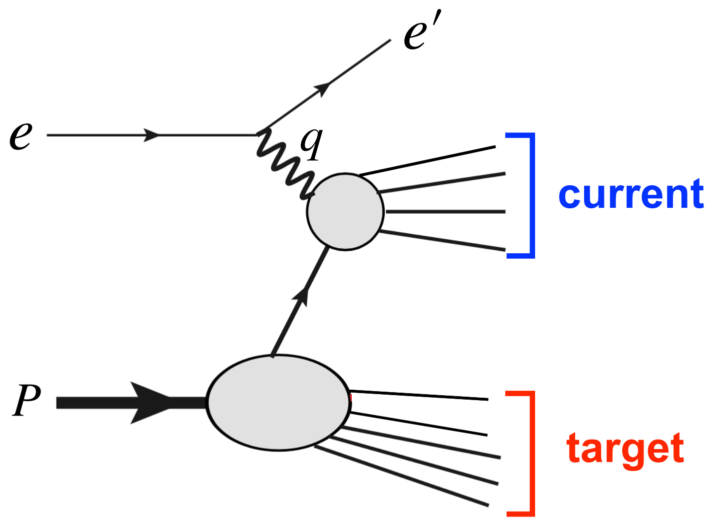
Twist-3 effects in SIDIS in Target Fragmentation Region

Xuan-Bo Tong The Chinese University of HongKong(ShenZhen)

Sep. 27th



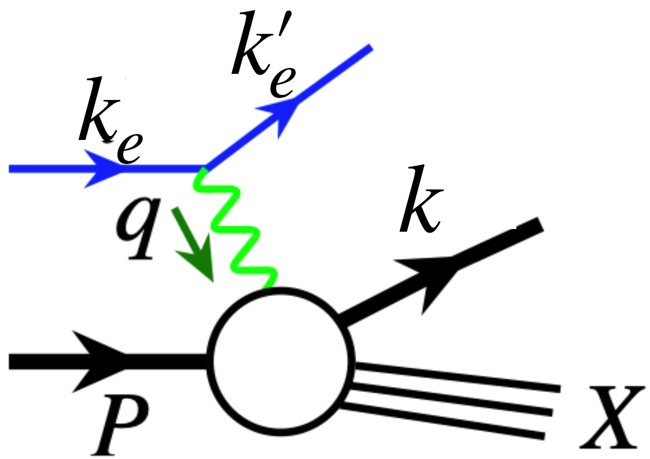
- **Introduction**
  - Factorizations and Kinematic regions of SIDIS
- **Twist-3 factorization of SIDIS in TFR**
- **Factorization of twist-2 fracture functions**

Based on JHEP 11 (2021) 038 and work in preparation with Kai-Bao Chen, Jian-Ping Ma

Introduction

Semi-Inclusive Deep Inelastic Scattering

$$e(k_e, \lambda_e) + h(P, s) \to e(k_e') + h'(k) + X$$



$$Q^2 = -q^2 \gg M^2$$

$$x_B = \frac{Q^2}{2p \cdot q}$$

- Various probes to the partonic structure of the hadrons:

  TMD distributions, nucleon spin partition, target fragmentation..

- Experiments: HEMERS, COMPASS, JLab, U.S. EIC, EICc…

- Cornerstone: QCD Factorizations by approximations in specific kinematics
- Different kinematics & Approximation accuracy $\Rightarrow$ Different factorizations and probes.

Factorizations and Kinematic regions of SIDIS

- Deep inelastic region allows a power expansion in (M/Q)

$$Q \gg M \quad \Rightarrow \quad \sigma(Q) = \sigma_2 + \left(\frac{M}{Q}\right)\sigma_3 + \left(\frac{M}{Q}\right)^2 \sigma_4 + \cdots$$

- Factorization theorem differs at different powers.
- Higher-power effects:
  (1) Improve the predictions
  (2) Generate unique physical effects

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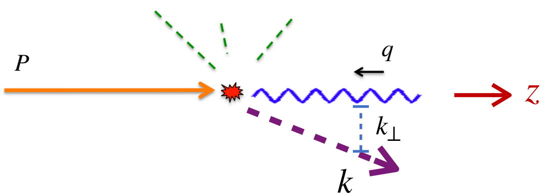
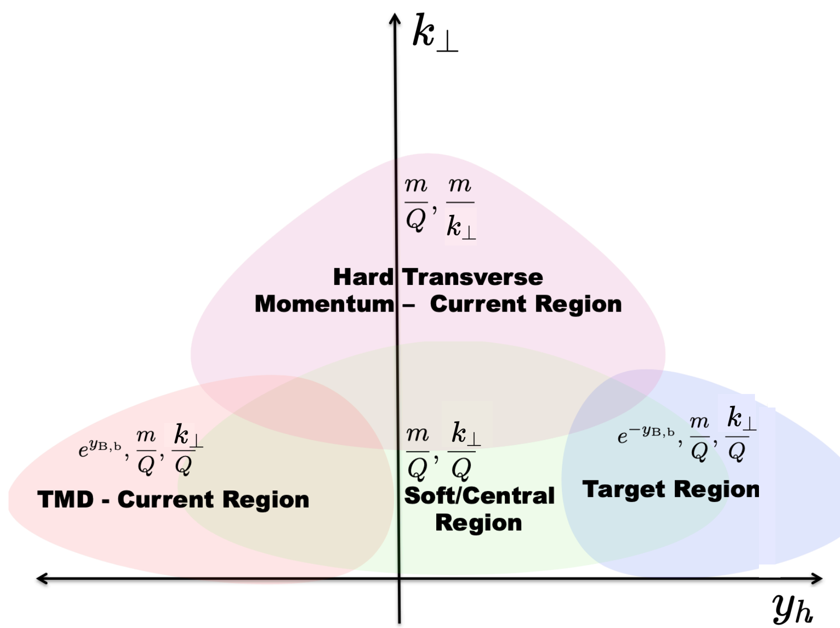
Photon-proton frame:



y_h : Rapidity

$k_\perp$: Transverse momentum



[Figure from M. Boglione *et al JHEP* 10 (2019) 122]

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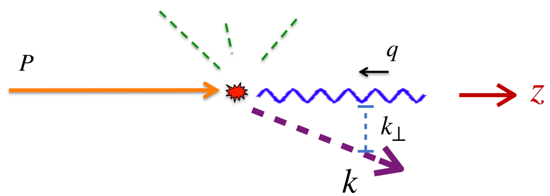

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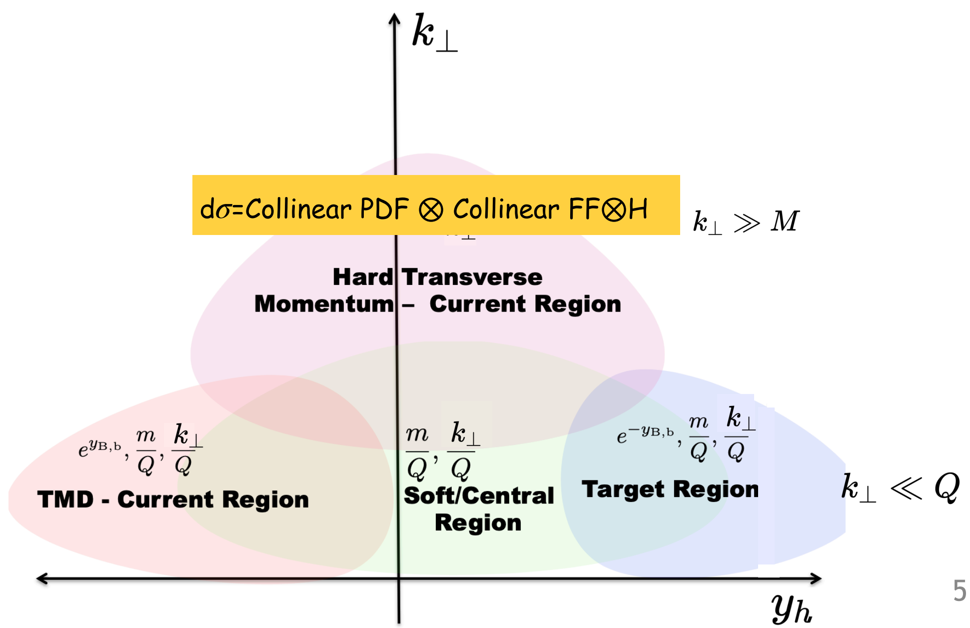

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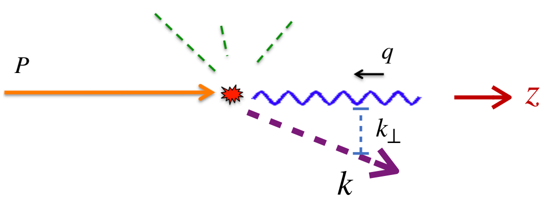

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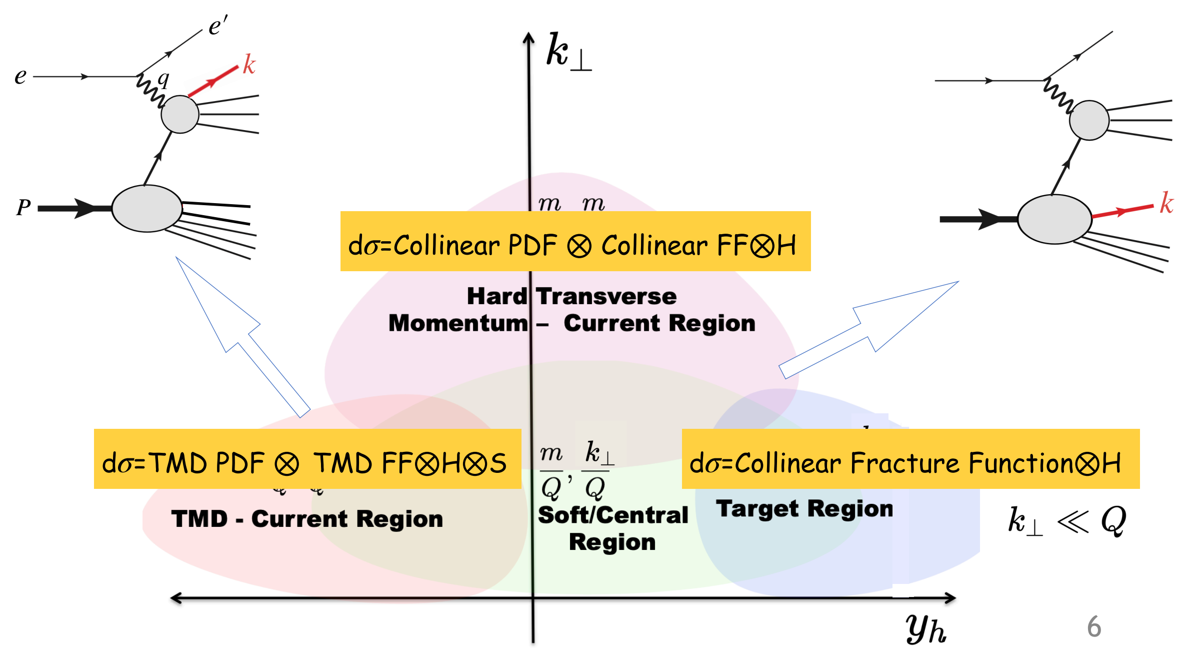

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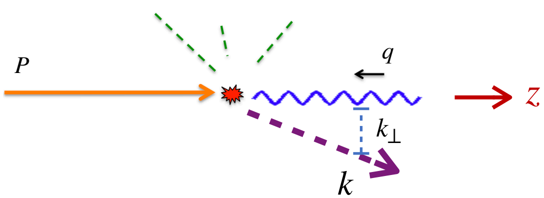

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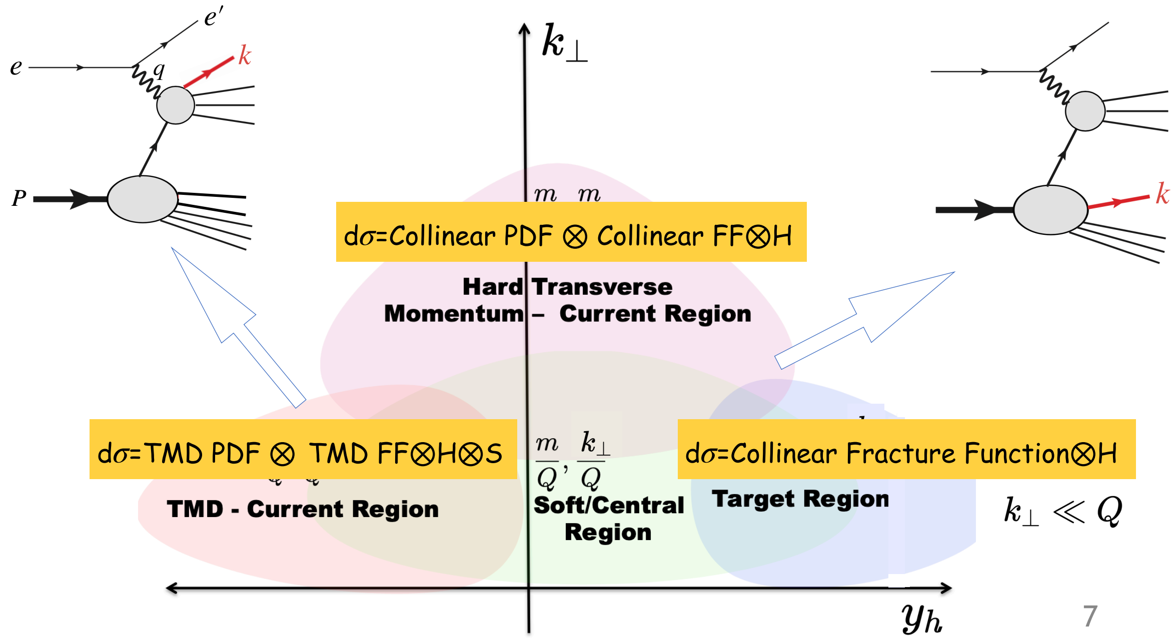

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- The TFR of SIDIS are relatively little studied (though see Hayward's talk, Kotzinian's talk, and references there)

e.g. Trentadue, Veneziano, Phys. Lett. B323 (1994) 201. Graudenz, Nucl. Phys. B432 (1994) 351
Berera, Soper, Phys.Rev.D 53 (1996) 6162-6179. Collins, Phys. Rev. D57 (1998) 3051
Grazzini, Trentadue, Veneziano, Nucl. Phys. B519 (1998) 394
Anselmino-Barone-Kotzinian, Phys. Lett. B 699 (2011) 108

⇒ Leading power, O($1/Q^0$)

Two questions on SIDIS in TFR

- **Available TFR studies focus on the leading power of M/Q.**
  What about the sub-leading power?

$$\sigma(Q) = \sigma_2 + \left(\frac{M}{Q}\right)\sigma_3 + \left(\frac{M}{Q}\right)^2 \sigma_4 + \cdots$$

Twist-2	$F_{UU,T} = x u_1$ $F_{LL} = x l_{1L}$ $F_{UT,T}^{\sin(\phi-\phi_S)} = x u_{1T}^{\perp} \lvert k_\perp \rvert / M$ $F_{LT}^{\cos(\phi-\phi_S)} = x l_{1T}^{\perp} \lvert k_\perp \rvert / M$

Only 4 of 18 structure functions at twist-2 at tree level

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Initial hadron: spin-1/2;
Final hadron: unpolarized

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Twist-3	$F_{UU}^{\cos\phi_h},\ F_{LU}^{\sin\phi_h},\ F_{LL}^{\cos\phi_h},\ F_{UL}^{\sin\phi_h}$ $F_{LT}^{\cos(2\phi_h-\phi_s)},\ F_{UT}^{\sin(2\phi_h-\phi_s)},\ F_{UT}^{\sin\phi_s},\ F_{LT}^{\sin\phi_s}$
Twist-4	$F_{UU,L},\ F_{UU}^{\cos 2\phi},\ \cdots$

Initial hadron: spin-1/2;
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Only 4 of 18 structure functions at twist-2 at tree level

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8 structure functions at twist-3 at tree level;

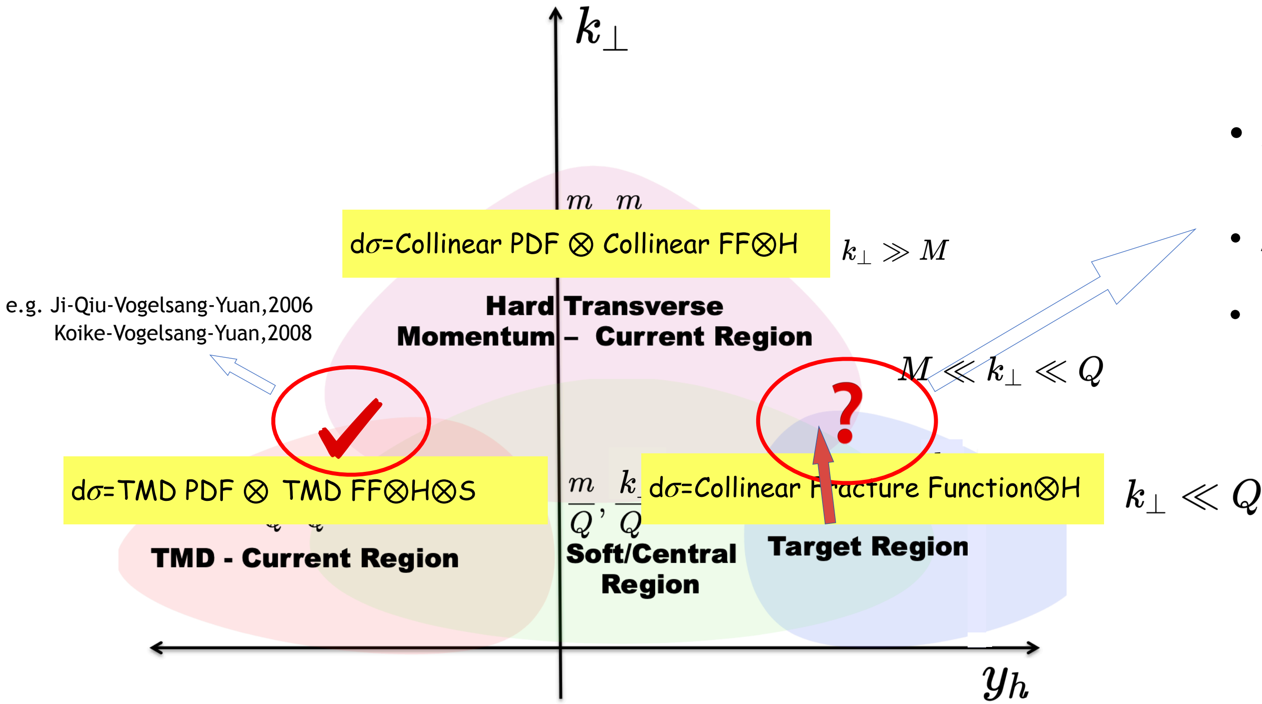
[Work in preparation with K.B Chen, J.P. Ma]

- **unpolarized target**; **longitudinal polarized target**; **transversely polarized target**;
- **4 of them(red, blue)are already accessible with data collected at CLAS12.**
  [see Hayward's talk]

Factorizations with twist-3 quark collinear fracture functions at tree level.

Two questions on SIDIS in TFR

- **How dose SIDIS in TFR behave in $M \ll k_\perp \ll Q$ region?**



[Figure from M. Boglione *et al JHEP* 10 (2019) 122]

- **An overlapping region that the two factorizations can both apply.**
- Are there unified descriptions?
- First step: from the TFR side

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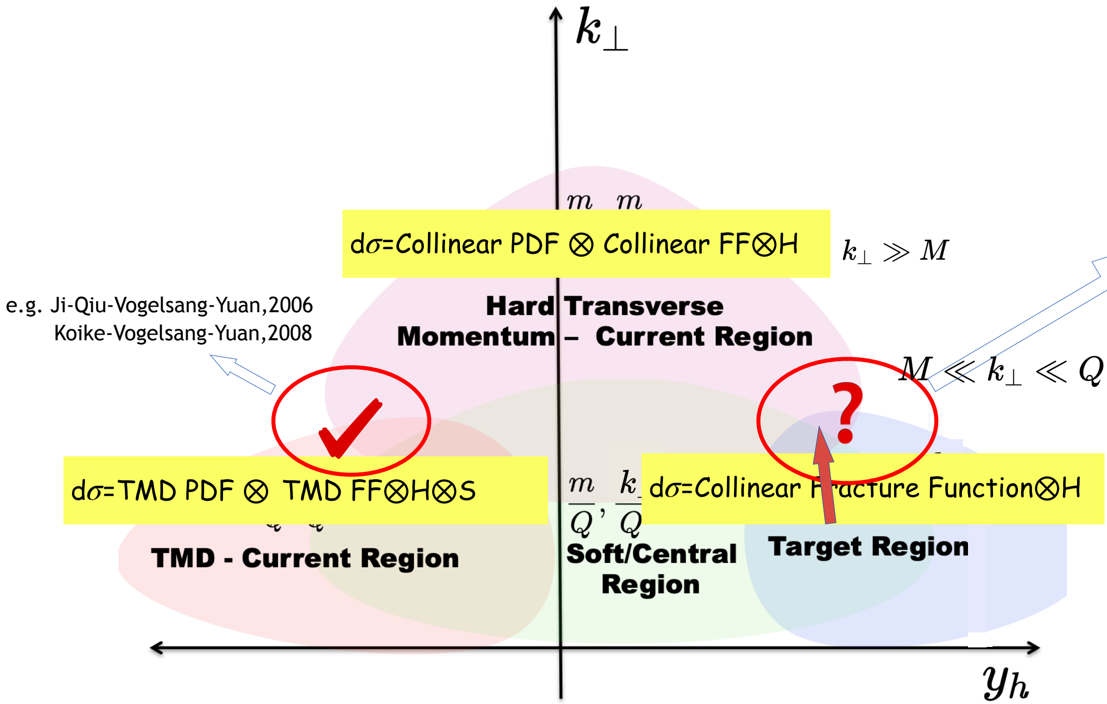

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- **$k_\perp$-dependence of FrF: perturbatively calculable with expansion in $(M/k_\perp)$**

$$\mathcal{F}(k_\perp) = \left(\frac{M^2}{\boldsymbol{k}_\perp^2}\right)\mathcal{F}_2 + \left(\frac{M^2}{\boldsymbol{k}_\perp^2}\right)^2 \mathcal{F}_3 + \cdots$$

Target spin: U/L T

Power behavior depends on the target polarization

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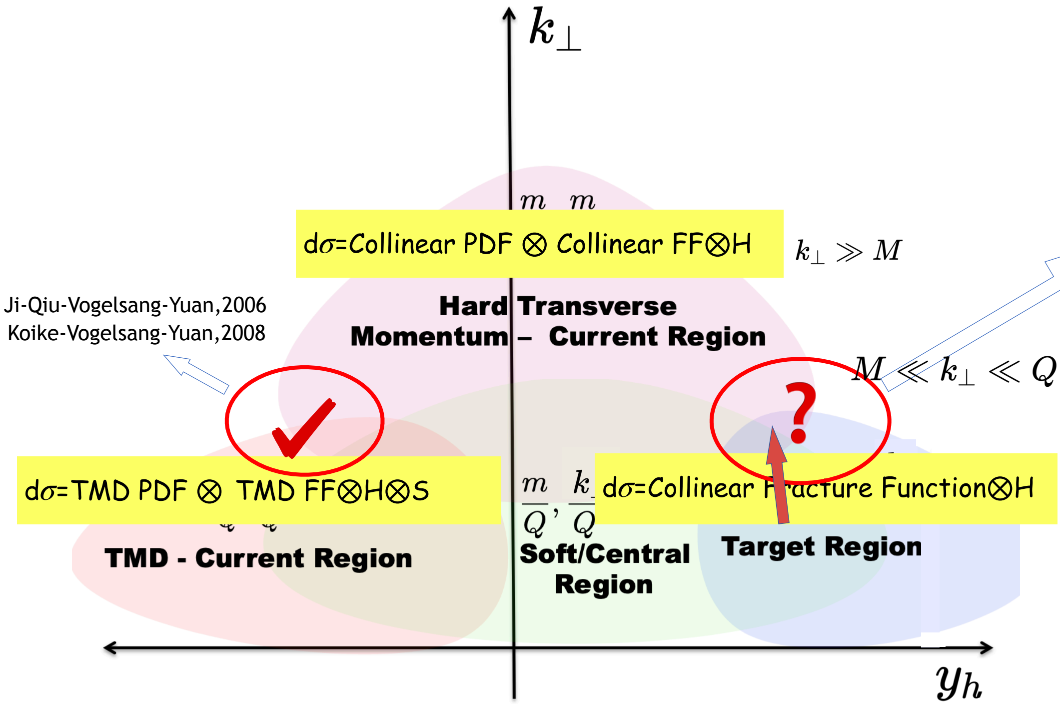

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- **Re-factorization at large transverse momentum:**

Collinear FrF≈Collinear (multi)parton distribution ⊗ Collinear FF⊗H

[K.B Chen, J.P. Ma, X.B. Tong JHEP 11 (2021) 038]

Target transverse spin effects are presented at Sud-leading Power with Twist-3 PDF .

What partonic structures can we learn?

- Twist-3 Factorization of SIDIS in TFR

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Multi-parton correlations (quantum interference)

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$$D_\perp = \partial_\perp + i g_s A$$

e.g. parton "transverse motion", quark-gluon correlation in presence of a final hadron

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Technique

Collinear expansion $\Longrightarrow$ Higher-power(twist) effects

—-widely applied in the CFR SIDIS or other hard processes

e.g. Ellis-Furmanski-.Petronzio, Nucl.Phys.B21229(1983);
Nucl.Phys.B207.1(1982).
Qiu, Phys.Rev.D42, 30(1990).
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Color gauge invariance.

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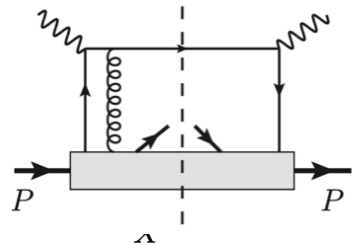

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$\rangle\langle X, h|\Gamma D_\perp \psi|\, P, s\rangle$

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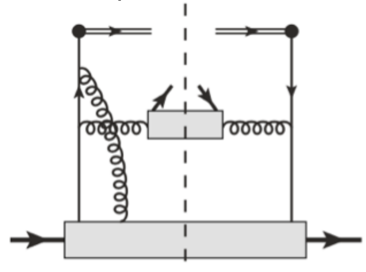

$^{+\perp}\Gamma\psi|\, P, s_\perp\rangle$

$F^{+\perp}F^{+\perp}|\, P, s_\perp\rangle$

e.g. quark-gluon correlation, three-gluon correlation…

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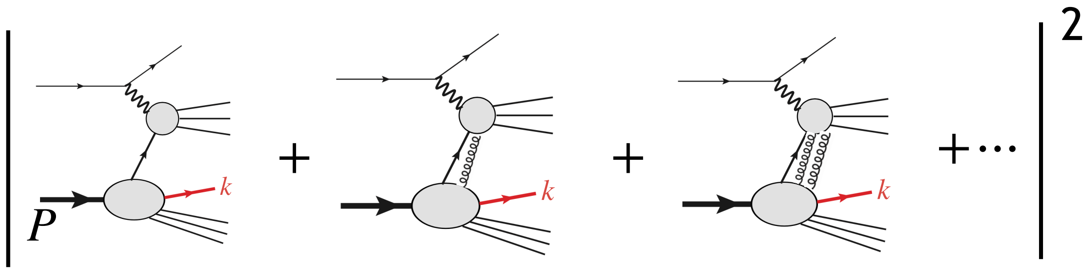
Twist-3 Factorization of SIDIS in TFR

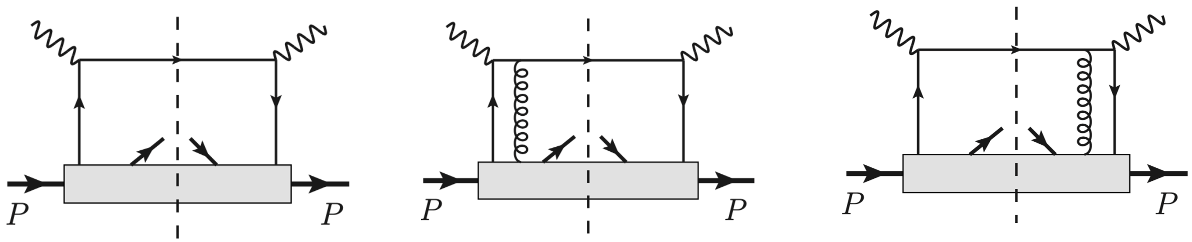
- Differential cross-section

$d\sigma \propto$



$$\frac{d\sigma}{dx_B dy d\xi d\psi d^2 k_\perp} = \frac{\alpha_{\rm em}^2 y}{2Q^4} L_{\mu\nu} W^{\mu\nu} \qquad \xi = \frac{k^+}{P^+}$$

- Hadron tensor(tree level) in TFR



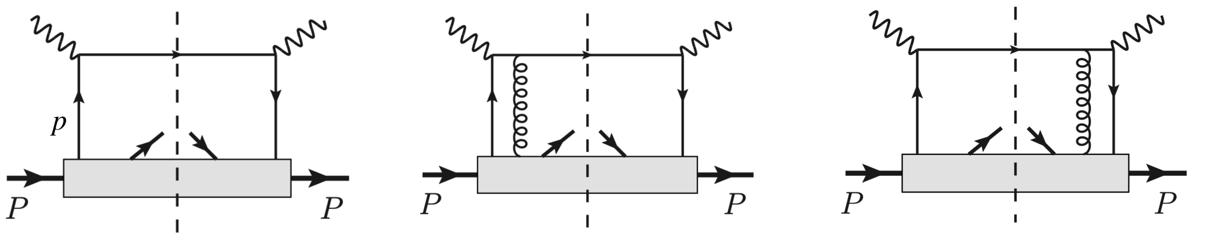
The partonic scatterings are in line with those in the inclusive DIS   e.g. Z.T. Liang and X.N. Wang (2007)

More solid conclusion was given on twist-2 factorizations to all-order-α_s, see Collins, Phys. Rev. D57 (1998) 305

The difference is the correlation matrix, which have a detected hadron!

Correlations

- Hadron tensor(tree level) in TFR



- Clear roles of the gluon field at next-leading power of Q:

$$A^\mu \sim (\underset{+}{1},\ \underset{-}{\lambda^2},\ \underset{\perp}{\lambda})Q \qquad \lambda = \frac{M}{Q}$$

(1) A^+: gauge link only

(2) $A^\perp$: "physical" degree , $\sum_X \langle P\,|\bar{\psi}|h,X\rangle\langle X,h|\Gamma A^\perp\psi|\,P\rangle \sim \lambda$ —— not gauge invariant

- Quarks are not exactly parallel to the incoming hadron beam, $p^\mu \sim (1,\ \lambda^2,\ \lambda)Q$

$$\langle p_\perp\rangle \sim \sum_X \langle P\,|\bar{\psi}|h,X\rangle\langle X,h|\Gamma\partial_\perp\psi|\,P\rangle \sim \lambda$$

Combined with $A^\perp$(Ward identity),

$$\sum_X \langle P,s|\bar{\psi}|h,X\rangle\langle X,h|\Gamma D_\perp\psi|P,s\rangle$$

- Other sub-leading contribution from Γ matrix,

$$\sum_X \langle P,s|\bar{\psi}|h,X\rangle\langle X,h|\Gamma\psi|P,s\rangle \qquad \Gamma = \gamma_\perp^\mu, \gamma_\perp^\mu\gamma_5$$

Correlations

- After the collinear expansion and organizations,

$$W_{\mu\nu} = W_{\mu\nu}^{(0)} + W_{\mu\nu}^{(1)}$$

$$W_{\mu\nu}^{(0)} = \mathrm{Tr}\left[h_{\mu\nu}^{(0)}\mathcal{M}\right] \ ,$$

$$h_{\mu\nu}^{(0)} = \gamma_\mu\gamma^+\gamma_\nu/2 \ ,$$

$$\underline{\mathcal{M}_{ij}}(x,\xi,k_\perp) = \frac{1}{2\xi(2\pi)^3}\int\frac{d\xi^-}{2\pi}e^{-ixP^+\xi^-}\sum_X\langle P,s|\bar\psi_j(\xi)\mathcal{L}_n^\dagger(\xi)|k;X\rangle\langle k;X|\mathcal{L}_n(0)\psi_i(0)|P,s\rangle\Big|_{\xi^+,\xi_\perp=0}$$

$$W_{\mu\nu}^{(1)} = \mathrm{Tr}\left[h_{\mu\nu,\alpha}^{(1)}\varphi_{ij}^\alpha\right] + \left(\mathrm{Tr}\left[h_{\nu\mu,\alpha}^{(1)}\varphi_{ij}^\alpha\right]\right)^*$$

$$h_{\mu\nu,\alpha}^{(1)} = \frac{1}{4q^-}\gamma_\mu\gamma^+\gamma_{\perp\alpha}\gamma^-\gamma_\nu$$

$$\underline{\varphi_{ij}^\alpha}(x,\xi,k_\perp) = \frac{1}{2\xi(2\pi)^3}\int\frac{d\xi^-}{2\pi}e^{-ixP^+\xi^-}\sum_X\langle P,S|\bar\psi_j(\xi)\mathcal{L}_n^\dagger(\xi)|k;X\rangle\langle k;X|\mathcal{L}_n(0)(\underline{-i\overrightarrow{D}_\perp^\alpha})(0)\psi_i(0)|P,S\rangle\Big|_{\xi^+,\xi_\perp=0}$$

- Two types of collinear correlations (Twist-3, chiral even)

$$\boxed{\begin{aligned}\mathcal{M}^{[\gamma^\mu]} &= \frac{1}{P^+}\left[k_\perp^\mu u^h - s_L\tilde{k}_\perp^\mu u_L^h - \tilde{s}_\perp^\mu M u_T - \frac{k_\perp^{\langle\mu}k_\perp^{\nu\rangle}}{M}\tilde{s}_{\perp\nu}u_T^h\right] ,\\ \mathcal{M}^{[\gamma^\mu\gamma_5]} &= \frac{1}{P^+}\left[\tilde{k}_{\perp\mu}l^h + s_L k_\perp^\mu l_L^h + s_\perp^\mu M l_T - \frac{k_\perp^{\langle\mu}k_\perp^{\nu\rangle}}{M}s_{\perp\nu}l_T^h\right] ,\end{aligned}}$$

$$\boxed{\begin{aligned}\varphi_\mu^{[\gamma^+]} &= k_{\perp\mu}u_d^h - s_L\tilde{k}_{\perp\mu}u_{dL}^h - M\tilde{s}_{\perp\mu}u_{dT} - \frac{k_{\perp\langle\mu}k_{\perp\beta\rangle}}{M}\tilde{s}_\perp^\beta u_{dT}^h \ ,\\ \varphi_\mu^{[\gamma^+\gamma_5]} &= i\left(\tilde{k}_{\perp\mu}l_d^h + s_L k_{\perp\mu}l_{dL}^h + M s_{\perp\mu}l_{dT} - \frac{k_{\perp\langle\mu}k_{\perp\beta\rangle}}{M}s_\perp^\beta l_{dT}^h\right)\end{aligned}}$$

Parametrizations are similar to TMD PDFs $(f \to u,\ g \to l,\ \perp \to h)$

$$\tilde{a}^\mu \equiv \epsilon_\perp^{\mu\nu}a_\nu$$

$$k_{\perp\langle\mu}k_{\perp\beta\rangle} \equiv k_{\perp\mu}k_{\perp\nu} - \tfrac{1}{2}k_\perp^2 g_{\mu\nu}^\perp$$

e.g. Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, (2007)

Equation of motion

- However, these distributions are not independent, one can use equation of motion $i\not{D}\psi = 0$ to relate

See the TMD case in e.g. S.Y. Wei et al Phys. Rev. D 95, 074017 (2017)

$$\mathcal{M}^{[\gamma^\mu]} = \frac{1}{P^+}\left[k_\perp^\mu u^h - s_L \tilde{k}_\perp^\mu u_L^h - \tilde{s}_\perp^\mu M u_T - \frac{k_\perp^{\langle\mu} k_\perp^{\nu\rangle}}{M}\tilde{s}_{\perp\nu} u_T^h\right],$$

$$\mathcal{M}^{[\gamma^\mu\gamma_5]} = \frac{1}{P^+}\left[\tilde{k}_{\perp\mu} l^h + s_L k_\perp^\mu l_L^h + s_\perp^\mu M l_T - \frac{k_\perp^{\langle\mu} k_\perp^{\nu\rangle}}{M} s_{\perp\nu} l_T^h\right],$$

$$\varphi_\mu^{[\gamma^+]} = k_{\perp\mu} u_d^h - s_L \tilde{k}_{\perp\mu} u_{dL}^h - M\tilde{s}_{\perp\mu} u_{dT} - \frac{k_{\perp\langle\mu} k_{\perp\beta\rangle}}{M}\tilde{s}_\perp^\beta u_{dT}^h,$$

$$\varphi_\mu^{[\gamma^+\gamma_5]} = i\left(\tilde{k}_{\perp\mu} l_d^h + s_L k_{\perp\mu} l_{dL}^h + M s_{\perp\mu} l_{dT} - \frac{k_{\perp\langle\mu} k_{\perp\beta\rangle}}{M} s_\perp^\beta l_{dT}^h\right)$$

- Unified formulas:

$$\boxed{x u_S^K} = \boxed{\mathrm{Re}\left(l_{dS}^K - u_{dS}^K\right)}$$

$$\boxed{x l_S^K} = \boxed{\mathrm{Im}\left(l_{dS}^K - u_{dS}^K\right)}$$

$S = \text{null},\ L \text{ or } T$

$K = \text{null or } \perp$

- The structure functions can be expressed in terms of the distributions with or without the covariant derivative —> Neat forms

Final results

- Eight structure functions contribute at twist-3(M/Q) in TFR, which are all missing at twist-2. [Work in preparation, K.B Chen, J.P. Ma, X.B. Tong]

$$F_{UU}^{\cos(\phi_h)} = -2\sum_a e_a^2\, x_B^2 \frac{|k_\perp|}{Q} u(x_B,\xi,k_\perp),$$

$$F_{LU}^{\sin(\phi_h)} = 2\sum_a e_a^2\, x_B^2 \frac{|k_\perp|}{Q} l(x_B,\xi,k_\perp)$$

$$F_{LL}^{\cos(\phi_h)} = -2\sum_a e_a^2\, x_B^2 \frac{|k_\perp|}{Q} l_L(x_B,\xi,k_\perp)$$

$$F_{UL}^{\sin(\phi_h)} = -2\sum_a e_a^2\, x_B^2 \frac{|k_\perp|}{Q} u_L(x_B,\xi,k_\perp)$$

$$F_{LT}^{\cos(2\phi_h-\phi_s)} = -\sum_a e_a^2\, x_B^2 \frac{|k_\perp^2|}{QM} l_T^h(x_B,\xi,k_\perp)$$

$$F_{UT}^{\sin(2\phi_h-\phi_s)} = -\sum_a e_a^2\, x_B^2 \frac{|k_\perp^2|}{QM} u_T^h(x_B,\xi,k_\perp)$$

$$F_{UT}^{\sin(\phi_s)} = -2\sum_a e_a^2\, x_B^2 \frac{M}{Q} u_T^h(x_B,\xi,k_\perp)$$

$$F_{LT}^{\sin(\phi_s)} = -2\sum_a e_a^2\, x_B^2 \frac{M}{Q} l_T^h(x_B,\xi,k_\perp)$$

- Probe eight twist-3 collinear quark fracture functions at tree level
  - Two with unpolarized target; Two with longitudinally polarized target; Four with transversely polarized target;
  - Four of them (red, blue) are already accessible with data collected at CLAS12. [see Hayward's talk]
  - Have no simple probability interpretation, but are fundamental ingredients in QCD
  - Give different types of azimuthal modulations /asymmetries

Twist-3 Factorization of SIDIS in TFR

- Compared with the single inclusive jet production in CFR :

e.g. S.Y. Wei et al Phys. Rev. D 95, 074017 (2017)

$$F_{UU}^{\cos(\phi_h)} = -2\sum_a e_a^2\, x_B^2 \frac{|k_\perp|}{Q} u(x_B,\xi,k_\perp),$$

$$F_{LU}^{\sin(\phi_h)} = 2\sum_a e_a^2\, x_B^2 \frac{|k_\perp|}{Q} l(x_B,\xi,k_\perp)$$

$$F_{LL}^{\cos(\phi_h)} = -2\sum_a e_a^2\, x_B^2 \frac{|k_\perp|}{Q} l_L(x_B,\xi,k_\perp)$$

$$F_{UL}^{\sin(\phi_h)} = -2\sum_a e_a^2\, x_B^2 \frac{|k_\perp|}{Q} \underline{u_L(x_B,\xi,k_\perp)}$$

Collinear fracture function

$$F_{UU}^{\cos(\phi_h)} = -2\sum_a e_a^2\, x_B^2 \frac{|k_\perp|}{Q} f(x_B,k_\perp),$$

$$F_{LU}^{\sin(\phi_h)} = 2\sum_a e_a^2\, x_B^2 \frac{|k_\perp|}{Q} g(x_B,k_\perp)$$

$$F_{LL}^{\cos(\phi_h)} = -2\sum_a e_a^2\, x_B^2 \frac{|k_\perp|}{Q} g_L(x_B,k_\perp)$$

$$F_{UL}^{\sin(\phi_h)} = -2\sum_a e_a^2\, x_B^2 \frac{|k_\perp|}{Q} \underline{f_L(x_B,k_\perp)}$$

TMD PDF

This corresponding is just due to

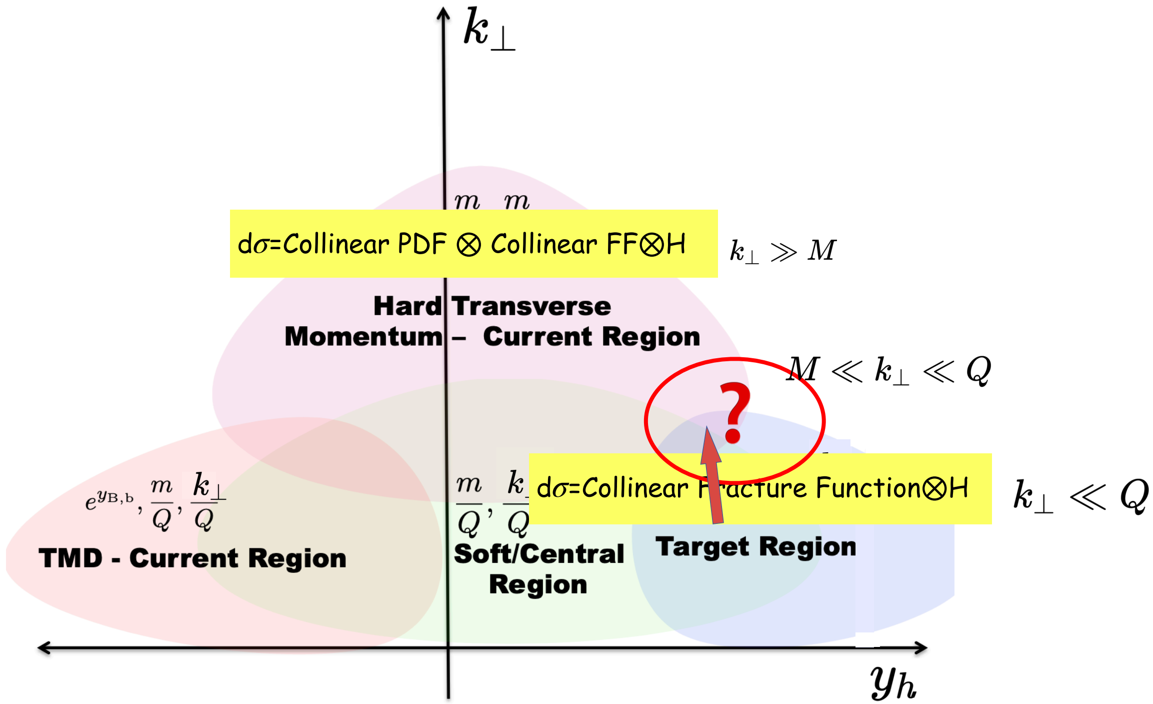
- "Similar" partonic-scattering forms at tree level
- "Similar" patermetrization forms between collinear FrFs and TMD PDFs

Have no physical implication, should fail beyond the tree level!

However, it is interesting to compare the twist-4 cases in the future.

Factorizations of the twist-2 collinear FrFs

- **How dose SIDIS in TFR behaves in $M \ll k_\perp \ll Q$ region?**



[Figure from M. Boglione *et al JHEP* 10 (2019) 122]

Try to understand this overlapping region from the collinear FrFs formalism.

Factorizations of the twist-2 collinear FrFs

- Focus on twist-2 quark collinear FrFs

$F_{UU,T} = xu_1$ Unplorized

$F_{LL} = xl_{1L}$ Double spin asymmetry (L, L)

$F_{UT,T}^{\sin(\phi-\phi_S)} = xu_{1T}^{\perp}|k_\perp|/M$ Single transverse spin asymmetry(SSA):

$F_{LT}^{\cos(\phi-\phi_S)} = xl_{1T}^{\perp}|k_\perp|/M$ Double spin asymmetry(L,T)

Factorizations of the twist-2 collinear FrFs

- Focus on twist-2 quark collinear FrFs

$$F_{UU,T} = xu_1 \quad \text{Unplorized}$$

$$F_{LL} = xl_{1L} \quad \text{Double longitudinal spin asymmetry}$$

$$F_{UT,T}^{\sin(\phi-\phi_S)} = xu_{1T}^{\perp}|k_\perp|/M \quad \text{Single transverse spin asymmetry(SSA):}$$

$$F_{LT}^{\cos(\phi-\phi_S)} = xl_{1T}^{\perp}|k_\perp|/M \quad \text{Double spin asymmetry(L,T)}$$

- Unlike the twist-3 case, the twist-2 factorization of SIDIS in TFR has been proved to all orders of α_s J.C. Collins Phys. Rev. D 57 (1998) 3051

dσ= Twist-2 Collinear FrF(x,ξ,$k_\perp$,μ) $\otimes$ H(Q,μ)+$O(M/Q)$+$O(k_\perp/Q)$

Factorizations of the twist-2 collinear FrFs

- Focus on twist-2 quark collinear FrFs

$$F_{UU,T} = x u_1 \quad \text{Unplorized}$$

$$F_{LL} = x l_{1L} \quad \text{Double longitudinal spin asymmetry}$$

$$F_{UT,T}^{\sin(\phi-\phi_S)} = x u_{1T}^{\perp} |k_\perp|/M \quad \text{Single transverse spin asymmetry(SSA):}$$

$$F_{LT}^{\cos(\phi-\phi_S)} = x l_{1T}^{\perp} |k_\perp|/M \quad \text{Double spin asymmetry(L,T)}$$

- Unlike the twist-3 case, the twist-2 factorization of SIDIS in TFR has been proved to all orders of α_s J.C. Collins Phys. Rev. D 57 (1998) 3051

dσ= Twist-2 Collinear FrF($x,\xi,k_\perp,\mu$) $\otimes$ H(Q,μ)+$O(M/Q)$+$O(k_\perp/Q)$

- Requires $M, k_\perp \ll Q$, but $k_\perp$ do not have to be order of M

- The intermediate region $M \ll k_\perp \ll Q$ implies a further factorization of FrFs with a power expansion in $M/k_\perp$.

Collinear FrF≈Collinear (multi)parton distribution $\otimes$ Collinear FF$\otimes$H

Factorizations of the twist-2 collinear FrFs

$$\mathcal{M}_{ij}(x,\xi,k_\perp) = \frac{1}{2N_c}\left(\gamma^-\right)_{ij}\left[u_1(x,\xi,k_\perp) + \epsilon_\perp^{\mu\nu} k_{\perp\mu} s_{\perp\nu} \frac{1}{M} u_{1T}^h(x,\xi,k_\perp)\right] + \frac{1}{2N_c}\left(\gamma_5\gamma^-\right)_{ij}\left[s_L l_1(x,\xi,k_\perp) + k_\perp \cdot s_\perp \frac{1}{M} l_{1T}^h(x,\xi,k_\perp)\right] + \cdots$$

- u_1, l_1 : Unpolarized or longitudinal spin

  $\frac{1}{k_\perp^2}$ $\otimes$ twist-2 FF $\otimes$ twist-2 parton distributions

- $u_{1T}^h,\ l_{1T}^h$: Transverse spin

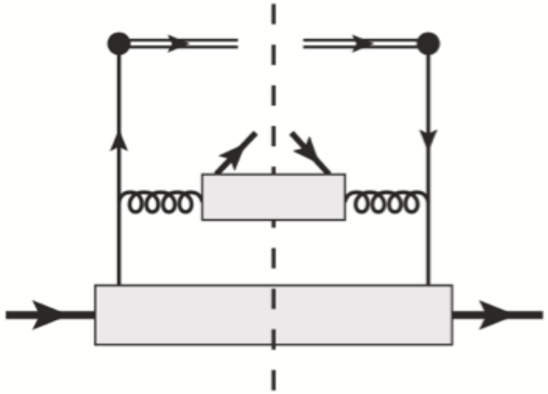
  $\frac{1}{(k_\perp^2)^2}$ $\otimes$ twist-2 FF $\otimes$ twist-3 multi-parton distributions

Helicity conservation -> No twist-3 FF $\otimes$ twist-2 distributions

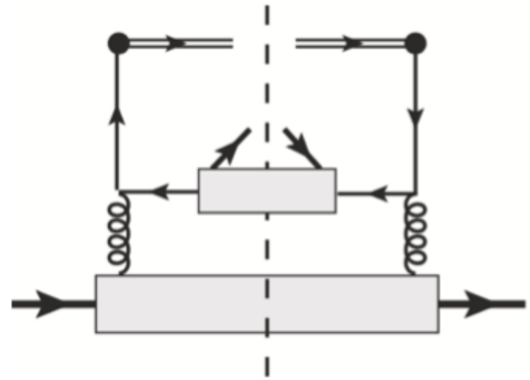
Factorizations of the twist-2 collinear FrFs

	$k_\perp \sim M$	$M \ll k_\perp \ll Q$
$F_{UU,T} = x u_1$ $F_{LL} = x l_{1L}$	Twist-2 effect	Twist-2 effect
$F_{UT,T}^{\sin(\phi-\phi_S)} = x u_{1T}^{\perp} \lvert k_\perp \rvert / M$ $F_{LT}^{\cos(\phi-\phi_S)} = x l_{1T}^{\perp} \lvert k_\perp \rvert / M$	Twist-2 effect	Twist-3 effect

Factorizations: Unpolarized/Longitudinal target



$\langle \psi(0)\psi(x) \rangle$



$\langle A_a^\mu(x) A_b^\nu(0) \rangle$

$$u_1(x,\xi,k_\perp) = g_s^2 \frac{1}{k_\perp^2} \int \frac{dz}{z^2} \left[2C_F d_g(z) q(y) \frac{z^2}{y^2}(x^2+y^2) + d_{\bar{q}}(z) g(y) \frac{\xi}{zy^3}(z^2x^2+\xi^2) \right]$$

$$l_1(x,\xi,k_\perp) = g_s^2 \frac{1}{k_\perp^2} \int \frac{dz}{z^2} \left[2C_F d_g(z) \Delta q(y) \frac{z^2}{y^2}(x^2+y^2) + d_{\bar{q}}(z) \Delta g(y) \frac{\xi}{y^2}(xz-\xi) \right]$$

$$y = x + \frac{\xi}{z}$$

Factorizations: transversely polarized target

$$\mathcal{M}_{ij}(x,\xi,k_\perp) = \frac{1}{2N_c}\left(\gamma^-\right)_{ij}\left[u_1(x,\xi,k_\perp) + \epsilon_\perp^{\mu\nu}k_{\perp\mu}s_{\perp\nu}\frac{1}{M}u_{1T}^h(x,\xi,k_\perp)\right]$$
$$+\frac{1}{2N_c}\left(\gamma_5\gamma^-\right)_{ij}\left[s_L l_1(x,\xi,k_\perp) + k_\perp\cdot s_\perp\frac{1}{M}l_{1T}^h(x,\xi,k_\perp)\right] + \cdots$$

- u_1, l_1 : Unpolarized or longitudinal spin

  $\frac{1}{k_\perp^2}$ ⊗ twist-2 FF ⊗ twist-2 parton distributions

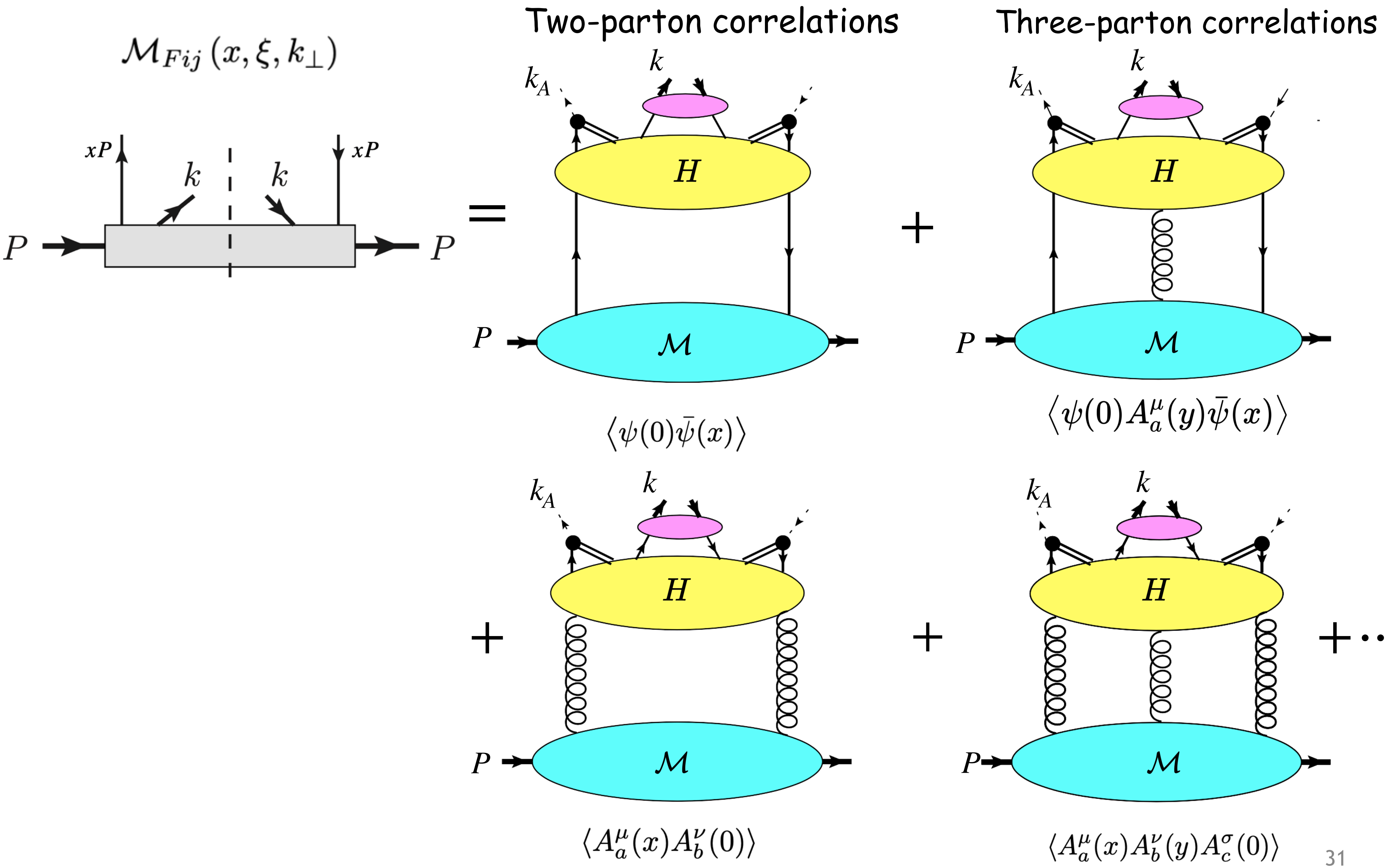
- $u_{1T}^h,\ l_{1T}^h$: Transverse spin

  $\frac{1}{(k_\perp^2)^2}$ ⊗ twist-2 FF ⊗ twist-3 multi-parton distributions

Twist-3 matching is none-trivial even at tree level !

It is not easy to obtain the gauge invariant results : multi-parton correlations

Factorizations: transversely polarized target



Factorizations: transversely polarized target

Set of Twist-3 distributions

A pair of quark fields: e.g., Chen-Ma- Zhang, Phys. Lett. B 754 (2016)

$$q_T(x)s_\perp^\mu = P^+\int\frac{d\lambda}{4\pi}e^{-ix\lambda P^+}\left\langle h_A\middle|\bar\psi(\lambda n)\mathcal{L}_n^\dagger(\lambda n)\gamma_\perp^\mu\gamma_5\mathcal{L}_n(0)\psi(0)\middle|h_A\right\rangle$$

$$-iq_\partial(x)s_\perp^\mu = \int\frac{d\lambda}{4\pi}e^{-ix\lambda P^+}\left\langle h_A\middle|\bar\psi(\lambda n)\mathcal{L}_n^\dagger(\lambda n)\gamma^+\gamma_5\partial_\perp^\mu(\mathcal{L}_n\psi)(0)\middle|h_A\right\rangle$$

A pair of quark fields with one gluon field strength tenor(Chirality even):

Qiu-Sterman, Phys. Rev. D 59 (1999) 014004

$$T_F(x_1,x_2)\,\epsilon_\perp^{\mu\nu}s_{\perp\nu} = g_s\int\frac{d\lambda_1 d\lambda_2}{4\pi}e^{-i\lambda_2(x_2-x_1)p^+-i\lambda_1x_1p^+}\left\langle h_A\left|\bar\psi(\lambda_1n)\,\gamma^+G^{+\mu}(\lambda_2n)\,\psi(0)\right|h_A\right\rangle$$

$$T_\Delta(x_1,x_2)\,s_\perp^\mu = -ig_s\int\frac{d\lambda_1 d\lambda_2}{4\pi}e^{-i\lambda_2(x_2-x_1)p^+-i\lambda_1x_1p^+}\left\langle h_A\left|\bar\psi(\lambda_1n)\,\gamma_5\gamma^+G^{+\mu}(\lambda_2n)\,\psi(0)\right|h_A\right\rangle$$

Factorizations: transversely polarized target

Set of Twist-3 distributions

Set of twist-3 gluon distributions: Ji, Phys. Lett. B 289, 137 (1992).

$$\frac{i^3 g_s}{P^+}\int\frac{d\lambda_1}{2\pi}\frac{d\lambda_2}{2\pi}e^{i\lambda_1 x_1 P^+ + i\lambda_2(x_2-x_1)P^+}\langle h_A|G^{a,+\alpha}(\lambda_1 n)G^{c,+\gamma}(\lambda_2 n)G^{b,+\beta}(0)|h_A\rangle$$
$$=\frac{N_c}{(N_c^2-1)(N_c^2-4)}d^{abc}O^{\alpha\beta\gamma}(x_1,x_2)-\frac{i}{N_c(N_c^2-1)}f^{abc}N^{\alpha\beta\gamma}(x_1,x_2),$$

F-type

From the Bose-symmetry and covariance Beppu-Koike-Tanaka-Yoshida, Phys. Rev. D 82 (2010) 054005

$$O^{\alpha\beta\gamma}(x_1,x_2)=-2i\Big[O(x_1,x_2)g_\perp^{\alpha\beta}\tilde{s}_\perp^\gamma+O(x_2,x_2-x_1)g_\perp^{\beta\gamma}\tilde{s}_\perp^\alpha+O(x_1,x_1-x_2)g_\perp^{\gamma\alpha}\tilde{s}_\perp^\beta\Big],$$

$$N^{\alpha\beta\gamma}(x_1,x_2)=-2i\Big[N(x_1,x_2)g_\perp^{\alpha\beta}\tilde{s}_\perp^\gamma-N(x_2,x_2-x_1)g_\perp^{\beta\gamma}\tilde{s}_\perp^\alpha-N(x_1,x_1-x_2)g_\perp^{\gamma\alpha}\tilde{s}_\perp^\beta\Big],$$

All the other twist-3 gluon distributions can be determined by

$$N(x_1,x_2)\quad O(x_1,x_2)$$

Factorizations: transversely polarized target

$$F_{UT,T}^{\sin(\phi-\phi_S)} = x u_{1T}^{h}(x_B, \xi, k_\perp)|k_\perp|/M$$

- Single transverse-spin asymmetry (T-odd effect)

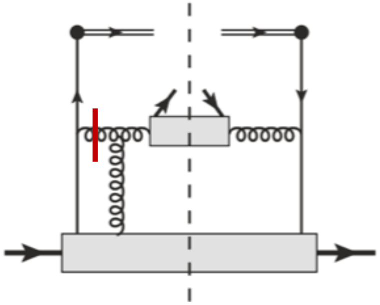
Time reversal invariance implies that none-zero SSA is induced by the interference of the amplitude of different phase

ETQS mechanism

Qiu-Sterman, Phys. Rev. D 59 (1999) 014004
Phys. Rev. Lett. 67 (1991) 2264
Efremov-Teryaev, Phys. Lett. B 150 (1985) 383

- Perturbative way to generate the phase in TFR:

$$\frac{1}{p^2 \pm i\epsilon} = P\frac{1}{p^2} \mp \underbrace{i\pi\delta(p^2)}_{\text{“pole”}}$$



Some propagators in FrFs' diagrams should go on shell

- Ward identity
- Constrain the partonic momenta: soft, hard

- Two-parton correlations do not contribute to SSA

SSA: Quark-gluon correlations

Hard pole: $$u_{1T}^h(x_B,\xi,k_\perp)/M\Big|_{HP} = g_s^2 \frac{N_c}{\left(k_\perp^2\right)^2} \int \frac{dz}{z^2} d_g(z) \frac{z^2}{y} \left(\xi T_\Delta(y,x) - (\xi + 2xz) T_F(y,x)\right)$$

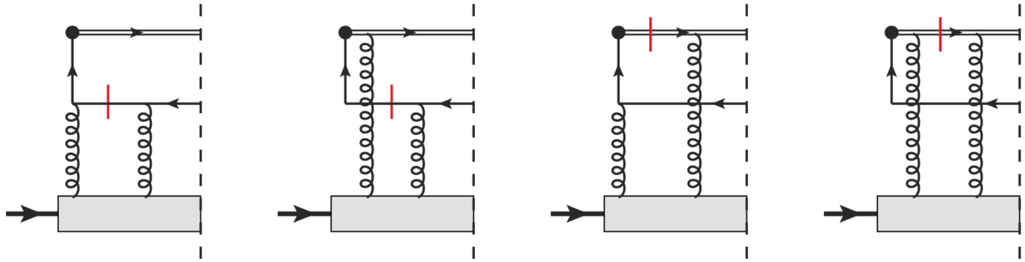
$$y = x + \xi/z.$$

Soft-fermion pole: $$u_{1T}^h(x_B,\xi,k_\perp)/M\Big|_{SFP} = g_s^2 \frac{1}{N_c} \frac{1}{\left(k_\perp^2\right)^2} \int \frac{dz}{z^2} d_g(z) \frac{x\xi z}{y^4} \Big[(xz - \xi) T_F(y,0) - (\xi + xz) T_\Delta(y,0)\Big]$$

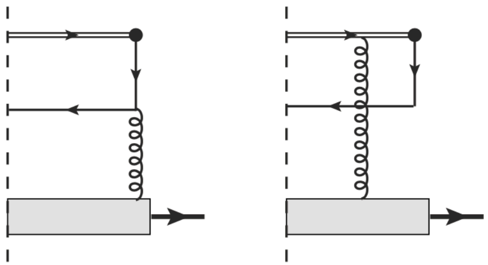
Soft gluon pole: $$u_{1T}^h(x_B,\xi,k_\perp)/M\Big|_{SGP} = \frac{g_s^2 N_c}{(k_\perp^2)^2} \int \frac{dz}{z^2} d_g(z) \frac{1}{y^3} \Big[z^3(y^3 + 3x^2 y - 2x^3) T_F(y,y) - y\xi z^2 (y^2 + x^2) \frac{\partial T_F(y,y)}{\partial y}\Big].$$

One integration on the twist-3 distribution

SSA: Three-gluon correlations



(a)



(b)

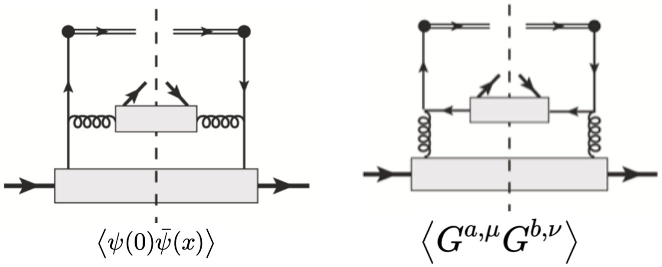
- Soft-gluon poles($k_1^+ = 0$):

$$
\begin{aligned}
& u_{1T}^h(x_B, \xi, k_\perp)/M|_{3G} \\
& \quad = \frac{4\pi g_s^4 \xi^4}{\left(k_\perp^2\right)^2} \int \frac{dz}{z^2} d_{\bar{q}}(z) \frac{1}{zy^5} \Big[2\big(\xi^2 + 2x^2z^2 - \xi x z\big)\big(N(y,y) - O(y,y)\big) \\
& \quad - 2\big(\xi^2 + 4x^2z^2 - 3\xi x z\big)\big(N(y,0) + O(y,0)\big) + y(\xi - xz)^2 \frac{d}{dy}\big(N(y,0) + O(y,0)\big) \\
& \quad - y\big(\xi^2 + x^2z^2\big) \frac{d}{dy}\big(N(y,y) - O(y,y)\big) \Big] \qquad y = x + \xi/z.
\end{aligned}
$$

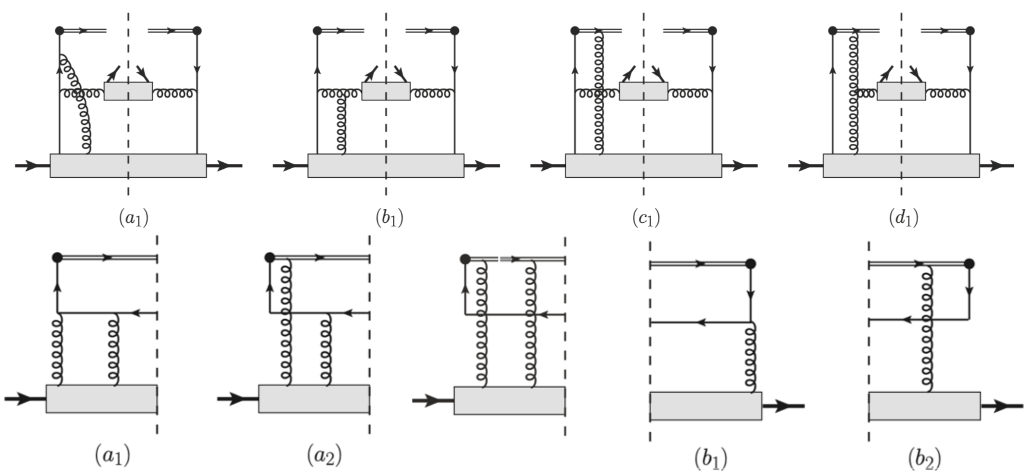
Factorizations: transversely polarized target

$$F_{LT}^{\cos(\phi-\phi_S)} = x l_{1T}^{\perp} |k_\perp|/M$$

- Double spin asymmetry (T-even effect)
  - Do not need a phase to get none-zero results
  - Two-parton correlations contribute



- A gauge invariant results are obtained with QCD equation of motions



Factorizations: transversely polarized target

DSA: Quark-gluon correlations

$$l_{1T}^h(x,\xi,k_\perp)/M\ \Big|_{q\bar{q}+qG\bar{q}} = \frac{1}{2(k_\perp^2)^2}\int\frac{dz}{z^2}d_g(z)\Big[\Big(H_{2p,T}(x,\xi)q_T(y)+H_{2p,\partial}(x,\xi)q_\partial(y)\Big)$$
$$+\frac{2}{\pi}\int dx_2\Big(T_F(y,x_2)H(x,\xi,x_2)+T_\Delta(y,x_2)H_A(x,\xi,x_2)\Big)\Big],$$

DSA: Pure Gluonic correlations

$$l_{1T}^h(x,\xi,k_\perp)/M\Big|_{2G+3G}$$
$$=\frac{-16\pi\alpha_s\xi^4}{(k_\perp^2)^2}\int\frac{dzdx_2}{z^2y^3}d_{\bar{q}}(z)\Big\{\frac{xz}{\pi y}T_F(x_2,x_2+y)-\frac{2xz}{y(y-x_2)}\Big[N(y,x_2)$$
$$-N(y-x_2,y)+2N(y-x_2,-x_2)\Big]+\frac{1}{x_2^2(y-x_2)}\Big(x_2\xi\Big[O(y-x_2,y)$$
$$-N(y-x_2,y)\Big]+(2\xi y+yx_2z-y^2z-\xi x_2)\Big[N(y,x_2)+O(y,x_2)\Big]$$
$$+y(z(y+x_2)-2\xi)\Big[N(y-x_2,-x_2)+O(y-x_2,-x_2)\Big]\Big)\Big\}.$$

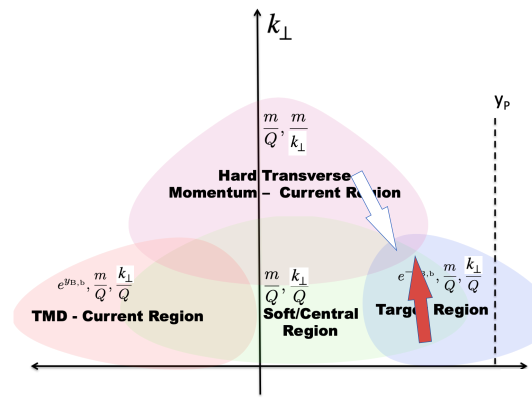
2 Integration on the twist-3 distribution, no derivative term

Summary&Outlook

- **Twist-3 factorization of SIDIS in TFR**
  - We made an attempt at tree level, found 8 structure functions factorized with 8 twist-3 quark collinear FrFs in a neat form.
  - Four of them are already accessible with data, models of these FrFs are needed.
  - Extension to twist-4 can also be done.

- **Factorization of twist-2 fracture functions**
  - Studied the $\Lambda_{QCD} \ll k_\perp \ll Q$ for SIDIS in the TFR at tree level.
  - One-loop extension.
  - To full understand the interplay between the target and current regions, further study on the current region at large forward rapidity is needed.
  - Numerical/Phenomenological studies;
  - May help the kinematic regime estimation in SIDIS.

e.g. M. Boglione et al, arXiv: 2201.12197;JHEP 10 (2019) 122;



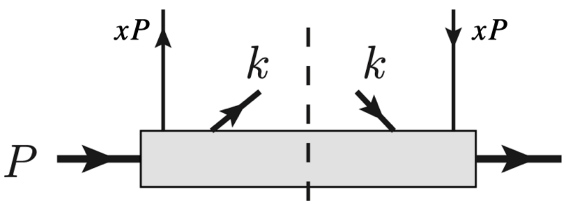
[Figure from M. Boglione *et al JHEP* 10 (2019) 122]

	$k_\perp \sim M$	$M \ll k_\perp \ll Q$
$F_{UU,T} = xu_1$ $F_{LL} = xl_{1L}$	Twist-2 effect	Twist-2 effect
$F_{UT,T}^{\sin(\phi-\phi_S)} = xu_{1T}^{\perp}\lvert k_\perp\rvert/M$ $F_{LT}^{\cos(\phi-\phi_S)} = xl_{1T}^{\perp}\lvert k_\perp\rvert/M$	Twist-2 effect	Twist-3 effect

Backup

Transversely polarized Fracture Function

$$\mathcal{M}_{Fij}\,(x,\xi,k_\perp)$$



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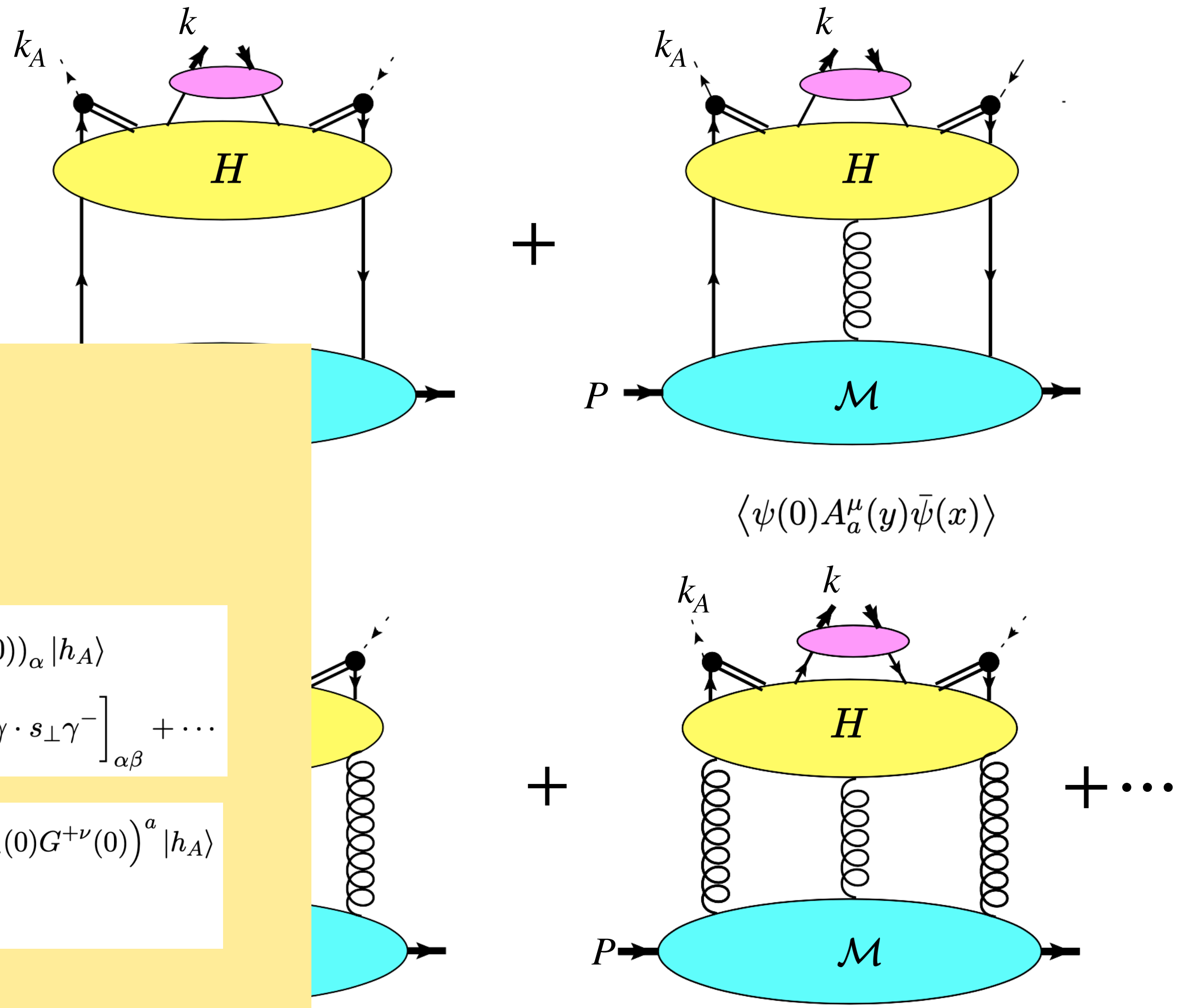
Two-parton correlations

Three-parton correlations



$$\langle \psi(0) A_a^\mu(y) \bar{\psi}(x) \rangle$$

$$\langle A_a^\mu(x) A_b^\nu(y) A_c^\sigma(0) \rangle$$

No twist-3 FF ⊗ twist-2 distributions

Chirality even

twist-2 quark/gluons distributions:

$$\int \frac{dy}{2\pi} e^{-iyxP^+} \langle h_A | \left(\bar{\psi}(yn) \mathcal{L}_n^\dagger(yn) \right)_\beta \left(\mathcal{L}_n(0)\psi(0) \right)_\alpha | h_A \rangle$$
$$= \frac{1}{2N_c} \left[q(x)\gamma^- + s_L \Delta q(x) \gamma_5 \gamma^- + h_1(x) \gamma_5 \gamma \cdot s_\perp \gamma^- \right]_{\alpha\beta} + \cdots$$

$$\frac{1}{xP^+} \int \frac{d\lambda}{2\pi} e^{-ix\lambda P^+} \langle h_A | \left(G^{+\mu}(\lambda n) \mathcal{L}_n^\dagger(\lambda n) \right)^a \left(\mathcal{L}_n(0) G^{+\nu}(0) \right)^a | h_A \rangle$$
$$= -\frac{1}{2} g_\perp^{\mu\nu} g(x) - \frac{i}{2} \epsilon_\perp^{\mu\nu} s_L \Delta g(x) + \cdots .$$

Twist-2 Factorization at $\Lambda_{QCD} \ll k_\perp \ll Q$

$$F_{q,h_B/h_A}(x,\xi,k_\perp) = \sum_{a,b}\int \frac{dydz}{yz^2} d_{h_B/b}(z) H_{ba}(x/y,\xi/(yz),k_\perp/z) f_{a/h_A}(y)$$

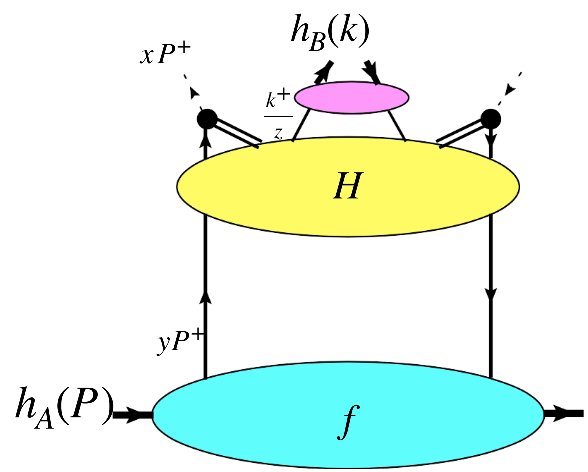
$$\Delta F_{q,h_B/h_A}(x,\xi,k_\perp) = \sum_{a,b}\int \frac{dydz}{yz^2} d_{h_B/b}(z) \Delta H_{ba}(x/y,\xi/(yz),k_\perp/z) \Delta f_{a/h_A}(y)$$

- 6 hard coefficients contribute at one loop level

  $$g/q, \quad q/q, \quad \bar{q}/q, \quad \bar{q}/g\,. \quad g/g, \quad \bar{q}/\bar{q}$$

- Over 70 independent perturbative diagrams contribute

- Evolution equation: Same as DGLAP Eq.



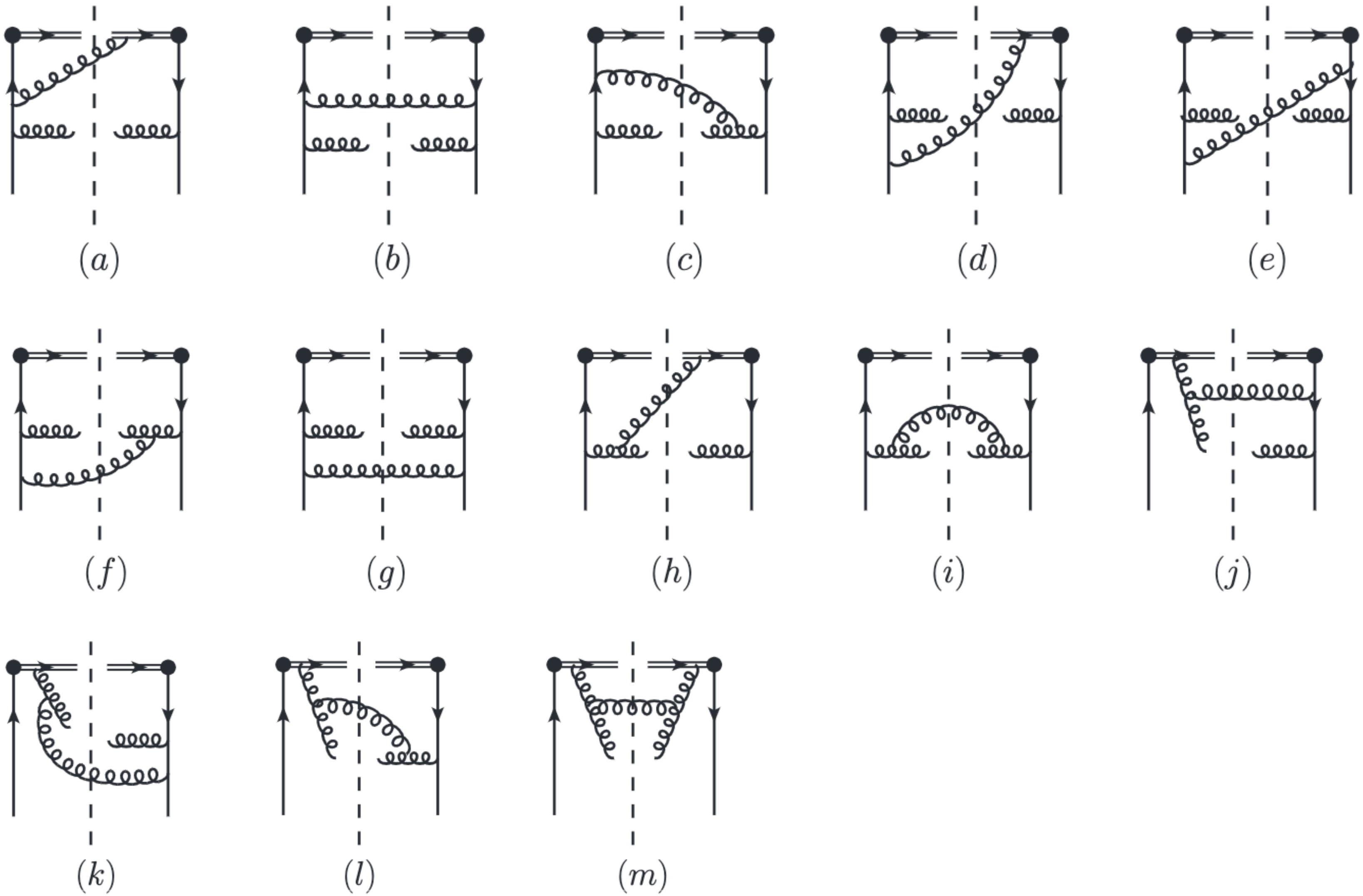


Figure 3: Real corrections to quark fracture functions from an initial quark to an observed final gluon, complex conjugate diagrams not shown.

Fracture Function

- Density matrix of collinear quark fracture function(FrF)

$$
\begin{aligned}
&\mathcal{M}_{Fij}\left(x, \xi, k_{\perp}\right) \\
&= \int \frac{d\lambda}{2\pi} e^{-ixP^{+}\lambda} \sum_{X} \langle h_A(P,s) | [\bar{\psi}(\lambda n)\mathcal{L}_n^{\dagger}(\lambda n)]_j | X h_B(k)\rangle \, \langle h_B(k) X \, | [\mathcal{L}_n(0)\psi(0)]_i | \, h_A(P,s)\rangle
\end{aligned}
$$

- Conditional parton distribution;
- "Hybrids" between the PDF and FF
- Kinematics variables

$$
x = \frac{p_a^{+}}{P^{+}} \quad \xi = \frac{k^{+}}{P^{+}} \quad \boldsymbol{k}_{\perp}
$$

- Color gauge invariance

$$
\mathcal{L}_n(x) = \mathrm{P}\exp\left\{ -ig_s \int_0^{\infty} d\lambda G^{+}(\lambda n + x)\right\}
$$

- Same evolution as the PDF—DGLAP Eq

- $k_{\perp}$integrated FrF: Trentadue-Veneziano Phys. Lett. B 323 (1994) 201
- $k_{\perp}$un-integrated FrF: Berera-Soper Phys. Rev. D 53 (1996) 6162
- TMD quark fracture function: Anselmino-Barone-Kotzinian, Phys. Lett. B
- Other applications: hadron collisions

e.g. Chen-Ma-Tong, JHEP 10 (2019) *285*

Ceccopieri -Trentadue, Phys. Lett. B 668 (2008) 319

Ceccopieri, Phys. Lett. B 703 (2011) 491

APPENDIX A: T-ODD EFFECTS

In this appendix we briefly review the well-known facts about T-odd effects[7] for completeness.

The scattering amplitude from the state i to the state f can be written as

$$S_{fi}=\delta_{fi}+i(2\pi)^4\delta^4(P_f-P_i)T_{fi}$$

in general. Unitarity of the S matrix reads

$$T_{fi}-T^*_{if}=iA_{fi} \tag{A1}$$

with

$$A_{fi}=\sum_n T^*_{nf}T_{ni}(2\pi)^4\delta^4(P_n-P_i)\ , \tag{A2}$$

where the sum over n runs over every possible state. We call A_{fi} the absorptive part of the amplitude T_{fi}.

It is straightforward to see that the unitarity relation can be rewritten as

$$|T_{fi}|^2=|T_{if}|^2-2\,\mathrm{Im}(T^*_{fi}A_{fi})-|A_{fi}|^2\ .$$

Let the states $\tilde{i}$ and $\tilde{f}$ be the states made from i and f, respectively, by reversing the directions of both three-momenta and spins. Subtracting $|T_{\tilde{f}\tilde{i}}|^2$ from both sides of Eq. (A3) we find

$$|T_{fi}|^2-|T_{\tilde{f}\tilde{i}}|^2=(|T_{if}|^2-|T_{\tilde{f}\tilde{i}}|^2)\\ -2\,\mathrm{Im}(T^*_{fi}A_{fi})-|A_{fi}|^2\ . \tag{A4}$$

Here T-odd effects are defined by the left-hand side of this equation. The first term in the right-hand side (RHS) vanishes if the interactions preserv time-reversal invariance (detailed balance). The remaining terms in the RHS include the absorptive part. In perturbation theory, the absorptive part of