# Fracture functions formalism for polarization effects in TFR of SIDIS

**Aram Kotzinian**

*YerPhI, Armenia & INFN, Torino*

# Outlook

- **Accessing LO (twist-2) nucleon structure in SIDIS**
- **Non-perturbative inputs Spin and Transverse Momentum Dependent (STMDs)**
  - Parton Distribution Functions in nucleon.
    - STMD PDF: SIDIS, DY
  - Parton Fragmentation Functions STMD FF: Hadron production in $e^+e^-$ annihilation (SIA), SIDIS, high $p_T$ hadron production in pp collisions
  - STMD Fracture Functions
  - String Fragmentation: LEPTO, PYTHIA

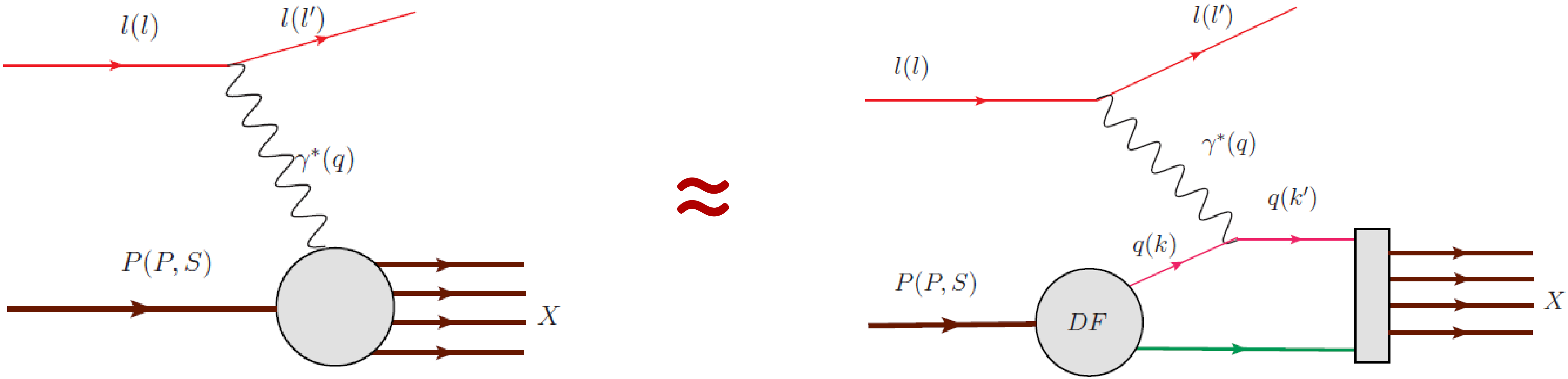
# Twist-2 TMD PDFs

| Nucleon Polarization \ Quark polarization | U | L | T |
|---|---|---|---|
| U | $f_1^q(x,k_T^2)$ | | $\frac{\epsilon_T^{ij} k_T{}^j}{M} h_1^{\perp q}(x,k_T^2)$ |
| L | | $S_L g_{1L}^q(x,k_T^2)$ | $S_L \frac{\mathbf{k}_T}{M} h_{1L}^{\perp q}(x,k_T^2)$ |
| T | $\frac{[\mathbf{k}_T \times \mathbf{S}_T]_3}{M} f_{1T}^{\perp q}(x,k^2)$ | $\frac{\mathbf{k}_T \cdot \mathbf{S}_T}{M} g_{1T}^{\perp q}(x,k_T^2)$ | $\mathbf{S}_T h_{1T}^q(x,k_T^2) + \frac{\mathbf{k}_T}{M}\frac{(\mathbf{k}_T \cdot \mathbf{S}_T)}{M} h_{1T}^{\perp q}(x,k_T^2)$ |

All azimuthal dependences are in prefactors. TMDs do not depend on them

# pQCD factorization: DIS

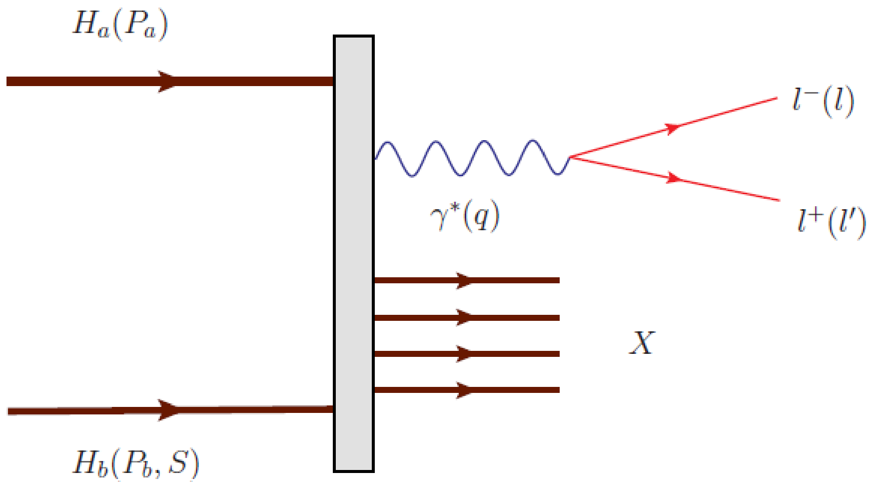
At large $Q^2 = -q^2$ the DIS can be described using QED lepton quark scattering cross section and nonperturbative input – colinear PDF $f_1^q(x)$: $d\sigma^{lN\to lhX} \sim \sum_q f_q\left(x, k_T^2\right) \otimes d\sigma^{lq\to l'q'}$



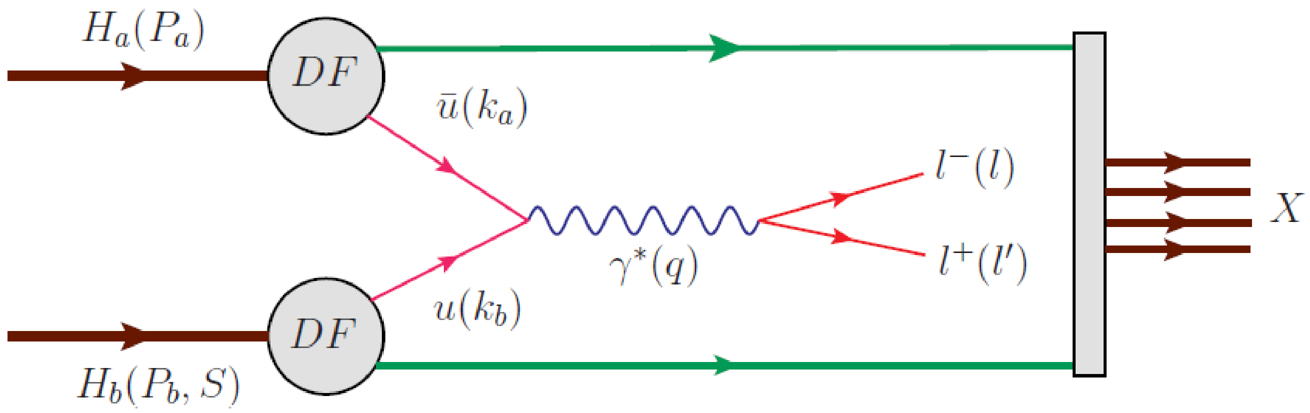

Access to nucleon unpolarized $f_1^{q+\bar{q}}(x)$ and longitudinally polarized $g_1^{q+\bar{q}}(x)$

leading twist colinear (transverse momentum integrated) PDFs

# pQCD TMD factorization: DY processes




$\approx$

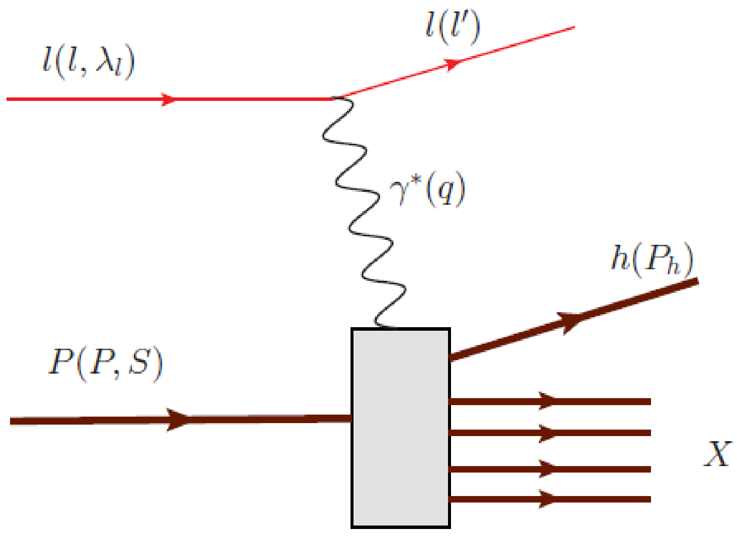



We can access to nucleon, pion and kaon TMD PDFs

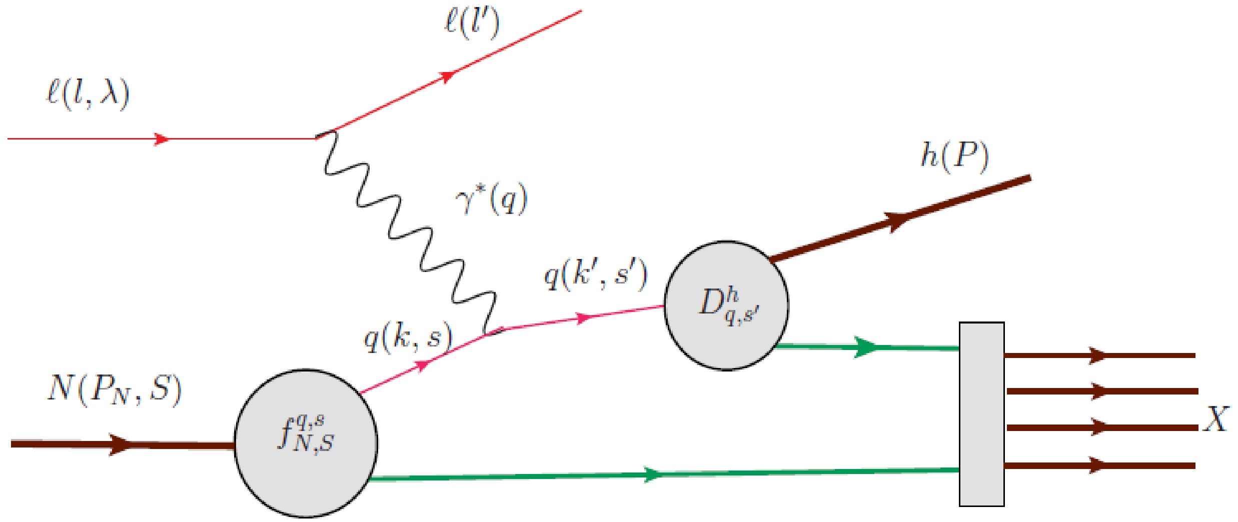
$f_1(x, k_T^2)$, $g_1(x, k_T^2)$, $h_1(x, k_T^2)$ and $h_1^{\perp}(x, k_T^2)$ leading twist PDFs

if we do not intergrate over transverse momentum of virtual photon

# QCD TMD factorization: SIDIS in CFR




$\approx$



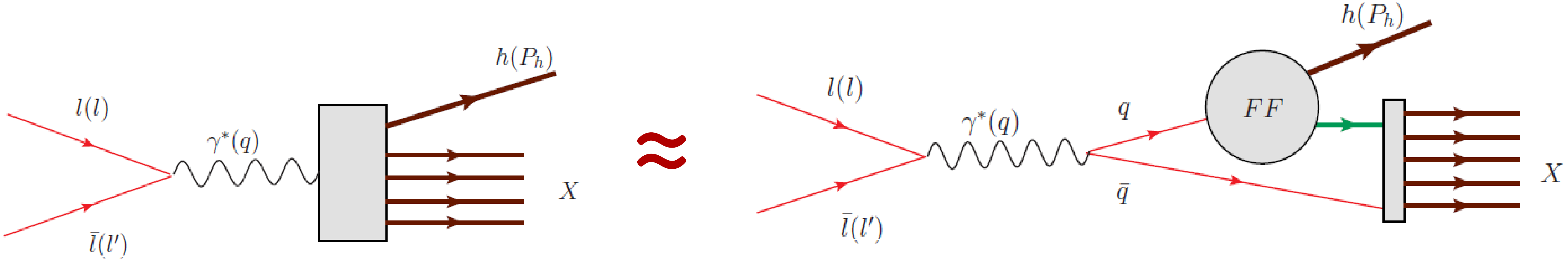

$$d\sigma^{lN\to lhX}=\sum_q f_q\left(x,k_T^2\right)\otimes d\sigma^{lq\to lq}\otimes D_h^q(x,\mathbf{k}_\mathbf{T};x_F,\mathbf{p}_\mathbf{T}^h)$$

Access to nucleon $f_1^q(x,k_T^2)$, $g_1^q(x,k_T^2)$ and $h_1^q(x,k_T^2)$,… leading twist TMD PDFs.

Additional nonperturbative input:

unpolarized TMD FFs $D_q^h(z,p_T^2)$ and Collins TMD FF $H_{1q}^h(z,p_T^2)$

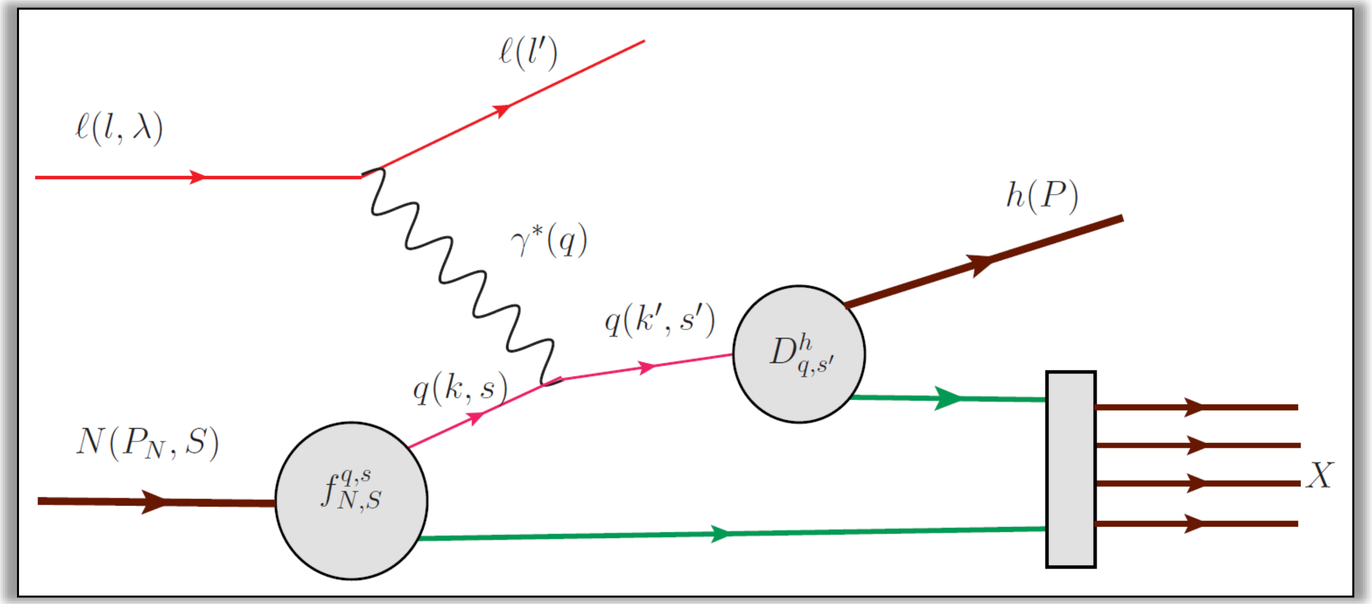
# QCD TMD factorization in semi-inclusive e+e- annihilation (SIA)




Access to $q+\bar{q}$ fragmentation finctions $D^h_{q+\bar{q}}(\mathrm{z}, \mathrm{p}_\perp^2)$

Two hadron production in opposite hemispheres: acces to Collins FF $H^h_{1q}(\mathrm{z}, \mathrm{p}_\perp^2)$

# SIDIS: CFR




$x_F > 0,\ \lambda -$ beam longitudinal polarization

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T} = f_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z,p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s'}_T}{m_h} H_1(z,p_T^2)$$

Measured in $e^+e^-$ semi-inclusive annihilation (SIA) to 2 back-to-back jets $e^+e^- \to h_1 h_2 + X$

# LO cross section in SIDIS CFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F>0)}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T}=\frac{\alpha^2 x}{yQ^2}\left(1+(1-y)^2\right)\times$$

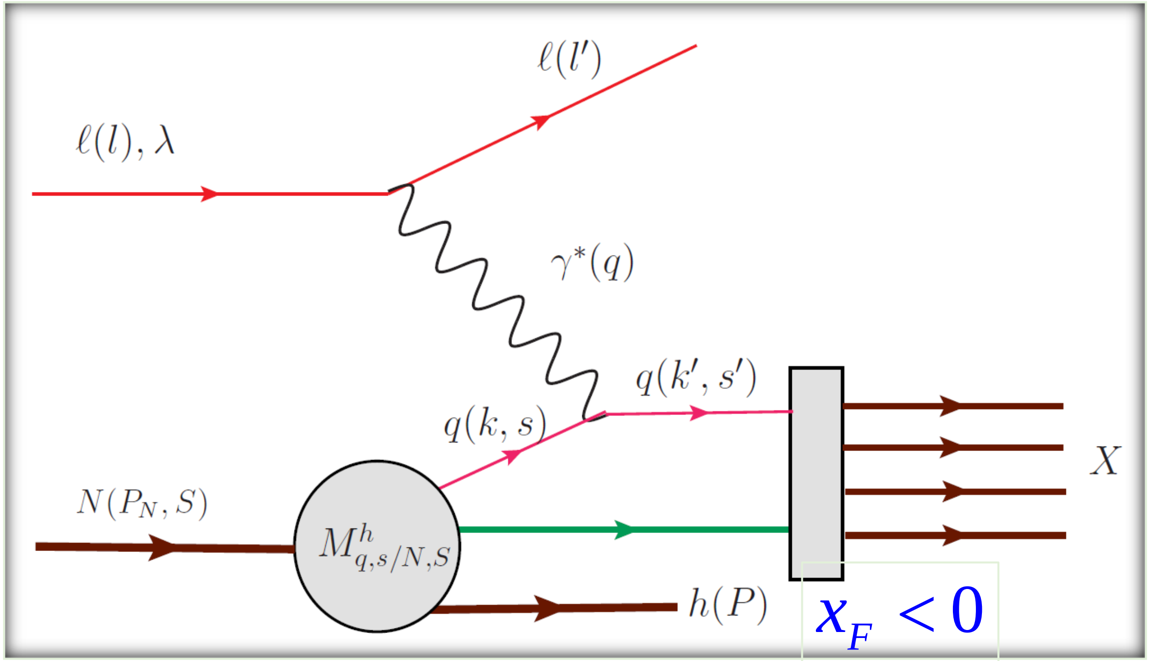
$$\times\left[\begin{array}{l}F_{UU,T}+D_{nn}(y)F_{UU}^{\cos 2\phi_h}\cos(2\phi_h)+\\ S_L D_{nn}(y)F_{UL}^{\sin 2\phi_h}\sin(2\phi_h)+\lambda S_L D_{ll}(y)F_{LL}+\\ S_T\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}\boxed{\sin(\phi_h-\phi_S)}+D_{nn}(y)\left(\begin{array}{l}F_{UT}^{\sin(\phi_h+\phi_S)}\boxed{\sin(\phi_h+\phi_S)}+\\ F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S)\end{array}\right)\right)+\\ \lambda S_T D_{ll}(y)F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S)\end{array}\right]$$

$$D_{ll}(y)=\frac{y(2-y)}{1+(1-y)^2},\qquad D_{nn}(y)=\frac{2(1-y)}{1+(1-y)^2}$$

At LO only 8 terms contributes out of 18 Structure Functions entering in the general expression of SIDIS cross section

6 azimuthal modulations, 4 terms are generated by Collins effect in fragmentation

# SIDIS: TFR




Trentadue, Veneziano 1994
Graudenz 1994
Collins 1998, 2000, 2002
de Florian, Sassot 1997, 1998
Grazzini, Trentadue, Veneziano 1998
Ceccopieri, Trentadue 2006, 2007, 2008
Sivers 2009
Ceccopieri , Mancusi 2013
Ceccopieri 2013
……..

$$\frac{d\sigma^{\ell(l)+N(P_N)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\zeta} = M^h_{q/N}\left(x, Q^2, \zeta\right) \otimes \frac{d\sigma^{\ell(l)+q(k)\to\ell(l')+q(k')}}{dQ^2},$$

$$\zeta = \frac{P^-}{P_N^-} \approx x_F(1-x)$$

Fracture function *M* is a Conditional Probability Distribution Function (CPDF) to observe the hadron h produced in target nucleon momentum direction in $\gamma^*$P CMS when hard probe interacts with parton carrying fraction x of nucleon momentum.

# Collinear Frac.Func.: application to HERA data, 1

D. de Florian, R. Sassot, Leading Proton Structure Function. PRD 58, 054003 (1998)

$$ep \to e'p'X: \quad \frac{d^3\sigma^p_{target}}{d\beta dQ^2 dx_P} = \frac{4\pi\alpha^2}{\beta Q^4}\left(1-y+\frac{y^2}{2}\right)M^h_p\left(\beta, Q^2, x_P\right), \quad \beta = \frac{x}{1-\zeta}, \quad \zeta = \frac{p_h^+}{p_N^+} \quad x_P = \zeta$$

$$\boxed{xM_q^{p/p}(\beta, Q_0^2, x_P) = N_s \beta^{a_s}(1-\beta)^{b_s}\{C_P \beta x_P^{\alpha_P} + C_{LP}(1-\beta)^{\gamma_{LP}}[1 + a_{LP}(1-x_P)^{\beta_{LP}}]\}}$$

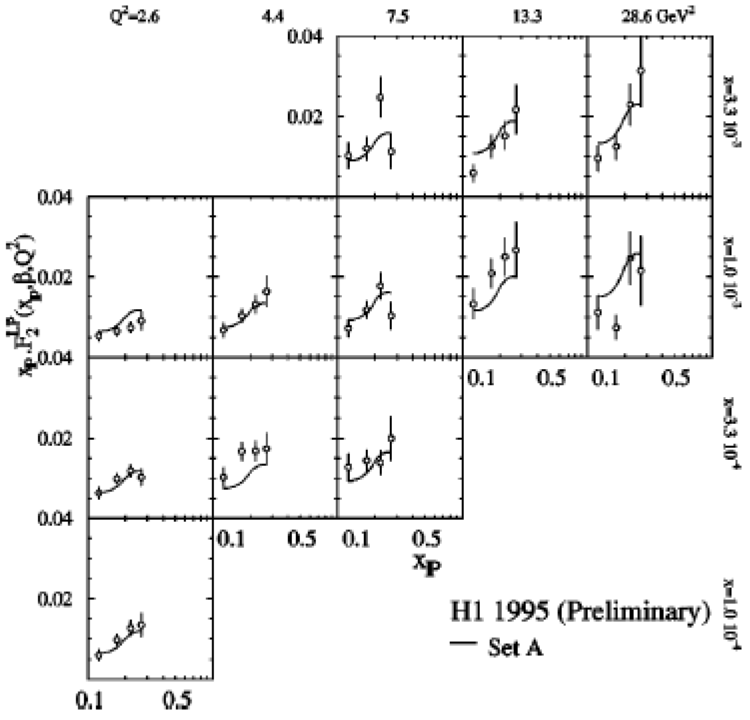



FIG. 2. H1 leading-proton data against the outcome of the fracture function parametrization (solid lines).

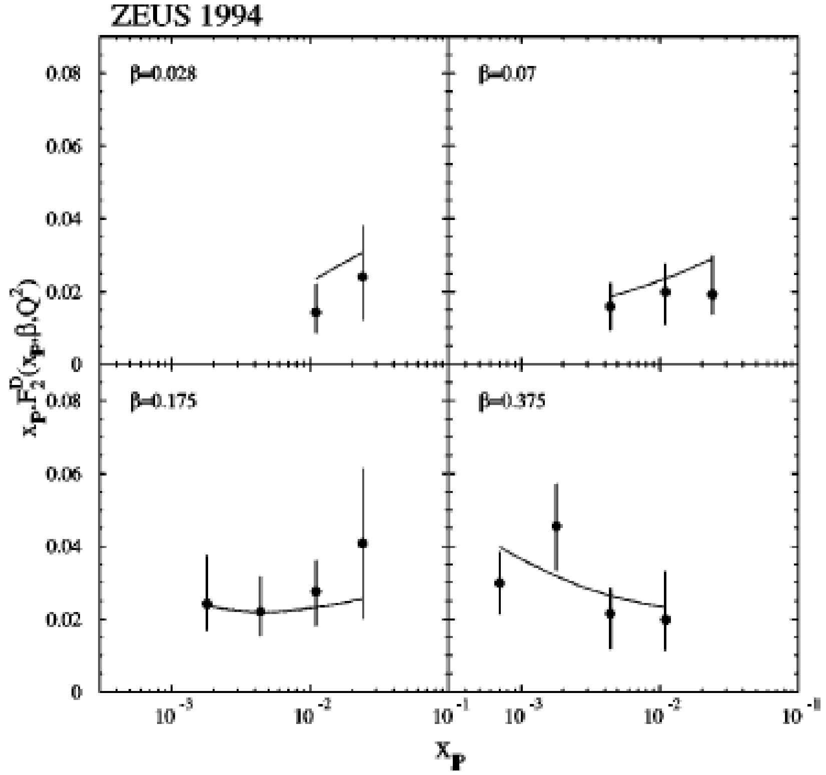
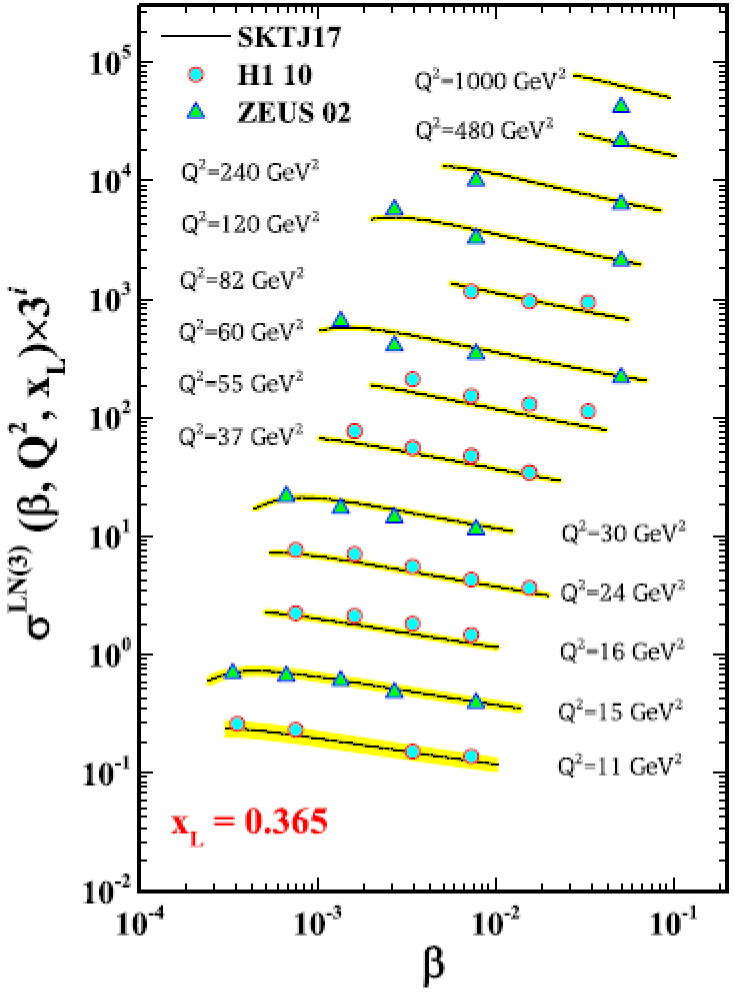



FIG. 8. ZEUS diffractive data, against the expectation coming from the fracture function parametrization (fit A).

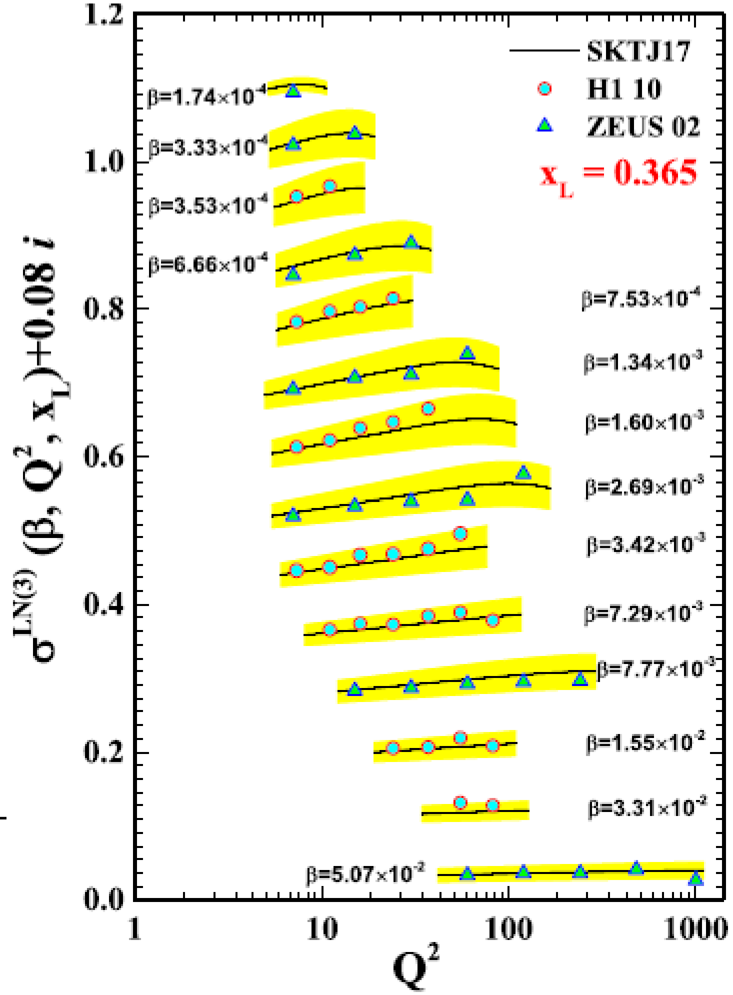
# Collinear Frac.Func.: application to HERA data, 2

Shoeibi *et al*, Neutron fracture functions. PRD 95, 074011 (2017)

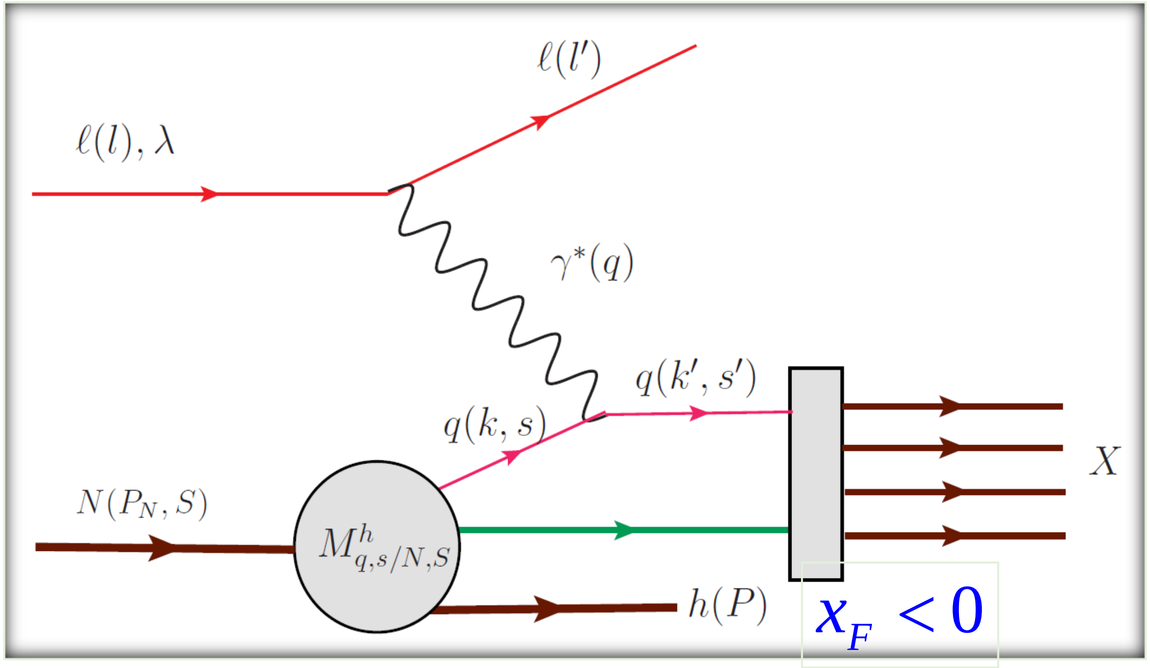
$$ep \to enX$$




$$x_L \simeq \frac{E_h}{E_P}, \quad \beta = \frac{x}{1-x_L}$$



# SIDIS TFR: Spin & TMD (STMD) Fracture Functions




**Anselmino, Barone and AK, PL B 699** (2011)108; **706** (2011)46; **713** (2012)317

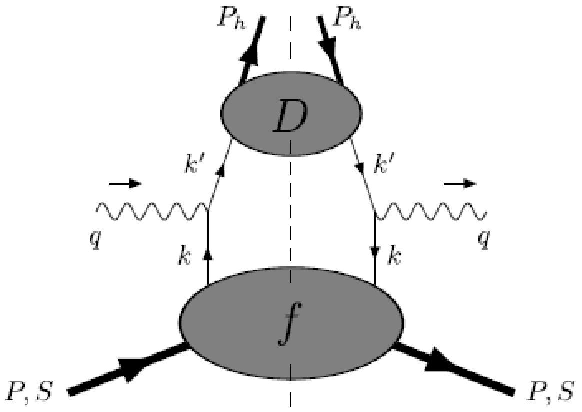
Nucleon and quark polarization are included, produced hadron and quark transverse momentum are not integrated over. Classification of twist-two Fracture Functions and cross sections expressions.

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,d\zeta\,d^2P_T} = M^h_{q,s/N,S}(x, k_T^2, \zeta, P_T^2, \mathbf{k}_T\cdot\mathbf{P}_T) \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2}$$

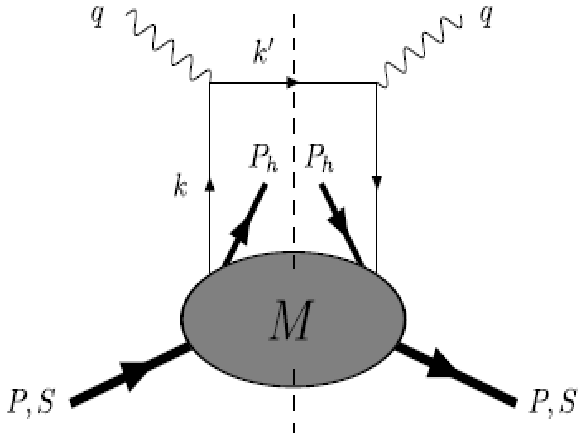
$$\mathbf{k}_T\cdot\mathbf{P}_T = k_T P_T \cos(\phi_h - \phi_q), \qquad \zeta = \frac{P^-}{P_N^-} \approx x_F(1-x)$$

# Quark correlator

**SIDIS**




**CFR**




**TFR**

$$\mathcal{M}^{[\Gamma]}(x_B,\vec{k}_\perp,\zeta,\vec{P}_{h\perp}) = \frac{1}{4\zeta}\int\frac{d\xi^+ d^2\xi_\perp}{(2\pi)^6} e^{\mathrm{i}(x_B P^- \xi^+ - \vec{k}_\perp\cdot\vec{\xi}_\perp)} \sum_X \int \frac{d^3 P_X}{(2\pi)^3 2E_X} \times$$

$$\times \langle P,S\,|\,\bar{\psi}(0)\Gamma\,|\,P_h,S_h;X\rangle\langle P_h,S_h;X\,|\,\psi(\xi^+,0,\vec{\xi}_\perp)\,|\,P,S\rangle$$

$$\Gamma = \gamma^-,\quad \gamma^-\gamma_5,\quad i\sigma^{i-}\gamma_5$$

Probabilistic interpretation at LO:

the conditional probabilities to find an unpolarized ($\Gamma = \gamma^-$), a longitudinally polarized ($\Gamma = \gamma^-\gamma_5$) or a transversely polarized ($\Gamma = \sigma^{i-}\gamma_5$) quark with longitudinal momentum fraction $x_{Bj}$ and transverse momentum $\boldsymbol{k}_\perp$ inside a nucleon fragmenting into a hadron carrying a fraction $\zeta$ of the nucleon longitudinal momentum and a transverse momentum $\boldsymbol{P}_{h\perp}$.

# STMD Fracture Functions for spinless hadron production

| Nucleon Polarization | Quark polarization: U | L | T |
|---|---|---|---|
| U | $\hat{u}_1$ | $\dfrac{\mathbf{k}_T \times \mathbf{P}_T}{m_N\, m_h}\,\hat{l}_1^{\perp h}$ | $\dfrac{\epsilon_T^{ij} P_T^{\,j}}{m_h}\,\hat{t}_1^{\,h} + \dfrac{\epsilon_T^{ij} k_T^{\,j}}{m_N}\,\hat{t}_1^{\,\perp}$ |
| L | $\dfrac{S_L(\mathbf{k}_T \times \mathbf{P}_T)}{m_N\, m_h}\,\hat{u}_{1L}^{\perp h}$ | $S_L\,\hat{l}_{1L}$ | $\dfrac{S_L \mathbf{P}_T}{m_h}\,\hat{t}_{1L}^{\,h} + \dfrac{S_L \mathbf{k}_T}{m_N}\,\hat{t}_{1L}^{\,\perp}$ |
| T | $\dfrac{\mathbf{P}_T \times \mathbf{S}_T}{m_h}\,\hat{u}_{1T}^{\,h} + \dfrac{\mathbf{k}_T \times \mathbf{S}_T}{m_N}\,\hat{u}_{1T}^{\,\perp}$ | $\dfrac{\mathbf{P}_T \cdot \mathbf{S}_T}{m_h}\,\hat{l}_{1T}^{\,h} + \dfrac{\mathbf{k}_T \cdot \mathbf{S}_T}{m_N}\,\hat{l}_{1T}^{\,\perp}$ | $\mathbf{S}_T \hat{t}_{1T} + \dfrac{\mathbf{P}_T(\mathbf{P}_T \cdot \mathbf{S}_T)}{m_h^2}\,\hat{t}_{1T}^{\,hh} + \dfrac{\mathbf{k}_T(\mathbf{k}_T \cdot \mathbf{S}_T)}{m_N^2}\,\hat{t}_{1T}^{\,\perp\perp} + \dfrac{\mathbf{P}_T(\mathbf{k}_T \cdot \mathbf{S}_T) - \mathbf{k}_T \cdot (\mathbf{P}_T \cdot \mathbf{S}_T)}{m_N m_h}\,\hat{t}_{1T}^{\,\perp h}$ |

At twist-2 there are 16 independent Fracture Functions depending on quark and TFR hadron momenta

$$x,\, k_T^2,\, \zeta,\, P_T^2,\, k_T P_T \cos(\phi_h - \phi_q)$$

Azimuthal dependences for different nucleon and quark polarizations appears not only in prefactors, as it was in the case of SIDIS in CFR, but also in the argument of fracture functions

The terms which contains the same prefactors as in SIDIS in CFR are marked in red

# STMD Fracture Functions for spinless hadron production

| | | Quark polarization | | |
|---|---|---|---|---|
| | | U | L | T |
| Nucleon Polarization | U | $\hat{u}_1$ | $\frac{\mathbf{k}_T \times \mathbf{P}_T}{m_N m_h} \hat{l}_1^{\perp h}$ | $\frac{\epsilon_T^{ij} P_T^j}{m_h} \hat{t}_1^h + \frac{\epsilon_T^{ij} k_T^j}{m_N} \hat{t}_1^\perp$ |
| | L | $\frac{S_L (\mathbf{k}_T \times \mathbf{P}_T)}{m_N m_h} \hat{u}_{1L}^{\perp h}$ | $S_L \hat{l}_{1L}$ | $\frac{S_L \mathbf{P}_T}{m_h} \hat{t}_{1L}^h + \frac{S_L \mathbf{k}_T}{m_N} \hat{t}_{1L}^\perp$ |
| | T | $\frac{\mathbf{P}_T \times \mathbf{S}_T}{m_h} \hat{u}_{1T}^h + \frac{\mathbf{k}_T \times \mathbf{S}_T}{m_N} \hat{u}_{1T}^\perp$ | $\frac{\mathbf{P}_T \cdot \mathbf{S}_T}{m_h} \hat{l}_{1T}^h + \frac{\mathbf{k}_T \cdot \mathbf{S}_T}{m_N} \hat{l}_{1T}^\perp$ | $\mathbf{S}_T \hat{t}_{1T} + \frac{\mathbf{P}_T (\mathbf{P}_T \cdot \mathbf{S}_T)}{m_h^2} \hat{t}_{1T}^{hh} + \frac{\mathbf{k}_T (\mathbf{k}_T \cdot \mathbf{S}_T)}{m_N^2} \hat{t}_{1T}^{\perp\perp} + \frac{\mathbf{P}_T (\mathbf{k}_T \cdot \mathbf{S}_T) - \mathbf{k}_T \cdot (\mathbf{P}_T \cdot \mathbf{S}_T)}{m_N m_h} \hat{t}_{1T}^{\perp h}$ |

$\hat{u}_1 \rightarrow$ unintegrated twist-2 fracture functions

$U, L, T$ subscrupts $\rightarrow$ unpolarized, longitudinal and transversely polarized nucleon

$\perp, h \rightarrow$ dependence on transverse momenum of quark and produced hadron

# Sum Rules connecting Fracture Functions to TMD PDFs

Integrating the fracture function over the longitudinal and transverse momentum of hadron in TFR and summing over all hadrons we recover the corresponding quark TMD DF

$$\sum_h \int \zeta d\zeta \int d^2 P_T \, \hat{u}_1 = (1-x) \, f_1(x, k_T^2)$$

# Similar TMD Sum Rules for all TMD PDFs

$$\sum_h \int \zeta d\zeta \int d^2P_T\, \hat{u}_1 = (1-x)\, f_1(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T \left( \hat{u}_{1T}^{\perp} + \frac{m_N}{m_h}\frac{\mathbf{k}_T\cdot\mathbf{P}}{k_T^2}\hat{u}_{1T}^{h} \right) = -(1-x)\, f_{1T}^{\perp}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T\, \hat{l}_{1L} = (1-x) g_{1L}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T \left( \hat{l}_{1T}^{\perp} + \frac{m_N}{m_h}\frac{\mathbf{k}_T\cdot\mathbf{P}}{k_T^2}\hat{l}_{1T}^{h} \right) = (1-x) g_{1T}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T \left( \hat{t}_{1L}^{\perp} + \frac{m_N}{m_h}\frac{\mathbf{k}_T\cdot\mathbf{P}}{k_T^2}\hat{t}_{1L}^{h} \right) = (1-x) h_{1L}^{\perp}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T \left( \hat{t}_{1}^{\perp} + \frac{m_N}{m_h}\frac{\mathbf{k}_T\cdot\mathbf{P}}{k_T^2}\hat{t}_{1}^{h} \right) = -(1-x) h_{1}^{\perp}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T \left( \hat{t}_{1T}^{\perp\perp} + \frac{m_N^2}{m_h^2}\frac{2(\mathbf{k}_T\cdot\mathbf{P})^2 - k_T^2P_T^2}{k_T^4}\hat{t}_{1T}^{hh} \right) = (1-x) h_{1T}^{\perp}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2P_T \left( \hat{t}_{1T} + \frac{k_T^2}{2m_N^2}\hat{t}_{1T}^{\perp\perp} + \frac{P_T^2}{2m_h^2}\hat{t}_{1T}^{hh} \right) = (1-x) h_1(x,k_T^2)$$

| Nucleon Polarization | Quark polarization: U | L | T |
|---|---|---|---|
| U | $\hat{u}_1$ | $\frac{\mathbf{k}_T\times\mathbf{P}_T}{m_N m_h}\hat{l}_1^{\perp h}$ | $\frac{\epsilon_T^{ij}P_T^{\,j}}{m_h}\hat{t}_1^{h} + \frac{\epsilon_T^{ij}k_T^{j}}{m_N}\hat{t}_1^{\perp}$ |
| L | $\frac{S_L(\mathbf{k}_T\times\mathbf{P}_T)}{m_N m_h}\hat{u}_{1L}^{\perp h}$ | $S_L\hat{l}_{1L}$ | $\frac{S_L\mathbf{P}_T}{m_h}\hat{t}_{1L}^{h} + \frac{S_L\mathbf{k}_T}{m_N}\hat{t}_{1L}^{\perp}$ |
| T | $\frac{\mathbf{P}_T\times\mathbf{S}_T}{m_h}\hat{u}_{1T}^{h} + \frac{\mathbf{k}_T\times\mathbf{S}_T}{m_N}\hat{u}_{1T}^{\perp}$ | $\frac{\mathbf{P}_T\cdot\mathbf{S}_T}{m_h}\hat{l}_{1T}^{h} + \frac{\mathbf{k}_T\cdot\mathbf{S}_T}{m_N}\hat{l}_{1T}^{\perp}$ | $\mathbf{S}_T\hat{t}_{1T} + \frac{\mathbf{P}_T(\mathbf{P}_T\cdot\mathbf{S}_T)}{m_h^2}\hat{t}_{1T}^{hh} + \frac{\mathbf{k}_T(\mathbf{k}_T\cdot\mathbf{S}_T)}{m_N^2}\hat{t}_{1T}^{\perp\perp} + \frac{\mathbf{P}_T(\mathbf{k}_T\cdot\mathbf{S}_T)-\mathbf{k}_T\cdot(\mathbf{P}_T\cdot\mathbf{S}_T)}{m_N m_h}\hat{t}_{1T}^{\perp h}$ |

| Nucleon Polarization | Quark polarization: U | L | T |
|---|---|---|---|
| U | $f_1^q(x,k_T^2)$ | | $\frac{\epsilon_T^{ij}k_T^{\,j}}{M}h_1^{\perp q}(x,k_T^2)$ |
| L | | $S_L g_{1L}^q(x,k_T^2)$ | $S_L\frac{\mathbf{k}_T}{M}h_{1L}^{\perp q}(x,k_T^2)$ |
| T | $\frac{[\mathbf{k}_T\times\mathbf{S}_T]_3}{M}f_{1T}^{\perp q}(x,k^2)$ | $\frac{\mathbf{k}_T\cdot\mathbf{S}_T}{M}g_{1T}^{\perp q}(x,k_T^2)$ | $\mathbf{S}_T h_{1T}^q(x,k_T^2) + \frac{\mathbf{k}_T}{M}\frac{(\mathbf{k}_T\cdot\mathbf{S}_T)}{M}h_{1T}^{\perp q}(x,k_T^2)$ |

# Quark transverse momentum integrated Fracture Functions

In single hadron production in TFR NO access to final quark transverse momentum and polarization

Quark transverse momentum integrates fracture functions market by tilde:

$$\tilde{u}_1(x_B,\zeta_2,P_{T2}^2) = \int d^2k_T\,\hat{u}_1\left(x_B,k_T^2,\zeta,P_{T1}^2,\mathbf{k}_T\cdot\mathbf{P}_{T1}\right)$$

$$\tilde{u}_{1T}^h(x_B,\zeta_2,P_{T2}^2) = \int d^2k_T\left\{\hat{u}_{1T}^h + \frac{m_2}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_{T2}}{P_{T2}^2}\hat{u}_{1T}^\perp\right\}$$

$$\tilde{l}_{1L}(x_B,\zeta_2,P_{T2}^2) = \int d^2k_T\,\hat{l}_{1L}$$

$$\tilde{l}_{1T}^h(x_B,\zeta_2,P_{T2}^2) = \int d^2k_T\left\{\hat{l}_{1T}^h + \frac{m_2}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_{T2}}{P_{T2}^2}\hat{l}_{1T}^\perp\right\}$$

# LO cross-section of single hadron production in TFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F<0)}{dx\,dQ^2\,d\phi_S\,d\zeta\,d^2P_T} = \frac{\alpha^2 x}{yQ^4}\left(1+(1-y)^2\right)\sum_q e_q^2 \times$$

$$\times\left[\begin{array}{l}\tilde{u}_1\left(x,\zeta,P_T^2\right) - S_T \boxed{\frac{P_T}{m_h}\tilde{u}_{1T}^h\left(x,\zeta,P_T^2\right)\sin(\phi_h-\phi_S)} + \\ \lambda y(2-y)\left(S_L\tilde{l}_{1L}\left(x,\zeta,P_T^2\right)+S_T\frac{P_T}{m_h}\tilde{l}_{1T}^h\left(x,\zeta,P_T^2\right)\cos(\phi_h-\phi_S)\right)\end{array}\right]$$

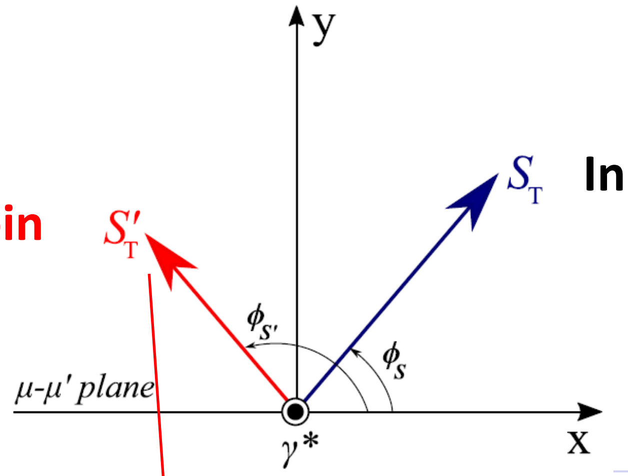
At LO (twist 2) only 4 terms out of 18 Structure Functions in SIDIS,
Only 2 azimuthal modulations

In single hadron production in TFR NO access to final quark transverse momentum and polarization $\Rightarrow$ No Collins-like $\sin(\phi_h+\phi_S)$ modulation

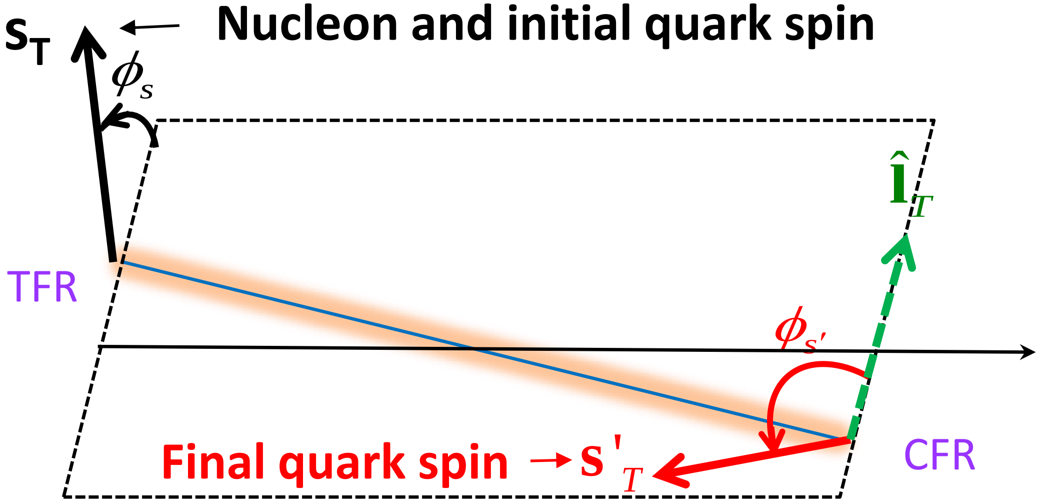
# Quark transverse spin in hard *l-q* scattering

AK, Transversity workshop,
Yerevan, 2009

Final quark spin




Initial quark spin



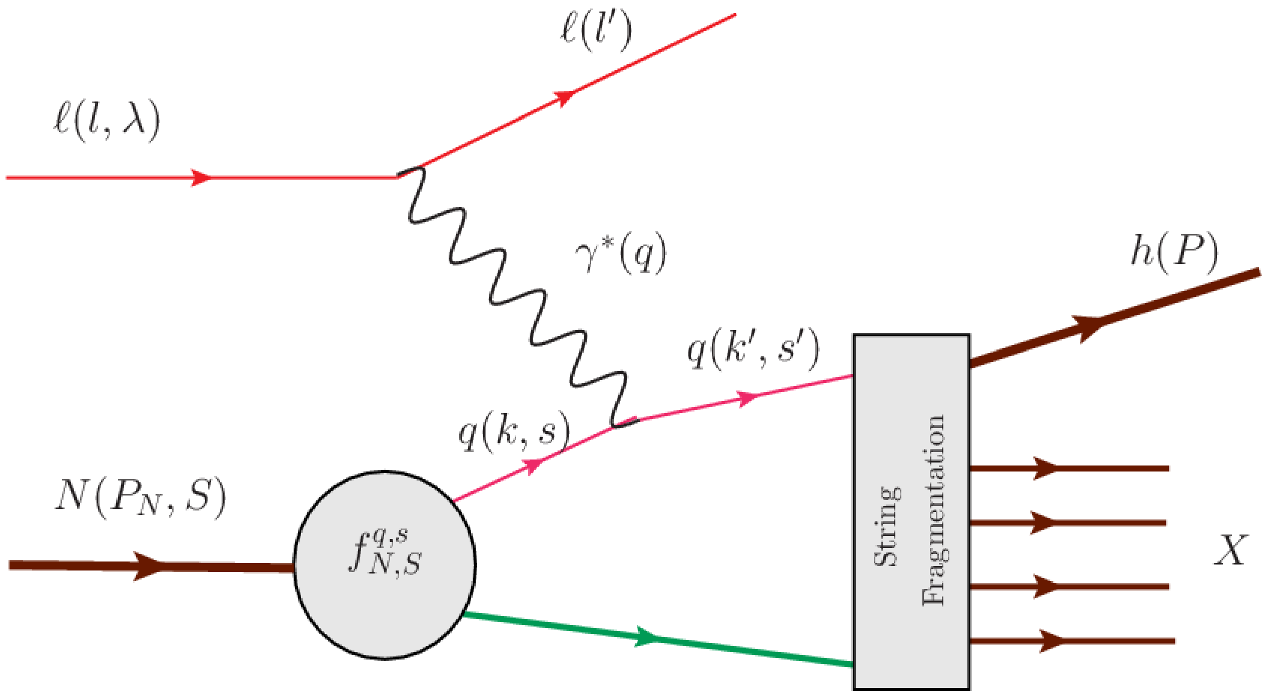

$$\text{QED: } lq \to l'q' \;\Rightarrow\; s'_T = D_{nn}(y)s_T, \quad D_{nn}(y) = \frac{2(1-y)}{1+(1-y)^2}, \quad \phi_{s'} = \pi - \phi_s$$

$$\text{CFR: } \left[\mathbf{s}'_T \times \mathbf{p}_T\right] \propto \sin(\phi_h - \phi_{s'}) = -\sin(\phi_h + \phi_s) \text{ Collins-like azimuthal modulation}$$

$$\text{TFR: } \left[\mathbf{s}_T \times \mathbf{p}_T\right] \propto \sin(\phi_h - \phi_s) \text{ Sivers-like azimuthal modulation}$$

If only one hadron in TFR of SIDIS is detected there is no final quark polarimetry.
→ STMD fracture functions depends on initial quark transverse polarization dependent fracture functions.
No Collins like modulation in TFR.

# Hadronization Function in MC event generators (LEPTO, PYTHIA)

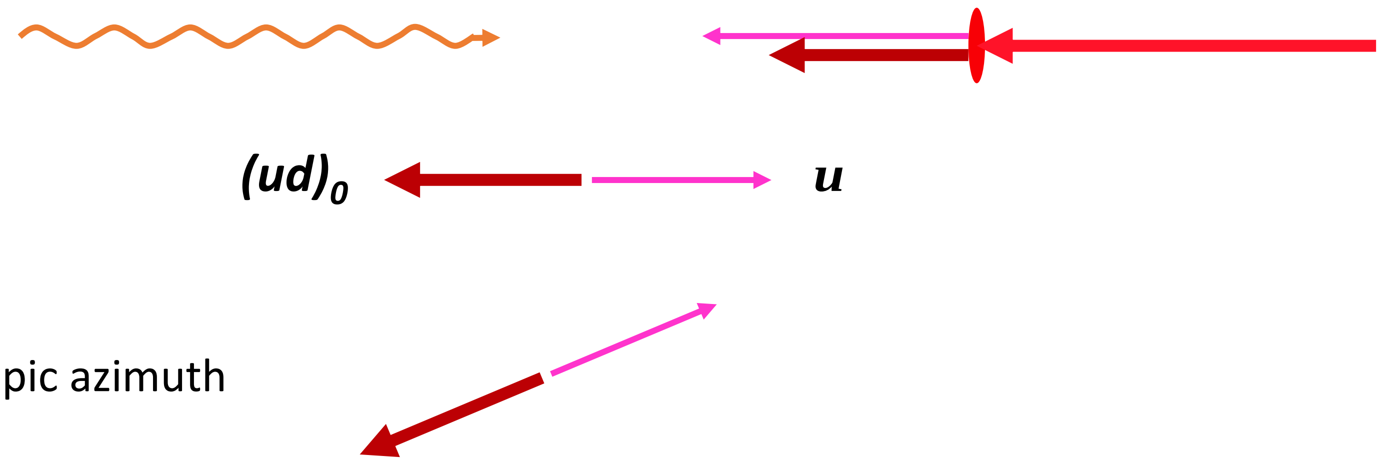



$$d\sigma^{lN\to lhX} = \sum_q f_q\left(x,k_T^2\right)\otimes d\sigma^{lq\to lq}\otimes H^q_{h/N}\left(x,\mathbf{k}_\mathbf{T};x_F,\mathbf{p}^h_\mathbf{T}\right)$$

**Hadronization Function modeled by Lund String Fragmentation describes both CFR and TFR**

# Quark dynamics in MC even generators

- Before
- After hard scattering
- Include $k_T$ with isotropic azimuth

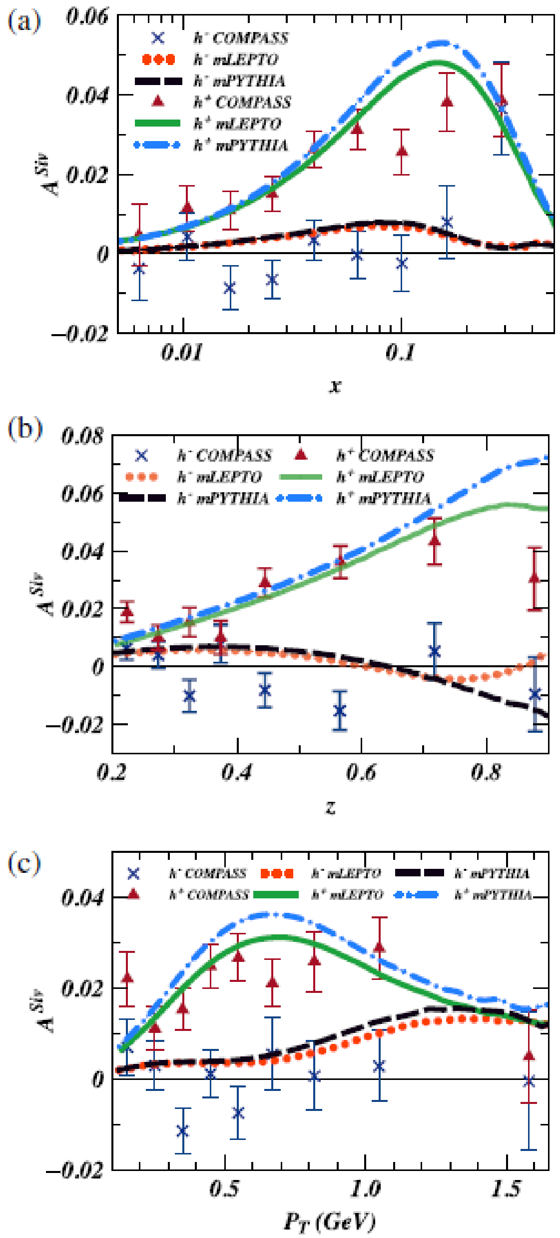



**We modified standard MC generators so that mLEPTO and mPYTHIA can generate $k_T$ with anisotropic azimuthal modulation according Sivers effect**

$$d\sigma^{lN\to lhX} = \sum_q \left( f_q(x,\mathrm{k}_T^2) + \frac{\mathbf{k_T}\times\mathbf{S_T}}{M} f_{qT}^{\perp}(x,\mathrm{k}_T^2) \right) \otimes d\sigma^{lq\to lq} \otimes H^q_{h/N}(x,\mathbf{k_T};x_F,\mathbf{p}^h_{\mathbf{T}})$$

# Sivers effect in the event generators

Matevosyan, AK, Aschenauer, Avakian, Thomas, PRD 92, 054028 (2015)



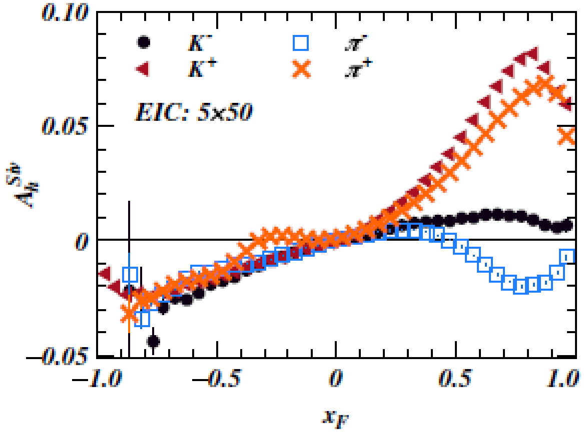




FIG. 13 (color online). EIC model SSAs for $5 \times 50$ SIDIS kinematics for charged pions and kaons versus $x_F$. The Sivers asymmetry is present both in the current and target fragmentation regions.

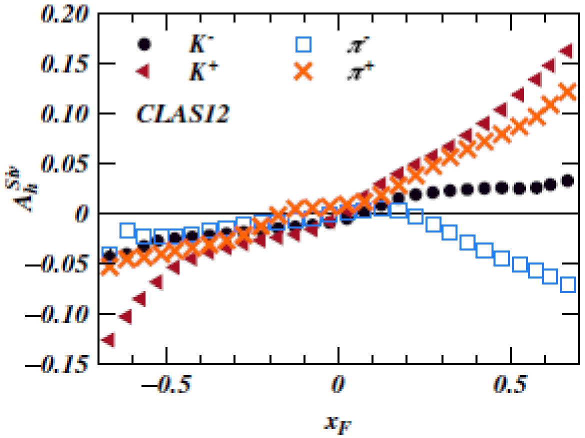
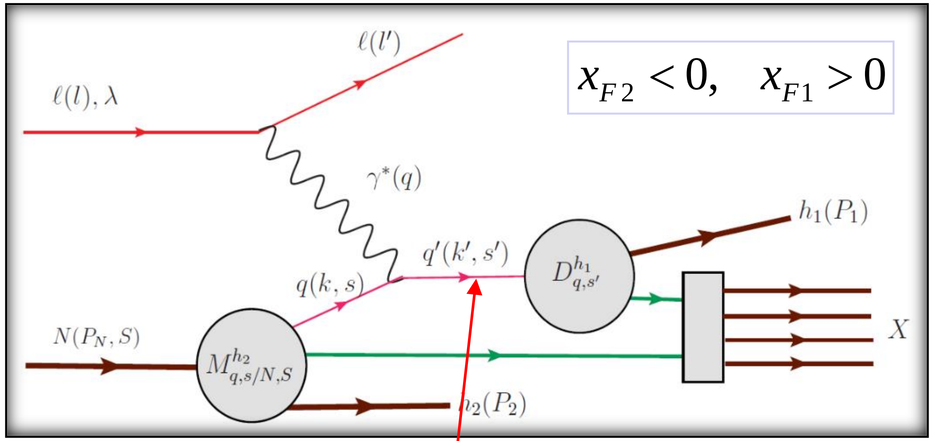


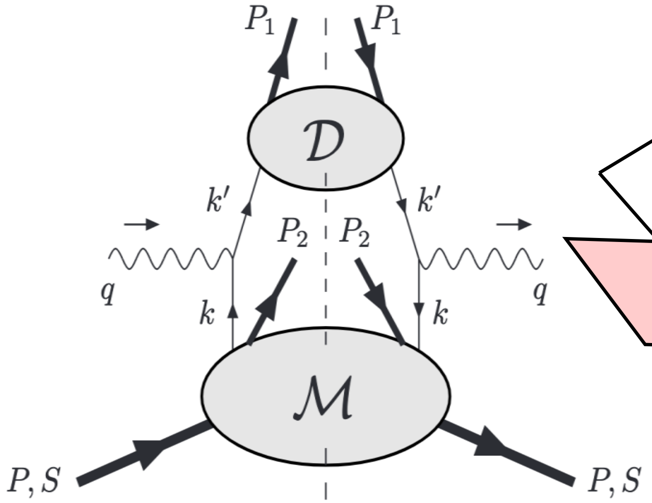

FIG. 17 (color online). Predictions for SSAs for charged pions and kaons versus $x_F$ at CLAS12. The Sivers asymmetry is present both in the current and target fragmentation regions.

Only correlation of target $\mathbf{S_T}$ and struck quark $\mathbf{k_T}$ is explicitly parametrized using Sivers PDFs. Then this correlation is transferred to produced hadrons via unpolarized string fragmentation .

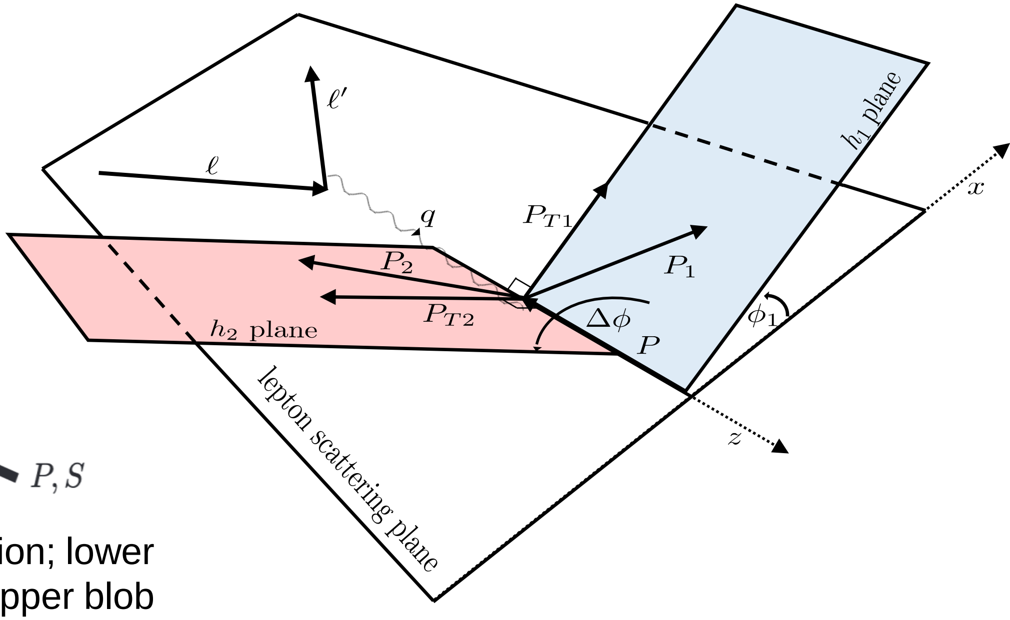
# Double hadron production in DIS (DSIDIS): TFR & CFR




Access to final quark transverse momentum and polarization




Handbag diagram for dihadron production; lower blob contains Fracture Functions and upper blob contains the FFs.




$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} = M^{h_2}_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z,p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s}'_T}{m_h} H_1(z,p_T^2), \quad \mathbf{s}'_T \text{ - fragmenting quark transverse polarization}$$

# Unintegrated DSIDIS LO cross-section: accessing quark polarization

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} =$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\left(\begin{array}{l}\hat{u}^{h_2}\otimes D_1^{h_1}+\lambda D_{ll}(y)\hat{l}^{h_2}\otimes D_1^{h_1} \\ +\hat{t}^{h_2}\otimes\dfrac{\mathbf{p}_T\times\mathbf{s}'_T}{m_{h_1}}H_1^{h_1}\end{array}\right)$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\left(\begin{array}{l}\sigma_{UU}+S_L\sigma_{UL}+S_T\sigma_{UT}+ \\ \lambda D_{ll}\left(\sigma_{LU}+S_L\sigma_{LL}+S_T\sigma_{LT}\right)\end{array}\right)$$

DSIDIS cross section is a sum of polarization independent, single and double spin dependent terms as in 1h SIDIS cross section.

$$D_{ll}(y) = \frac{y(2-y)}{1+(1-y)^2}$$

Back-to-back two hadrons production provides access to all 16 twist-2 $k_T$-unintegrated fracture functions (see additional slides)

# DSIDIS azimuthal modulations

AK @ DIS2011

$$\sigma_{UU} = F_0^{\hat{u}\cdot D_1} - D_{nn}\left( \frac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_1^\perp \cdot H_1} \cos(2\phi_1) + \frac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_1^h \cdot H_1} \cos(\phi_1+\phi_2) + \left( \frac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_1^\perp \cdot H_1} + \frac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_1^h \cdot H_1} \right) \cos(2\phi_2) \right)$$

$$D_{nn}(y) = \frac{2(1-y)}{1+(1-y)^2}$$

$$F_{k1}^{\hat{M}\cdot D} = C\left[ \hat{M}\cdot D \frac{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})(\mathbf{P}_{T2}\cdot\mathbf{k}) - (\mathbf{P}_{T1}\cdot\mathbf{k})\mathbf{P}_{T2}^2}{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2 - \mathbf{P}_{T1}^2\mathbf{P}_{T2}^2} \right]$$

$$C[\hat{M}\cdot D w] = \sum_a e_a^2 \int d^2k_T\, d^2p_T\, \delta^{(2)}(z\mathbf{k}_T + \mathbf{p}_T - \mathbf{P}_{T1}) \hat{M}_a(x,\zeta,k_T^2,P_{T2}^2,\mathbf{k}_T\cdot\mathbf{P}_{T2}) D_a(z,p_T^2) w$$

Structure functions $F_{\dots}^{\hat{u}\cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2$ and $(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})$

$$\mathbf{P}_{T1}\cdot\mathbf{P}_{T2} = P_{T1}P_{T2}\cos(\Delta\phi), \text{ with } \Delta\phi = \phi_1 - \phi_2$$

# $A_{LU}$ asymmetry, 1

Anselmino, Barone and AK, PLB **713** (2012) 317

$$\boxed{\sigma_{LU} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)}$$

$$\sigma_{UU} = F_0^{\hat{u}\cdot D_1} - D_{nn}\left( \frac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_1^{\perp}\cdot H_1} \cos(2\phi_1) + \frac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_1^{h}\cdot H_1} \cos(\phi_1 + \phi_2) + \left( \frac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_1^{\perp}\cdot H_1} + \frac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_1^{h}\cdot H_1} \right) \cos(2\phi_2) \right)$$

| | | Quark polarization | | |
|---|---|---|---|---|
| | | U | L | T |
| Nucleon Polarization | U | | * | |
| | L | | | |
| | T | | | |

$F_{...}^{\hat{u}\cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2$ and $(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})$

$\mathbf{P}_{T1}\cdot\mathbf{P}_{T2} = P_{T1}P_{T2}\cos(\Delta\phi)$, with $\Delta\phi = \phi_1 - \phi_2$

Choosing as independent angles $\Delta\phi$ and $\phi_2$ $(\phi_1 = \Delta\phi + \phi_2)$

and integrating $\sigma_{UU}$ over $\phi_2$ we eliminate all terms

proportional to $D_{NN} \Rightarrow$

# $A_{LU}$ asymmetry, 2

$$\mathcal{A}_{LU} = -\frac{y(1-\frac{y}{2})}{(1-y+\frac{y^2}{2})}\frac{\mathcal{F}_{LU}^{\sin\Delta\phi}}{\mathcal{F}_{UU}}\sin\Delta\phi$$

$$= -\frac{|\boldsymbol{P}_{1\perp}||\boldsymbol{P}_{2\perp}|}{m_N m_2}\frac{y(1-\frac{y}{2})}{(1-y+\frac{y^2}{2})}\frac{\mathcal{C}[w_5\hat{l}_1^{\perp h}D_1]}{\mathcal{C}[\hat{u}_1 D_1]}\sin\Delta\phi.$$

$$A_{LU} = \frac{\int d\phi_2 \sigma_{LU}}{\int d\phi_2 \sigma_{UU}} = -\frac{P_{T1}P_{T2}}{m_2 m_N}\frac{F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1}\left(x,z,\zeta,P_{T1}^2,P_{T2}^2,\cos(\Delta\phi)\right)}{F_0^{\hat{u}\cdot D_1}\left(x,z,\zeta,P_{T1}^2,P_{T2}^2,\cos(\Delta\phi)\right)}\sin(\Delta\phi)$$

Expected leading-twist asymmetry is proportional to *sin(Δϕ)*

# $A_{LU}$ @ CLAS12, (1)

Timothy B. Hayward, H. Avakian and A.Kotzinian et al, CLASS Collaboration. arXiv:2208.05086v1 [hep-ex] 10 Aug 2022

- Observed non-zero asymmetries are the first experimental confirmation of possible spin-orbit correlations between hadrons produced simultaneously in the CFR and TFR.
- Observed linear dependence on the product of transverse momenta is consistent with expectations.

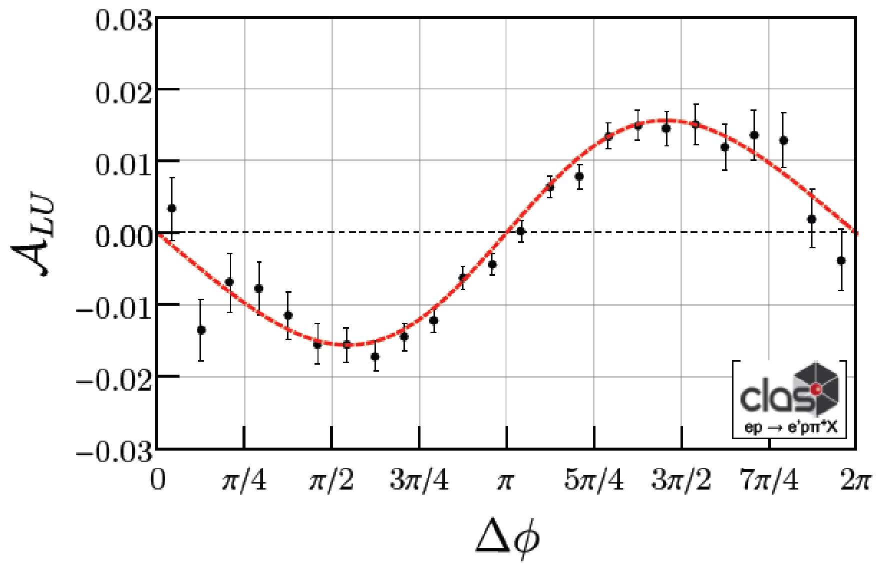



FIG. 2. The beam spin asymmetry, $\mathcal{A}_{LU}$, as a function of $\Delta\phi$ and integrated over all other kinematics for the entire data set. A clear $\sin(\Delta\phi)$ dependence is observed with small $\sin(2\Delta\phi)$ contributions.

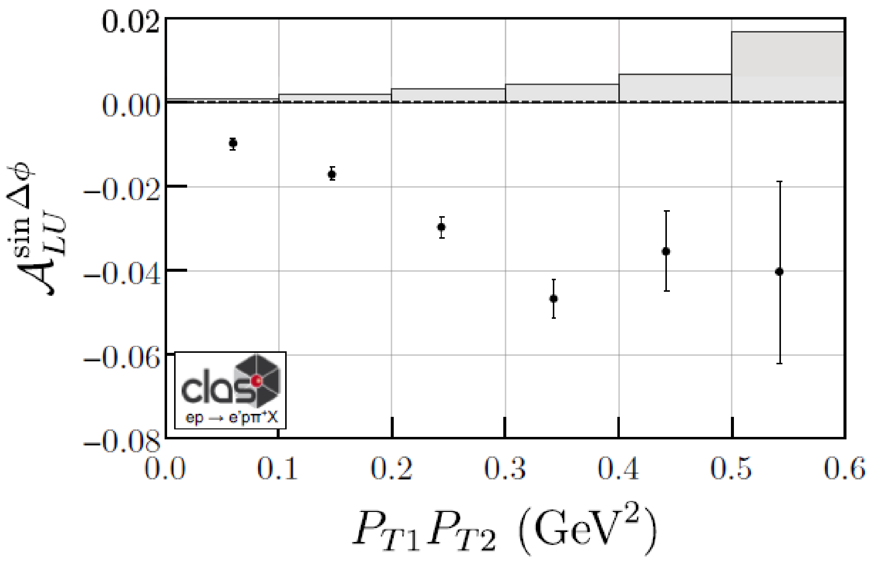
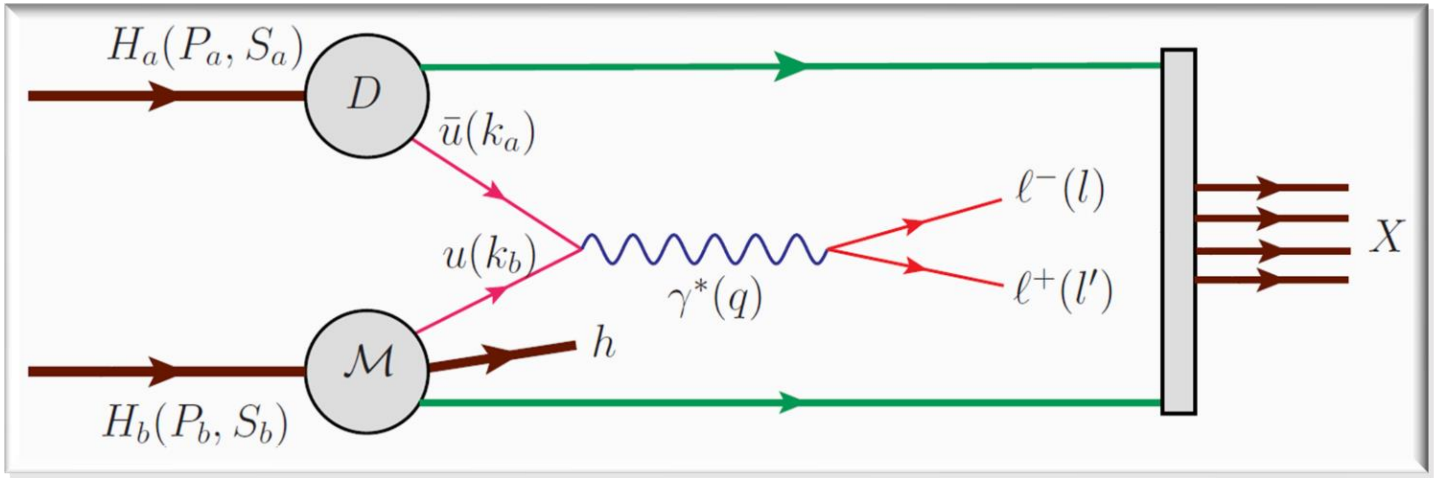


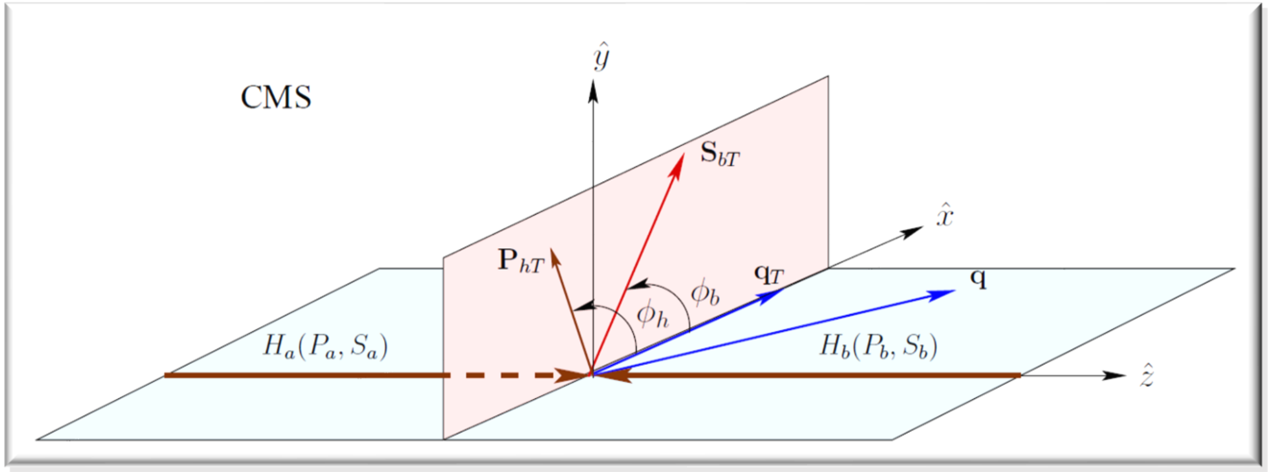

FIG. 3. The measured $\mathcal{A}_{LU}^{\sin\Delta\phi}$ asymmetry as a function of $P_{T1}P_{T2}$. Thin black bars indicate statistical uncertainties and wide gray bars represent systematic uncertainties.

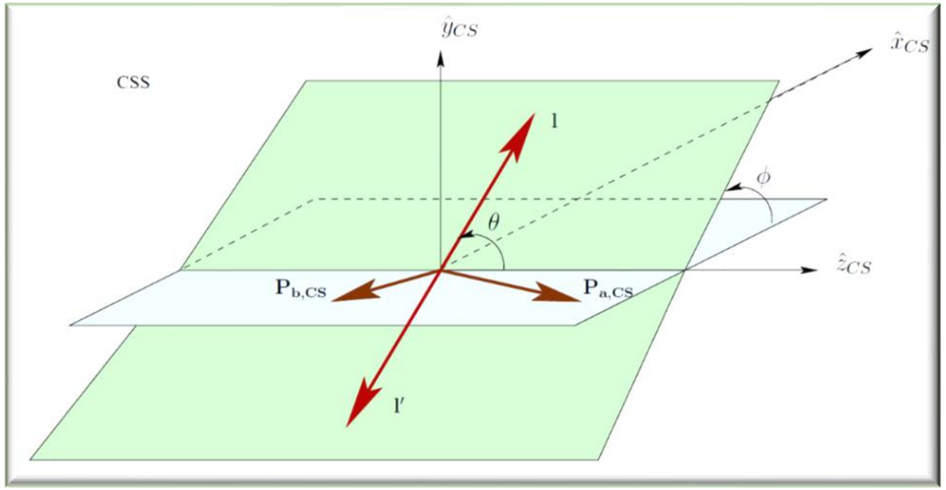
# Polarized SIDY




STAR@RHIC?

**Kinematics as in Arnold.Metz.Schlegel,** PhysRevD.79.034005



# Conclusions

- Frac. Funs: A new members of the polarized TMDs family -- 16 LO STMD fracture functions
- For hadron produced in the TFR of SIDIS, only 4 $k_T$-integrated fracture functions of unpolarized and longitudinally polarized quarks are accessible at twist-two
  - SSA contains only a Sivers-type modulation $\sin(\phi_h-\phi_S)$ but no Collins-type $\sin(\phi_h+\phi_S)$ or $\sin(3\phi_h-\phi_S)$. The eventual observation of Collins-type asymmetry will indicate that LO factorized approach fails and long-range correlations between the struck quark polarization and $P_T$ of produced in TFR hadron might be important.
- DSIDIS cross section at LO contains 2 azimuthal independent and 20 azimuthally modulated terms. Access to all 16 STMD fracture functions.
  - The first b2b $\sigma_{LU}$ asymmetry measurement at JLAB12 shows significant effect
- Polarized SIDY cross section ($p + p \to l^+l^- + h + X$) at LO contains 2 azimuthal independent, 20 lepton plane azimuthal angle independent and 52 lepton plane azimuthal angle dependent terms. In total – 74 terms. Access to all 16 STMD fracture functions. See additional slides.
- The ideal place to test the fracture functions formalism and measure these new nonperturbative objects are JLab12 (24), EIC facilities and COMPAS using Camera detector

# Additional slides

# LO double hadron production cross section

AK @ DIS2011

$$\sigma_{UU} = F_0^{\hat{u}\cdot D_1} - D_{nn}\left(\begin{array}{l} \dfrac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_1^{\perp}\cdot H_1}\cos(2\phi_1) \\ +\dfrac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_1^{h}\cdot H_1}\cos(\phi_1+\phi_2) \\ +\left(\dfrac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_1^{\perp}\cdot H_1} + \dfrac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_1^{h}\cdot H_1}\right)\cos(2\phi_2) \end{array}\right)$$

$$\boxed{\sigma_{LU} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1}\sin(\phi_1-\phi_2)}$$

# $\sigma_{UL}$

$$\sigma_{UL} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{u}_{1L}^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$

$$+D_{nn}\left( \frac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_{1L}^{\perp}\cdot H_1} \sin(2\phi_1) + \frac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_{1L}^{h}\cdot H_1} \sin(\phi_1 + \phi_2) + \left( \frac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_{1L}^{\perp}\cdot H_1} + \frac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_{1L}^{h}\cdot H_1} \right) \sin(2\phi_2) \right)$$

# $\sigma_{UT}$

$$
\begin{aligned}
\sigma_{UT} = & -\frac{P_{T1}}{m_N} F_{k1}^{\hat{u}_{1T}^{\perp}\cdot D_1} \sin(\phi_1-\phi_S) - \left( \frac{P_{T2}}{m_2} F_0^{\hat{u}_{1T}^{h}\cdot D_1} + \frac{P_{T2}}{m_N} F_{k2}^{\hat{u}_{1T}^{\perp}\cdot D_1} \right) \sin(\phi_2-\phi_S) \\
& + D_{nn}(y) \Bigg[ \left( \frac{P_{T1}}{m_1} F_{p1}^{\hat{t}_{1T}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1m_2^2} F_{p1}^{\hat{t}_{1T}^{hh}\cdot H_1} - \frac{P_{T1}P_{T2}^2}{2m_1m_2m_N} F_{kp3}^{\hat{t}_{1T}^{\perp h}\cdot H_1} + \frac{P_{T1}^3}{2m_1m_N^2} F_{kkp1}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1m_N^2} F_{kkp4}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T1}}{m_1m_N^2} F_{kkp5}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_1+\phi_S) \\
& + \left( \frac{P_{T2}}{m_1} F_{p2}^{\hat{t}_{1T}\cdot H_1} + \frac{P_{T2}^3}{2m_1m_2^2} F_{p2}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T1}^2P_{T2}}{2m_1m_2m_N} F_{kp1}^{\hat{t}_{1T}^{\perp h}\cdot H_1} + \frac{P_{T2}}{m_1m_2m_N} F_{kp4}^{\hat{t}_{1T}^{\perp h}\cdot H_1} + \frac{P_{T1}^2P_{T2}}{2m_1m_N^2} F_{kkp2}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T2}^3}{2m_1m_N^2} F_{kkp3}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T2}}{m_1m_N^2} F_{kkp6}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_2+\phi_S) \\
& + \frac{P_{T1}^3}{2m_1m_N^2} F_{kkp1}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \sin(3\phi_1-\phi_S) \\
& + \left( \frac{P_{T2}^3}{2m_1m_2^2} F_{p2}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T2}^3}{2m_1m_N^2} F_{kkp3}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(3\phi_2-\phi_S) \\
& + \left( \frac{P_{T1}P_{T2}^2}{2m_1m_2^2} F_{p1}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1m_N^2} F_{kkp4}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_1+2\phi_2-\phi_S) \\
& - \frac{P_{T1}^2P_{T2}}{2m_1m_2m_N} F_{kp1}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \sin(2\phi_1-\phi_2+\phi_S) \\
& - \frac{P_{T1}P_{T2}^2}{2m_1m_2m_N} F_{kp3}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \sin(\phi_1-2\phi_2-\phi_S) \\
& + \frac{P_{T1}^2P_{T2}}{2m_1m_N^2} F_{kkp2}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \sin(2\phi_1+\phi_2-\phi_S) \Bigg]
\end{aligned}
$$

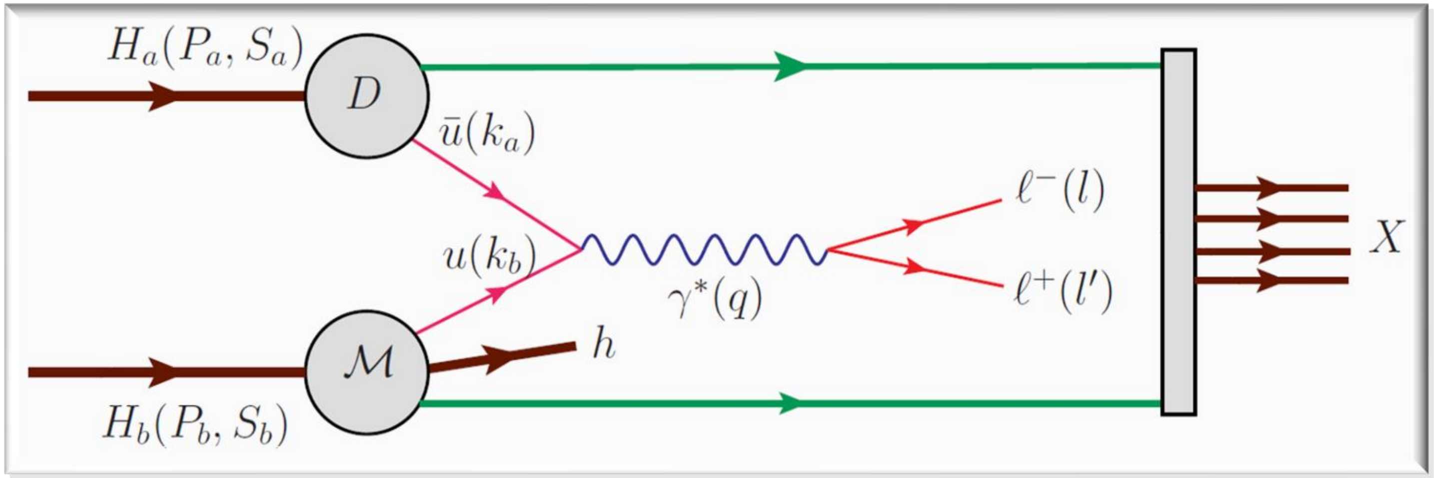
# $\sigma_{LU}$, $\sigma_{LL}$, $\sigma_{LT}$

$$\boxed{\sigma_{LU} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)}$$

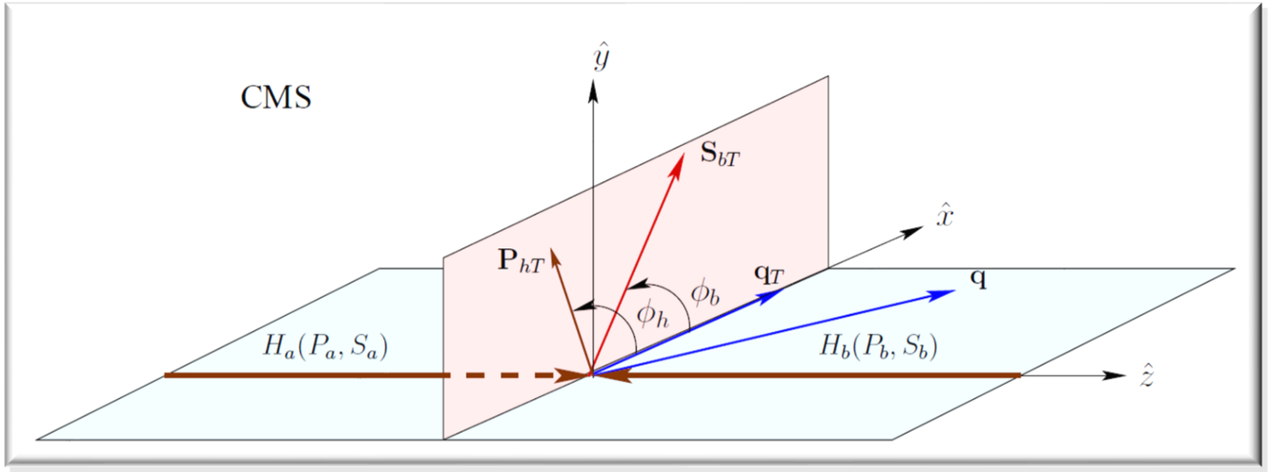
$$\sigma_{LL} = F_0^{\hat{l}_1 \cdot D_1}$$

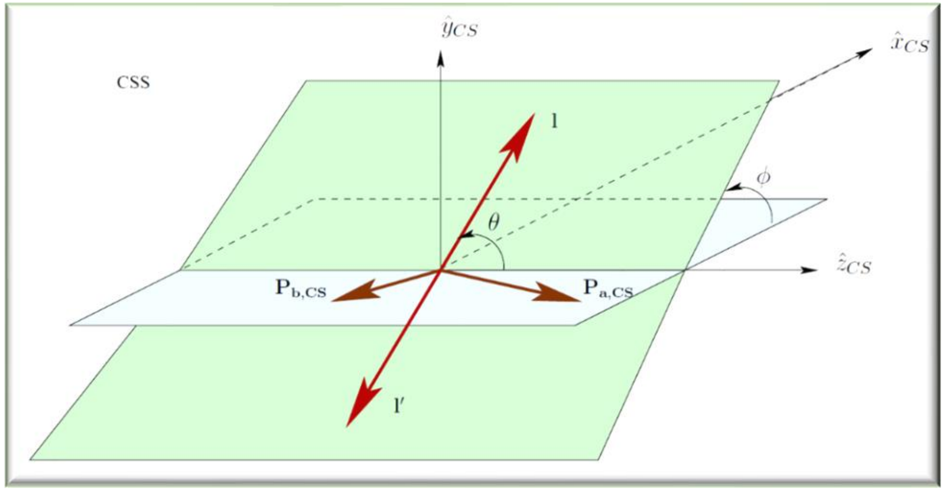
$$\sigma_{LT} = \frac{P_{T1}}{m_N} F_{k1}^{\hat{l}_{1T}^{\perp}\cdot D_1} \cos(\phi_1 - \phi_S) + \left( \frac{P_{T2}}{m_2} F_0^{\hat{l}_{1T}^{h}\cdot D_1} + \frac{P_{T2}}{m_N} F_{k2}^{\hat{l}_{1T}^{\perp}\cdot D_1} \right) \cos(\phi_2 - \phi_S)$$

# SIDY




**Kinematics as in Arnold.Metz.Schlegel,** PhysRevD.79.034005



# SIDY cross section

$$\frac{d\sigma}{d^4q\,d\Omega\,d\zeta\,d^2P_T} = \frac{\alpha_{em}^2 x_a x_b}{2q^4}\frac{1}{N_c}\sum_q e_q^2 \int d^2\vec{k}_{aT}\, d^2\vec{k}_{bT}\, \delta^{(2)}\left(\vec{q}_T - \vec{k}_{aT} - \vec{k}_{bT}\right)\times$$

$$\times\left(\begin{array}{l} \left(1+\cos^2\theta\right)\left(\Phi^{q[\gamma^+]}\overline{\mathcal{M}}^{q[\gamma^-]} + \Phi^{q[\gamma^+\gamma_5]}\overline{\mathcal{M}}^{q[\gamma^-\gamma_5]}\right) \\ +\sin^2\theta\left(\begin{array}{l}\cos 2\phi\left(\delta^{i1}\delta^{j1} - \delta^{i2}\delta^{j2}\right) \\ +\sin 2\phi\left(\delta^{i1}\delta^{j2} + \delta^{i2}\delta^{j1}\right)\end{array}\right)\Phi^{q[i\sigma^{i+}\gamma_5]}\overline{\mathcal{M}}^{q[i\sigma^{j-}\gamma_5]} \\ +\{\Phi \leftrightarrow \overline{\Phi}, \overline{\mathcal{M}} \leftrightarrow \mathcal{M}\} + \mathcal{O}(1/q) \end{array}\right)$$

$$= \frac{\alpha_{em}^2 x_a x_b}{2q^4}\left(\begin{array}{l} \sigma_{UU} + S_{bL}\sigma_{UL} + S_{bT}\sigma_{UT} \\ +S_{aL}\sigma_{LU} + S_{aL}S_{bL}\sigma_{LL} + S_{aL}S_{bT}\sigma_{LT} \\ +S_{aT}\sigma_{TU} + S_{aT}S_{bL}\sigma_{TL} + S_{aT}S_{bT}\sigma_{TT} \end{array}\right)$$

# $\sigma_{UU}$

$$\sigma_{UU} = \left(1+\cos^2\theta\right) F_{UU}$$

$$-\sin^2\theta \left[ \begin{array}{l} F_{UU}^{\cos(2\phi)} \cos(2\phi) \\ +F_{UU}^{\cos(2\phi-\phi_h)} \cos(2\phi-\phi_h) \\ +F_{UU}^{\cos(2\phi-2\phi_h)} \cos(2\phi-2\phi_h) \end{array} \right]$$

$$F_{UU} = F_0^{f_1 \cdot \hat{u}_1}$$

$$F_{UU}^{\cos(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_1^\perp \cdot \hat{u}_{1T}^\perp}$$

$$F_{UU}^{\cos(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_1^\perp \cdot \hat{u}_{1T}^h}$$

$$F_{UU}^{\cos(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_1^\perp \cdot \hat{u}_{1T}^h} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_1^\perp \cdot \hat{u}_{1T}^\perp} \hat{u}_{1T}^\perp$$

| | | Quark polarization | | |
|---|---|---|---|---|
| | | U | L | T |
| Nucleon Polarization | U | * | | * |
| | L | | | |
| | T | | | |

# $\sigma_{UL}$

Detected hadron originates from longitudinally polarized nucleon remnant (hadron b in reaction diagram).

$$\sigma_{UL} = \left(1+\cos^2\theta\right) F_{LU}^{\sin(\phi_h)} \sin\left(\phi_h\right) - \sin^2\theta \left[ F_{LU}^{\sin(2\phi-2\phi_h)} \sin\left(2\phi - 2\phi_h\right) + F_{LU}^{\sin(2\phi-\phi_h)} \sin\left(2\phi-\phi_h\right) + F_{LU}^{\sin(2\phi)} \sin(2\phi) \right]$$

$$\boxed{F_{UL}^{\sin(\phi_h)} = \frac{P_T q_T}{m M_b} F_{b2}^{f_1 \cdot \hat{l}_1^{\perp h}}}$$

| Nucleon Polarization \ Quark polarization | U | L | T |
|---|---|---|---|
| U | | * | * |
| L | | | |
| T | | | |

After integration over lepton scattering plane azimuth alangle $\phi$

only first term survives and gives access to the fracture function $\hat{l}_1^{\perp h}$

# Convolutions & tensorial decomposition

$$C[\hat{M}\cdot Dw]=\sum_a e_a^2\int d^2k_T\, d^2p_T\, \delta^{(2)}(z\mathbf{k}_T+\mathbf{p}_T-\mathbf{P}_{T1})\hat{M}_a(x,\zeta,k_T^2,P_{T2}^2,\mathbf{k}_T\cdot\mathbf{P}_{T2})D_a(z,p_T^2)w$$

$$C[\hat{M}\cdot D]=F_0^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^i]=P_{T1}^i F_{k1}^{\hat{M}\cdot D}+P_{T2}^i F_{k2}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot D\,p^i]=P_{T1}^i F_{p1}^{\hat{M}\cdot D}+P_{T2}^i F_{p2}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^i k^j]=P_{T1}^i P_{T1}^j F_{kk1}^{\hat{M}\cdot D}+P_{T2}^i P_{T2}^j F_{kk2}^{\hat{M}\cdot D}+\delta^{ij}F_{kk3}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^i p^j]=P_{T1}^i P_{T1}^j F_{kp1}^{\hat{M}\cdot D}+P_{T2}^i P_{T2}^j F_{kp2}^{\hat{M}\cdot D}+\left(P_{T1}^i P_{T2}^j-P_{T1}^j P_{T2}^i\right)F_{kp3}^{\hat{M}\cdot D}+\delta^{ij}F_{kp4}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^i k^j p^k]=P_{T1}^i P_{T1}^j P_{T1}^k F_{kkp1}^{\hat{M}\cdot D}+P_{T1}^i P_{T1}^j P_{T2}^k F_{kkp2}^{\hat{M}\cdot D}+P_{T2}^i P_{T2}^j P_{T2}^k F_{kkp3}^{\hat{M}\cdot D}$$
$$+P_{T2}^i P_{T2}^j P_{T1}^k F_{kkp4}^{\hat{M}\cdot D}+P_{T1}^k\delta^{ij}F_{kkp5}^{\hat{M}\cdot D}+P_{T2}^k\delta^{ij}F_{kkp6}^{\hat{M}\cdot D}$$

where $F_{\dots}^{\hat{M}\cdot D}$ depend on $x,z,\zeta,P_{T1}^2,P_{T2}^2,(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})$

# Structure functions

$$F_{k1}^{\hat{M}\cdot D}=C\left[\hat{M}\cdot D\,\frac{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})(\mathbf{P}_{T2}\cdot\mathbf{k})-(\mathbf{P}_{T1}\cdot\mathbf{k})\mathbf{P}_{T2}^2}{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2}\right]$$

$$F_{k2}^{\hat{M}\cdot D}=C\left[\hat{M}\cdot D\,\frac{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})(\mathbf{P}_{T1}\cdot\mathbf{k})-(\mathbf{P}_{T2}\cdot\mathbf{k})\mathbf{P}_{T1}^2}{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2}\right]$$

$$F_{kk1}^{\hat{M}\cdot D}=C\left[\hat{M}\cdot D\,\frac{\left(-2(\mathbf{P}_{T1}\cdot\mathbf{k})^2+\mathbf{k}^2\mathbf{P}_{T1}^2\right)\mathbf{P}_{T2}^4+\left(2(\mathbf{P}_{T2}\cdot\mathbf{k})^2-\mathbf{k}^2\mathbf{P}_{T2}^2\right)\left(2(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}{4(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2\left((\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}\right]$$

$$F_{kk2}^{\hat{M}\cdot D}=C\left[\hat{M}\cdot D\,\frac{\left(2(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)(\mathbf{P}_{T1}\cdot\mathbf{k})^2+\mathbf{P}_{T1}^2\left(\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2-(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2\right)\mathbf{k}^2-(\mathbf{P}_{T2}\cdot\mathbf{k})^2\,\mathbf{P}_{T1}^4}{2(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2\left((\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}\right]$$

$$F_{kk3}^{\hat{M}\cdot D}=C\left[\hat{M}\cdot D\,\frac{\left((\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2+\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)\mathbf{k}^2-(\mathbf{P}_{T2}\cdot\mathbf{k})^2\,\mathbf{P}_{T1}^2-(\mathbf{P}_{T1}\cdot\mathbf{k})^2\,\mathbf{P}_{T2}^2}{2(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2}\right]$$

$$F_{kp1}^{\hat{M}\cdot D}=C\left[\hat{M}\cdot D\left(\frac{\left(-2(\mathbf{P}_{T1}\cdot\mathbf{k})(\mathbf{P}_{T1}\cdot\mathbf{p})+(\mathbf{k}\cdot\mathbf{p})\mathbf{P}_{T1}^2\right)\mathbf{P}_{T2}^4}{4(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2\left((\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^4-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}+\frac{\left(2(\mathbf{P}_{T2}\cdot\mathbf{k})(\mathbf{P}_{T2}\cdot\mathbf{p})-(\mathbf{k}\cdot\mathbf{p})\mathbf{P}_{T2}^2\right)\left(2(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}{4(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2\left((\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^4-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}\right)\right]$$

# $\sigma_{UU}$

$$\sigma_{UU} = \left(1+\cos^2\theta\right) F_{UU}$$

$$-\sin^2\theta \left[ \begin{array}{l} F_{UU}^{\cos(2\phi)} \cos(2\phi) \\ +F_{UU}^{\cos(2\phi-\phi_h)} \cos(2\phi-\phi_h) \\ +F_{UU}^{\cos(2\phi-2\phi_h)} \cos(2\phi-2\phi_h) \end{array} \right]$$

$$F_{UU} = F_0^{f_1 \cdot \hat{u}_1}$$

$$F_{UU}^{\cos(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_1^\perp \cdot \hat{u}_{1T}^\perp}$$

$$F_{UU}^{\cos(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_1^\perp \cdot \hat{u}_{1T}^h}$$

$$F_{UU}^{\cos(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_1^\perp \cdot \hat{u}_{1T}^h} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_1^\perp \cdot \hat{u}_{1T}^\perp} \hat{u}_{1T}^\perp$$

| | | Quark polarization | | |
|---|---|---|---|---|
| | | U | L | T |
| Nucleon Polarization | U | * | | * |
| | L | | | |
| | T | | | |

# $\sigma_{UL}$

$$\sigma_{UL} = \left(1+\cos^2\theta\right) F_{UL}^{\sin(\phi_h)} \sin(\phi_h)$$

$$-\sin^2\theta \left[ \begin{array}{l} F_{UL}^{\sin(2\phi)} \sin(2\phi) \\ +F_{UL}^{\sin(2\phi-\phi_h)} \sin(2\phi-\phi_h) \\ +F_{UL}^{\sin(2\phi-2\phi_h)} \sin(2\phi-2\phi_h) \end{array} \right]$$

| | | Quark polarization | | |
|---|---|---|---|---|
| | | U | L | T |
| Nucleon Polarization | U | | | |
| | L | * | | * |
| | T | | | |

# $\sigma_{UT}$

$$\sigma_{UT} = \left(1+\cos^2\theta\right)\left[\begin{array}{l} F_{UT}^{\sin(\phi_b-\phi_h)}\sin\left(\phi_b-\phi_h\right) \\ +F_{UT}^{\sin(\phi_b)}\sin\left(\phi_b\right) \end{array}\right]$$

$$-\sin^2\theta\left[\begin{array}{l} F_{UT}^{\sin(2\phi+\phi_b)}\sin\left(2\phi+\phi_b\right) \\ +F_{UT}^{\sin(2\phi-\phi_b+\phi_h)}\sin\left(2\phi-\phi_b+\phi_h\right) \\ +F_{UT}^{\sin(2\phi+\phi_b-\phi_h)}\sin\left(2\phi+\phi_b-\phi_h\right) \\ +F_{UT}^{\sin(2\phi+\phi_b-3\phi_h)}\sin\left(2\phi+\phi_b-3\phi_h\right) \\ +F_{UT}^{\sin(2\phi-\phi_b-2\phi_h)}\sin\left(2\phi-\phi_b-2\phi_h\right) \\ +F_{UT}^{\sin(2\phi+\phi_b-2\phi_h)}\sin\left(2\phi+\phi_b-2\phi_h\right) \\ +F_{UT}^{\sin(2\phi-\phi_b-\phi_h)}\sin\left(2\phi-\phi_b-\phi_h\right) \\ +F_{UT}^{\sin(2\phi-\phi_b)}\sin\left(2\phi-\phi_b\right) \end{array}\right]$$

| | | Quark polarization | | |
|---|---|---|---|---|
| | | U | L | T |
| Nucleon Polarization | U | | | |
| | L | | | |
| | T | * | | * |

# $\sigma_{LU}$

$$\sigma_{LU} = \left(1+\cos^2\theta\right) F_{LU}^{\sin(\phi_h)} \sin\left(\phi_h\right) - \sin^2\theta \left[ \begin{array}{l} F_{LU}^{\sin(2\phi-2\phi_h)} \sin\left(2\phi-2\phi_h\right) \\ +F_{LU}^{\sin(2\phi-\phi_h)} \sin\left(2\phi-\phi_h\right) \\ +F_{LU}^{\sin(2\phi)} \sin(2\phi) \end{array} \right]$$

$$\boxed{F_{LU}^{\sin(\phi_h)} = \frac{P_T q_T}{m M_b} F_{b2}^{f_1 \cdot \hat{l}_1^{\perp h}}}$$

| Nucleon Polarization | Quark polarization: U | L | T |
|---|---|---|---|
| U | | * | * |
| L | | | |
| T | | | |

After integration over azimuthal angle of lepton scattering plane only first term survives and gives access to $\hat{l}_1^{\perp h}$ fracture function

# $\sigma_{LL}$

$$\sigma_{LL} = \left(1 + \cos^2\theta\right) F_{LL} + \sin^2\theta \left[ F_{LL}^{\cos(2\phi - 2\phi_h)} \cos\left(2\phi - 2\phi_h\right) + F_{LL}^{\cos(2\phi - \phi_h)} \cos\left(2\phi - \phi_h\right) + F_{LL}^{\cos(2\phi)} \cos(2\phi) \right]$$

| | | Quark polarization | | |
|---|---|---|---|---|
| | | U | L | T |
| Nucleon Polarization | U | | | |
| | L | | * | * |
| | T | | | |

# $\sigma_{LT}$

$$\sigma_{LT} = \left(1+\cos^2\theta\right)\left[\begin{array}{l} F_{LT}^{\cos(\phi_b-\phi_h)}\cos\left(\phi_b-\phi_h\right) \\ +F_{LT}^{\cos(\phi_b)}\cos\left(\phi_b\right) \end{array}\right]$$

$$+\sin^2\theta\left[\begin{array}{l} F_{LT}^{\cos(2\phi-\phi_b+\phi_h)}\cos\left(2\phi-\phi_b+\phi_h\right) \\ +F_{LT}^{\cos(2\phi+\phi_b)}\cos\left(2\phi+\phi_b\right)+ \\ F_{LT}^{\cos(2\phi+\phi_b-\phi_h)}\cos\left(2\phi+\phi_b-\phi_h\right) \\ +F_{LT}^{\cos(2\phi+\phi_b-3\phi_h)}\cos\left(2\phi+\phi_b-3\phi_h\right) \\ +F_{LT}^{\cos(2\phi-\phi_b-2\phi_h)}\cos\left(2\phi-\phi_b-2\phi_h\right) \\ +F_{LT}^{\cos(2\phi+\phi_b-2\phi_h)}\cos\left(2\phi+\phi_b-2\phi_h\right) \\ +F_{LT}^{\cos(2\phi-\phi_b-\phi_h)}\cos\left(2\phi-\phi_b-\phi_h\right) \\ +F_{LT}^{\cos(2\phi-\phi_b)}\cos\left(2\phi-\phi_b\right) \end{array}\right]$$

| | | Quark polarization | | |
|---|---|---|---|---|
| | | U | L | T |
| Nucleon Polarization | U | | | |
| | L | | | |
| | T | | * | * |

# $\sigma_{TU}$

$$
\begin{aligned}
\sigma_{TU} = {} & \left(1+\cos^2\theta\right)\left[\begin{array}{l}
F_{TU}^{\sin\left(\phi_a-\phi_h\right)}\sin\left(\phi_a-\phi_h\right) \\
+F_{TU}^{\sin\left(\phi_a\right)}\sin\left(\phi_a\right) \\
+F_{TU}^{\sin\left(\phi_a+\phi_h\right)}\sin\left(\phi_a+\phi_h\right) \\
+F_{TU}^{\sin\left(\phi_a-2\phi_h\right)}\sin\left(\phi_a-2\phi_h\right)
\end{array}\right] \\
& +\sin^2\theta\left[\begin{array}{l}
F_{TU}^{\sin\left(2\phi-\phi_a-\phi_h\right)}\sin\left(2\phi-\phi_a-\phi_h\right) \\
+F_{TU}^{\sin\left(2\phi-\phi_a\right)}\sin\left(2\phi-\phi_a\right) \\
+F_{TU}^{\sin\left(2\phi+\phi_a-3\phi_h\right)}\sin\left(2\phi+\phi_a-3\phi_h\right)+ \\
F_{TU}^{\sin\left(2\phi+\phi_a-\phi_h\right)}\sin\left(2\phi+\phi_a-\phi_h\right) \\
+F_{TU}^{\sin\left(2\phi+\phi_a-2\phi_h\right)}\sin\left(2\phi+\phi_a-2\phi_h\right) \\
+F_{TU}^{\sin\left(2\phi+\phi_a\right)}\sin\left(2\phi+\phi_a\right)
\end{array}\right]
\end{aligned}
$$

| | | Quark polarization | | |
|---|---|---|---|---|
| | | U | L | T |
| Nucleon Polarization | U | * | * | * |
| | L | | | |
| | T | | | |

# $\sigma_{TL}$

$$\sigma_{TL} = \left(1+\cos^2\theta\right)\left[\begin{array}{l} F_{TL}^{\cos(\phi_a-\phi_h)}\cos(\phi_a-\phi_h) \\ +F_{TL}^{\cos(\phi_a)}\cos(\phi_a) \\ +F_{TL}^{\cos(\phi_a+\phi_h)}\cos(\phi_a+\phi_h) \\ +F_{TL}^{\cos(\phi_a-2\phi_h)}\cos(\phi_a-2\phi_h) \end{array}\right]$$

$$+\sin^2\theta\left[\begin{array}{l} F_{TL}^{\cos(2\phi+\phi_a-3\phi_h)}\cos(2\phi+\phi_a-3\phi_h) \\ +F_{TL}^{\cos(2\phi+\phi_a-2\phi_h)}\cos(2\phi+\phi_a-2\phi_h) \\ +F_{TL}^{\cos(2\phi+\phi_a)}\cos(2\phi+\phi_a) \\ +F_{TL}^{\cos(2\phi-\phi_a-\phi_h)}\cos(2\phi-\phi_a-\phi_h) \\ +F_{TL}^{\cos(2\phi-\phi_a)}\cos(2\phi-\phi_a) \end{array}\right]$$

| | | Quark polarization | | |
|---|---|---|---|---|
| | | U | L | T |
| Nucleon Polarization | U | | | |
| | L | | * | * |
| | T | | | |

# $\sigma_{TT}$

$$\sigma_{TT} = \left(1+\cos^2\theta\right)\left[\begin{array}{l} F_{TT}^{\cos(\phi_a-\phi_b)}\cos(\phi_a-\phi_b) \\ +F_{TT}^{\cos(\phi_a+\phi_b-2\phi_h)}\cos(\phi_a+\phi_b-2\phi_h) \\ +F_{TT}^{\cos(\phi_a-\phi_b+\phi_h)}\cos(\phi_a-\phi_b+\phi_h) \\ +F_{TT}^{\cos(\phi_a+\phi_b-\phi_h)}\cos(\phi_a+\phi_b-\phi_h) \\ +F_{TT}^{\cos(\phi_a+\phi_b)}\cos(\phi_a+\phi_b) \\ +F_{TT}^{\cos(\phi_a-\phi_b-\phi_h)}\cos(\phi_a-\phi_b-\phi_h) \end{array}\right] + \sin^2\theta\left[\begin{array}{l} F_{TT}^{\cos(2\phi-\phi_a-\phi_b-\phi_h)}\cos(2\phi-\phi_a-\phi_b-\phi_h) \\ +F_{TT}^{\cos(2\phi-\phi_a-\phi_b+\phi_h)}\cos(2\phi-\phi_a-\phi_b+\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a+\phi_b)}\cos(2\phi+\phi_a+\phi_b) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b+\phi_h)}\cos(2\phi+\phi_a-\phi_b+\phi_h)+ \\ F_{TT}^{\cos(2\phi-\phi_a+\phi_b)}\cos(2\phi-\phi_a+\phi_b) \\ +F_{TT}^{\cos(2\phi-\phi_a+\phi_b-2\phi_h)}\cos(2\phi-\phi_a+\phi_b-2\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a+\phi_b-4\phi_h)}\cos(2\phi+\phi_a+\phi_b-4\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b-3\phi_h)}\cos(2\phi+\phi_a-\phi_b-3\phi_h) \\ +F_{TT}^{\cos(2\phi-\phi_a-\phi_b)}\cos(2\phi-\phi_a-\phi_b) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b-2\phi_h)}\cos(2\phi+\phi_a-\phi_b-2\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a+\phi_b-2\phi_h)}\cos(2\phi+\phi_a+\phi_b-2\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b-\phi_h)}\cos(2\phi+\phi_a-\phi_b-\phi_h) \\ +F_{TT}^{\cos(2\phi+\phi_a-\phi_b)}\cos(2\phi+\phi_a-\phi_b) \end{array}\right]$$

| | | Quark polarization | | |
|---|---|---|---|---|
| | | U | L | T |
| Nucleon Polarization | U | | | |
| | L | | | |
| | T | * | * | * |

# Structure functions of $\sigma_{UU}$, $\sigma_{UL}$, $\sigma_{UT}$

$$F_{UU} = F_0^{f_1\cdot\hat{u}_1}$$

$$F_{UU}^{\cos(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_1^\perp\cdot\hat{u}_{1T}^h}$$

$$F_{UU}^{\cos(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_1^\perp\cdot\hat{u}_{1T}^h}$$

$$F_{UU}^{\cos(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_1^\perp\cdot\hat{u}_{1T}^h} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_1^\perp\cdot\hat{u}_{1T}^\perp}$$

$$F_{UL}^{\sin(\phi_h)} = \frac{P_T q_T}{m M_b} F_{b2}^{f_1\cdot\hat{l}_1^{\perp h}}$$

$$F_{UL}^{\sin(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_1^\perp\cdot\hat{l}_{1T}^\perp}$$

$$F_{UL}^{\sin(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_1^\perp\cdot\hat{l}_{1T}^h}$$

$$F_{UL}^{\sin(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_1^\perp\cdot\hat{l}_{1T}^h} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_1^\perp\cdot\hat{l}_{1T}^\perp}$$

$$F_{UT}^{\sin(\phi_b-\phi_h)} = \frac{P_T}{m} F_0^{f_1\cdot\hat{t}_1^h} + \frac{P_T}{M_b} F_{b1}^{f_1\cdot\hat{t}_1^\perp}, \qquad F_{UT}^{\sin(\phi_b)} = \frac{q_T}{M_b} F_{b2}^{f_1\cdot\hat{t}_1^\perp}$$

$$F_{UT}^{\sin(2\phi-\phi_b+\phi_h)} = -\frac{P_T q_T^2}{2m M_a M_b} F_{ab2}^{h_1^\perp\cdot\hat{t}_1^{\perp h}}, \qquad F_{UT}^{\sin(2\phi+\phi_b)} = \frac{q_T^3}{2M_a M_b^2} F_{abb3}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}}, \qquad F_{UT}^{\sin(2\phi+\phi_b-\phi_h)} = \frac{P_T q_T^2}{2M_a M_b^2} F_{abb4}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}}$$

$$F_{UT}^{\sin(2\phi+\phi_b-3\phi_h)} = \frac{P_T^3}{2m^2 M_a} F_{a1}^{h_1^\perp\cdot\hat{t}_{1T}^{hh}} + \frac{P_T^3}{2M_a M_b^2} F_{abb1}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}}, \qquad F_{UT}^{\sin(2\phi-\phi_b-2\phi_h)} = \frac{P_T^2 q_T}{2m M_a M_b} F_{ab3}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp h}}$$

$$F_{UT}^{\sin(2\phi+\phi_b-2\phi_h)} = \frac{P_T^2 q_T}{2m^2 M_a} F_{a2}^{h_1^\perp\cdot\hat{t}_{1T}^{hh}} + \frac{P_T^2 q_T}{2M_a M_b^2} F_{abb2}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}}$$

$$F_{UT}^{\sin(2\phi-\phi_b-\phi_h)} = \left(\begin{array}{l} \dfrac{P_T}{M_a} F_{a1}^{h_1^\perp\cdot\hat{t}_{1T}} + \dfrac{P_T^3}{2m^2 M_a} F_{a1}^{h_1^\perp\cdot\hat{t}_{1T}^{hh}} + \dfrac{P_T^3}{2M_a M_b^2} F_{abb1}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}} + \dfrac{P_T q_T^2}{2m M_a M_b} F_{ab2}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp h}} \\ + \dfrac{P_T q_T^2}{2M_a M_b^2} F_{abb4}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}} + \dfrac{P_T}{m M_a M_b} F_{ab4}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp h}} + \dfrac{P_T}{M_a M_b^2} F_{abb5}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}} \end{array}\right)$$

$$F_{UT}^{\sin(2\phi-\phi_b)} = \left(\begin{array}{l} \dfrac{q_T}{M_a} F_{a2}^{h_1^\perp\cdot\hat{t}_{1T}} + \dfrac{P_T^2 q_T}{2m^2 M_a} F_{a2}^{h_1^\perp\cdot\hat{t}_{1T}^{hh}} + \dfrac{q_T^3}{2M_a M_b^2} F_{abb3}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}} \\ - \dfrac{P_T^2 q_T}{2m M_a M_b} F_{ab3}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp h}} + \dfrac{P_T^2 q_T}{2M_a M_b^2} F_{abb2}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}} + \dfrac{q_T}{M_a M_b^2} F_{abb6}^{h_1^\perp\cdot\hat{t}_{1T}^{\perp\perp}} \end{array}\right)$$

# Structure functions of $\sigma_{LU}$, $\sigma_{LL}$

$$F_{LU}^{\sin(\phi_h)} = \frac{P_T q_T}{m M_b} F_{b2}^{g_{1L}\cdot \hat{u}_{1L}^{\perp h}}$$

$$F_{LU}^{\sin(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_{1L}^{\perp}\cdot \hat{u}_{1T}^{h}} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_{1L}^{\perp}\cdot \hat{u}_{1T}^{\perp}}$$

$$F_{LU}^{\sin(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_{1L}^{\perp}\cdot \hat{u}_{1T}^{h}}$$

$$F_{LU}^{\sin(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_{1L}^{\perp}\cdot \hat{u}_{1T}^{\perp}}$$

$$F_{LL} = F_0^{g_{1L}\cdot \hat{l}_{1L}}$$

$$F_{LL}^{\cos(2\phi-2\phi_h)} = \frac{P_T^2}{m M_a} F_{a1}^{h_{1L}^{\perp}\cdot \hat{l}_{1T}^{h}} + \frac{P_T^2}{M_a M_b} F_{ab1}^{h_{1L}^{\perp}\cdot \hat{l}_{1T}^{\perp}}$$

$$F_{LL}^{\cos(2\phi-\phi_h)} = \frac{P_T q_T}{m M_a} F_{a2}^{h_{1L}^{\perp}\cdot \hat{l}_{1T}^{h}}$$

$$F_{LL}^{\cos(2\phi)} = \frac{q_T^2}{M_a M_b} F_{ab2}^{h_{1L}^{\perp}\cdot \hat{l}_{1T}^{\perp}}$$

# Structure functions of $\sigma_{LT}$

$$F_{LT}^{\cos(\phi_b-\phi_h)} = \frac{P_T}{m} F_0^{g_{1L}\cdot \hat{t}_{1L}^{h}} + \frac{P_T}{M_b} F_{b1}^{g_{1L}\cdot \hat{t}_{1L}^{\perp}}$$

$$F_{LT}^{\cos(\phi_b)} = \frac{q_T}{M_b} F_{b2}^{g_{1L}\cdot \hat{t}_{1L}^{\perp}}$$

$$F_{LT}^{\cos(2\phi-\phi_b+\phi_h)} = -\frac{P_T q_T^2}{2mM_a M_b} F_{ab2}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}^{\perp h}}$$

$$F_{LT}^{\cos(2\phi+\phi_b)} = \frac{q_T^3}{2M_a M_b^2} F_{abb3}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}^{\perp\perp}}$$

$$F_{LT}^{\cos(2\phi+\phi_b-\phi_h)} = \frac{P_T q_T^2}{2M_a M_b^2} F_{abb4}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}^{\perp\perp}}$$

$$F_{LT}^{\cos(2\phi+\phi_b-3\phi_h)} = \frac{P_T^3}{2m^2 M_a} F_{a1}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}^{hh}} + \frac{P_T^3}{2M_a M_b^2} F_{abb1}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}^{\perp\perp}}$$

$$F_{LT}^{\cos(2\phi-\phi_b-2\phi_h)} = \frac{P_T^2 q_T}{2mM_a M_b} F_{ab3}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}^{\perp h}}$$

$$F_{LT}^{\cos(2\phi+\phi_b-2\phi_h)} = \frac{P_T^2 q_T}{2m^2 M_a} F_{a2}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}^{hh}} + \frac{P_T^2 q_T}{2M_a M_b^2} F_{abb2}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}^{\perp\perp}}$$

$$F_{LT}^{\cos(2\phi-\phi_b-\phi_h)} = \left( \begin{array}{l} \frac{P_T}{M_a} F_{a1}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}} + \frac{P_T^3}{2m^2 M_a} F_{a1}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}^{hh}} + \frac{P_T^3}{2M_a M_b^2} F_{abb1}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}^{\perp\perp}} + \frac{P_T q_T^2}{2mM_a M_b} F_{ab2}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}^{\perp h}} \\ + \frac{P_T q_T^2}{2M_a M_b^2} F_{abb4}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}^{\perp\perp}} + \frac{P_T}{mM_a M_b} F_{ab4}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}^{\perp h}} + \frac{P_T}{M_a M_b^2} F_{abb5}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}^{\perp\perp}} \end{array} \right)$$

$$F_{LT}^{\cos(2\phi-\phi_b)} = \left( \begin{array}{l} \frac{q_T}{M_a} F_{a2}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}} + \frac{P_T^2 q_T}{2m^2 M_a} F_{a2}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}^{hh}} + \frac{q_T^3}{2M_a M_b^2} F_{abb3}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}^{\perp\perp}} - \frac{P_T^2 q_T}{2mM_a M_b} F_{ab3}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}^{\perp h}} \\ + \frac{P_T^2 q_T}{2M_a M_b^2} F_{abb2}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}^{\perp\perp}} + \frac{q_T}{M_a M_b^2} F_{abb6}^{h_{1L}^{\perp}\cdot \hat{t}_{1T}^{\perp\perp}} \end{array} \right)$$

# Structure functions of $\sigma_{TU}$

$$F_{TU}^{\sin(\phi_a-\phi_h)} = -\frac{P_T}{M_a}F_{a1}^{f_{1T}^{\perp}\cdot\hat{u}_1} - \frac{P_T q_T^2}{2mM_aM_b}F_{ab2}^{g_{1T}\cdot\hat{u}_{1L}^{\perp h}} - \frac{P_T}{mM_aM_b}F_{ab4}^{g_{1T}\cdot\hat{u}_{1L}^{\perp h}}$$

$$F_{TU}^{\sin(\phi_a)} = -\frac{q_T}{M_a}F_{a2}^{f_{1T}^{\perp}\cdot\hat{u}_1} + \frac{P_T^2 q_T}{2mM_aM_b}F_{ab3}^{g_{1T}\cdot\hat{u}_{1L}^{\perp h}}$$

$$F_{TU}^{\sin(\phi_a+\phi_h)} = \frac{P_T q_T^2}{2mM_aM_b}F_{ab2}^{g_{1T}\cdot\hat{u}_{1L}^{\perp h}}$$

$$F_{TU}^{\sin(\phi_a-2\phi_h)} = -\frac{P_T^2 q_T}{2mM_aM_b}F_{ab3}^{g_{1T}\cdot\hat{u}_{1L}^{\perp h}}$$

$$F_{TU}^{\sin(2\phi-\phi_a-\phi_h)} = -\frac{P_T}{M_b}F_{b1}^{h_1\cdot\hat{u}_{1T}^{\perp}} - \frac{P_T}{m}F_0^{h_1\cdot\hat{u}_{1T}^{h}}$$

$$F_{TU}^{\sin(2\phi-\phi_a)} = -\frac{q_T F_{b2}^{h_1\cdot\hat{u}_{1T}^{\perp}}}{M_b}$$

$$F_{TU}^{\sin(2\phi+\phi_a-3\phi_h)} = -\frac{P_T^3}{2mM_a^2}F_{aa1}^{h_{1T}^{\perp}\cdot\hat{u}_{1T}^{h}} - \frac{P_T^3}{2M_a^2M_b}F_{aab1}^{h_{1T}^{\perp}\cdot\hat{u}_{1T}^{\perp}}$$

$$F_{TU}^{\sin(2\phi+\phi_a-\phi_h)} = -\frac{P_T q_T^2}{2mM_a^2}F_{aa2}^{h_{1T}^{\perp}\cdot\hat{u}_{1T}^{h}} - \frac{P_T q_T^2}{2M_a^2M_b}F_{aab4}^{h_{1T}^{\perp}\cdot\hat{u}_{1T}^{\perp}}$$

$$F_{TU}^{\sin(2\phi+\phi_a-2\phi_h)} = -\frac{P_T^2 q_T}{2M_a^2M_b}F_{aab2}^{h_{1T}^{\perp}\cdot\hat{u}_{1T}^{\perp}}$$

$$F_{TU}^{\sin(2\phi+\phi_a)} = -\frac{q_T^3}{2M_a^2M_b}F_{aab3}^{h_{1T}^{\perp}\cdot\hat{u}_{1T}^{\perp}}$$

# Structure functions of $\sigma_{TL}$

$$F_{TL}^{\cos(\phi_a-\phi_h)} = \frac{P_T}{M_a} F_{a1}^{g_{1T}\cdot\hat{l}_{1L}} - \frac{P_T q_T^2}{2mM_a M_b} F_{ab2}^{f_{1T}^{\perp}\cdot\hat{u}_{1L}^{\perp h}} - \frac{P_T}{mM_a M_b} F_{ab4}^{f_{1T}^{\perp}\cdot\hat{u}_{1L}^{\perp h}}$$

$$F_{TL}^{\cos(\phi_a)} = \frac{q_T}{M_a} F_{a2}^{g_{1T}\cdot\hat{l}_{1L}} + \frac{P_T^2 q_T}{2mM_a M_b} F_{ab3}^{f_{1T}^{\perp}\cdot\hat{u}_{1L}^{\perp h}}$$

$$F_{TL}^{\cos(\phi_a+\phi_h)} = \frac{P_T q_T^2}{2mM_a M_b} F_{ab2}^{f_{1T}^{\perp}\cdot\hat{u}_{1L}^{\perp h}}$$

$$F_{TL}^{\cos(\phi_a-2\phi_h)} = -\frac{P_T^2 q_T}{2mM_a M_b} F_{ab3}^{f_{1T}^{\perp}\cdot\hat{u}_{1L}^{\perp h}}$$

$$F_{TL}^{\cos(2\phi+\phi_a-3\phi_h)} = \frac{P_T^3}{2mM_a^2} F_{aa1}^{h_{1T}^{\perp}\cdot\hat{l}_{1T}^{h}} + \frac{P_T^3}{2M_a^2 M_b} F_{aab1}^{h_{1T}^{\perp}\cdot\hat{l}_{1T}^{\perp}}$$

$$F_{TL}^{\cos(2\phi+\phi_a-2\phi_h)} = \frac{P_T^2 q_T}{2M_a^2 M_b} F_{aab2}^{h_{1T}^{\perp}\cdot\hat{l}_{1T}^{\perp}}$$

$$F_{TL}^{\cos(2\phi+\phi_a)} = \frac{q_T^3}{2M_a^2 M_b} F_{aab3}^{h_{1T}^{\perp}\cdot\hat{l}_{1T}^{\perp}}$$

$$F_{TL}^{\cos(2\phi-\phi_a-\phi_h)} = \frac{P_T}{M_b} F_{b1}^{h_1\cdot\hat{l}_{1T}^{\perp}} + \frac{P_T}{m} F_0^{h_1\cdot\hat{l}_{1T}^{h}}$$

$$F_{TL}^{\cos(2\phi-\phi_a)} = \frac{q_T}{M_b} F_{b2}^{h_1\cdot\hat{l}_{1T}^{\perp}}$$

# Structure functions $\sigma_{TT}$

$$F_{TT}^{\cos(\phi_a-\phi_b)} = \begin{pmatrix} -\frac{P_T^2}{2mM_a}F_{a1}^{f_{1T}^{\perp}\cdot\hat{t}_1^h} + \frac{P_T^2}{2mM_a}F_{a1}^{g_{1T}\cdot\hat{t}_{1L}^h} - \frac{P_T^2}{2M_aM_b}F_{ab1}^{f_{1T}^{\perp}\cdot\hat{t}_1^{\perp}} + \frac{P_T^2}{2M_aM_b}F_{ab1}^{g_{1T}\cdot\hat{t}_{1L}^{\perp}} \\ -\frac{q_T^2}{2M_aM_b}F_{ab2}^{f_{1T}^{\perp}\cdot\hat{t}_1^{\perp}} + \frac{q_T^2}{2M_aM_b}F_{ab2}^{g_{1T}\cdot\hat{t}_{1L}^{\perp}} - \frac{1}{M_aM_b}F_{ab4}^{f_{1T}^{\perp}\cdot\hat{t}_1^{\perp}} + \frac{1}{M_aM_b}F_{ab4}^{g_{1T}\cdot\hat{t}_{1L}^{\perp}} \end{pmatrix}$$

$$F_{TT}^{\cos(\phi_a+\phi_b-2\phi_h)} = \frac{P_T^2}{2mM_a}F_{a1}^{f_{1T}^{\perp}\cdot\hat{t}_1^h} + \frac{P_T^2}{2mM_a}F_{a1}^{g_{1T}\cdot\hat{t}_{1L}^h} + \frac{P_T^2}{2M_aM_b}F_{ab1}^{f_{1T}^{\perp}\cdot\hat{t}_1^{\perp}} + \frac{P_T^2}{2M_aM_b}F_{ab1}^{g_{1T}\cdot\hat{t}_{1L}^{\perp}}$$

$$F_{TT}^{\cos(\phi_a-\phi_b+\phi_h)} = -\frac{P_Tq_T}{2mM_a}F_{a2}^{f_{1T}^{\perp}\cdot\hat{t}_1^h} + \frac{P_Tq_T}{2mM_a}F_{a2}^{g_{1T}\cdot\hat{t}_{1L}^h} + \frac{P_Tq_T}{2M_aM_b}F_{ab3}^{f_{1T}^{\perp}\cdot\hat{t}_1^{\perp}} - \frac{P_Tq_T}{2M_aM_b}F_{ab3}^{g_{1T}\cdot\hat{t}_{1L}^{\perp}}$$

$$F_{TT}^{\cos(\phi_a+\phi_b-\phi_h)} = \frac{P_Tq_T}{2mM_a}F_{a2}^{f_{1T}^{\perp}\cdot\hat{t}_1^h} + \frac{P_Tq_T}{2mM_a}F_{a2}^{g_{1T}\cdot\hat{t}_{1L}^h}, \quad F_{TT}^{\cos(\phi_a+\phi_b)} = \frac{q_T^2}{2M_aM_b}F_{ab2}^{f_{1T}^{\perp}\cdot\hat{t}_1^{\perp}} + \frac{q_T^2}{2M_aM_b}F_{ab2}^{g_{1T}\cdot\hat{t}_{1L}^{\perp}}$$

$$F_{TT}^{\cos(\phi_a-\phi_b-\phi_h)} = \frac{P_Tq_T}{2M_aM_b}F_{ab3}^{g_{1T}\cdot\hat{t}_{1L}^{\perp}} - \frac{P_Tq_T}{2M_aM_b}F_{ab3}^{f_{1T}^{\perp}\cdot\hat{t}_1^{\perp}}, \quad F_{TT}^{\cos(2\phi-\phi_a-\phi_b-\phi_h)} = \frac{P_Tq_T}{2mM_b}F_{b2}^{h_1\cdot\hat{t}_{1T}^{\perp h}}$$

$$F_{TT}^{\cos(2\phi-\phi_a-\phi_b+\phi_h)} = -\frac{P_Tq_T}{2mM_b}F_{b2}^{h_1\cdot\hat{t}_{1T}^{\perp h}}, \quad F_{TT}^{\cos(2\phi+\phi_a+\phi_b)} = \frac{q_T^4}{4M_a^2M_b^2}F_{aabb3}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}}, \quad F_{TT}^{\cos(2\phi+\phi_a-\phi_b+\phi_h)} = -\frac{P_Tq_T^3}{4mM_a^2M_b}F_{aab3}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp h}}$$

$$F_{TT}^{\cos(2\phi-\phi_a+\phi_b)} = \frac{q_T^2}{2M_b^2}F_{bb2}^{h_1\cdot\hat{t}_{1T}^{\perp\perp}}, \quad F_{TT}^{\cos(2\phi-\phi_a+\phi_b-2\phi_h)} = \frac{P_T^2}{2m^2}F_0^{h_1\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{P_T^2}{2M_b^2}F_{bb1}^{h_1\cdot\Delta_T\hat{M}_T^{\perp\perp}}$$

$$F_{TT}^{\cos(2\phi+\phi_a+\phi_b-4\phi_h)} = \frac{P_T^4}{4m^2M_a^2}F_{aa1}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp h}} + \frac{P_T^4}{4M_a^2M_b^2}F_{aabb1}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}}, \quad F_{TT}^{\cos(2\phi+\phi_a-\phi_b-3\phi_h)} = \frac{P_T^3q_T}{4mM_a^2M_b}F_{aab2}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp h}}$$

$$F_{TT}^{\cos(2\phi-\phi_a-\phi_b)} = F_0^{h_1\cdot\Delta_T\hat{M}_T} + \frac{P_T^2}{2m^2}F_0^{h_1\cdot\Delta_T\hat{M}_T^{hh}} + \frac{P_T^2}{2M_b^2}F_{bb1}^{h_1\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{q_T^2}{2M_b^2}F_{bb2}^{h_1\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{1}{M_b^2}F_{bb3}^{h_1\cdot\hat{t}_{1T}^{\perp\perp}}$$

$$F_{TT}^{\cos(2\phi+\phi_a-\phi_b-2\phi_h)} = \frac{P_T^2}{2M_a^2}F_{aa1}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}} + \frac{P_T^4}{4m^2M_a^2}F_{aa1}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T^{hh}} + \frac{P_T^4}{4M_a^2M_b^2}F_{aabb1}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{P_T^2}{2M_a^2M_b^2}F_{aabb5}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{P_T^2q_T^2}{4M_a^2M_b^2}F_{aabb2}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}}$$

$$F_{TT}^{\cos(2\phi+\phi_a+\phi_b-2\phi_h)} = \frac{P_T^2q_T^2}{4m^2M_a^2}F_{aa2}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{hh}} + \frac{P_T^2q_T^2}{4M_a^2M_b^2}F_{aabb2}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{P_T^2q_T^2}{4M_a^2M_b^2}F_{aabb4}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}}$$

$$F_{TT}^{\cos(2\phi+\phi_a-\phi_b-\phi_h)} = \frac{P_Tq_T^3}{4mM_a^2M_b}F_{aab3}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp h}} - \frac{P_T^3q_T}{4mM_a^2M_b}F_{aab2}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp h}}$$

$$F_{TT}^{\cos(2\phi+\phi_a-\phi_b)} = \frac{q_T^2}{2M_a^2}F_{aa2}^{h_{1T}^{\perp}\cdot\Delta_T\hat{M}_T} + \frac{P_T^2q_T^2}{4m^2M_a^2}F_{aa2}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{hh}} + \frac{q_T^4}{4M_a^2M_b^2}F_{aabb3}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{P_T^2q_T^2}{4M_a^2M_b^2}F_{aabb4}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}} + \frac{q_T^2}{2M_a^2M_b^2}F_{aabb6}^{h_{1T}^{\perp}\cdot\hat{t}_{1T}^{\perp\perp}}$$