QCD and the quark model of hadrons

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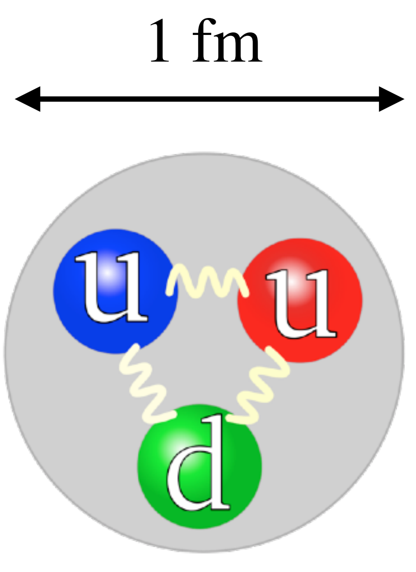
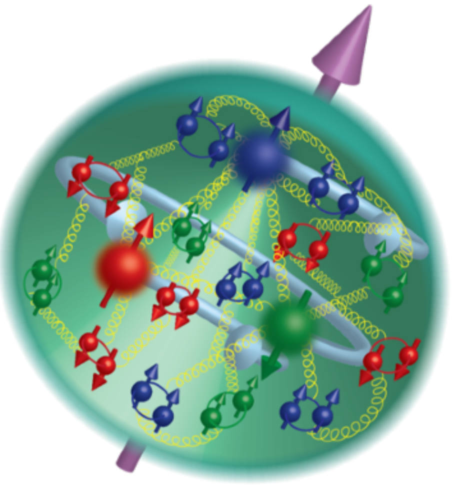
Opportunities with JLab Energy and Luminosity Upgrade
26-30 September 2022 at ECT*, Trento

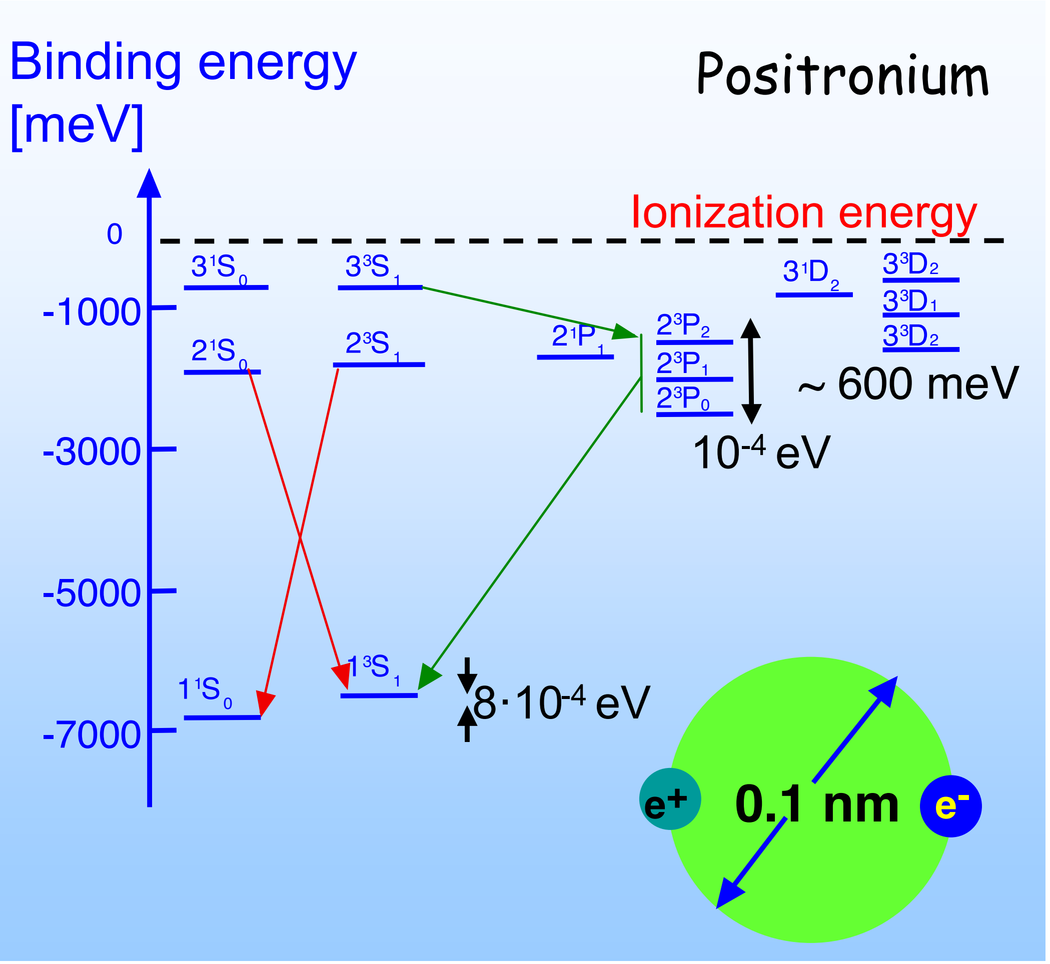
How can something so complicated as a proton look so simple?

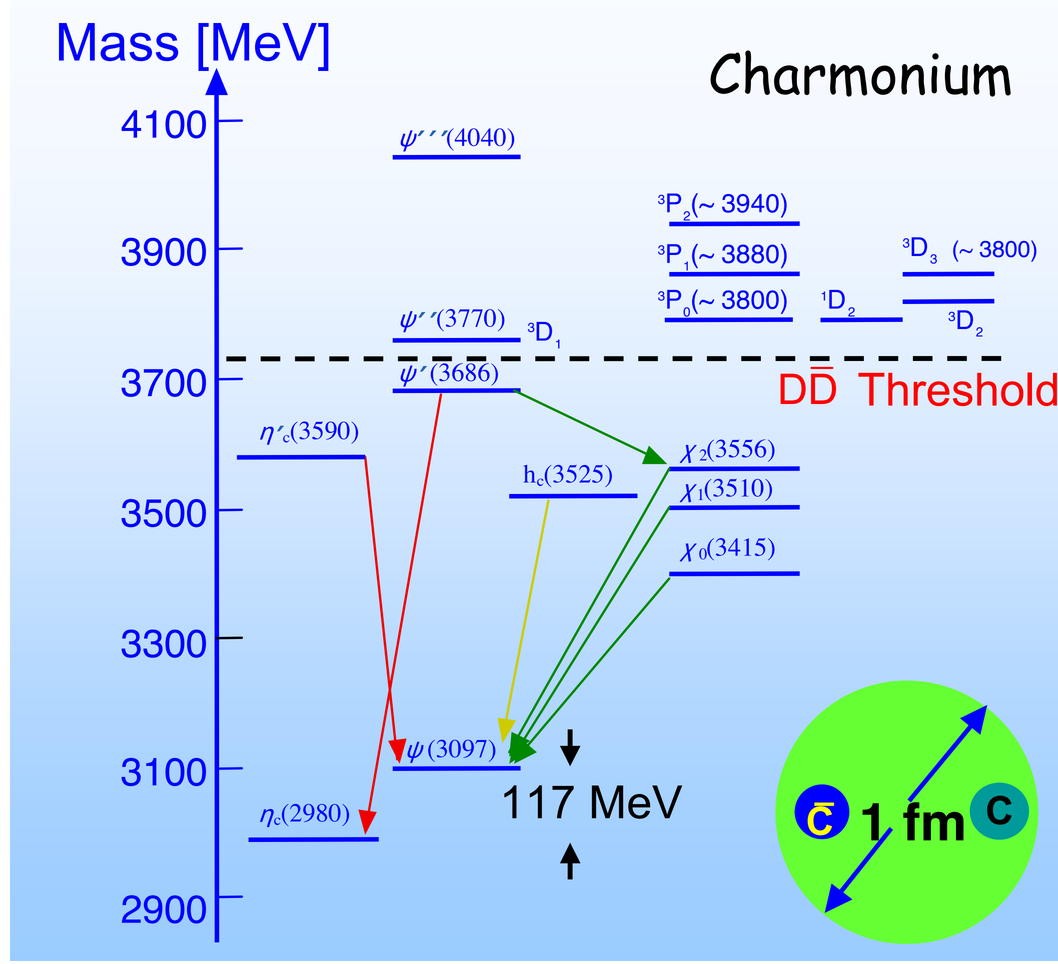




Can this be explained / excluded in QCD?

Quarkonia are like atoms with confinement





$$V(r) = -\frac{\alpha}{r}$$

$$V(r) = V'r - \frac{4}{3}\frac{\alpha_s}{r} \qquad (1980)$$

E. Eichten, S. Godfrey, H. Mahlke and J. L. Rosner, Rev. Mod. Phys. **80** (2008) 1161

"The J/ψ is the Hydrogen atom of QCD"

Bound states from the QFT action

Bound states are omitted from QFT textbooks

Cf. Careful derivations of the perturbative S-matrix

The Schrödinger equation does not follow from the QED action

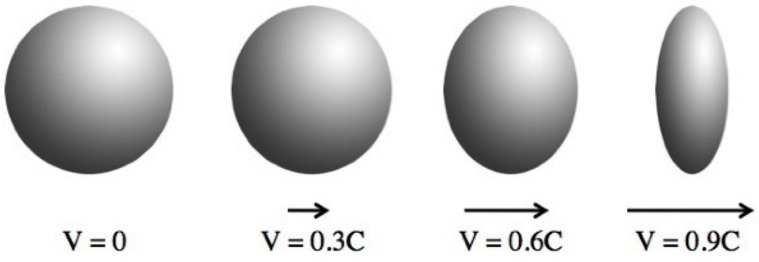
Caswell & Lepage (1975)

For states at rest: Assumed initial state for an EFT (NRQED, NRQCD)

What about moving atoms and hadrons: ?

Atoms in motion are pictured as classically contracted

Their QM wave functions are not calculated



M. Järvinen, PRD 71 (2005)

In our quest for hadrons:

Need textbook level derivations of bound states starting from $\mathcal{L}_{QFT}$

Take the features of hadrons at face value

Relativistic $q\bar{q}$ spectrum

$n^{2s+1}\ell_J$	J^{PC}	$I=1$ $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	$I=\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $\bar{u}s$	$I=0$ f'	$I=0$ f
1^1S_0	0^{-+}	$\boldsymbol{\pi}$	$\boldsymbol{K}$	$\boldsymbol{\eta}$	$\boldsymbol{\eta'}(958)$
1^3S_1	1^{--}	$\boldsymbol{\rho(770)}$	$\boldsymbol{K^*(892)}$	$\boldsymbol{\phi(1020)}$	$\boldsymbol{\omega(782)}$
1^1P_1	1^{+-}	$\boldsymbol{b_1(1235)}$	$\boldsymbol{K_{1B}}^\dagger$	$\boldsymbol{h_1(1415)}$	$\boldsymbol{h_1(1170)}$
1^3P_0	0^{++}	$\boldsymbol{a_0(1450)}$	$\boldsymbol{K_0^*(1430)}$	$\boldsymbol{f_0(1710)}$	$\boldsymbol{f_0(1370)}$
1^3P_1	1^{++}	$\boldsymbol{a_1(1260)}$	$\boldsymbol{K_{1A}}^\dagger$	$\boldsymbol{f_1(1420)}$	$\boldsymbol{f_1(1285)}$
1^3P_2	2^{++}	$\boldsymbol{a_2(1320)}$	$\boldsymbol{K_2^*(1430)}$	$\boldsymbol{f_2'(1525)}$	$\boldsymbol{f_2(1270)}$
1^1D_2	2^{-+}	$\boldsymbol{\pi_2(1670)}$	$\boldsymbol{K_2(1770)}^\dagger$	$\boldsymbol{\eta_2(1870)}$	$\boldsymbol{\eta_2(1645)}$
1^3D_1	1^{--}	$\boldsymbol{\rho(1700)}$	$\boldsymbol{K^*(1680)}^\ddagger$		$\boldsymbol{\omega(1650)}$
1^3D_2	2^{--}		$\boldsymbol{K_2(1820)}^\dagger$		
1^3D_3	3^{--}	$\boldsymbol{\rho_3(1690)}$	$\boldsymbol{K_3^*(1780)}$	$\boldsymbol{\phi_3(1850)}$	$\boldsymbol{\omega_3(1670)}$
1^3F_4	4^{++}	$\boldsymbol{a_4(1970)}$	$\boldsymbol{K_4^*(2045)}$	$f_4(2300)$	$\boldsymbol{f_4(2050)}$
1^3G_5	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$		
2^1S_0	0^{-+}	$\boldsymbol{\pi(1300)}$	$K(1460)$	$\boldsymbol{\eta(1475)}$	$\boldsymbol{\eta(1295)}$
2^3S_1	1^{--}	$\boldsymbol{\rho(1450)}$	$\boldsymbol{K^*(1410)}^\ddagger$	$\boldsymbol{\phi(1680)}$	$\boldsymbol{\omega(1420)}$
2^3P_1	1^{++}	$\boldsymbol{a_1(1640)}$			
2^3P_2	2^{++}	$\boldsymbol{a_2(1700)}$	$K_2^*(1980)$	$\boldsymbol{f_2(1950)}$	$f_2(1640)$

Particle Data Group

No gluons, sea quarks

α_s is perturbative

and

High-mass excitations

$\Rightarrow$ Confining potential: α_s^0, instantaneous

Dramatic consequences: A perturbative expansion for hadrons, as for atoms

The scale of Confinement: $\Lambda_{QCD} \simeq 1\ \mathrm{fm}^{-1}$

Cornell $Q\bar{Q}$ potential $V(r) = V'r - \frac{4}{3}\frac{\alpha_s}{r}$

E. Eichten et al
PRD **21** (1980) 203

$\alpha_s \approx 0.39$ Perturbative analysis of quarkonia

$V' \approx 0.18\ \mathrm{GeV}^2$ Confinement scale: Not in $\mathcal{L}_{QCD}$
Must be added without changing $\mathcal{L}_{QCD}$

Do QCD interactions depend on something beyond $\mathcal{L}_{QCD}$?

Yes: Boundary conditions

The instantaneous gauge potential

Gauge theories do have instantaneous interactions:
Although their action is local, the gauge may be fixed non-locally

The lack of $\partial_0 A^0$ and $\nabla\cdot\boldsymbol{A}$ in $F_{\mu\nu}F^{\mu\nu}$ means that A^0 and $\boldsymbol{A}_L$ do not propagate

The values of A^0 and $\boldsymbol{A}_L$ are determined by the choice of gauge

Covariant gauge fixing: $\mathscr{L}_{GF} = (\partial_\mu A^\mu)^2$ adds the missing derivatives

This hides the instantaneous potential, obscures bound state dynamics.

Temporal gauge: $A^0(t,\boldsymbol{x}) = 0$

Canonical quantisation is straightforward: $A^0 = \partial_0 A^0 = 0$

$$\left[E^i(t,\boldsymbol{x}), A^j(t,\boldsymbol{y})\right] = i\delta^{ij}\delta(\boldsymbol{x}-\boldsymbol{y})$$

$E^i = -\,\partial_0 A^i$ Electric field

$A^0(t,\boldsymbol{x}) = 0$ is preserved under time-independent gauge transformations, which are generated by the operator of “Gauss’ law”: Willemsen (1978)

$$\frac{\delta \mathcal{S}_{QED}}{\delta A^0(x)} = \partial_i E^i(x) - e\psi^\dagger\psi(x)$$

Does not vanish as an operator since $A^0 = 0$

Physical states must be invariant under all gauge transformations:

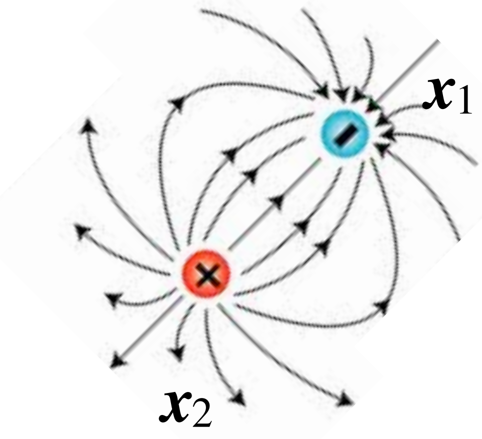
$$\frac{\delta \mathcal{S}_{QED}}{\delta A^0(x)}\,|phys\rangle = 0$$

$\boxed{\text{Determines } \nabla\cdot\boldsymbol{E}_L \text{ from the charges in each state}}$

The classical, instantaneous field E_L

$$\frac{\delta \mathcal{S}_{QED}}{\delta A^0(x)}\,|phys\rangle = 0$$

e^+e^- :



The electric field $\boldsymbol{E}_L$ is a classical field.

$\boldsymbol{E}_L$ can bind $q\bar{q}$ Fock states strongly, without pair creation.

Allows valence dominance even for relativistic hadrons

Temporal gauge in QCD: $A_a^0 = 0$

The temporal gauge constraint determines $\nabla \cdot \boldsymbol{E}_{L,a}$ for each state:

$$\frac{\delta \mathcal{S}_{QCD}}{\delta A_a^0(x)} = \partial_i E_a^i(x) + g f_{abc} A_b^i E_c^i - g\psi^\dagger T^a \psi(x)$$

$$\frac{\delta \mathcal{S}_{QCD}}{\delta A_a^0(x)} \, |phys\rangle = 0$$

Include a homogeneous solution for $\boldsymbol{E}_{L,a}$: $\nabla \cdot \boldsymbol{E}_{L,a}(\boldsymbol{x}) = 0$

$\boxed{\text{Introduces the QCD scale from a boundary condition}}$

Translation and rotation symmetry impose tight constraints

Works only for color singlet states

The confining potential in the Hamiltonian

$$E^i_{L,a}(\boldsymbol{x})\,|phys\rangle = -\partial^x_i \int d\boldsymbol{y}\left[\kappa\,\boldsymbol{x}\cdot\boldsymbol{y} + \frac{g}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\right]\mathcal{E}_a(\boldsymbol{y})\,|phys\rangle$$

where $\mathcal{E}_a(\boldsymbol{y}) = -f_{abc}A^i_b E^i_c(\boldsymbol{y}) + \psi^\dagger T^a\psi(\boldsymbol{y})$

$$\mathcal{H}_V \equiv \tfrac{1}{2}\int d\boldsymbol{x}\sum_a \boldsymbol{E}^a_L\cdot\boldsymbol{E}^a_L$$

$$= \int d\boldsymbol{y}d\boldsymbol{z}\Big\{\boldsymbol{y}\cdot\boldsymbol{z}\Big[\tfrac{1}{2}\kappa^2\int d\boldsymbol{x} + g\kappa\Big] + \tfrac{1}{2}\frac{\alpha_s}{|\boldsymbol{y}-\boldsymbol{z}|}\Big\}\mathcal{E}_a(\boldsymbol{y})\mathcal{E}_a(\boldsymbol{z})$$

The field energy $\propto$ volume of space is irrelevant only if it is universal.

This relates the normalisation $\varkappa$ of all Fock components,

leaving a universal scale $\Lambda = O(\alpha_s{}^0)$ as the single parameter.

Meson q$\bar{\text{q}}$ Fock state potential

$$|q(\boldsymbol{x}_1)\bar{q}(\boldsymbol{x}_2)\rangle \equiv \sum_A \bar{\psi}^A(\boldsymbol{x}_1)\,\psi^A(\boldsymbol{x}_2)\,|0\rangle$$ globally color singlet

$$\mathcal{H}_V \equiv \tfrac{1}{2}\int d\boldsymbol{x}\sum_a \boldsymbol{E}_L^a\cdot\boldsymbol{E}_L^a$$ $$\mathcal{H}_V\,|0\rangle = 0$$

$$\mathcal{H}_V\,|q\bar{q}\rangle = V_{q\bar{q}}\,|q\bar{q}\rangle$$

$$V_{q\bar{q}}(\boldsymbol{x}_1,\boldsymbol{x}_2) = \Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2| - C_F\frac{\alpha_s}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}$$ Cornell potential

This potential is valid also for relativistic $q\bar{q}$ Fock states, in any frame

The universal vacuum energy density is $E_\Lambda = \dfrac{\Lambda^4}{2g^2C_F}$

Baryon Fock state potential

Baryon: $$|q(\boldsymbol{x}_1)q(\boldsymbol{x}_2)q(\boldsymbol{x}_3)\rangle \equiv \sum_{A,B,C} \epsilon_{ABC}\psi_A^\dagger(\boldsymbol{x}_1)\,\psi_B^\dagger(\boldsymbol{x}_2)\,\psi_C^\dagger(\boldsymbol{x}_3)\,|0\rangle$$

$$V_{qqq}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3) = \Lambda^2 d_{qqq}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3) - \frac{2}{3}\,\alpha_s\Big(\frac{1}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}+\frac{1}{|\boldsymbol{x}_2-\boldsymbol{x}_3|}+\frac{1}{|\boldsymbol{x}_3-\boldsymbol{x}_1|}\Big)$$

$$d_{qqq}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3) \equiv \frac{1}{\sqrt{2}}\sqrt{(\boldsymbol{x}_1-\boldsymbol{x}_2)^2+(\boldsymbol{x}_2-\boldsymbol{x}_3)^2+(\boldsymbol{x}_3-\boldsymbol{x}_1)^2}$$

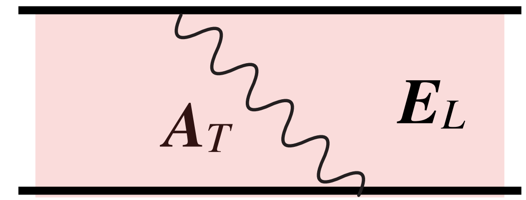
When two of the quarks coincide the potential reduces to the $q\bar{q}$ potential:

$$V_{qqq}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_2) = \Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2| - \frac{4}{3}\frac{\alpha_s}{|\boldsymbol{x}_1-\boldsymbol{x}_2|} = V_{q\bar{q}}(\boldsymbol{x}_1,\boldsymbol{x}_2)$$

Analogous potentials are obtained for any globally color singlet quark and gluon Fock state, such as $q\bar{q}g$ and gg.

The qg$\bar{\text{q}}$ potential

A q$\bar{\text{q}}$ state, with the emission of a transverse gluon:



$$|q(\boldsymbol{x}_1)g(\boldsymbol{x}_g)\bar{q}(\boldsymbol{x}_2)\rangle \equiv \sum_{A,B,b} \bar{\psi}_A(\boldsymbol{x}_1)\, A_b^j(\boldsymbol{x}_g) T_{AB}^b \psi_B(\boldsymbol{x}_2)\, |0\rangle$$

$$V_{qgq}^{(0)}(\boldsymbol{x}_1, \boldsymbol{x}_g, \boldsymbol{x}_2) = \frac{\Lambda^2}{\sqrt{C_F}}\, d_{qgq}(\boldsymbol{x}_1, \boldsymbol{x}_g, \boldsymbol{x}_2)$$

(universal Λ)

$$d_{qgq}(\boldsymbol{x}_1, \boldsymbol{x}_g, \boldsymbol{x}_2) \equiv \sqrt{\tfrac{1}{4}(N - 2/N)(\boldsymbol{x}_1 - \boldsymbol{x}_2)^2 + N(\boldsymbol{x}_g - \tfrac{1}{2}\boldsymbol{x}_1 - \tfrac{1}{2}\boldsymbol{x}_2)^2}$$

$$V_{qgq}^{(1)}(\boldsymbol{x}_1, \boldsymbol{x}_g, \boldsymbol{x}_2) = \tfrac{1}{2}\,\alpha_s \Big[\frac{1}{N}\, \frac{1}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|} - N\Big(\frac{1}{|\boldsymbol{x}_1 - \boldsymbol{x}_g|} + \frac{1}{|\boldsymbol{x}_2 - \boldsymbol{x}_g|}\Big)\Big]$$

When q and g coincide:

$$V_{qgq}^{(0)}(\boldsymbol{x}_1 = \boldsymbol{x}_g, \boldsymbol{x}_2) = \Lambda^2 |\boldsymbol{x}_1 - \boldsymbol{x}_2| = V_{q\bar{q}}^{(0)}$$

$$V_{qgq}^{(1)}(\boldsymbol{x}_1 = \boldsymbol{x}_g, \boldsymbol{x}_2) = V_{q\bar{q}}^{(1)}$$

The gg potential

A "glueball" component: $$|g(\boldsymbol{x}_1)g(\boldsymbol{x}_2)\rangle \equiv \sum_a A_a^i(\boldsymbol{x}_1)\, A_a^j(\boldsymbol{x}_2)\,|0\rangle$$

has the potential $$V_{gg} = \sqrt{\frac{N}{C_F}}\,\Lambda^2\,|\boldsymbol{x}_1-\boldsymbol{x}_2| - N\,\frac{\alpha_s}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}$$

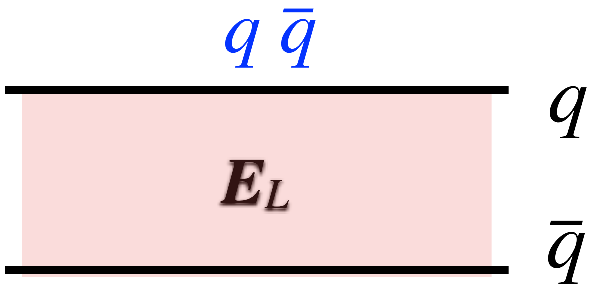
This agrees with the qgq̄ potential where the quarks coincide:

$$V_{gg}(\boldsymbol{x},\boldsymbol{x}_g) = V_{qg\bar{q}}(\boldsymbol{x},\boldsymbol{x}_g,\boldsymbol{x})$$

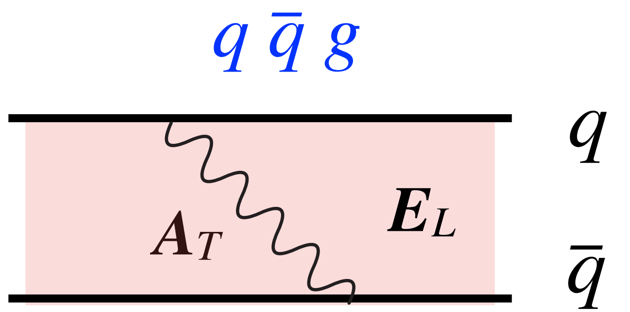
It is straightforward to work out the instantaneous potential for any Fock state.

Bound Fock expansion for mesons in $A^0=0$ gauge

The perturbative expansion in α_s starts from the $|q\bar{q}\rangle$ Fock state, bound by the $O(\alpha_s^0)$ instantaneous potential $V_{q\bar{q}}$:



$O(\alpha_s)$ corrections include states with transverse gluons and quark pairs, determined perturbatively by $H_{QCD}\,|q\bar{q}\rangle$



Each Fock component of the bound state includes its $O(\alpha_s^0)$ instantaneous potential.

This Fock expansion is valid in any frame, and is formally exact at $O(\alpha_s^\infty)$.

$\mathcal{O}\left(\alpha_s^0\right)$ $q\bar{q}$ bound states

An $\mathcal{O}\left(\alpha_s^0\right)$ meson state with $\boldsymbol{P}=0$ and wave function Φ:

$$|M\rangle = \sum_{A,B;\alpha,\beta} \int d\boldsymbol{x}_1 d\boldsymbol{x}_2\, \bar{\psi}_\alpha^A(t=0,\boldsymbol{x}_1)\delta^{AB}\Phi_{\alpha\beta}(\boldsymbol{x}_1-\boldsymbol{x}_2)\psi_\beta^B(t=0,\boldsymbol{x}_2)\,|0\rangle$$

The (rest frame) bound state condition $H\,|M\rangle = M\,|M\rangle$ gives

$$\left[i\gamma^0\boldsymbol{\gamma}\cdot\overrightarrow{\boldsymbol{\nabla}} + m\gamma^0\right]\Phi(\boldsymbol{x}) + \Phi(\boldsymbol{x})\left[i\gamma^0\boldsymbol{\gamma}\cdot\overleftarrow{\boldsymbol{\nabla}} - m\gamma^0\right] = \left[M - V(|\boldsymbol{x}|)\right]\Phi(\boldsymbol{x})$$

where $\boldsymbol{x} \equiv \boldsymbol{x}_1 - \boldsymbol{x}_2$ and $V(\boldsymbol{x}) = \Lambda^2|\boldsymbol{x}|$ at $\mathcal{O}\left(\alpha_s^0\right)$

In the non-relativistic limit ($m \gg \Lambda$) this reduces to the Schrödinger equation.

$\Longrightarrow$ The quarkonium phenomenology with the Cornell potential.

Example: $-\eta_P = \eta_C = (-1)^j$ states at O(α_s^0)

$$\Phi_{-+}(\boldsymbol{x}) = \Big[\frac{2}{M-V}(i\boldsymbol{\alpha}\cdot\overrightarrow{\boldsymbol{\nabla}} + m\gamma^0) + 1\Big]\gamma_5\, F_1(r)Y_{j\lambda}(\hat{\boldsymbol{x}})$$

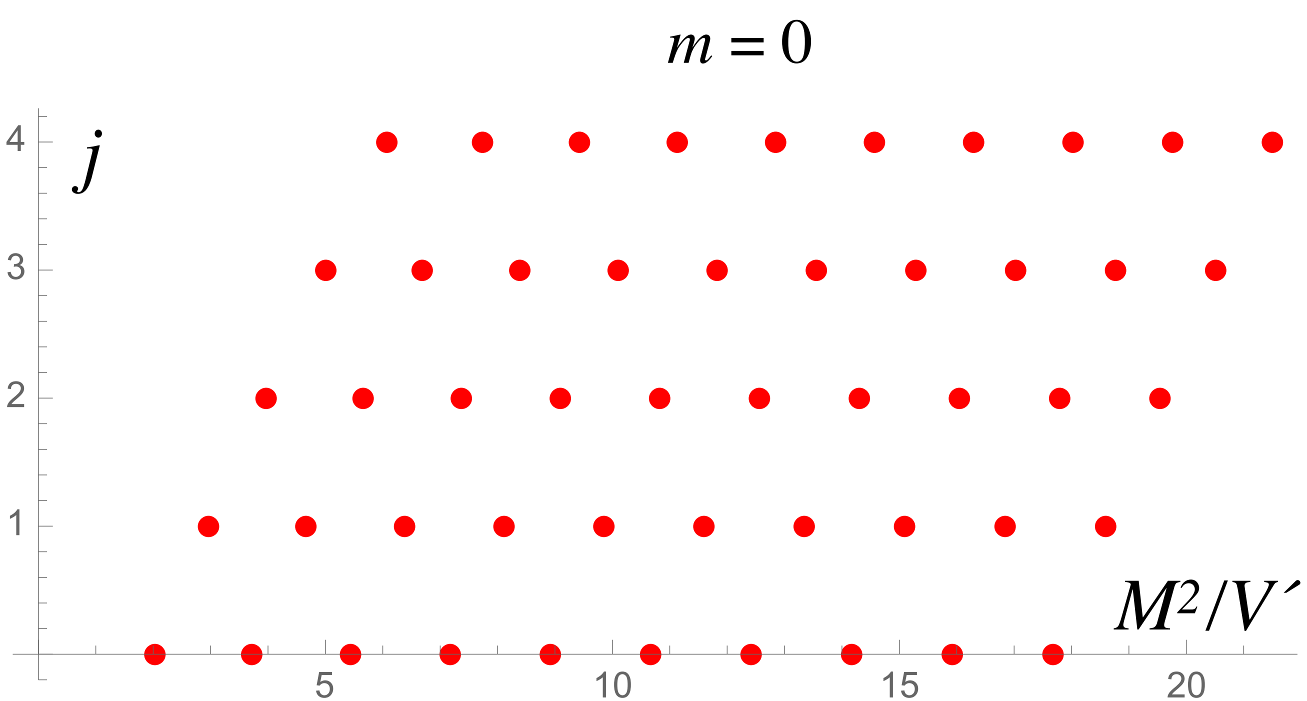
Radial equation: $F_1'' + \Big(\frac{2}{r} + \frac{V'}{M-V}\Big)F_1' + \Big[\tfrac{1}{4}(M-V)^2 - m^2 - \frac{j(j+1)}{r^2}\Big]F_1 = 0$

Regularity of the wave function determines the bound state masses M

Mass spectrum:

Linear Regge trajectories with daughters

Spectrum similar to dual models



QFT dynamics at $\mathcal{O}\left(\alpha_s^0\right)$

In a perturbative expansion each order in α_s, including $O(\alpha_s^0)$, must have all features required by field theory.

Boost covariance of bound state with CM momentum $\boldsymbol{P}$:

$$|M,P\rangle = \frac{1}{\sqrt{N_c}}\sum_{A,B}\int d\boldsymbol{x}_1 d\boldsymbol{x}_2\, \bar{\psi}^A(\boldsymbol{x}_1)e^{i\boldsymbol{P}\cdot(\boldsymbol{x}_1+\boldsymbol{x}_2)/2}\delta^{AB}\Phi^{(\boldsymbol{P})}(\boldsymbol{x}_1-\boldsymbol{x}_2)\psi^B(\boldsymbol{x}_2)\,|0\rangle$$

$$i\boldsymbol{\nabla}\cdot\left\{\boldsymbol{\alpha},\Phi^{(\boldsymbol{P})}(\boldsymbol{x})\right\} - \tfrac{1}{2}\boldsymbol{P}\cdot\left[\boldsymbol{\alpha},\Phi^{(\boldsymbol{P})}(\boldsymbol{x})\right] + m\left[\gamma^0,\Phi^{(\boldsymbol{P})}(\boldsymbol{x})\right] = \left[E-V(\boldsymbol{x})\right]\Phi^{(\boldsymbol{P})}(\boldsymbol{x})$$

Implies $E=\sqrt{M^2+\boldsymbol{P}^2}$

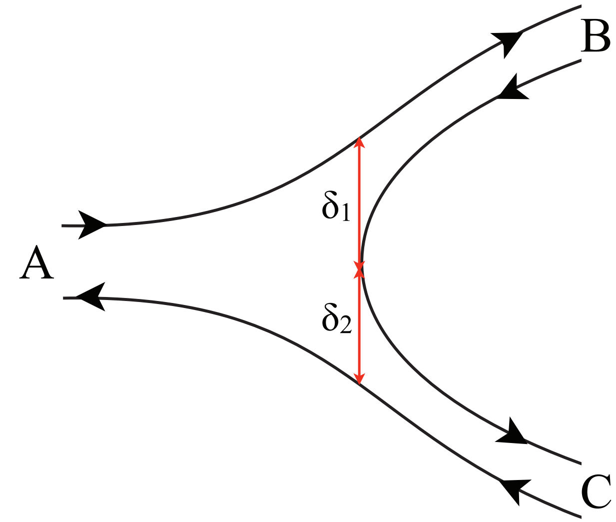
Relates $\Phi^{(P)}(\tau)$ at a common value of $\tau \equiv (E-V'r)^2 - \boldsymbol{P}^2$

EM form factor: Gauge invariant

Poincaré invariant (checked in D = 1+1)

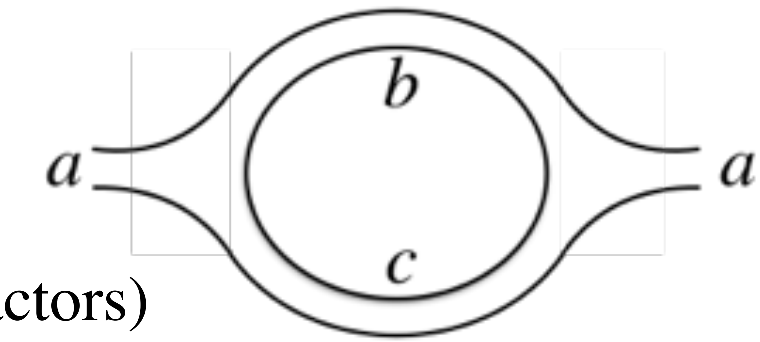
QFT dynamics at $\mathcal{O}\left(\alpha_s^0\right)$

String breaking: Quark pairs created in $V(r)$:



Bound state overlap determined by their wf's

Hadron loops: Required by unitarity



Dihadron component of hadron wf (cf. form factors)

A brave new $\alpha_s{}^0$ world given to us by QFT

It should have all required features

Summary

The QCD scale Λ_{QCD} can be introduced only via a boundary condition

In temporal gauge ($A^0 = 0$) the constituents instantaneously determine $\nabla\cdot\boldsymbol{E}_L$

Including a homogeneous solution for $\boldsymbol{E}_L$ gives confinement in QCD

Expanding around free quarks and gluons may never give confinement.

A Bound Fock expansion: Formally exact when summed to all orders in α_s

$O(\alpha_s^0)$ "Born term" provides a non-trivial, consistent(?) hadron dynamics

A formulation of bound states at the same level as the perturbative S-matrix.

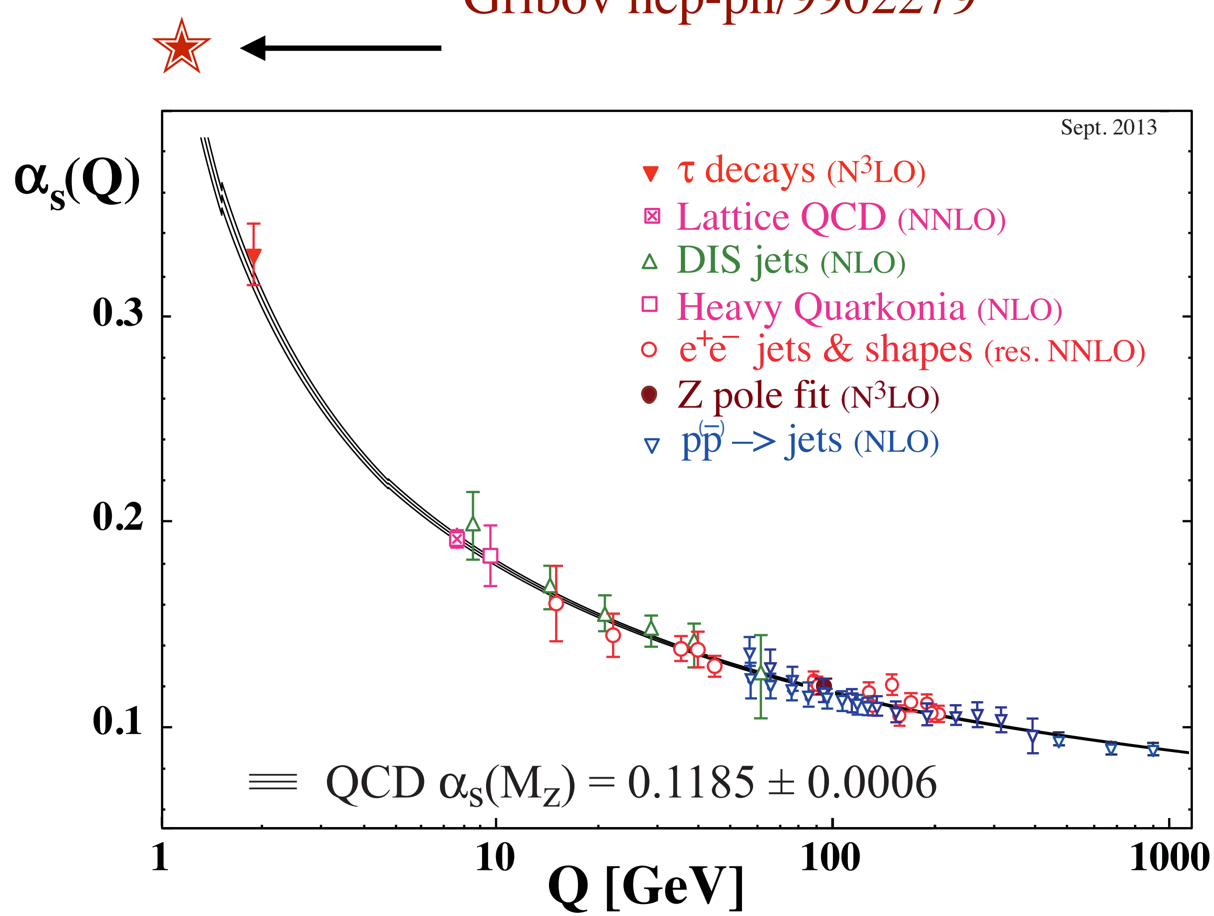
More details in:
PH 2101.06721

Back-up slides

The perturbative α_s

$\alpha_s(0) < 1$ if the strong coupling stops running around Q ~ 1 GeV

$\alpha_s^{crit} \approx 0.43$ Gribov hep-ph/9902279



QED atoms (are not) in QFT textbooks

Bound states are not discussed in today's textbooks. The last exception:

C. Itzykson and J.-B. Zuber: Quantum Field Theory (1980)

10-3 HYPERFINE SPLITTING IN POSITRONIUM

It should not be concluded that relativistic weak binding corrections cannot be obtained for two-body systems that agree with experiment. On the contrary, the positronium states give an example of a successful agreement. This will serve to illustrate the theory. To be completely fair, we should admit that accurate predictions require some artistic gifts from the practitioner. As yet no systematic method has been devised to obtain the corrections in a completely satisfactory way.

I & Z do not discuss the Schrödinger equation and its relation to the QED action.

The situation has not improved qualitatively.

The art of atoms

Review paper: Rev. Mod. Phys. **57** (1985) 723

Recoil effects in the hyperfine structure of QED bound states

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"In spite of the statement in the preceding paragraph that bound-state theory is nonperturbative, it is possible to make use of small parameters such as α and m_e/m_A (where m_A is the mass of the nucleus) to develop expressions in increasing orders of smallness. However, the nonperturbative nature of the expansion shows up in non-analytic dependence on these parameters (such as logarithms). As indicated in the preceding paragraph, there is an art in developing a theoretical expression in this manner."