



UNIVERSITÀ DI PAVIA



Istituto Nazionale di Fisica Nucleare

TMDs extraction

Framework and Validation

Matteo Cerutti

Available Global fits

	Accuracy	SIDIS	DY	Z production	N of points	χ^2/N_{data}
Pavia 2017 arXiv:1703.10157	NLL	✓	✓	✓	8059	1.55
SV 2019 arXiv:1912.06532	N^3LL^-	✓	✓	✓	1039	1.06
MAPTMD22	N^3LL^-	✓	✓	✓	2031	1.06

Bacchetta, Bertone, Bissolotti, Bozzi, MC, Piacenza, Radici, Signori arXiv: 2206.07598

Unpolarized TMD structure

Collins, "Foundations of Perturbative QCD"

Fourier space

$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^2} e^{i \mathbf{b}_T \cdot \mathbf{k}_\perp} f_1^q(x, k_\perp^2; \mu_f, \zeta_f)$$

$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = [C \otimes f_1](x, \mu_{b_*}) e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu} (\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_f}}{\mu})} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}} \right)^{K_{\text{resum}} + g_K} f_{1NP}(x, b_T^2; \zeta_f, Q_0)$$

Scales $\mu_f = Q$ $\mu_{b_*} = \frac{2e^{-\gamma_E}}{b_*}$

$\zeta_f = Q^2$

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perturbative Sudakov form factor

$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = [C \otimes f_1](x, \mu_{b_*}) e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu} (\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_f}}{\mu})} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}} \right)^{K_{\text{resum}} + g_K} f_{1NP}(x, b_T^2; \zeta_f, Q_0)$$

collinear PDF

matching coefficients (perturbative)

Collins-Soper kernel (perturbative and nonperturbative)

nonperturbative part of TMD

Scales

$$\mu_f = Q \quad \mu_{b_*} = \frac{2e^{-\gamma_E}}{b_*}$$
$$\zeta_f = Q^2$$

MAPTMD22 – Parameterization of TMDs

$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{k_\perp^2}{g^{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g^{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g^{1C}}} \right)$$

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$$g_1(x) = N_1 \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma}$$

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11 parameters for TMD PDF
+ 1 for NP evolution + 9 for TMD FF
= 21 free parameters

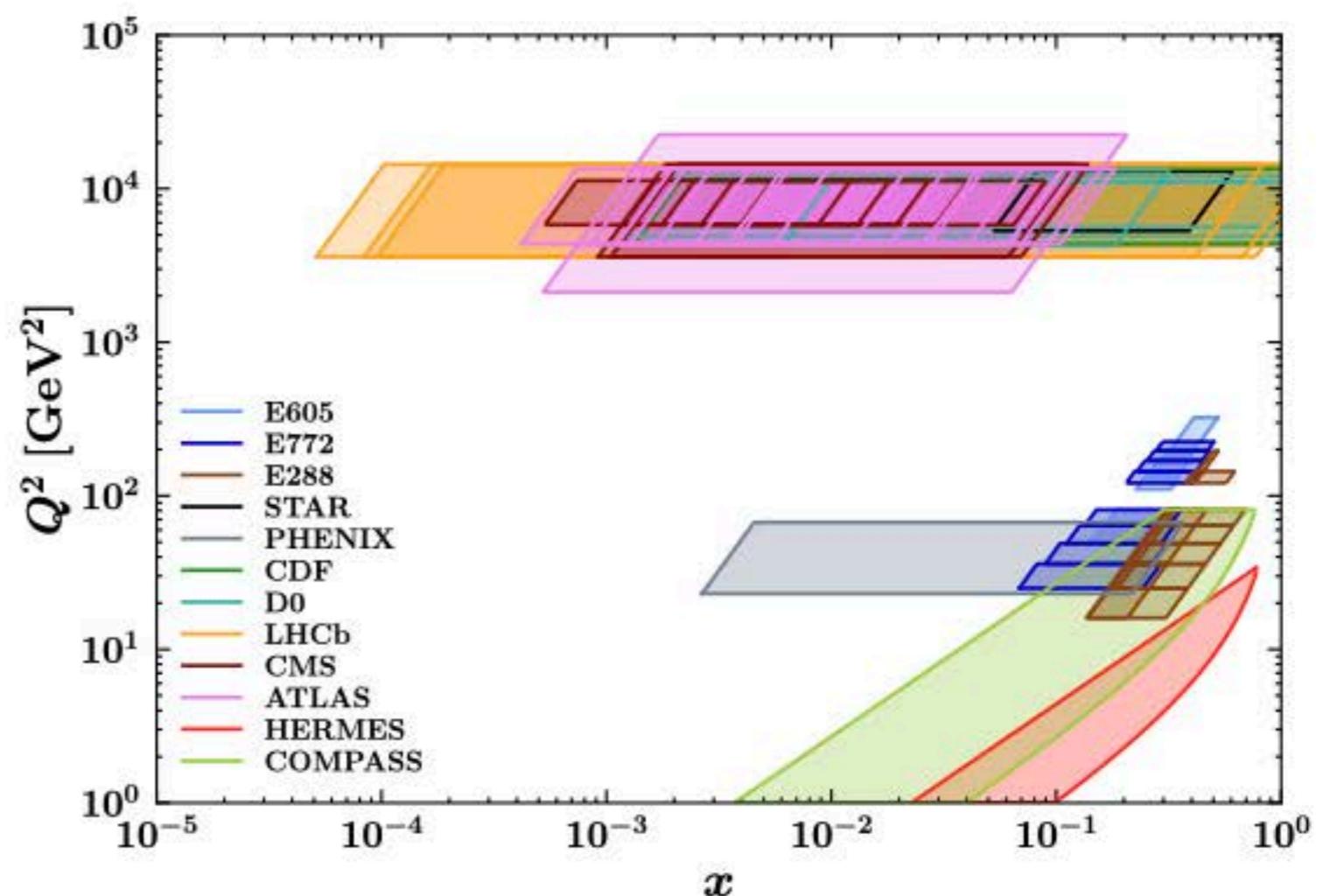
MAPTMD22 – Datasets included

Drell–Yan

484 experimental points

SIDIS

1547 experimental points



Total: 2031 fitted experimental points

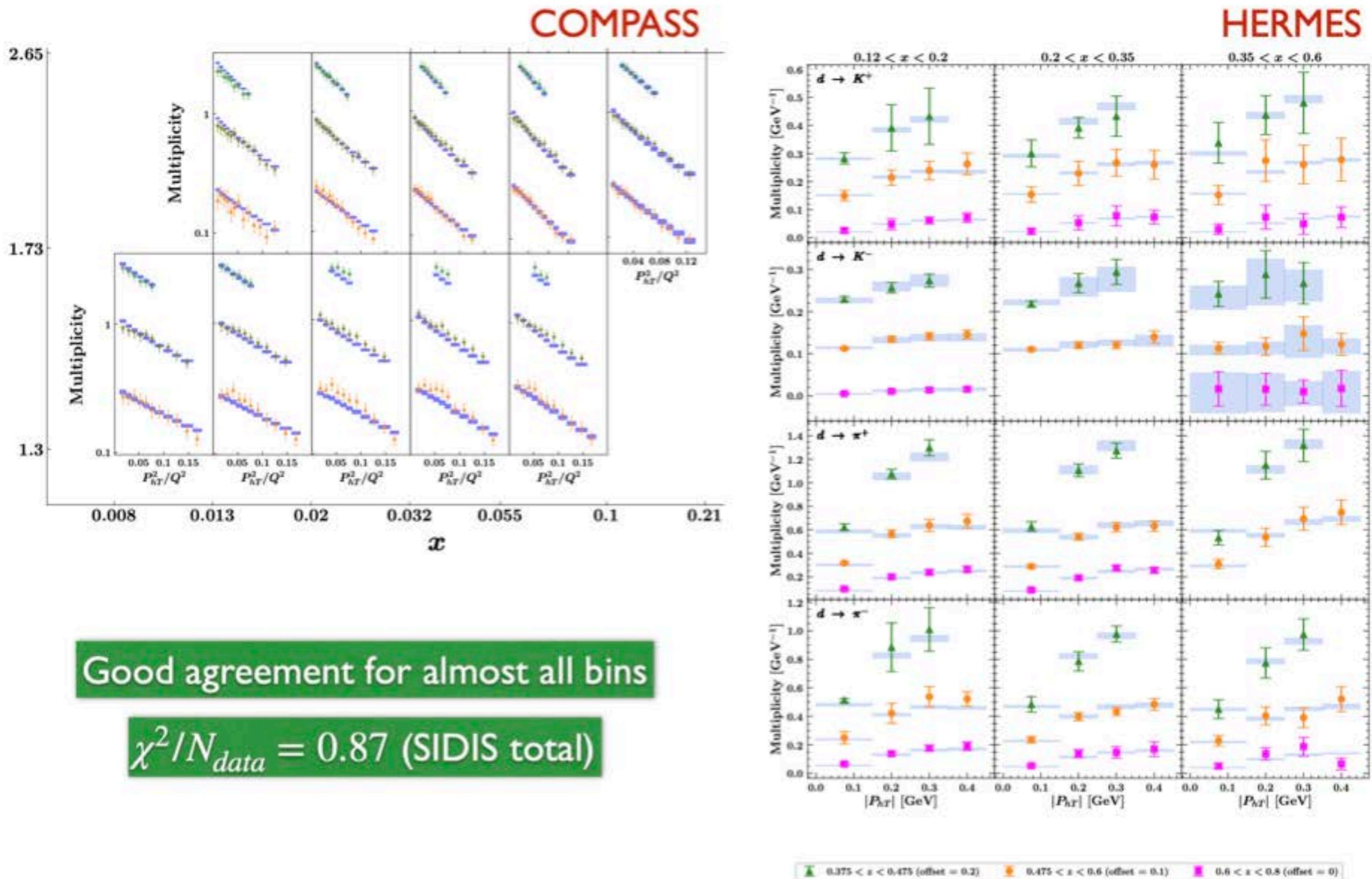


BEAST 1: NORMALIZATION

Slide by A. Bacchetta

see talk on Monday

MAPTMD22 – Quality of the fit



MAPTMD22 – Results

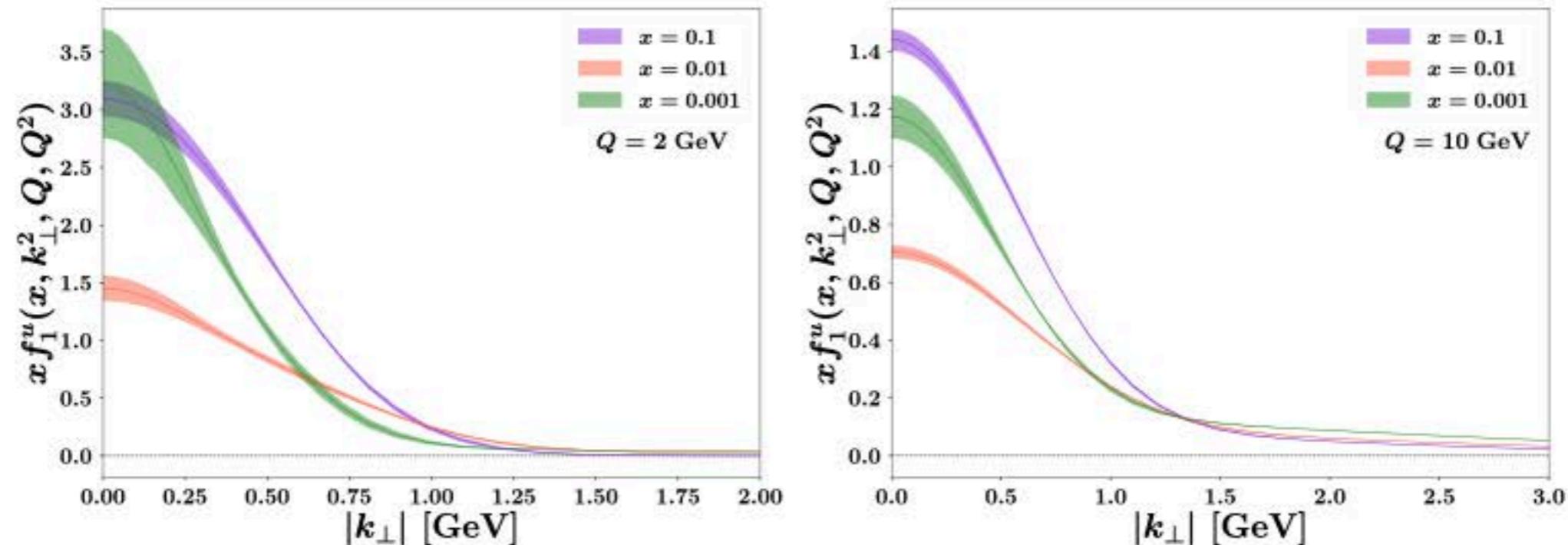


FIG. 13: The TMD PDF of the up quark in a proton at $\mu = \sqrt{\zeta} = Q = 2 \text{ GeV}$ (left panel) and 10 GeV (right panel) as a function of the partonic transverse momentum $|k_\perp|$ for $x = 0.001, 0.01$ and 0.1 . The uncertainty bands represent the 68% CL.

MAPTMD22 – Results

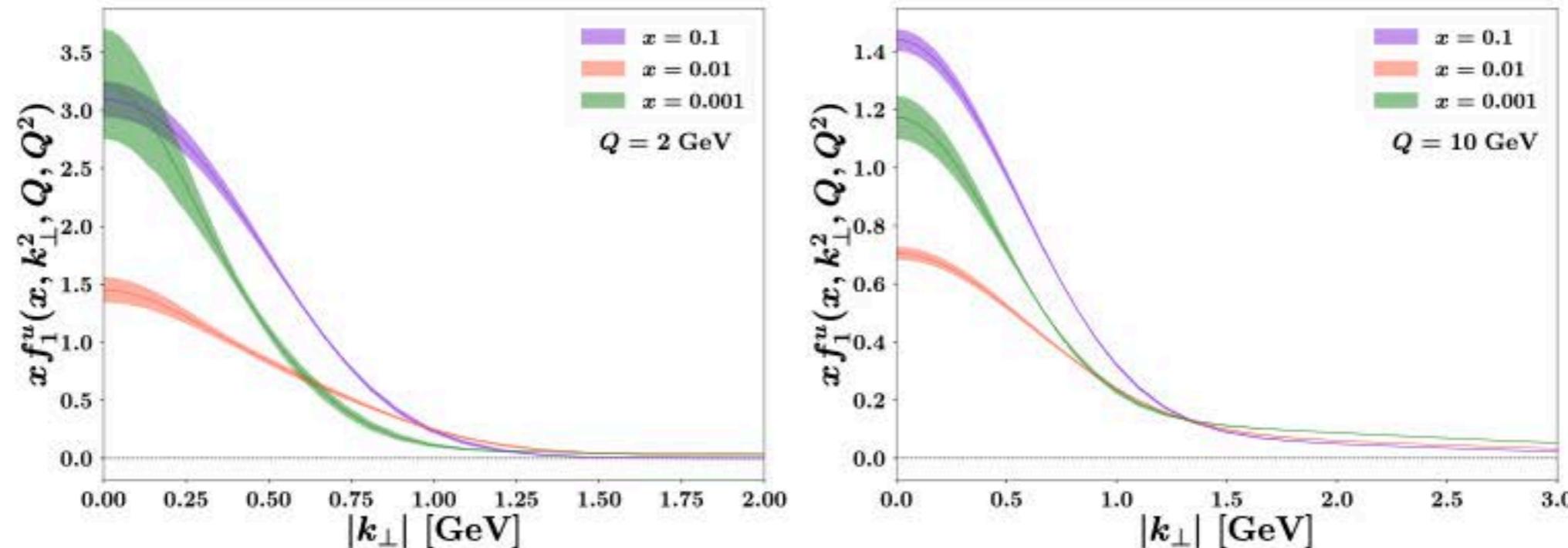


FIG. 13: The TMD PDF of the up quark in a proton at $\mu = \sqrt{\zeta} = Q = 2$ GeV (left panel) and 10 GeV (right panel) as a function of the partonic transverse momentum $|k_\perp|$ for $x = 0.001, 0.01$ and 0.1 . The uncertainty bands represent the 68% CL.

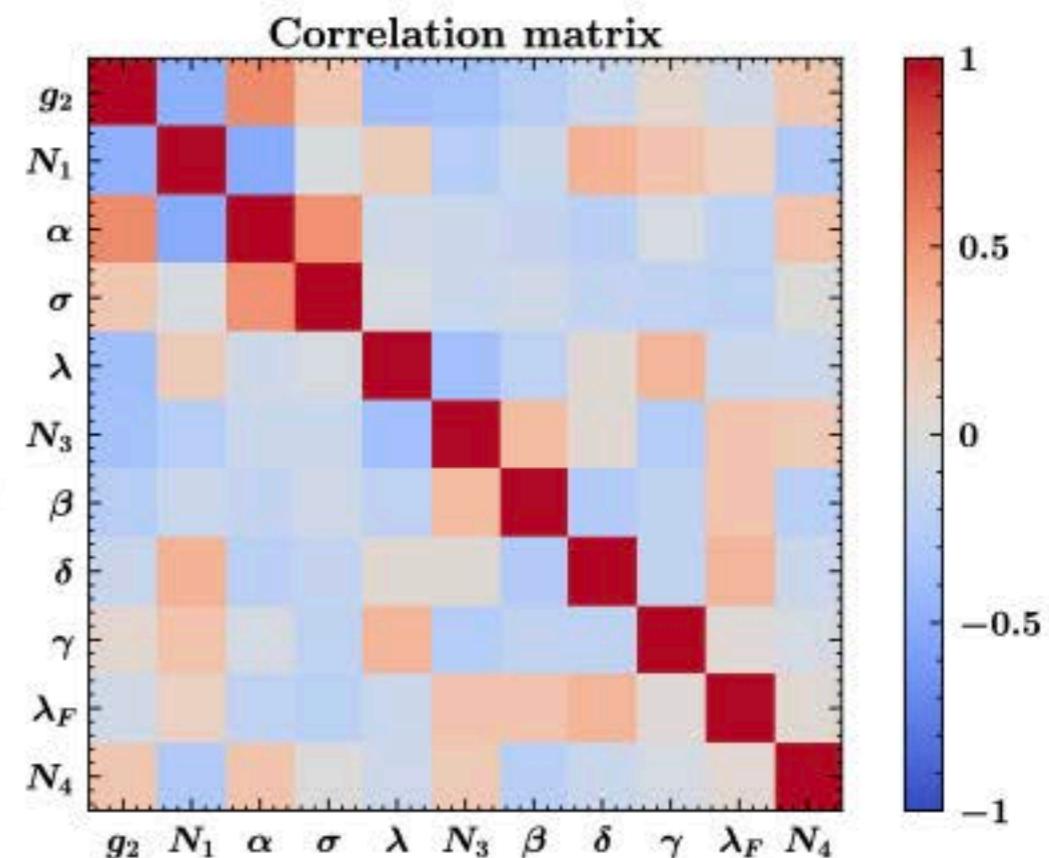
As usual, the rigidity of the functional form plays a role
and probably leads to underestimated bands

What about new data sets?



EIC impact studies with PV17

Parameter	Average over replicas
g_2	0.1171 ± 0.0145
N_1	0.283 ± 0.0368
α	2.2393 ± 1.2967
σ	-0.1416 ± 0.0959
λ	0.2548 ± 0.2549
N_3	0.2203 ± 0.0222
β	2.9304 ± 0.9978
δ	0.1175 ± 0.0506
γ	2.4736 ± 0.1649
λ_F	7.5475 ± 3.2037
N_4	0.0318 ± 0.0068



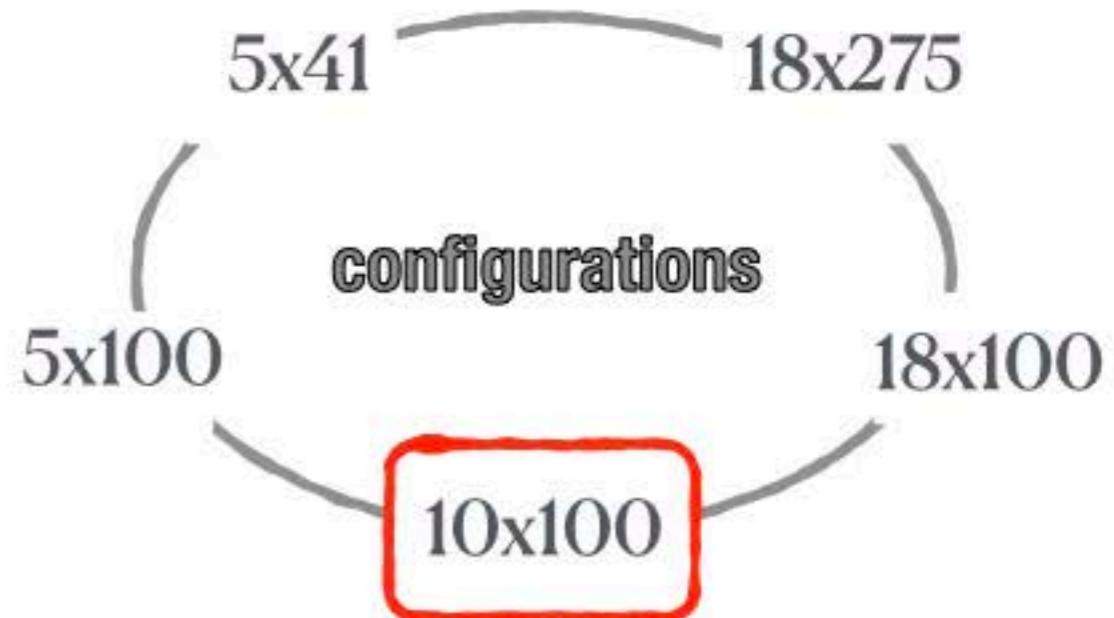
with **NangaParbat**
a **new fit** with uncertainties
similar to **PV17**
 $\chi^2_{\text{d.o.f.}} = 1.14 \pm 0.06$

~ 2000 fitted data

EIC impact studies with PV17

generation of pseudo data

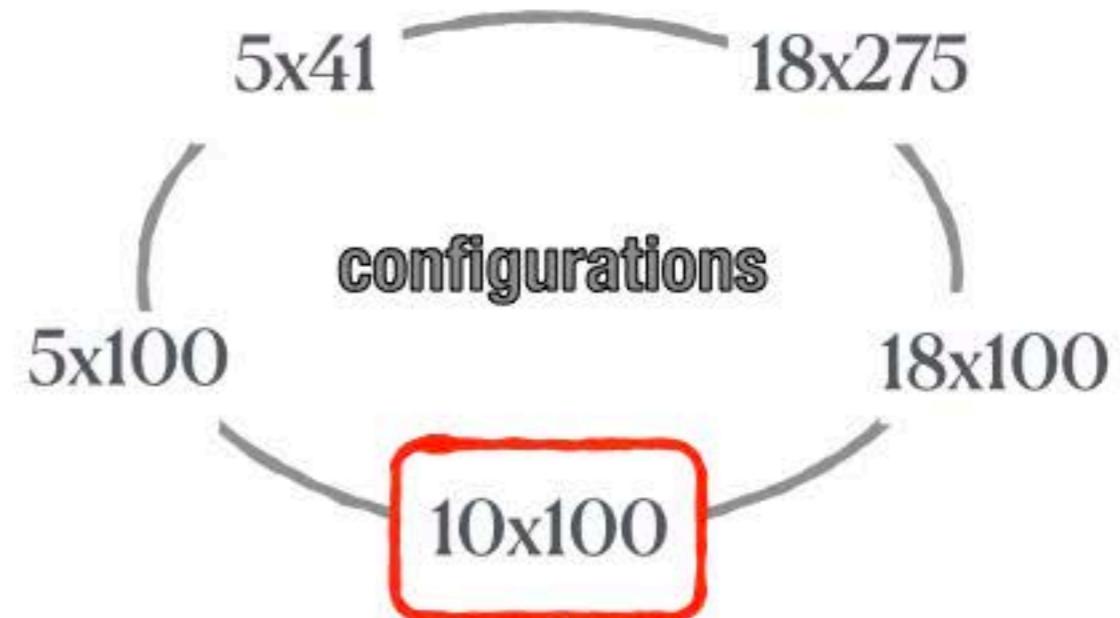
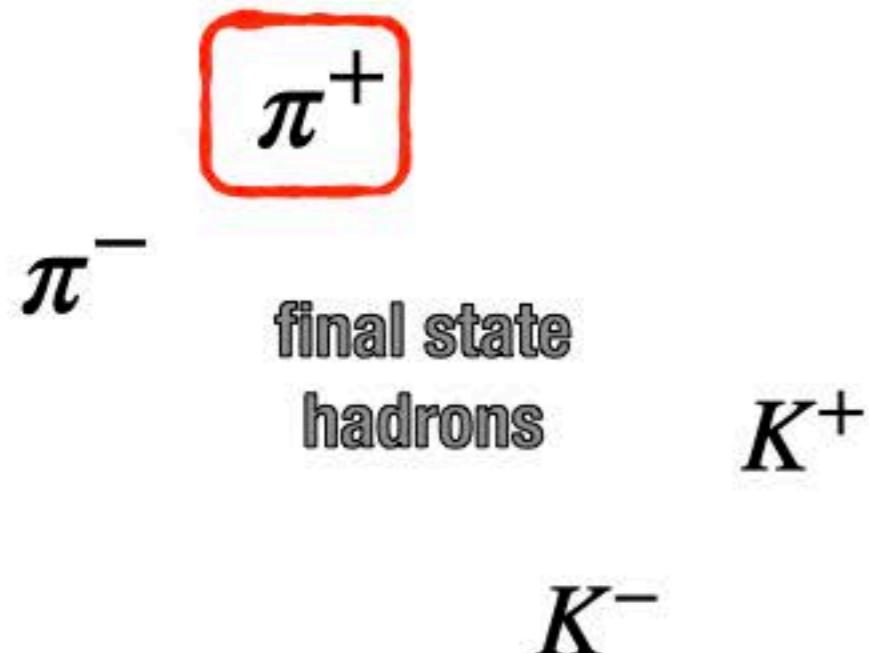
π^+
 π^-
final state
hadrons K^+
 K^-



```
Name,unpol.10x100_pip_ACC_opt8_cut
Comment,Ralf's pseudo data for EIC.
Reference,Ralf
Process type,SIDIS
Number of points,3837
Number of uncorr.errors,2
Number of corr.errors,0
Number of norm.errors,1
List of norm.errors (relative),0.03
Total cross-section nomalized,False
List of points
Point id,process
id,s[GeV^2],<Q>[GeV],Qmin[GeV],Qmax[GeV],<x>,xMin,xMax,<z>,zMin,zMax,<pT>[GeV],pTMin[GeV],pTMax[GeV],xSec,Uncorr.Err.0,Uncorr.Err.1,Th.Factor,FiducialCuts,yMin,yMax,W2min[GeV^2],W2max[GeV^2],TargetMass[GeV],ProductMass[GeV]
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EIC impact studies with PV17

generation of pseudo data



central value of pseudo data obtained

using average parameters of the PV17 baseline fit

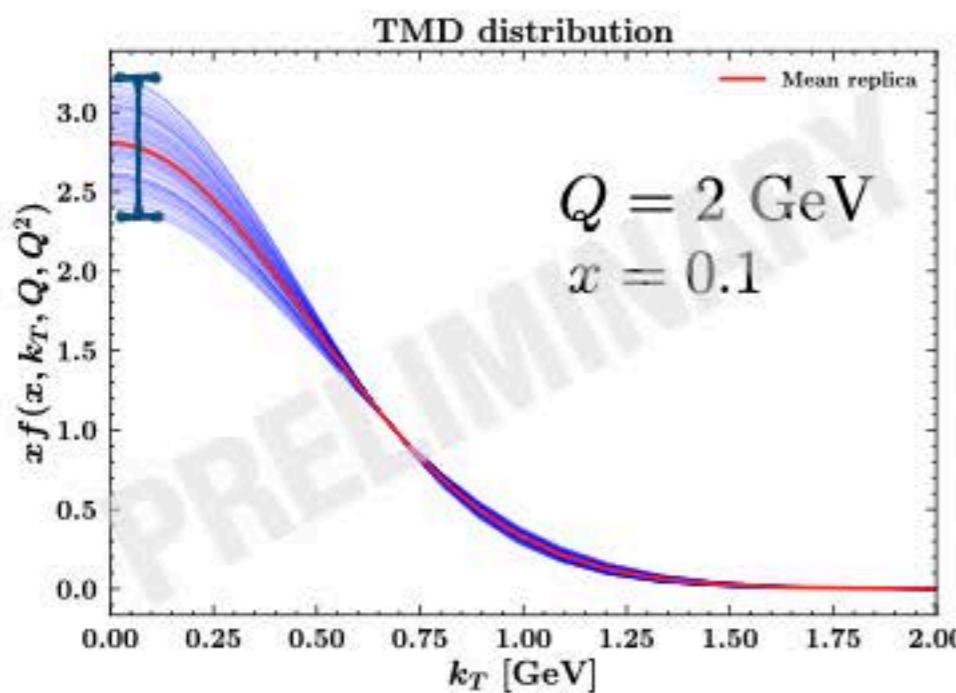
~ 2500
pseudodata points

uncertainties of pseudo data

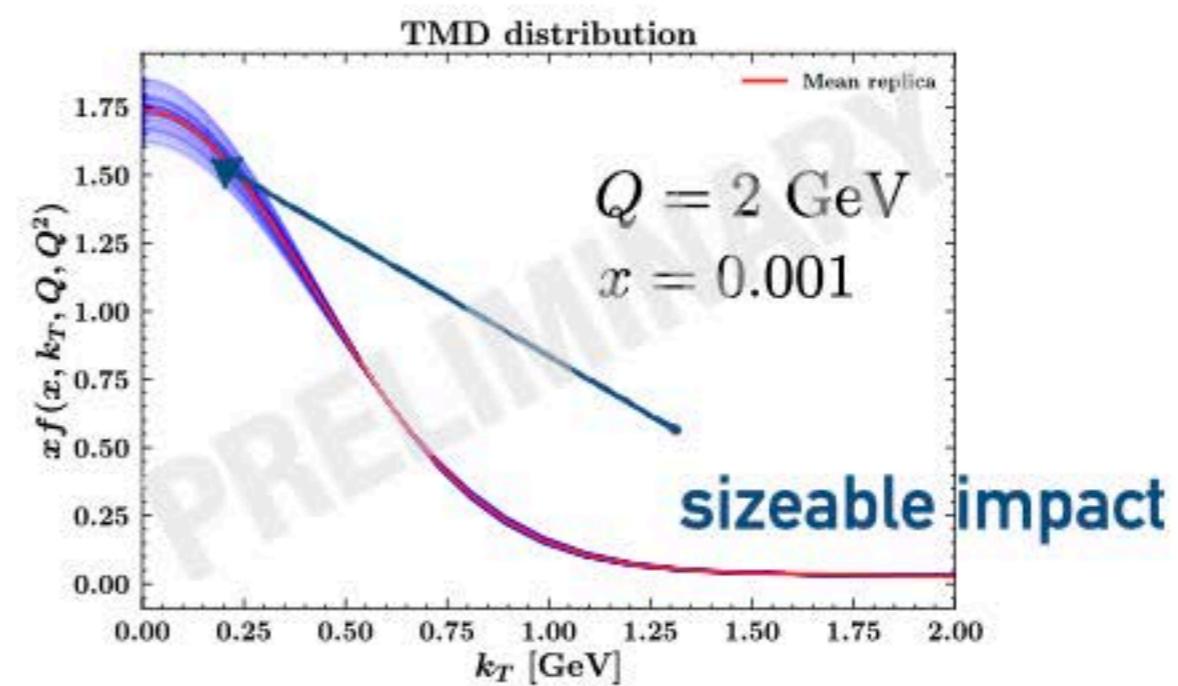
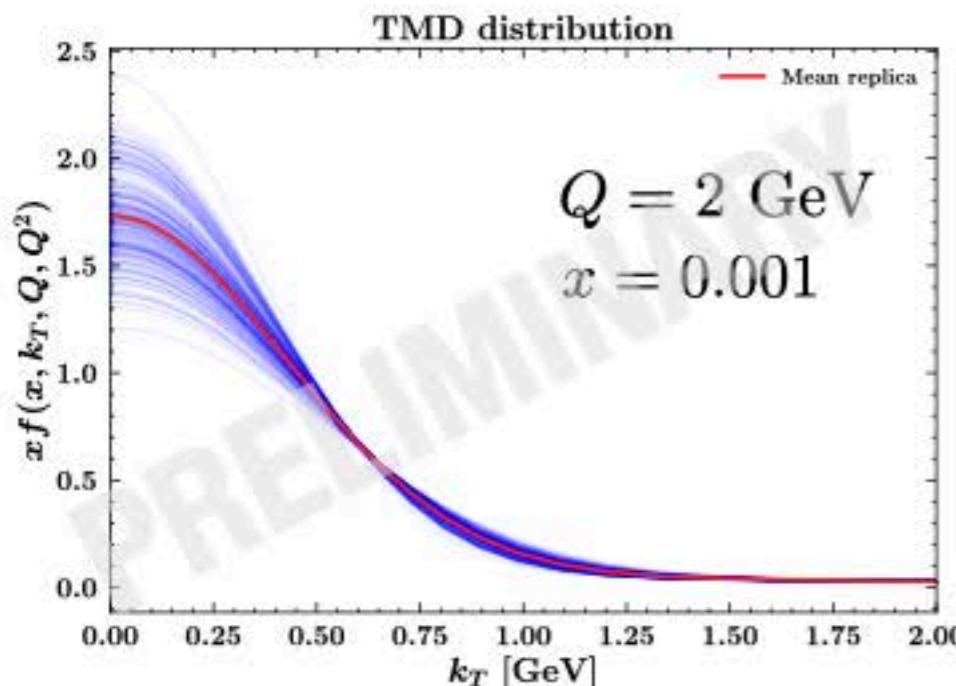
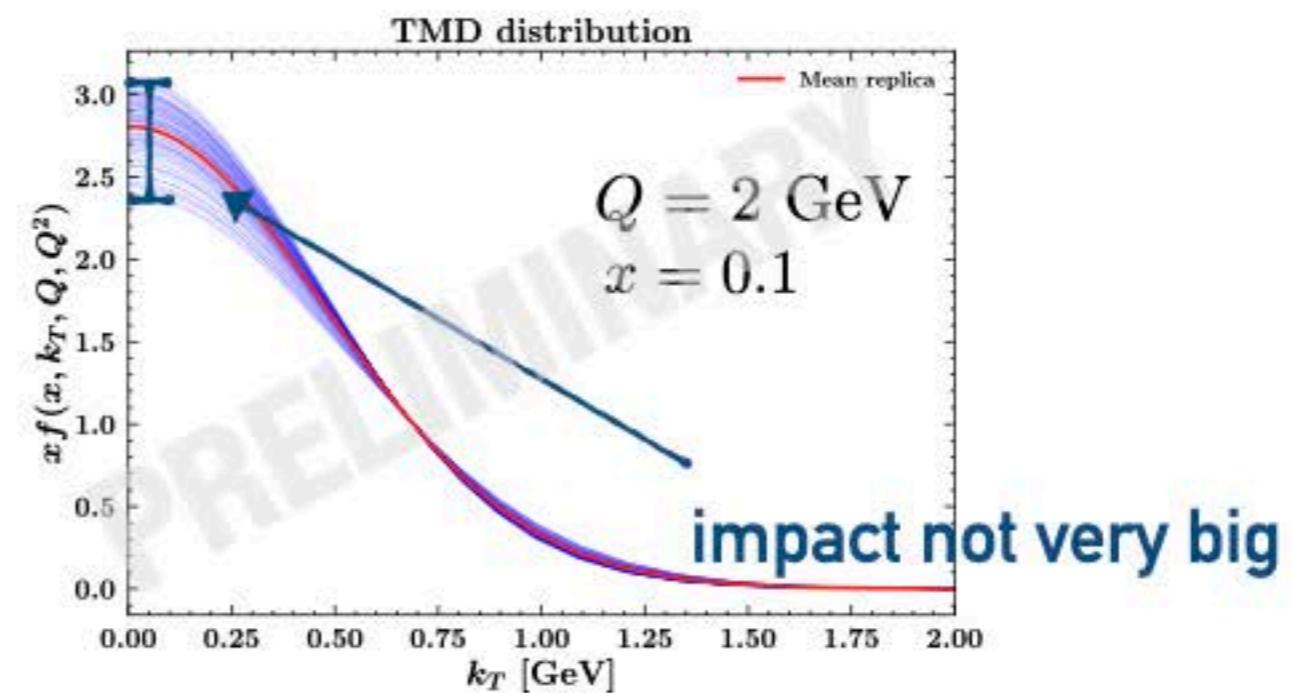
are given by simulations done
by the EIC SIDIS working group

EIC impact studies with PV17

PV17 baseline



PV17 baseline + EIC



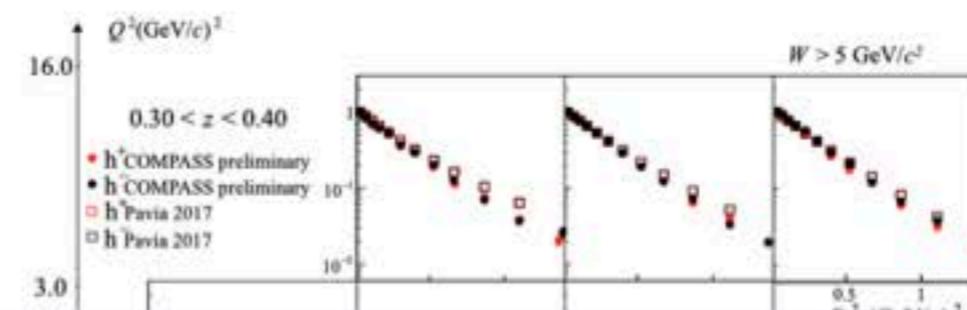
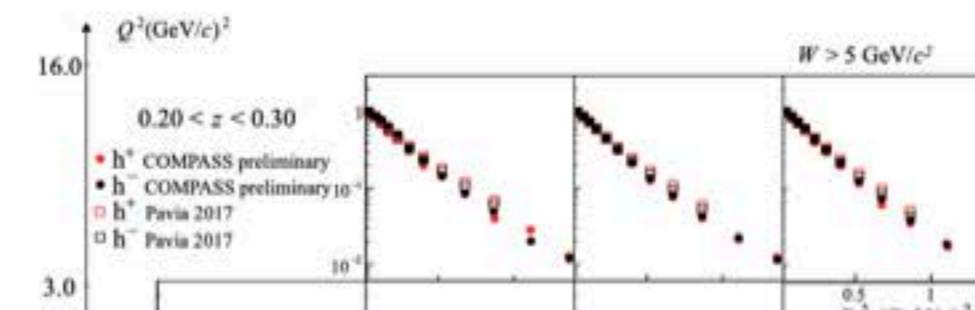
COMPASS Proton predictions with PV17

Measurement on LH₂: Results for the P_T -distributions

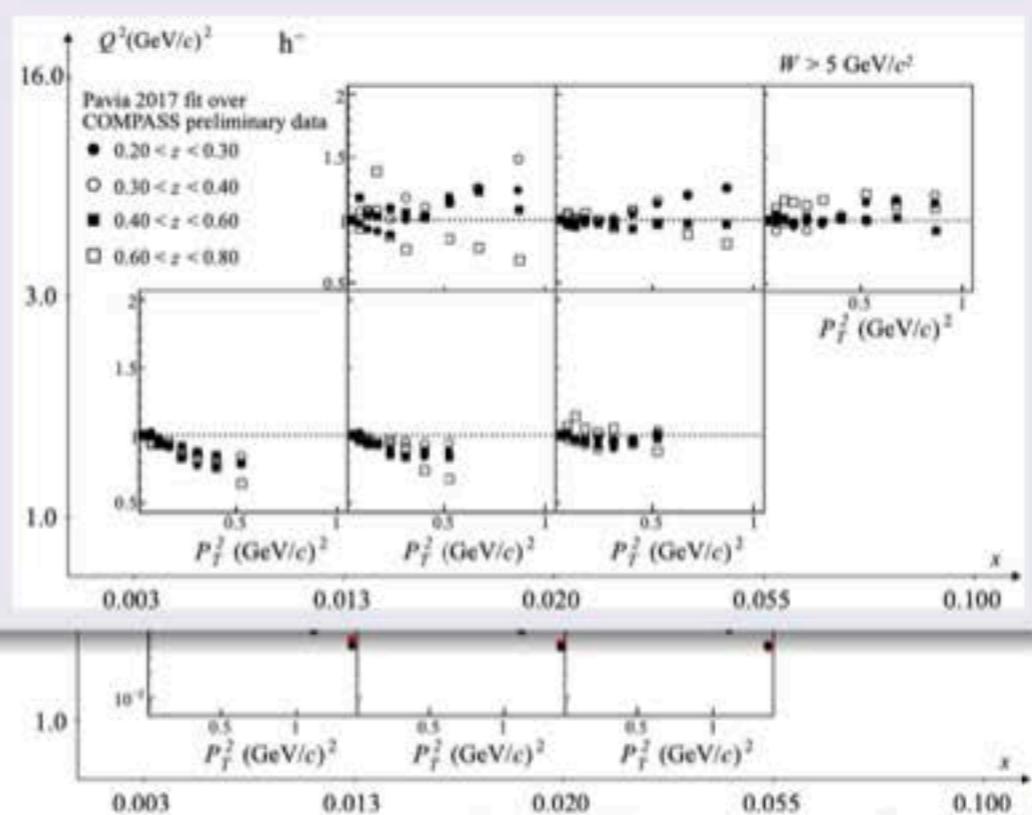
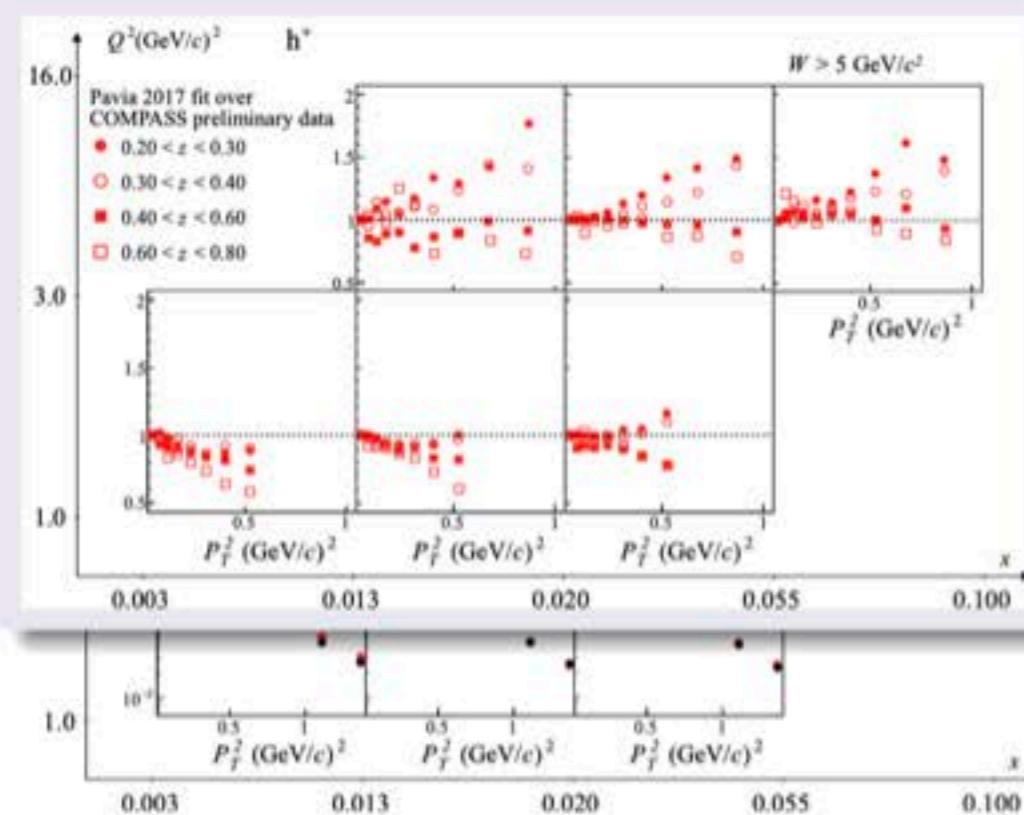


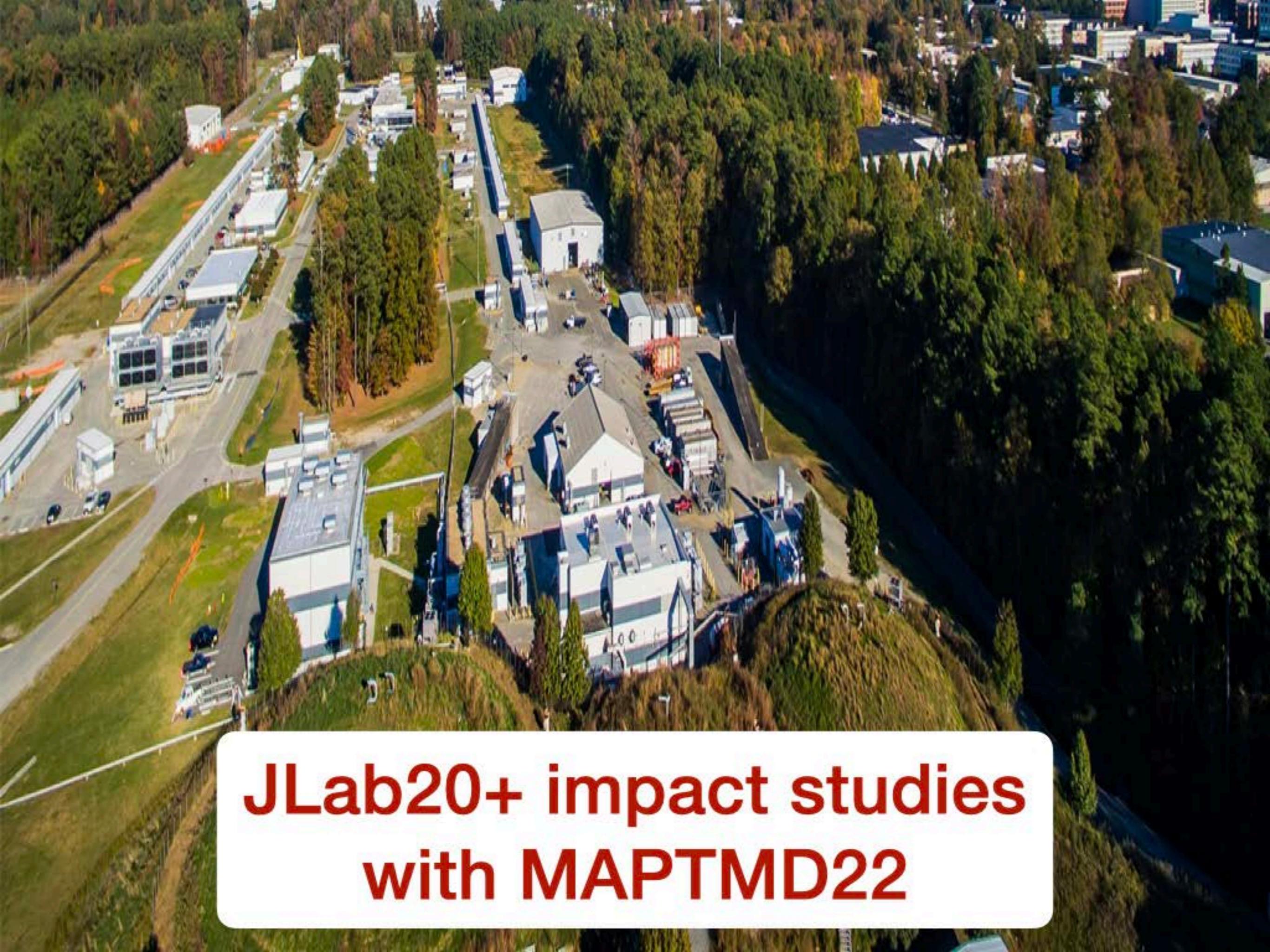
Comparison with Pavia 2017 fit [A. Bacchetta *et al.*, JHEP 06 (2017) 081]

- SIDIS ep(D) $\rightarrow e\pi^\pm(K^\pm)X$ (HERMES)
- SIDIS $\mu D \rightarrow \mu h X$ (COMPASS)
- Drell-Yan (E228, E605)
- Z boson production (CDF, D0)



The ratio of the prediction from the fit over the preliminary data:





**JLab20+ impact studies
with MAPTMD22**

JLab20+ Impact Study

1000000	1	1	1	1	1	1.70	0.06	0.05	0.06	0.67	28
1000001	2	1	1	1	2	1.74	0.06	0.06	0.16	0.69	76
1000002	3	1	1	1	3	1.71	0.06	0.07	0.26	0.68	119
1000003	4	1	1	1	4	1.68	0.06	0.07	0.35	0.67	159
1000004	5	1	1	1	5	1.69	0.06	0.06	0.45	0.67	205
1000005	6	1	1	1	6	1.71	0.06	0.06	0.55	0.68	280
1000006	7	1	1	1	7	1.70	0.06	0.06	0.65	0.68	297
1000007	8	1	1	1	8	1.71	0.06	0.06	0.75	0.67	375
1000008	9	1	1	1	9	1.70	0.06	0.06	0.85	0.68	413
1000009	10	1	1	1	10	1.71	0.06	0.07	0.95	0.68	449
1000010	11	1	1	1	11	1.72	0.06	0.06	1.05	0.68	462
1000011	12	1	1	1	12	1.69	0.06	0.06	1.15	0.67	548
1000012	13	1	1	1	13	1.70	0.06	0.06	1.25	0.68	592
1000013	14	1	1	1	14	1.71	0.06	0.06	1.35	0.68	655
1000014	15	1	1	1	15	1.70	0.06	0.06	1.45	0.68	723
1000015	16	1	1	1	16	1.70	0.06	0.06	1.55	0.68	788
1000016	17	1	1	1	17	1.70	0.06	0.06	1.65	0.67	816
1000017	18	1	1	1	18	1.69	0.06	0.06	1.75	0.67	883
1000018	19	1	1	1	19	1.70	0.06	0.06	1.85	0.68	968
1000019	20	1	1	1	20	1.70	0.06	0.06	1.95	0.68	929

Kinematics

19 bins in Q^2 : [1, 20] GeV 2 ($\Delta Q^2 = 1$ GeV 2)

20 bins in x : [0.04, 0.84] ($\Delta x = 0.04$)

20 bins in z : [0.05, 0.95] ($\Delta z = 0.05$)

80 bins in q_T : [0.05, 7.95] GeV ($\Delta q_T = 0.1$ GeV)

JLab20+ Impact Study

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Pseudodata generation

Central value obtained using **average parameters** of MAPTMD22 baseline fit

Uncertainties of pseudodata

Stat $1/\sqrt{N}$

Sys ? (1 – 5%)

JLab20+ Impact Study

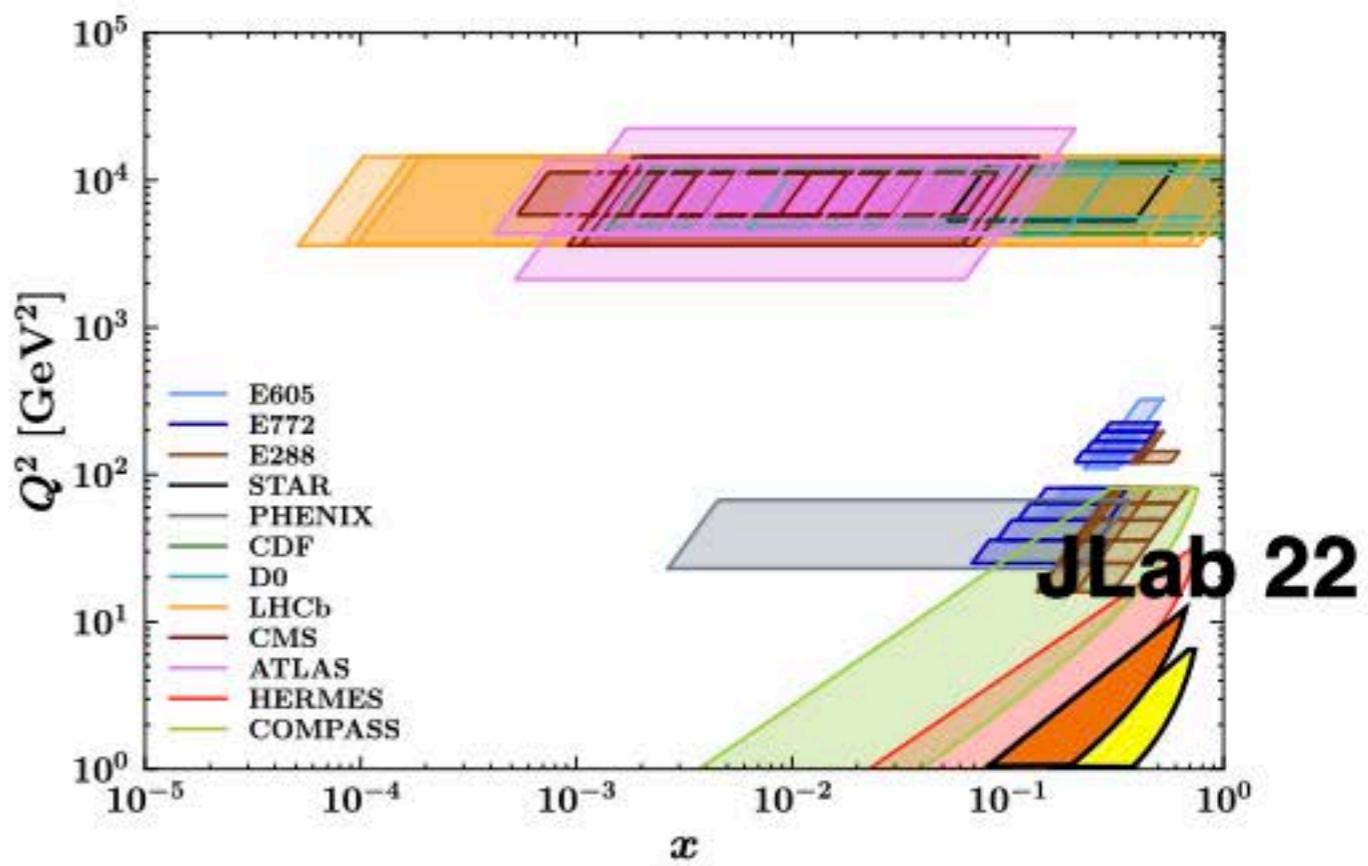
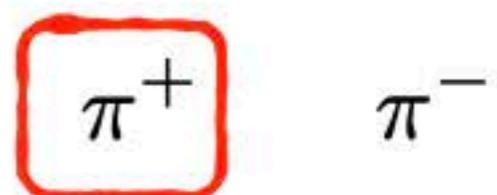
Included dataset

$$Q^2 > 1.4 \text{ GeV}^2$$

$$0.2 < z < 0.7$$

$$P_{hT} < \min [\min [0.2 Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$

Final-state hadrons



JLab20+ Impact Study

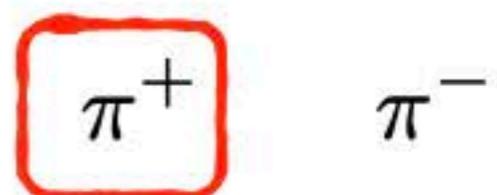
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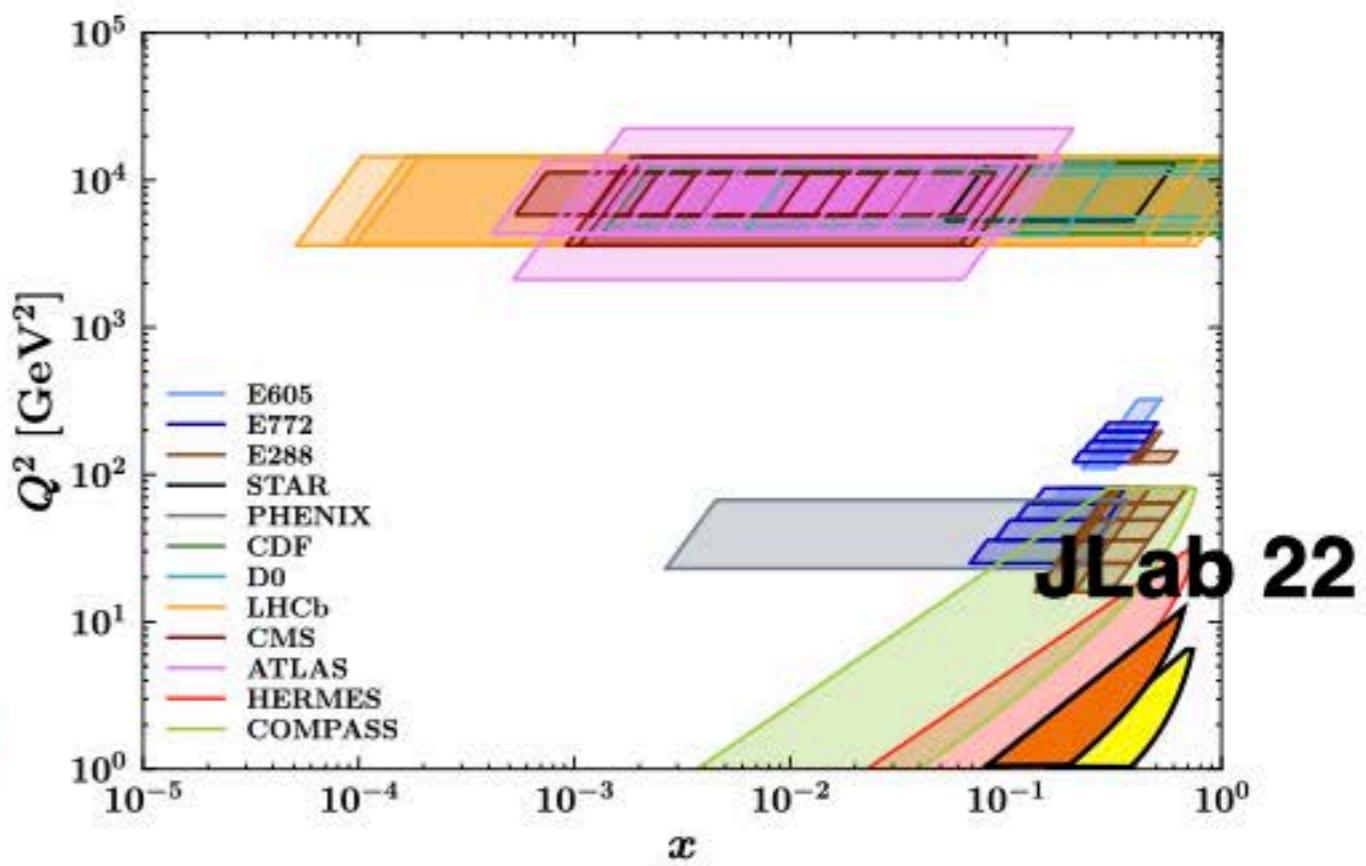


~ 2000 MAPTMD22

+

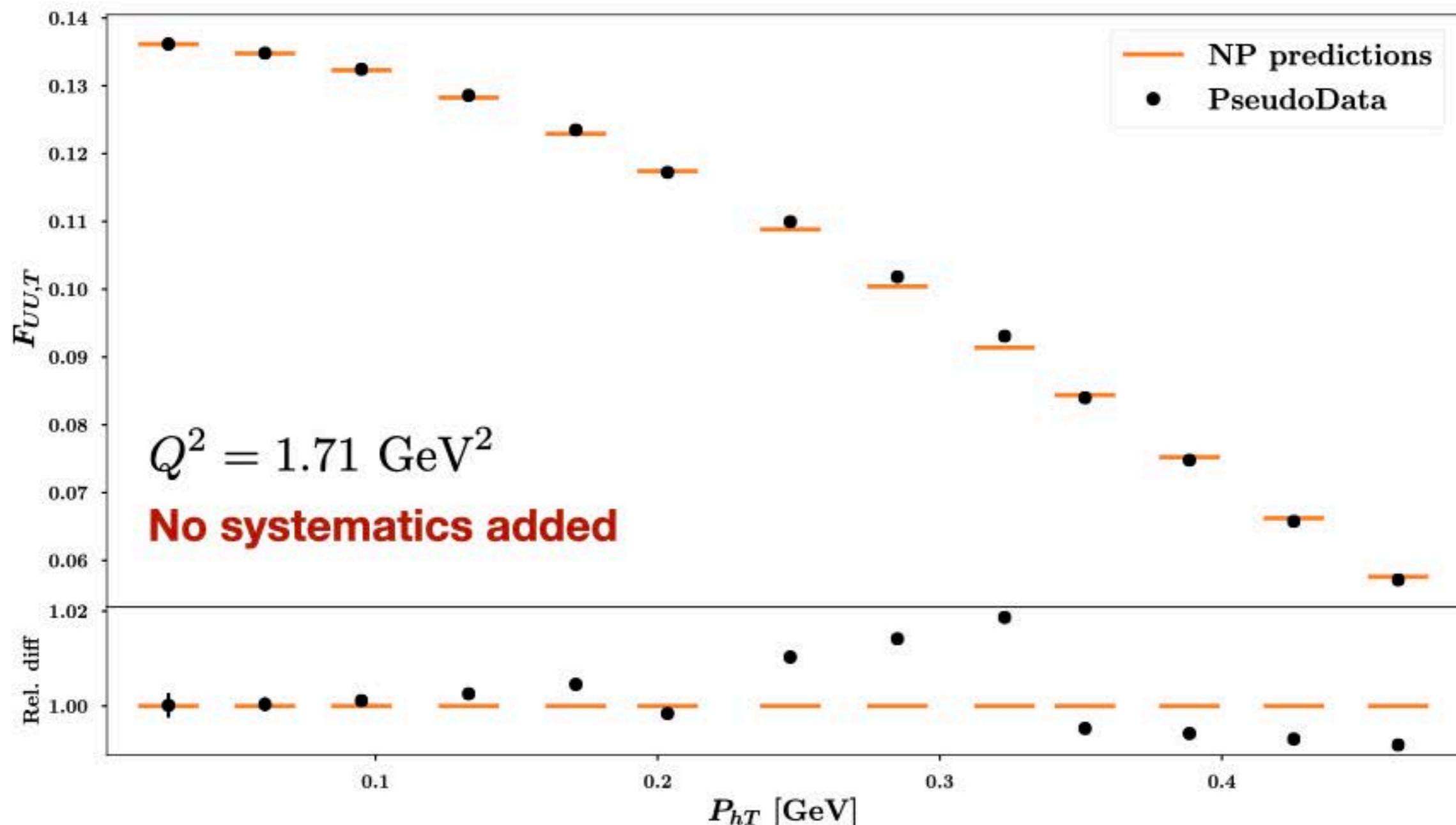
~ 25000 JLab 20+
pseudodata

Increasing of an order of magnitude!



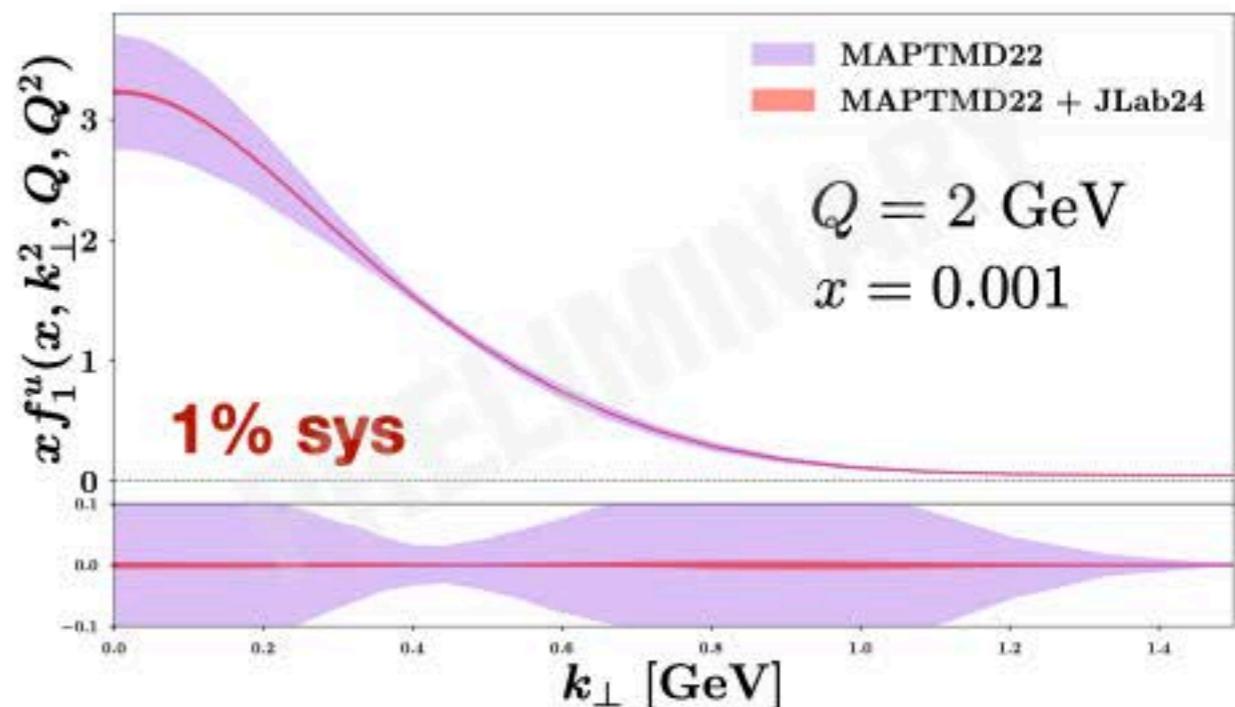
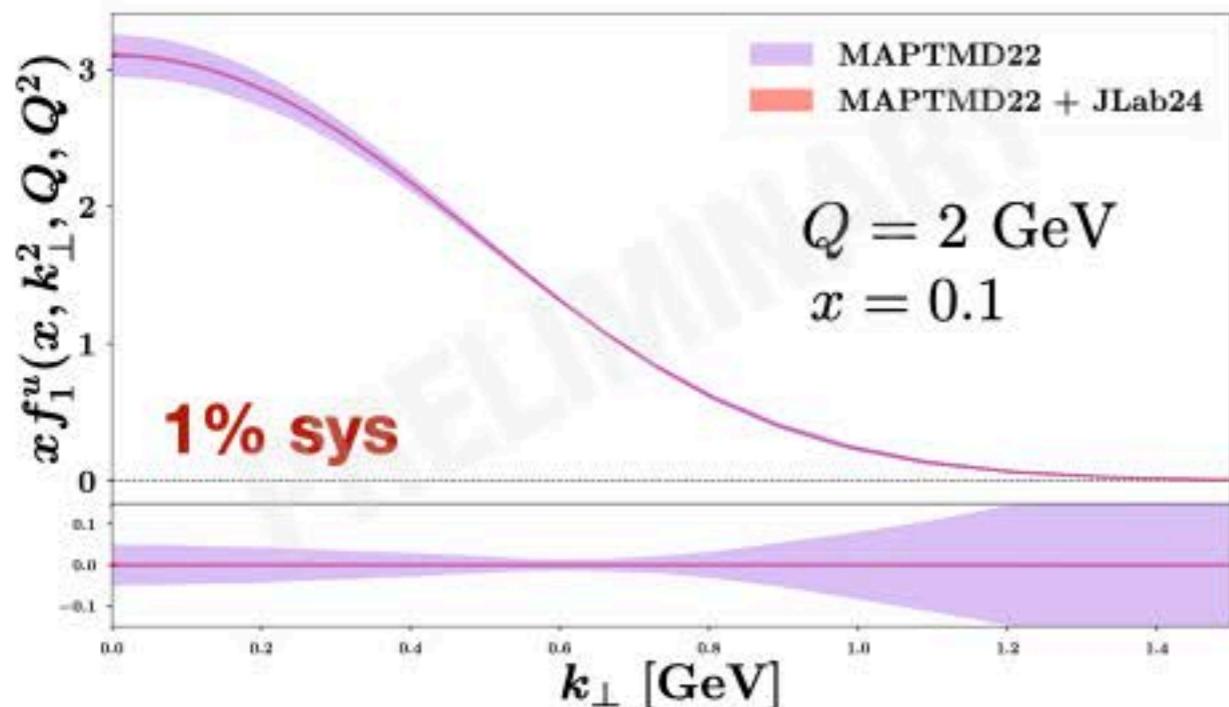
JLab20+ Impact Study

Effect of systematic uncertainties on pseudodata



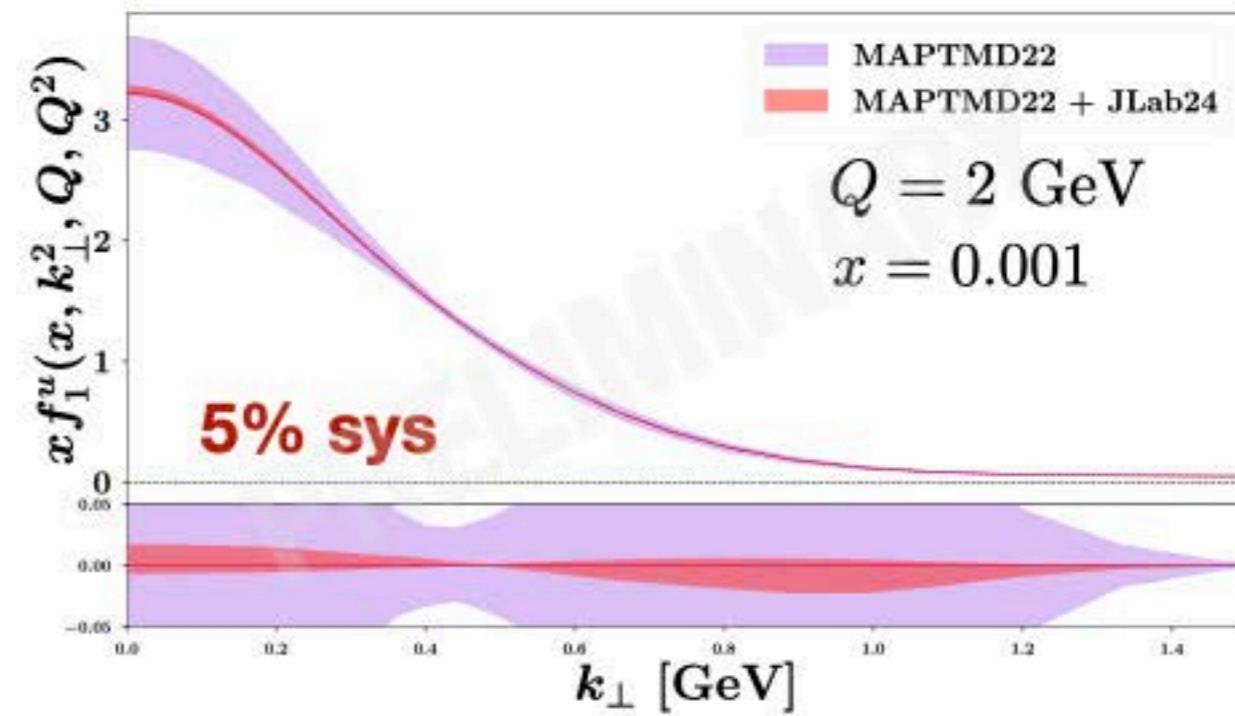
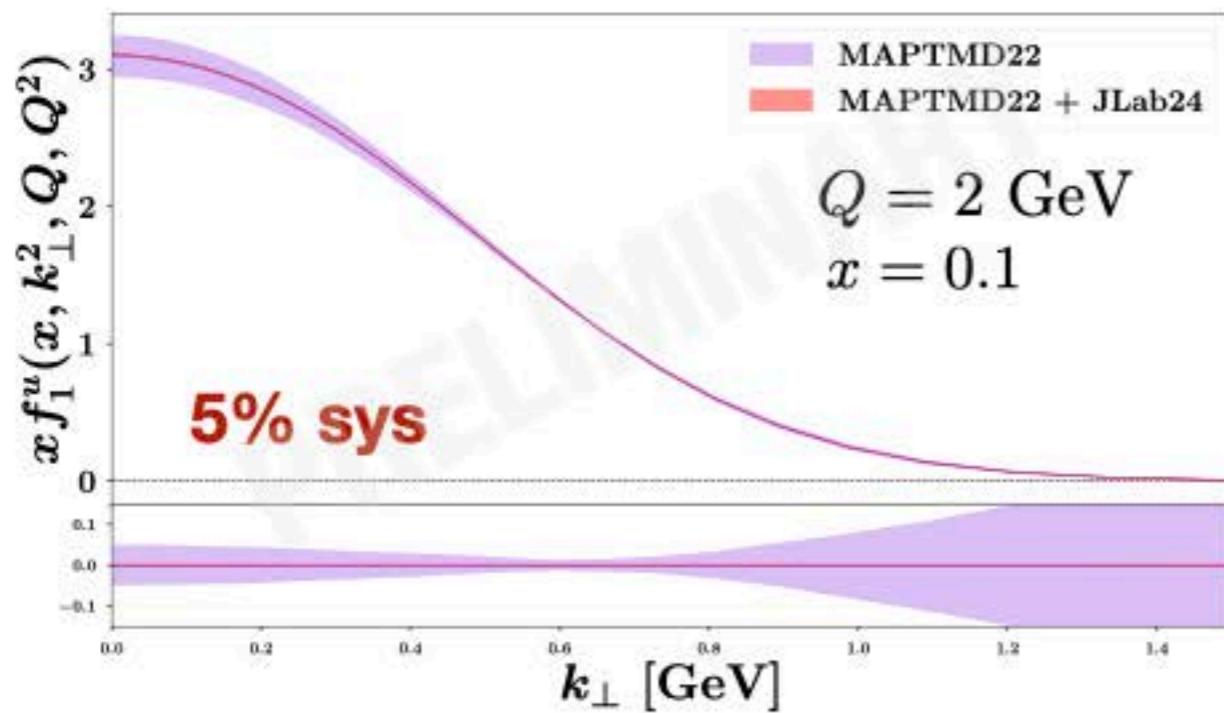
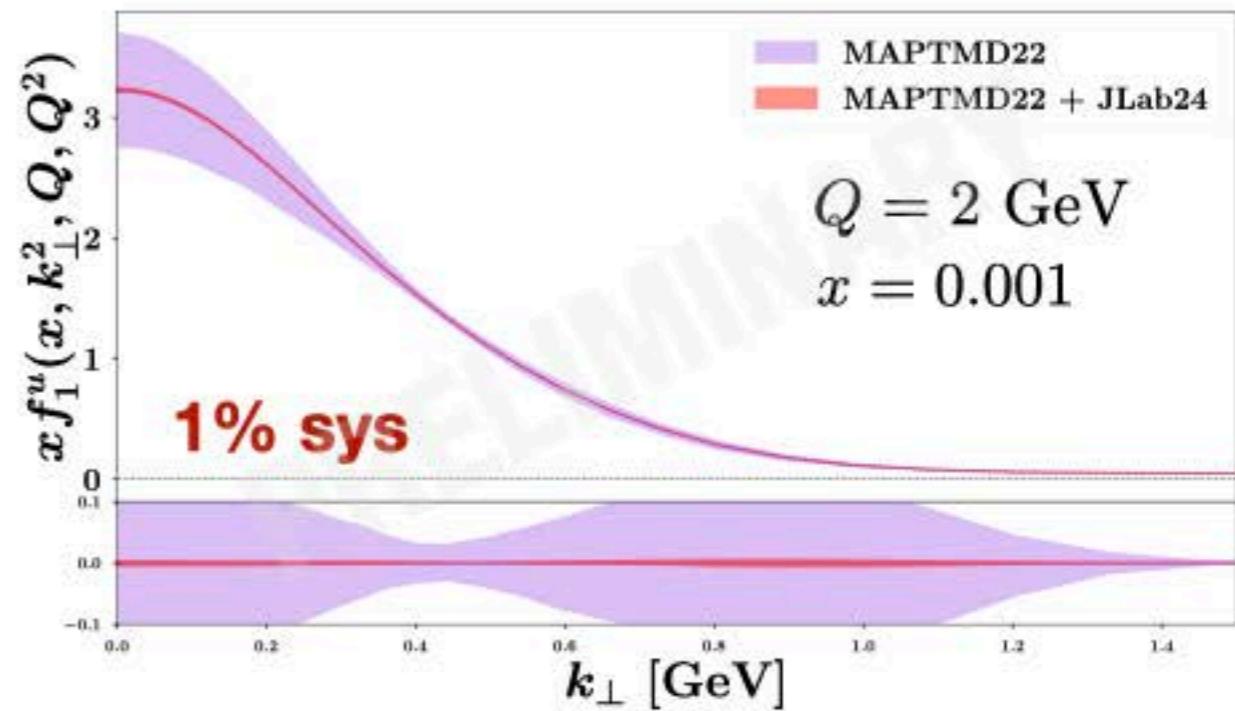
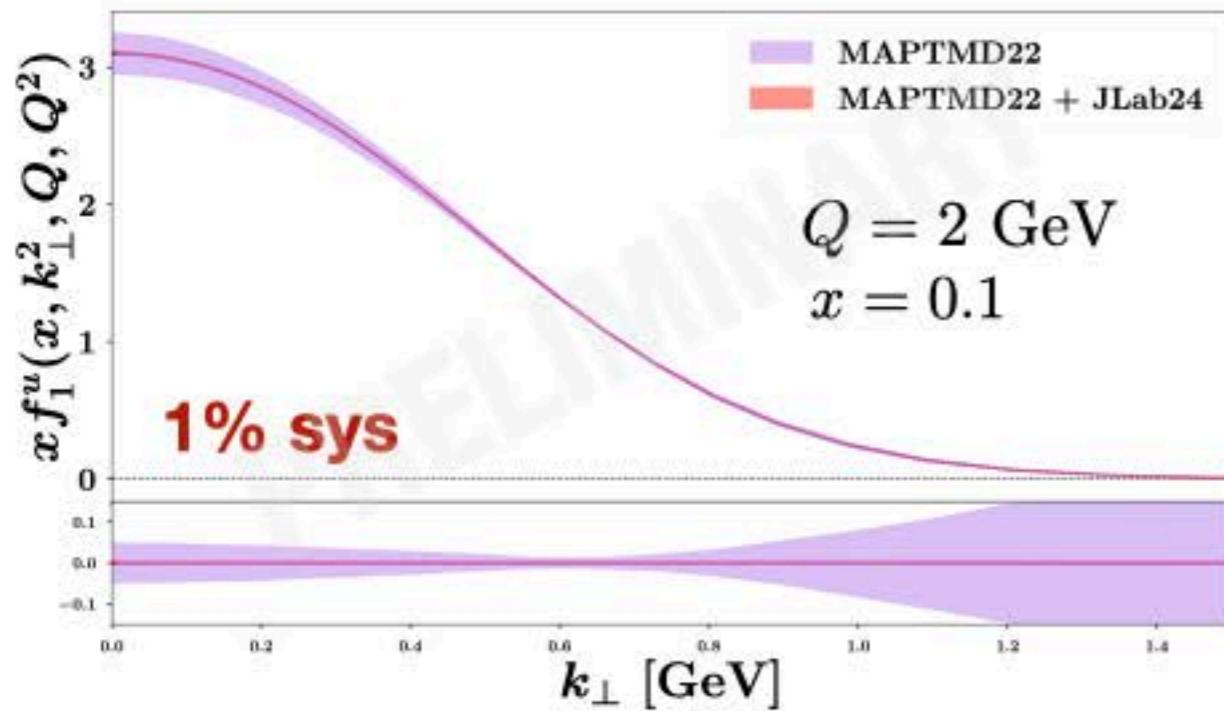
JLab20+ Impact Study

PRELIMINARY results

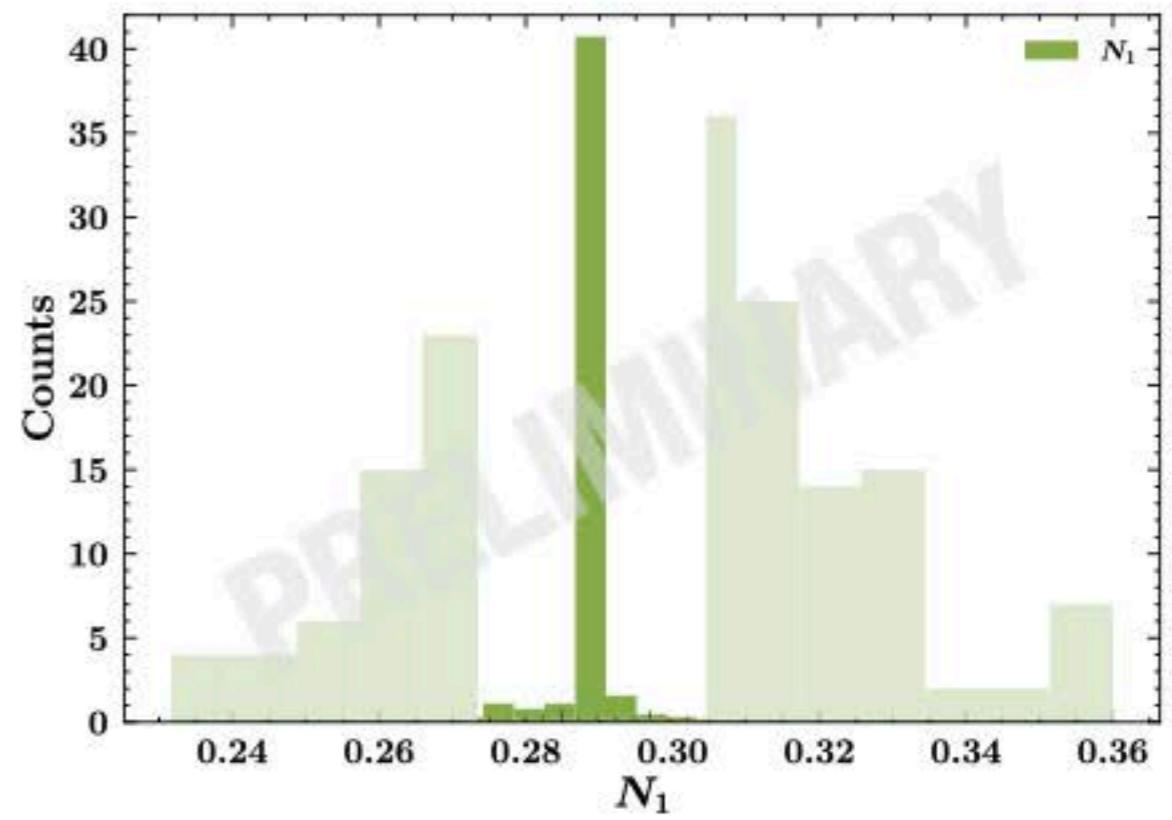
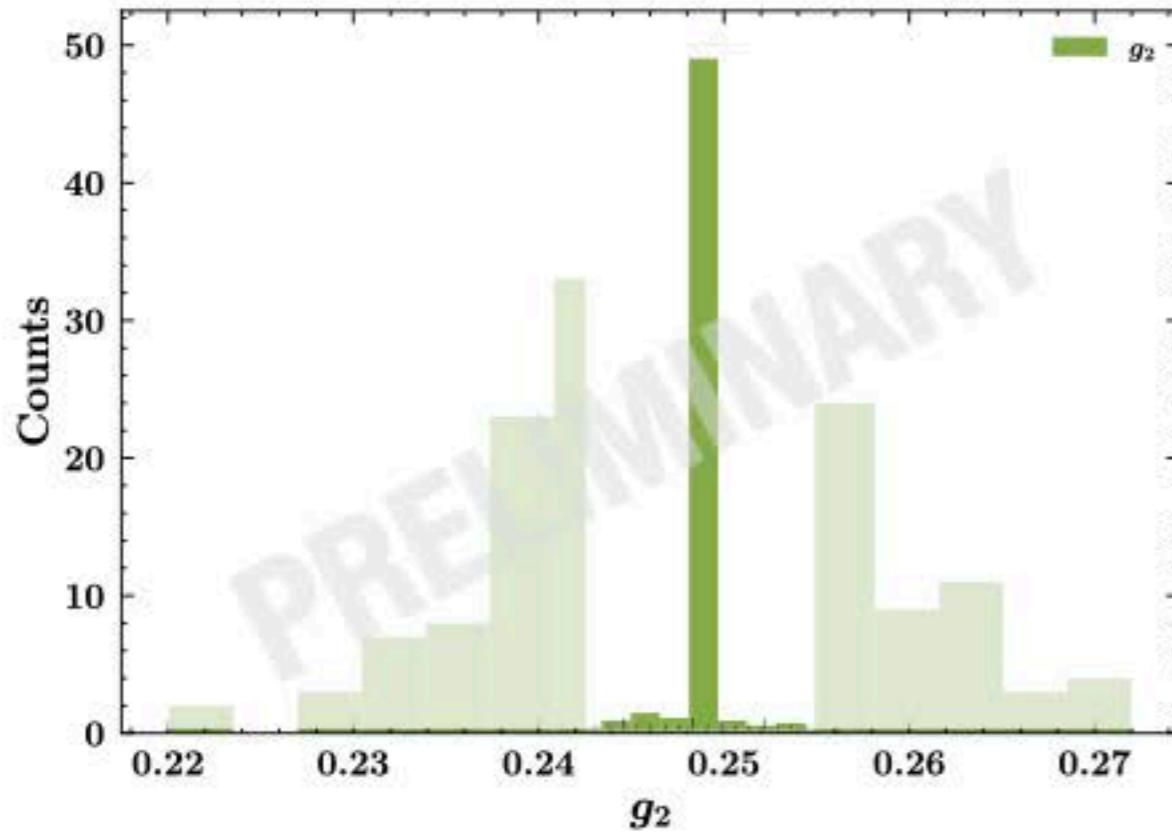


JLab20+ Impact Study

PRELIMINARY results



JLab20+ Impact Study



JLab20+ Impact Study

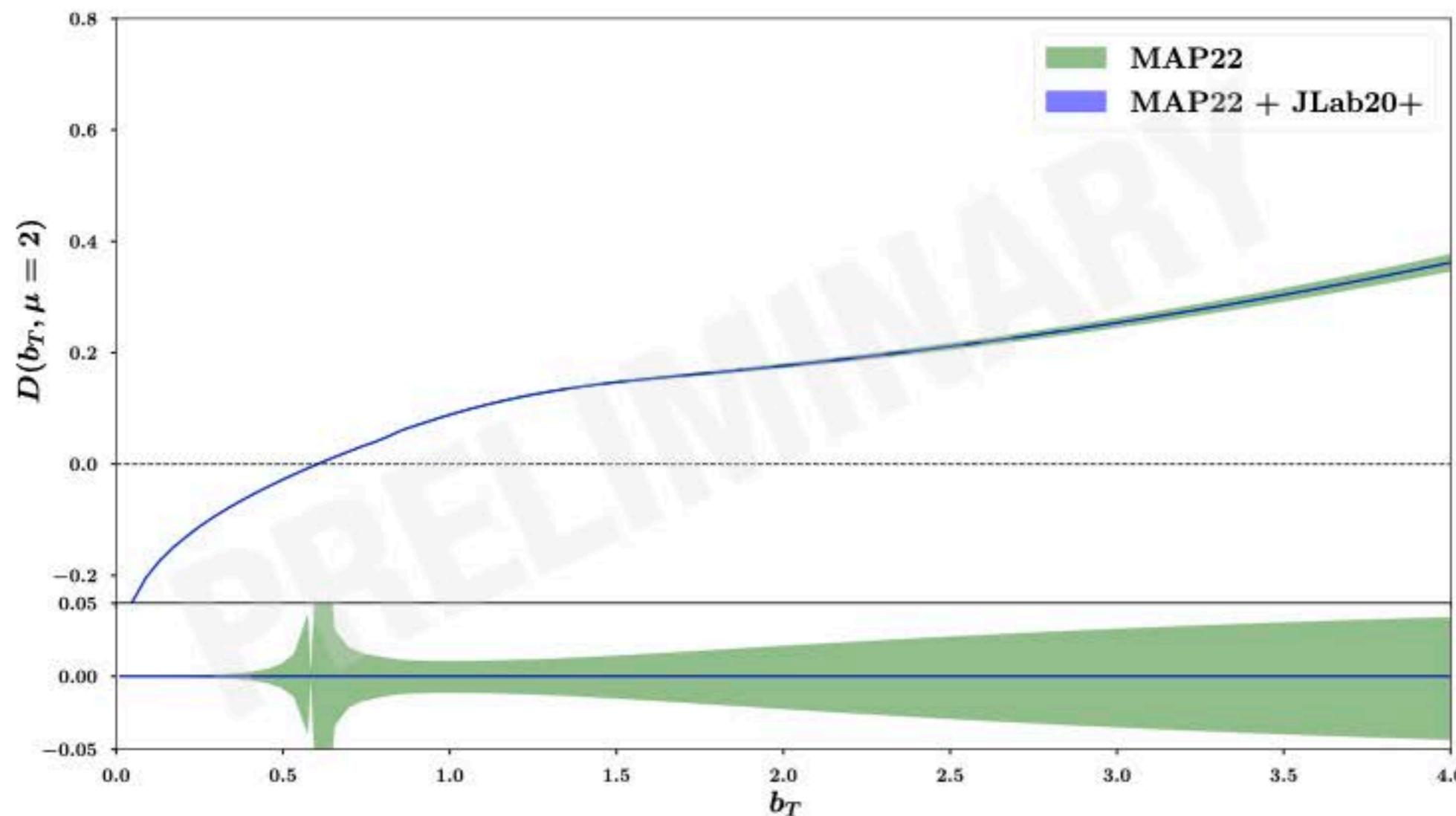
Kernel of the rapidity evolution equation

$$\frac{\partial \ln \hat{f}_1(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = K(b_T, \mu) \quad K(b_T, \mu_{b_*}) = K(b_*, \mu_{b_*}) + g_K(b_T)$$

JLab20+ Impact Study

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Outlook and Conclusions

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- **MAPTMD22**: global extraction of unpolarized quark TMDs

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Outlook and Conclusions

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- Old studies with PV17 show impact of EIC only at low-x
- **Preliminary** study on JLab20+ shows impact at both low- and high-x
 - number of data?*
 - flexibility of TMD model?*

Outlook and Conclusions

- **MAPTMD22**: global extraction of unpolarized quark TMDs
- Old studies with PV17 show impact of EIC only at low-x
- **Preliminary** study on JLab20+ shows impact at both low- and high-x
 - number of data?*
 - flexibility of TMD model?*
- We have to understand the role of systematic errors and the methodology

BACKUP SLIDES

TMD factorization – Logarithmic counting

Orders in powers of

TMD factorization – Logarithmic counting

Orders in powers of α_S

Hard factor and matching coefficient

Ingredients in perturbative Sudakov form factor

Accuracy	H and C	K and γ_F	γ_K	PDFs/FFs and α_S evol.
LL	0	-	1	-
NLL	0	1	2	LO
NLL'	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL'	2	2	3	NNLO
N^3LL^-	2	3	4	NNLO (NLO FF)
N^3LL	2	3	4	NNLO

TMD factorization – Logarithmic counting

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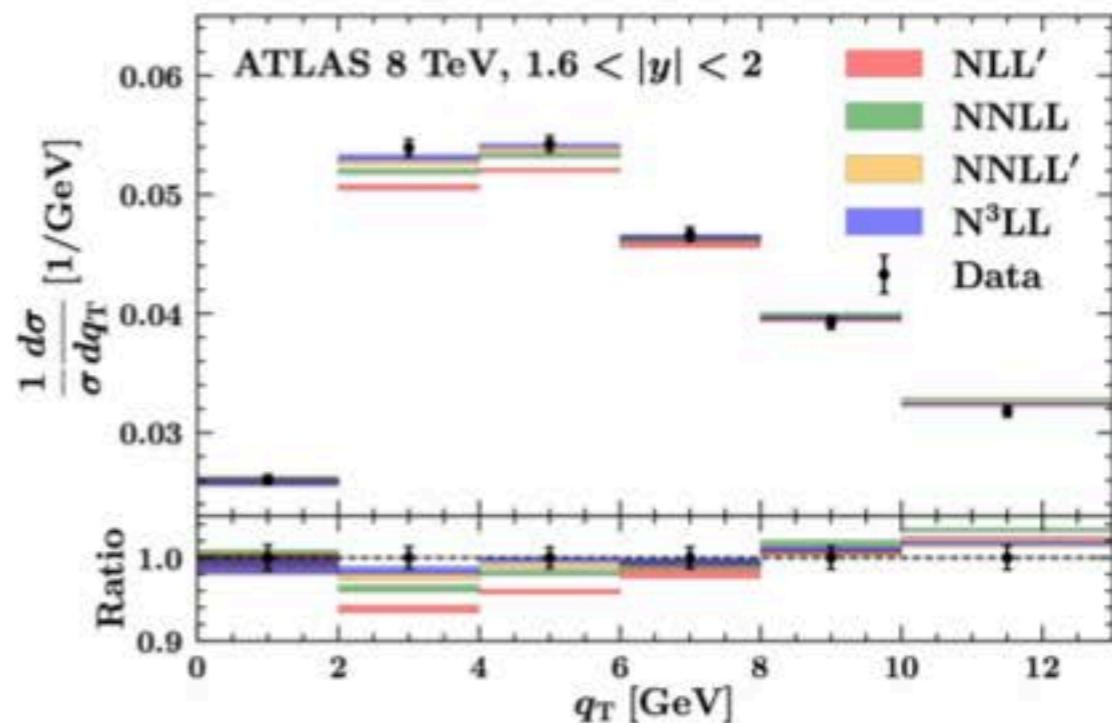
$N^3 LL = N^3 LL$ but with NLO collinear FF

MAPTMD22 – Normalization of SIDIS

MAPTMD22 – Normalization of SIDIS

High-Energy Drell-Yan beyond NLL

$$Q \sim 100 \text{ GeV}$$



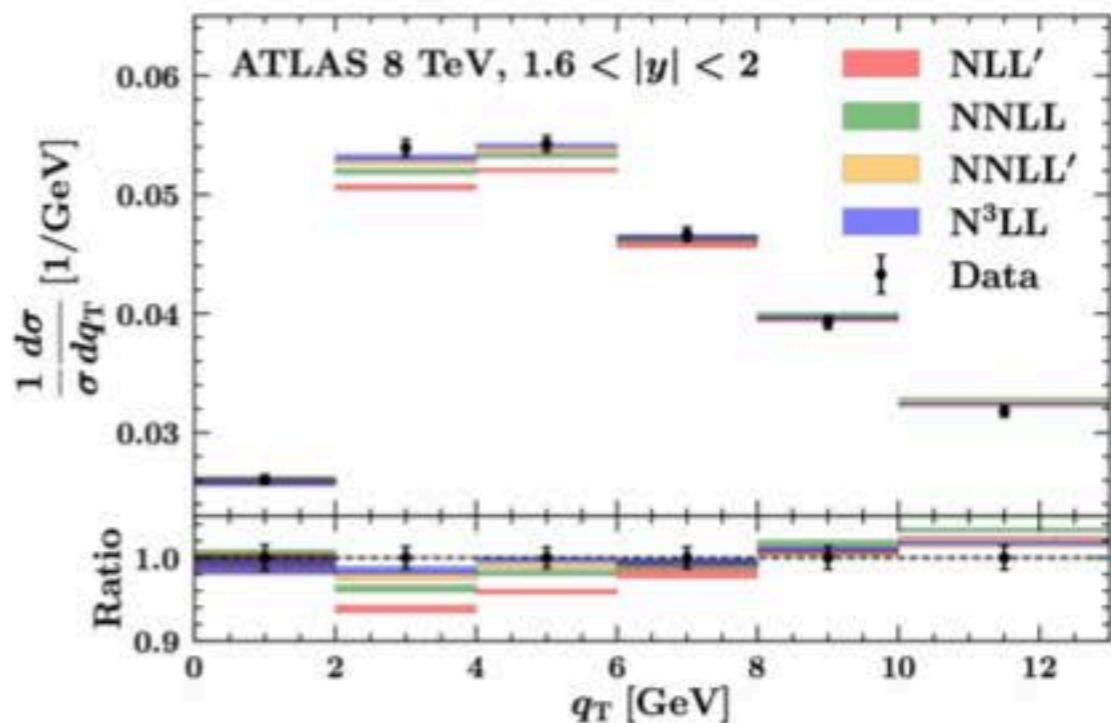
Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici,

MAPTMD22 – Normalization of SIDIS

SIDIS multiplicities beyond NLL

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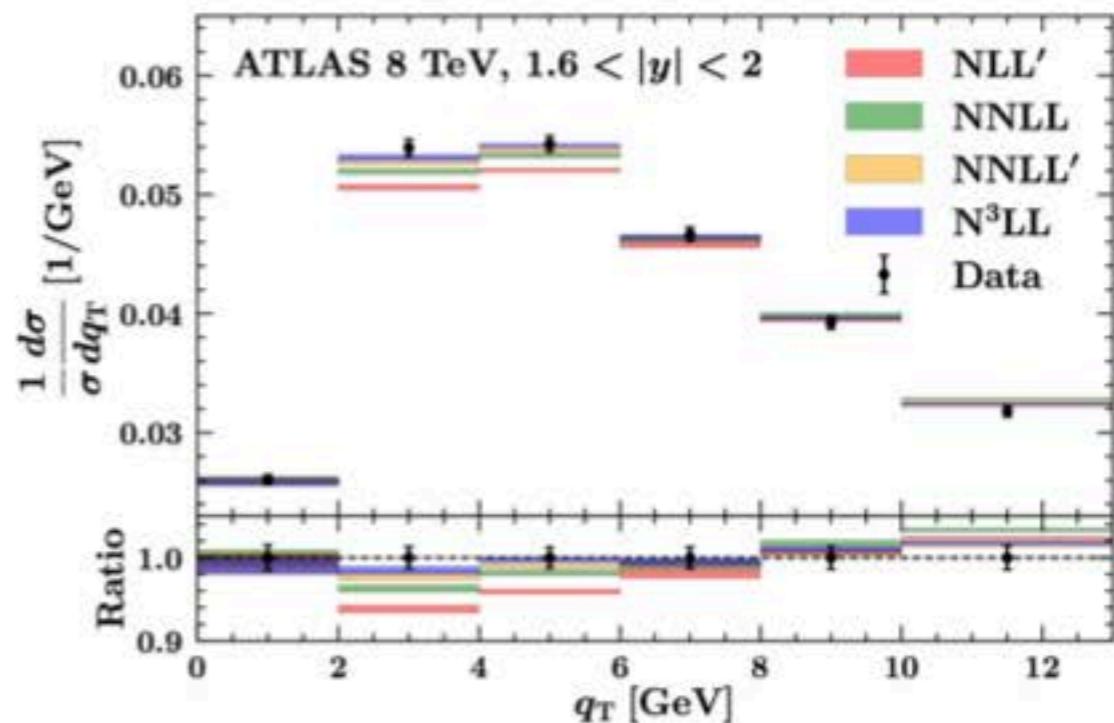
MAPTMD22 – Normalization of SIDIS

SIDIS multiplicities beyond NLL

$Q \sim 2 \text{ GeV}$

High-Energy Drell-Yan beyond NLL

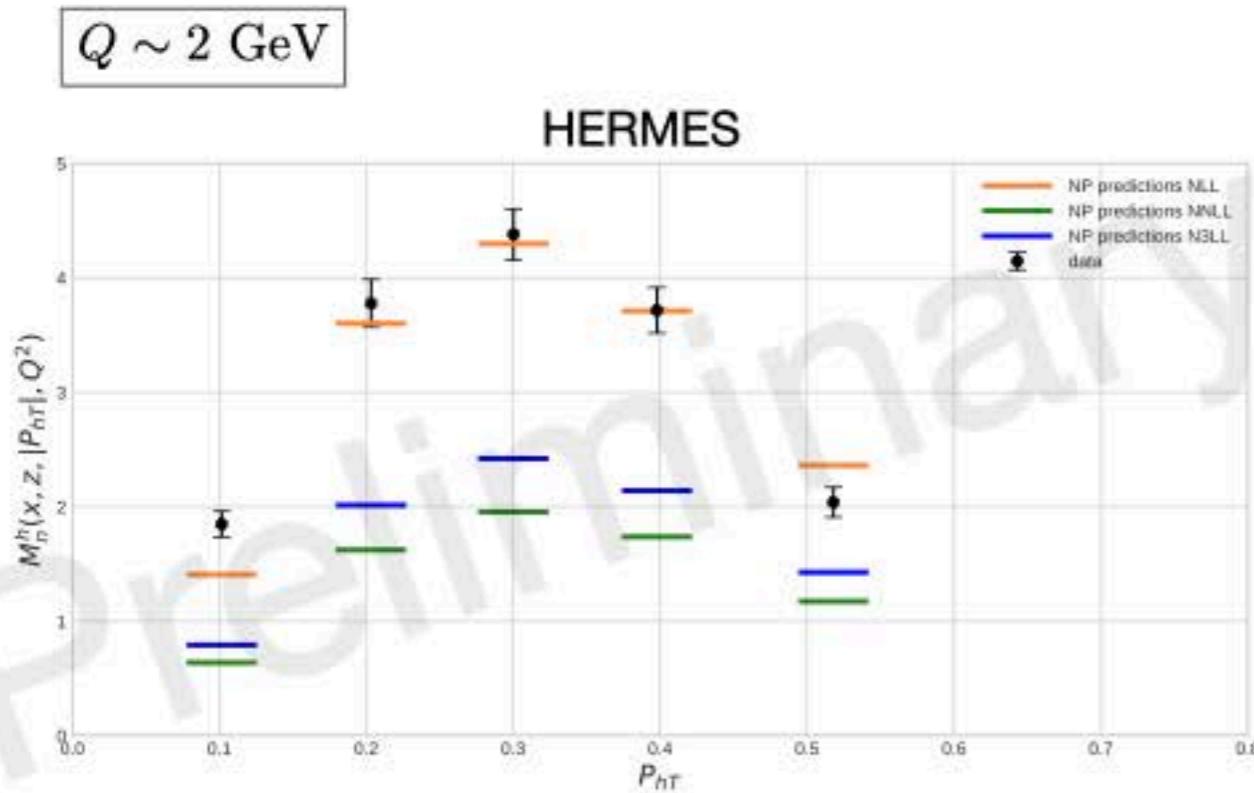
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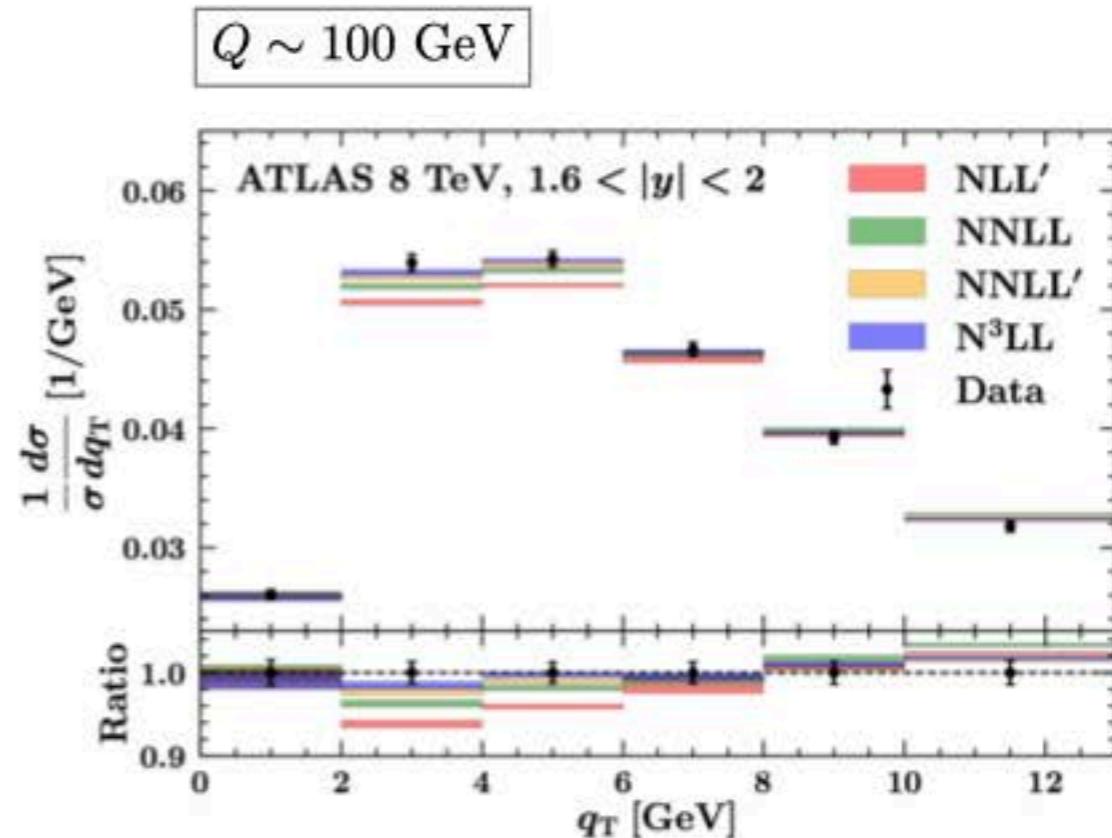
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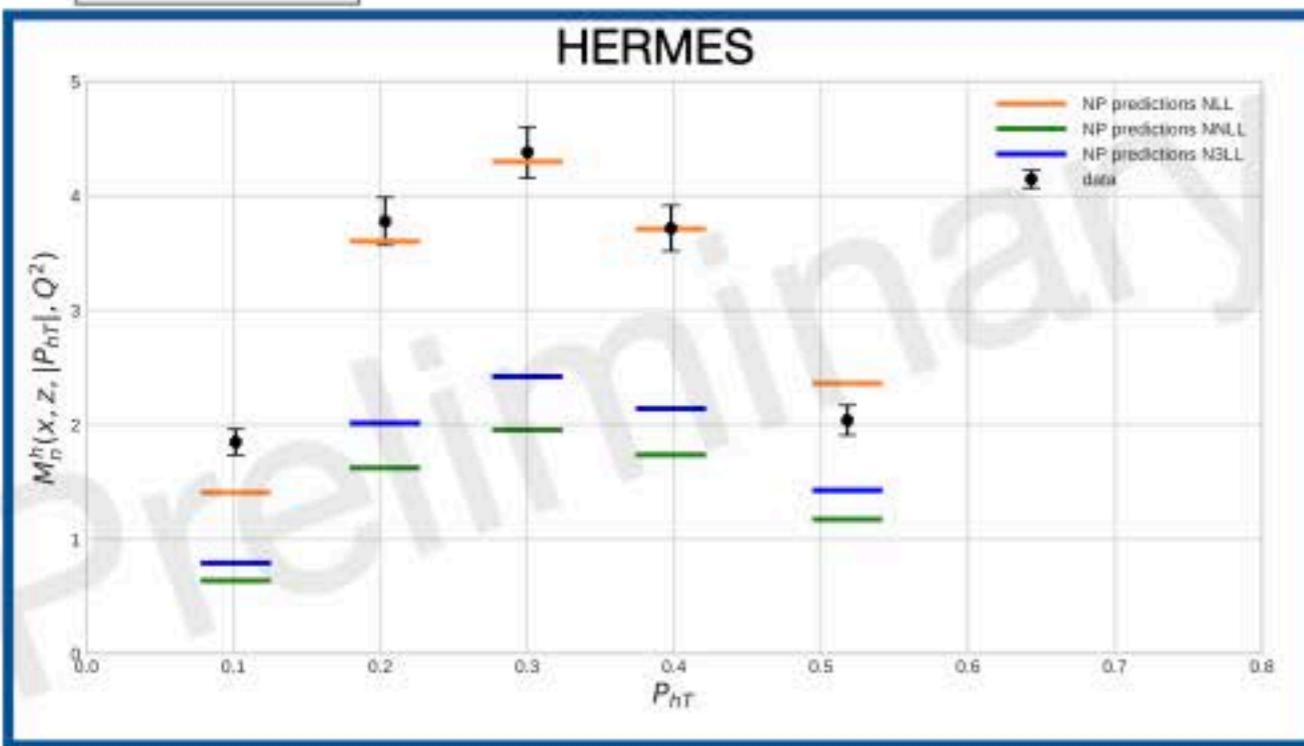


Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici,

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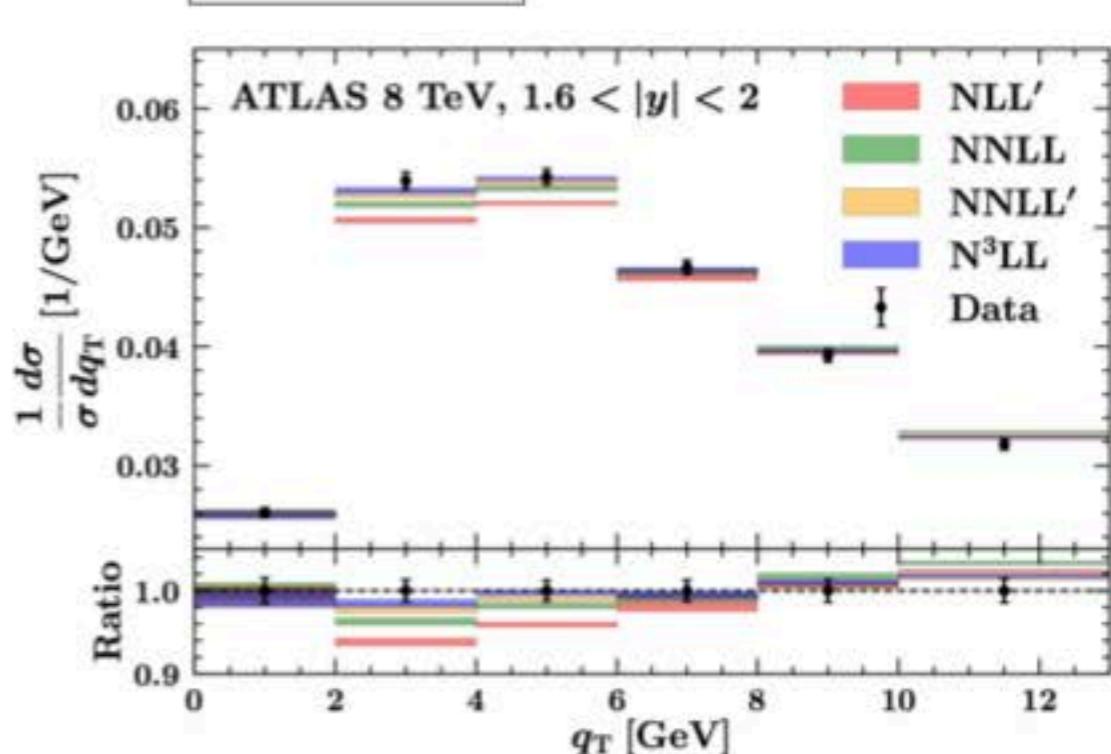
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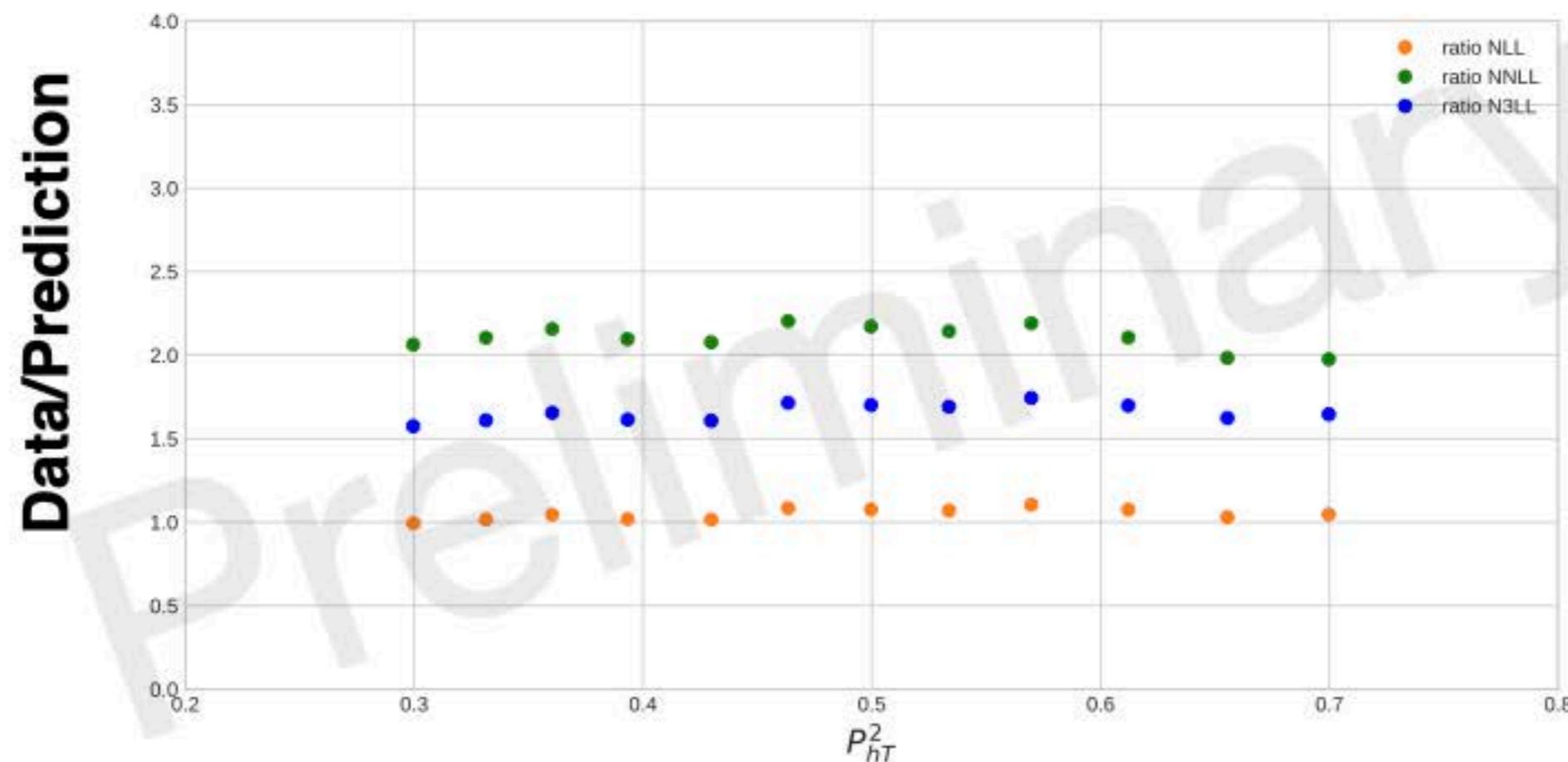
The description considerably worsens at higher orders!!

Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici,

MAPTMD22 – Normalization of SIDIS

COMPASS multiplicities (one of many bins)

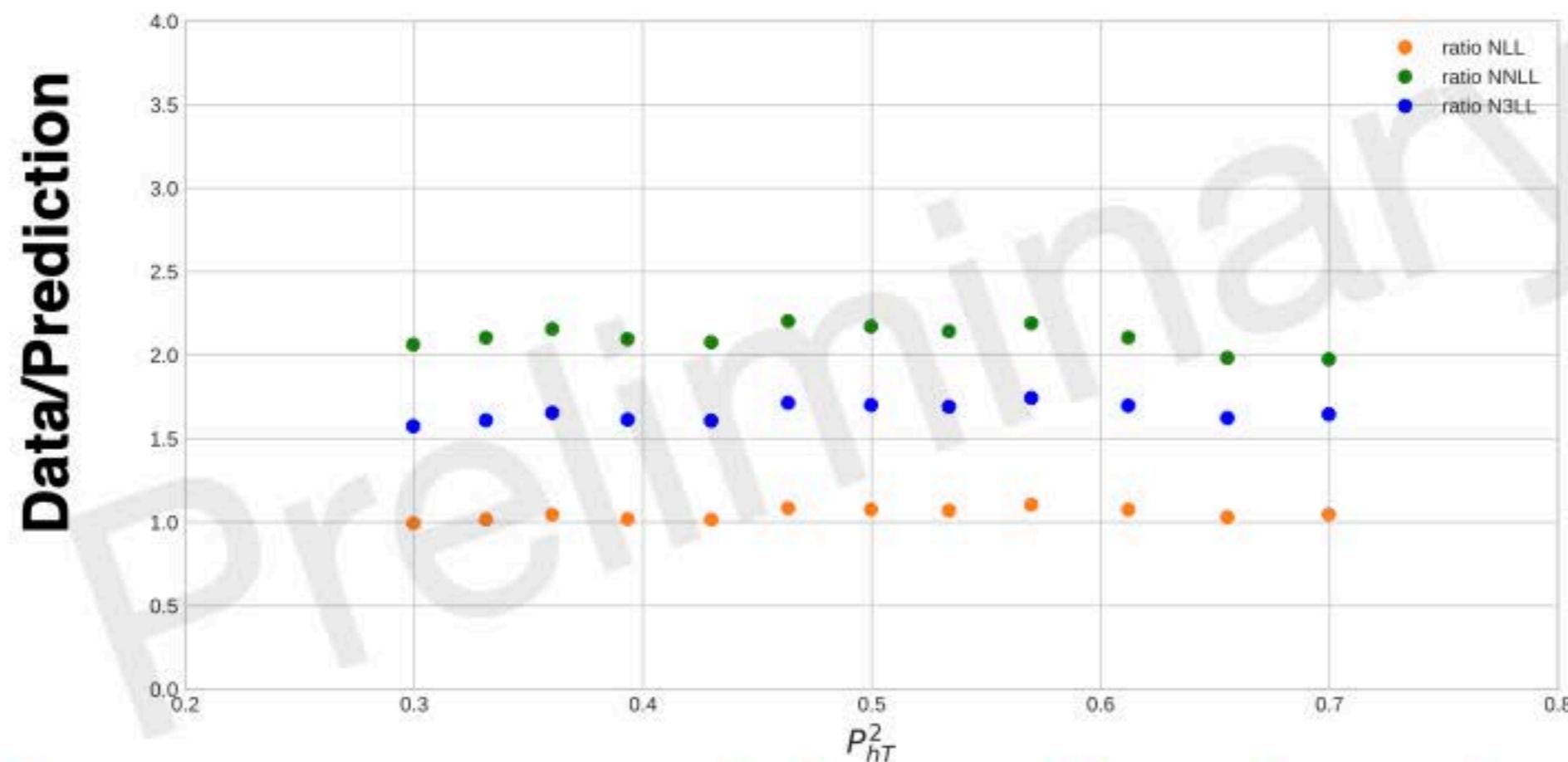
J.O. Gonzalez-Hernandez, PoS DIS2019



MAPTMD22 – Normalization of SIDIS

COMPASS multiplicities (one of many bins)

J.O. Gonzalez-Hernandez, PoS DIS2019



The discrepancy amounts to an almost constant factor!!

MAPTMD22 – Normalization of SIDIS

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SIDIS multiplicity $M(x, z, P_{hT}, Q) = \frac{d\sigma}{dxdQ \cancel{dzdP_{hT}}} \Big/ \frac{d\sigma}{dxdQ}$

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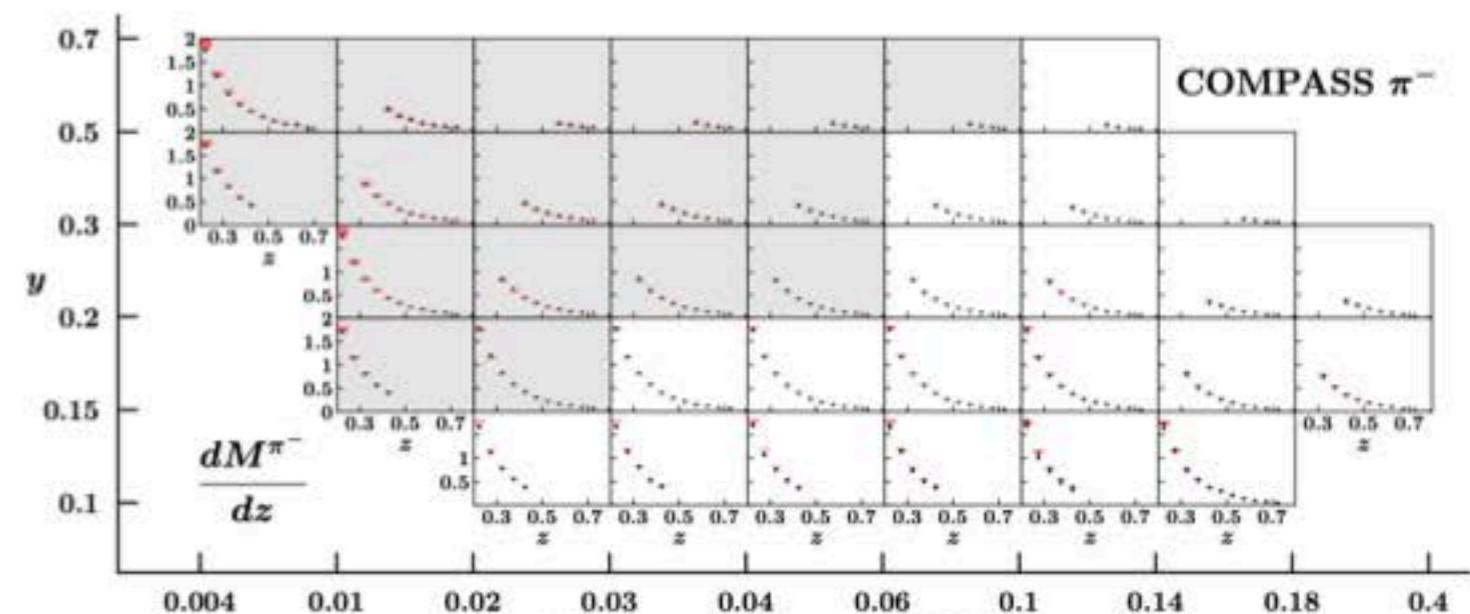
Collinear SIDIS cross section $\frac{d\sigma}{dx dQ \cancel{dz}}$

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No problems of normalization!!



*Khalek, Bertone, Nocera, arXiv:
2105.08725*

MAPTMD22 – Normalization of SIDIS

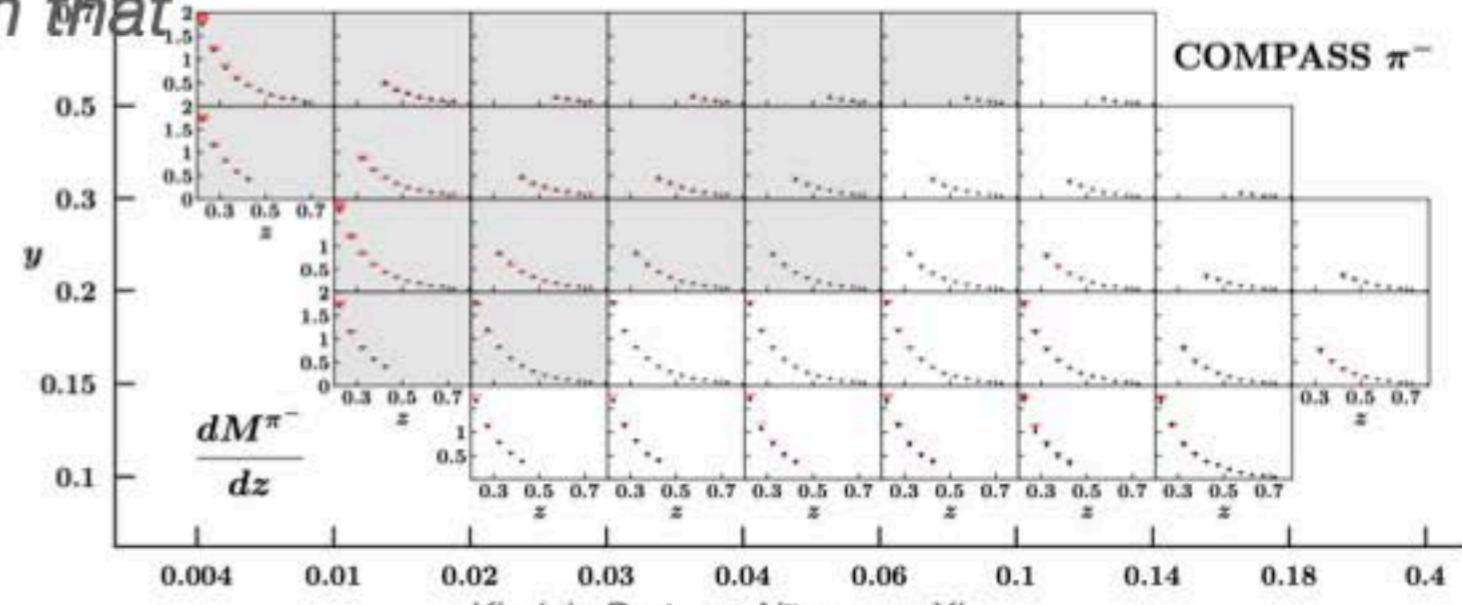
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Normalization of prediction such that

$$\int dP_{hT} \frac{d\sigma}{dx dQ dz dP_{hT}} = \frac{d\sigma}{dx dQ dz}$$



Khalek, Bertone, Nocera, arXiv:
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MAPTMD22 – Normalization of SIDIS

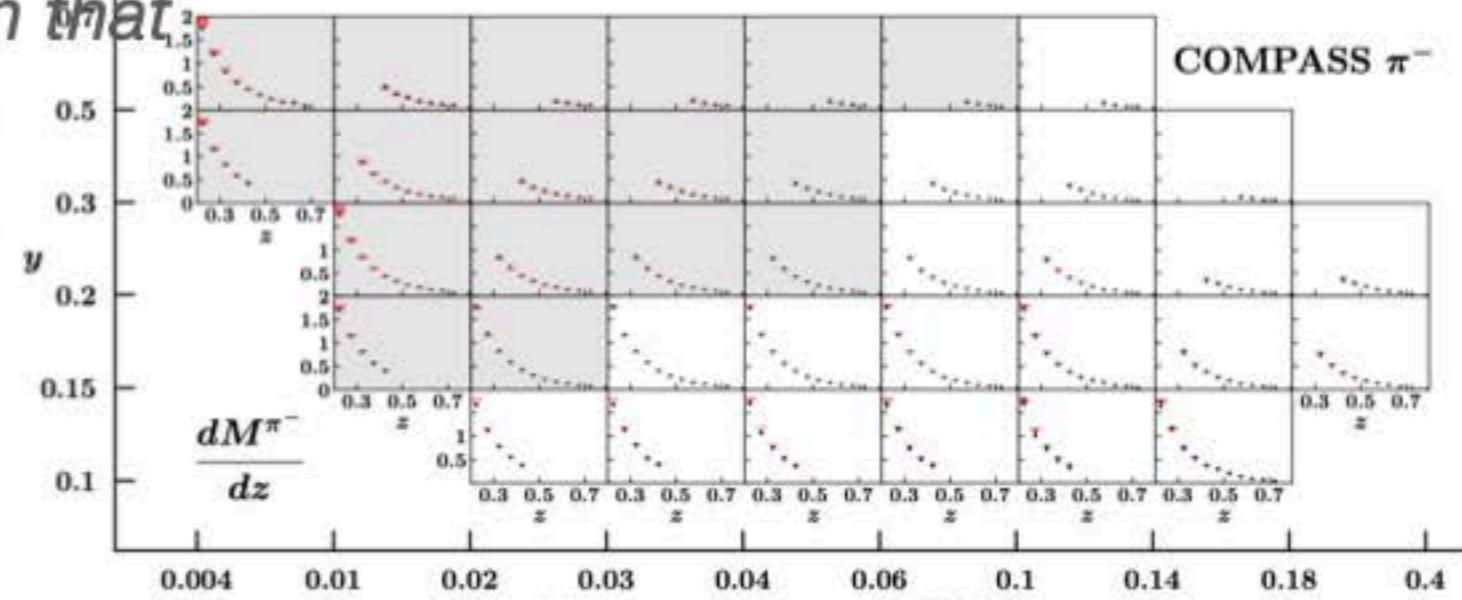
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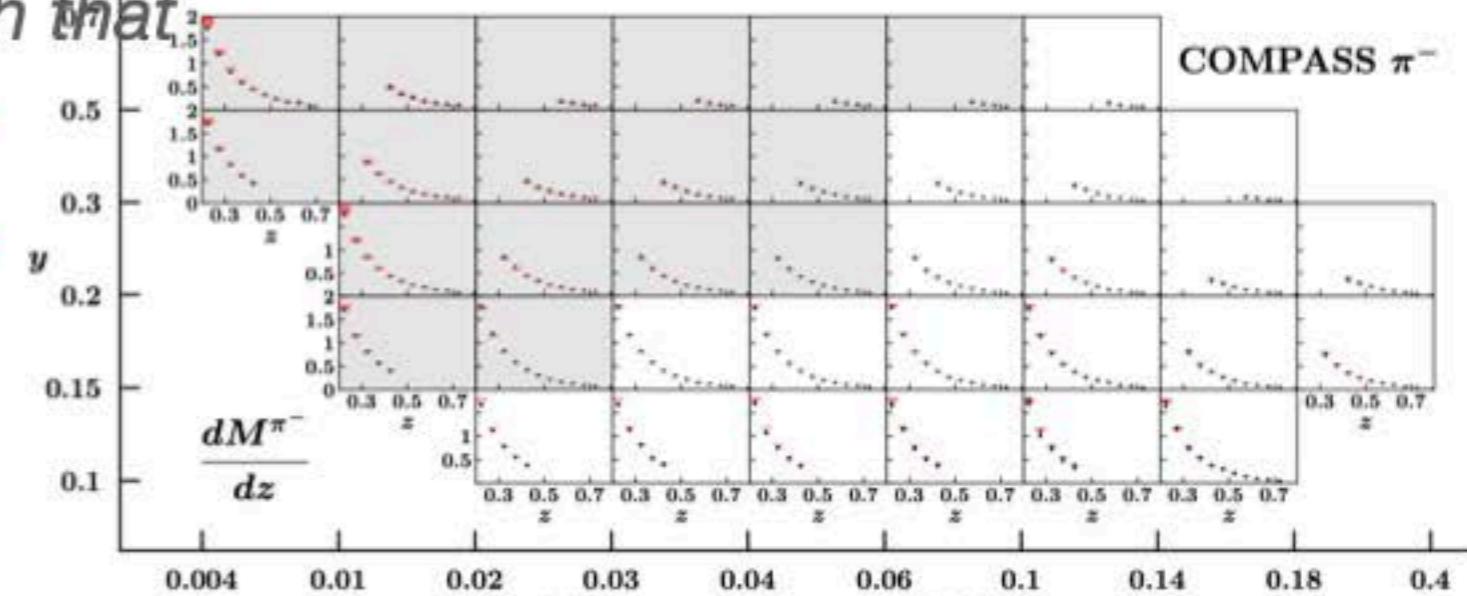
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$$M(x, z, P_{hT}, Q) = w(x, z, Q) \frac{d\sigma}{dx dQ dz dP_{hT}} / \frac{d\sigma}{dx dQ}$$



Khalek, Bertone, Nocera, arXiv:
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MAPTMD22 – Normalization of SIDIS

$$\text{SIDIS multiplicity} \quad M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ dz dP_{hT}} \Bigg/ \frac{d\sigma}{dx dQ}$$

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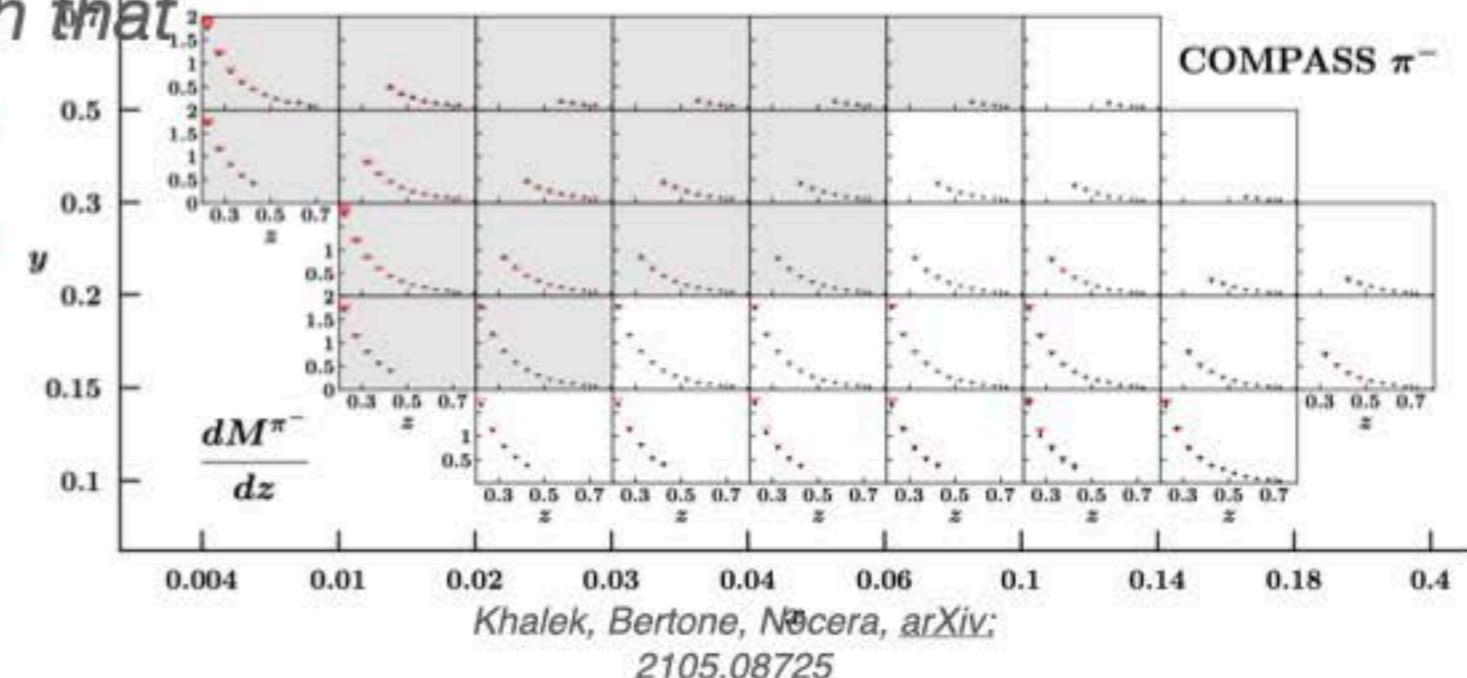
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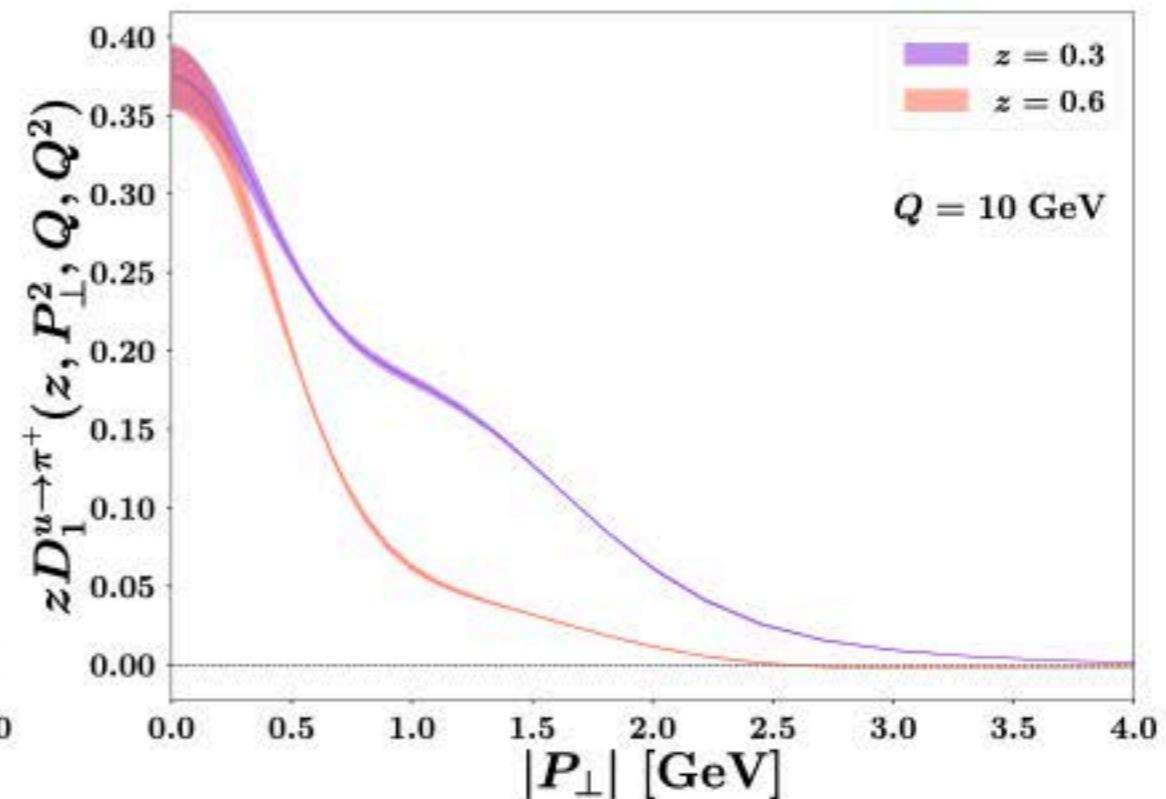
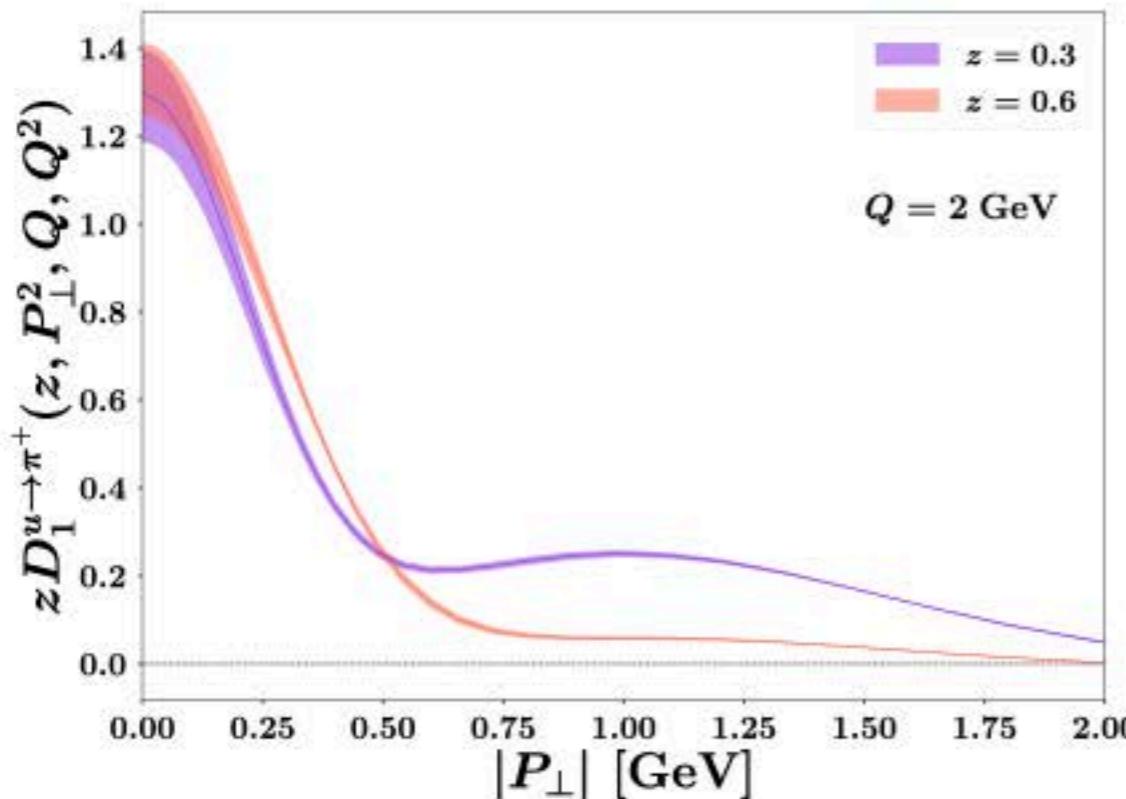
$$M(x, z, P_{hT}, Q) = \boxed{w(x, z, Q)} \frac{d\sigma}{dx dQ dz dP_{hT}} \Bigg/ \frac{d\sigma}{dx dQ}$$

Independent of the fitting parameters!!



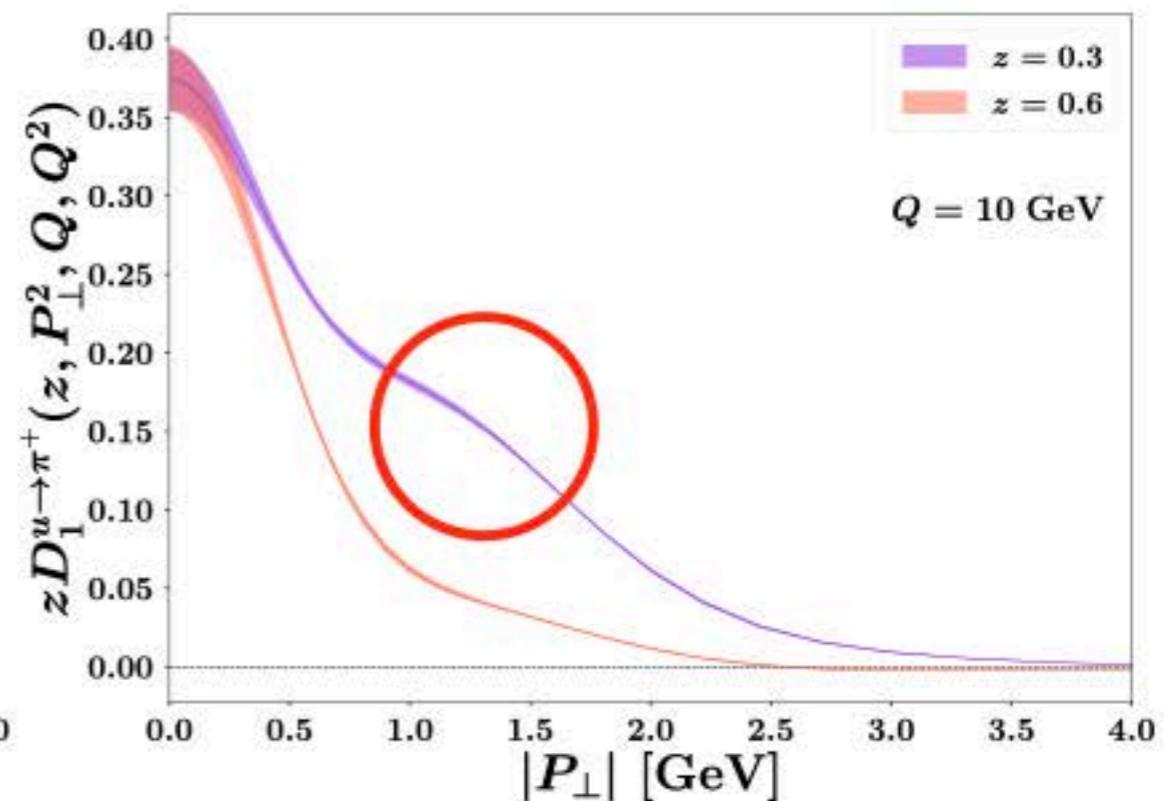
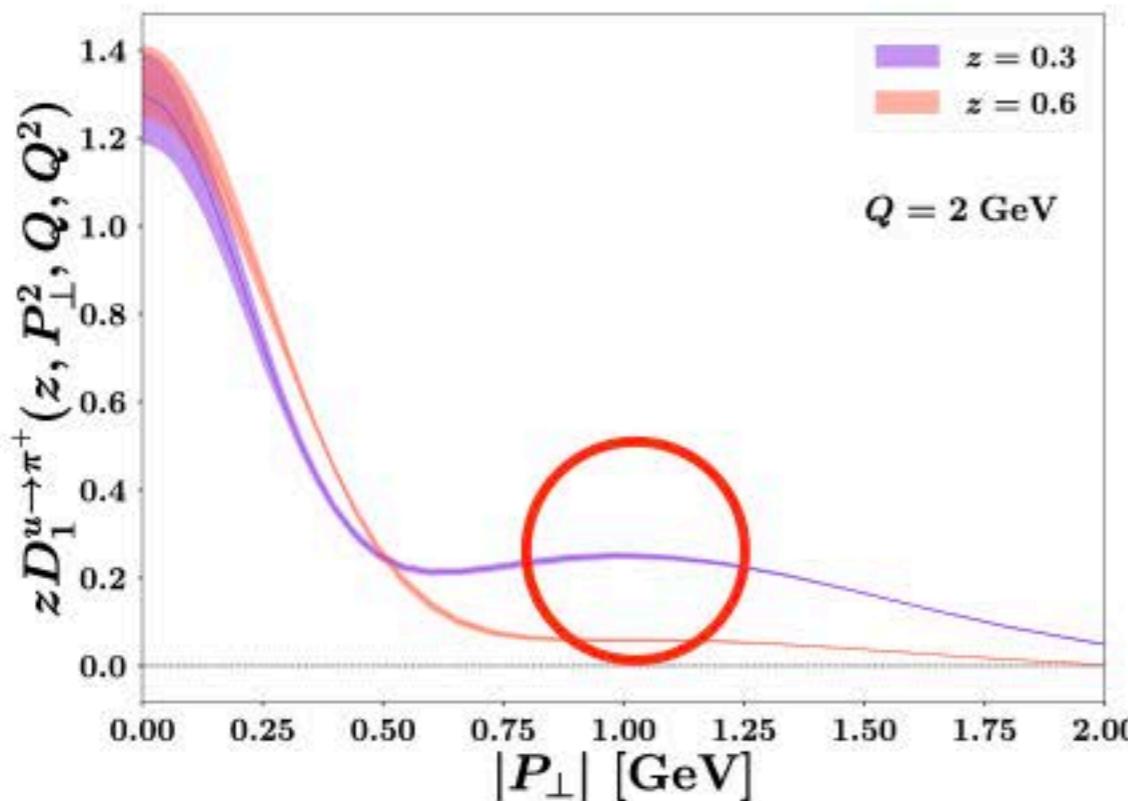
MAPTMD22 – Output of the fit

Visualisation of TMD FFs



MAPTMD22 – Output of the fit

Visualisation of TMD FFs



MAPTMD22 – Output of the fit

Collins-Soper kernel

MAPTMD22 – Output of the fit

Collins-Soper kernel

Kernel of the rapidity evolution equation

$$\frac{\partial \ln \hat{f}_1(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = K(b_T, \mu)$$

$$K(b_T, \mu_{b_*}) = K(b_*, \mu_{b_*}) + g_K(b_T)$$

MAPTMD22 – Output of the fit

Collins-Soper kernel

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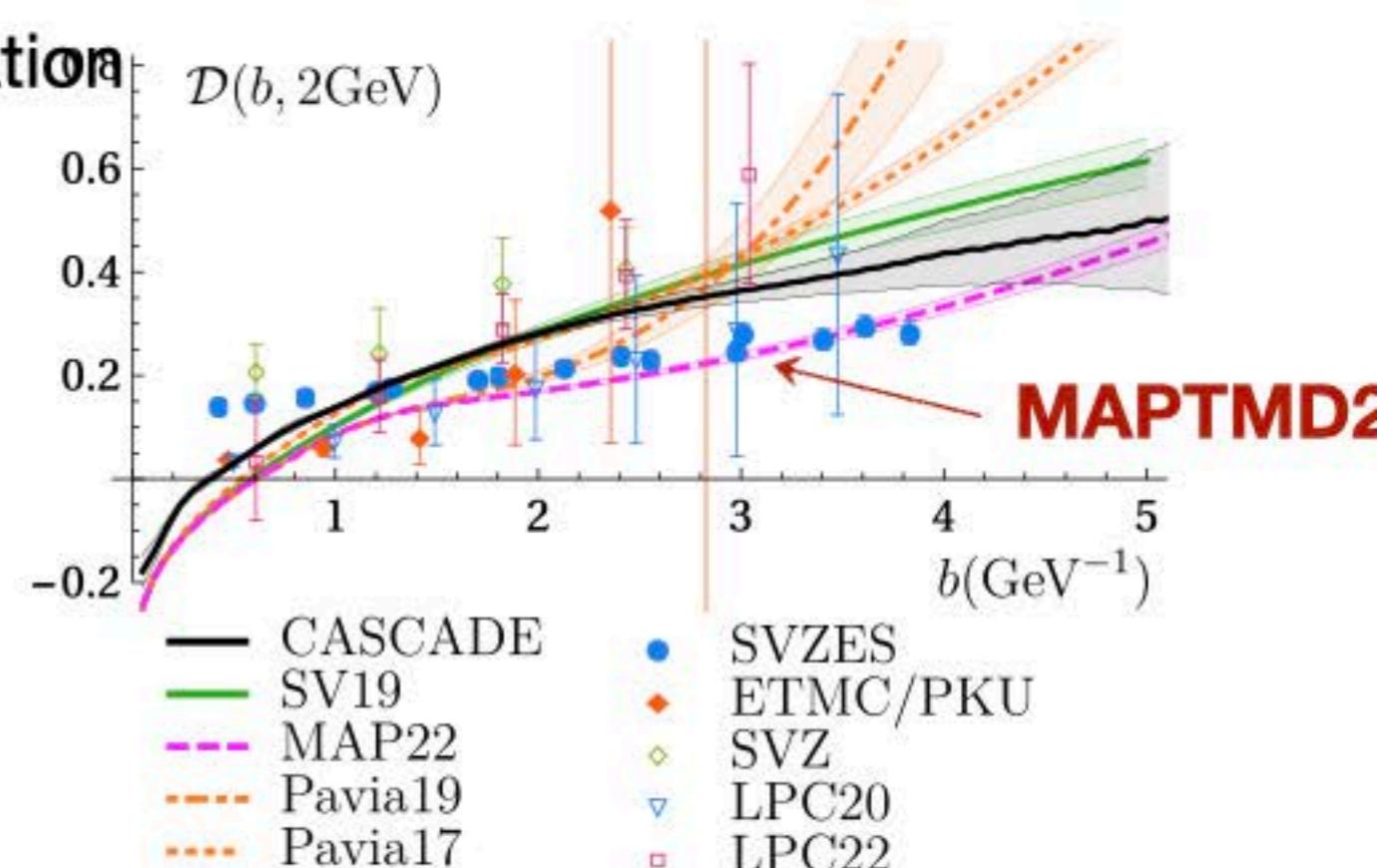
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↓ ↓
to be fitted

↓
perturbatively calculable



Martinez, Vladimirov,
arXiv:2206.01105

Logarithmic Accuracy

	Sudakov form factor	Matching coefficient
LL	$\alpha_S^n \ln^{2n} \left(\frac{Q^2}{\mu_b^2} \right)$	\tilde{C}^0
NLL	$\alpha_S^n \ln^{2n} \left(\frac{Q^2}{\mu_b^2} \right), \quad \alpha_S^n \ln^{2n-1} \left(\frac{Q^2}{\mu_b^2} \right)$	\tilde{C}^0
NLL'	$\alpha_S^n \ln^{2n} \left(\frac{Q^2}{\mu_b^2} \right), \quad \alpha_S^n \ln^{2n-1} \left(\frac{Q^2}{\mu_b^2} \right)$	$(\tilde{C}^0 + \alpha_S \tilde{C}^1)$
the difference between the two is NNLL:		$\alpha_S^n \ln^{2n-2} \left(\frac{Q^2}{\mu_b^2} \right)$

Logarithmic Accuracy

$$S_{\text{pert}}(\mu_b, Q) = 1 + \sum_{k=0}^{\infty} \sum_{n=1+[k/2]}^{\infty} \left(\frac{\alpha_S(Q)}{4\pi} \right)^n \sum_{k=1}^{2n} L^{2n-k} R^{(n,2n-k)} \quad L = \ln \left(\frac{Q^2}{\mu_b^2} \right)$$

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Non-mixed terms in collinear SIDIS cross section

$$\begin{aligned} \frac{d\sigma^h}{dx dQ^2 dz} \Big|_{O(\alpha_s^1)} = & \sigma_0 \sum_{ff'} \frac{e_f^2}{z^2} (\delta_{f'f} + \delta_{f'g}) \frac{\alpha_s}{\pi} \left\{ \left[D_1^{h/f'} \otimes C_1^{f'f} \otimes f_1^{f/N} \right] (x, z, Q) \right. \\ & \left. + \frac{1-y}{1+(1-y)^2} \left[D_1^{h/f'} \otimes C_L^{f'f} \otimes f_1^{f/N} \right] (x, z, Q) \right\}, \end{aligned}$$

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$$\begin{aligned} C_1^{qq} = & \frac{C_F}{2} \left\{ -8\delta(1-x)\delta(1-z) \right. \\ & + \delta(1-x) \left[P_{qq}(z) \ln \frac{Q^2}{\mu_F^2} + L_1(z) + L_2(z) + (1-z) \right] \\ & + \delta(1-z) \left[P_{qq}(x) \ln \frac{Q^2}{\mu_F^2} + L_1(x) - L_2(x) + (1-x) \right] \\ & \left. + 2 \frac{1}{(1-x)_+} \frac{1}{(1-z)_+} - \frac{1+z}{(1-x)_+} - \frac{1+x}{(1-z)_+} + 2(1+xz) \right\}, \end{aligned}$$

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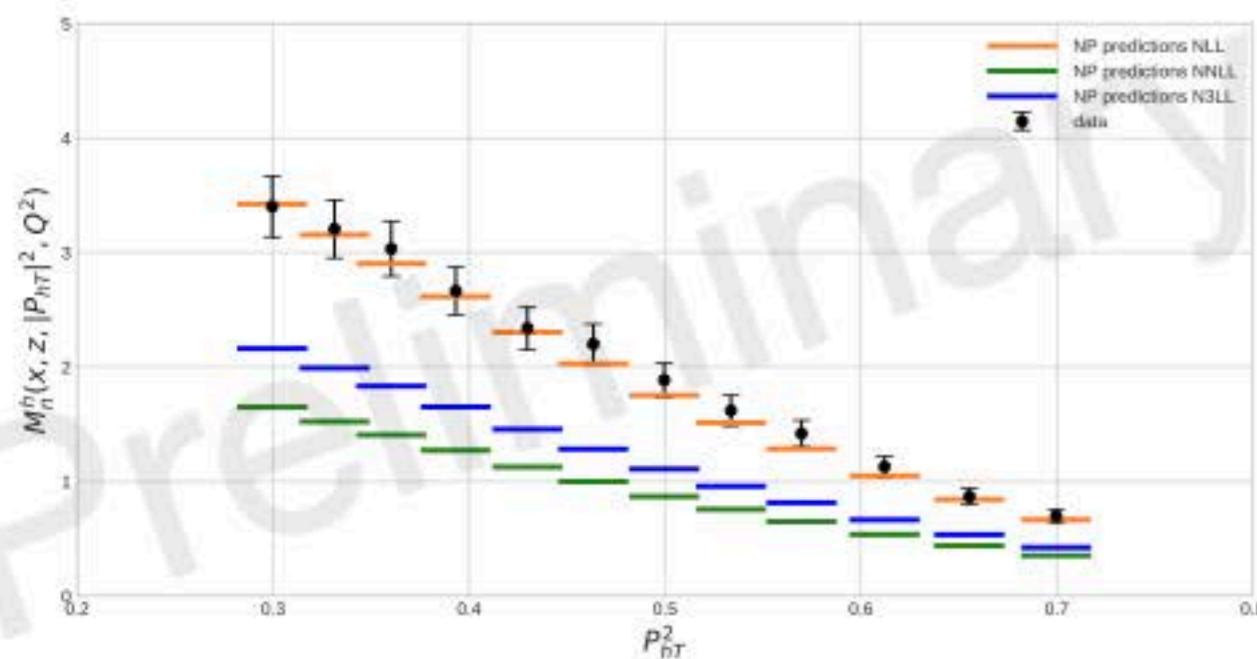
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Source of W-term suppression

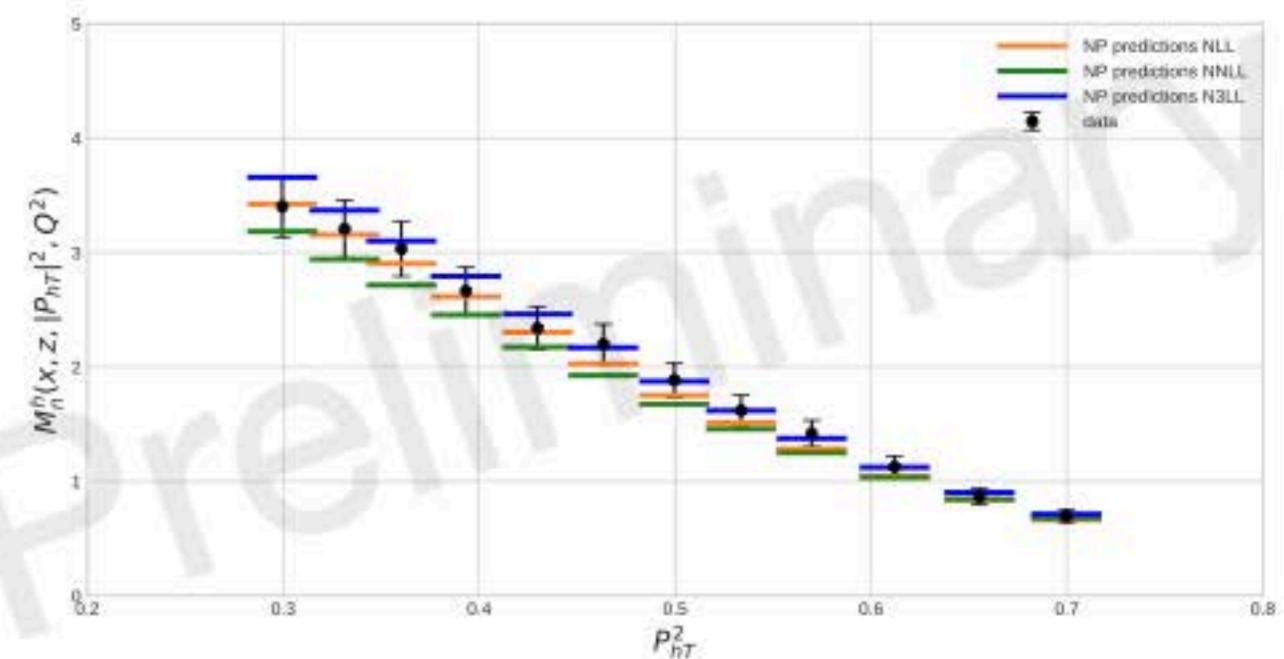
Present situation at low Q

COMPASS multiplicity

Full Hard Factor



Hard Factor = 1

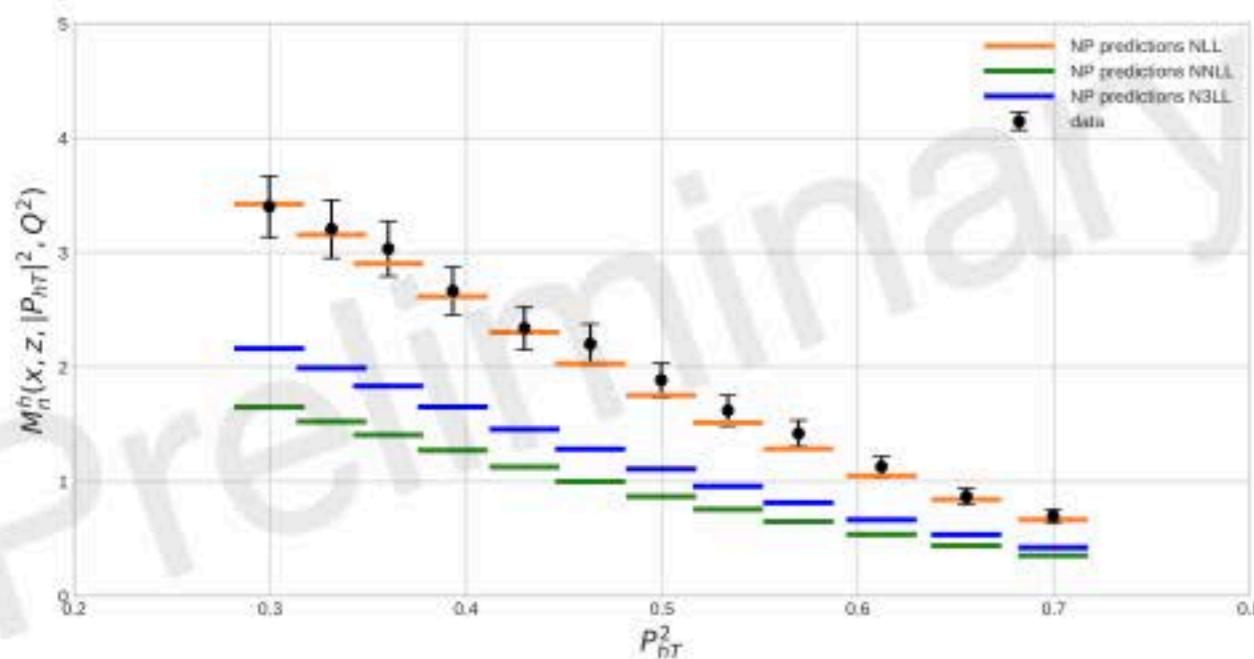


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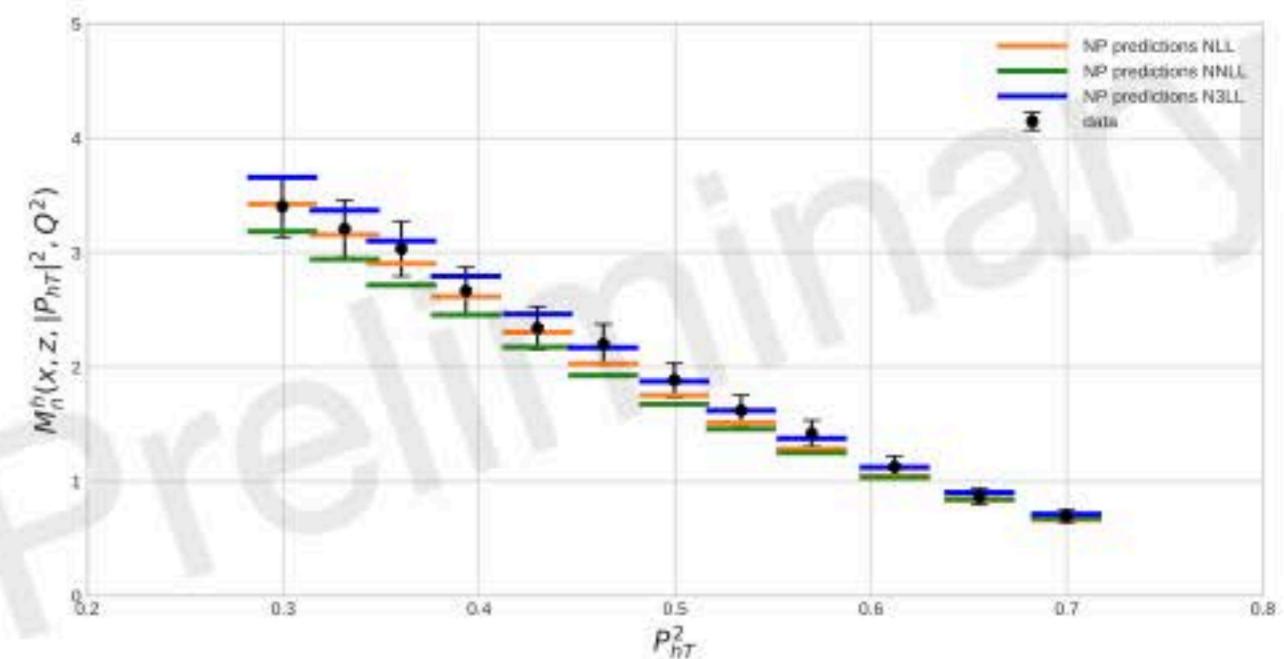
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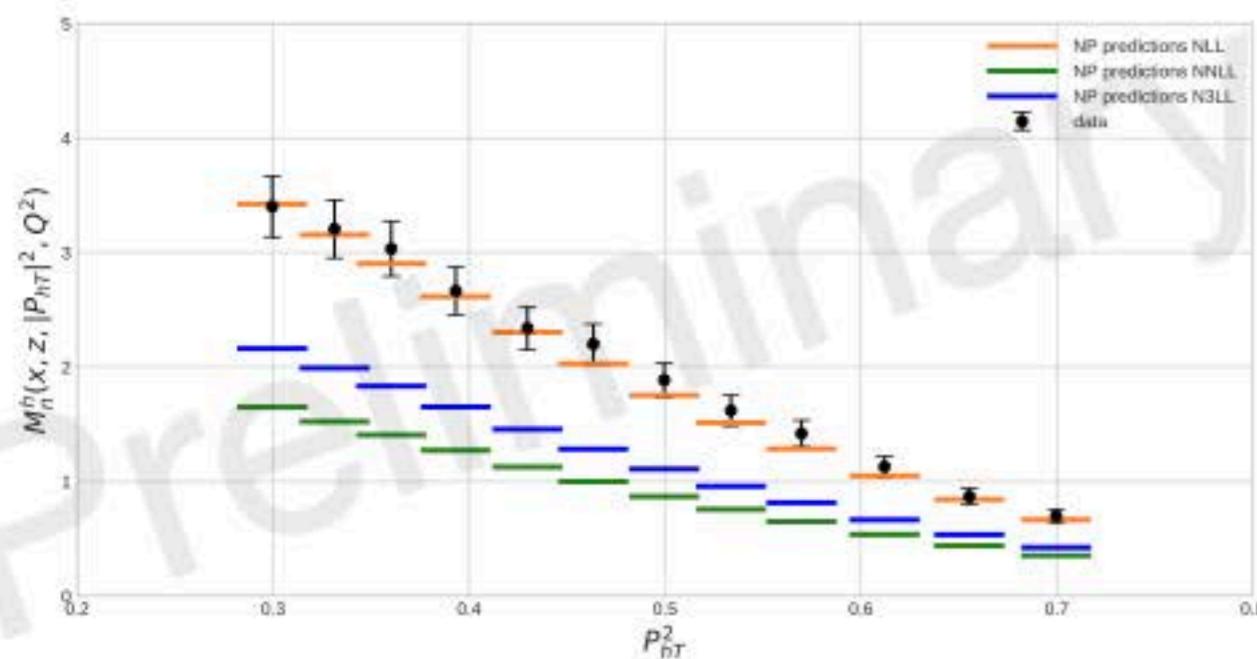


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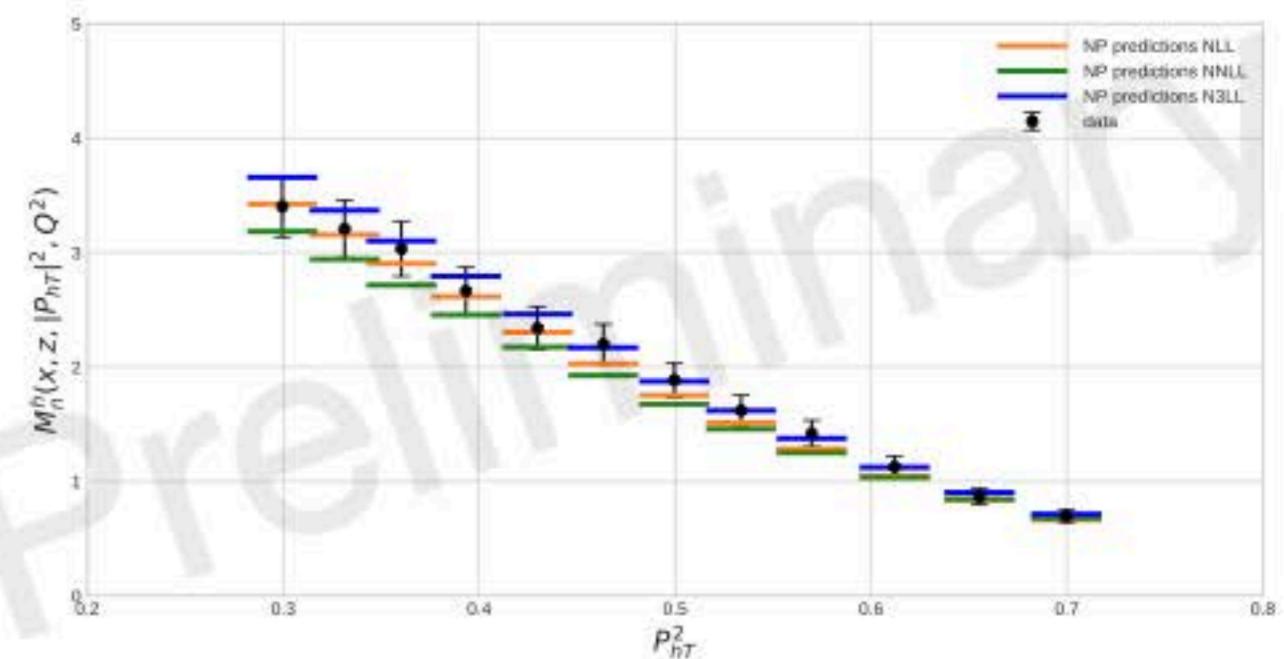
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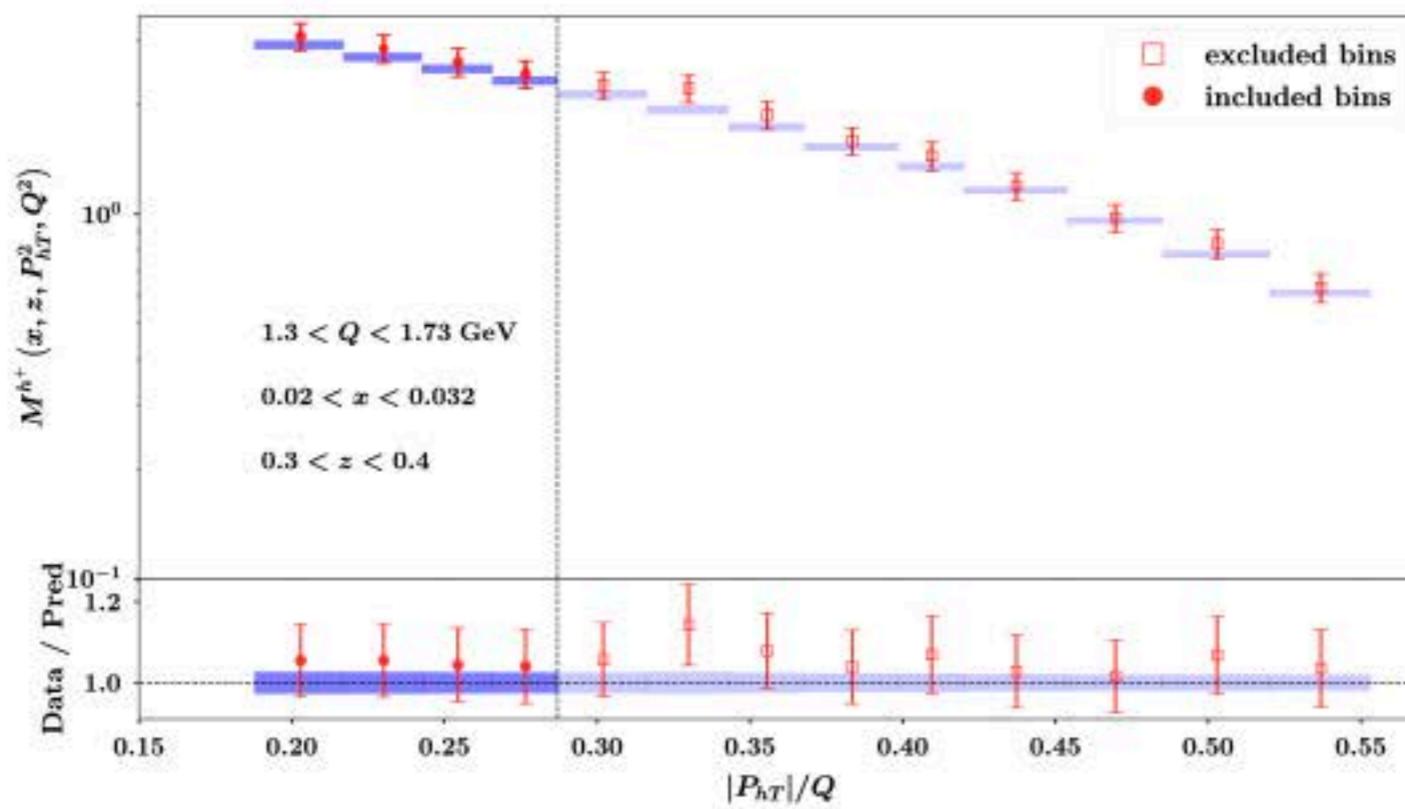


MAPTMD22 – SIDIS data selection

COMPASS multiplicities (one of many bins)

$$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$

$$P_{hT}|_{\max} = \min[0.2Q, 0.7zQ] + 0.5 \text{ GeV}$$

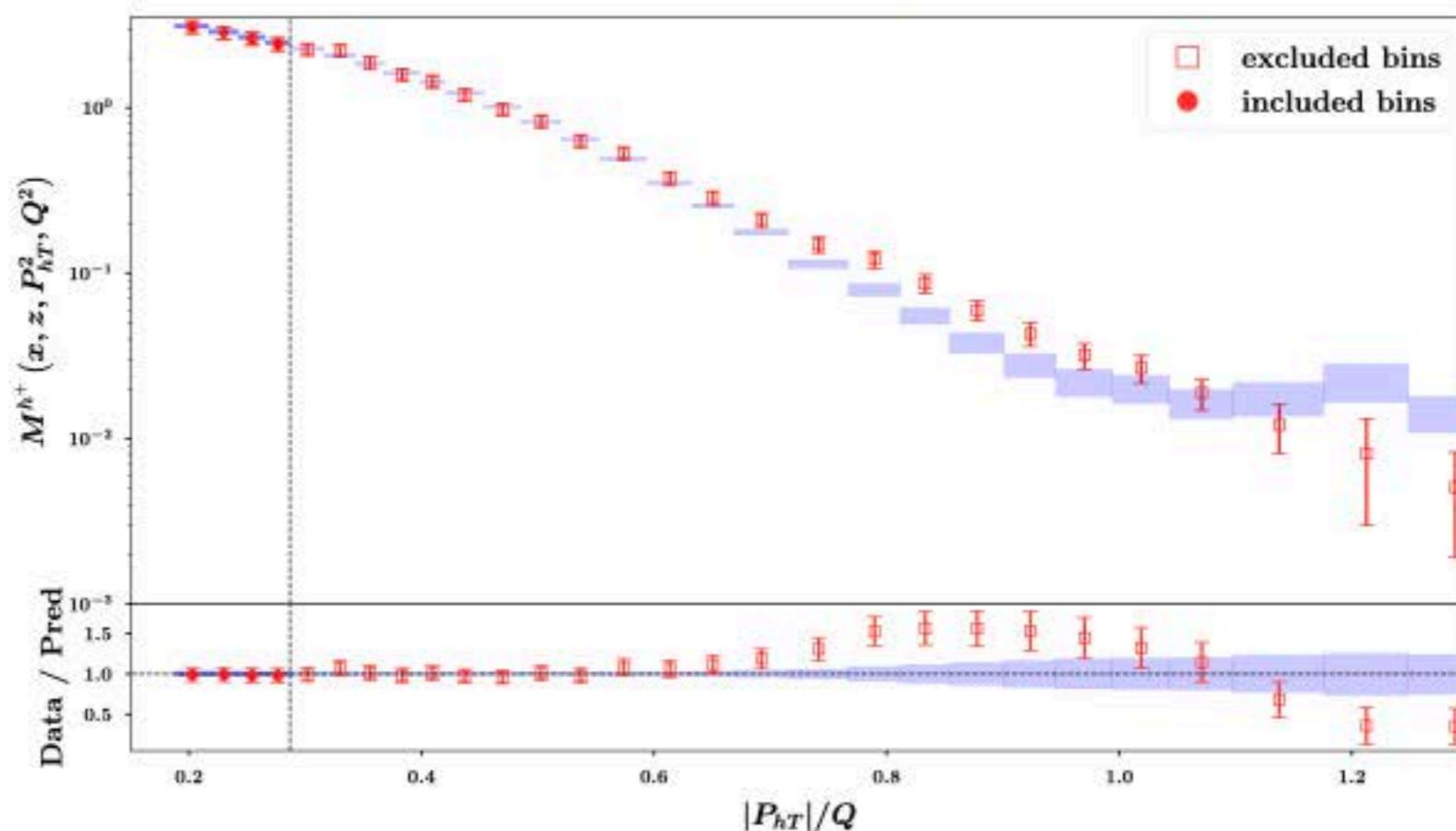


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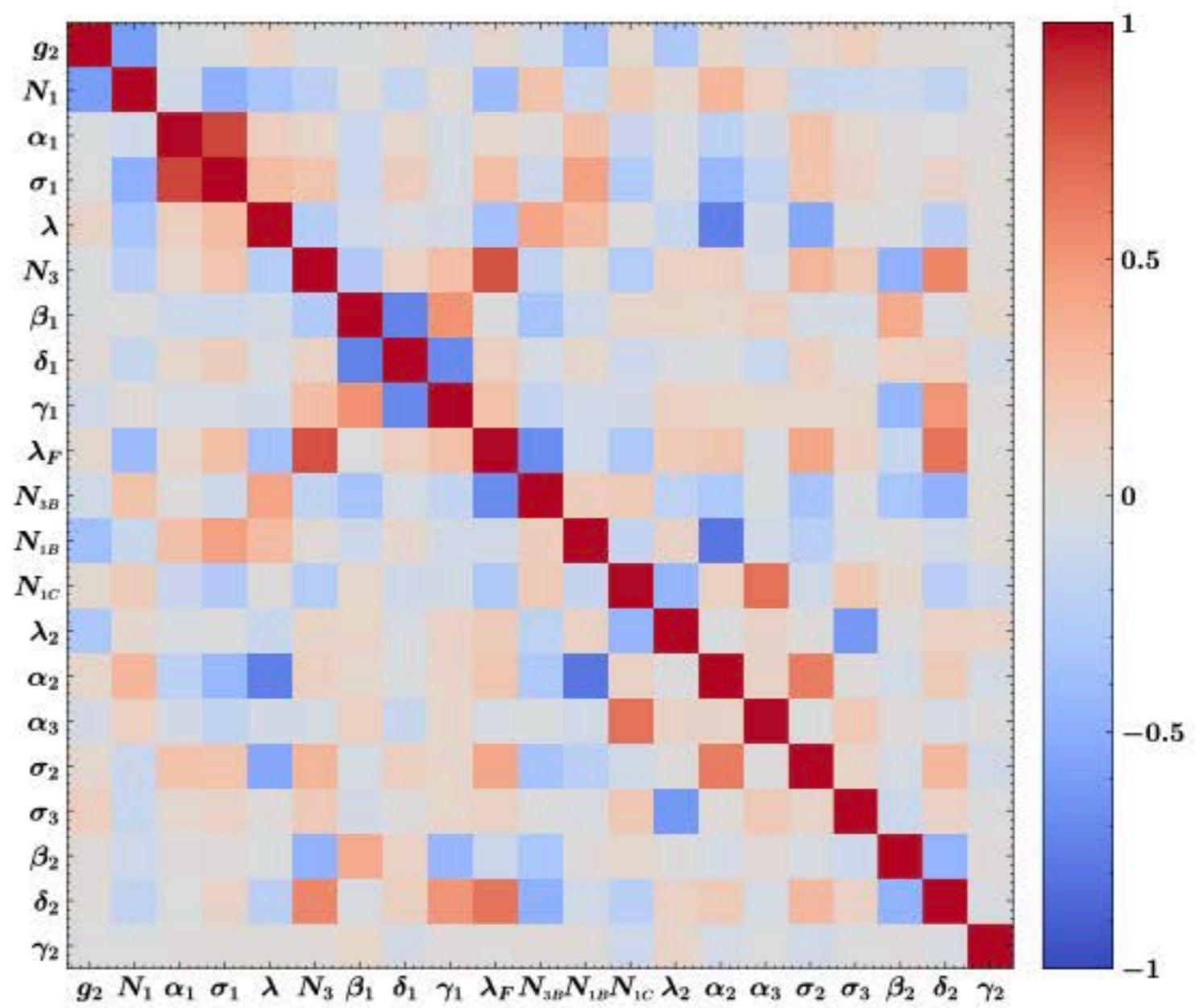
$$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$

Total number of points



Results of the baseline fit

Error propagation
↓
250 Montecarlo
replicas



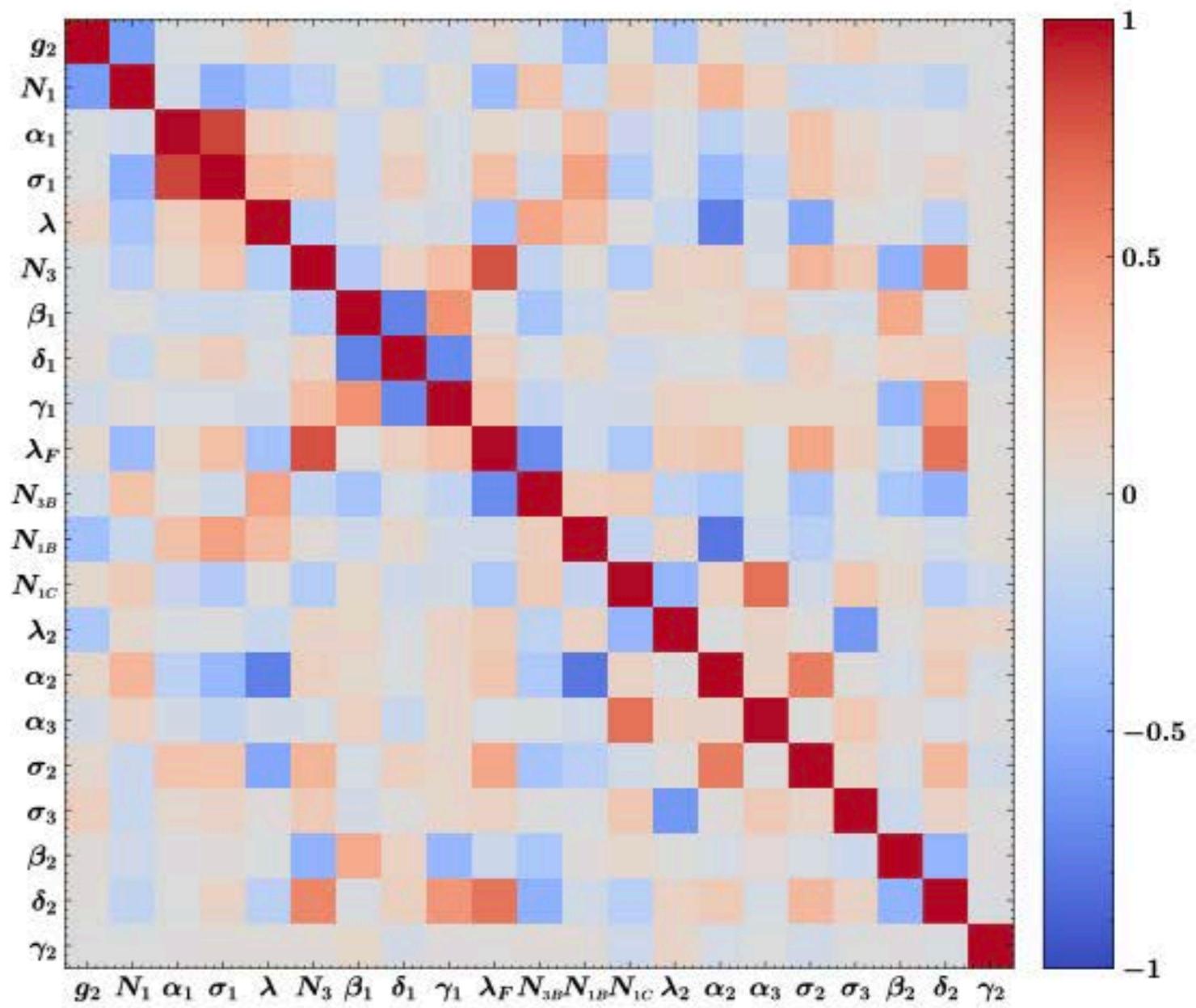
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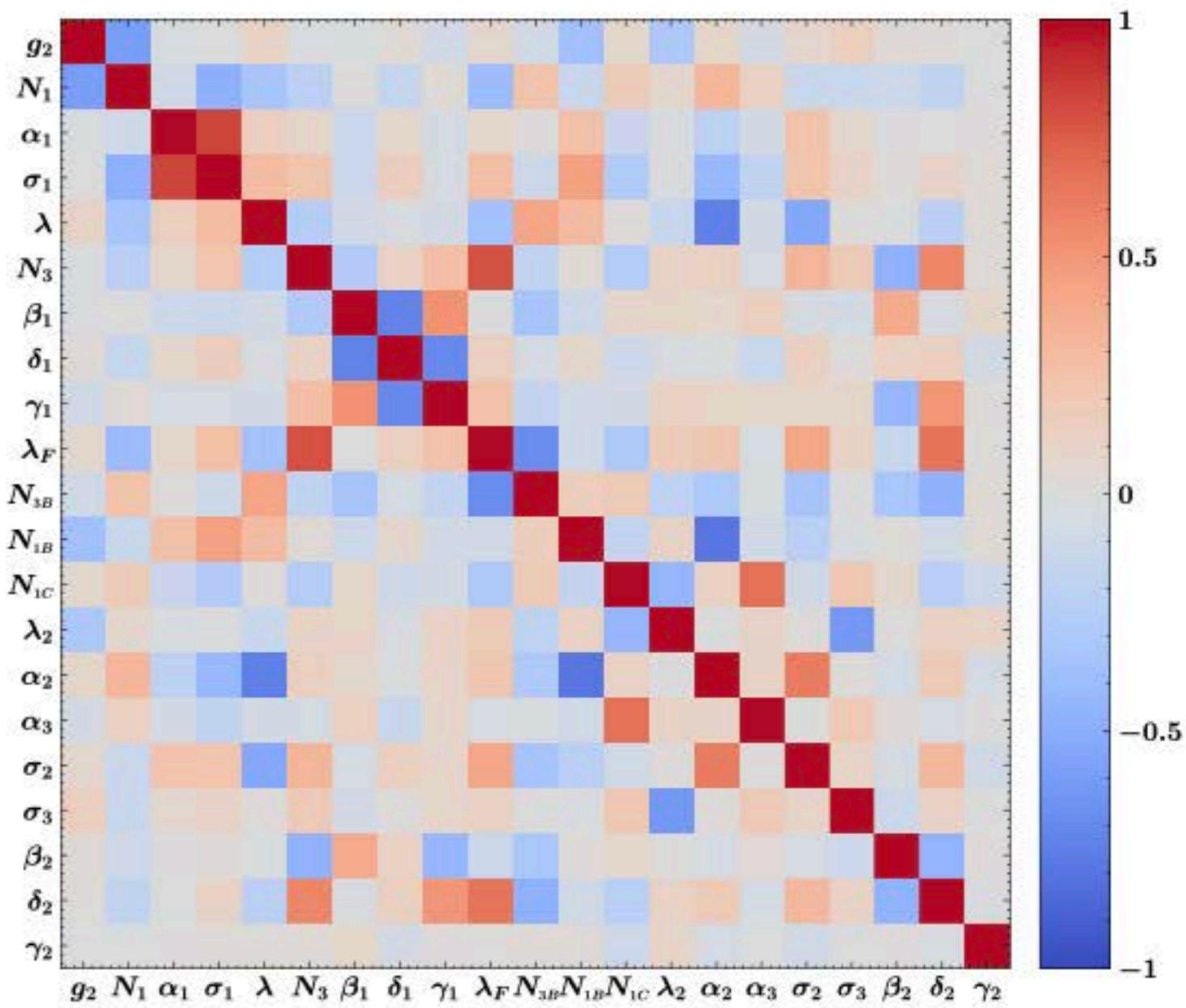
Correlation matrix

Hints of the
appropriateness of
the chosen
functional form



Results of the baseline fit

Parameter	Average over replicas
g_2 [GeV]	0.248 ± 0.008
N_1 [GeV 2]	0.316 ± 0.025
α_1	1.29 ± 0.19
σ_1	0.68 ± 0.13
λ [GeV $^{-1}$]	1.82 ± 0.29
N_3 [GeV 2]	0.0055 ± 0.0006
β_1	10.23 ± 0.29
δ_1	0.0094 ± 0.0012
γ_1	1.406 ± 0.084
λ_F [GeV $^{-2}$]	0.078 ± 0.011
N_{3B} [GeV 2]	0.2167 ± 0.0055
N_{1B} [GeV 2]	0.134 ± 0.017
N_{1C} [GeV 2]	0.0130 ± 0.0069
λ_2 [GeV $^{-1}$]	0.0215 ± 0.0058
α_2	4.27 ± 0.31
α_3	4.27 ± 0.13
σ_2	0.455 ± 0.050
σ_3	12.71 ± 0.21
β_2	4.17 ± 0.13
δ_2	0.167 ± 0.006
γ_2	0.0007 ± 0.0110



Impact of EIC

PV17 baseline

Average over replicas
0.1171 ± 0.0145
0.283 ± 0.0368
2.2393 ± 1.2967
-0.1416 ± 0.0959
0.2548 ± 0.2549
0.2203 ± 0.0222
2.9304 ± 0.9978
0.1175 ± 0.0506
2.4736 ± 0.1649
7.5475 ± 3.2037
0.0318 ± 0.0068

reduction by
factor 10

PV17 baseline
+ EIC

Parameter

g_2

λ

N_3

β

δ

γ

λ_F

N_4

Average over replicas

0.119 ± 0.0025

0.2814 ± 0.0362

2.3882 ± 0.5448

-0.1445 ± 0.0134

0.3061 ± 0.4085

0.2122 ± 0.0157

2.6773 ± 0.3861

0.1099 ± 0.0358

2.4643 ± 0.12

5.3198 ± 2.0531

0.0346 ± 0.0048

non - perturbative
evolution