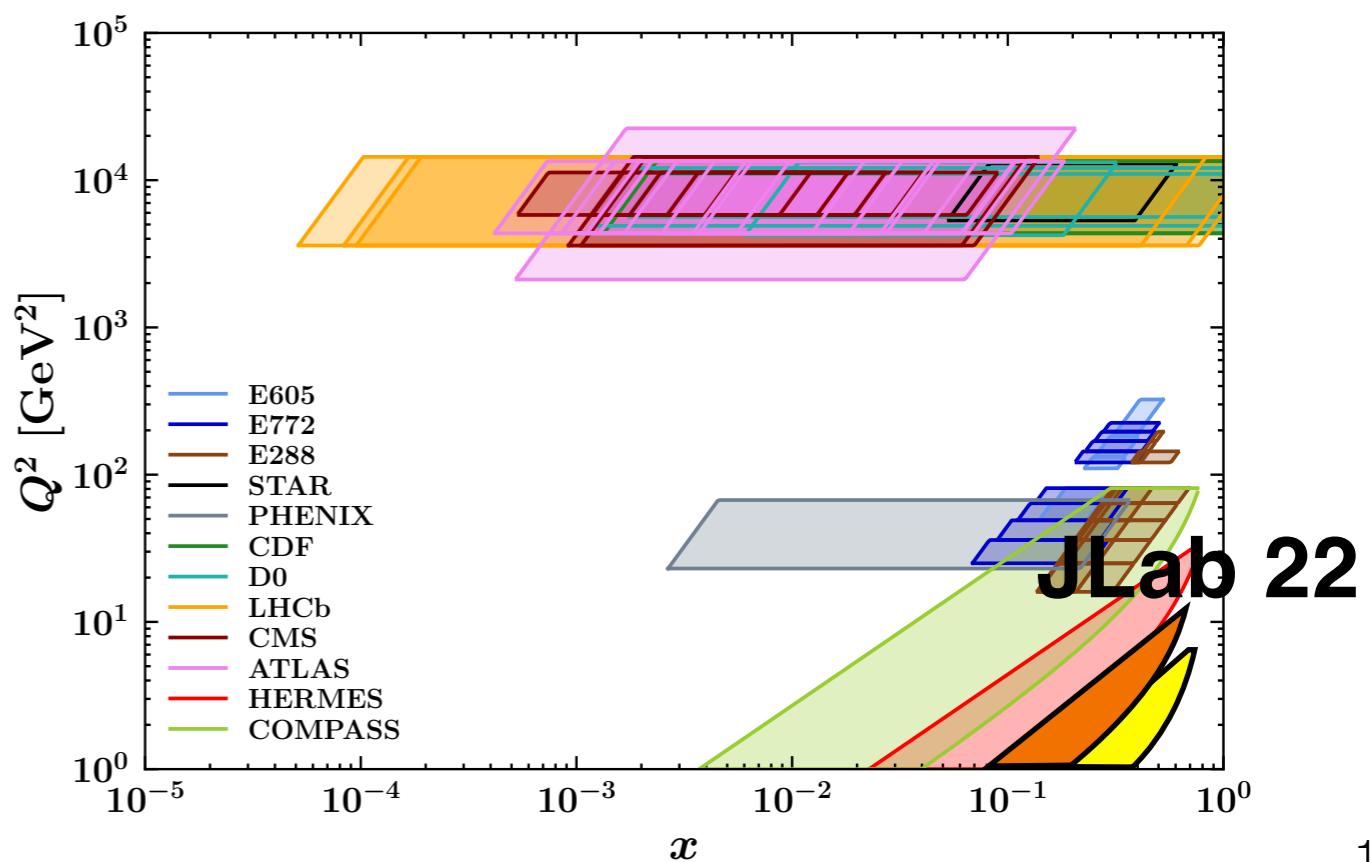
$$Q^2 > 1.4 \text{ GeV}^2$$

$$0.2 < z < 0.7$$

$$P_{hT} < \min [\min [0.2 Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$

Final-state hadrons

π^+ π^-



JLab20+ Impact Study

Included dataset

$$Q^2 > 1.4 \text{ GeV}^2$$

$$0.2 < z < 0.7$$

$$P_{hT} < \min [\min [0.2 Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$

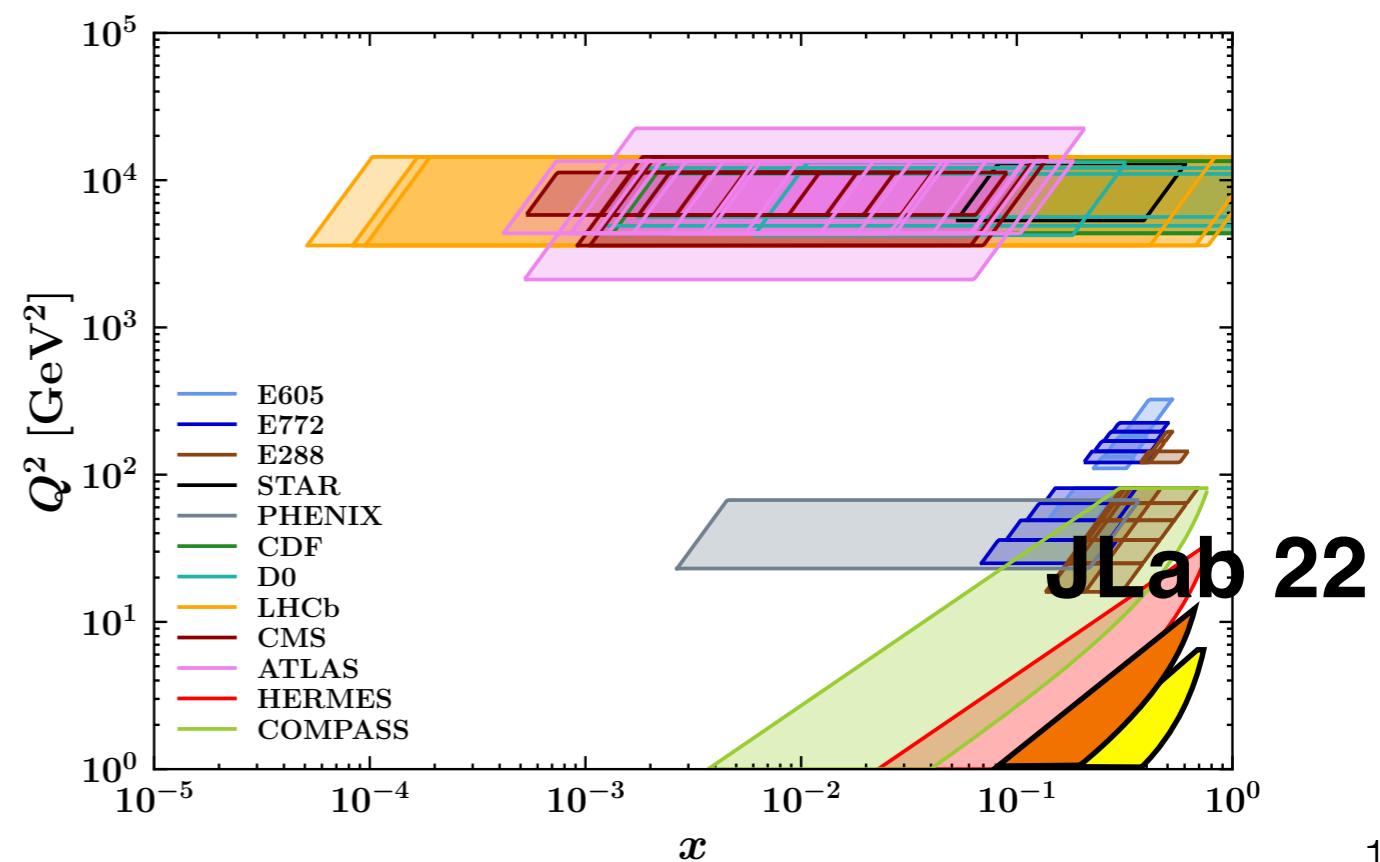
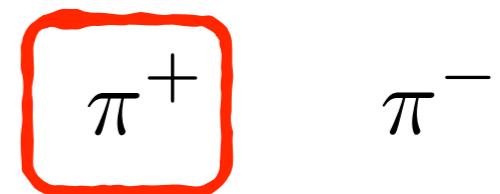
~ 2000 MAPTMD22

+

~ 25000 JLab 20+
pseudodata

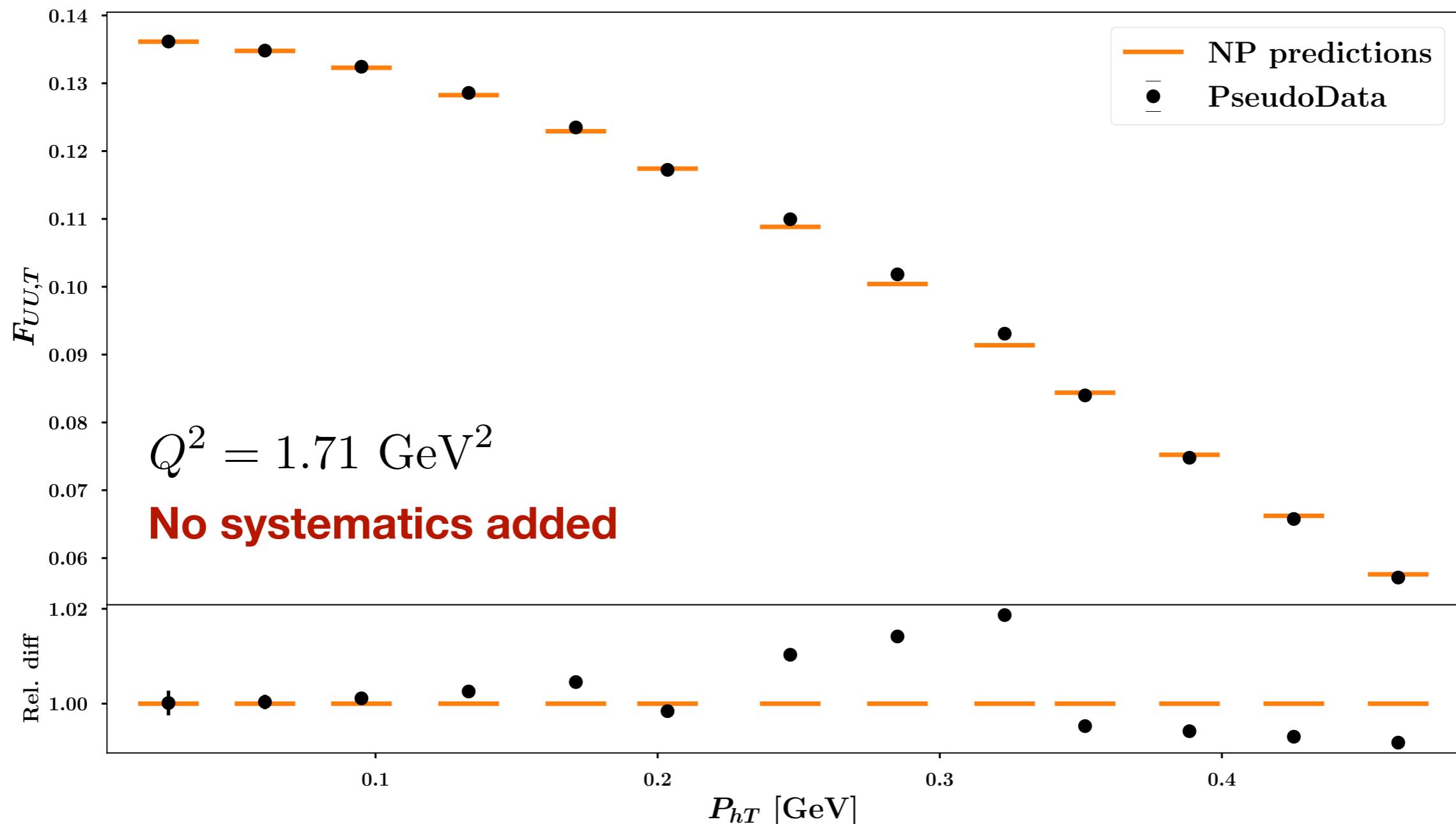
Increasing of an order of magnitude!

Final-state hadrons



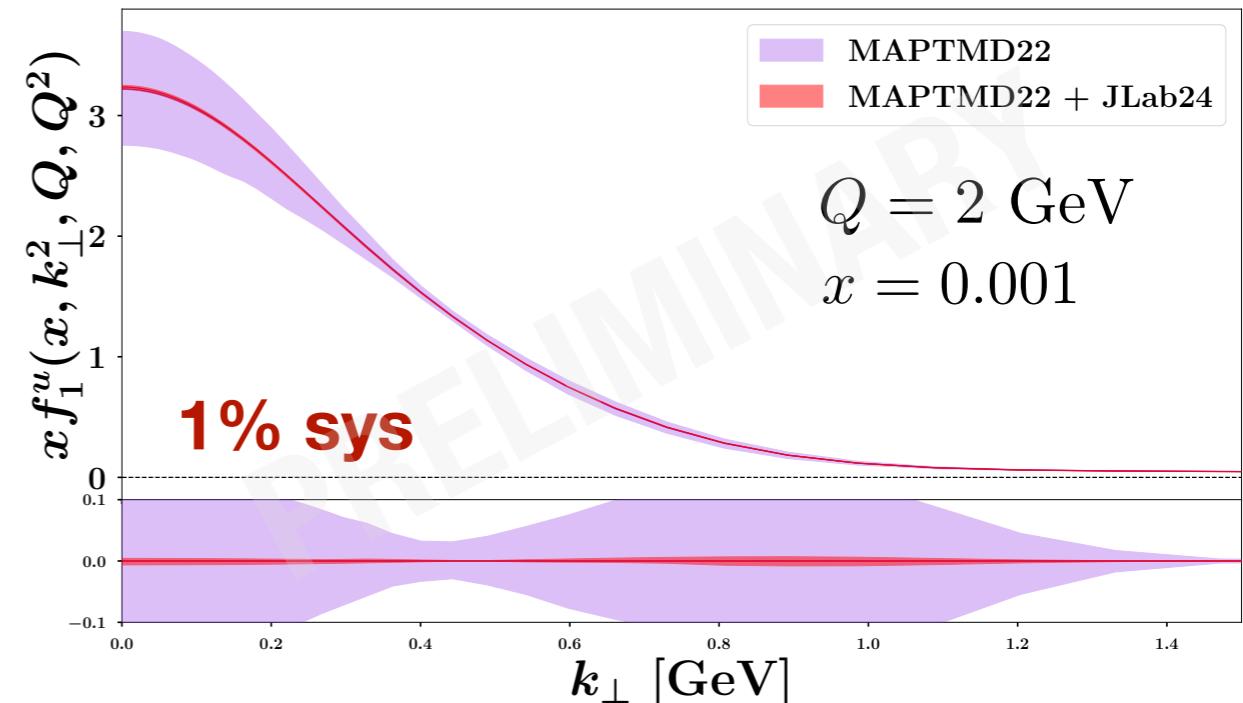
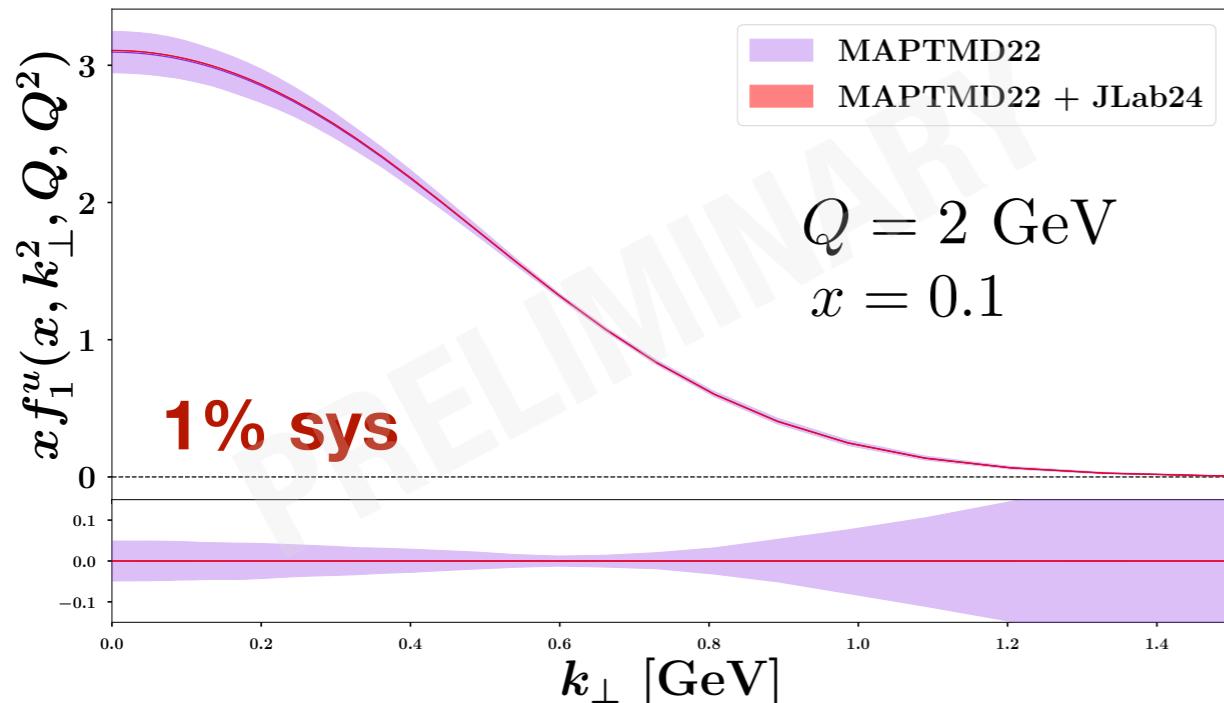
JLab20+ Impact Study

Effect of systematic uncertainties on pseudodata



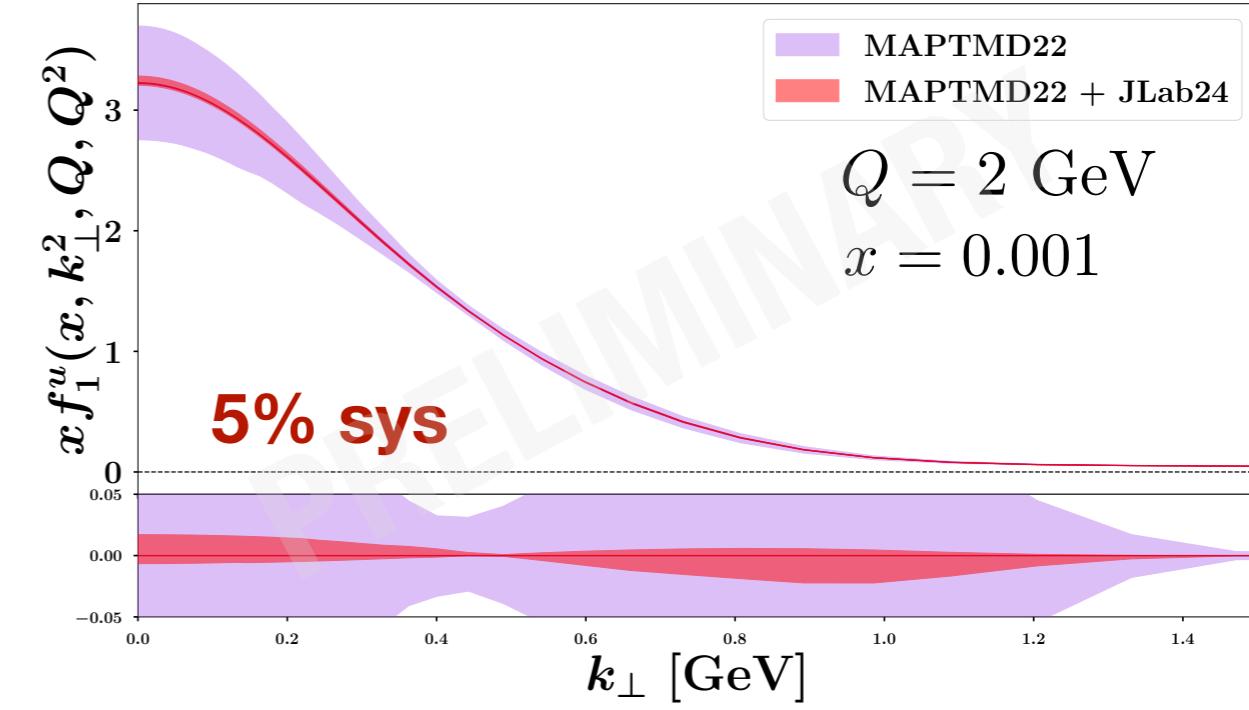
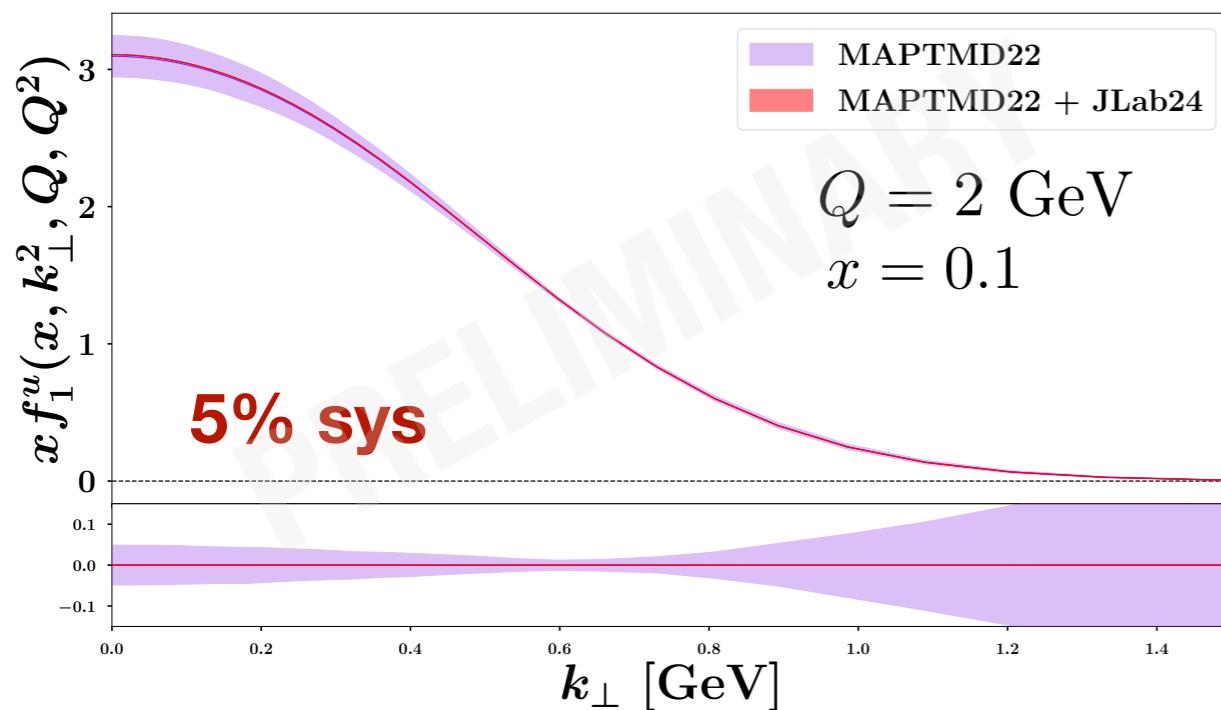
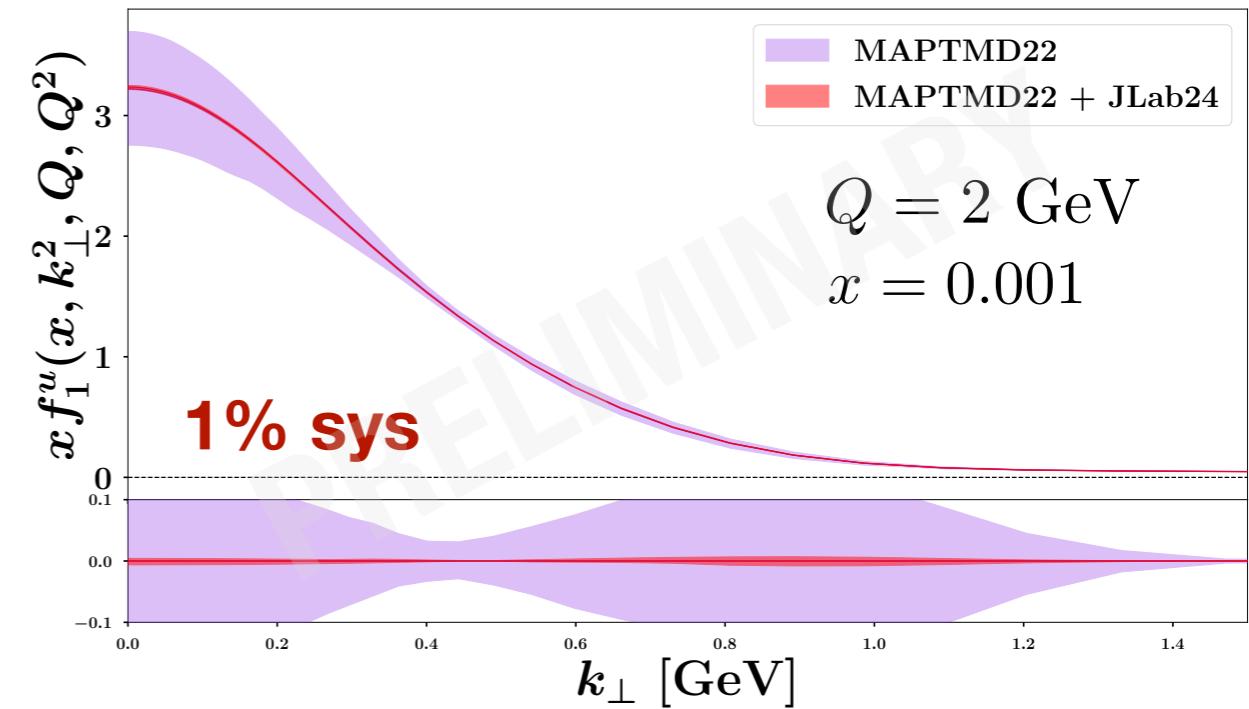
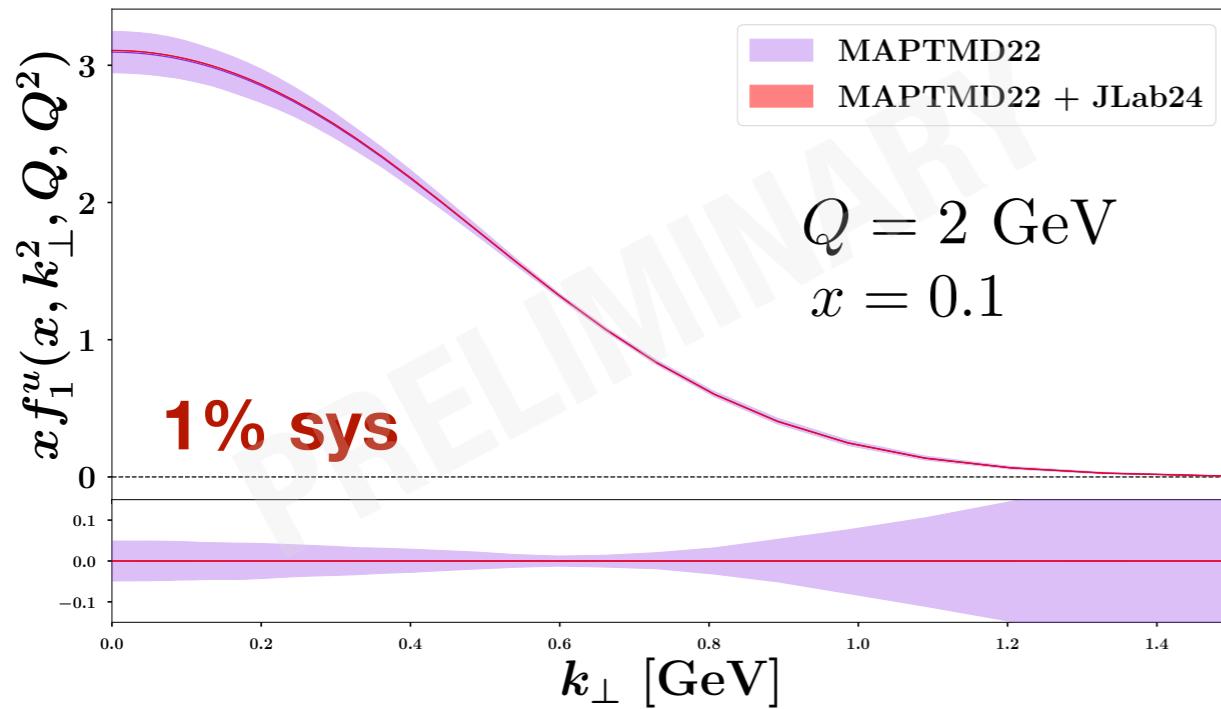
JLab20+ Impact Study

PRELIMINARY results

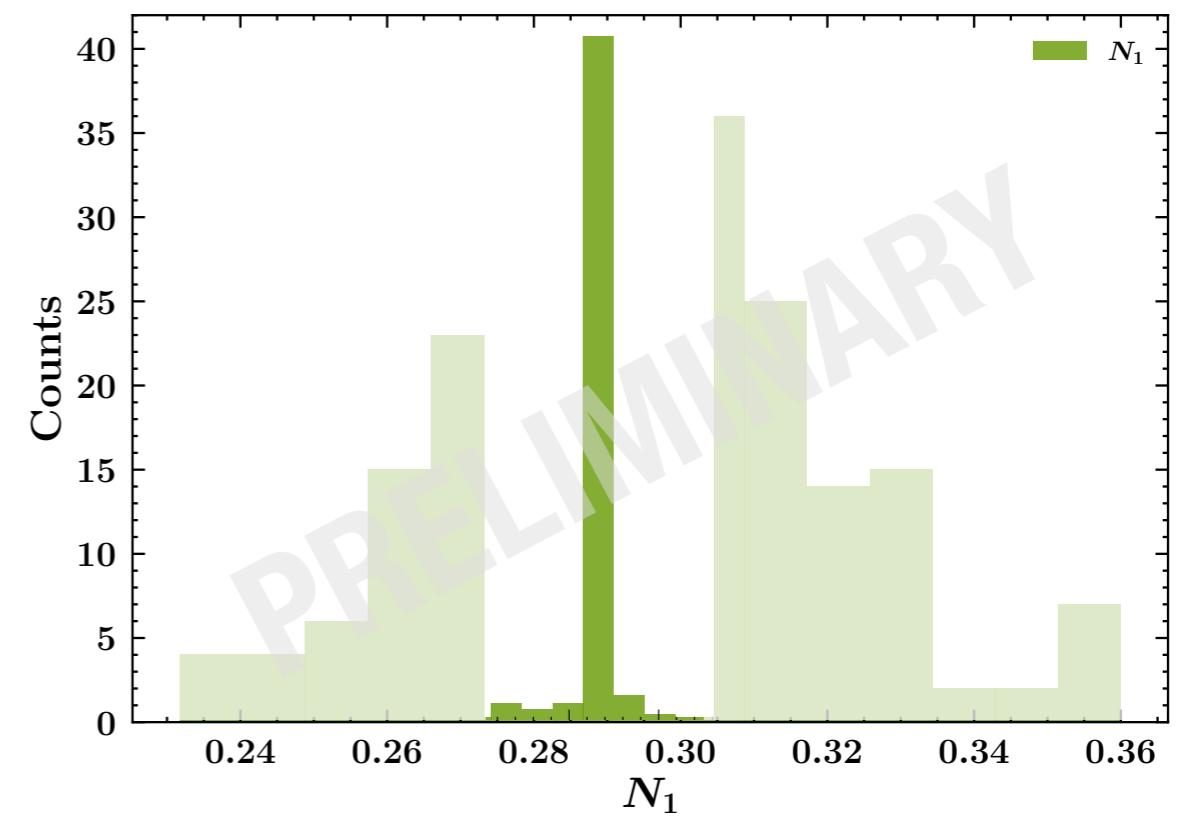
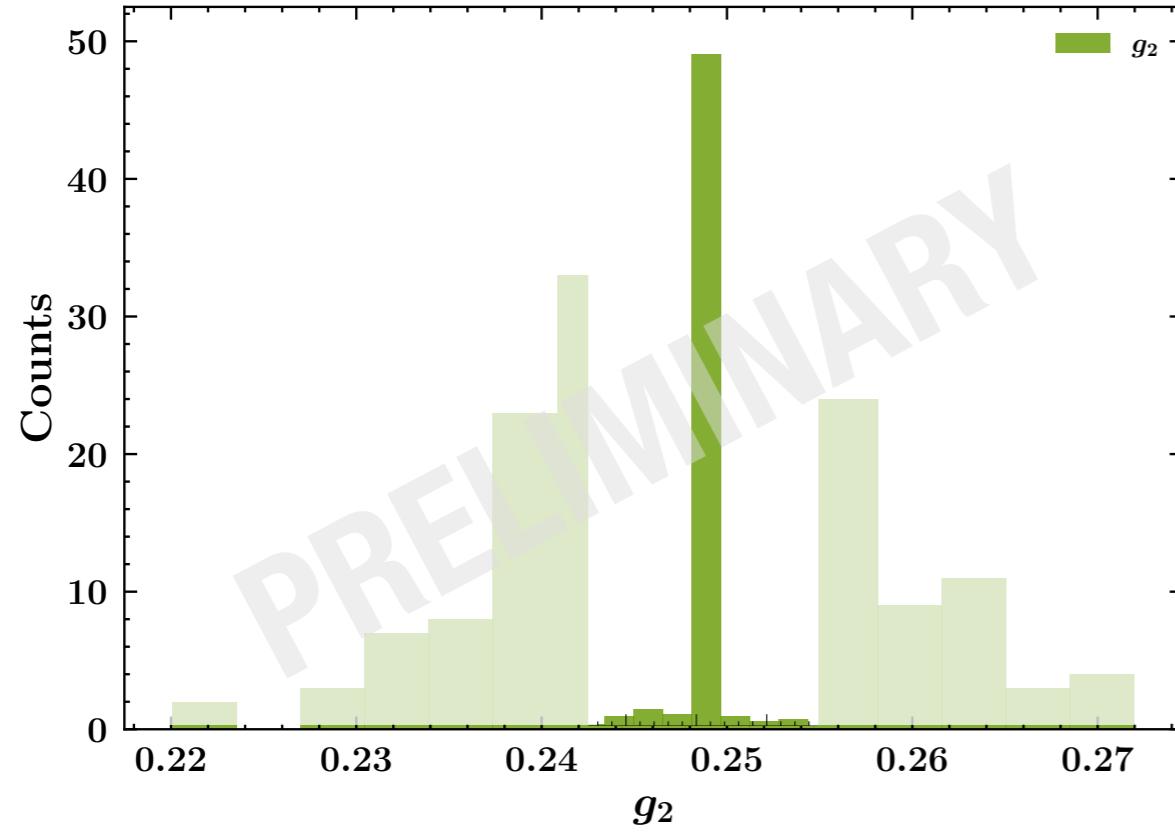


JLab20+ Impact Study

PRELIMINARY results



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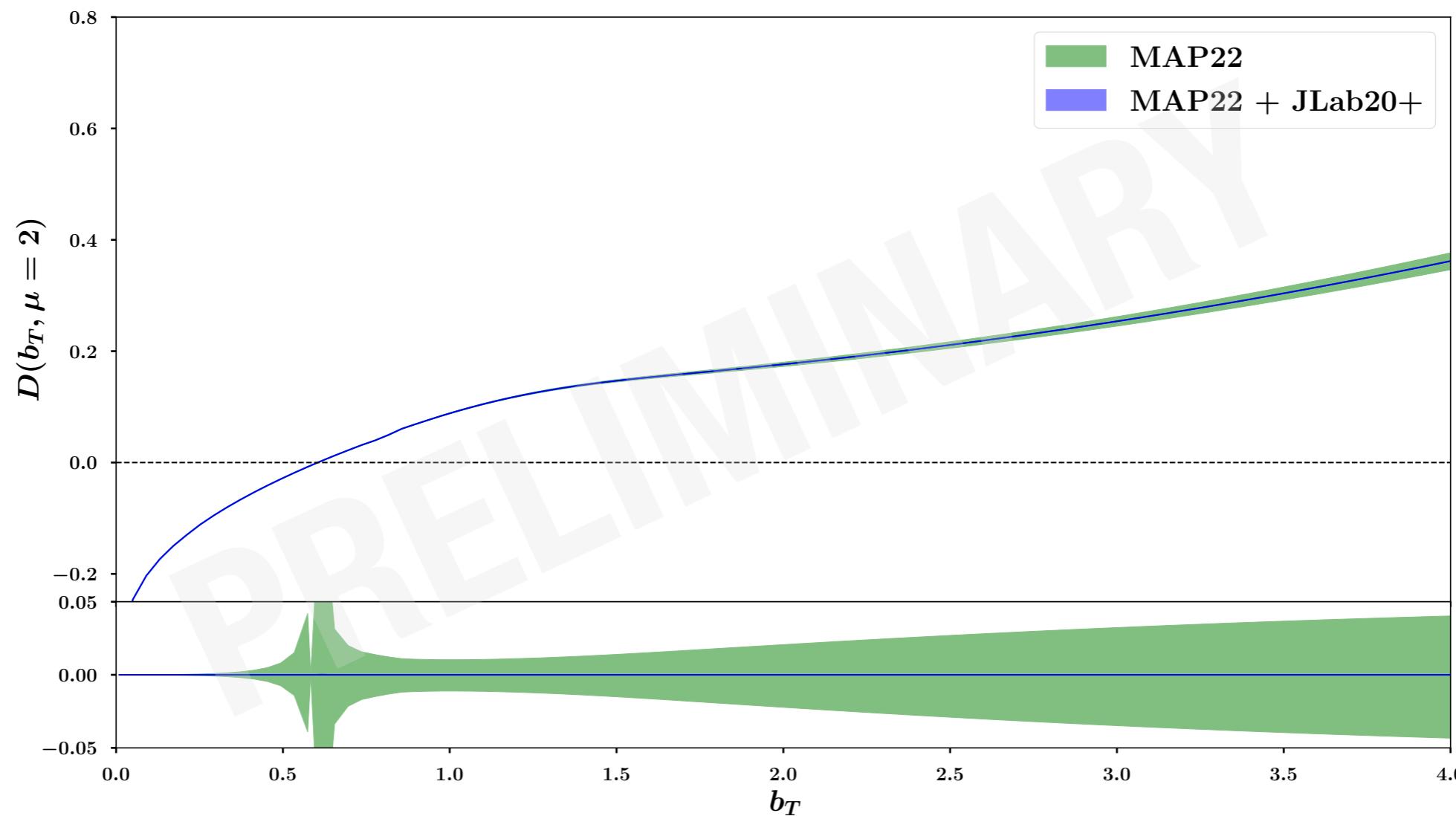
Kernel of the rapidity evolution equation

$$\frac{\partial \ln \hat{f}_1(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = K(b_T, \mu) \quad K(b_T, \mu_{b_*}) = K(b_*, \mu_{b_*}) + g_K(b_T)$$

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Outlook and Conclusions

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 - flexibility of TMD model?*

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- Old studies with PV17 show impact of EIC only at low-x
- **Preliminary** study on JLab20+ shows impact at both low- and high-x
 - number of data?*
 - flexibility of TMD model?*
- We have to understand the role of systematic errors and the methodology

BACKUP SLIDES

TMD factorization – Logarithmic counting

Orders in powers of

TMD factorization – Logarithmic counting

Orders in powers of

Hard factor and
matching
coefficient

Ingredients in
perturbative Sudakov
form factor

Accuracy	H and C	K and γ_F	γ_K	PDFs/FFs and α_S evol.
LL	0	-	1	-
NLL	0	1	2	LO
NLL'	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL'	2	2	3	NNLO
N^3LL^-	2	3	4	NNLO (NLO FF)
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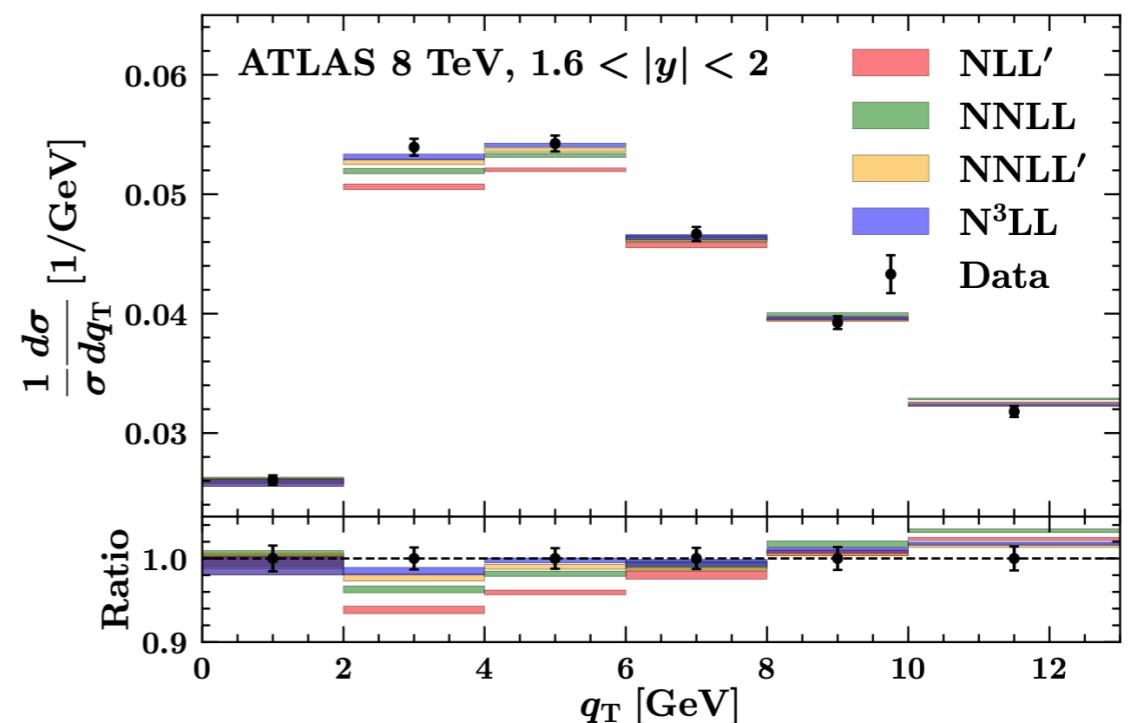
$N^3 LL = {}^3 N LL$ but with NLO collinear FF

MAPTMD22 – Normalization of SIDIS

MAPTMD22 – Normalization of SIDIS

High-Energy Drell-Yan beyond N

$$Q \sim 100 \text{ GeV}$$

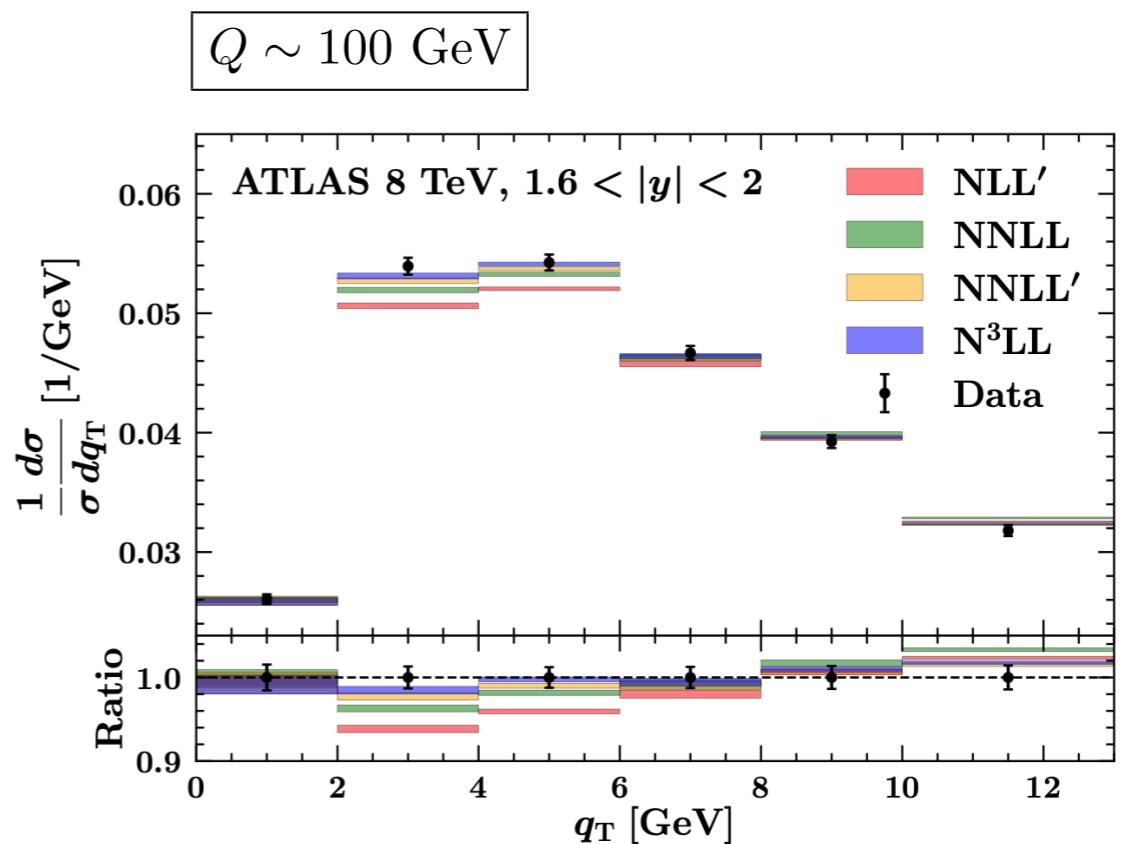


Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici,

MAPTMD22 – Normalization of SIDIS

SIDIS multiplicities beyond NLL

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Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici,

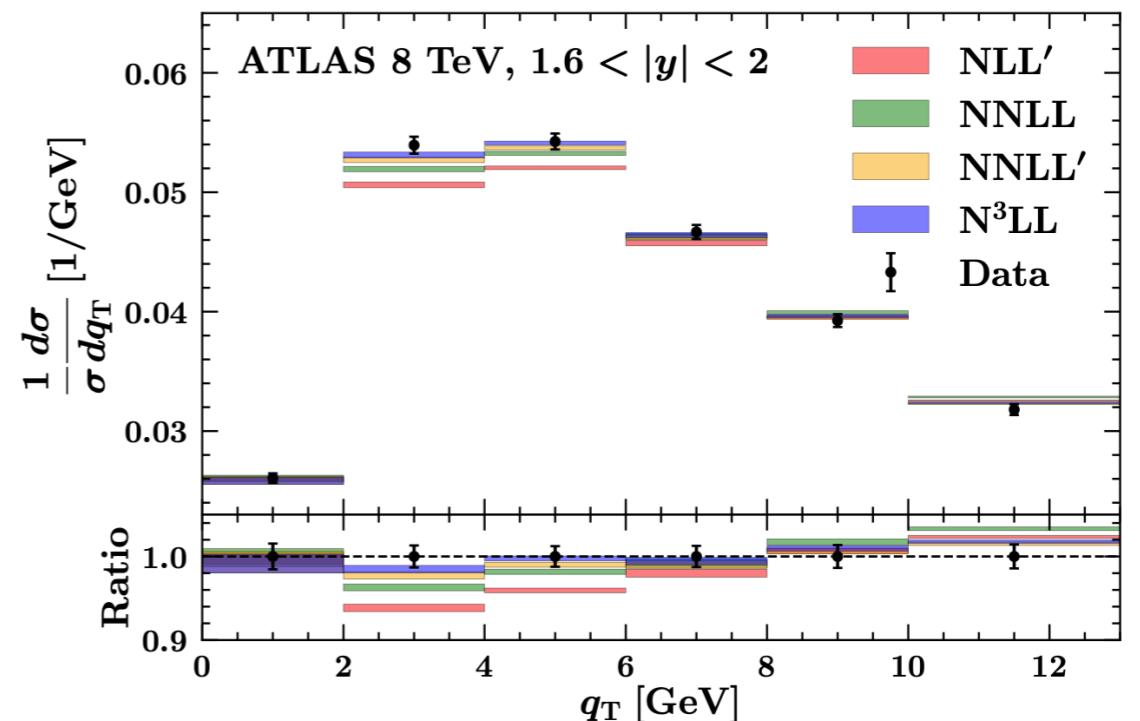
MAPTMD22 – Normalization of SIDIS

SIDIS multiplicities beyond NLL

$Q \sim 2 \text{ GeV}$

High-Energy Drell-Yan beyond NLL

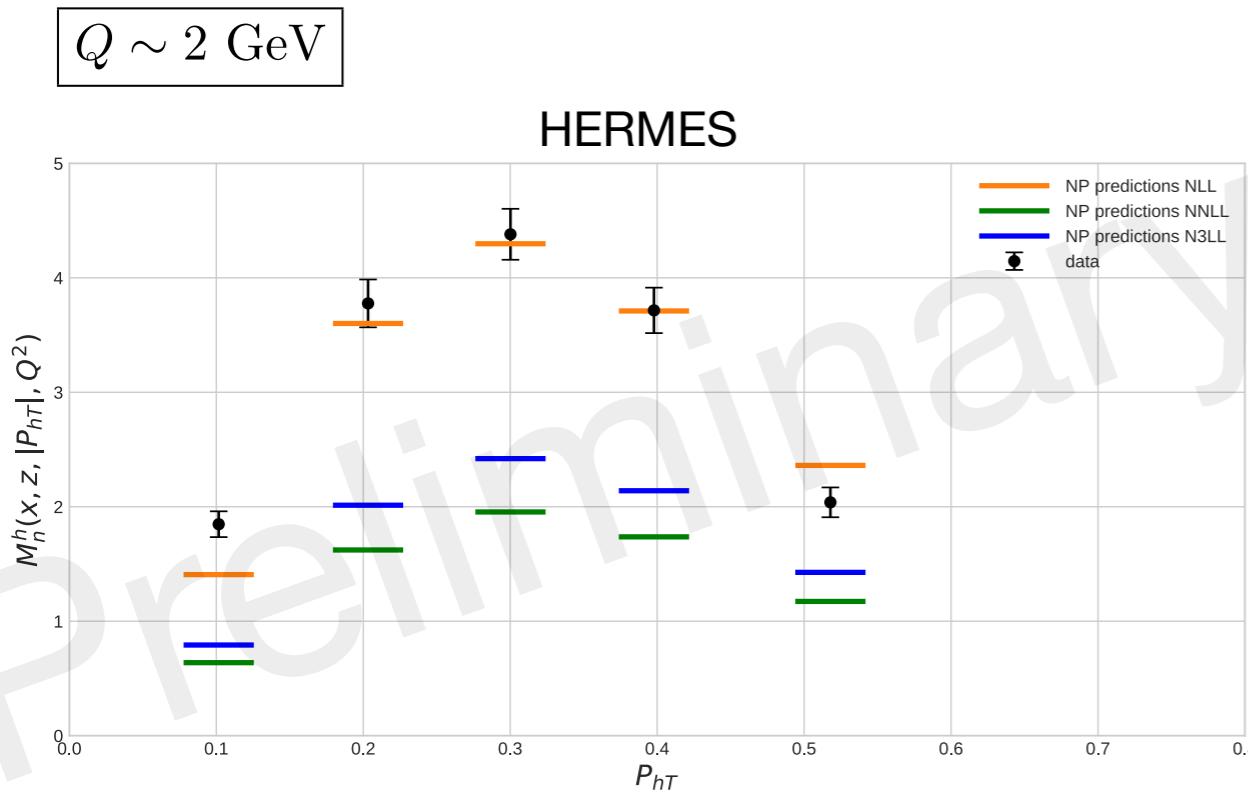
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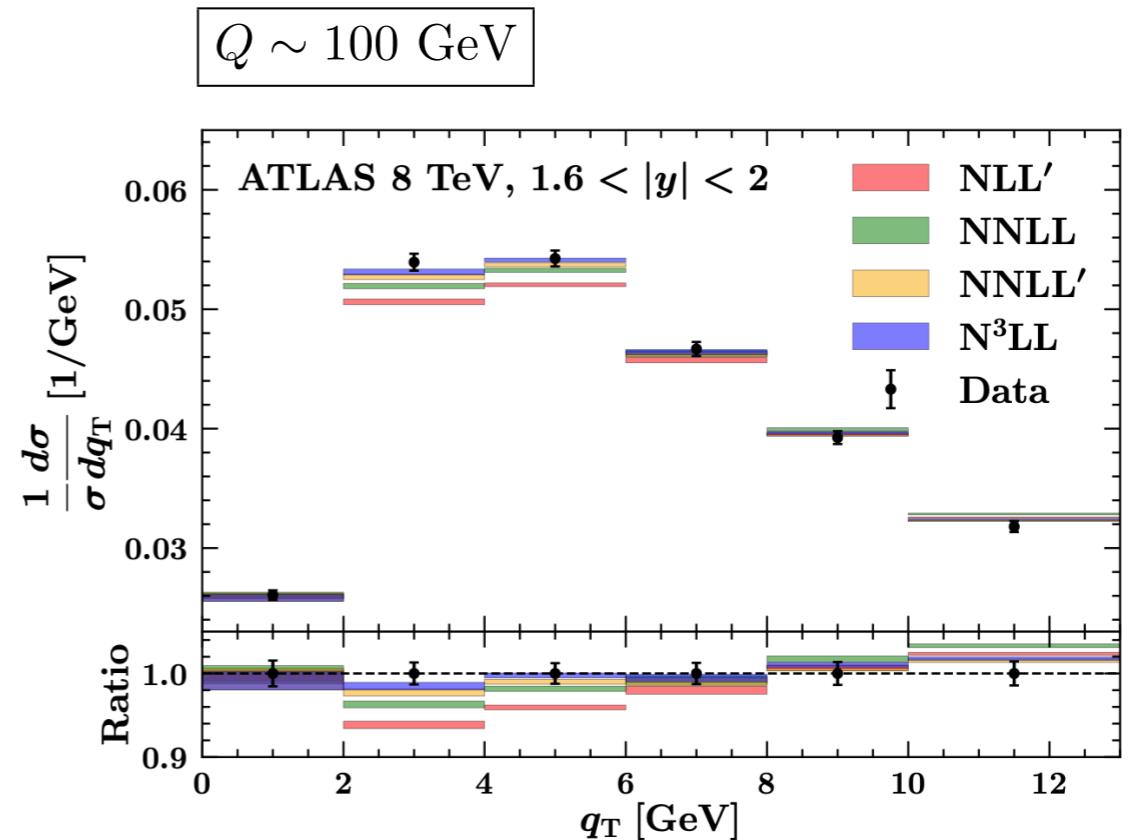
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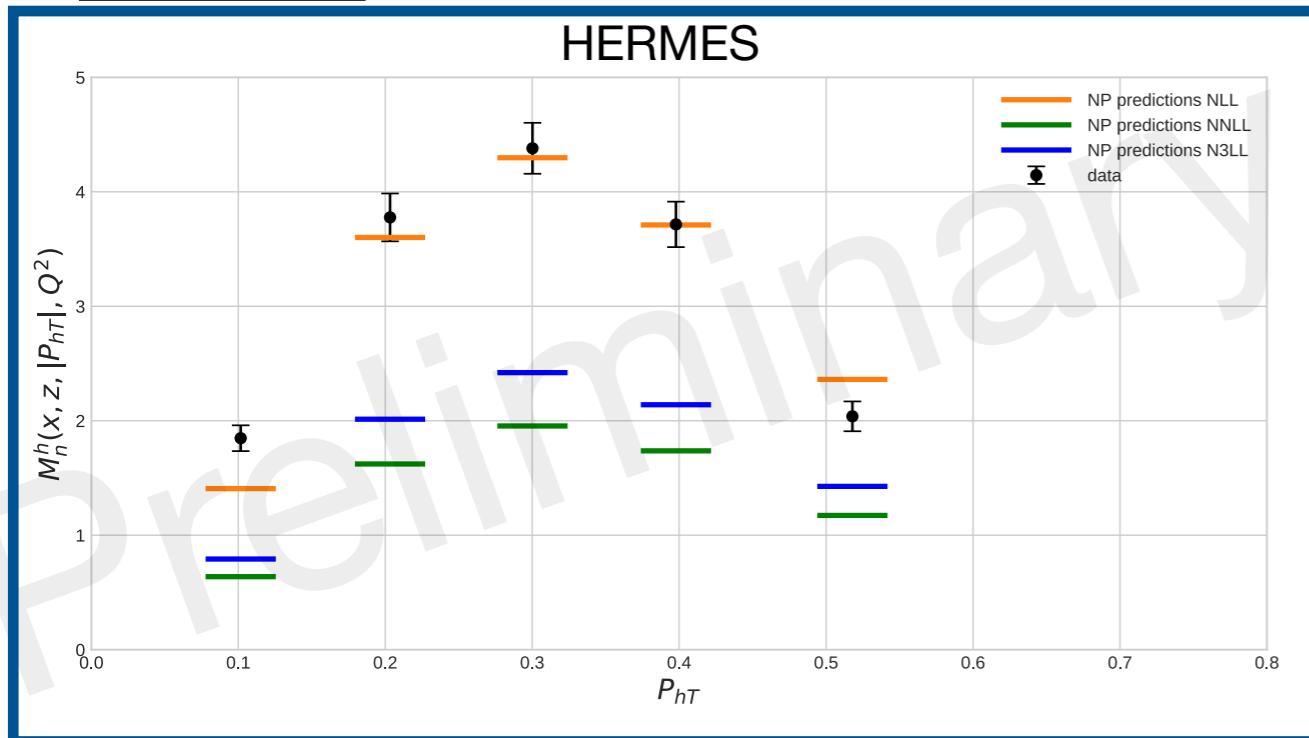


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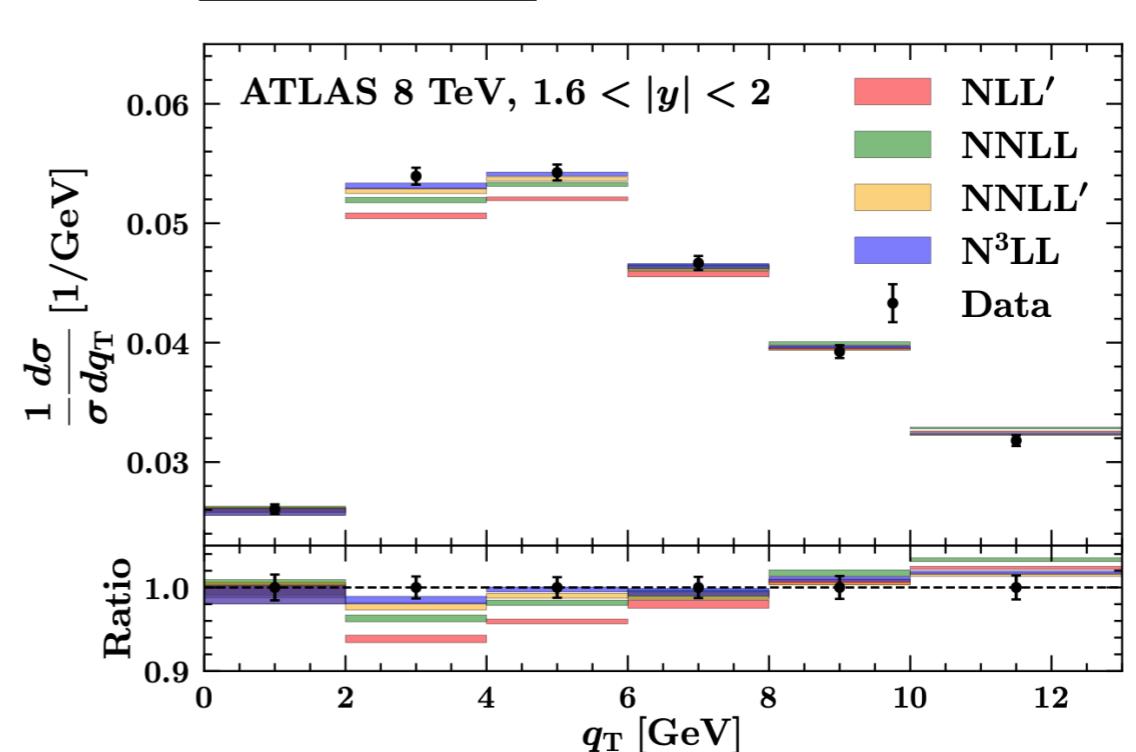
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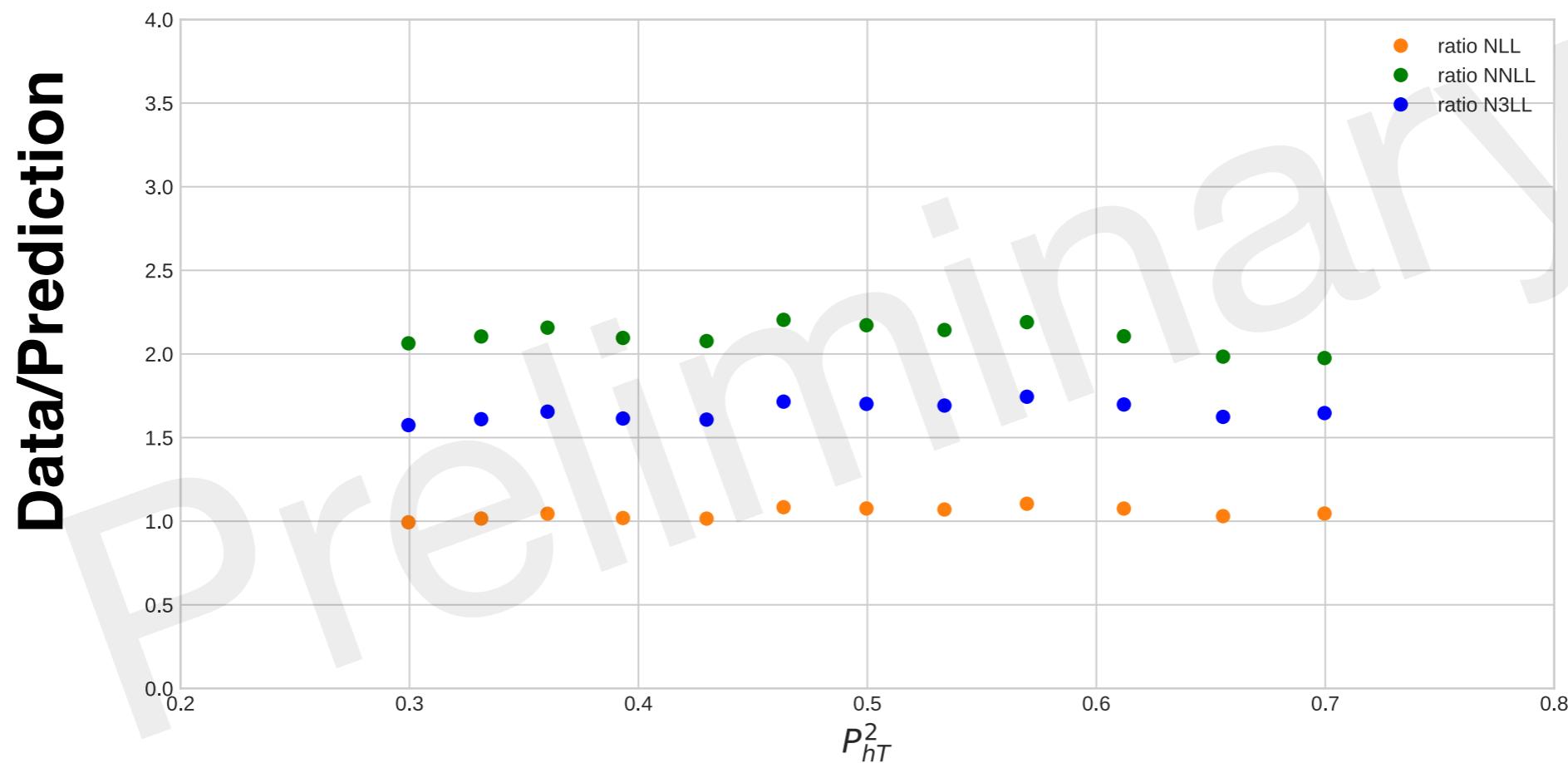
The description considerably worsens at higher orders!!

Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici,

MAPTMD22 – Normalization of SIDIS

COMPASS multiplicities (one of many bins)

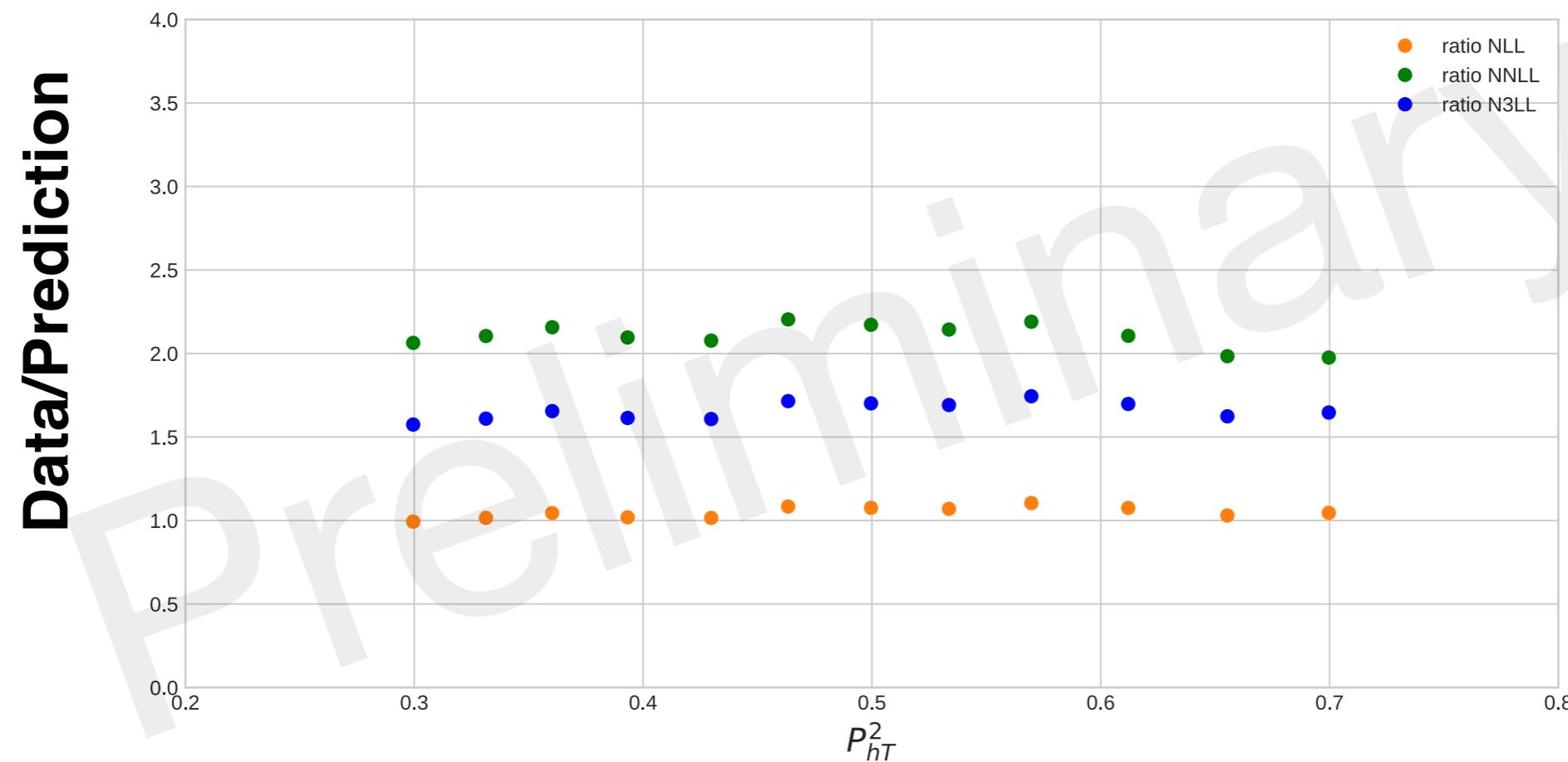
J.O. Gonzalez-Hernandez, PoS DIS2019



MAPTMD22 – Normalization of SIDIS

COMPASS multiplicities (one of many bins)

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The discrepancy amounts to an almost constant factor!!

MAPTMD22 – Normalization of SIDIS

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SIDIS multiplicity $M(x, z, P_{hT}, Q) = \frac{d\sigma}{dxdQ dz dP_{hT}} \Big/ \frac{d\sigma}{dxdQ}$

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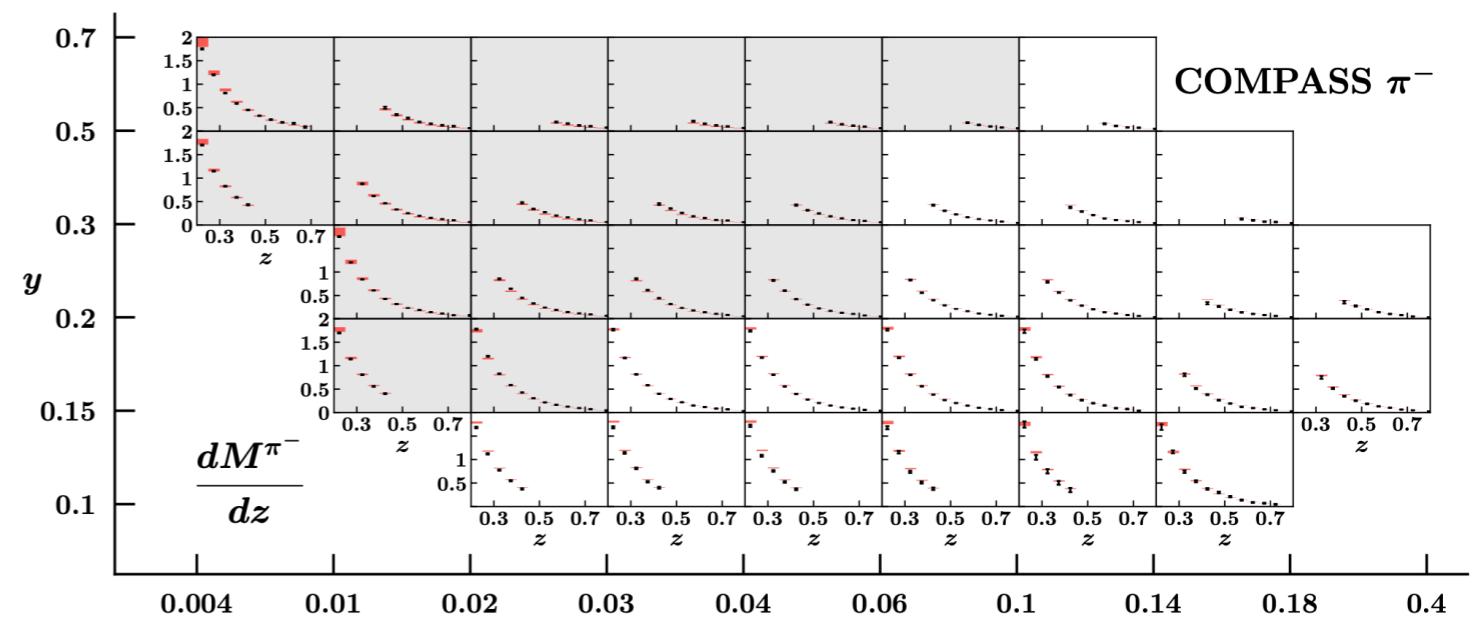
Collinear SIDIS cross section $\frac{d\sigma}{dxdQ \cancel{dz}}$

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No problems of normalization!!



*Khalek, Bertone, Nocera, arXiv:
2105.08725*

MAPTMD22 – Normalization of SIDIS

SIDIS multiplicity

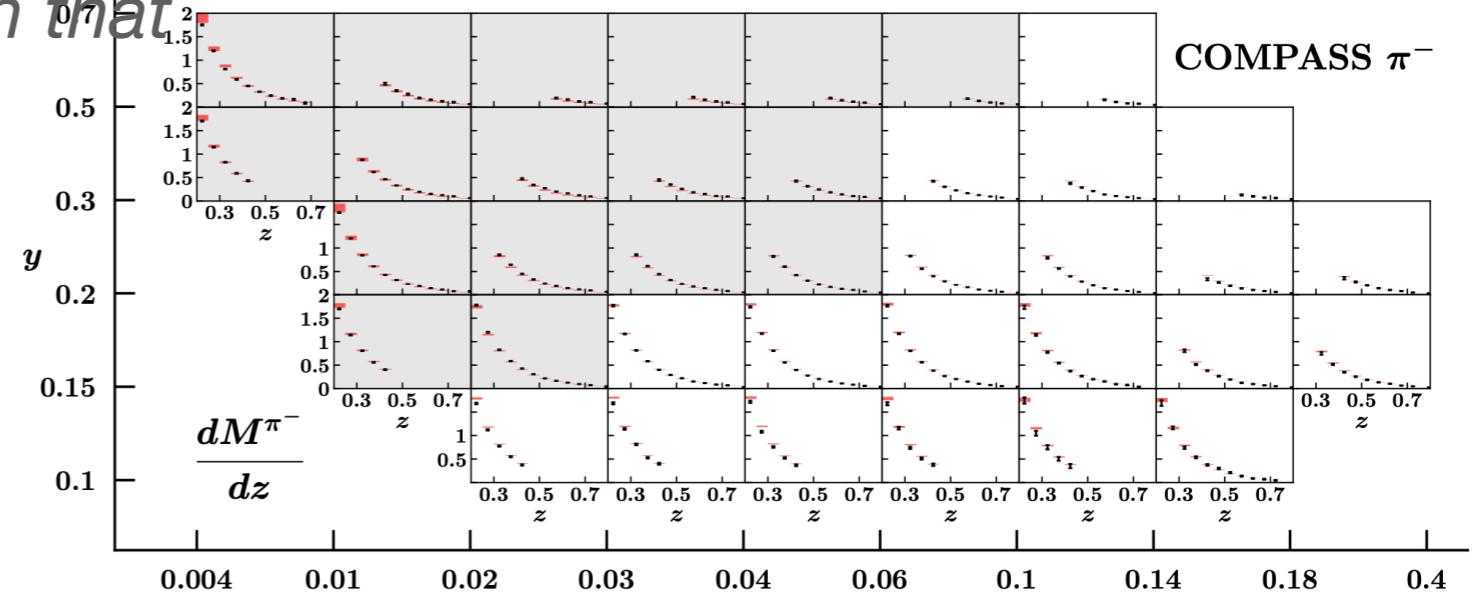
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Normalization of prediction such that

$$\int dP_{hT} \frac{d\sigma}{dx dQ dz dP_{hT}} = \frac{d\sigma}{dx dQ dz}$$



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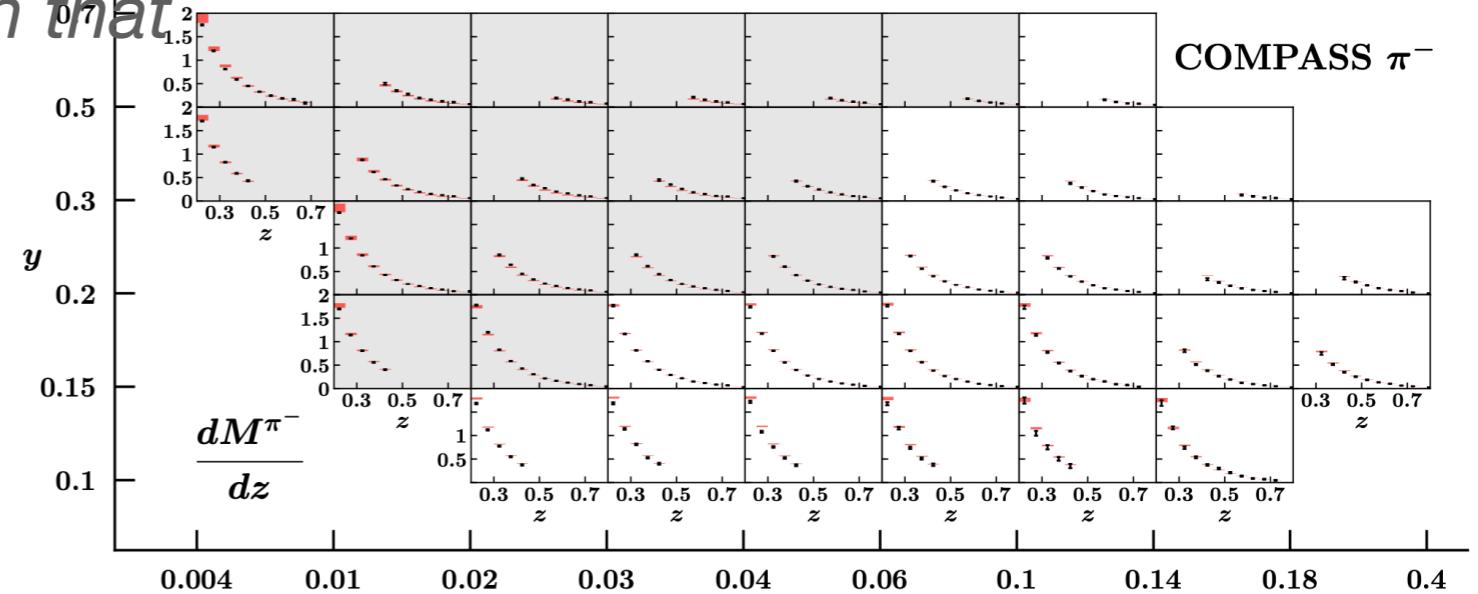
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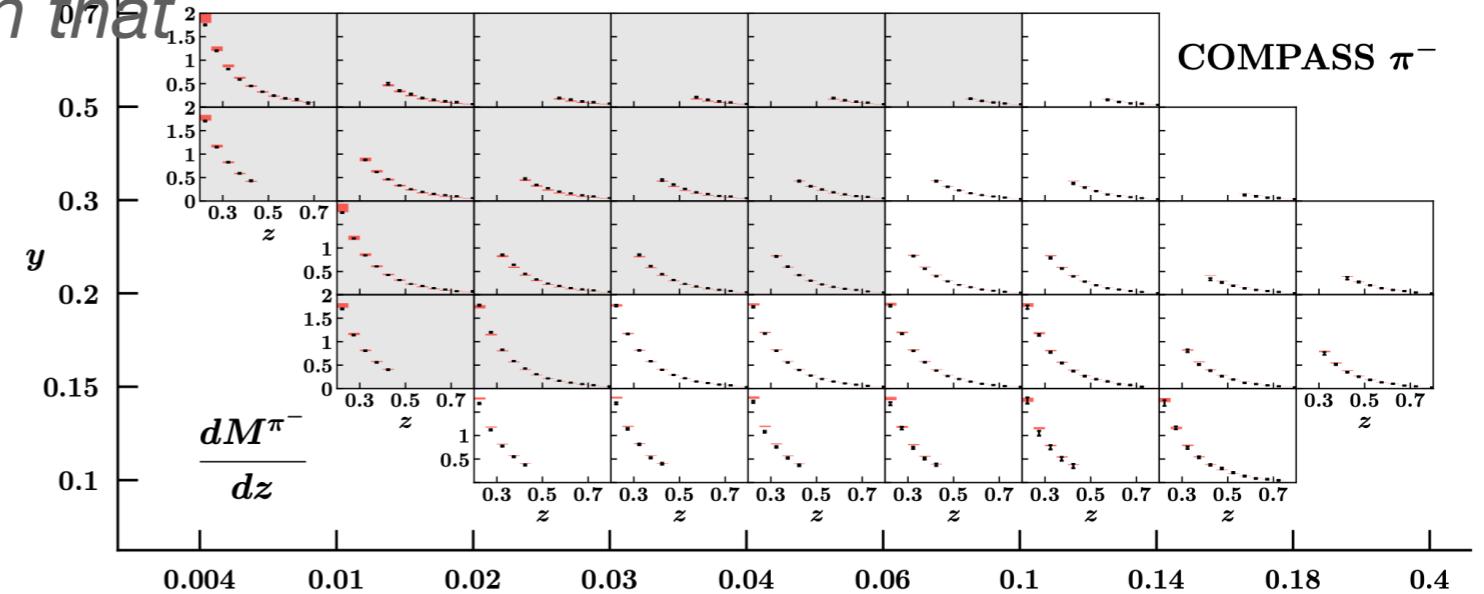
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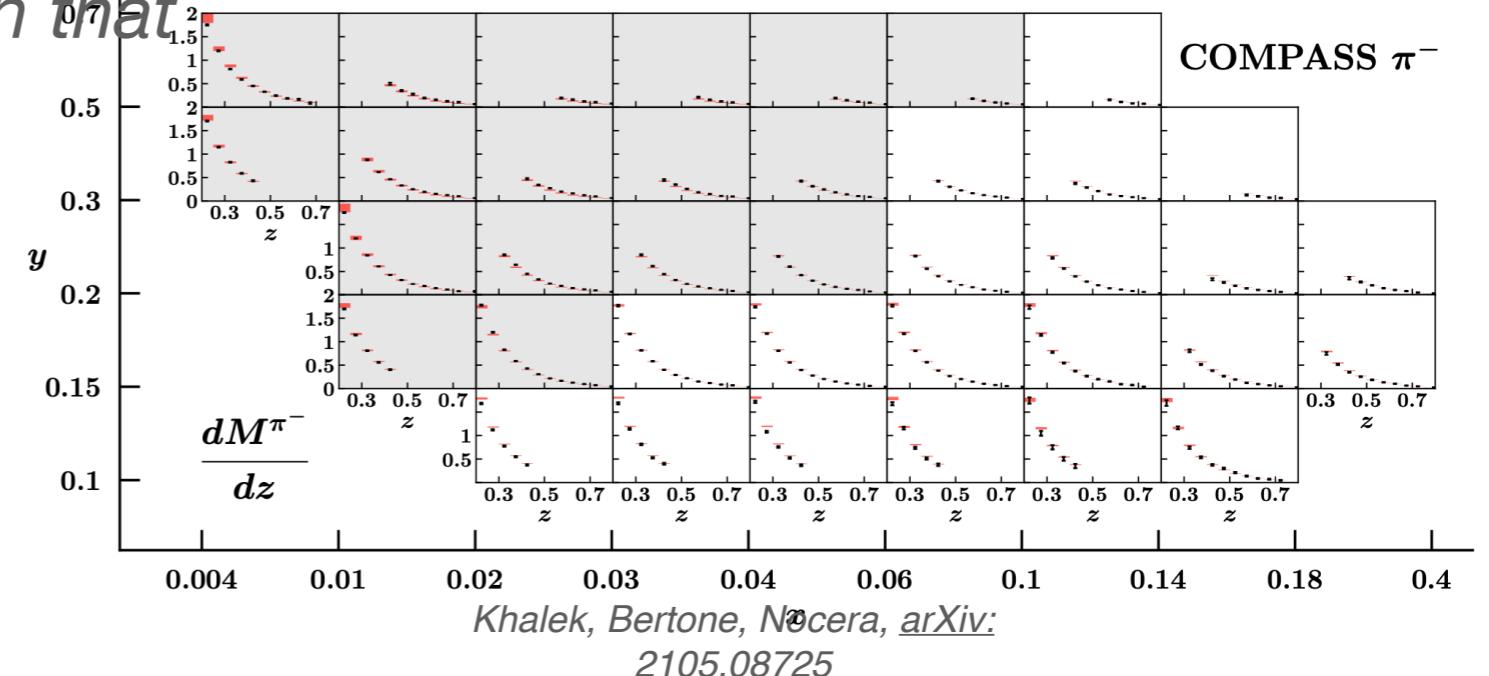
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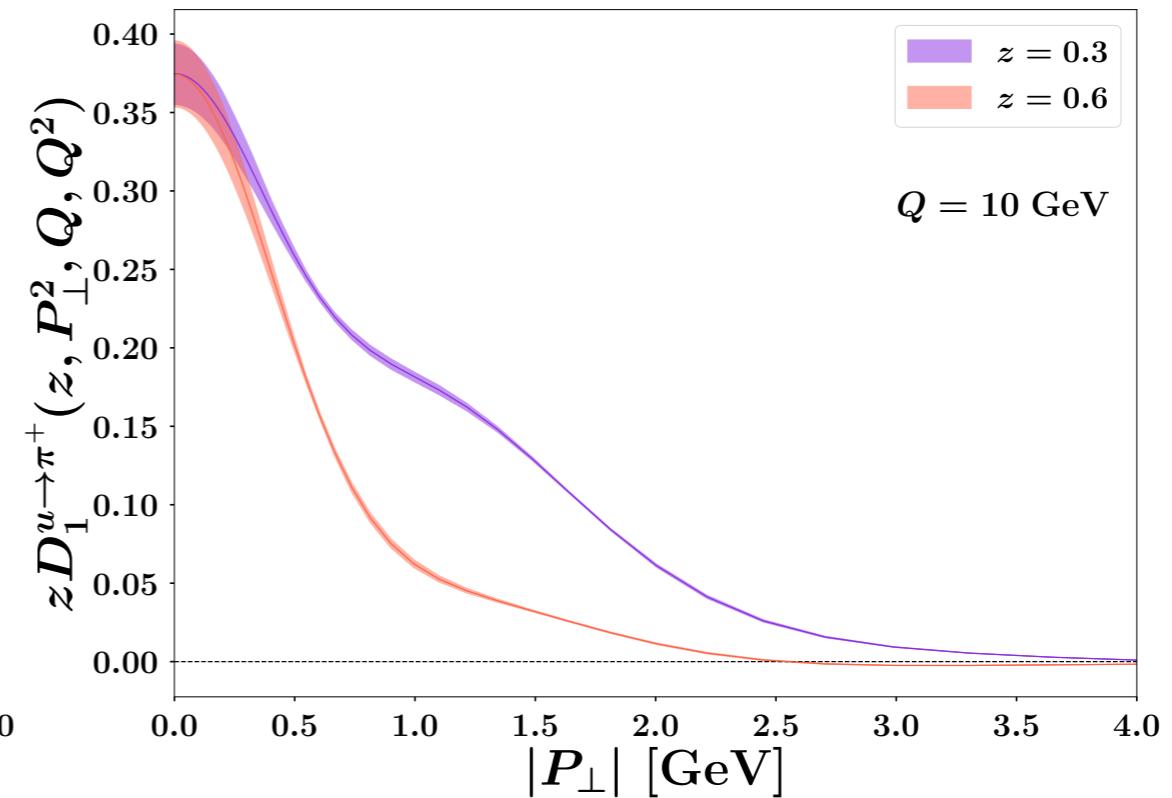
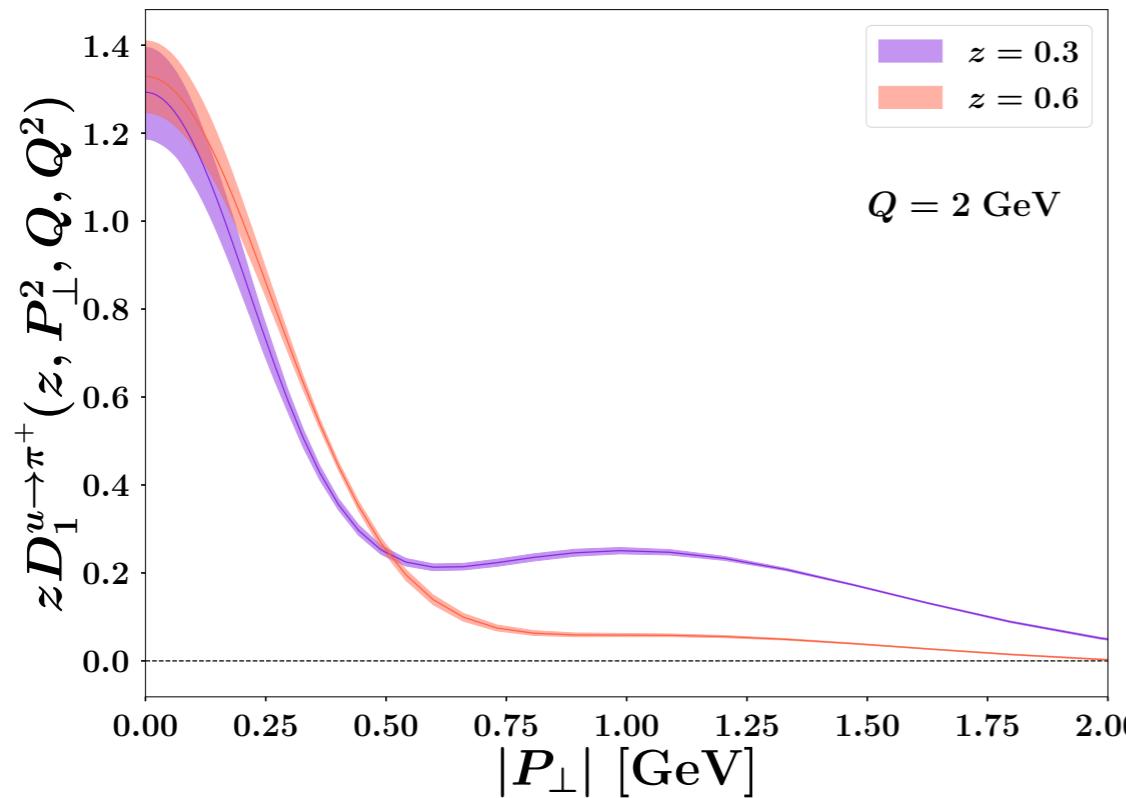
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Independent of the fitting parameters!!



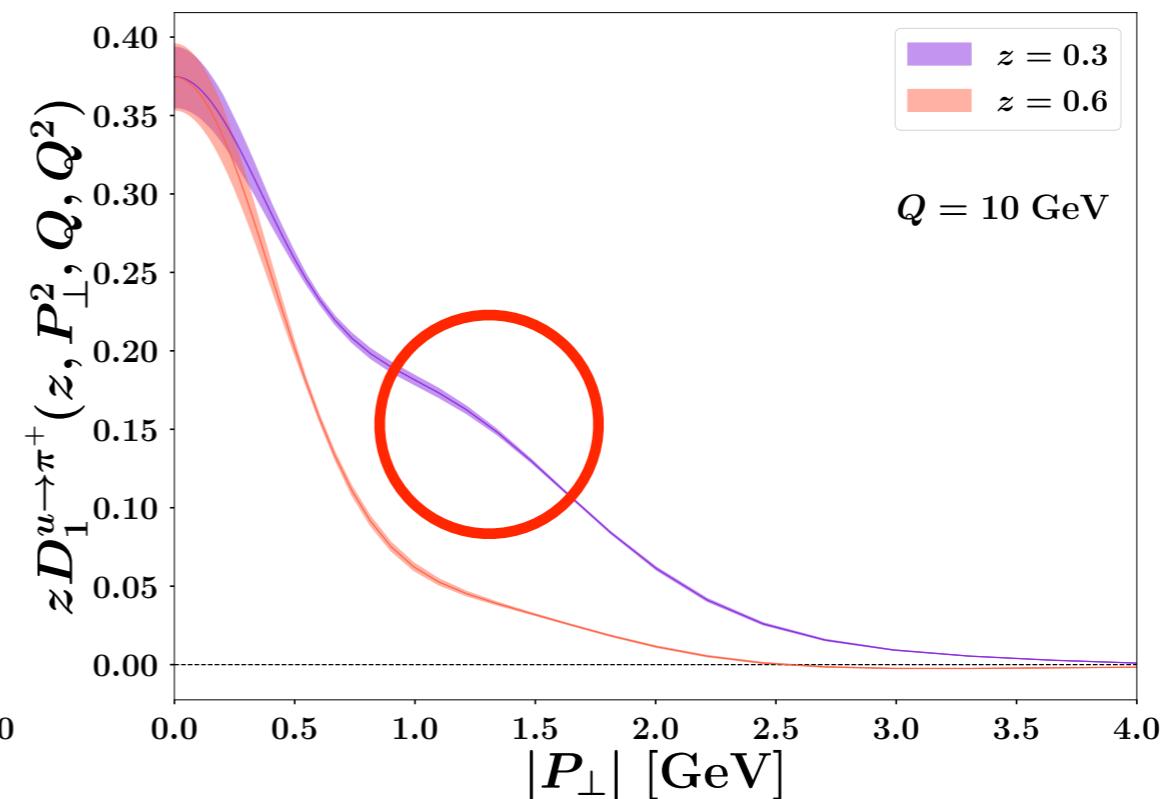
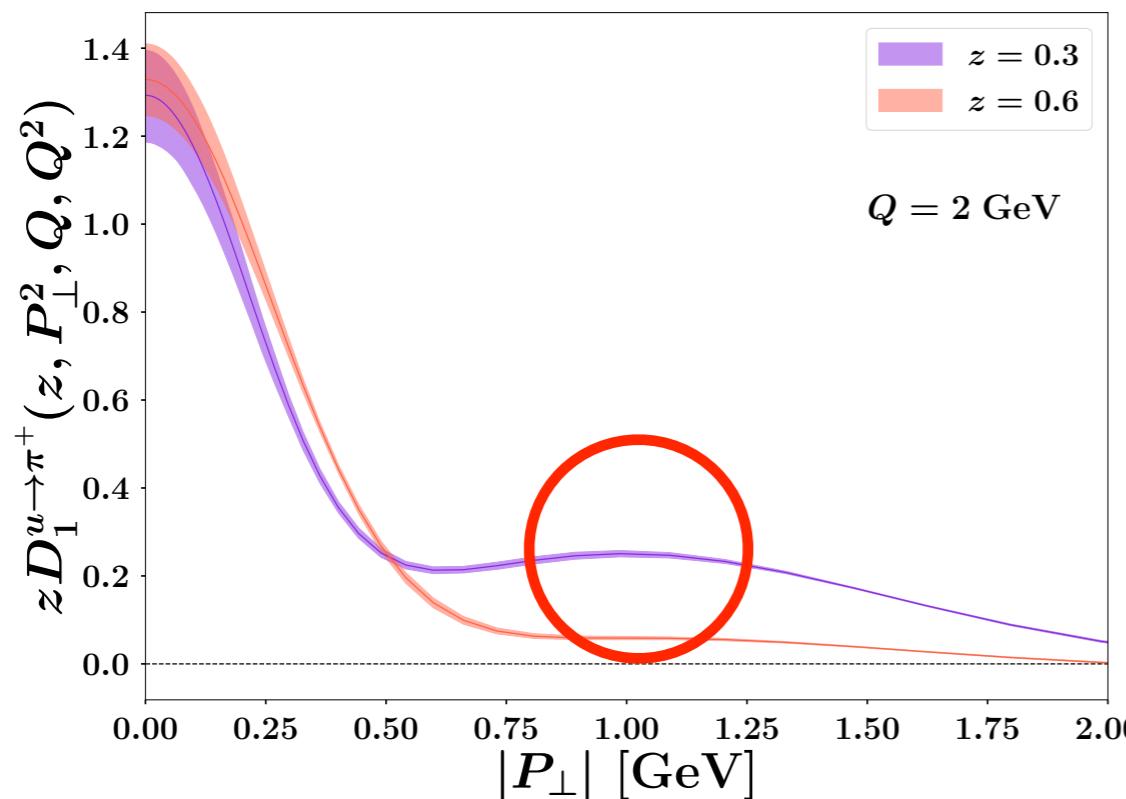
MAPTMD22 – Output of the fit

Visualisation of TMD FFs



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Visualisation of TMD FFs



MAPTMD22 – Output of the fit

Collins-Soper kernel

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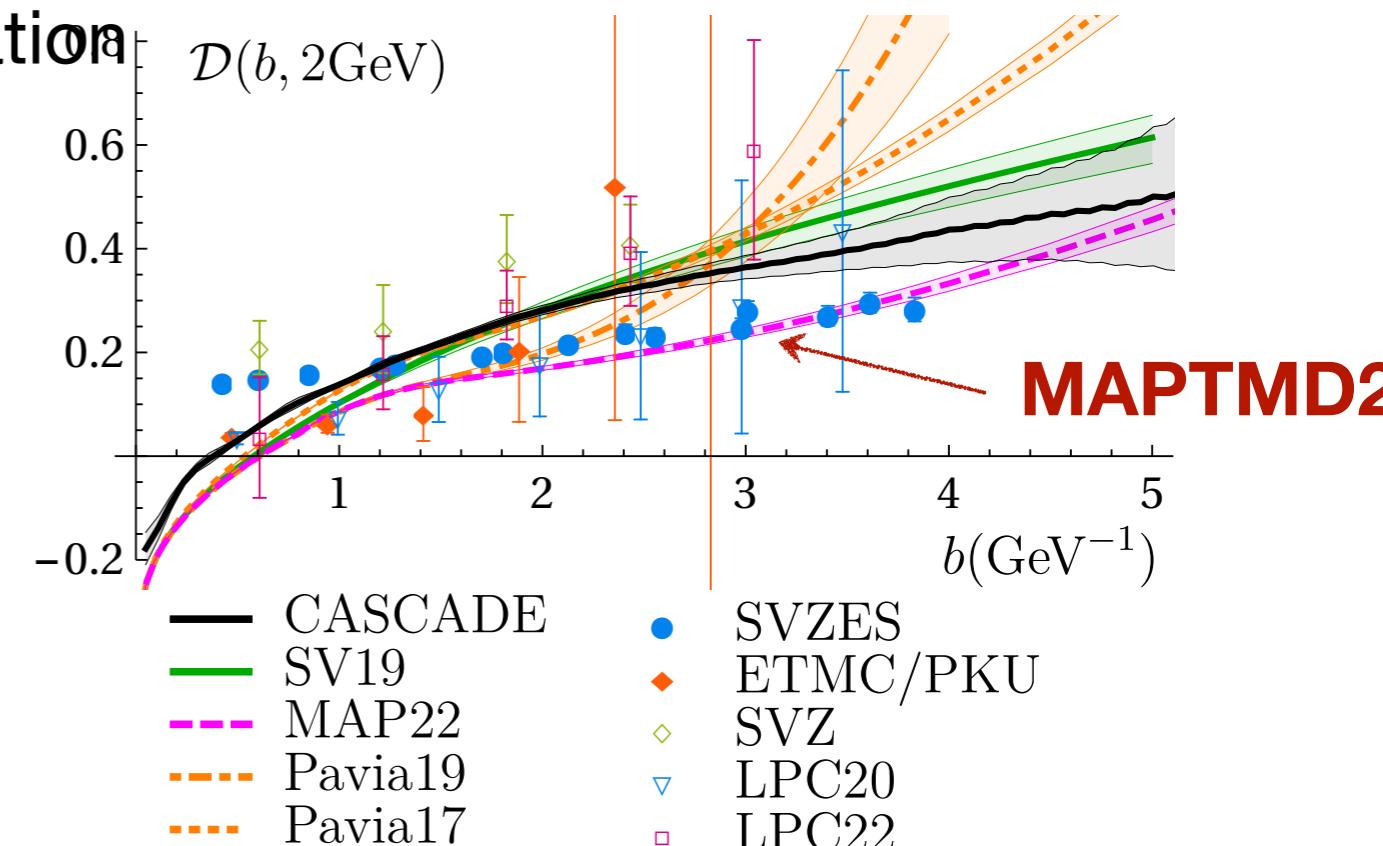
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↓ ↓
to be fitted

perturbatively calculable



*Martinez, Vladimirov,
arXiv:2206.01105*

Logarithmic Accuracy

	Sudakov form factor	Matching coefficient
LL	$\alpha_S^n \ln^{2n} \left(\frac{Q^2}{\mu_b^2} \right)$	\tilde{C}^0
NLL	$\alpha_S^n \ln^{2n} \left(\frac{Q^2}{\mu_b^2} \right), \quad \alpha_S^n \ln^{2n-1} \left(\frac{Q^2}{\mu_b^2} \right)$	\tilde{C}^0
NLL'	$\alpha_S^n \ln^{2n} \left(\frac{Q^2}{\mu_b^2} \right), \quad \alpha_S^n \ln^{2n-1} \left(\frac{Q^2}{\mu_b^2} \right)$	$(\tilde{C}^0 + \alpha_S \tilde{C}^1)$
the difference between the two is NNLL:		$\alpha_S^n \ln^{2n-2} \left(\frac{Q^2}{\mu_b^2} \right)$

Logarithmic Accuracy

- $S_{\text{pert}}(\mu_b, Q) = 1 + \sum_{k=0}^{\infty} \sum_{n=1+[k/2]}^{\infty} \left(\frac{\alpha_S(Q)}{4\pi} \right)^n \sum_{k=1}^{2n} L^{2n-k} R^{(n,2n-k)}$ $L = \ln \left(\frac{Q^2}{\mu_b^2} \right)$

Sudakov form factor

LL $\alpha_S^n \ln^{2n} \left(\frac{Q^2}{\mu_b^2} \right)$

Matching coefficient

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$$(\tilde{C}^0 + \alpha_S \tilde{C}^1)$$

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Non-mixed terms in collinear SIDIS cross section

$$\begin{aligned} \frac{d\sigma^h}{dx dQ^2 dz} \Big|_{O(\alpha_s^1)} = & \sigma_0 \sum_{ff'} \frac{e_f^2}{z^2} (\delta_{f'f} + \delta_{f'g}) \frac{\alpha_s}{\pi} \left\{ \left[D_1^{h/f'} \otimes C_1^{f'f} \otimes f_1^{f/N} \right] (x, z, Q) \right. \\ & \left. + \frac{1-y}{1+(1-y)^2} \left[D_1^{h/f'} \otimes C_L^{f'f} \otimes f_1^{f/N} \right] (x, z, Q) \right\}, \end{aligned}$$

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$$\begin{aligned} C_1^{qq} &= \frac{C_F}{2} \left\{ -8\delta(1-x)\delta(1-z) \right. \\ &\quad + \delta(1-x) \left[P_{qq}(z) \ln \frac{Q^2}{\mu_F^2} + L_1(z) + L_2(z) + (1-z) \right] \\ &\quad + \delta(1-z) \left[P_{qq}(x) \ln \frac{Q^2}{\mu_F^2} + L_1(x) - L_2(x) + (1-x) \right] \\ &\quad \left. + 2 \frac{1}{(1-x)_+} \frac{1}{(1-z)_+} - \frac{1+z}{(1-x)_+} - \frac{1+x}{(1-z)_+} + 2(1+xz) \right\}, \end{aligned}$$

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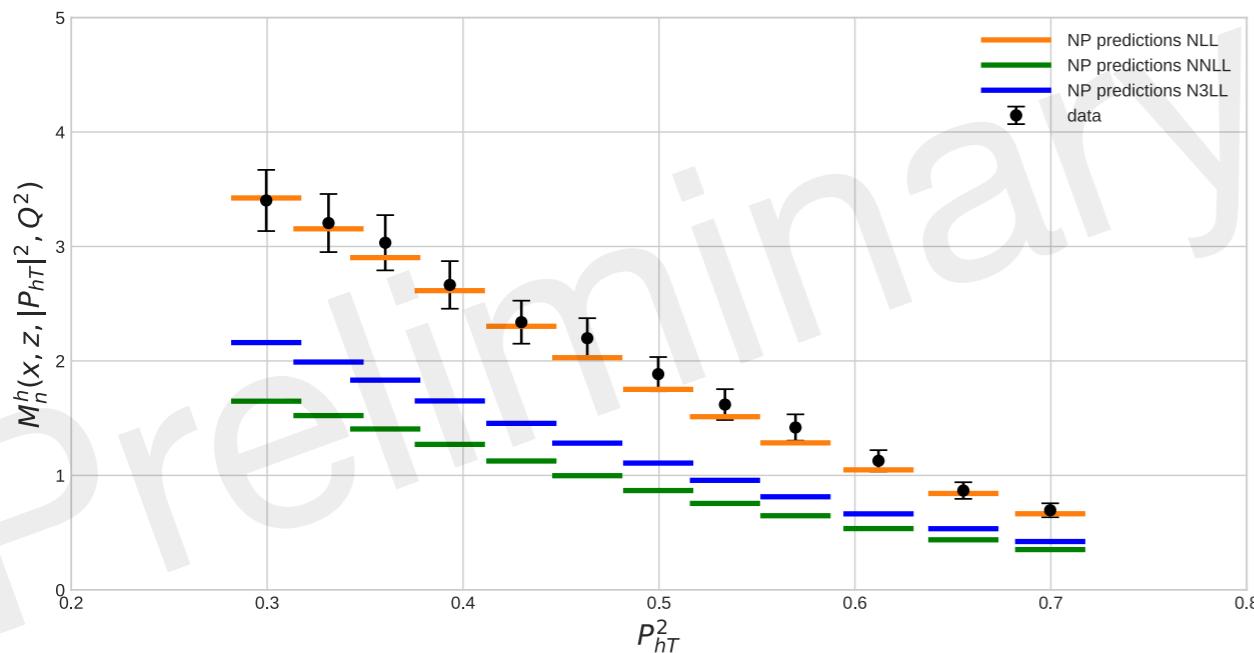
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Source of W-term suppression

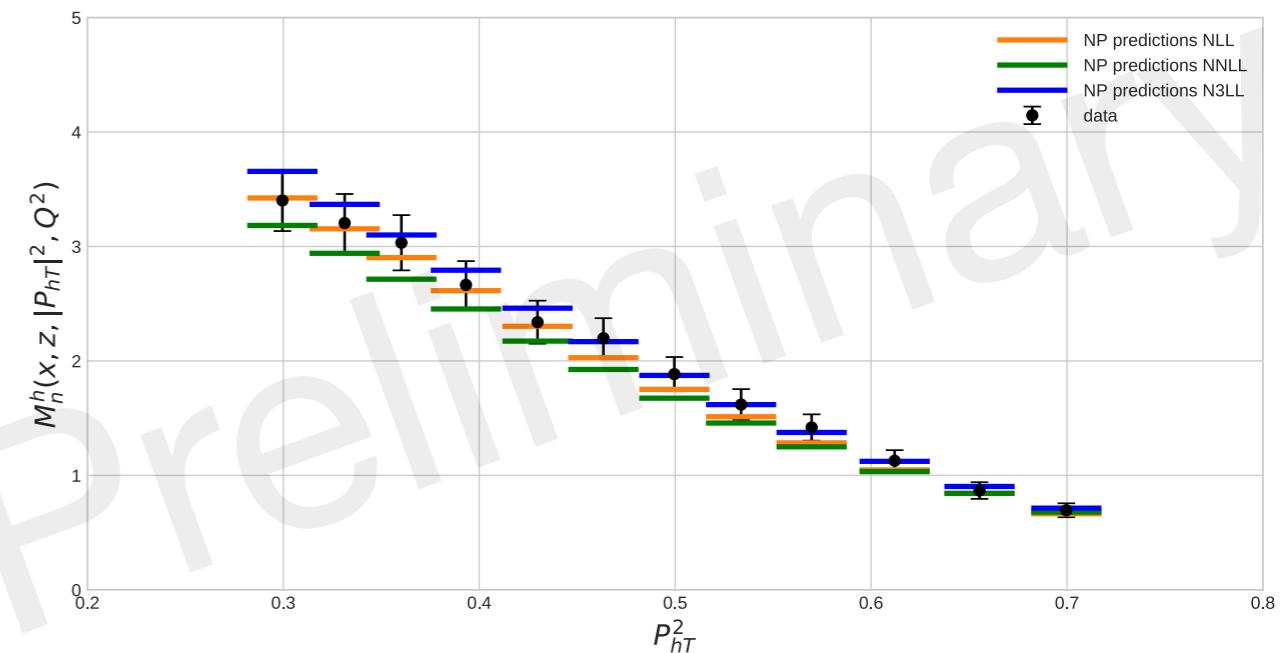
Present situation at low Q

COMPASS multiplicity

Full Hard Factor



Hard Factor = 1

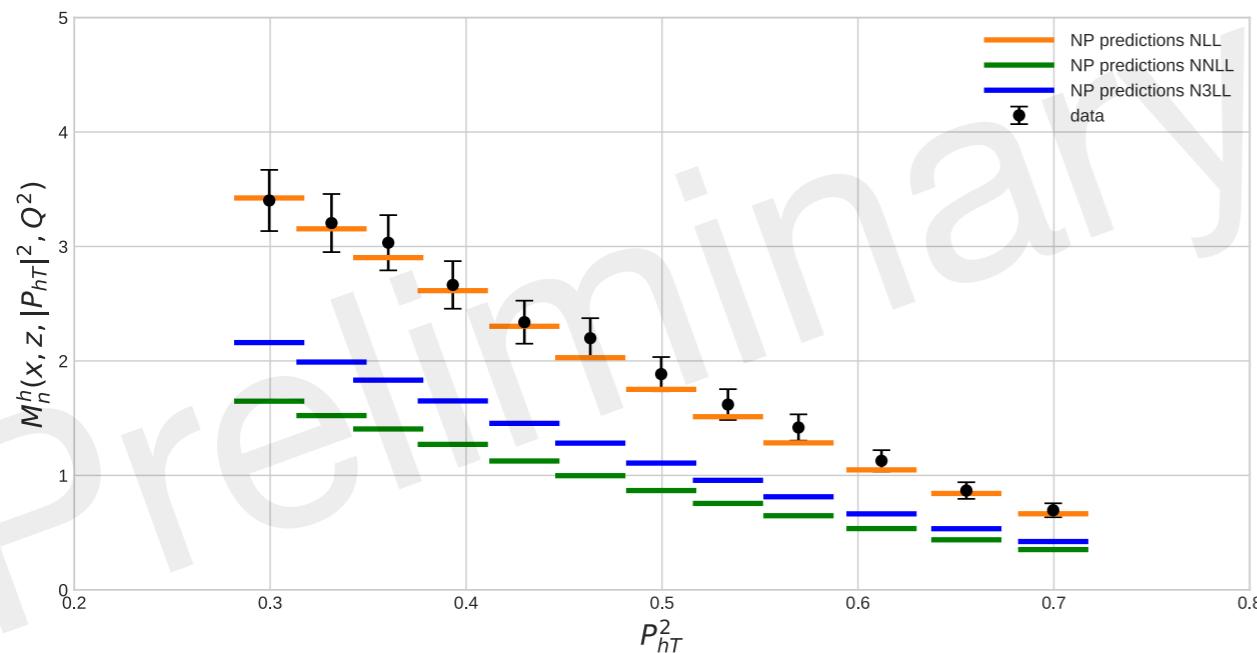


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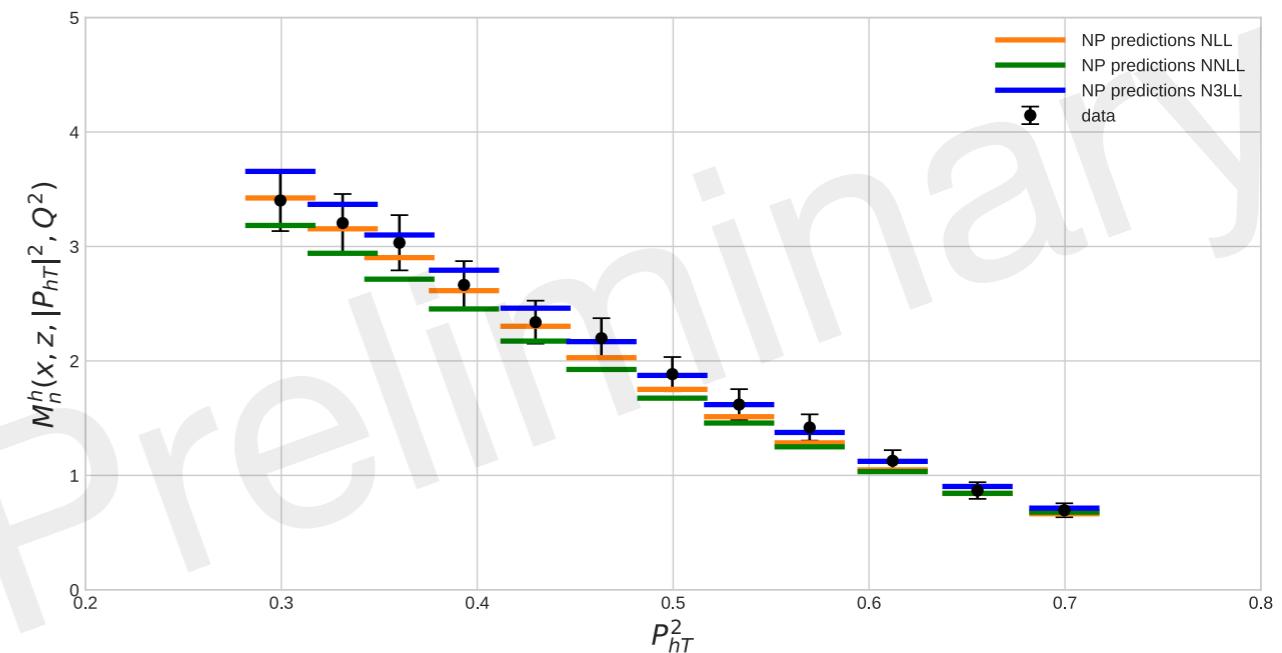
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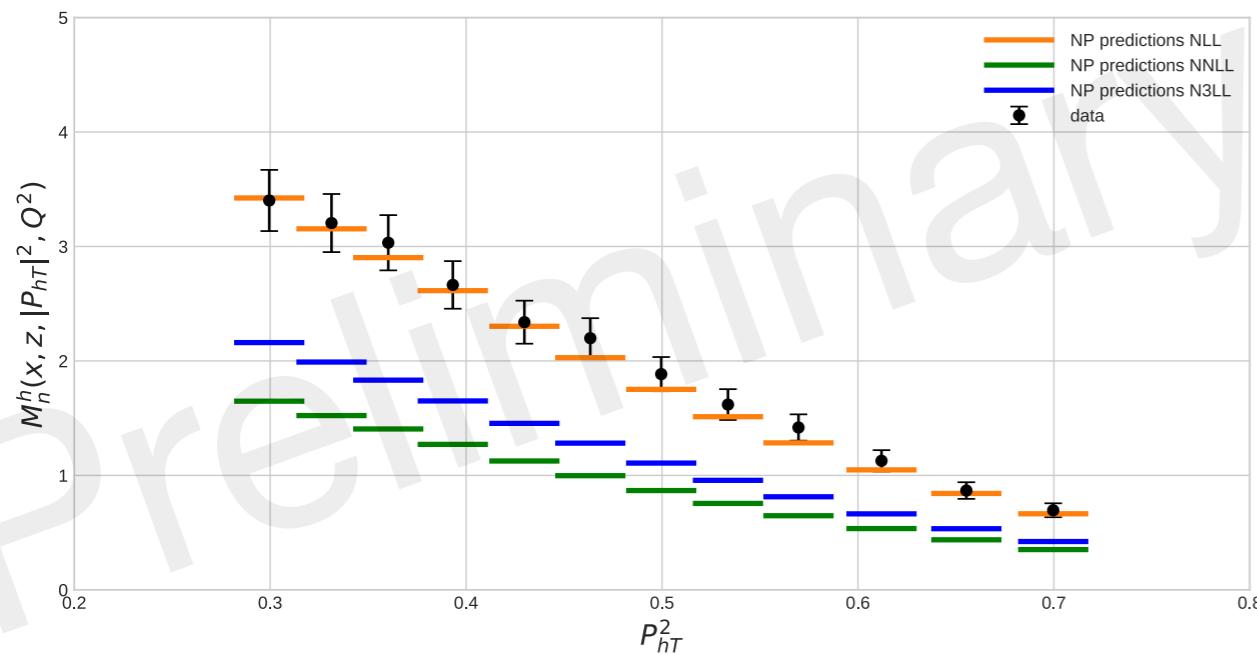


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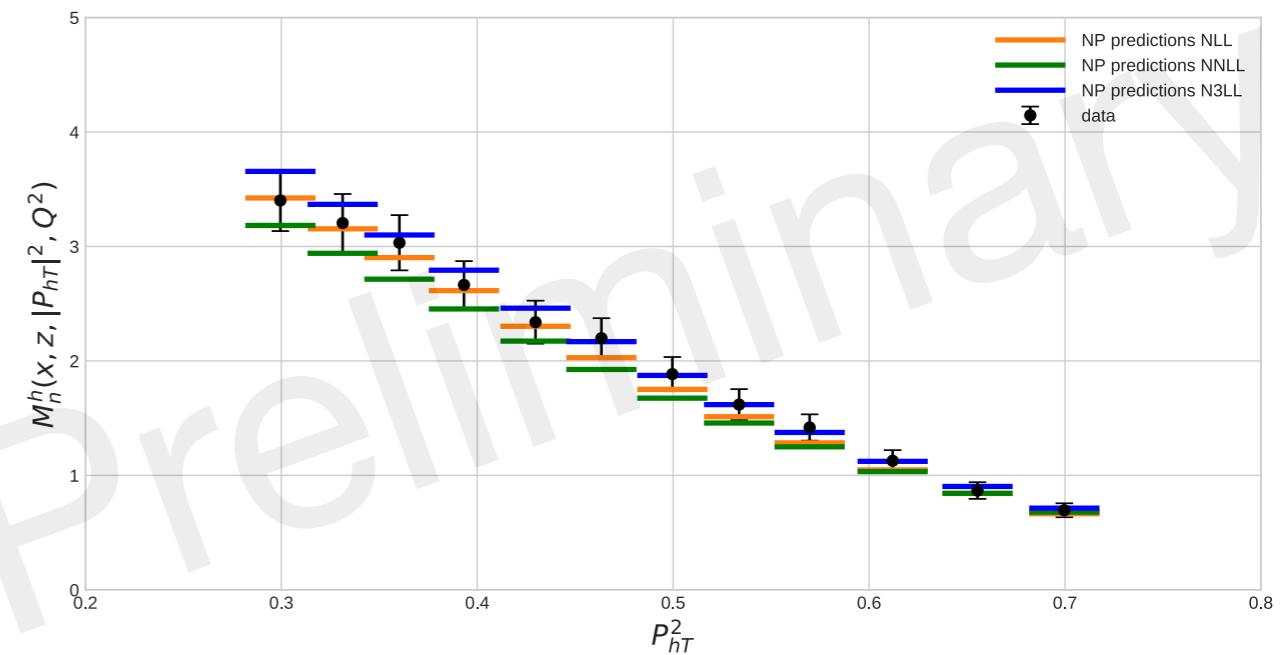
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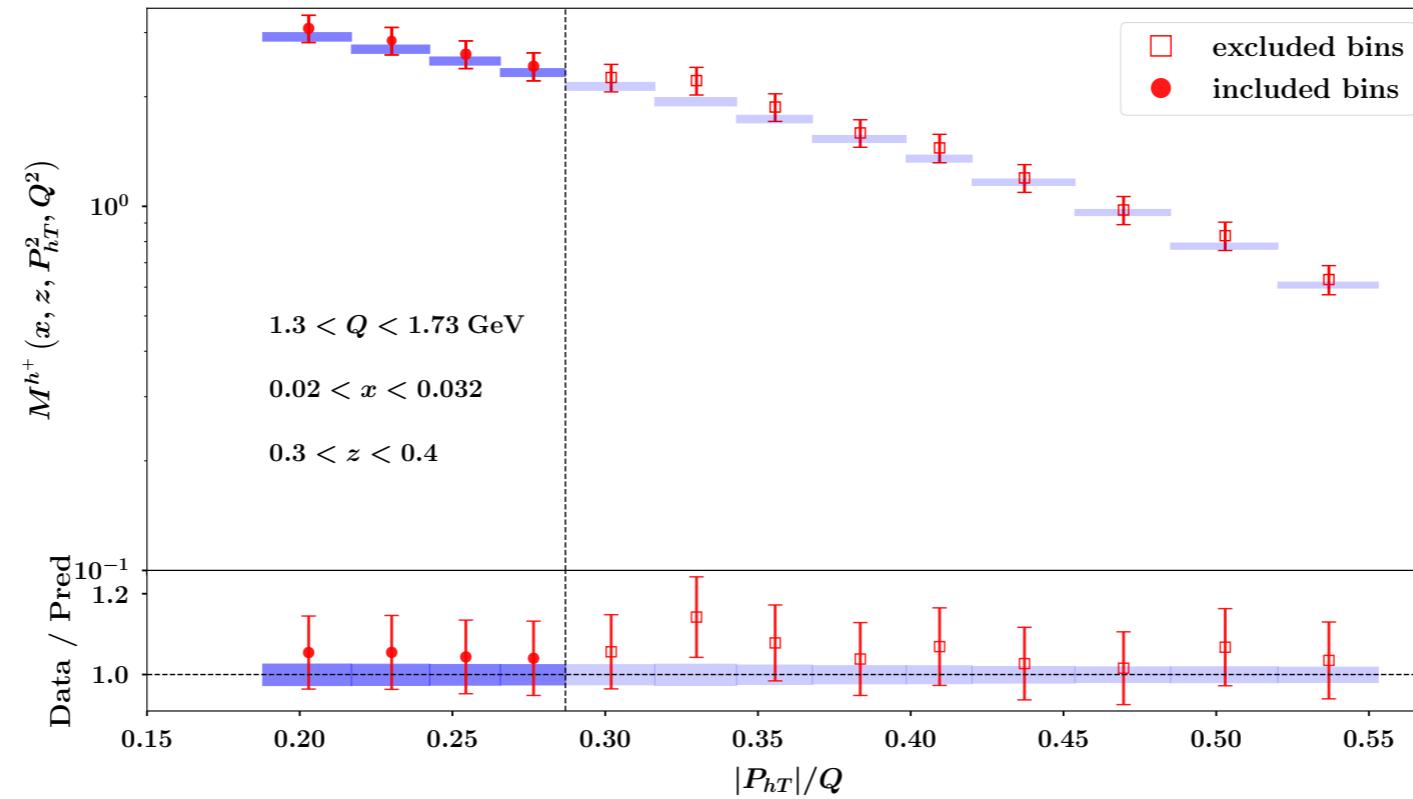


MAPTMD22 – SIDIS data selection

COMPASS multiplicities (one of many bins)

$$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$

$$P_{hT}|_{\max} = \min[0.2Q, 0.7zQ] + 0.5 \text{ GeV}$$

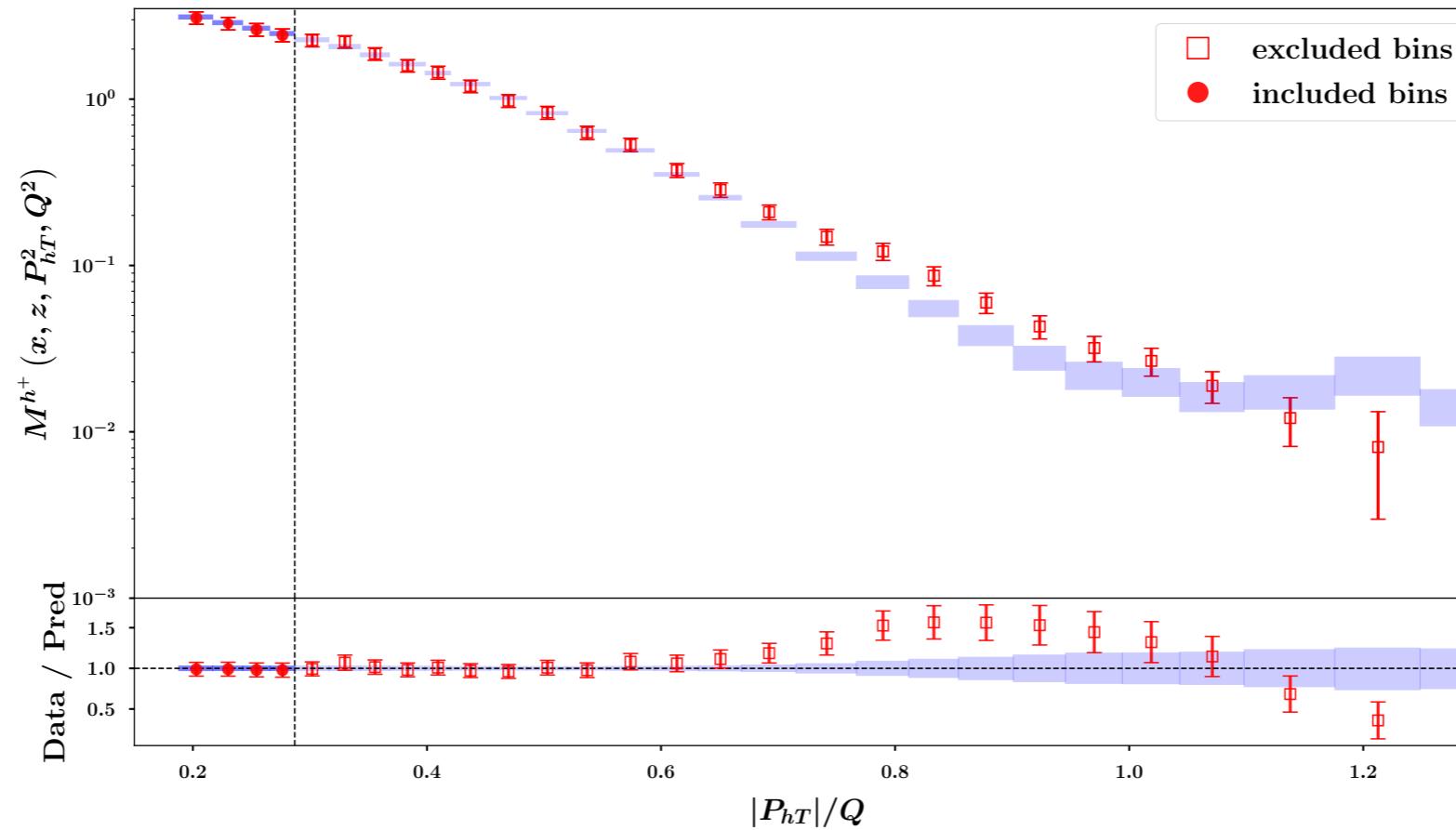


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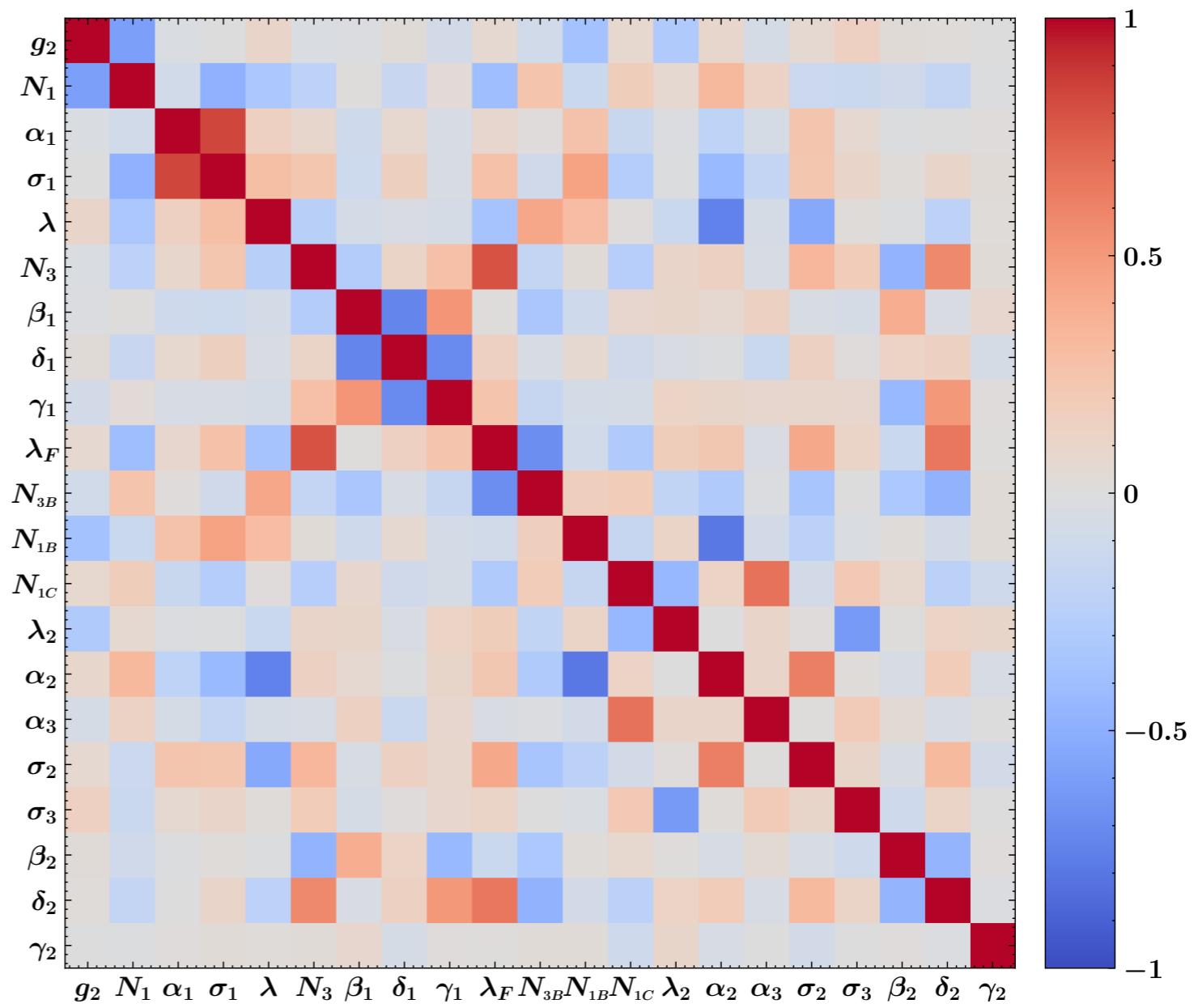
$$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$

Total number of points



Results of the baseline fit

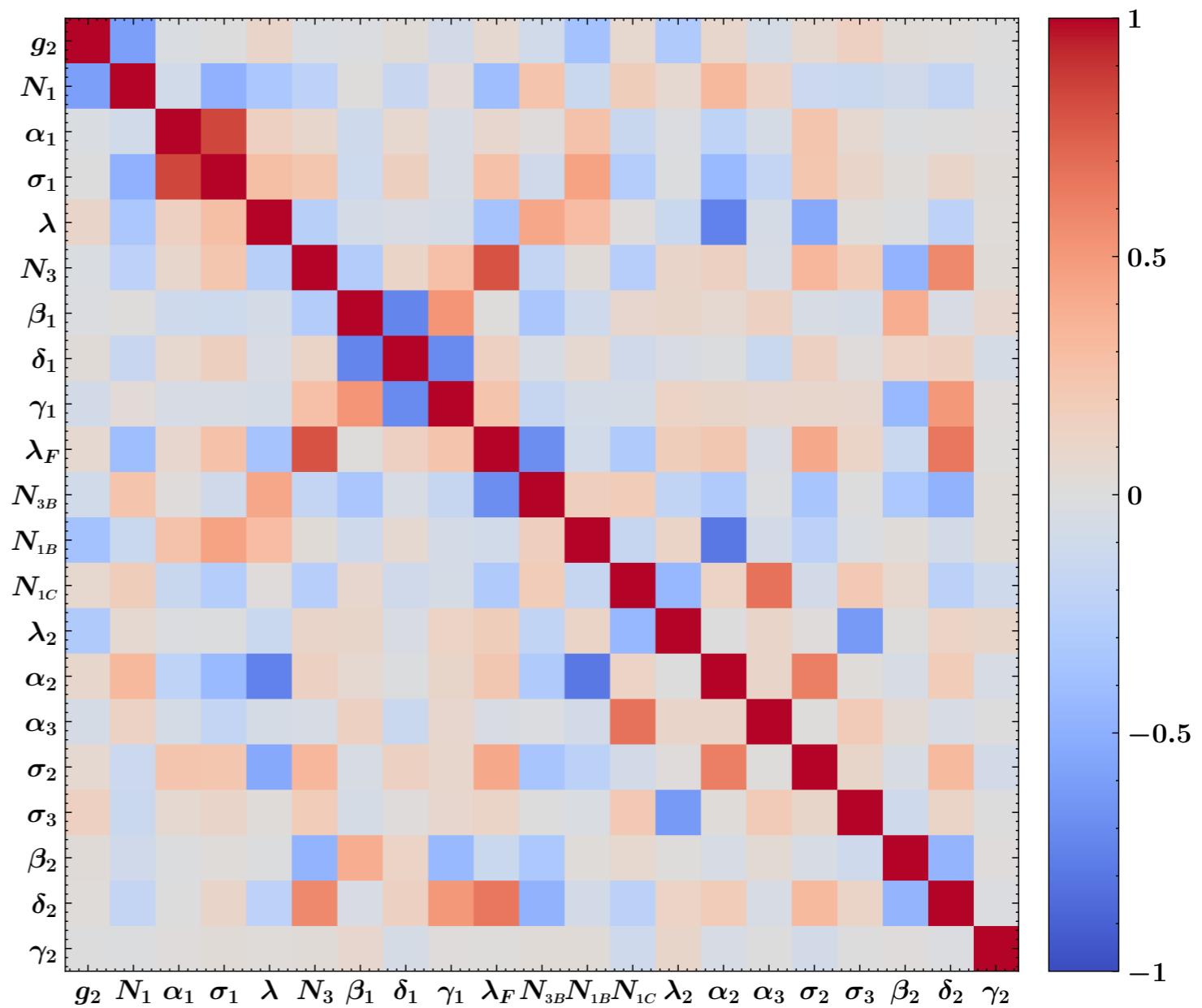
Error propagation
↓
250 Montecarlo
replicas



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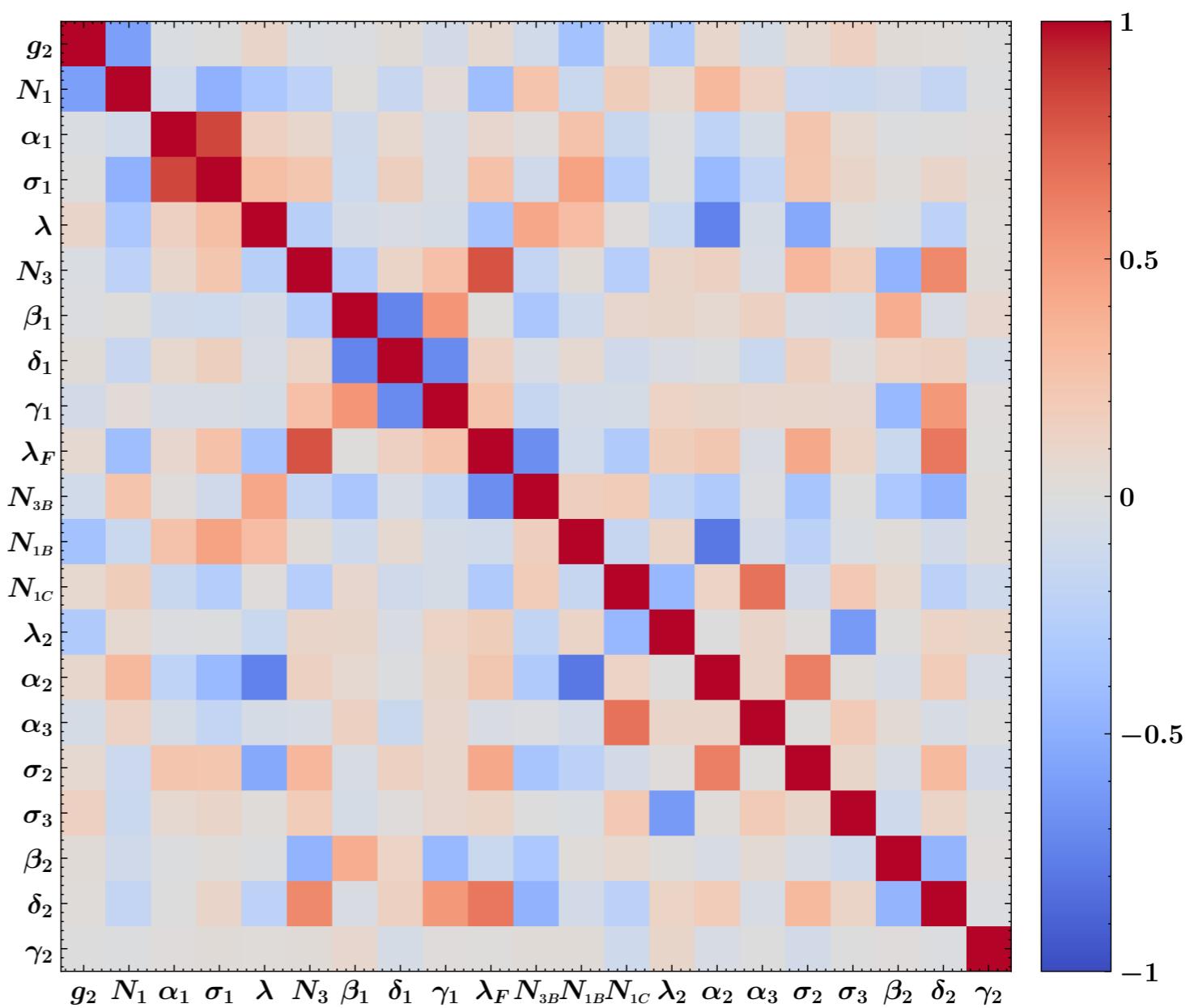
Error propagation
↓
250 Montecarlo
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Correlation matrix
↓
Hints of the
appropriateness of
the chosen
functional form



Results of the baseline fit

Parameter	Average over replicas
g_2 [GeV]	0.248 ± 0.008
N_1 [GeV 2]	0.316 ± 0.025
α_1	1.29 ± 0.19
σ_1	0.68 ± 0.13
λ [GeV $^{-1}$]	1.82 ± 0.29
N_3 [GeV 2]	0.0055 ± 0.0006
β_1	10.23 ± 0.29
δ_1	0.0094 ± 0.0012
γ_1	1.406 ± 0.084
λ_F [GeV $^{-2}$]	0.078 ± 0.011
N_{3B} [GeV 2]	0.2167 ± 0.0055
N_{1B} [GeV 2]	0.134 ± 0.017
N_{1C} [GeV 2]	0.0130 ± 0.0069
λ_2 [GeV $^{-1}$]	0.0215 ± 0.0058
α_2	4.27 ± 0.31
α_3	4.27 ± 0.13
σ_2	0.455 ± 0.050
σ_3	12.71 ± 0.21
β_2	4.17 ± 0.13
δ_2	0.167 ± 0.006
γ_2	0.0007 ± 0.0110



Impact of EIC

PV17 baseline

Average over replicas
0.1171 ± 0.0145
0.283 ± 0.0368
2.2393 ± 1.2967
-0.1416 ± 0.0959
0.2548 ± 0.2549
0.2203 ± 0.0222
2.9304 ± 0.9978
0.1175 ± 0.0506
2.4736 ± 0.1649
7.5475 ± 3.2037
0.0318 ± 0.0068

reduction by factor 10

PV17 baseline + EIC

Parameter
g_2
λ
N_3
β
δ
γ
λ_F
N_4

non - perturbative evolution

Average over replicas
0.119 ± 0.0025
0.2814 ± 0.0362
2.3882 ± 0.5448
-0.1445 ± 0.0134
0.3061 ± 0.4085
0.2122 ± 0.0157
2.6773 ± 0.3861
0.1099 ± 0.0358
2.4643 ± 0.12
5.3198 ± 2.0531
0.0346 ± 0.0048