TMD extraction framework and validation tests

&

the role of large kT

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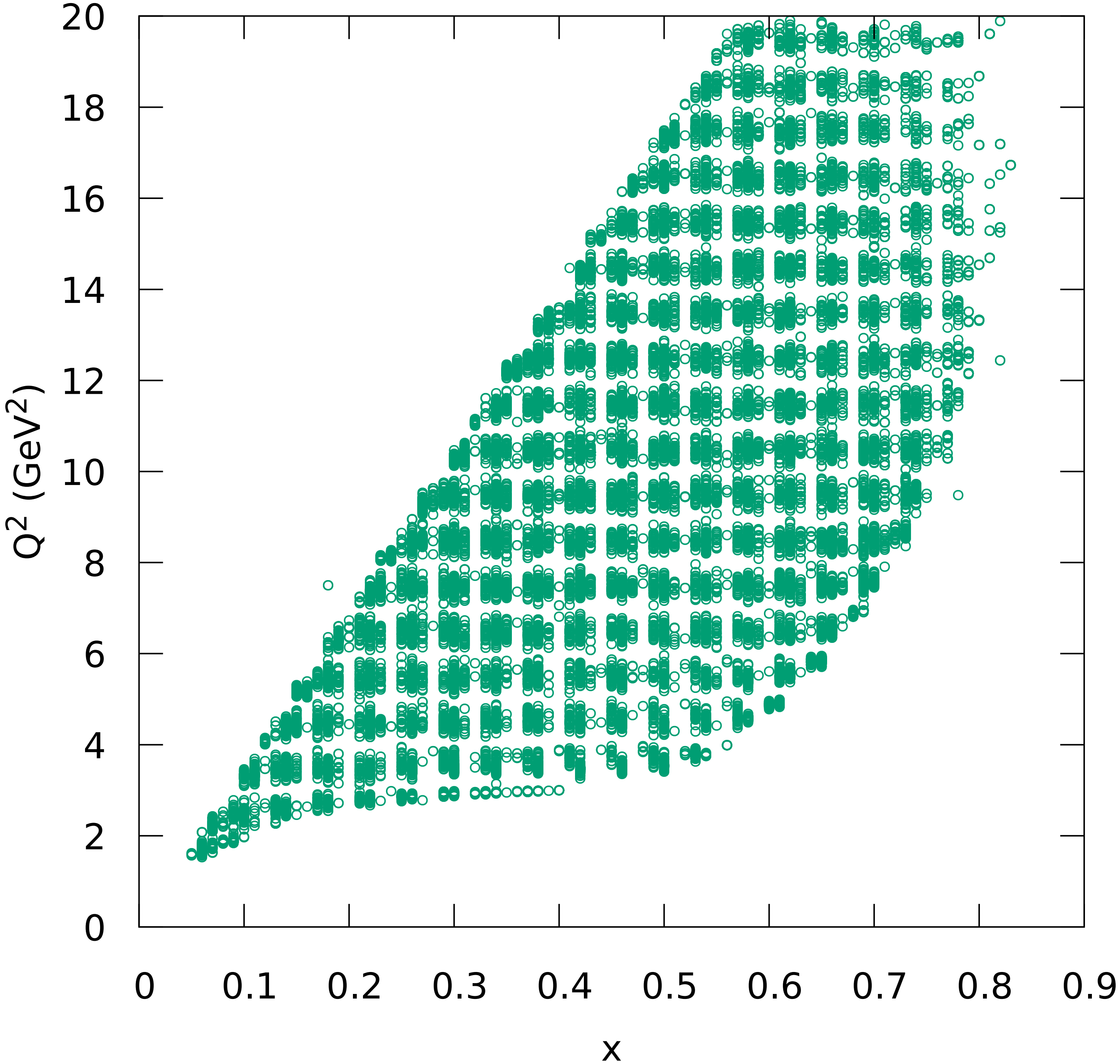
Based on:

J.O. Gonzalez-Hernandez(Turin U.), T.C. Rogers(Old Dominion U. and Jefferson Lab), N. Sato(Jefferson Lab) (May 11, 2022)
Phys.Rev.D 106 (2022) 3, 034002 • e-Print: 2205.05750 [hep-ph]

& ongoing work with

Tommaso Rainaldi and T. C. Rogers

New opportunities with JLab upgrade



High luminosity
(precision)

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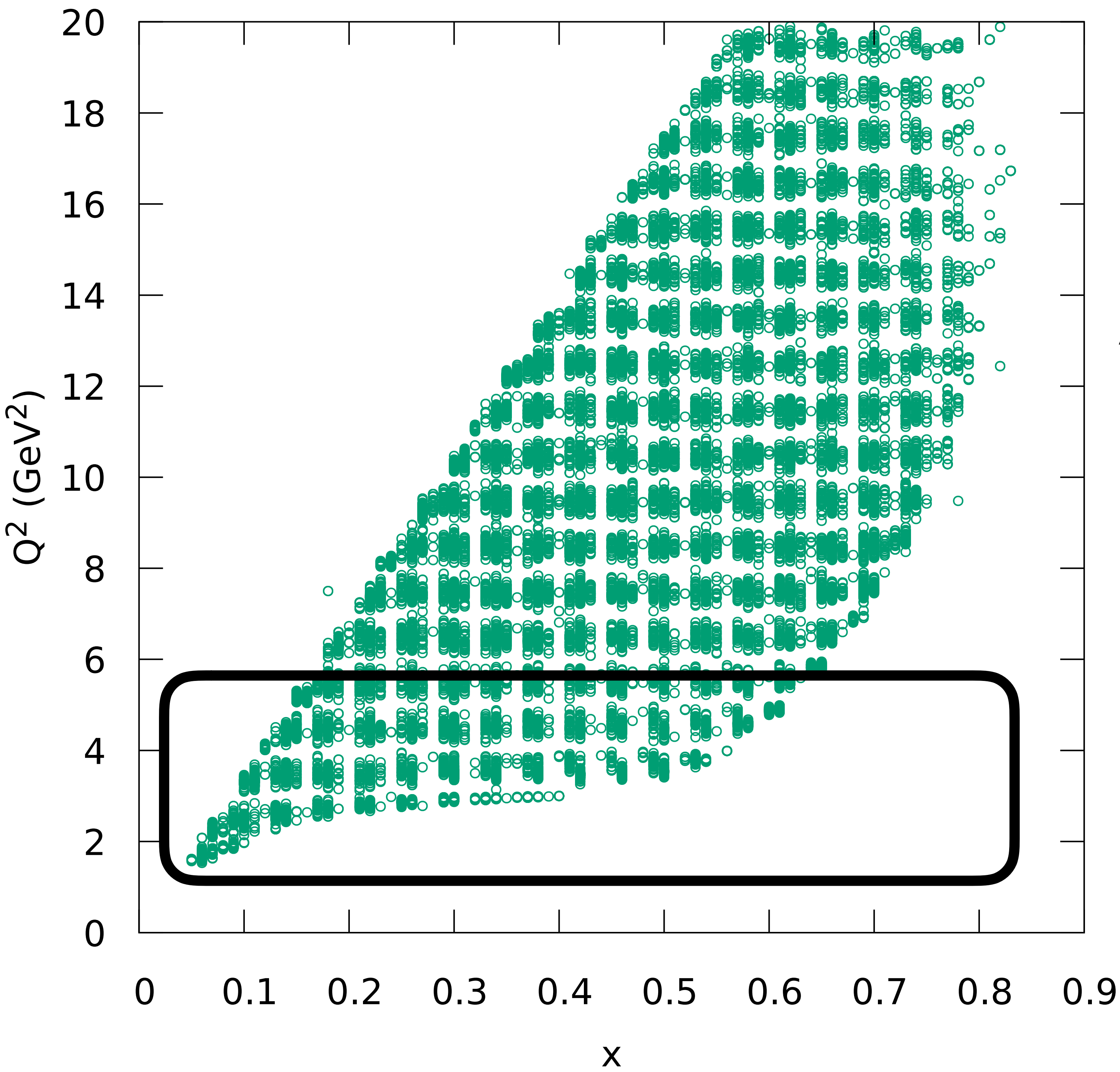

High luminosity
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+
Access to sufficiently
small Q (accuracy)

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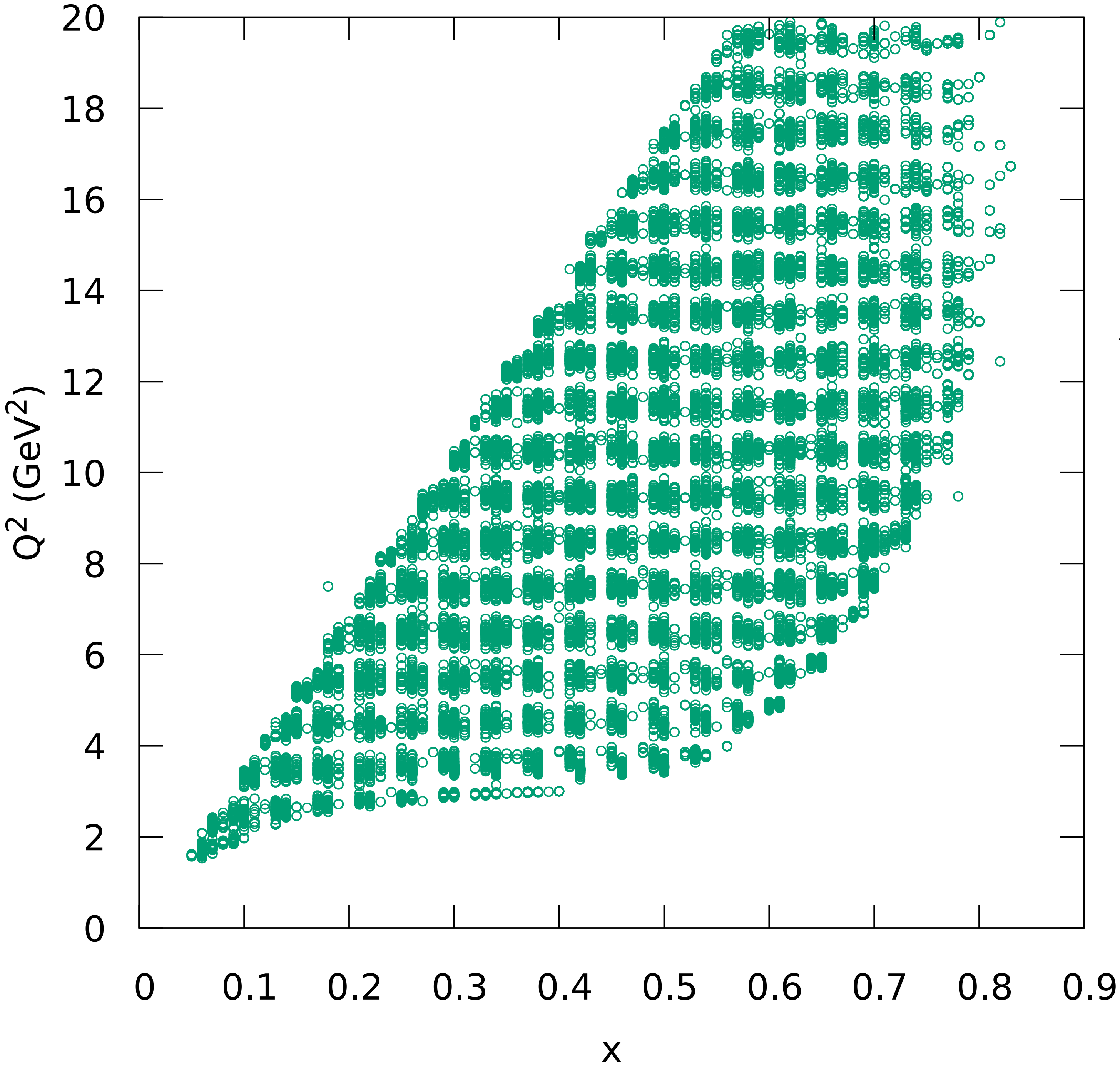

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Multidimensional
binning
(TMD evolution, test
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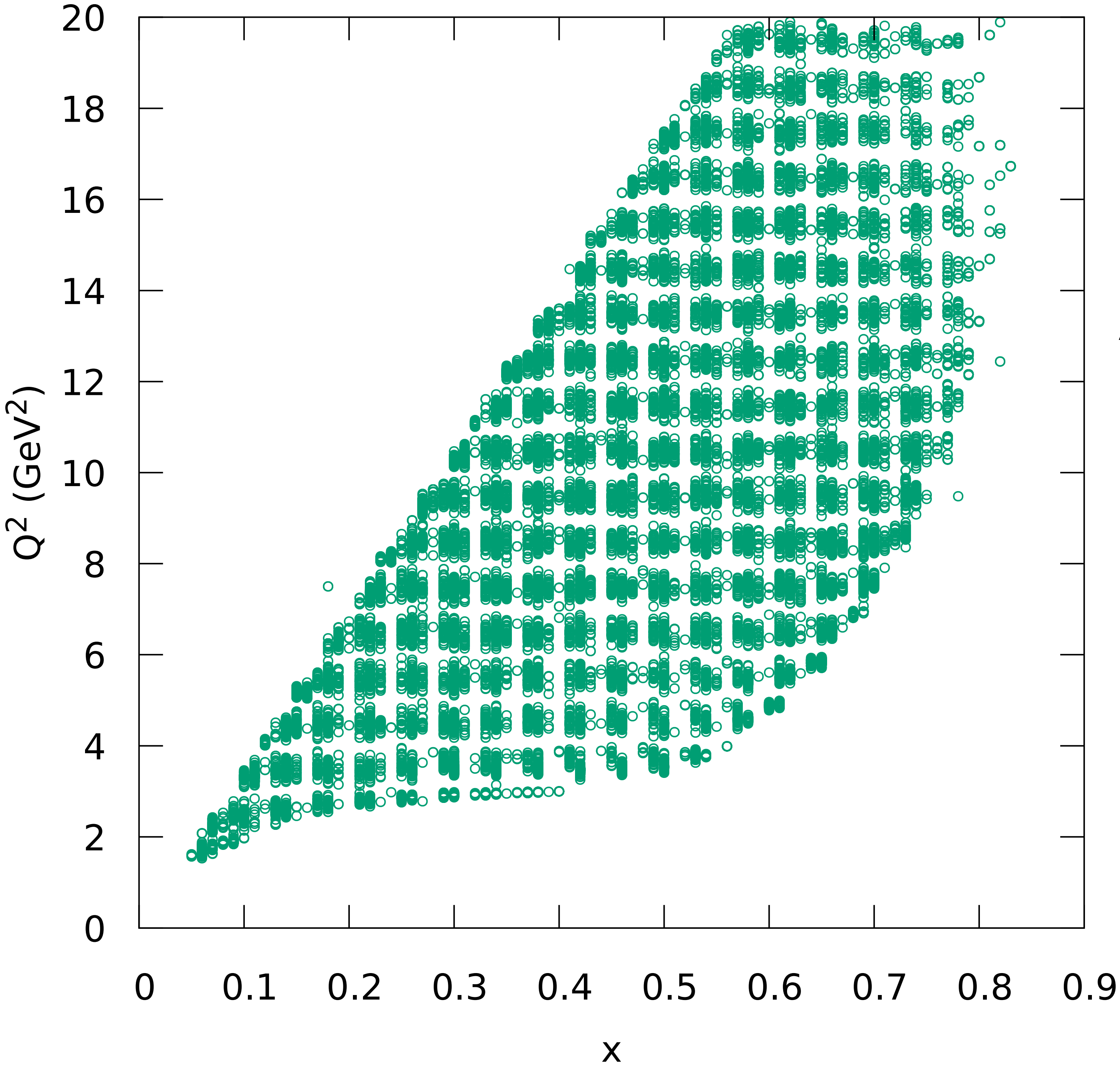

I will focus on these

Access to sufficiently small Q (accuracy)

+

Multidimensional binning

(TMD evolution, test of factorization)

Possible issues/sources of error in determining TMDs

J.O. Gonzalez-Hernandez(Turin U.), T.C. Rogers(Old Dominion U. and Jefferson Lab), N. Sato(Jefferson Lab) (May 11, 2022)
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1. The choices of models, assumptions, or approximations used to describe nonperturbative contributions,
2. The neglect of power suppressed corrections to factorization, like the last term in Eq.
3. Truncation of high powers of α_s in perturbative parts of the calculation and,
4. In phenomenological applications involving fits, whatever assumptions and approximations are used at the level of fit extractions.

$$Q^2 \frac{\mathrm{d}\sigma}{\mathrm{d}z_A\,\mathrm{d}z_B\,\mathrm{d}q_\mathrm{T}^2}$$

$$= H(\mu_Q; C_2) \int \mathrm{d}^2\boldsymbol{k}_{A\mathrm{T}}\ \mathrm{d}^2\boldsymbol{k}_{B\mathrm{T}}\, D_A\left(z_A, z_A\boldsymbol{k}_{A\mathrm{T}}; \mu_Q, Q^2\right) D_B\left(z_B, z_B\boldsymbol{k}_{B\mathrm{T}}; \mu_Q, Q^2\right) \delta^{(2)}\left(\boldsymbol{q}_\mathrm{T} - \boldsymbol{k}_{A\mathrm{T}} - \boldsymbol{k}_{B\mathrm{T}}\right)$$

$$+ Y \quad (q_\mathrm{T}, Q; \mu_Q) + O\left(m/Q\right) \ .$$

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are models consistent with pQCD?

$$
\begin{aligned}
&Q^2 \frac{\mathrm{d}\sigma}{\mathrm{d}z_A\,\mathrm{d}z_B\,\mathrm{d}q_\mathrm{T}^2} \\
&= H(\mu_Q; C_2) \int \mathrm{d}^2\boldsymbol{k}_{A\mathrm{T}}\,\mathrm{d}^2\boldsymbol{k}_{B\mathrm{T}}\, D_A\left(z_A, z_A\boldsymbol{k}_{A\mathrm{T}}; \mu_Q, Q^2\right) D_B\left(z_B, z_B\boldsymbol{k}_{B\mathrm{T}}; \mu_Q, Q^2\right) \delta^{(2)}\left(\boldsymbol{q}_\mathrm{T} - \boldsymbol{k}_{A\mathrm{T}} - \boldsymbol{k}_{B\mathrm{T}}\right) \\
&\quad + Y\quad \left(q_\mathrm{T}, Q; \mu_Q\right) + O\left(m/Q\right)\,.
\end{aligned}
$$

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How well can we access nonperturbative effects?

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Large kT behaviour of TMDs is important

$$Q^2 \frac{\mathrm{d}\sigma}{\mathrm{d}z_A\,\mathrm{d}z_B\,\mathrm{d}q_\mathrm{T}^2}$$
$$= H(\mu_Q;C_2)\int \mathrm{d}^2\boldsymbol{k}_{A\mathrm{T}}\,\mathrm{d}^2\boldsymbol{k}_{B\mathrm{T}}\,D_A\left(z_A, z_A\boldsymbol{k}_{A\mathrm{T}};\mu_Q,Q^2\right)D_B\left(z_B,z_B\boldsymbol{k}_{B\mathrm{T}};\mu_Q,Q^2\right)\delta^{(2)}\left(\boldsymbol{q}_\mathrm{T}-\boldsymbol{k}_{A\mathrm{T}}-\boldsymbol{k}_{B\mathrm{T}}\right)$$
$$+Y\quad(q_\mathrm{T},Q;\mu_Q)+O\left(m/Q\right)\,.$$

5. Neglecting the Y term

“W-term” usually written as

WOPE (pQCD)

$$W(q_{\mathrm{T}}, Q) = H(\mu_Q; C_2) \int \frac{\mathrm{d}^2 \boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} \, e^{-i\boldsymbol{q}_{\mathrm{T}} \cdot \boldsymbol{b}_{\mathrm{T}}} \boxed{\tilde{D}_A(z_A, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}^2) \tilde{D}_B(z_B, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}^2)}$$
$$\times \exp\left\{ 2 \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[\gamma(\alpha_s(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(\alpha_s(\mu')) \right] + \ln \frac{Q^2}{\mu_{b_*}^2} \tilde{K}(b_*; \mu_{b_*}) \right\}$$
$$\times \exp\left\{ -g_A(z_A, b_{\mathrm{T}}) - g_B(z_B, b_{\mathrm{T}}) - g_K(b_{\mathrm{T}}) \ln \left(\frac{Q^2}{Q_0^2} \right) \right\} \, .$$

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$$\times \exp\left\{2\int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu'); 1) - \ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu'))\right] + \ln\frac{Q^2}{\mu_{b_*}^2}\tilde{K}(b_*; \mu_{b_*})\right\}$$
$$\times \exp\left\{\boxed{-g_A(z_A, b_\mathrm{T}) - g_B(z_B, b_\mathrm{T}) - g_K(b_\mathrm{T})}\ln\left(\frac{Q^2}{Q_0^2}\right)\right\}\,.$$

Models characterizing nonperturbative behaviour

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& \times \exp\left\{ 2\int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[\gamma(\alpha_s(\mu'); 1) - \ln\frac{Q}{\mu'} \gamma_K(\alpha_s(\mu')) \right] + \ln\frac{Q^2}{\mu_{b_*}^2} \tilde{K}(b_*; \mu_{b_*}) \right\} \\
& \times \exp\left\{ -g_A(z_A, b_\mathrm{T}) - g_B(z_B, b_\mathrm{T}) - g_K(b_\mathrm{T}) \ln\left(\frac{Q^2}{Q_0^2}\right) \right\} .
\end{aligned}
$$

Models characterizing nonperturbative behaviour

Transition from small to large bT

$$\boldsymbol{b}_*(b_\mathrm{T}) = \frac{\boldsymbol{b}_\mathrm{T}}{\sqrt{1 + b_\mathrm{T}^2/b_\mathrm{max}^2}} .$$

Scale setting in the OPE

$$\mu_{b_*} \equiv C_1/b_* .$$

Exact definition of W does not depend on the shape of b* nor on the value of bmax

$$g_K(b_T) \equiv \tilde{K}(b_*, \mu) - \tilde{K}(b_T, \mu)$$

$$-g_A(z, \boldsymbol{b}_T) \equiv \ln\left(\frac{\tilde{D}_A(z, \boldsymbol{b}_T; \mu_{Q_0}, Q_0^2)}{\tilde{D}_A(z, \boldsymbol{b}_*; \mu_{Q_0}, Q_0^2)}\right)$$

When modelling g-functions, should only allow for mild dependence on b* and bmax

g-functions must be constrained explicitly at small bT (large kT)

Exact definition of W does not depend on the shape of b* nor on the value of bmax

$$g_K(b_{\mathrm{T}}) \equiv \tilde{K}(b_*, \mu) - \tilde{K}(b_{\mathrm{T}}, \mu)$$

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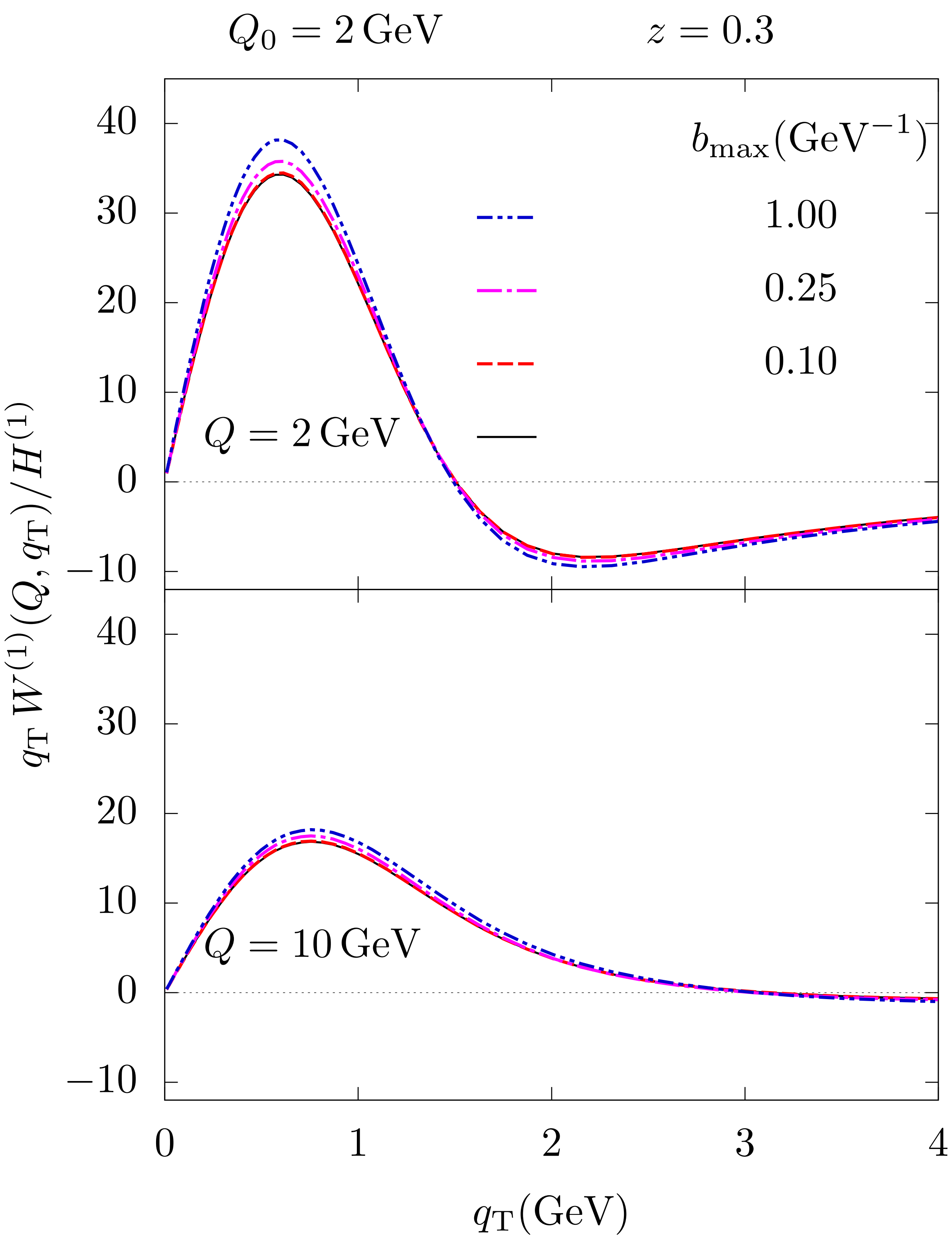
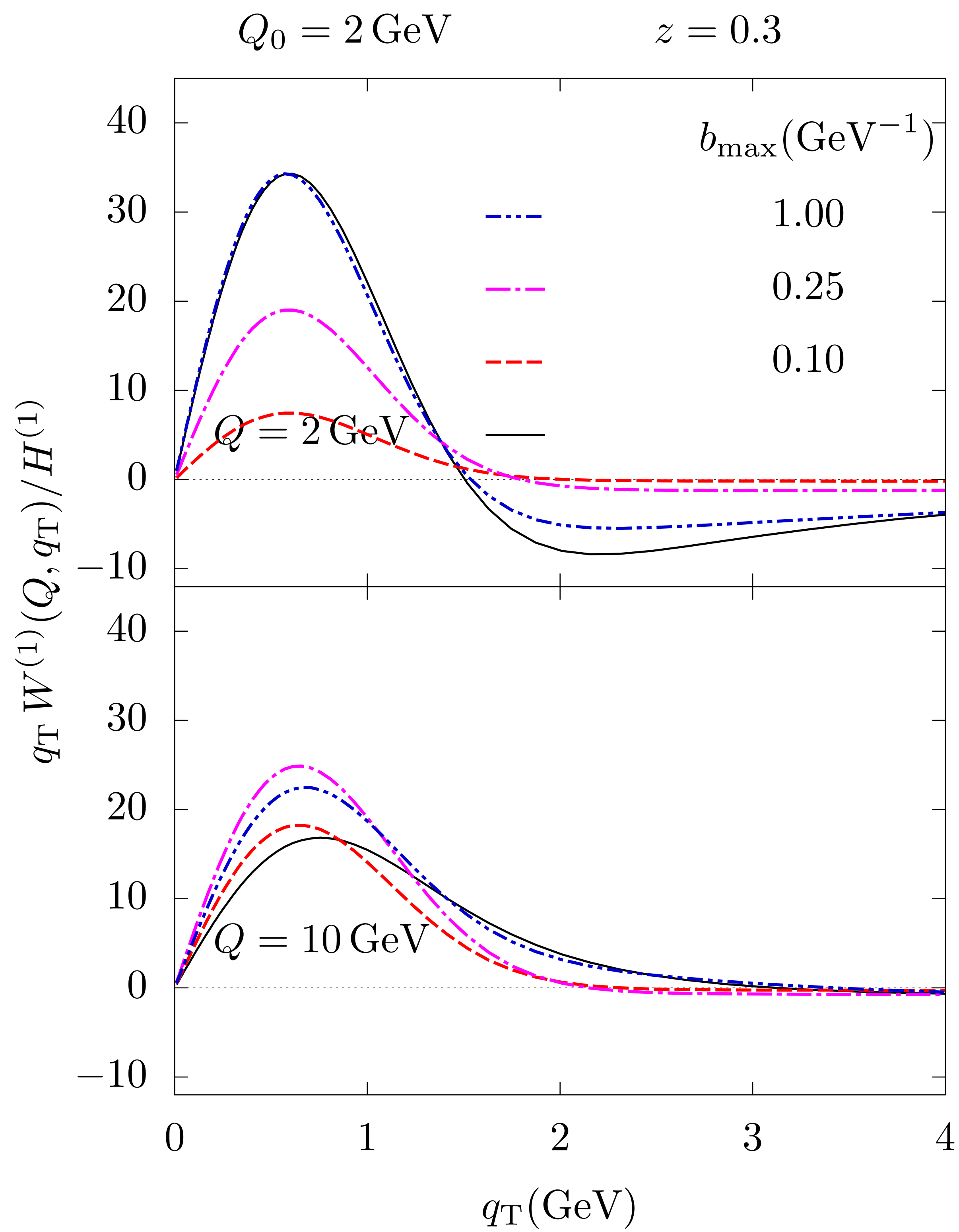
When modelling g-functions, should only allow for mild dependence on b* and bmax

In both cases, using WOPE

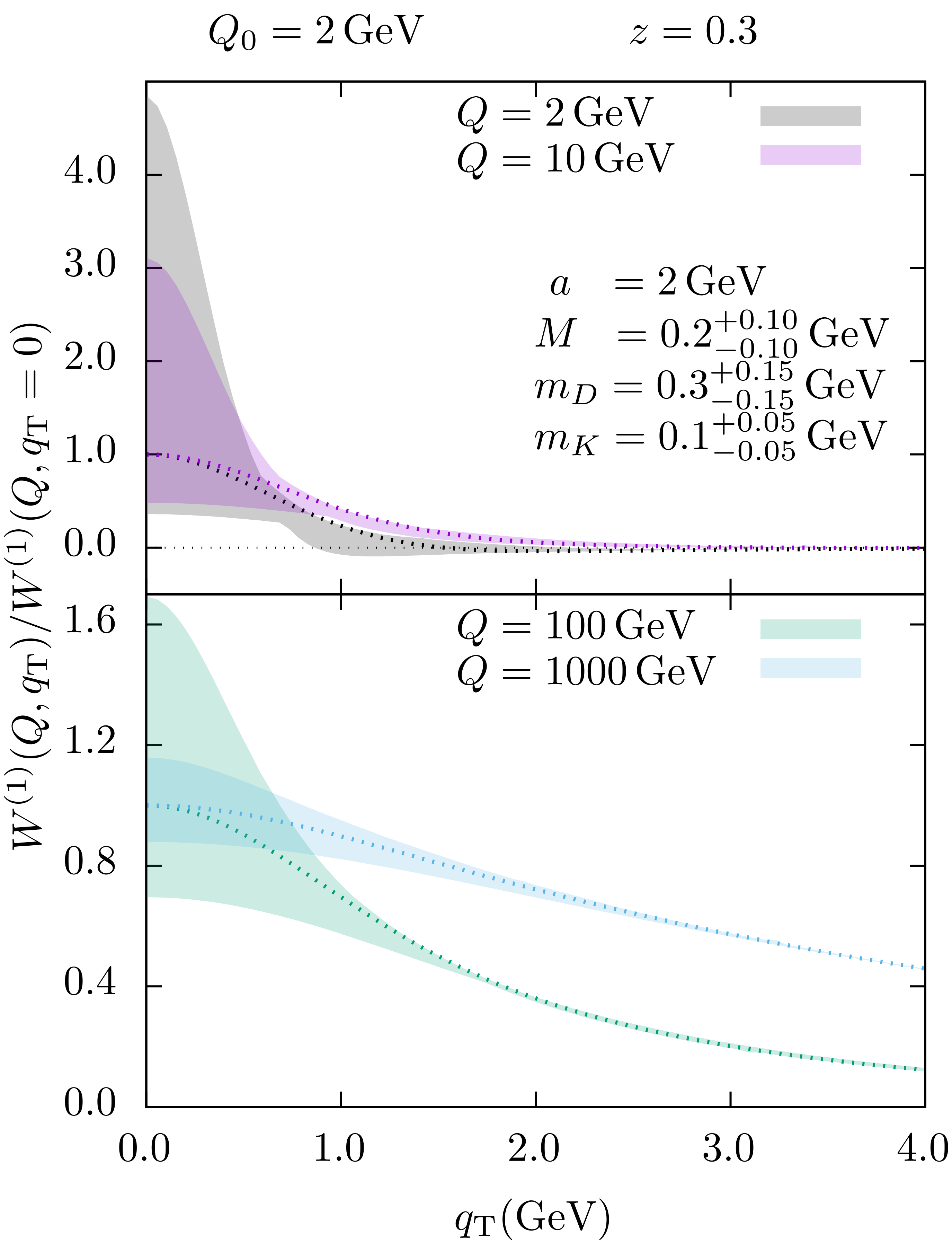
Example: e+e- —> h h



Explicitly constraining g-functions

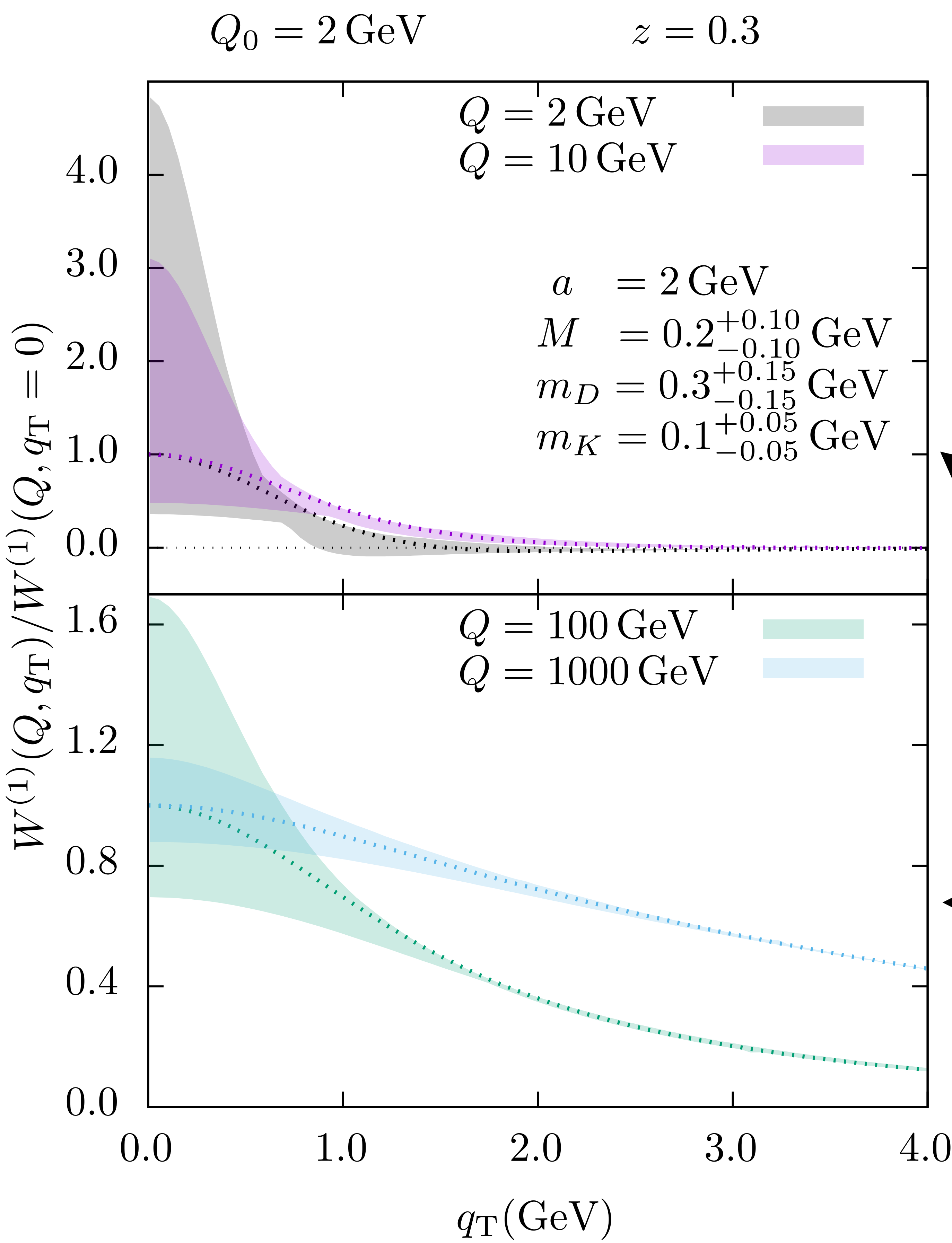


Unconstrained g-functions



Relative variation of W
w.r.t
mass parameters

TMDs parametrized
at Q0, then evolved.

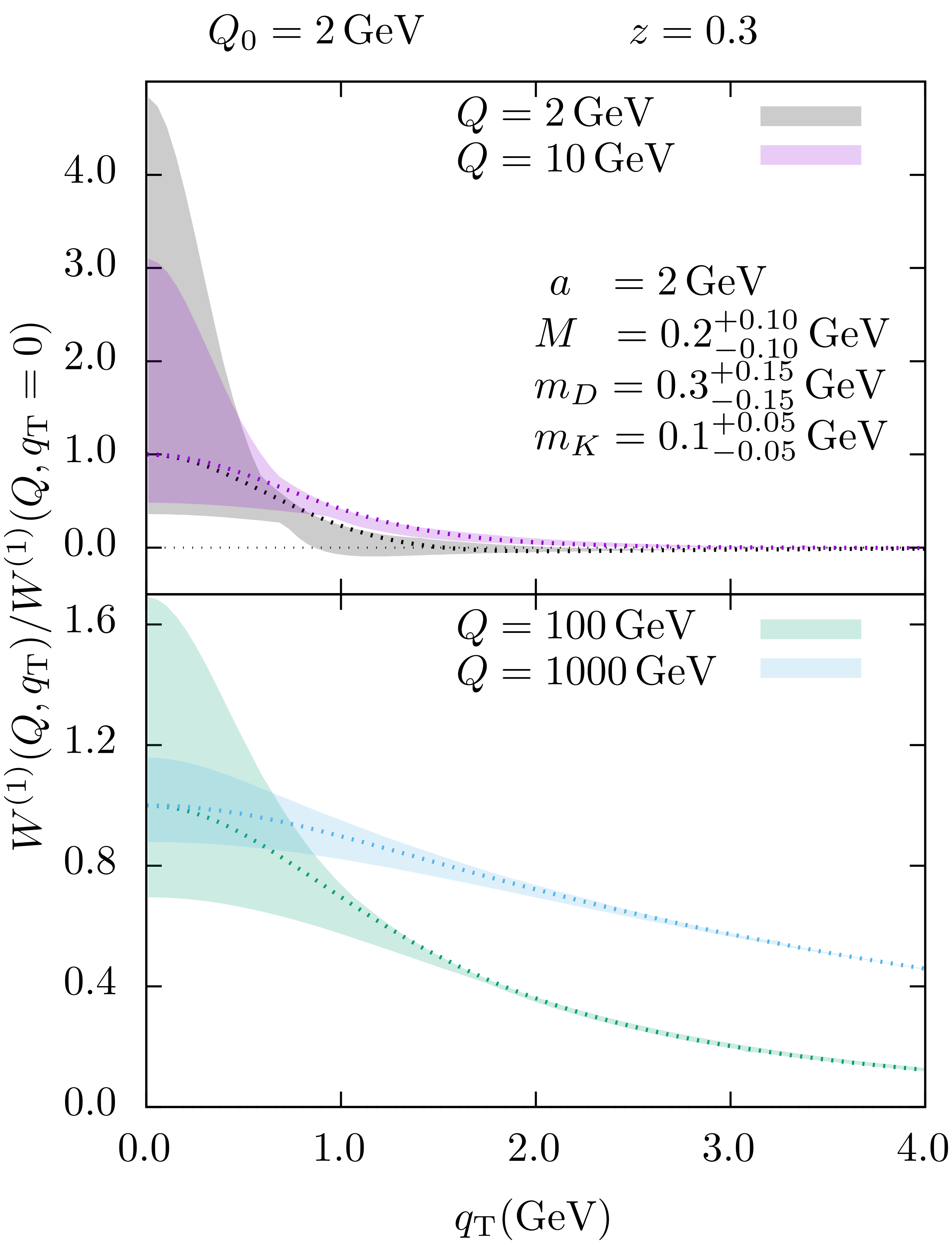


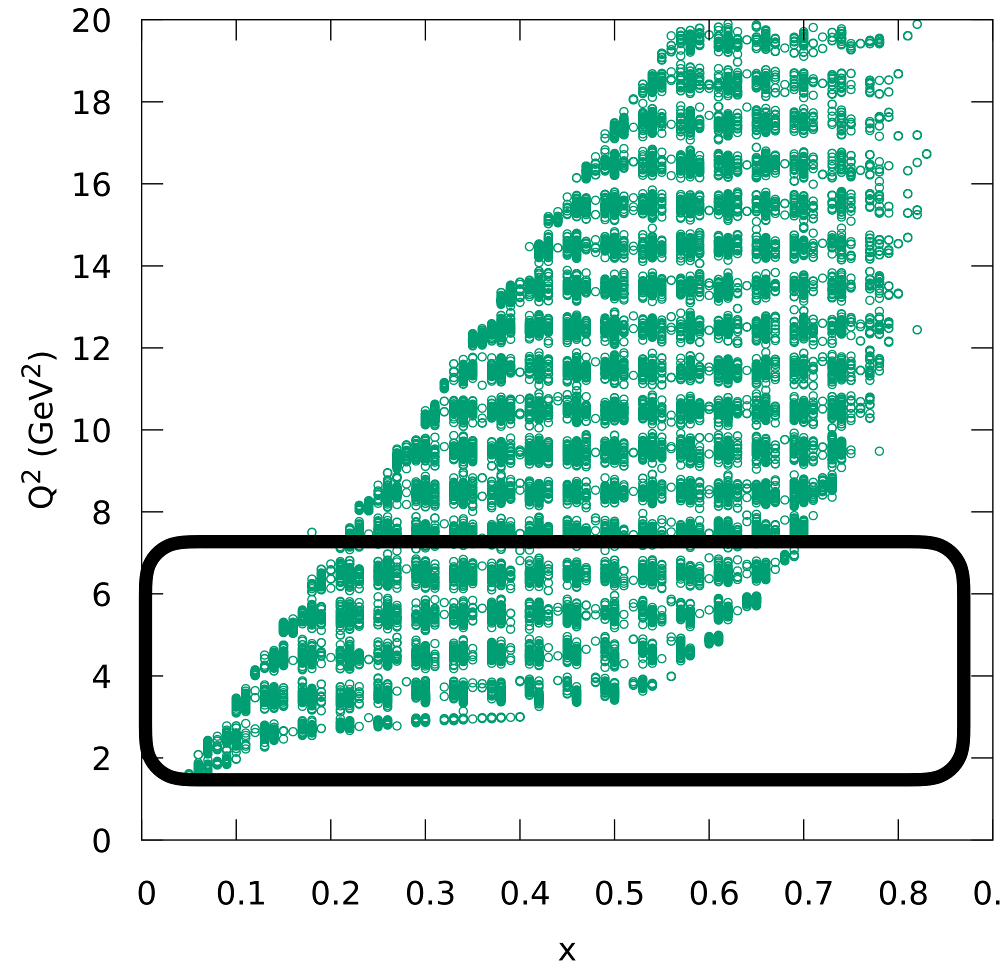
Relative variation of W w.r.t mass parameters

TMDs parametrized at Q0, then evolved.

Extract TMDs here

Predict this





Important test:
What is Q_0?

an idea:
(good old) predictions,
the ultimate test.

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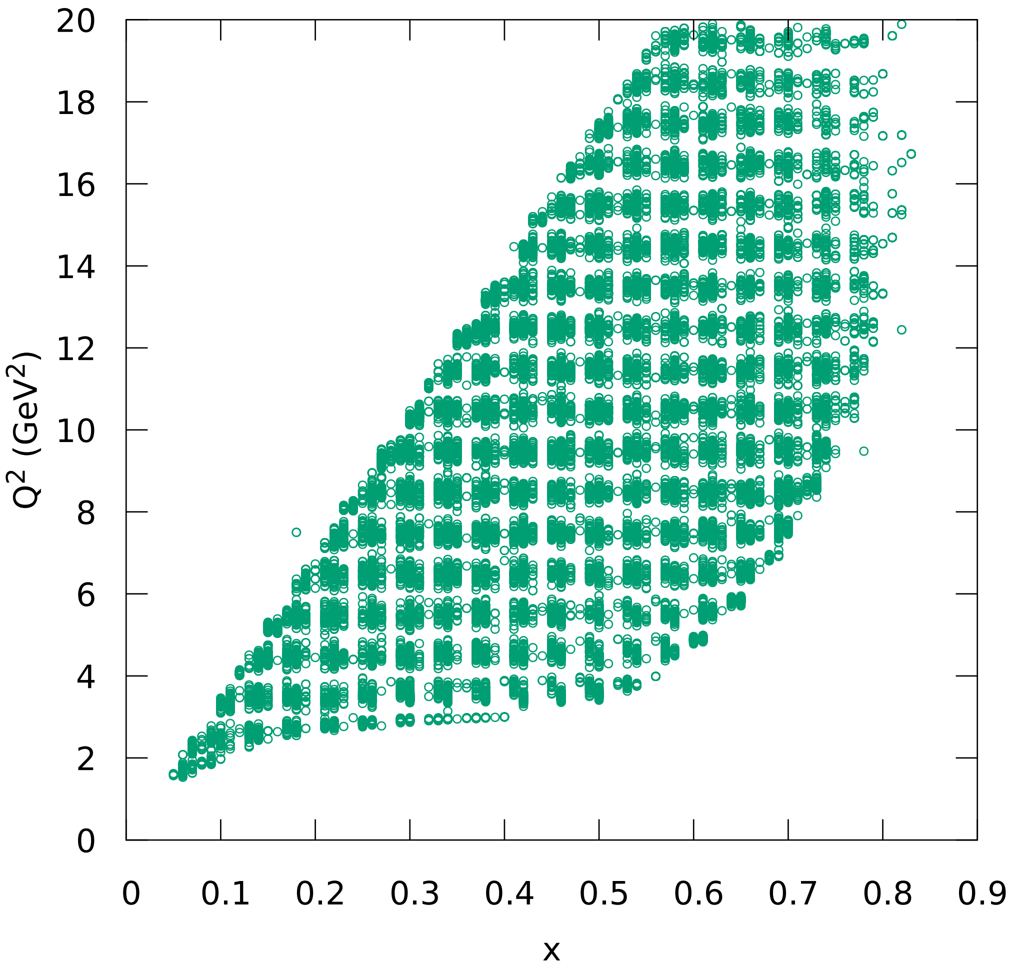
A) Constrain models

B) Extract TMDs & CS kernel at $Q \approx Q_0$

C) Evolve to higher energies and test extraction.



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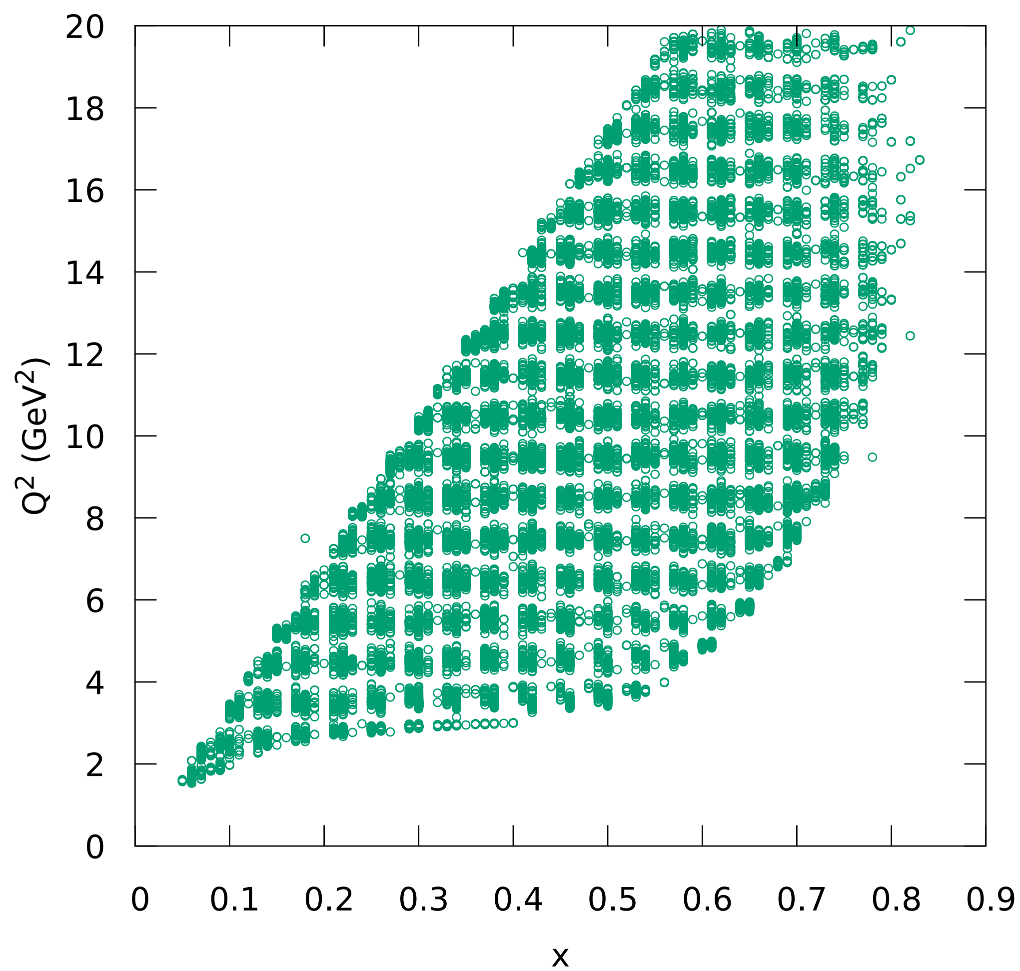
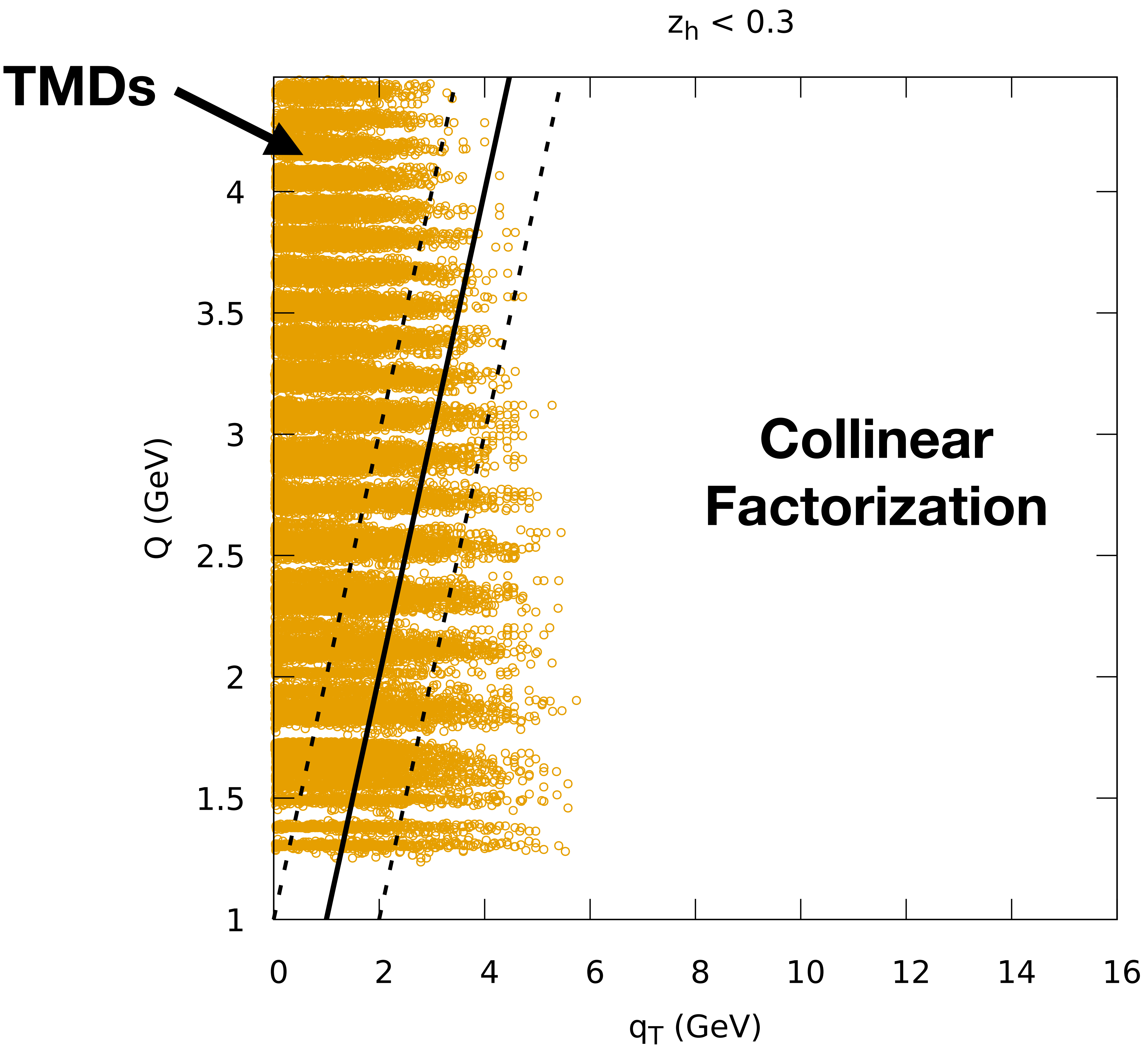
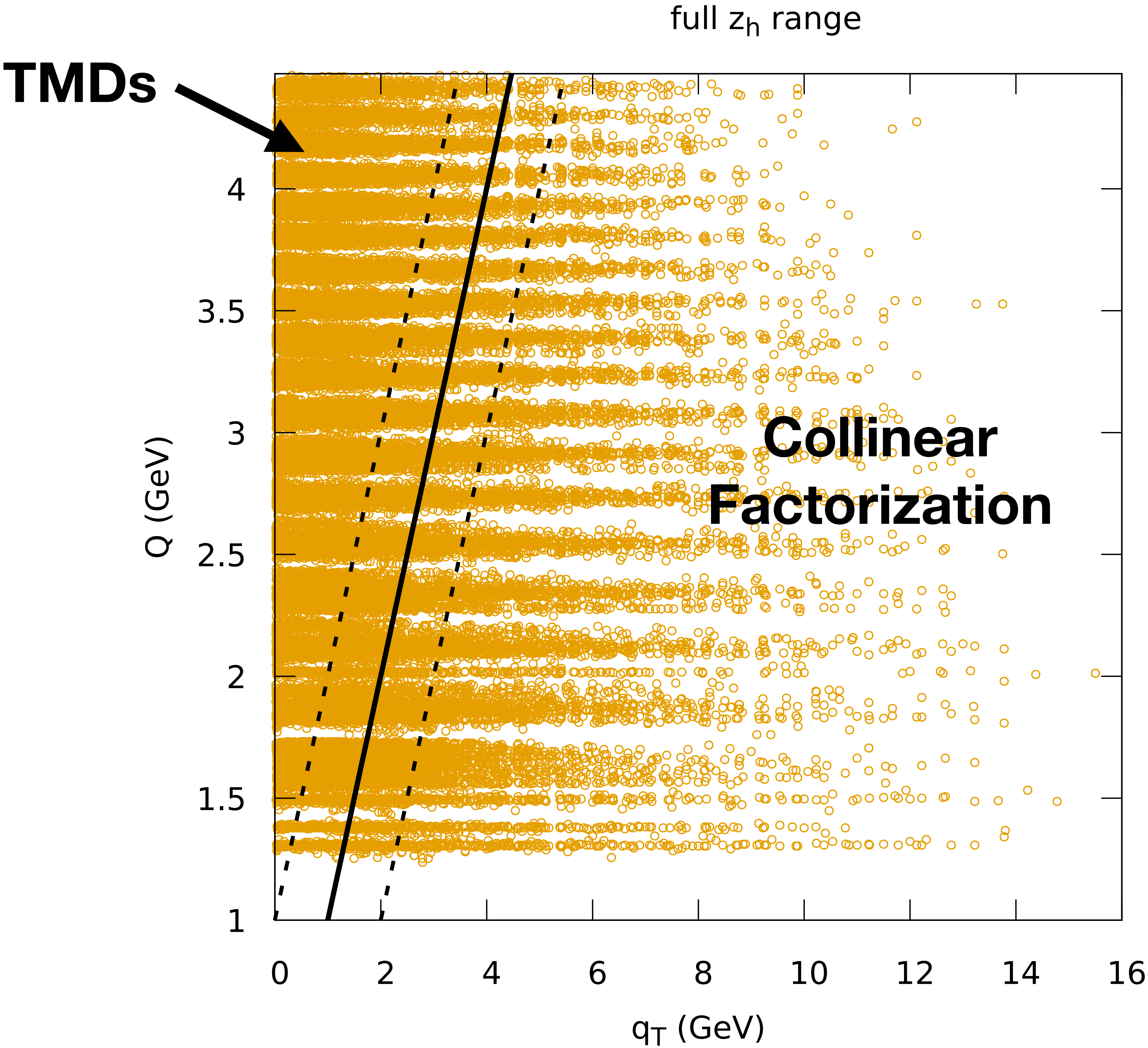


High luminosity (precision)
+
Access to sufficiently small Q (accuracy)



$z_h < 0.3$
TMDs
Collinear
Factorization
Q (GeV)
1
1.5
2
2.5
3
3.5
4
0
2
4
6
8
10
12
14
16
q_T (GeV)



How small can z_h be?

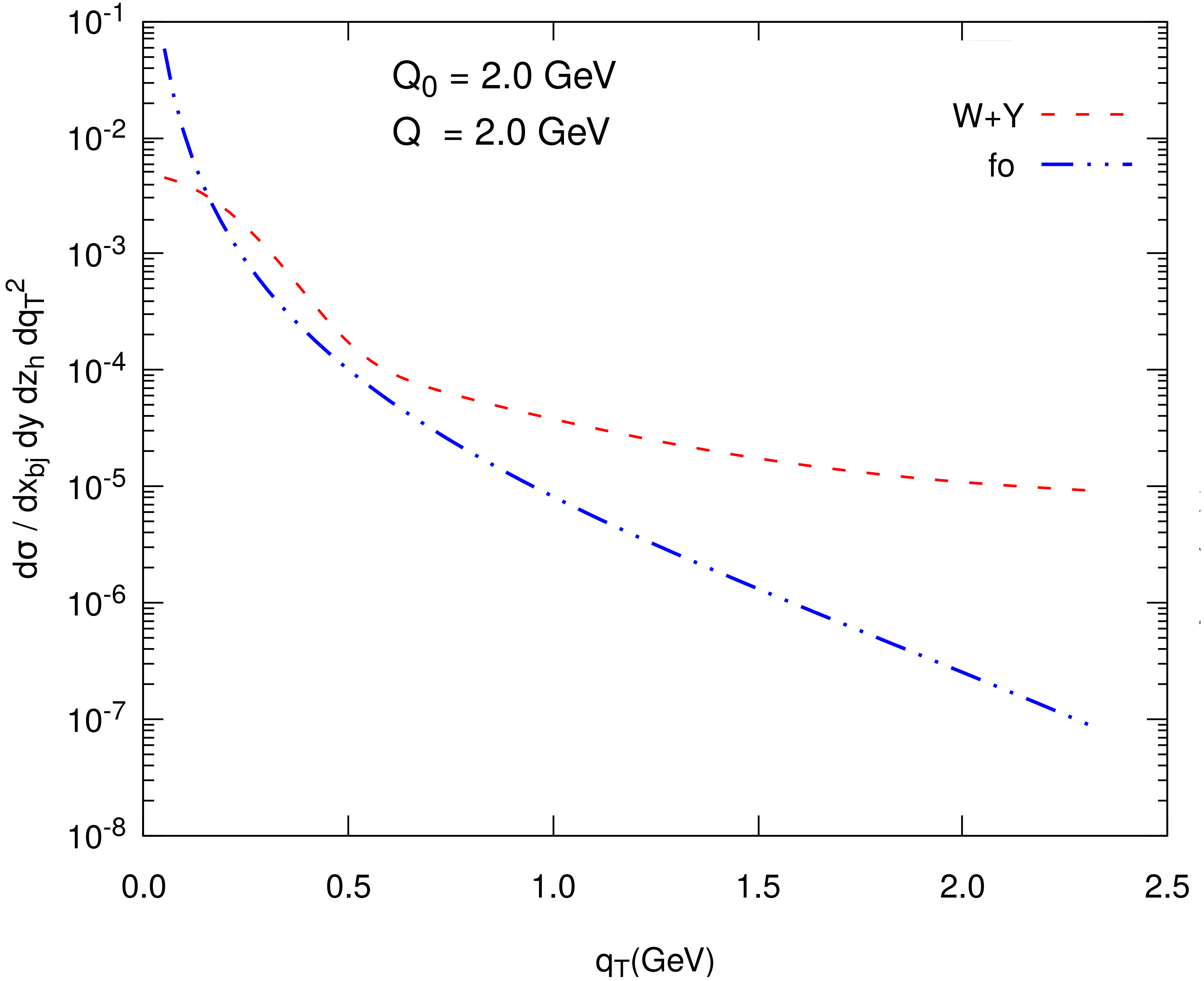
Why should we care about collinear factorization?
(large qT)

- important test of pQCD (e. g. what is Q_0?)
- Small qT limit should match the large qT behaviour of W-term (TMDs)

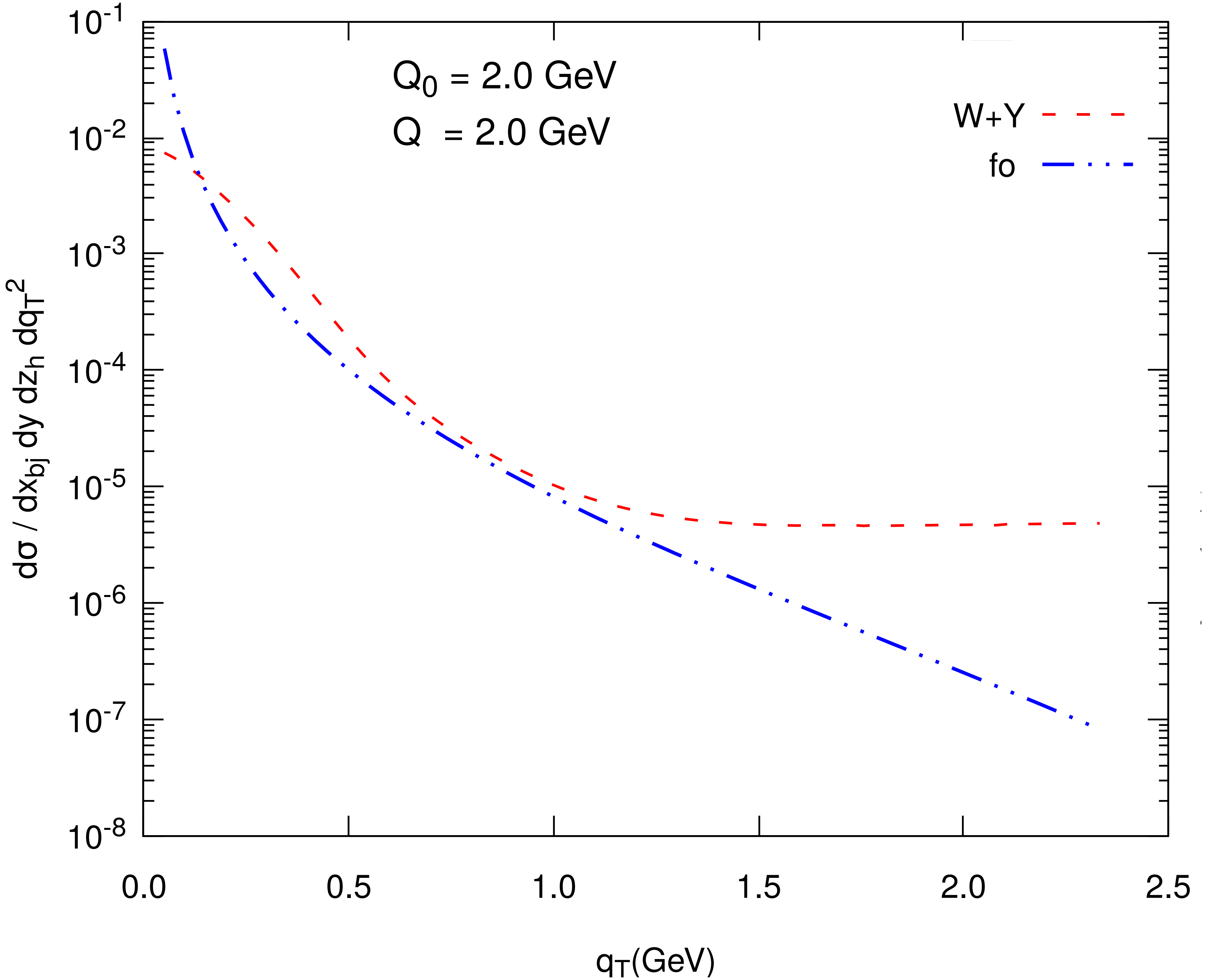
This last one is **not** only a formal issue, it has consequences in phenomenology

SIDIS example

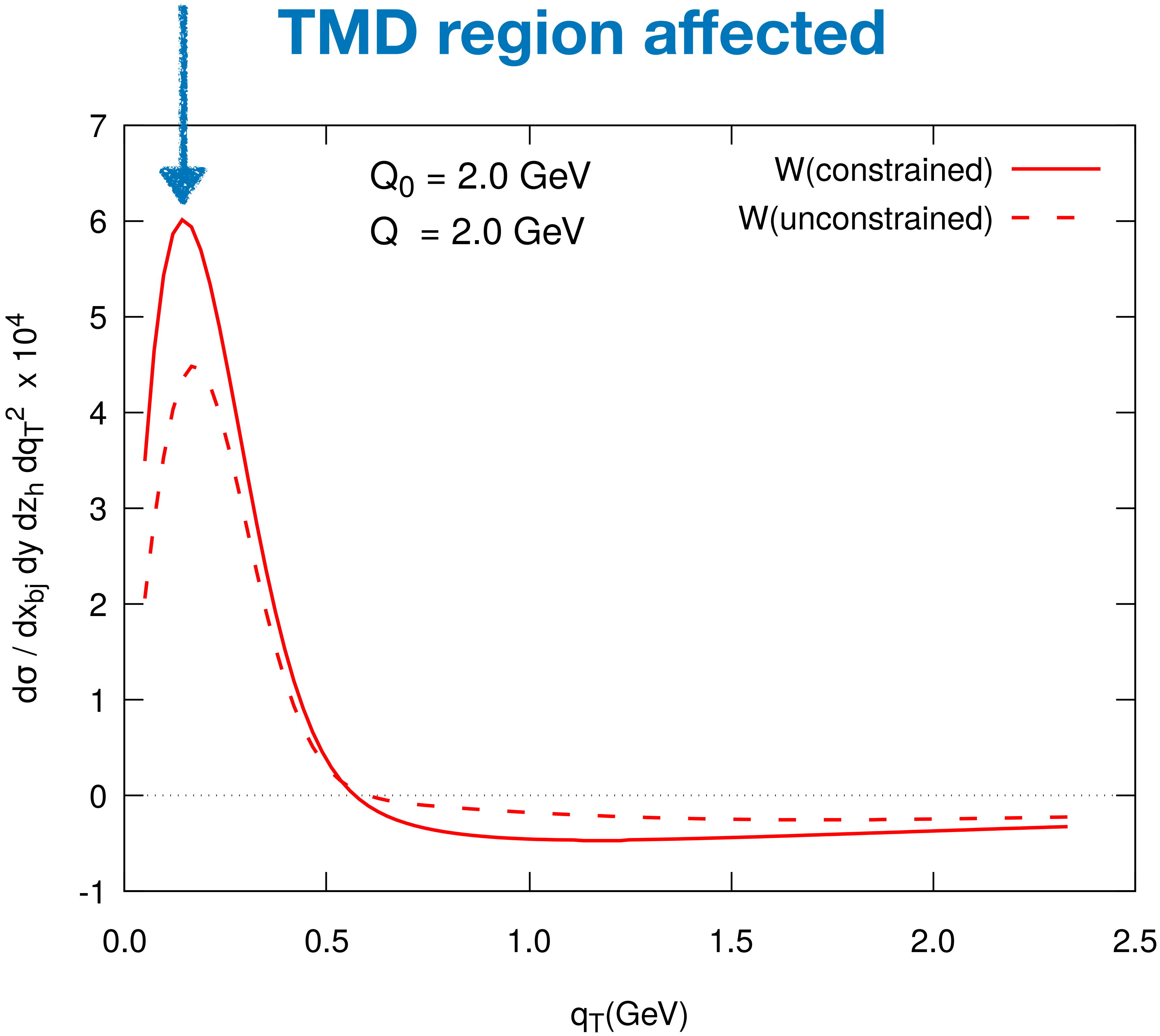
Ignoring large kT constraints

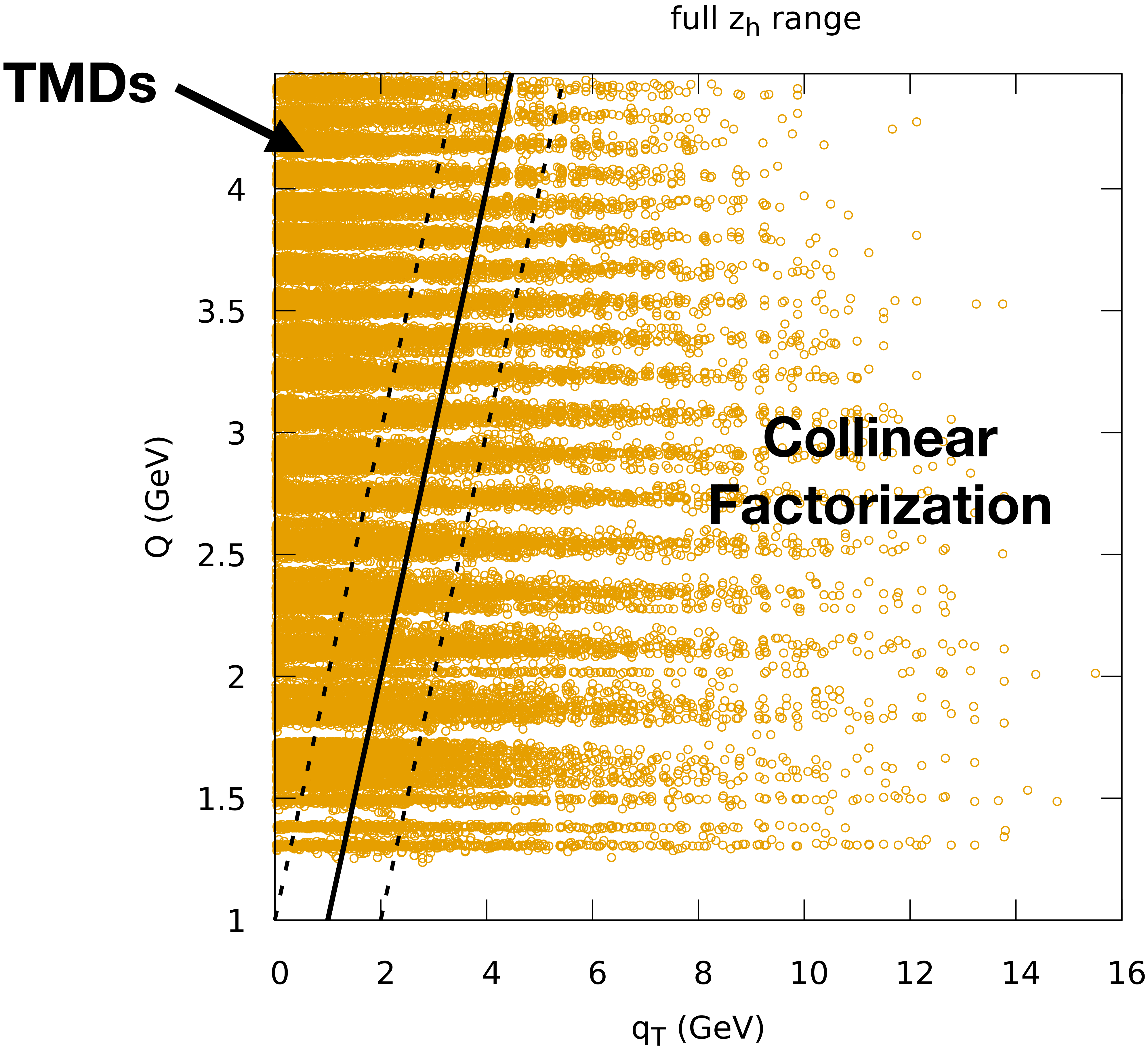


With large kT constraints



TMD region affected





How small can z_h be?

Summarizing some of these ideas:

- Binning in qT is most useful.
- Different cuts at small zh may help.
- More nonperturbative information at smaller
- scales. How low can we go? (What is Q_0?)
- Should not disregard large qT data.
- A good alternative to global fits: extract at $Q \approx Q_0$
- and test at higher energies.

Summarizing some of these ideas:

- Binning in qT is most useful.
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- A good alternative to global fits: extract at $Q \approx Q_0$ and test at higher energies.

JLab 24-GeV upgrade

- Ideal for both precision and accuracy

Thanks

Backup (recent work on CS kernel)

TMDs from e+e- -> h X

Formalism

$$
\begin{aligned}
&\widetilde{D}_{1,\,h/f}(z_h,\,b_\mathrm{T};\,Q,\,\tau\,Q^2) = \\
&\quad \frac{1}{z_h^2}\Bigg(d_{h/f}(z_h,\,\mu_{b_*}) + \frac{\alpha_S(\mu_{b_*})}{4\pi}\int_{z_h}^1 \frac{dz}{z}\,\left[d_{h/f}(z_h/z,\,\mu_{b_*})\,z^2\,\mathcal{C}^{[1]}_{q/q}(z,b_*;\mu_{b_*},\mu^2_{b_*}) + d_{h/g}(z_h/z,\,\mu_{b_*})\,z^2\,\mathcal{C}^{[1]}_{g/q}(z,b_*;\mu_{b_*},\mu^2_{b_*})\right]\Bigg) \\
&\quad \times \exp\left\{\log\frac{Q}{\mu_{b_*}}\,g_1(\lambda) + g_2(\lambda) + \frac{1}{4}\,\log\tau\,\left[g_2^K(\lambda) + \frac{1}{\log\frac{Q}{\mu_{b_*}}}\,g_3^K(\lambda)\right]\right\} \\
&\quad \times M_\mathrm{D}(z,b_\mathrm{T})\,\exp\left\{-\frac{1}{4}\,g_\mathrm{K}(b_\mathrm{T})\,\log\left(\frac{Q^2}{M_H^2}\,\tau\right)\right\}, \qquad (3)
\end{aligned}
$$

$$
\begin{aligned}
M_\mathrm{D}^\mathrm{sqrt}(z,b_\mathrm{T}) &= M_\mathrm{D}(z,b_\mathrm{T})\,\sqrt{M_\mathrm{S}(b_\mathrm{T})}, \\
g_\mathrm{K}^\mathrm{sqrt}(b_\mathrm{T}) &= \frac{1}{2} g_\mathrm{K}(b_\mathrm{T}),
\end{aligned}
$$

TMDs from e+e- -> h X

models

$$M_{\mathrm{D}} = \frac{2^{2-p}(b_{\mathrm{T}}M_0)^{p-1}}{\Gamma(p-1)}K_{p-1}(b_{\mathrm{T}}M_0) \;\times\; F(b_{\mathrm{T}}, z_h)$$

ID	M_{D} model	parameters
I	$F = \left(\frac{1+\log\left(1+(b_{\mathrm{T}}M_z)^2\right)}{1+(b_{\mathrm{T}}M_z)^2}\right)^q$ $M_z = -M_1 \log(z_h)$	$M_0,\ M_1$ $p = 1.51,\ q = 8$
II	$F = 1$ $M_z = M_\pi \frac{1}{z\, f(z)^2}\sqrt{\frac{3}{1-f(z)}}$ $p_z = 1 + \frac{3}{2}\frac{f(z)}{1-f(z)}$ $f(z) = 1-(1-z)^\beta, \quad \beta = \frac{1-z_0}{z_0}$	z_0

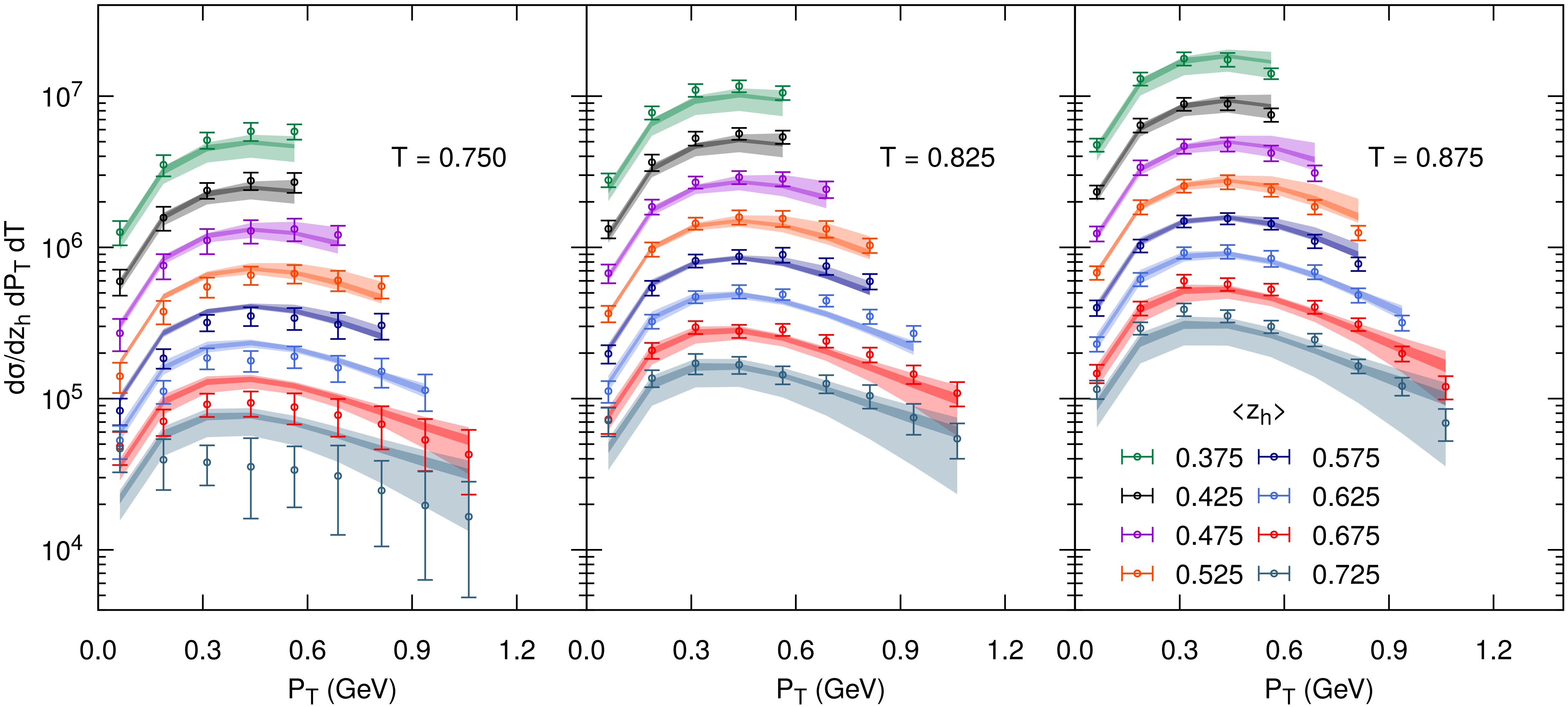
g_{K} model		
A	$g_{\mathrm{K}} = \log\left(1+(b_{\mathrm{T}}M_{\mathrm{K}})^{p_{\mathrm{K}}}\right)$	$M_{\mathrm{K}},\ p_{\mathrm{K}}$
B	$g_{\mathrm{K}} = M_{\mathrm{K}} b_{\mathrm{T}}^{(1-2p_{\mathrm{K}})}$	$M_{\mathrm{K}},\ p_{\mathrm{K}}$

TMDs from e+e- -> h X

Fit results

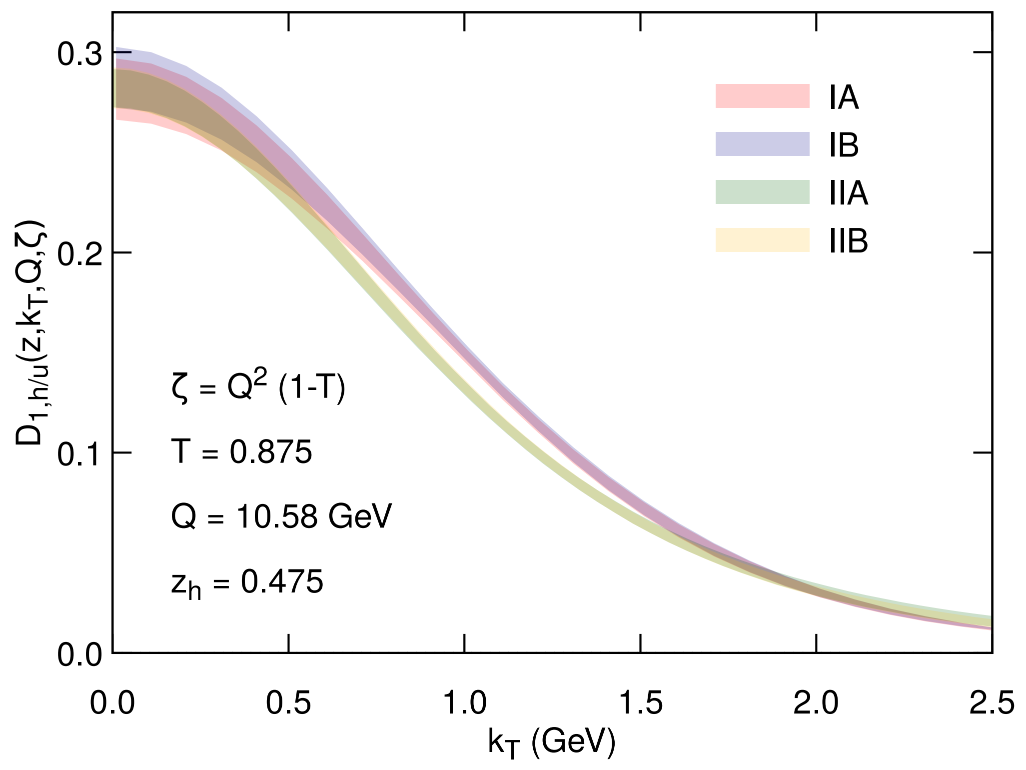
$q_{\mathrm{T}}/Q < 0.15$ (pts = 168)		
	IA	IB
$\chi^2_{\mathrm{d.o.f.}}$	1.25	1.19
$M_0(\mathrm{GeV})$	$0.300^{+0.075}_{-0.062}$	$0.003^{+0.089}_{-0.003}$
$M_1(\mathrm{GeV})$	$0.522^{+0.037}_{-0.041}$	$0.520^{+0.027}_{-0.040}$
p^*	1.51	1.51
q^*	8	8
$M_{\mathrm{K}}(\mathrm{GeV})$	$1.305^{+0.139}_{-0.146}$	$0.904^{+0.037}_{-0.086}$
p^*_{K}	0.609	0.229

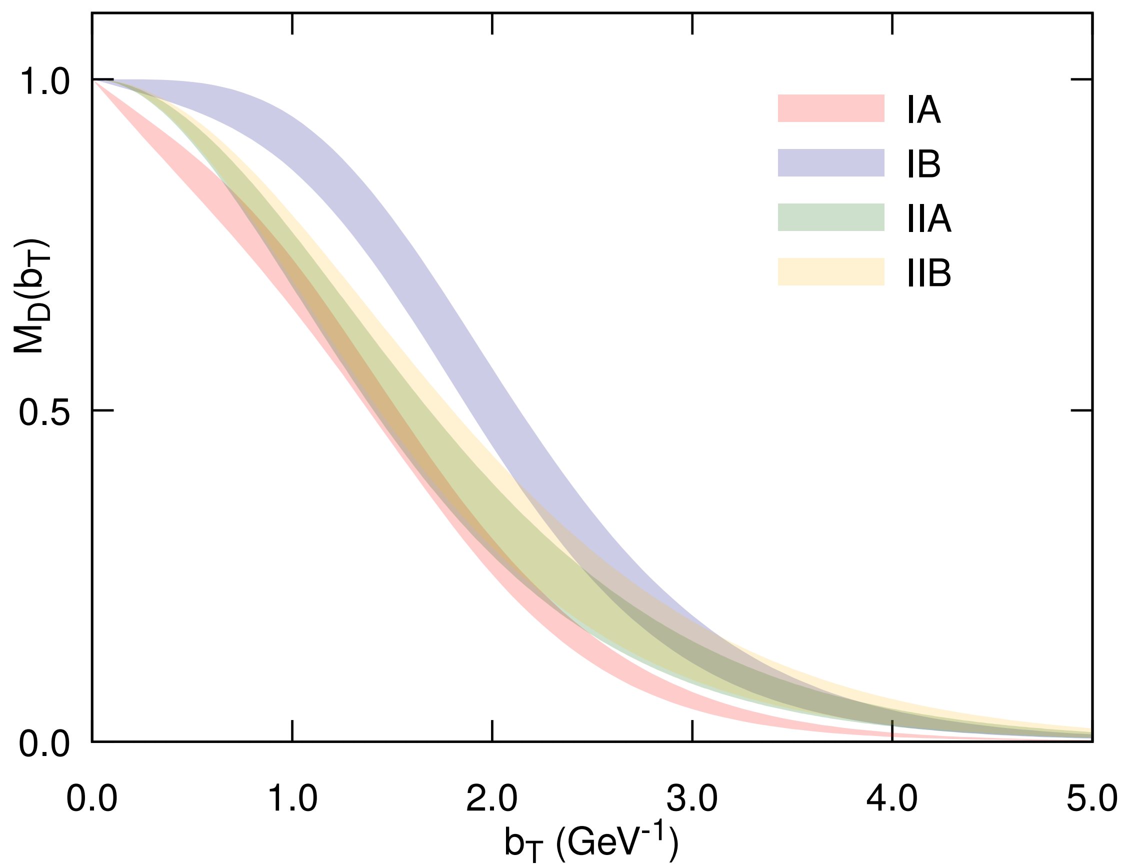
$q_{\mathrm{T}}/Q < 0.15$ (pts = 168)		
	IIA	IIB
$\chi^2_{\mathrm{d.o.f.}}$	1.35	1.33
z_0	$0.574^{+0.039}_{-0.041}$	$0.556^{+0.047}_{-0.051}$
$M_{\mathrm{K}}(\mathrm{GeV})$	$1.633^{+0.103}_{-0.105}$	$0.687^{+0.114}_{-0.171}$
p_k	$0.588^{+0.127}_{-0.141}$	$0.293^{+0.047}_{-0.038}$

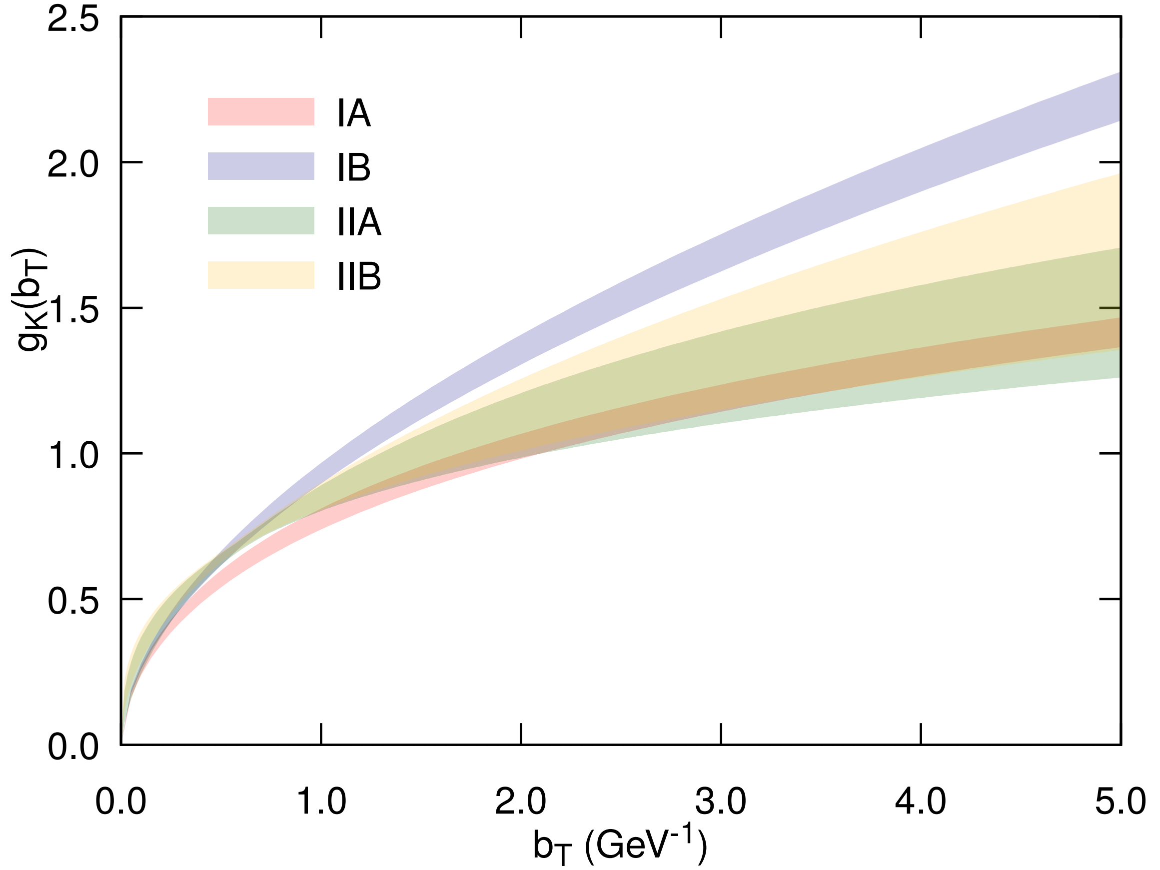


dσ/dz_h dP_T dT
T = 0.750
T = 0.825
T = 0.875
⟨z_h⟩
0.375
0.425
0.475
0.525
0.575
0.625
0.675
0.725
P_T (GeV)

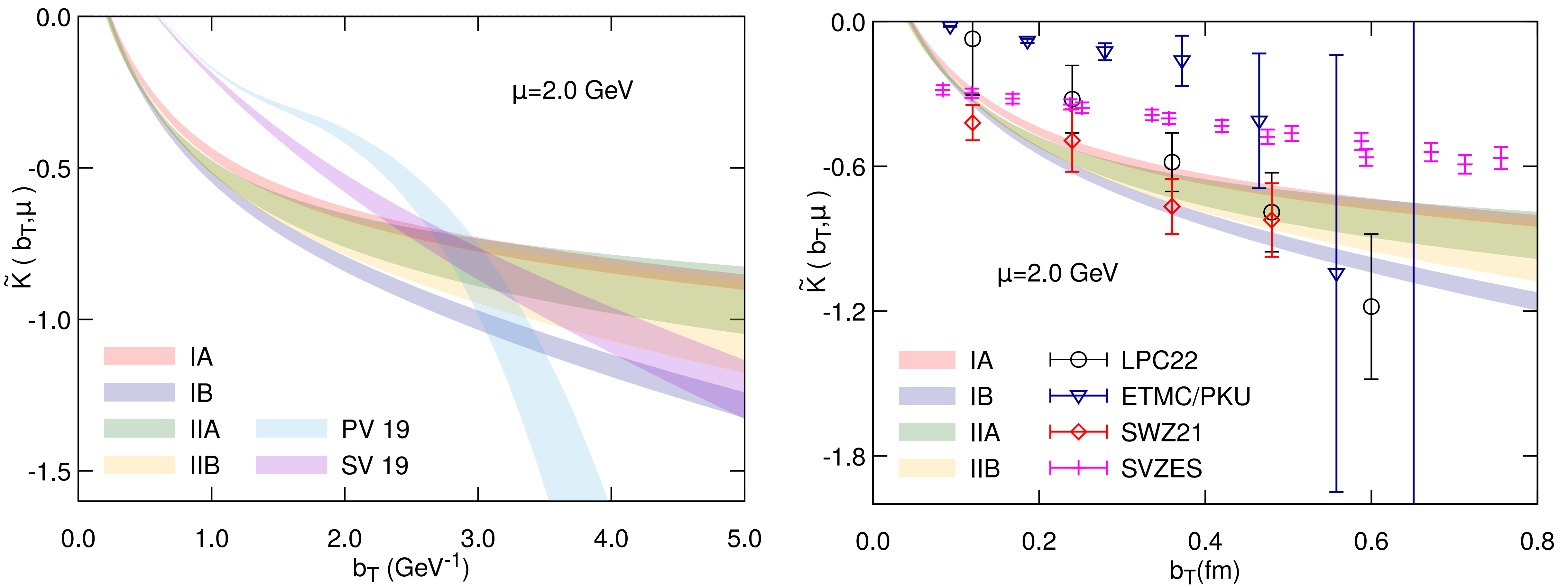
TMDs from e+e- -> h X







CS kernel



μ=2.0 GeV
$\tilde{K}(b_T,\mu)$
b_T (GeV^{-1})
0.0
-0.5
-1.0
-1.5
0.0
1.0
2.0
3.0
4.0
5.0
IA
IB
IIA
IIB
PV 19
SV 19
μ=2.0 GeV
$\tilde{K}(b_T,\mu)$
b_T(fm)
0.0
-0.6
-1.2
-1.8
0.0
0.2
0.4
0.6
0.8
IA
IB
IIA
IIB
LPC22
ETMC/PKU
SWZ21
SVZES