

Probing Non-Nucleonic Components in Nuclei

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Opportunities with JLab Energy and Luminosity Upgrade
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Nuclear Forces at Extreme Dynamics:

(dynamics of Cold Dense Nuclear Matter)

1. Non-Nucleonic Components in Nuclei
2. Super-Fast Quarks in Nuclei
3. Three Nucleon Short Range Correlations
4. Multi-Parameter EMC effects
5. Color Transparency Phenomenon

1. Non-Nucleonic Components in the Nuclei

One of the outstanding issues of strong interaction physics is the understanding of the dynamics of transition between hadronic to quark-gluon phases of matter.

- Such transitions at high temperature is relevant to the evolution of the universe after the big bang and can be studied experimentally in heavy ion collisions.
- Transitions at low (near zero) temperatures and high densities ("cold-dense" transitions) are relevant for superdense nuclear matter that can exist at the cores of neutron stars and can set up the limits of matter density before it collapses to the black hole.

However, the direct exploration of "cold-dense" transitions is severely restricted.

Currently the accepted ways of investigating such transitions are;

(1) Studying nuclear medium modification of quark-gluon structure of bound nucleons: EMC effect.

Few progresses were made in understanding of this phenomena for past 40 years

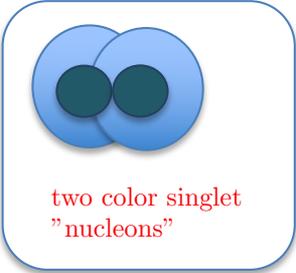
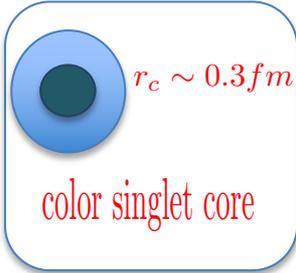
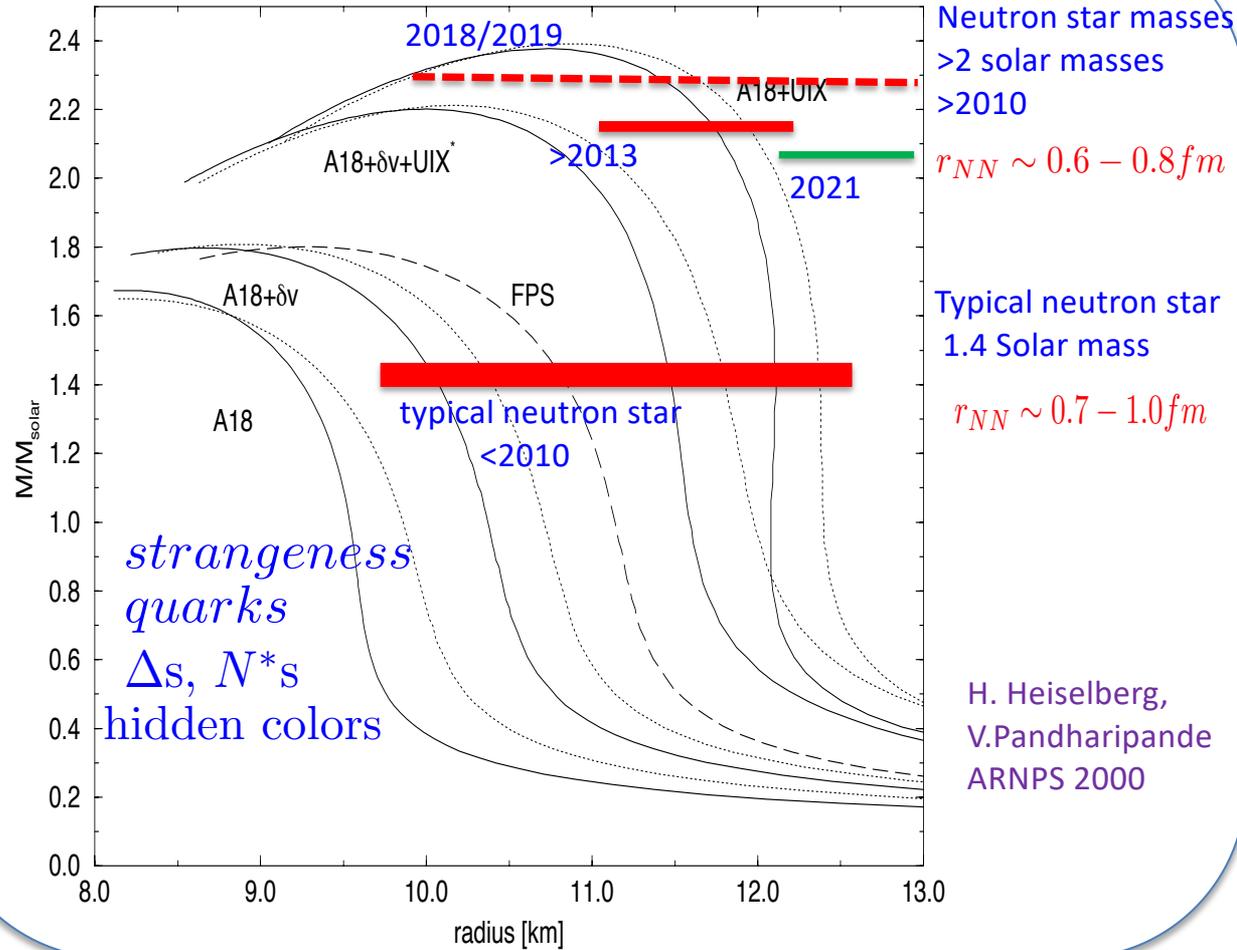
- observation of the dependence of the effect on local nuclear density*
- importance of Short-Range Nucleonic Correlations*

In all these cases the role of the hadronic to quark-gluon transition is not clearly understood.

(2) Studying the implications of the transition of baryonic matter to the quark matter in the cores of neutron stars. Situation is even more unclear.

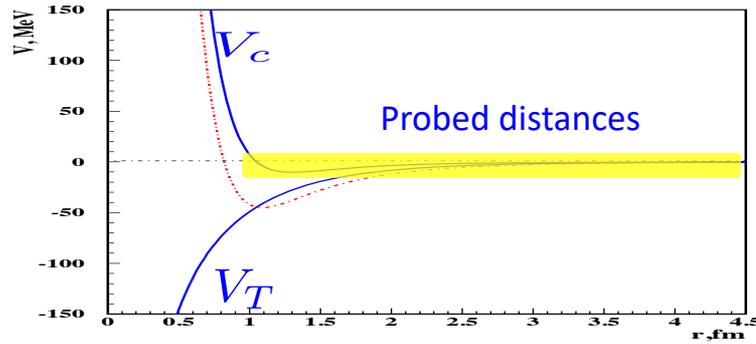
With the observation of unexpectedly large neutron star masses ($2.08M_{\odot}$) it was expected that if such stars would have radii, $R < 10\text{km}$ it will be indicative of large quark matter component in their cores.

"Unreasonable" Persistence of Nucleons

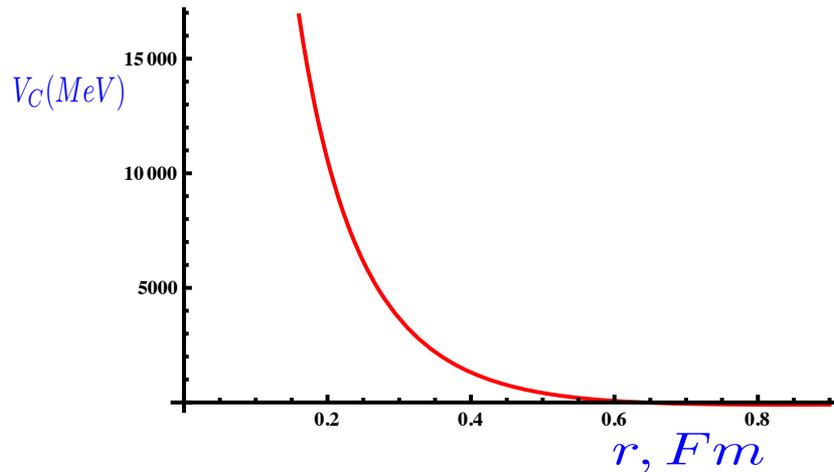


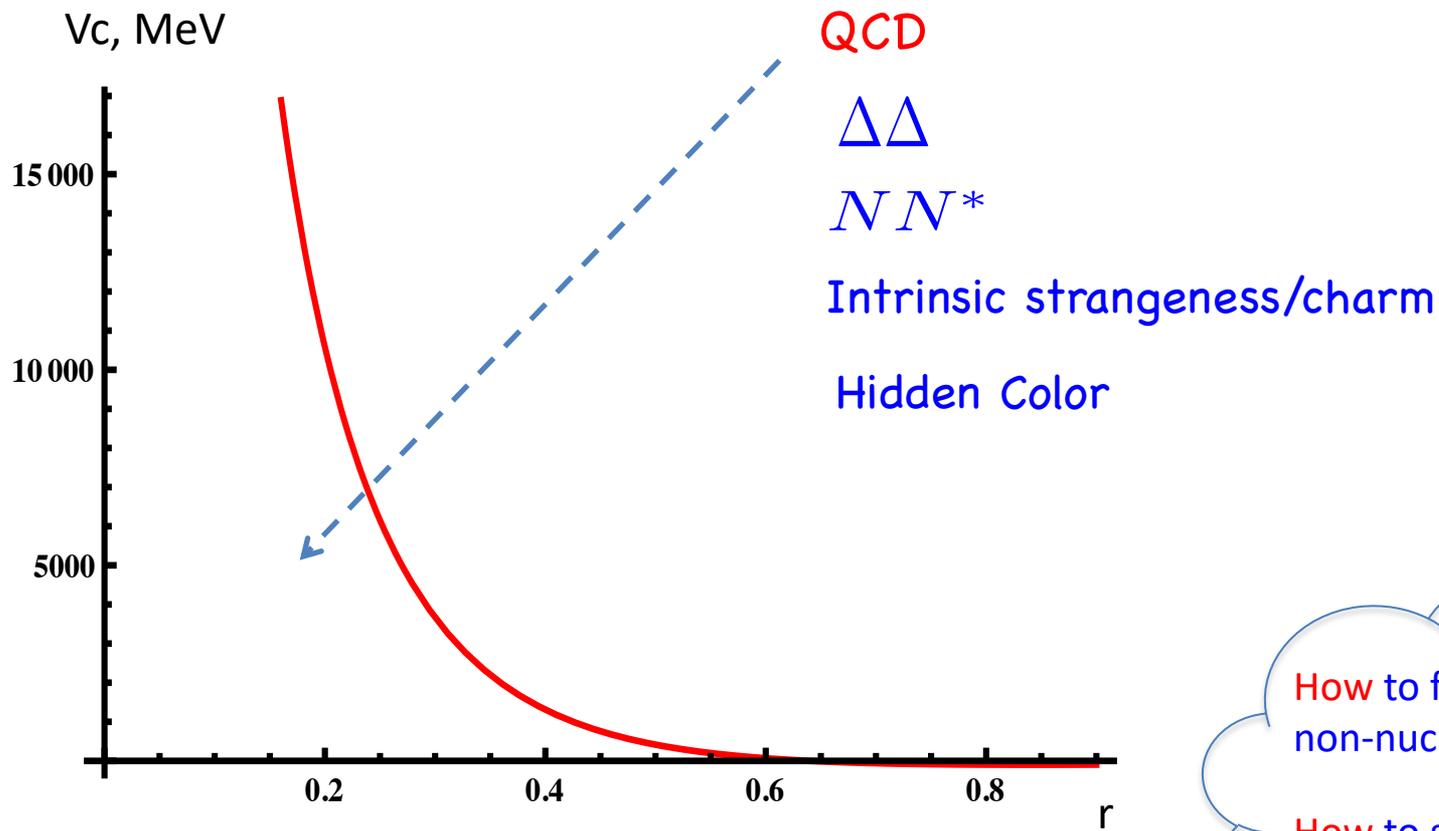
H. Heiselberg,
 V.Pandharipande
 ARNPS 2000

Currently: Probed NN structure up to $> 0.8\text{fm}$



Next: NN – Repulsive Core





~80% hidden color
 Brodsky, Ji, Lepage, PRL 83

How to find the presence of non-nucleonic components

How to study the dynamics of NN interaction at these distances

For the Deuteron it means, at Short Distances

$$\Psi_d = \Psi_{pn} + \Psi_{\Delta\Delta} + \Psi_{NN^*} + \Psi_{hc} \cdots + \Psi_{p\Lambda^0 K^0} \cdots$$

$$\Psi_{hc} = \Psi_{N_c, N_c}$$

The NN repulsive core can be due to the orthogonality of

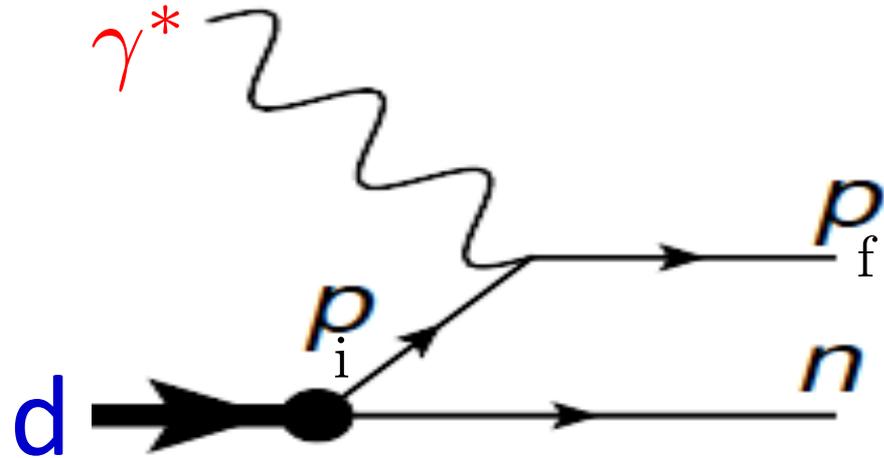
$$\langle \Psi_{NonNucleonic} | \Psi_{NN} \rangle = 0$$

Probing NN interaction at short distances

Considering reaction: $e + d \rightarrow e' + p_f + n$

$$|p_i| = |p_f - q| \leq 550 \text{ MeV}/c$$

$$|p_i| = |p_f - q| > 550 \text{ MeV}/c$$



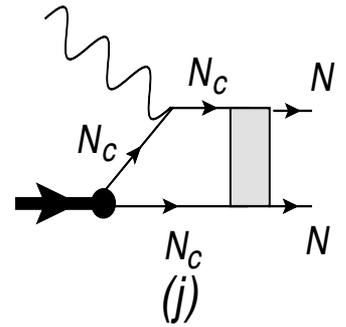
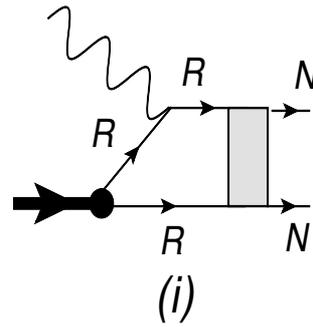
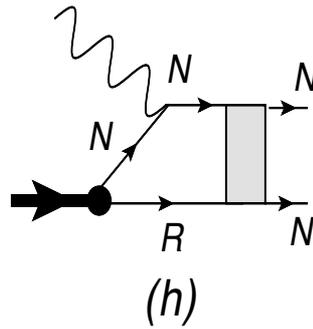
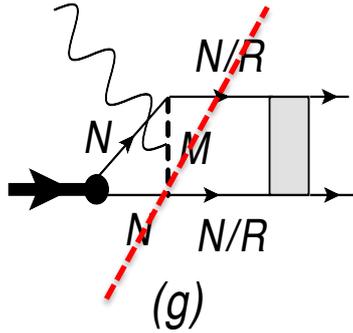
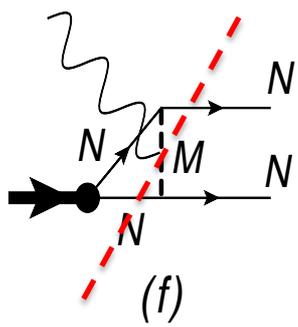
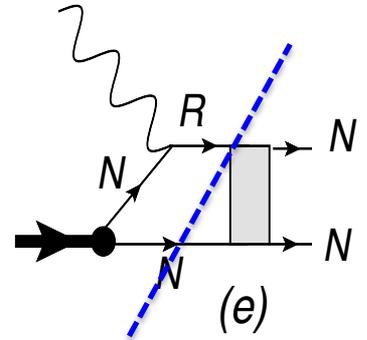
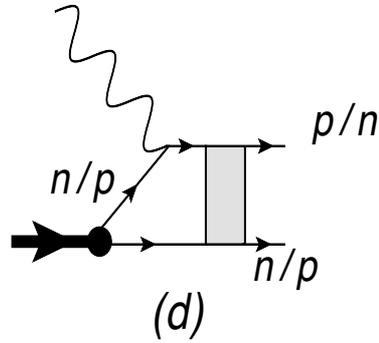
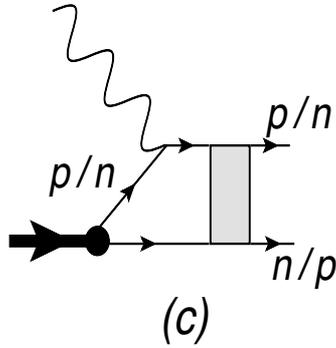
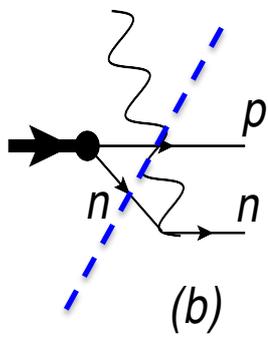
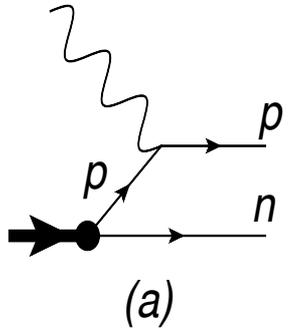
Some Paradigm Shift

Our current mindset about deuteron is fully non-relativistic, the observation that it has total spin, $J=1$ and parity, $P=+$, together with the relation that for non-relativistic wave function, $P=(-1)^l$, one concludes that the deuteron consists of S- and D- partial waves for proton-neutron system.

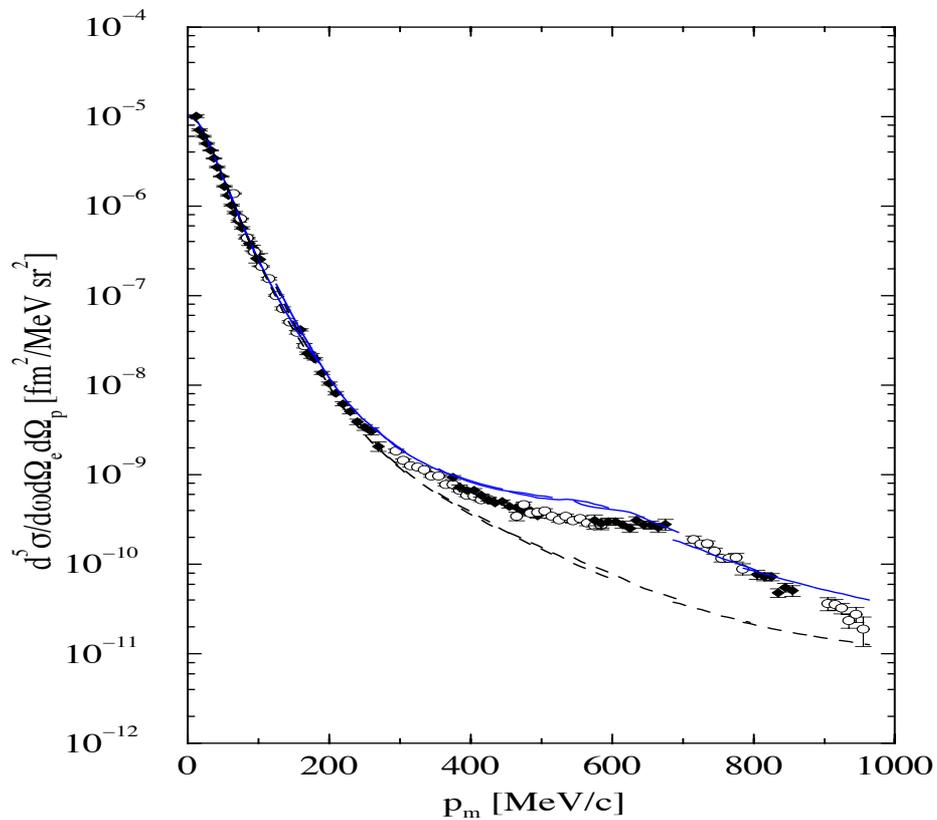
Paradigm Shift: The above reaction at high Q^2 , measures the probability of observing proton and neutron in the deuteron at very large relative momenta. In such a formulation the deuteron is not a composite system consisting of proton and neutron but it is a composite pseudo - vector ($J=1, P=+$) "particle" from which one extracts proton and neutron.

How such a proton and neutron produced at such extremal conditions is related to the dynamical structure of Light-Front deuteron wave function, which may include internal elastic $pn \rightarrow pn$ as well as inelastic $\Delta\Delta \rightarrow pn$, $N^*N \rightarrow pn$ or $N_C N_C \rightarrow pn$ transitions.

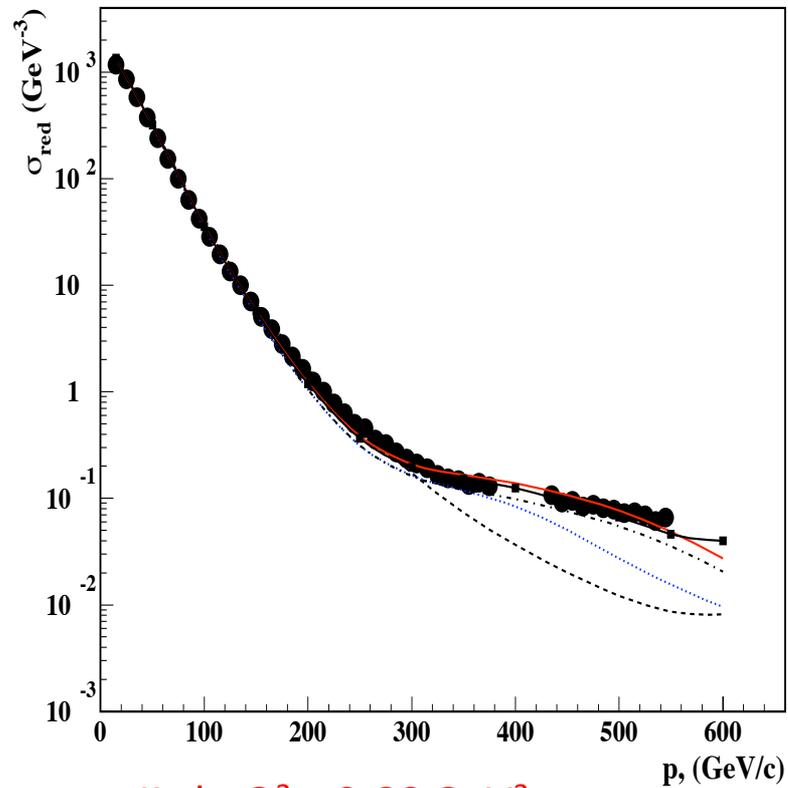
Considering reaction: $e + d \rightarrow e' + p_f + n$



Impossibility to Probe Deuteron at Small Distances at low Q^2

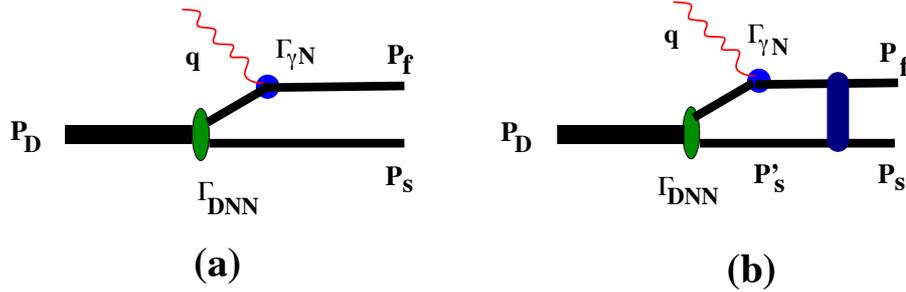


Mainz, $Q^2 = 0.33 \text{ GeV}^2$



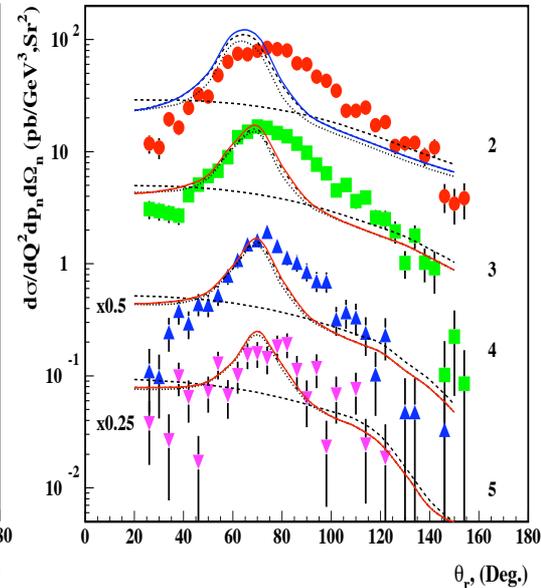
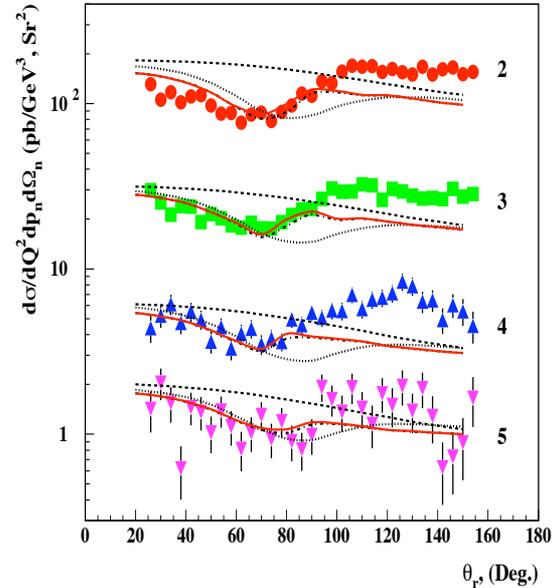
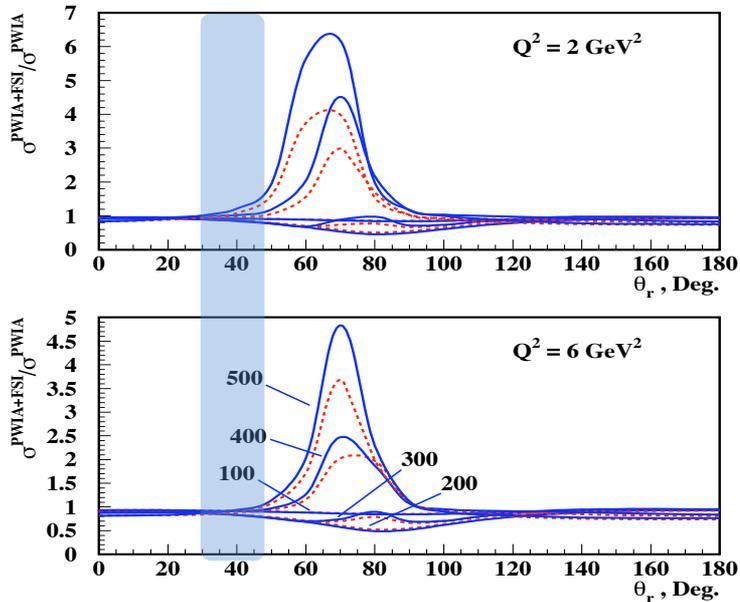
JLab, $Q^2 = 0.66 \text{ GeV}^2$

At Large $Q^2 > 1-2 \text{ GeV}^2$ Eikonal Regime is Established)

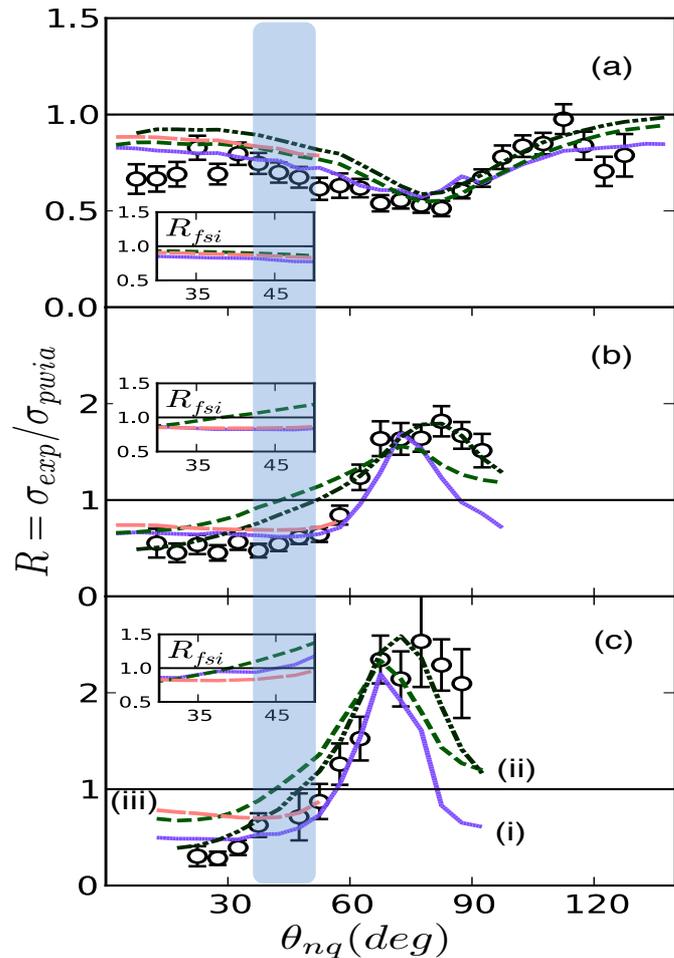


For the case of
 $e + d \rightarrow e' + p_f + p_s$

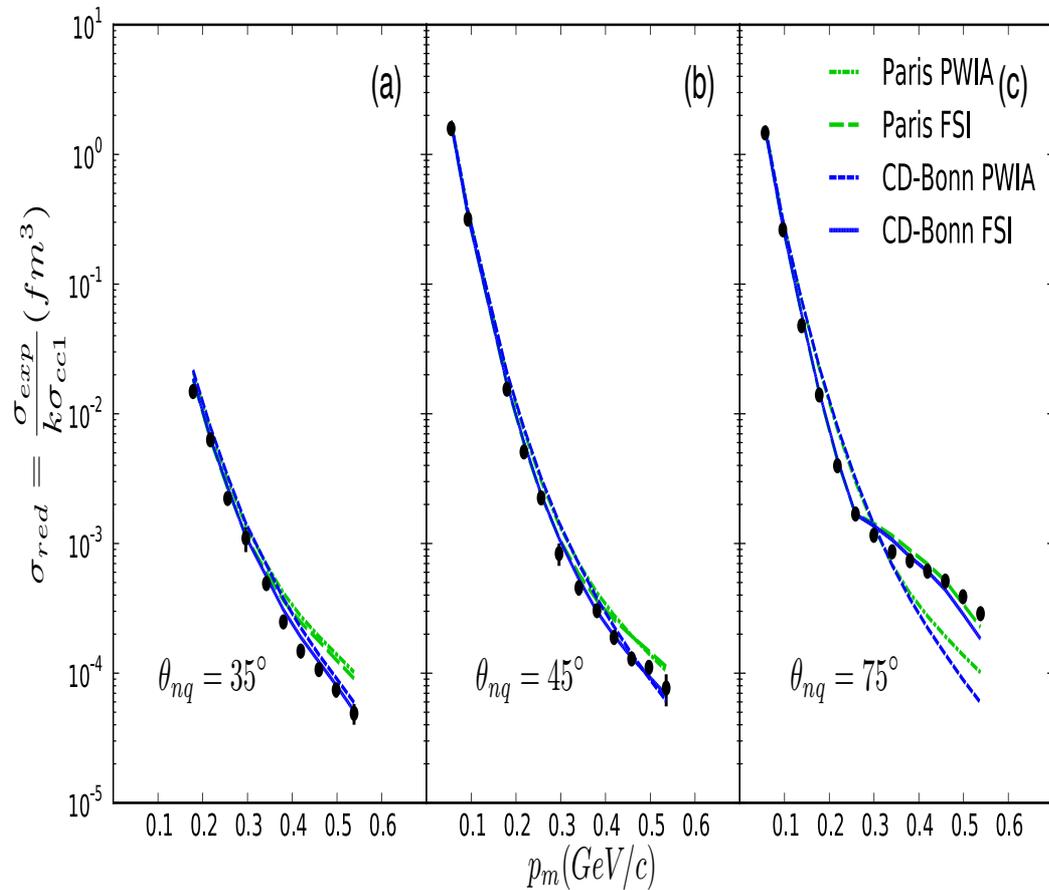
K.Egiyan et al 2008



Probing Deuteron at Small Distances at large Q^2



JLab, $Q^2 = 3.5 \text{ GeV}^2$



Boeglin et al PRL 2011, deuteron probed at up to 550 MeV/c

New Structure in the Deuteron and possible non-nucleonic components

M.S & Frank Vera ArXiv 2022

Paradigm shift:

- consider a deuteron not a nucleus that consist of proton and neutron
- but **pseudovector composite particle** from which we **extract** proton and neutron
- Light-Front Deuteron wave function

$$\psi_d^{\lambda_d}(\alpha_i, p_{\perp}, \lambda_1 \lambda_2) = - \frac{\bar{u}(p_2, \lambda_2) \bar{u}(p_1, \lambda_1) \Gamma_d \chi^{\lambda_d}}{\frac{1}{2} \left(m_d^2 - 4 \frac{m_N^2 + p_{\perp}^2}{\alpha_i (2 - \alpha_i)} \right) \sqrt{2} (2\pi)^3}$$

- Absorbing the energy denominator into the vertex function and using crossing symmetry

$$\psi_d^{\mu}(\alpha_i, p_{\perp}, \lambda_1, \lambda_2) = - \bar{u}(p_2, \lambda_2) \Gamma_d^{\mu}(k) \frac{(i\gamma_2 \gamma_0)}{\sqrt{2}} \bar{u}(p_1, \lambda_1)^T = - \sum_{\lambda'_1} \bar{u}(p_1, \lambda_1) \Gamma_d^{\mu} \gamma_5 \frac{\epsilon_{\lambda_1, \lambda'_1}}{\sqrt{2}} u(p_1, \lambda'_1)$$

$$\psi_d^\mu(\alpha_i, p_\perp, \lambda_1, \lambda_2) = -\bar{u}(p_2, \lambda_2) \Gamma_d^\mu(k) \frac{(i\gamma_2 \gamma_0)}{\sqrt{2}} \bar{u}(p_1, \lambda_1)^T = -\sum_{\lambda'_1} \bar{u}(p_1, \lambda_1) \Gamma_d^\mu \gamma_5 \frac{\epsilon^{\lambda_1, \lambda'_1}}{\sqrt{2}} u(p_1, \lambda'_1)$$

- Γ_d^μ - is a four-vector, which can be constructed in a most general form satisfying time reversal, parity and charge conjugate symmetries

- Because the deuteron is a bound system, in addition to on-shell p_1 and p_2 four momenta one introduces

$$\Delta^\mu \equiv p_1^\mu + p_2^\mu - p_d^\mu \equiv (\Delta^-, \Delta^+, \Delta_\perp) = (\Delta^-, 0, 0)$$

$$\Delta^- = p_1^- + p_2^- - p_d^- = \frac{m_N^2 + k_\perp^2}{p_1^+} + \frac{m_N^2 + k_\perp^2}{p_2^+} - \frac{M_d^2}{p_d^+} = \frac{1}{p_d^+} \left[\frac{4(m_N^2 + k_\perp^2)}{\alpha_1(2 - \alpha_1)} - M_d^2 \right] = \frac{4}{p_d^+} \left[m_N^2 - \frac{M_d^2}{4} + k^2 \right]$$

- Constructed vertex:

$$\Gamma_d^\mu = \Gamma_1 \gamma^\mu + \Gamma_2 \frac{(p_1 - p_2)^\mu}{2m_N} + \Gamma_3 \frac{\Delta^\mu}{2m_N} + \Gamma_4 \frac{(p_1 - p_2)^\mu \not{\Delta}}{4m_N^2} + i\Gamma_5 \frac{1}{4m_N^3} \gamma_5 \epsilon^{\mu\nu\rho\gamma} (p_d)_\nu (p_1 - p_2)_\rho (\Delta)_\gamma + \Gamma_6 \frac{\Delta^\mu \not{\Delta}}{4m_N^2}$$

High Momentum Transfer Kinematics

For large Q^2 limit, Light-Front momenta for the reaction are chosen as follows:

$$p_d^\mu \equiv (p_d^-, p_d^+, p_{d\perp}) = \left(\frac{Q^2}{x\sqrt{s}} \left[1 + \frac{x}{\tau} - \sqrt{1 + \frac{x^2}{\tau}} \right], \frac{Q^2}{x\sqrt{s}} \left[1 + \frac{x}{\tau} + \sqrt{1 + \frac{x^2}{\tau}} \right], 0_\perp \right)$$
$$q^\mu \equiv (q^-, q^+, q_\perp) = \left(\frac{Q^2}{x\sqrt{s}} \left[1 - x + \sqrt{1 + \frac{x^2}{\tau}} \right], \frac{Q^2}{x\sqrt{s}} \left[1 - x - \sqrt{1 + \frac{x^2}{\tau}} \right], 0_\perp \right)$$

where $s = (q + p_d)^2$, $\tau = \frac{Q^2}{M_d^2}$ and $x = \frac{Q^2}{M_d q_0}$, with q_0 being virtual photon energy in the deuteron rest frame.

- One observes that for fixed x , $p_d^+ \sim \sqrt{Q^2} \gg m_N$

$$\Delta^\mu \equiv p_1^\mu + p_2^\mu - p_d^\mu \equiv (\Delta^-, \Delta^+, \Delta_\perp) = (\Delta^-, 0, 0),$$

where

$$\begin{aligned} \Delta^- &= p_1^- + p_2^- - p_d^- = \frac{m_N^2 + k_\perp^2}{p_1^+} + \frac{m_N^2 + k_\perp^2}{p_2^+} - \frac{M_d^2}{p_d^+} \\ &= \frac{1}{p_d^+} \left[\frac{4(m_N^2 + k_\perp^2)}{\alpha_1(2 - \alpha_1)} - M_d^2 \right] = \frac{4}{p_d^+} \left[m_N^2 - \frac{M_d^2}{4} + k^2 \right]. \end{aligned}$$

In high Q^2 limit $\frac{\Delta^-}{2m_N} \ll 1$

$$\begin{aligned} \Gamma_d^\mu &= \Gamma_1 \gamma^\mu + \Gamma_2 \frac{(p_1 - p_2)^\mu}{2m_N} + \Gamma_3 \frac{\cancel{\Delta^\mu}}{2m_N} + \Gamma_4 \frac{(p_1 - p_2)^\mu \cancel{\Delta}}{4m_N^2} \\ &+ i\Gamma_5 \frac{1}{4m_N^3} \gamma_5 \epsilon^{\mu\nu\rho\gamma} (p_d)_\nu (p_1 - p_2)_\rho (\Delta)_\gamma + \Gamma_6 \frac{\cancel{\Delta^\mu \Delta}}{4m_N^2} \end{aligned}$$

Consider: $\epsilon^{\mu,+,\perp,-} p_{d,-} k_{\perp} \Delta_{+}$

Since: $p_{d,-} = \frac{1}{2} p_d^+$ and $\Delta_{+} = \frac{1}{2} \Delta^{-}$ then $p_d^+ \Delta^{-} = p_d^+ \frac{1}{p_d^+} \left[\frac{4(m_N^2 + k_{\perp}^2)}{\alpha_1(2-\alpha_1)} - M_d^2 \right] = \left[\frac{4(m_N^2 + k_{\perp}^2)}{\alpha_1(2-\alpha_1)} - M_d^2 \right]$

$\epsilon^{\mu,+,\perp,-} p_{d,-} k_{\perp} \Delta_{+} = \frac{1}{4} \epsilon^{\mu,+,\perp,-} p_d^+ k_{\perp} \Delta^{-}$ **Leading Order!**

$$\Gamma_d^{\mu} = \Gamma_1 \gamma^{\mu} + \Gamma_2 \frac{(p_1 - p_2)^{\mu}}{2m_N} + \cancel{\Gamma_3 \frac{\Delta^{\mu}}{2m_N}} + \Gamma_4 \frac{(p_1 - p_2)^{\mu} \cancel{\Delta}}{4m_N^2} + i\Gamma_5 \frac{1}{4m_N^3} \gamma_5 \epsilon^{\mu\nu\rho\gamma} (p_d)_{\nu} (p_1 - p_2)_{\rho} (\Delta)_{\gamma} + \Gamma_6 \frac{\cancel{\Delta^{\mu}} \cancel{\Delta}}{4m_N^2}$$

$$\psi_d^{\lambda_d}(\alpha_i, k_{\perp}) = - \sum_{\lambda_2, \lambda_1, \lambda_1'} \bar{u}(-k, \lambda_2) \left\{ \Gamma_1 \gamma^{\mu} + \Gamma_2 \frac{\tilde{k}^{\mu}}{m_N} + \sum_{i=1}^2 i\Gamma_5 \frac{1}{8m_N^3} \epsilon^{\mu+i-} p_d'^+ k_i \Delta'^- \right\} \gamma_5 \frac{\epsilon_{\lambda_1, \lambda_1'}}{\sqrt{2}} u(k, \lambda_1') s_{\mu}^{\lambda_d}$$

where $\tilde{k}^{\mu} = (0, k_z, k_{\perp})$

$$\psi_d^{\lambda_d}(\alpha_i, k_\perp) = - \sum_{\lambda_2, \lambda_1, \lambda'_1} \bar{u}(-k, \lambda_2) \left\{ \Gamma_1 \gamma^\mu + \Gamma_2 \frac{\tilde{k}^\mu}{m_N} + \sum_{i=1}^2 i \Gamma_5 \frac{1}{8m_N^3} \epsilon^{\mu+i-} p_d'^+ k_i \Delta'^- \right\} \gamma_5 \frac{\epsilon_{\lambda_1, \lambda'_1}}{\sqrt{2}} u(k, \lambda'_1) s_\mu^{\lambda_d}$$

$$\psi_d^{\lambda_d}(\alpha_1, k_t, \lambda_1, \lambda_2) = \sum_{\lambda'_1} \phi_{\lambda_2}^\dagger \sqrt{E_k} \left[\frac{U(k)}{\sqrt{4\pi}} \sigma \mathbf{s}_d^{\lambda_d} - \frac{W(k)}{\sqrt{4\pi} \sqrt{2}} \left(\frac{3(\sigma \mathbf{k})(\mathbf{k} \mathbf{s}_d^\lambda)}{k^2} - \sigma \mathbf{s}_d^\lambda \right) + \right. \\ \left. (-1)^{\frac{1+\lambda_d}{2}} P(k) Y_1^{\lambda_d}(\theta, \phi) \delta^{1, |\lambda_d|} \right] \frac{\epsilon_{\lambda_1, \lambda'_1}}{\sqrt{2}} \phi_{\lambda'_1}$$

$$U(k) = \frac{2\sqrt{4\pi}\sqrt{E_k}}{3} \left[\Gamma_1 \left(2 + \frac{m_N}{E_k} \right) + \Gamma_2 \frac{k^2}{m_N E_k} \right]$$

$$W(k) = \frac{2\sqrt{4\pi}\sqrt{2E_k}}{3} \left[\Gamma_1 \left(1 - \frac{m_N}{E_k} \right) - \Gamma_2 \frac{k^2}{m_N E_k} \right]$$

Where: $Y_1^\pm(\theta, \phi) = \mp i \sqrt{\frac{3}{4\pi}} \sum_{i=1}^2 \frac{(k \times s_d^{\pm 1})_z}{k}$

$$P(k) = \sqrt{4\pi} \frac{\Gamma_5(k) \sqrt{E_k}}{\sqrt{3}} \frac{k^3}{m_N^3}$$

fully relativistic: in addition to $\frac{k^{l=1}}{m_N}$ term
has additional $\frac{k^2}{m_N^2}$ term

Light Front Density Matrix and Momentum Distribution

$$\psi_d^{\lambda_d}(\alpha_1, k_t, \lambda_1, \lambda_2) = \sum_{\lambda'_1} \phi_{\lambda'_1}^\dagger \sqrt{E_k} \left[\frac{U(k)}{\sqrt{4\pi}} \sigma \mathbf{s}_d^{\lambda_d} - \frac{W(k)}{\sqrt{4\pi}\sqrt{2}} \left(\frac{3(\sigma \mathbf{k})(\mathbf{k} \mathbf{s}_d^{\lambda_d})}{k^2} - \sigma \mathbf{s}_d^{\lambda_d} \right) + (-1)^{\frac{1+\lambda_d}{2}} P(k) Y_1^{\lambda_d}(\theta, \phi) \delta^{1,|\lambda_d|} \right] \frac{\epsilon_{\lambda_1, \lambda'_1}}{\sqrt{2}} \phi_{\lambda'_1}$$

$$\rho_d(\alpha, k_\perp) = \frac{n_d(k, k_\perp)}{2-\alpha}$$

$$n_d(k, k_\perp) = \frac{1}{3} \sum_{\lambda_d=-1}^1 |\psi_d^{\lambda_d}(\alpha, k_\perp)|^2 = \frac{1}{4\pi} \left(U(k)^2 + W(k)^2 + \frac{k_\perp^2}{k^2} P^2(k) \right)$$

Baryonic and Momentum Sum Rules $\int \rho_d(\alpha, k_\perp) \frac{d\alpha}{\alpha} = 1$ and $\int \alpha \rho_d(\alpha, k_\perp) \frac{d\alpha}{\alpha} = 1$

$$\int (U(k)^2 + W(k)^2 + \frac{2}{3} P^2(k)) k^2 dk = 1.$$

Non-Nucleonic Components and the New Structure

$$n_d(k, k_{\perp}) = \frac{1}{3} \sum_{\lambda_d=-1}^1 |\psi_d^{\lambda_d}(\alpha, k_{\perp})|^2 = \frac{1}{4\pi} \left(U(k)^2 + W(k)^2 + \frac{k_{\perp}^2}{k^2} P^2(k) \right)$$

- Momentum distribution depends on k_{\perp} separately
- *This is impossible for non-relativistic quantum mechanics of the deuteron since in this case the potential of the interaction is real (no inelasticities) and the solution of Lippmann-Schwinger (or Schroedinger) equation for partial S- and D-waves satisfies "angular condition", according to which the momentum distribution in unpolarized deuteron depends on the magnitude of relative momentum only.*
- On the other hand, in the relativistic domain the definition of the interaction potential is not straightforward to allow to use quantum-mechanical arguments in claiming that momentum distribution should satisfy the angular condition (i.e. depends on magnitude of k only).

- However, for the Light-Front, there is a remarkable theorem (Frankfurt and Strikman, 1990) which states that if **one considers only pn component in the deuteron**, then for most acceptable forms of NN potential – constructed from elastic pn → pn scattering, the angular condition should be satisfied also for LF momentum distribution.

$$T_{NN}(\alpha_i, k_{i\perp}, \alpha_f, k_{f,\perp}) \equiv T_{NN}(k_{i,z}, k_{i\perp}, k_{f,z}, k_{f,\perp}) = V(k_{i,z}, k_{i\perp}, k_{f,z}, k_{f,\perp}) + \int V(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m,\perp}) \times \frac{d^3 k_m}{(2\pi)^3 \sqrt{m^2 + k_m^2}} \frac{T_{NN}(k_{m,z}, k_{m,\perp}, k_{f,z}, k_{f,\perp})}{4(k_m^2 - k_f^2)}$$

- The realization of the angular condition for relativistic case will require that light-front potential to satisfy a condition

$$V(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m,\perp}) = V(\vec{k}_i^2, (\vec{k}_m - \vec{k}_i)^2)$$

- Lorentz invariance for on-shell NN amplitude requires

$$T_{NN}^{on\ shell}(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m,\perp}) = T_{NN}^{on\ shell}(\vec{k}_i^2, (\vec{k}_m - \vec{k}_i)^2)$$

- Existence of the Born term indicates that

$$T_{NN}^{on\ shell}(\vec{k}_i^2, (\vec{k}_m - \vec{k}_i)^2) = V_{NN}^{on\ shell}(\vec{k}_i^2, (\vec{k}_m - \vec{k}_i)^2) + \int V_{NN}(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m,\perp}) \times \frac{d^3 k_m}{(2\pi)^3 \sqrt{m^2 + k_m^2}} \frac{T_{NN}(k_{m,z}, k_{m\perp}, k_{f,z}, k_{f,\perp})}{4(k_m^2 - k_f^2)}$$

- Iterating the equation before around the on-shell kinematics point.

$$T_{NN}(\alpha_i, k_{i\perp}, \alpha_f, k_{f,\perp}) \equiv T_{NN}(k_{i,z}, k_{i\perp}, k_{f,z}, k_{f,\perp}) = V(k_{i,z}, k_{i\perp}, k_{f,z}, k_{f,\perp}) + \int V(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m,\perp}) \times \frac{d^3 k_m}{(2\pi)^3 \sqrt{m^2 + k_m^2}} \frac{T_{NN}(k_{m,z}, k_{m\perp}, k_{f,z}, k_{f,\perp})}{4(k_m^2 - k_f^2)}$$

- will result in:

$$T_{NN}(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m,\perp}) = T_{NN}(\vec{k}_i^2, (\vec{k}_m - \vec{k}_i)^2)$$

$$V_{NN}(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m,\perp}) = V_{NN}(\vec{k}_i^2, (\vec{k}_m - \vec{k}_i)^2) \quad \text{for the general case}$$

- V_{NN} – analytic function of angular momentum and it does not diverge exponentially in the complex-angular momentum space it was shown that also for the off-shell case

- For Non-nucleonic components no such iteration can be done

$$T_{NN}(k_{i,z}, k_{i,\perp}, k_{f,z}, k_{f,\perp}) = \int V_{NN^*}(k_{i,z}, k_{i,\perp}, k_{m,z}, k_{m,\perp}) \frac{d^3 k_m}{(2\pi)^3 \sqrt{m_m^2 + k_m^2}} \frac{T_{N^*N}(k_{m,z}, k_{m,\perp}, k_{f,z}, k_{f,\perp})}{4(k_m^2 - k_f^2 + m_m^2 - m_N^2)}$$

- transition amplitudes such as $T_{\Delta\Delta \rightarrow NN}$, $T_{N^*,N \rightarrow NN}$ or $T_{N^c, N^c \rightarrow NN}$ where $N^c N^c$ represents a hidden color component in the deuteron could not be described with any combination of interaction potentials that satisfies angular condition

- if Γ_5 term is not zero then it should originate from non-nucleonic component in the deuteron.

- Our prediction is that the observation of LF momentum distribution depending on the center of mass k and k_\perp separately will indicate the presence of non-nucleonic component in the deuteron

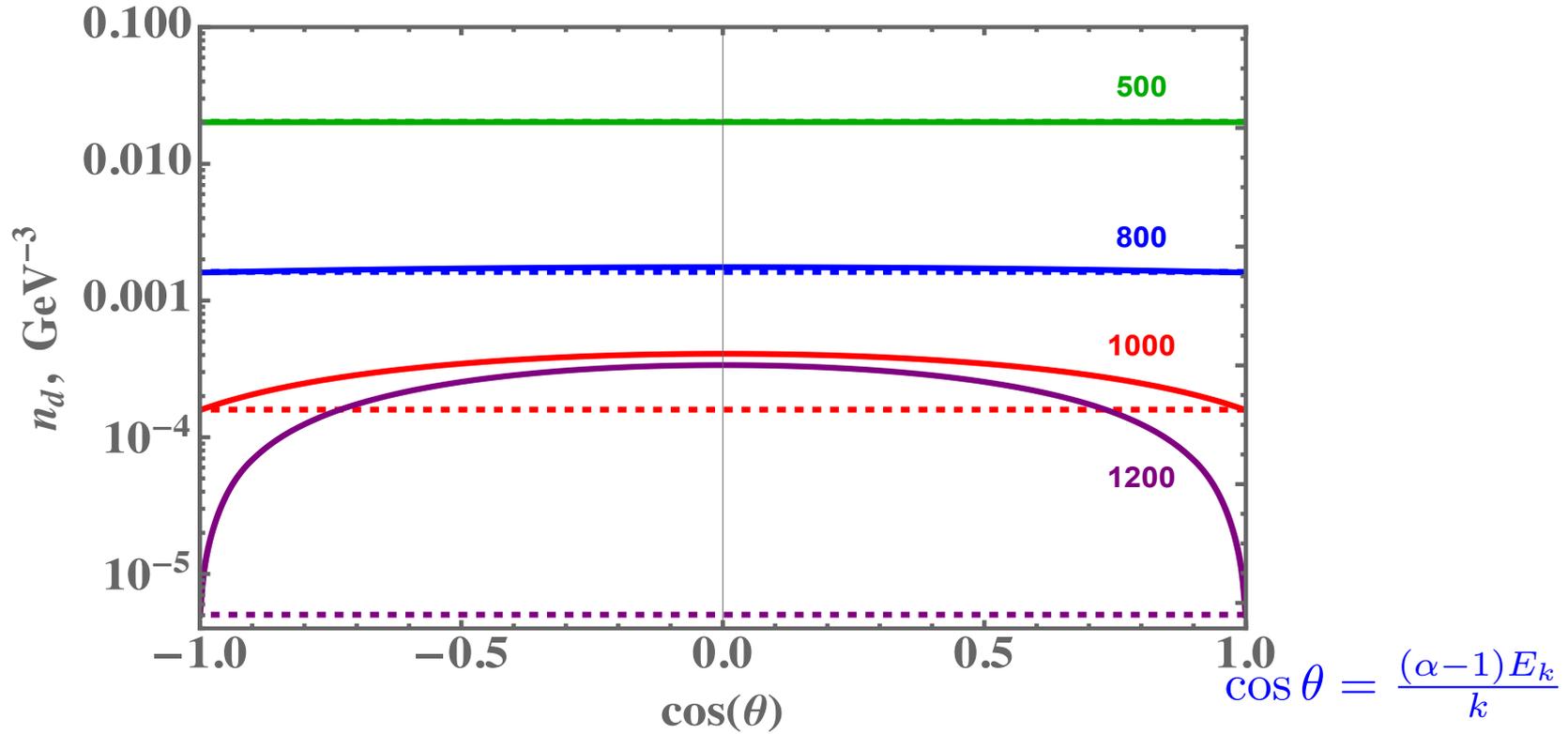
Estimate of the effect

$$n_d(k, k_\perp) = \frac{1}{3} \sum_{\lambda_d=-1}^1 |\psi_d^{\lambda_d}(\alpha, k_\perp)|^2 = \frac{1}{4\pi} \left(U(k)^2 + W(k)^2 + \frac{k_\perp^2}{k^2} P^2(k) \right)$$

$$P(k) = \sqrt{4\pi} \frac{\Gamma_5(k) \sqrt{E_k}}{\sqrt{3}} \frac{k^3}{m_N^3}$$

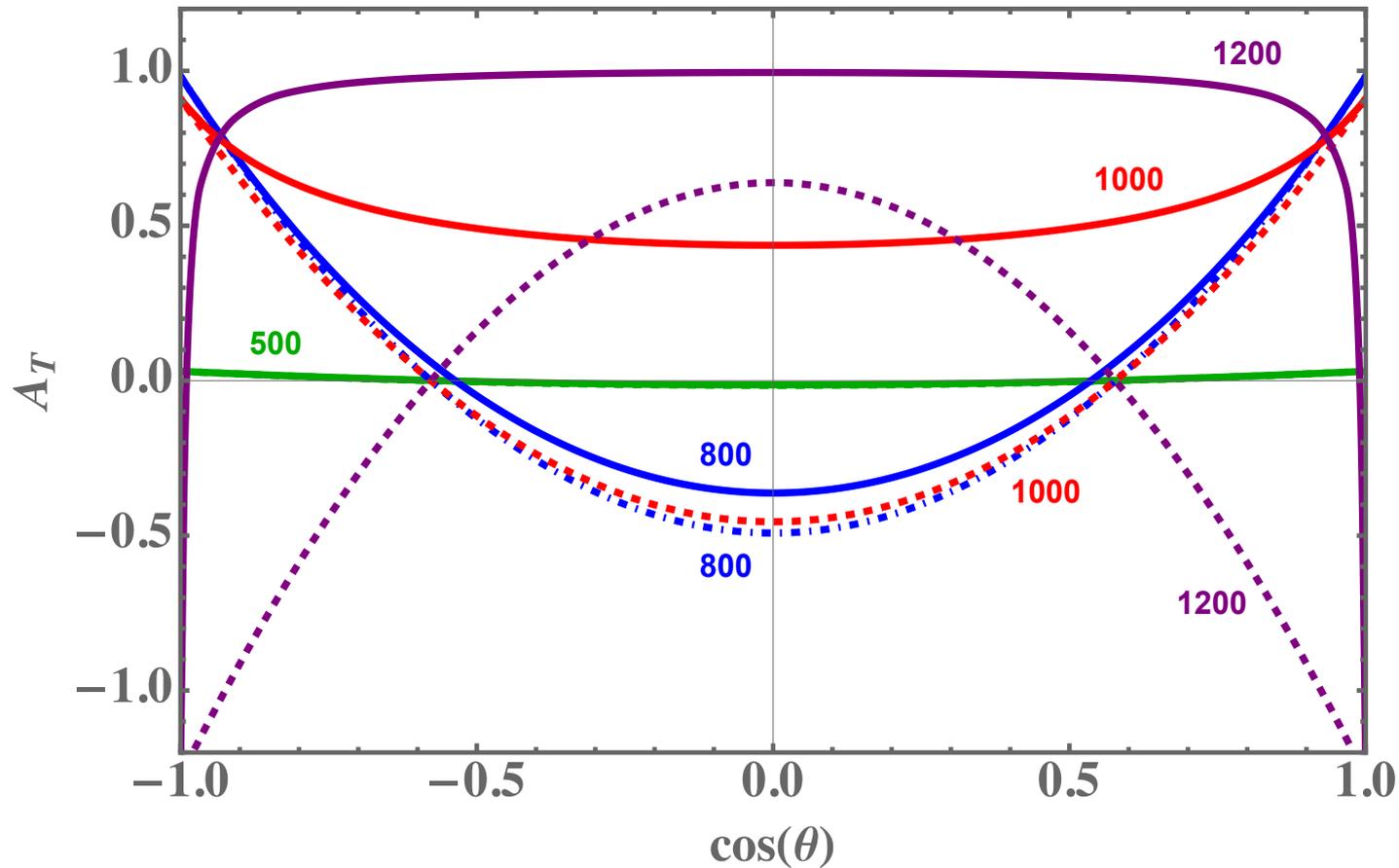
$$\Gamma_5(k) = \frac{A}{(1 + \frac{k^2}{0.71})^2}$$

A is estimated by assuming 1% contribution to the total normalization.



Estimate of the effect

$$A_T = \frac{n_d^{\lambda_d=1}(k, k_\perp) + n_d^{\lambda_d=-1}(k, k_\perp) - 2n_d^{\lambda_d=0}(k, k_\perp)}{n_d(k, k_\perp)}$$

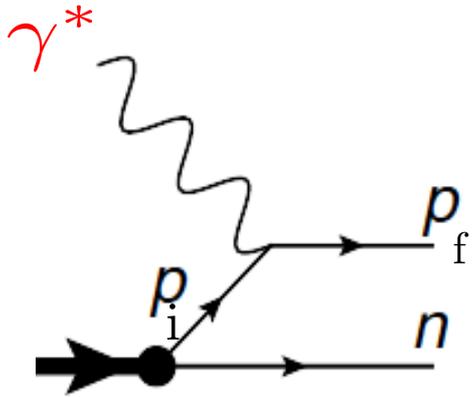


$$\cos \theta = \frac{(\alpha - 1) E_k}{k}$$

Possibility of Experimental Verification

Considering reaction: $e + d \rightarrow e' + p_f + n$

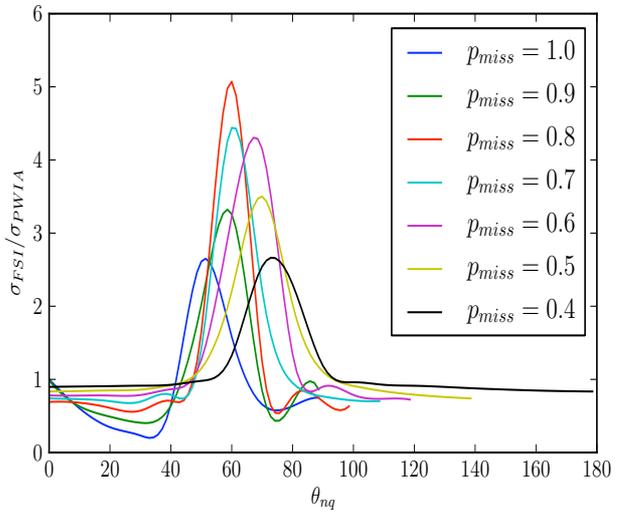
$$|p_i| = |p_f - q| \gtrsim 800 \text{ MeV}/c$$



PAC-36, 2010

■ E12-10-003 ($p_m \square 300 \text{ MeV}$): “Deuteron Electro-Disintegration at Very High Missing Momentum”

Rating: B+



data are essential to constrain further theory developments. Overall the experiment was viewed very highly; the lower rating simply reflects the likelihood that the data will not reveal any particular surprise and that their impact may thus be limited to experts in the field.

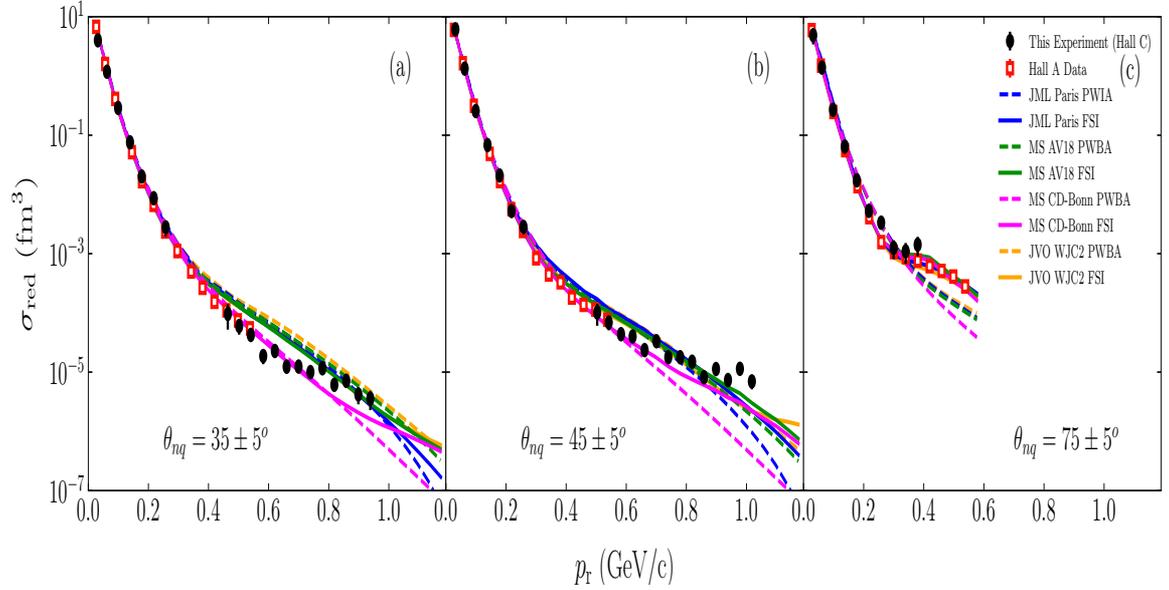
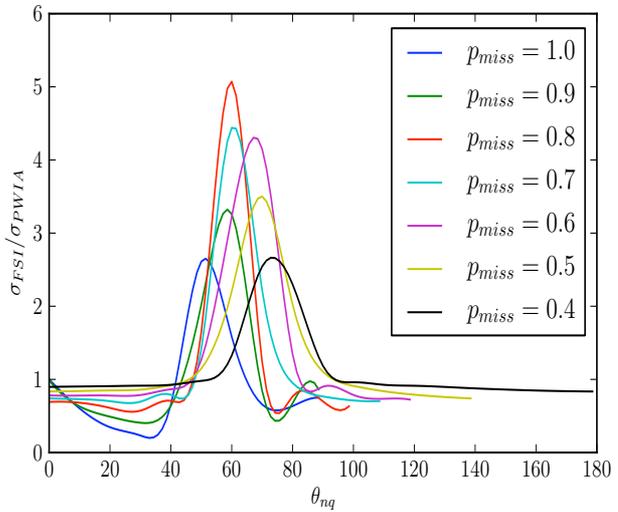
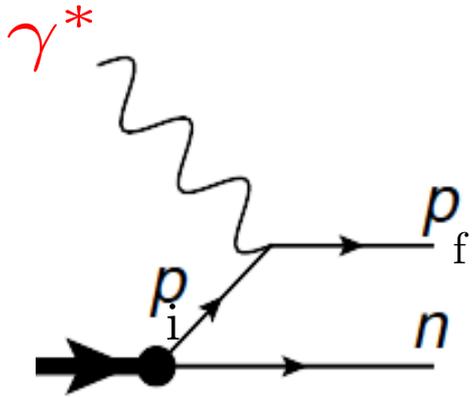
Possibility of Experimental Verification

Considering reaction: $e + d \rightarrow e' + p_f + n$

$$|p_i| = |p_f - q| \gtrsim 800 \text{ MeV}/c$$

3-days of commissioning measurement,

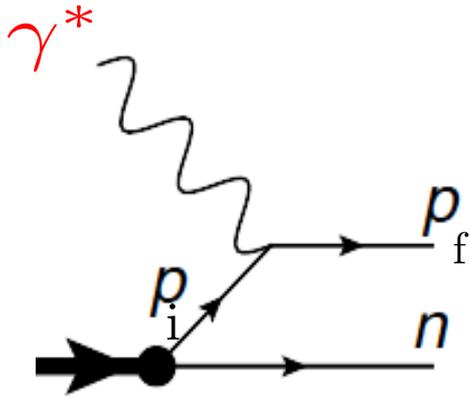
JLab experiment $Q^2 = 4 \text{ GeV}^2$



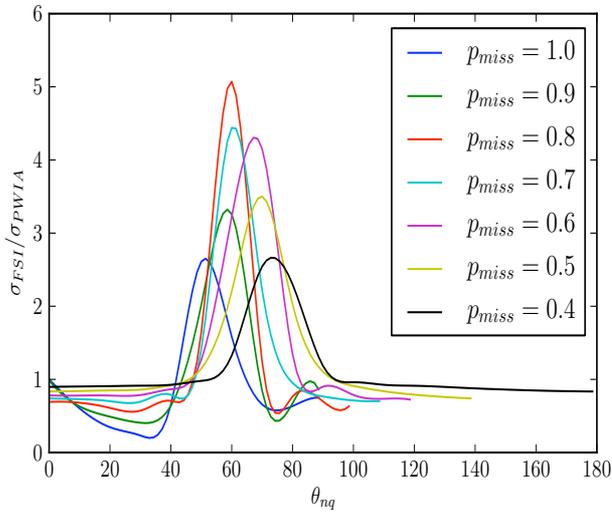
Possibility of Experimental Verification

Considering reaction: $e + d \rightarrow e' + p_f + n$

$$|p_i| = |p_f - q| \gtrsim 800 \text{ MeV}/c$$



PAC-49, 2021



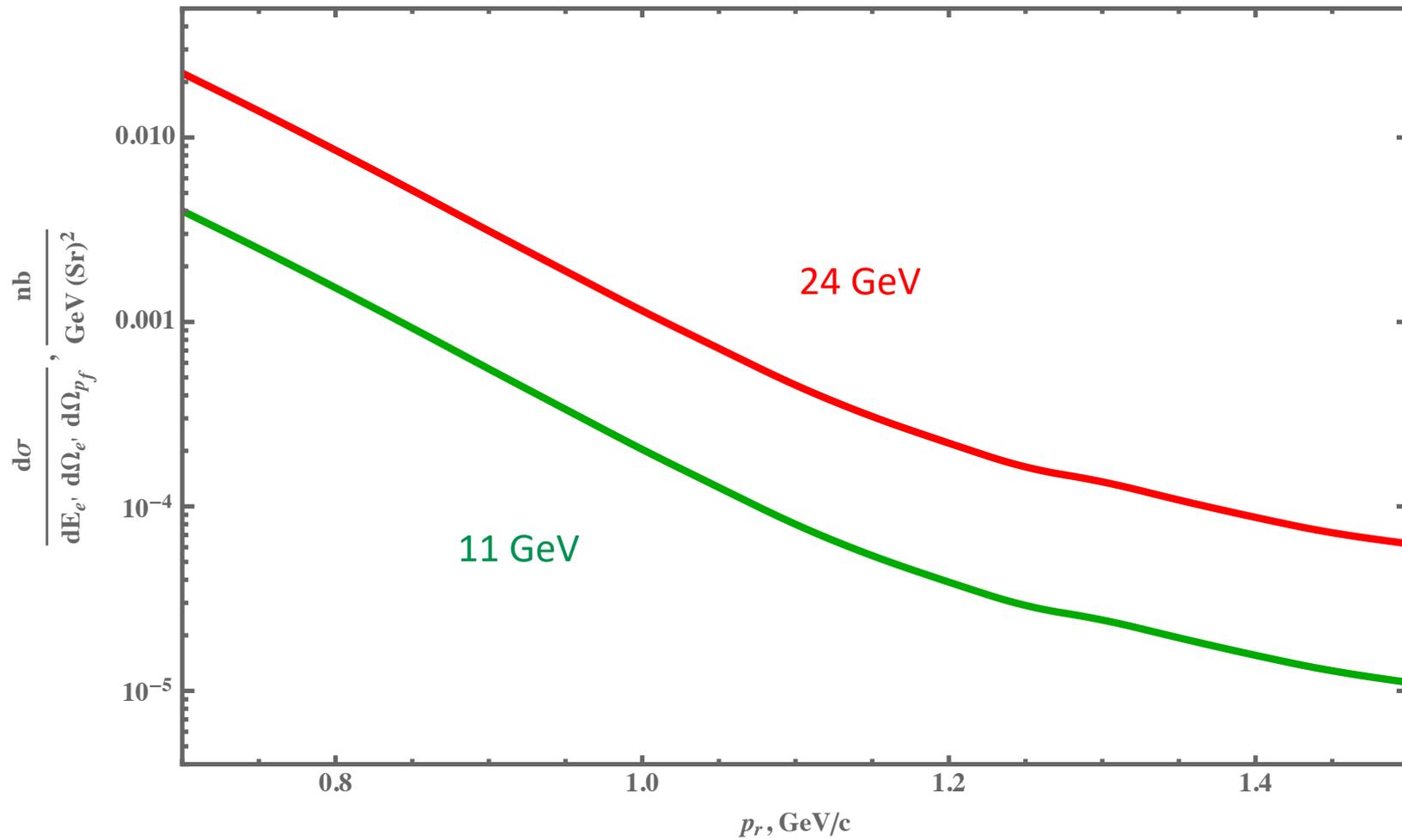
PAC 49 SUMMARY OF JEOPARDY RECOMMENDATIONS

Number	Contact Person	Title	Hall	Previously Approved Days	Days Already Rec'd	Days Awarded	PAC Decision
E12-09-011	Tanja Horn	Studies of the L-T Separated Kaon Electroproduction Cross Section from 5-11 GeV	C	40	32	8	Remain active
E12-10-003	W. Boeglin	Deuteron Electro-Disintegration at Very High Missing Momentum	C	21	3	18	Upgrade Rating to A-

1) Is there any new information that would affect the scientific importance or impact of the Experiment since it was originally proposed?

PAC 36 graded the proposal with B+ because, even though the physics motivation was viewed highly, the foreseen impact of the result was judged to be limited. The results of the three days commissioning in April 2018, published in Physical Review Letters 125, 262501 (2020), exhibit an unexpected behavior when compared with theoretical calculations. Therefore, the expected impact of future data has increased.

$$Q^2 = 5 \text{ GeV}^2 \text{ d}(e,e'p)n$$



Outlook on Experimental Verification of the Effect

- analysis of the experiment will require careful account for competing nuclear effects most importantly **final state interactions**
- **If angular dependence is found** it will motivate new area of research
 - a: modeling non-nucleonic components in the deuteron,
 - b: understanding their origin and nature
 - c: evaluating parameters that can be used for Equation of State of high density Nuclear Matter
- **If no angular dependence is found,**
 - a: nucleonic degrees persist at very high density fluctuations
 - b: non-nucleonic components conspire to preserve angular condition
 - c: theory was wrong

2. Probing SuperFast Quarks in Nuclei

Studies of nuclear partonic distributions at $x > 1$

Bjorken $x = \frac{Q^2}{2m_N \nu}$

- $x > 1$ requires a momentum transfer from the nearby nucleon or the quark from the nearby nucleon.
- $x > 1$ "super-fast quarks"

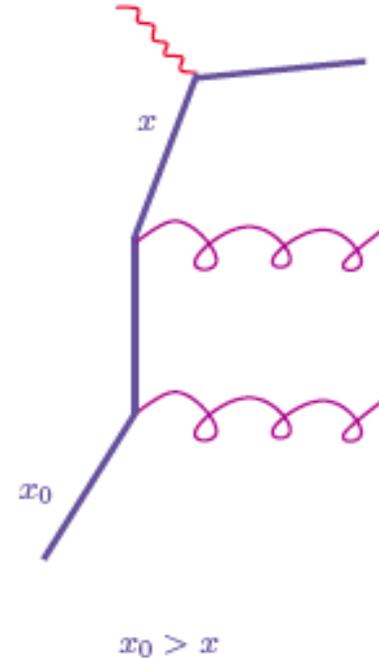
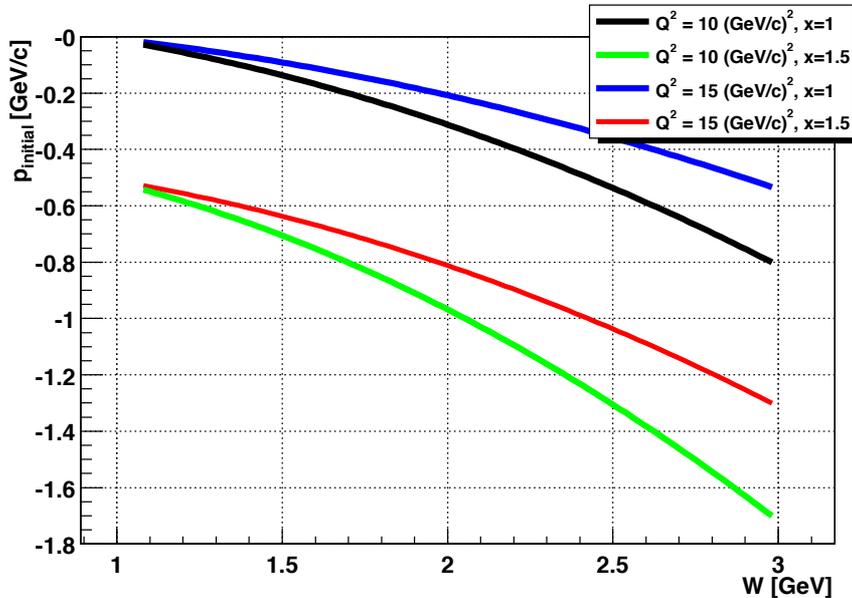
2. SuperFast quarks – short distance probes in nuclei

$$x = \frac{Q^2}{2m_N q_0} > 1$$

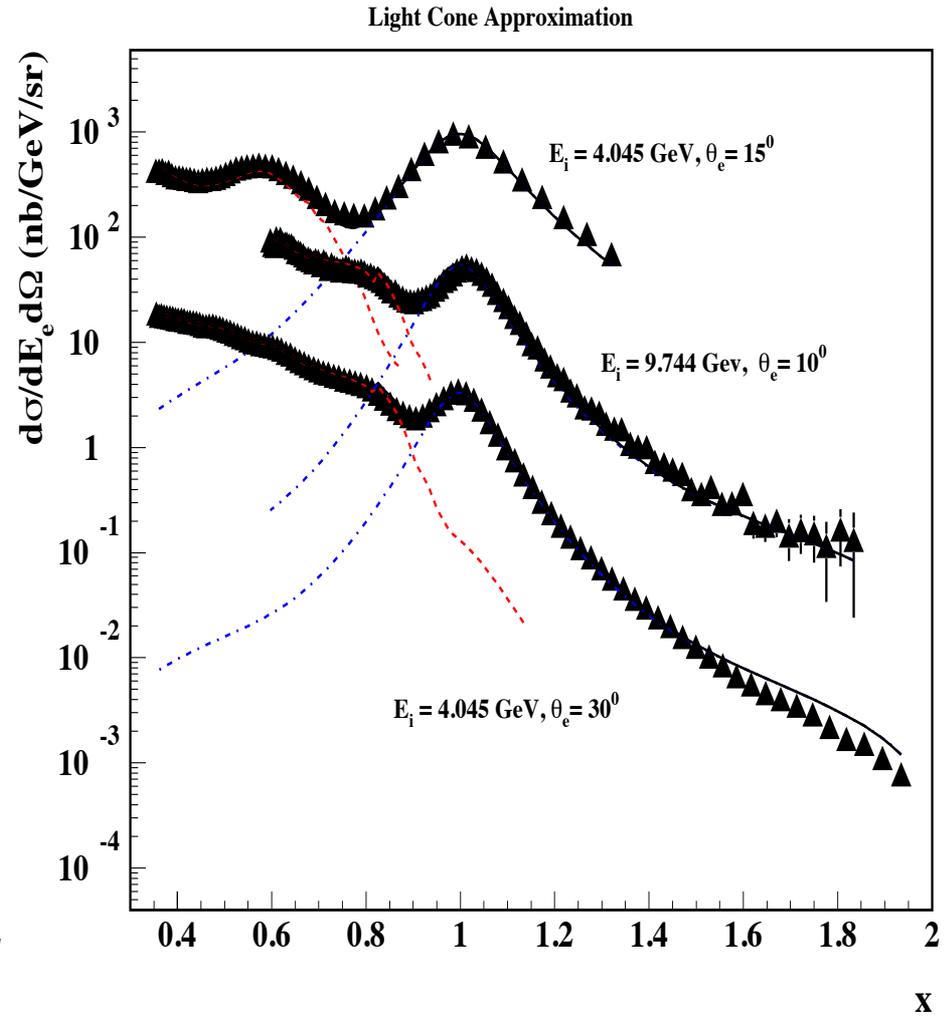
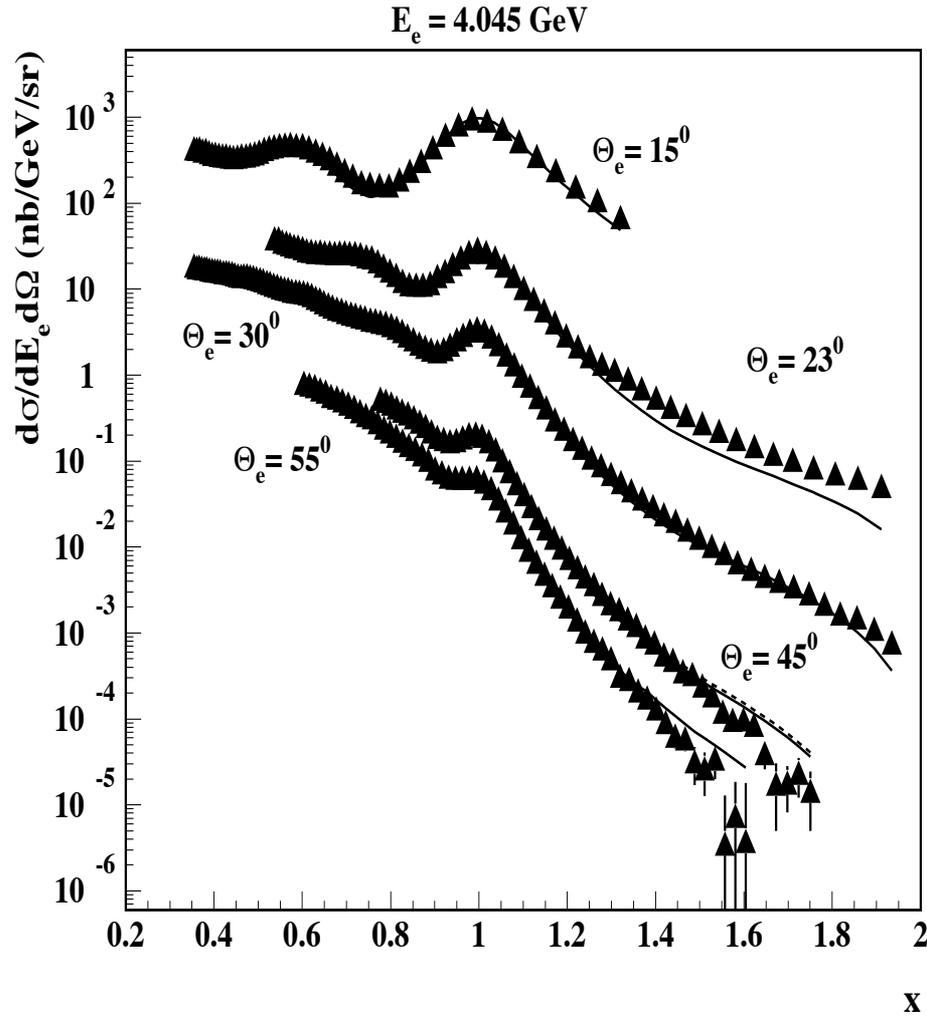
Two factors driving nucleons close together

Kinematic $p_{min} \equiv p_z = m_N \left(1 - x - x \left[\frac{W_N^2 - m_N^2}{Q^2} \right] \right)$

Dynamical: QCD evolution



Inclusive $d(e, e')X$



Existing Experiments:

1. BCDMS Collaboration 1994 (CERN): $52 \leq Q^2 \leq 200 \text{ GeV}^2$
2. CCFR Collaboration 2000 (FermiLab): $Q^2 = 120 \text{ GeV}^2$
3. E02-019 Experiment 2010 (JLab) $Q_{AV}^2 = 7.4 \text{ GeV}^2$
4. New Approved Experiments at JLab12: $e + A \rightarrow e' + X$, $Q^2 \geq 10 \text{ GeV}^2$
5. Alternative Studies at LHC: $p+A \rightarrow 2 \text{ jets} + X$
6. Electron Ion Collider: $\gamma + A \rightarrow e' + X$,
 $e + A \rightarrow e' + jet/N/h + X$, $x_{Bj} > 1$, $Q^2 \geq 20 \text{ GeV}^2$
 $\gamma + A \rightarrow jet_f/h_f + jet_b/h_b + X$ $x_h > 1$

1. BCDMS Collaboration 1994 (CERN): Z.Phys C63 1994

Structure function of Carbon in deep-inelastic scattering of 200GeV muons

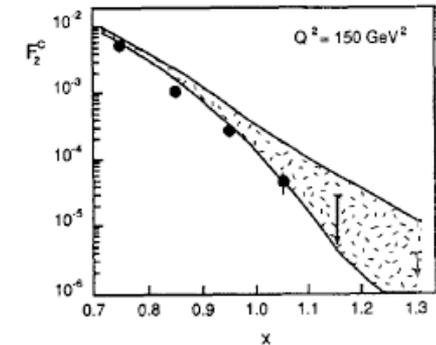
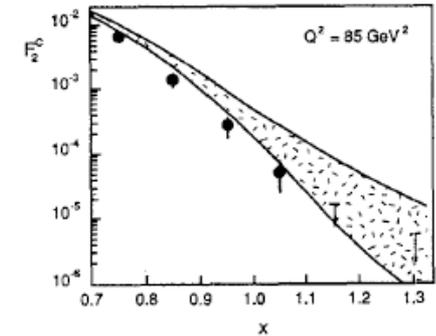
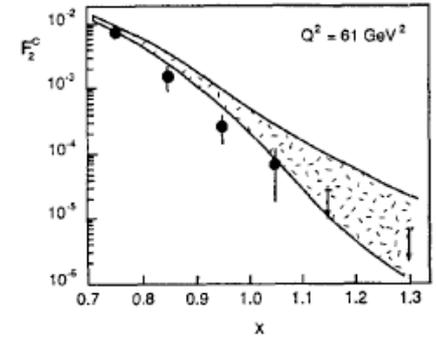
$$Q^2 = 61, 85 \text{ and } 150 \text{ GeV}^2$$

$$x = 0.85, 0.95, 1.05, 1.15 \text{ and } 1.3$$

$$F_{2A}(x, Q^2) = F_{2A}(x_0 = 0.75, Q^2) e^{-s(x-0.75)}$$

$$s = 16.5 \pm 0.6$$

More than Fermi Gas but very marginal high momentum component



2. CCFR Collaboration 2000 (FermiLab):

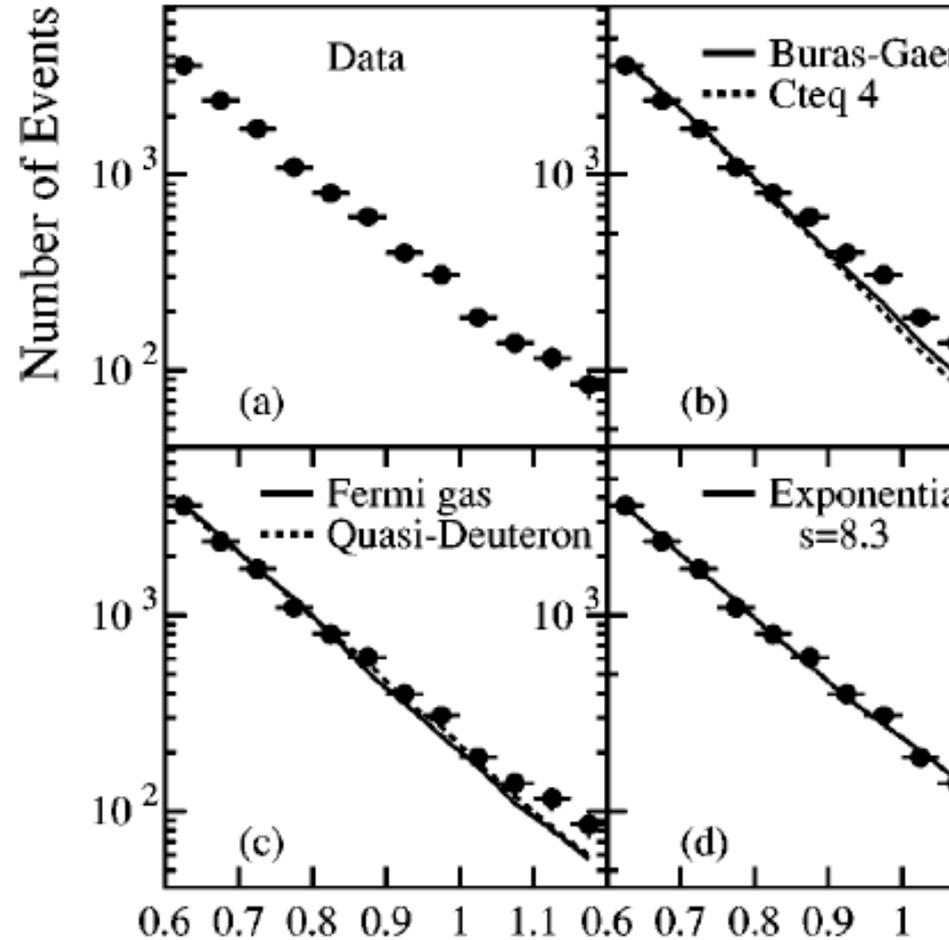
Phys. Rev. D61 2000

Using the neutrino and antineutrino beams in which structure function of Iron was measured in the charged current sector for average

$$Q^2 = 120 \text{ GeV}^2 \text{ and } 0.6 \leq x \leq 1.2.$$

$$F_{2A} \sim e^{-s(x-x_0)}$$

$$s = 8.3 \pm 0.7(\text{stat}) \pm 0.7(\text{sys})$$



3. E02-019 Experiment 2010 (JLab)

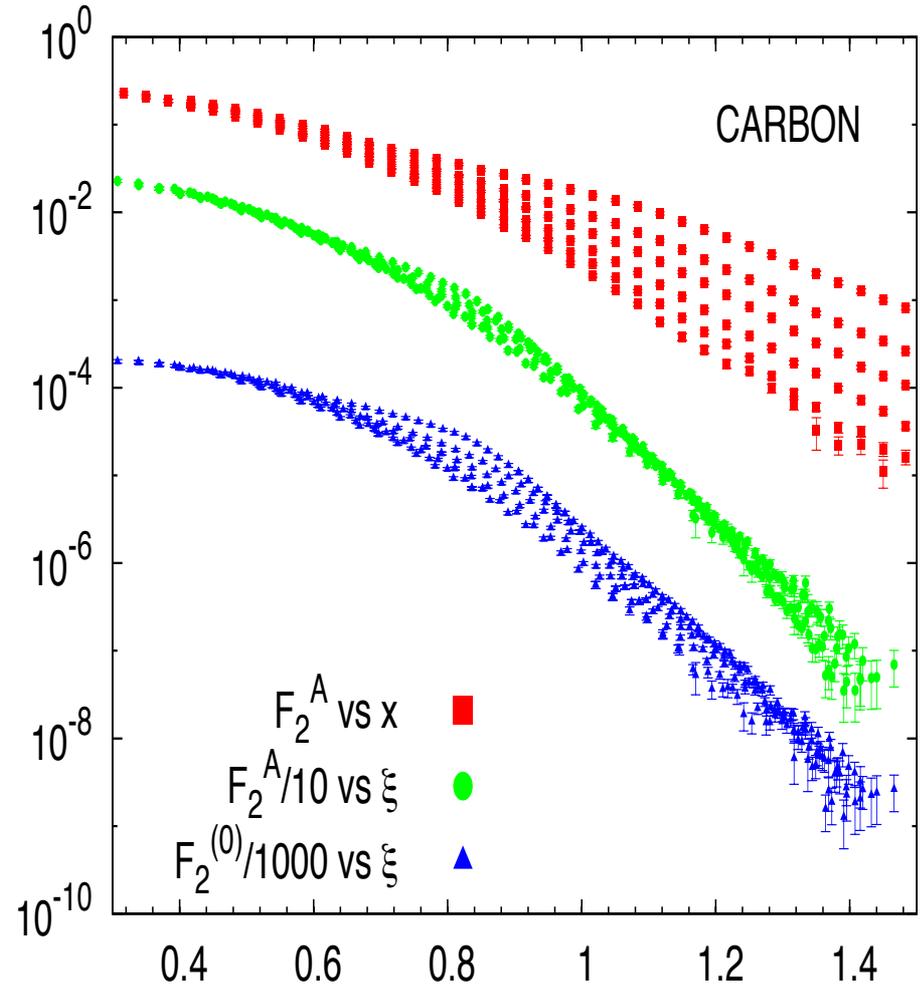
Fomin, Arrington, Phys.Rev.Lett 204 2010

(ee') scattering of

^2H , ^3He , ^4He , ^9Be , ^{12}C , ^{64}Cu and ^{197}Au

$$6 < Q^2 < 9 \text{ GeV}^2$$

$$\xi = \frac{2x}{(1+r)} \text{ where } r = \sqrt{1 + \frac{4M_N^2 x^2}{Q^2}}$$



QCD Evolution Equation for Nuclear Partonic Distributions

Adam Freese, MS
ArXiv 2015

$$\frac{dq_{i,A}(x, Q^2)}{d \log Q^2} = \frac{\alpha_s}{2\pi} \left\{ 2 \left(1 + \frac{4}{3} \log \left(1 - \frac{x}{A} \right) \right) q_{i,A}(x, Q^2) \right. \\ \left. + \frac{4}{3} \int_{x/A}^1 \frac{dz}{1-z} \left(\frac{1+z^2}{z} q_{i,A}\left(\frac{x}{z}, Q^2\right) - 2q_{i,A}(x, Q^2) \right) + \int_{x/A}^1 dz \frac{(1-z)^2 + z^2}{2z} G_A\left(\frac{x}{z}, Q^2\right) \right\}$$

$$F_{2A}(x, Q^2) = \sum_i e_i^2 x q_{i,A}(x, Q^2),$$

$$\frac{dF_{2A}(x, Q^2)}{d \log Q^2} = \frac{\alpha_s}{2\pi} \left\{ 2 \left(1 + \frac{4}{3} \log \left(1 - \frac{x}{A} \right) \right) F_{2,A}(x, Q^2) \right. \\ \left. + \frac{4}{3} \int_{x/A}^1 \frac{dz}{1-z} \left(\frac{1+z^2}{z} F_{2A}\left(\frac{x}{z}, Q^2\right) - 2F_{2A}(x, Q^2) \right) + \frac{f_Q}{2} \int_{x/A}^1 dz [(1-z)^2 + z^2] \frac{x}{z} G_A\left(\frac{x}{z}, Q^2\right) \right\}$$

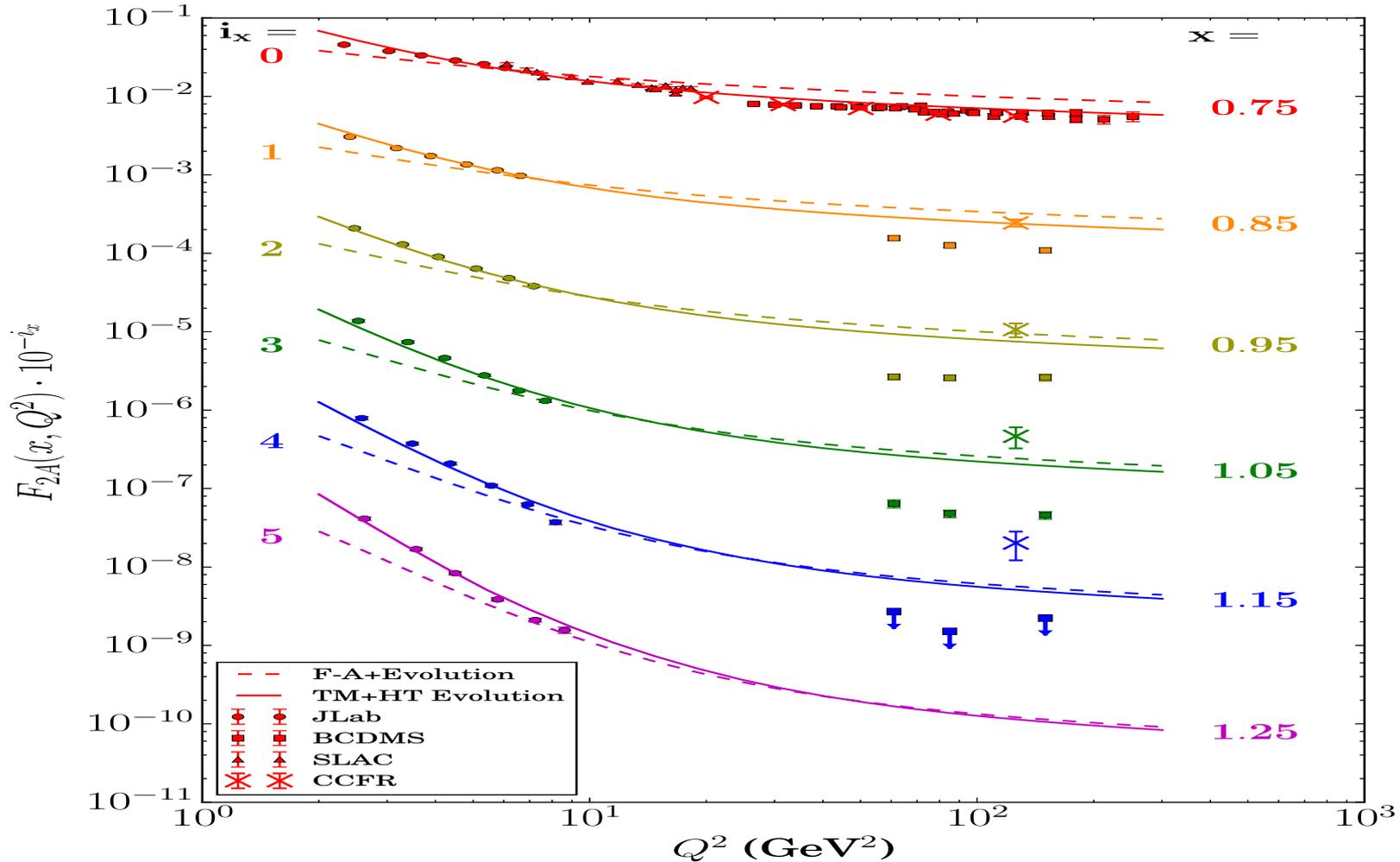
Neglecting $G_A(x, Q^2)$

$$\frac{dF_{2A}(x, Q^2)}{d \log Q^2} = \frac{\alpha_s}{2\pi} \left\{ 2 \left(1 + \frac{4}{3} \log\left(1 - \frac{x}{A}\right) \right) F_{2,A}(x, Q^2) + \frac{4}{3} \int_{x/A}^1 \frac{dz}{1-z} \left(\frac{1+z^2}{z} F_{2A}\left(\frac{x}{z}, Q^2\right) - 2F_{2A}(x, Q^2) \right) \right\}$$

Using input $F_{2A}^{(0)}(\xi, Q^2)$ from JLab analysis at $Q^2 = 7.4 \text{ GeV}^2$

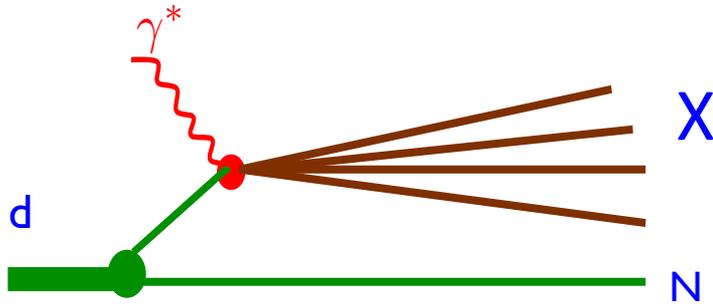
and calculate the evolution to Q^2 region of CCFR and BCDMS

$$Q^2 = 120 \text{ GeV}^2 \quad 52 \leq Q^2 \leq 200 \text{ GeV}^2$$



- Dynamics of generation of superfast quarks in nuclei

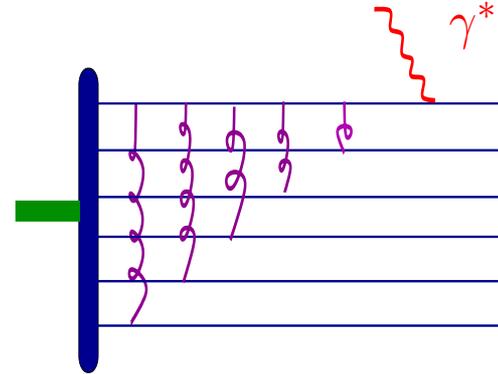
1. Convolution Model



$$F_{2d} = \int_x^2 \rho_d^N(\alpha, p_t) F_{2N}\left(\frac{x}{\alpha}, Q^2\right) \frac{d^2\alpha}{\alpha} d^2p_t$$

$$x_N = \frac{x}{\alpha}$$

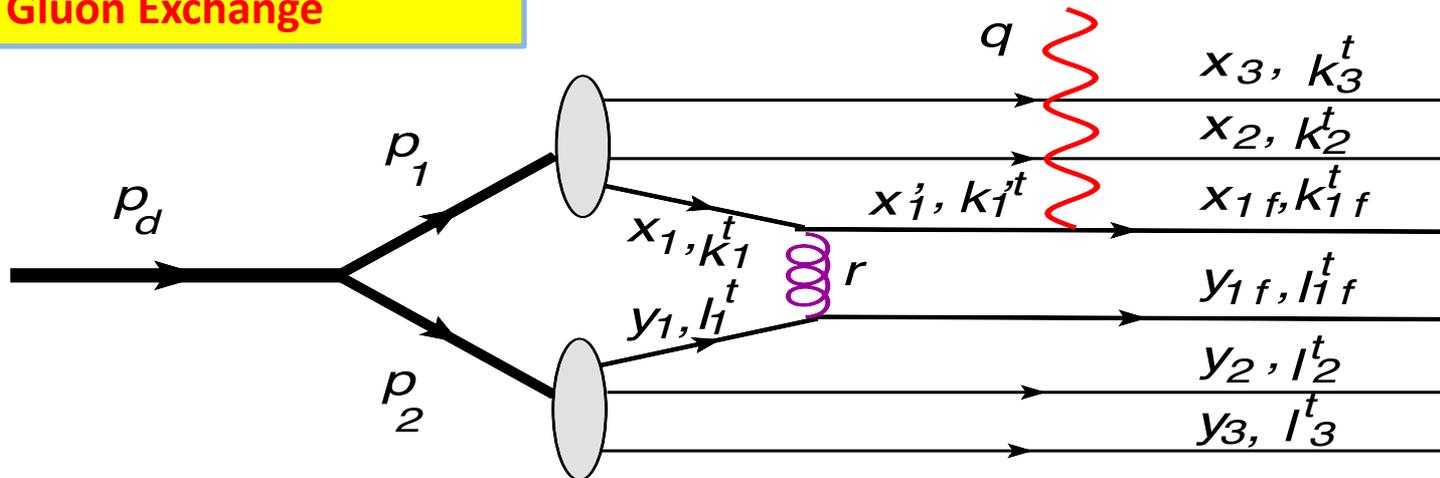
2. Six-Quark Model



$$F_{2D} = F_{2,(6q)} \sim \left(1 - \frac{x}{2}\right)^{10}$$

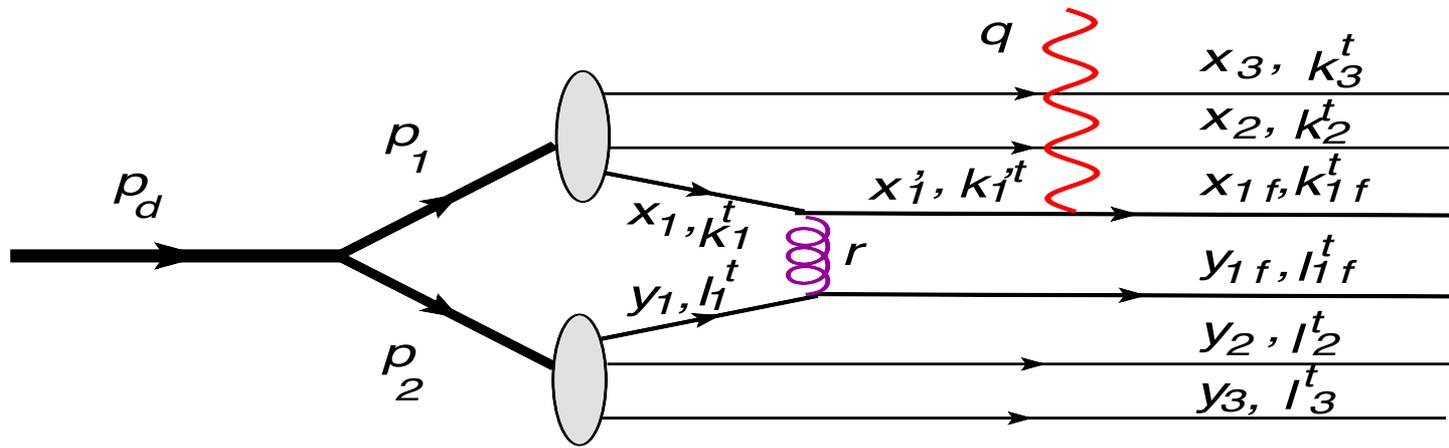
Hard Gluon Exchange

MS in progress



$$A^\sigma = \sum_{h_1, h_2} \int \frac{d\alpha}{\alpha} \frac{d^2 p_2}{2(2\pi)^3}$$

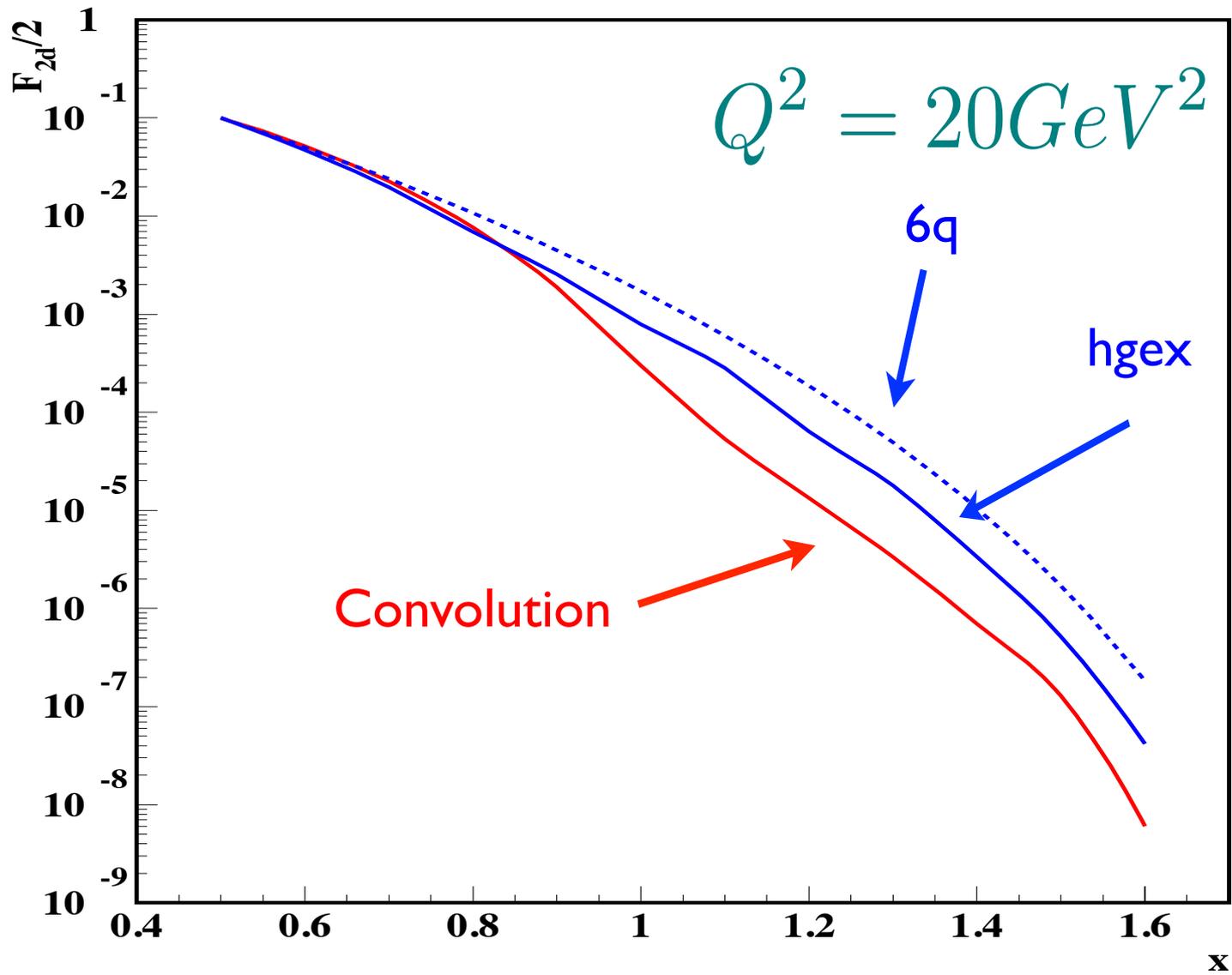
$$\left\{ \sum_{\eta_1, \lambda_1} H^\sigma_{(\eta_{1f}, \eta_1), (\lambda_{1f}, \lambda_1)} \frac{\psi_N^{h_1}(k_1, \eta_1; k_2, \eta_2; k_3, \eta_3)}{x_1 \sqrt{2(2\pi)^3}} \frac{\psi_N^{h_2}(l_1, \lambda_1; l_2, \lambda_2; l_3, \lambda_3)}{y_1 \sqrt{2(2\pi)^3}} \right\} \frac{\Psi_d^{h_1, h_2, m_d}(p_1, p_2)}{(1 - \alpha) \sqrt{2(2\pi)^3}}$$



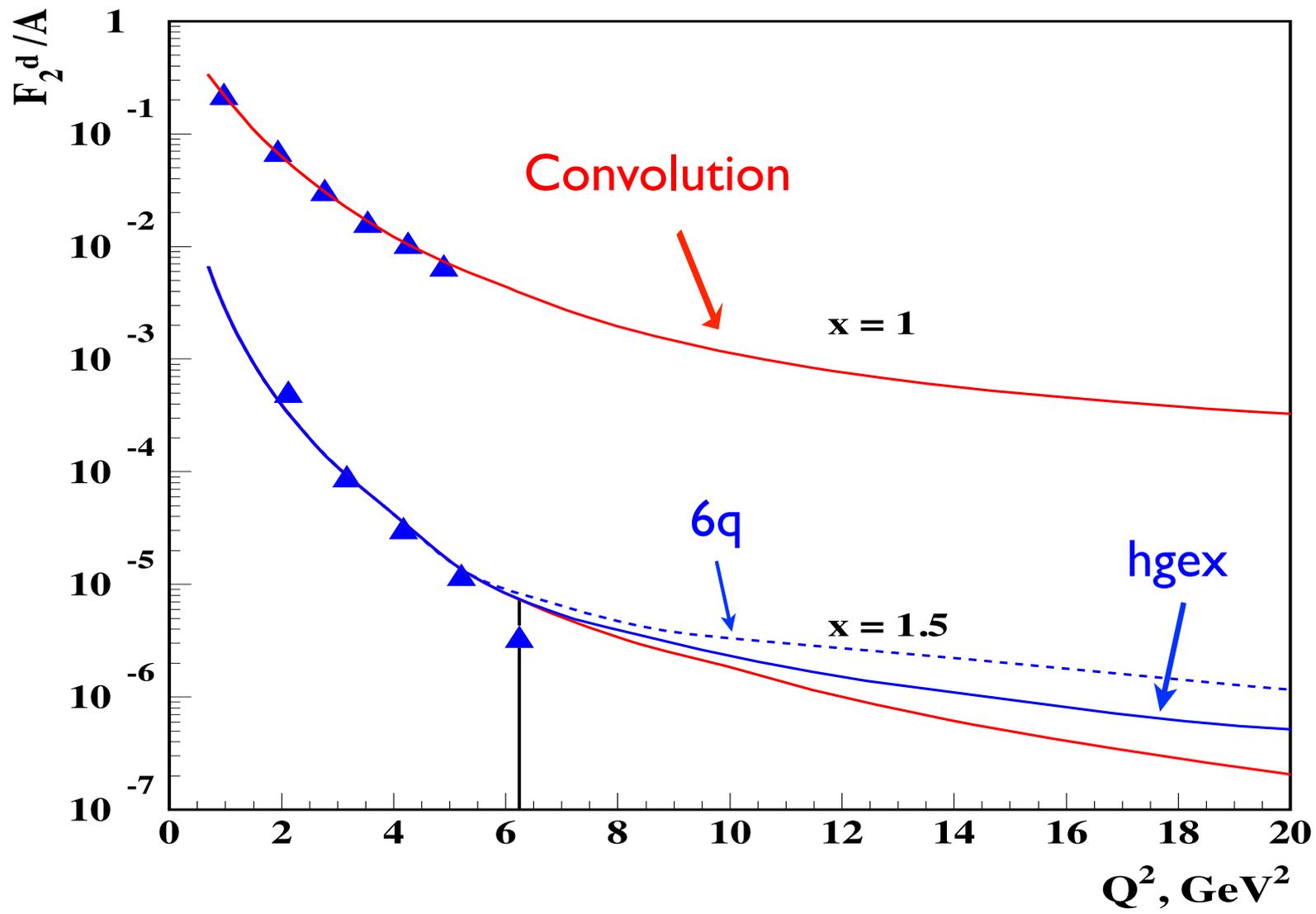
$$F_{2d}(x_{Bj}, Q^2) = \sum_{i,j} x_{Bj} e_i^2 \int dx_1 dy_1 \frac{d^2 l_{1f,t}}{2(2\pi)^3} \frac{8\alpha_{QCD}}{l_{1f,t}^4} f_i(x_1, Q^2) f_j(y_1, l_{1f,t}^2) \times$$

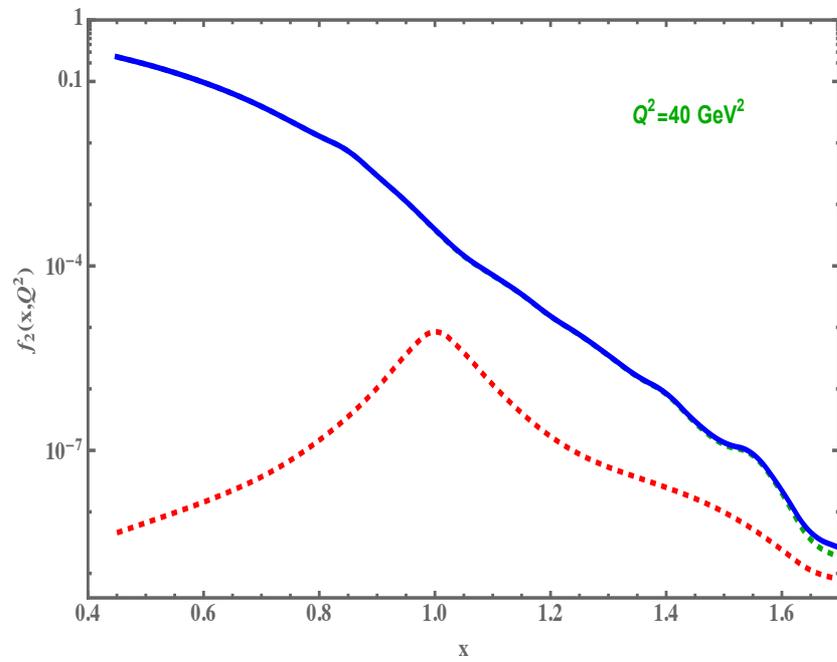
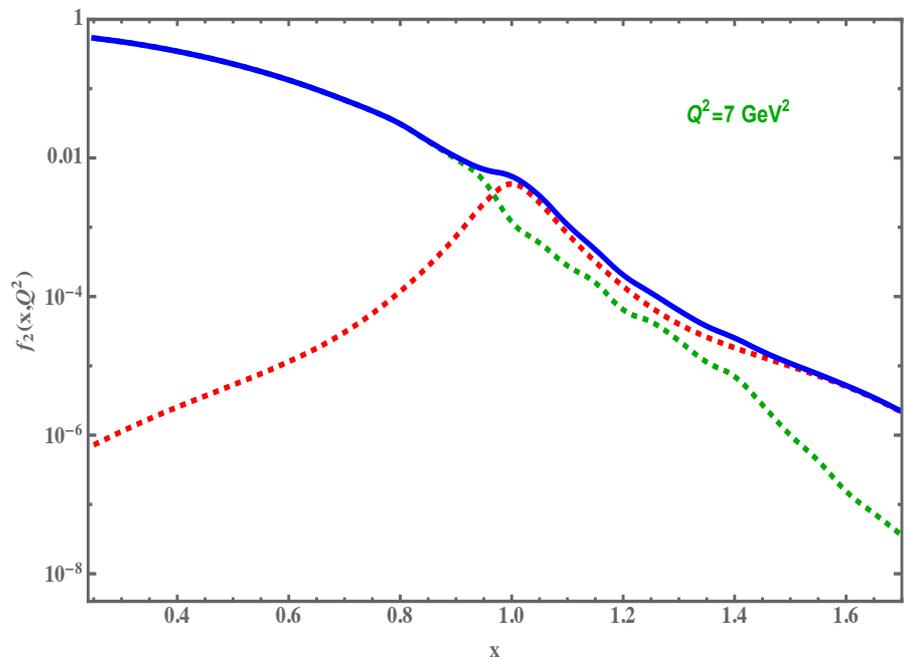
$$\frac{1}{y_1^2} \left[1 - \frac{x_{Bj}}{x_1 + y_1} \right]^2 \Theta(x_1 + y_1 - x_{Bj}) \left[\sum_{h_1, h_2} \int \frac{\Psi_d(\alpha, p_t)}{\alpha(1-\alpha)} \frac{d\alpha}{\sqrt{2(2\pi)^3}} \frac{d^2 p_t}{(2\pi)^2} \right]^2$$

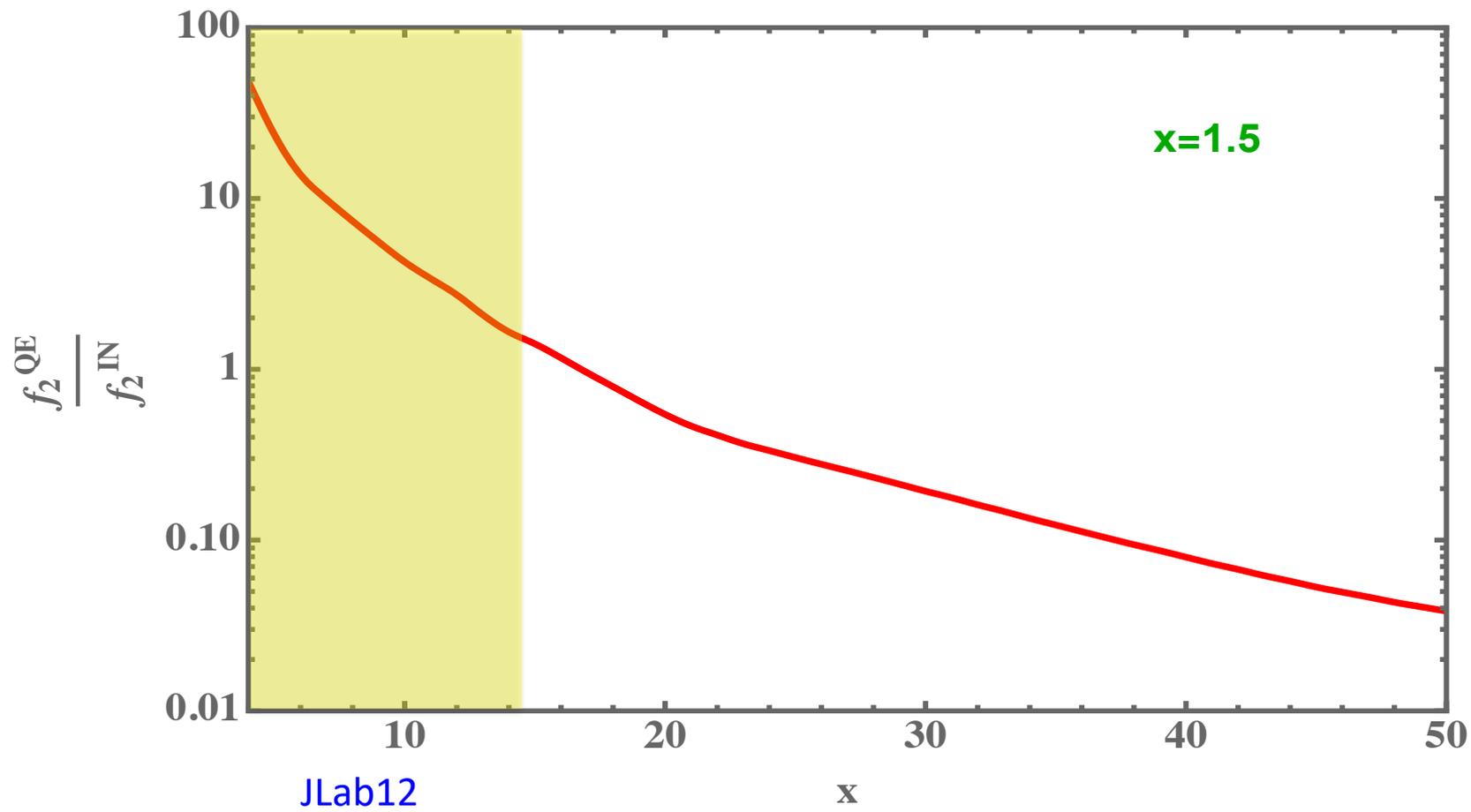
where $x_{Bj} = \frac{Q^2}{2m_N \nu}$.



$d(e, e') X$







Conclusions and Outlook

- Dedicated studies of deuteron will allow for the first time to probe the NN core
- Anisotropic Momentum Distribution will Indicate the onset of Non-Nucleonic Component in the NN - system
- DIS structure function of superfast quarks will allow to identify the mechanism of NN- interaction at very short short

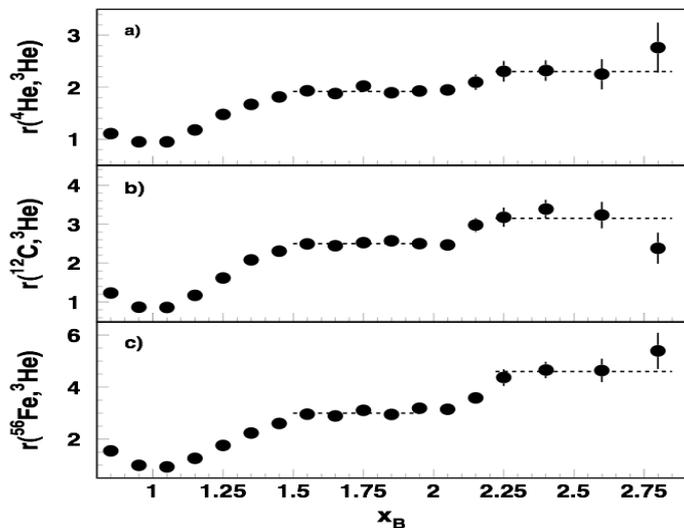
Additional Slides

(3) Probing Three Nucleon Short Range Correlations

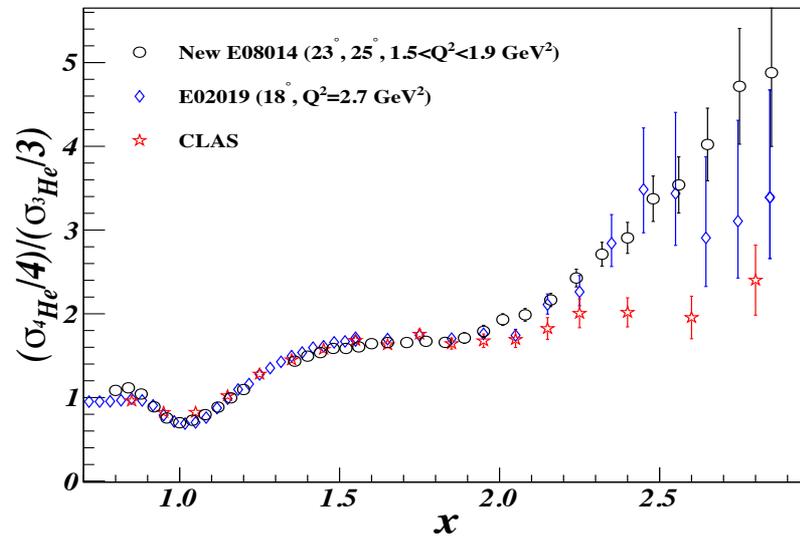
Looking for the Plateau in Inclusive Cross Section Ratios $x > 2$

$$\text{For } 1 < x < 2 \quad R \approx \frac{a_2(A_1)}{a_2(A_2)}$$

$$\text{For } 2 < x < 3 \quad R \approx \frac{a_3(A_1)}{a_3(A_2)}$$



Egiyan, et al PRL 2006

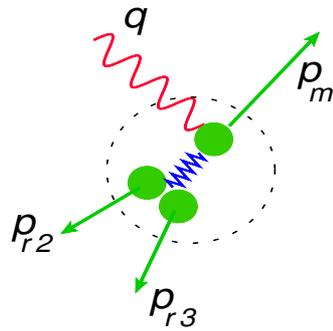


Z. Ye, et al, 2017

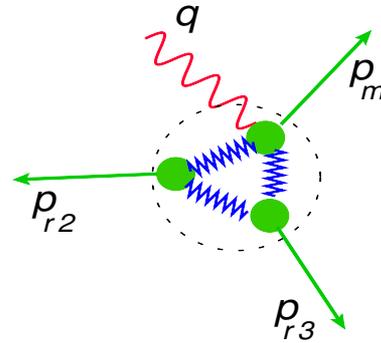
3N SRCs:

Proper Variables of 3N SRC are

- the Light Front Momentum Fraction: $\alpha = \frac{p_N^+}{p_{3N}^+}$
- transverse momentum: p_{\perp}



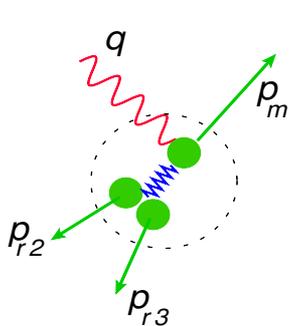
(a)



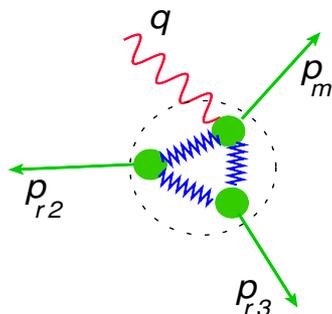
(b)

3N SRCs in Inclusive $A(e,e')X$ Reactions

$$\sigma_{eA} = \sum \sigma_{eN} \cdot \rho_A(\alpha) \quad \text{where} \quad \rho_A(\alpha) = \int \rho_A(\alpha, p_{\perp}) d^2 p_{\perp}$$

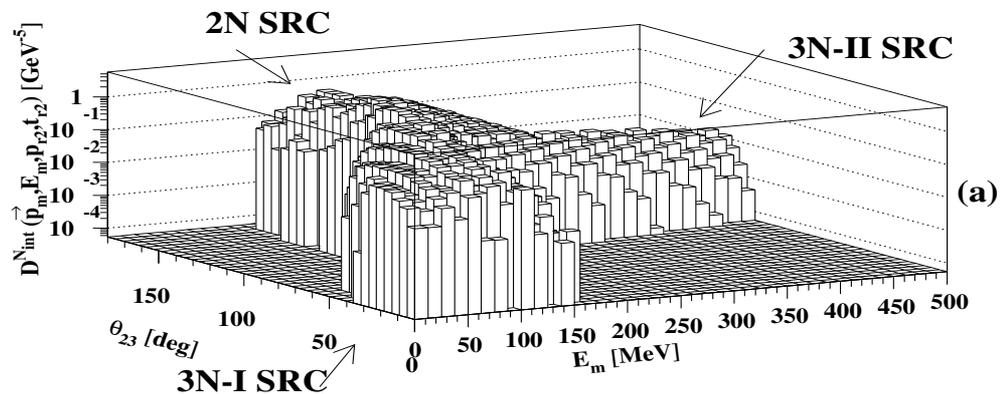


(a)

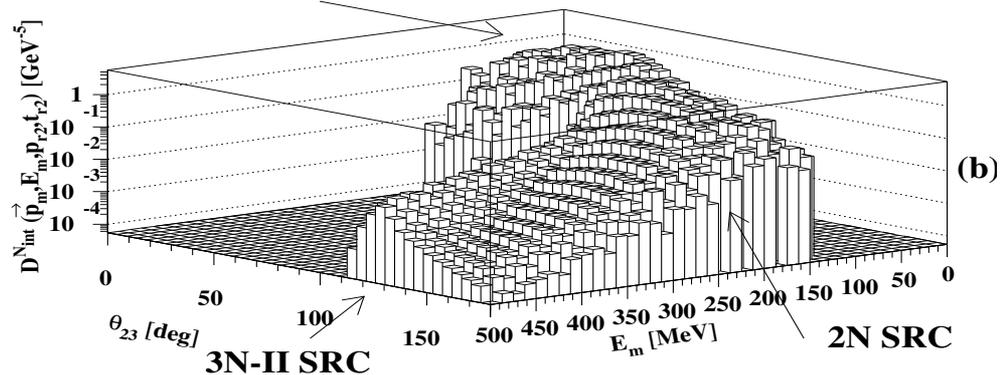


(b)

M.S. Abrahamyan, Frankfurt,
Strikman, Phys. Rev. C 2005



(a)



(b)

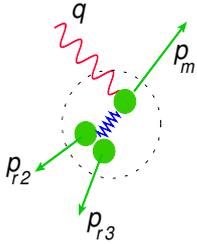
3N SRC model

$$\sigma_{eA} = \sum \sigma_{eN} \cdot \rho_A(\alpha) \quad \text{where} \quad \rho_A(\alpha) = \int \rho_A(\alpha, p_\perp) d^2 p_\perp$$

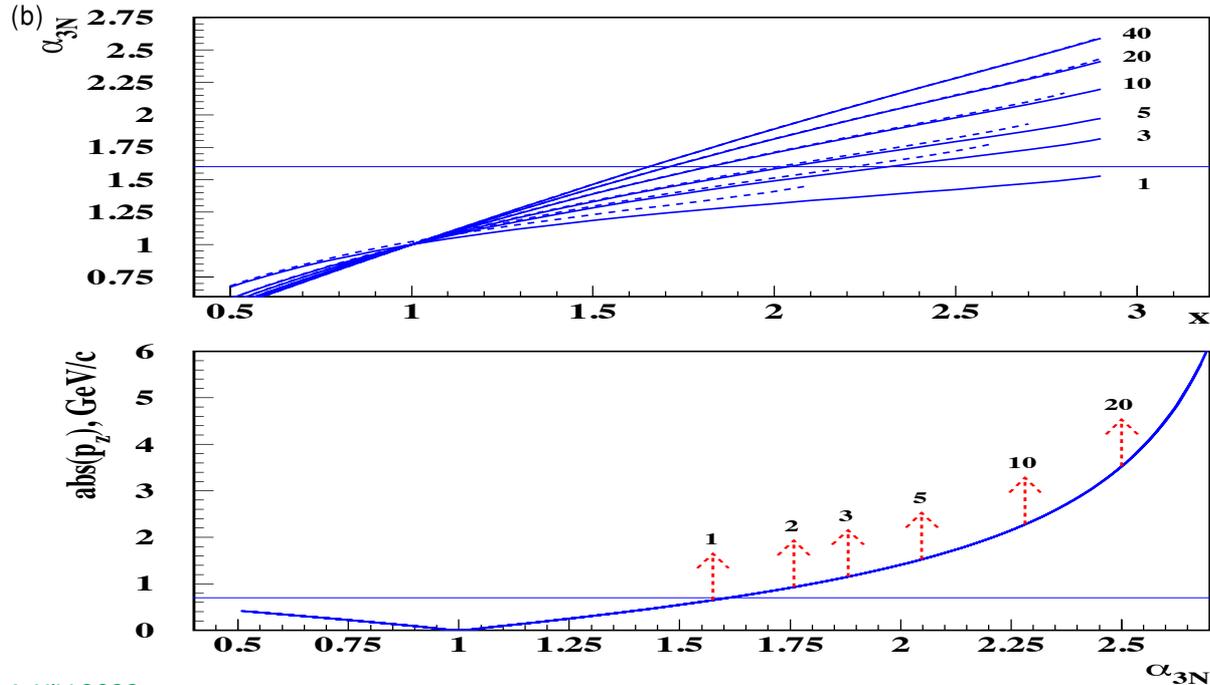
$$1.6 \leq \alpha_{3N} < 3$$

$$q + 3m_N = p_f + p_s$$

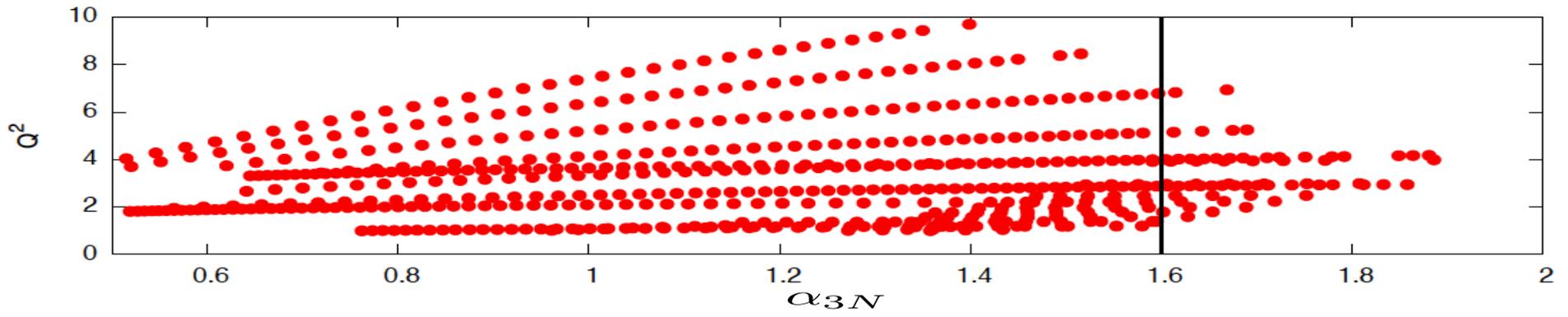
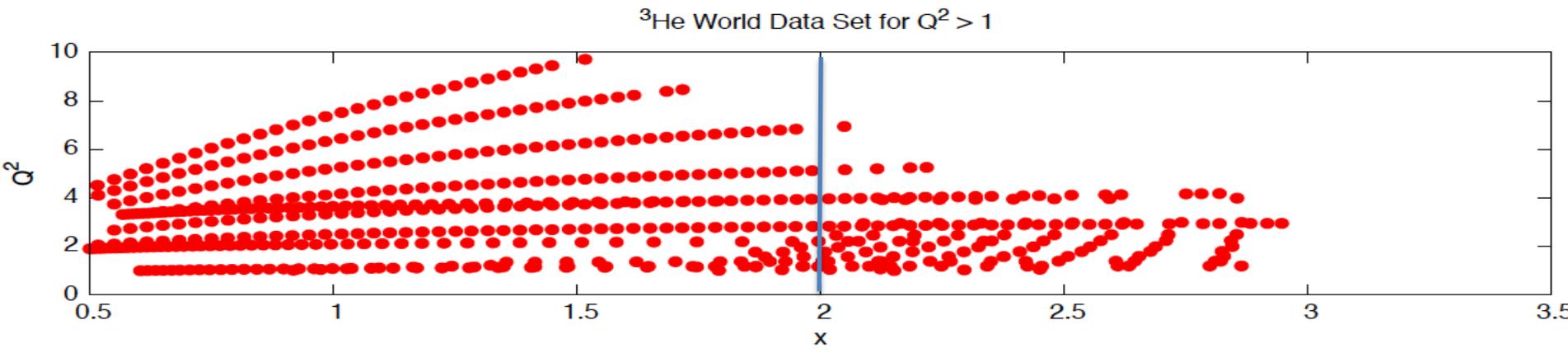
$$= 3 - \frac{q_- + 3m_N}{2m_N} \left[1 + \frac{m_S^2 - m_N^2}{W_{3N}^2} + \sqrt{\left(1 - \frac{(m_S + m_n)^2}{W_{3N}^2}\right) \left(1 - \frac{(m_S - m_n)^2}{W_{3N}^2}\right)} \right] (1)$$



(a)

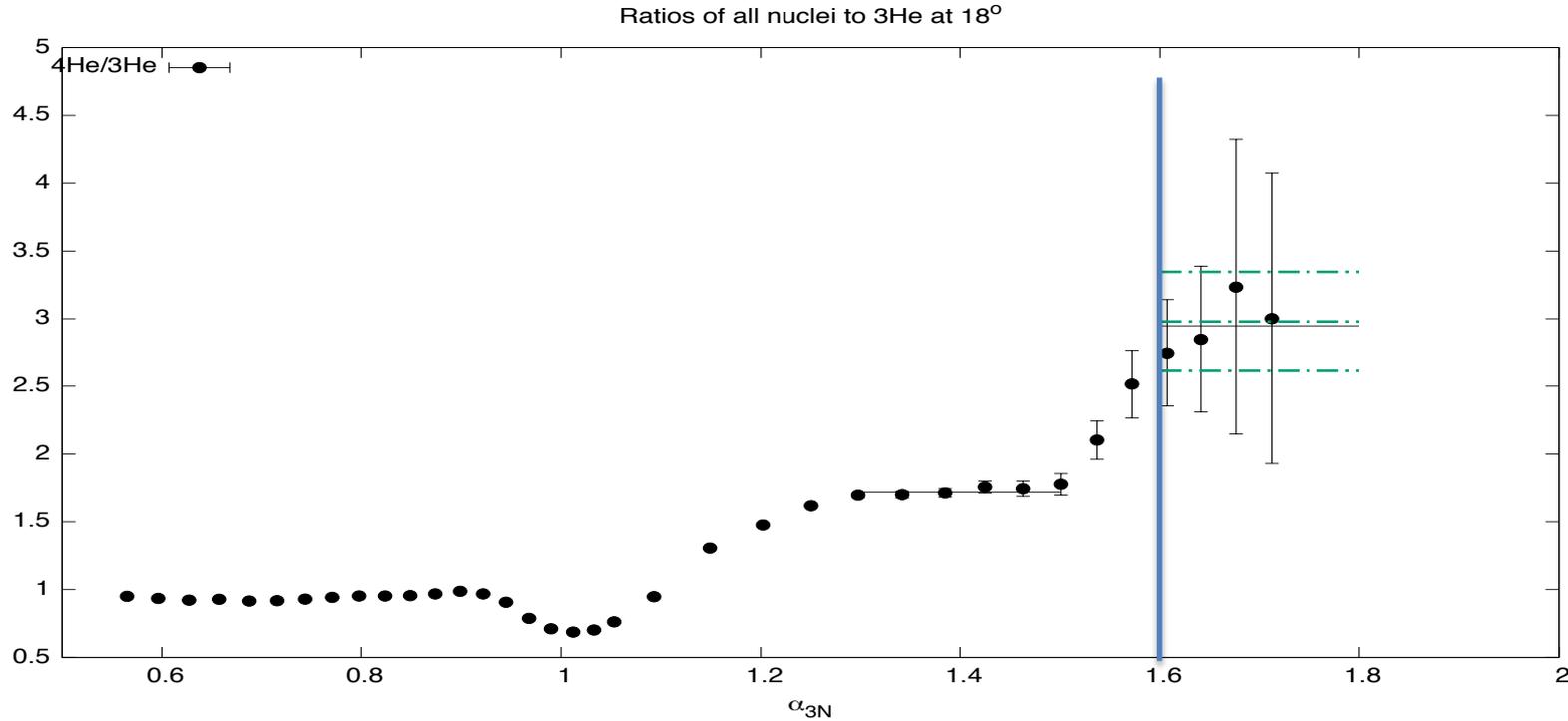


3N SRC model $\sigma_{eA} = \sum \sigma_{eN} \cdot \rho_A(\alpha)$ where $\rho_A(\alpha) = \int \rho_A(\alpha, p_{\perp}) d^2 p_{\perp}$
 $1.6 \leq \alpha_{3N} < 3$



3N SRCs

$$\sigma_{eA} = \sum \sigma_{eN} \cdot \rho_A(\alpha) \quad \text{where} \quad \rho_A(\alpha) = \int \rho_A(\alpha, p_{\perp}) d^2 p_{\perp}$$
$$1.6 \leq \alpha_{3N} < 3$$

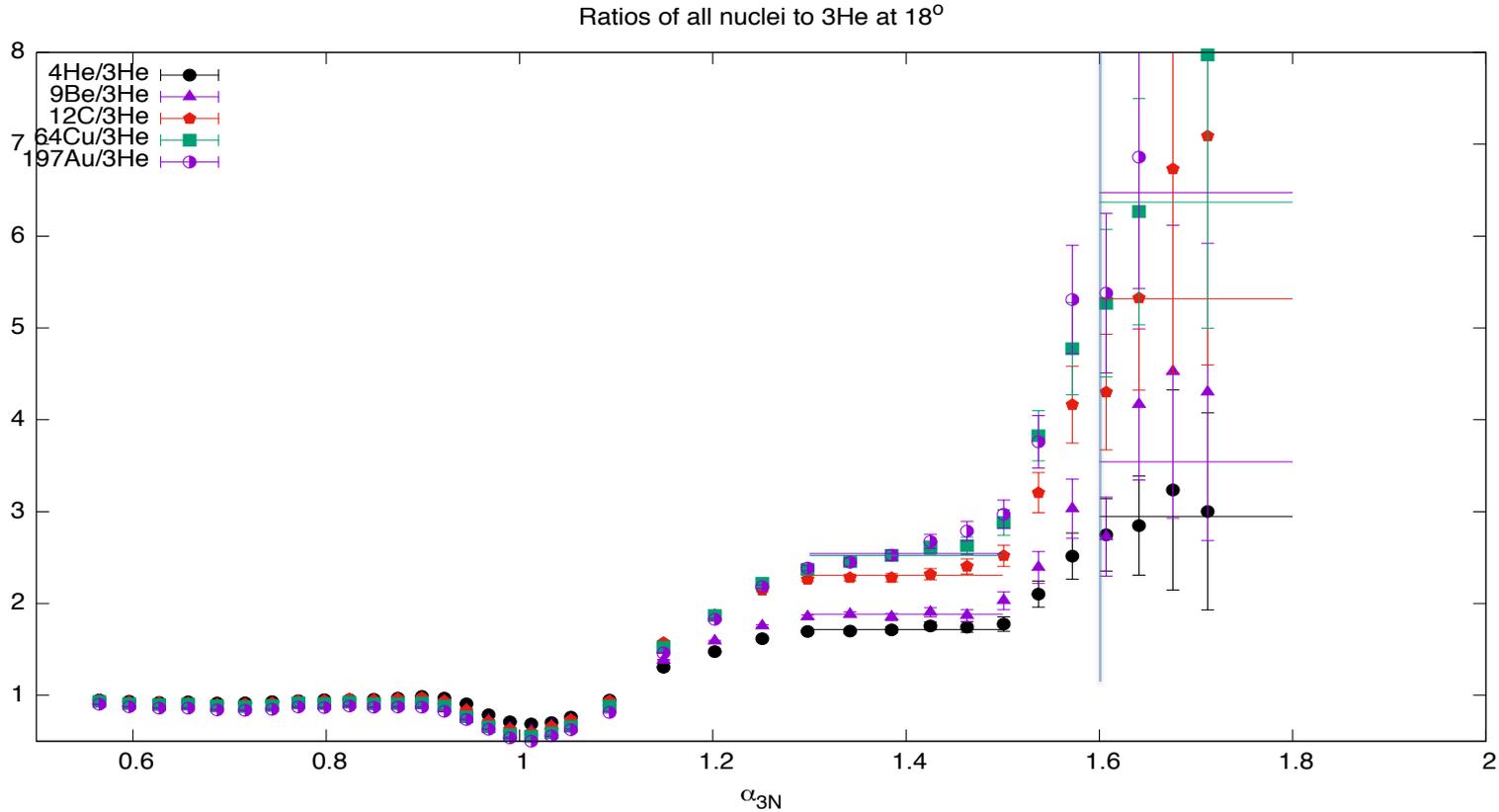


JLab - E02019 - Data

3N SRC scaling

$$\sigma_{eA} = \sum \sigma_{eN} \cdot \rho_A(\alpha) \quad \text{where} \quad \rho_A(\alpha) = \int \rho_A(\alpha, p_{\perp}) d^2 p_{\perp}$$

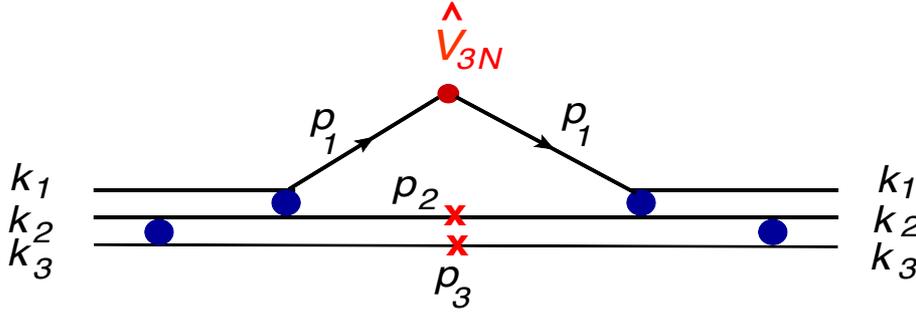
$$1.6 \leq \alpha_{3N} < 3$$



3N SRC: Light-Cone Momentum Fraction Distribution

A.Freese, M.S., M.Strikman, Eur. Phys. J 2015

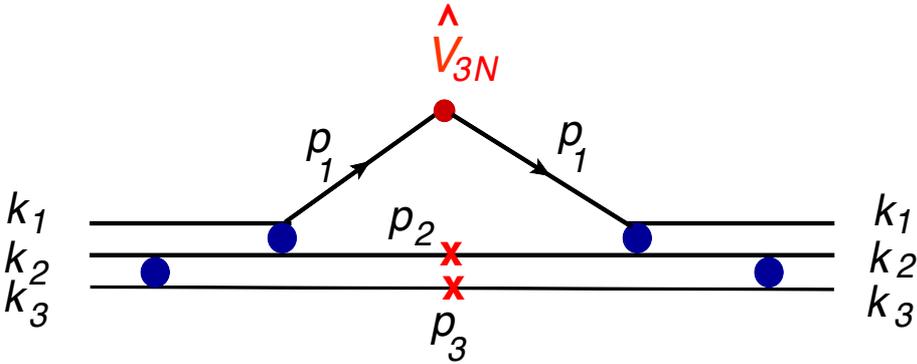
O. Artiles M.S. Phys. Rev. C 2016



$$\begin{aligned}
 P_{A,3N}^N(\alpha_1, p_{1,\perp}, s_1, \tilde{M}_N) &= \sum_{s_2, s_3, s_{2'}, \tilde{s}_{2'}} \int \bar{u}(k_1) \bar{u}(k_2) \bar{u}(k_3) \Gamma_{NN \rightarrow NN}^\dagger \frac{u(p_{2'}, \tilde{s}_{2'}) \bar{u}(p_{2'}, \tilde{s}_{2'})}{p_{2'}^2 - M_N^2} \\
 &\times \Gamma_{NN \rightarrow NN}^\dagger \frac{u(p_1, s_1)}{p_1^2 - M_N^2} u(p_2, s_2) \left[2\alpha_1^2 \delta(\alpha_1 + \alpha_2 + \alpha_3 - 3) \delta^2(p_{1\perp} + p_{2\perp} + p_{3\perp}) \delta(\tilde{M}_N^2 - M_N^{3N,2}) \right] \\
 &\times \bar{u}(p_2, s_2) \frac{\bar{u}(p_1, s_1)}{p_1^2 - M_N^2} \Gamma_{NN \rightarrow NN} \frac{u(p_{2'}, s_{2'}) \bar{u}(p_{2'}, s_{2'})}{p_{2'}^2 - M_N^2} u(p_3, s_3) \bar{u}(p_3, s_3) \Gamma_{NN \rightarrow NN}^\dagger u(k_1) u(k_2) u(k_3) \\
 &\times \frac{d\alpha_2}{\alpha_2} \frac{d^2 p_{2\perp}}{2(2\pi)^3} \frac{d\alpha_3}{\alpha_3} \frac{d^2 p_{3\perp}}{2(2\pi)^3}, \tag{1}
 \end{aligned}$$

$$\begin{aligned}
 P_{A,3N}^N(\alpha_1, p_{1,\perp}, \tilde{M}_N) &= \int \frac{3 - \alpha_3}{2(2 - \alpha_3)^2} \boxed{\rho_{NN}(\beta_3, p_{3\perp}) \rho_{NN}(\beta_1, \tilde{k}_{1\perp})} 2\delta(\alpha_1 + \alpha_2 + \alpha_3 - 3) \\
 &\delta^2(p_{1\perp} + p_{2\perp} + p_{3\perp}) \delta(\tilde{M}_N^2 - M_N^{3N,2}) d\alpha_2 d^2 p_{2\perp} d\alpha_3 d^2 p_{3\perp}, \tag{1}
 \end{aligned}$$

3N SRC: Light-Cone Momentum Fraction Distribution



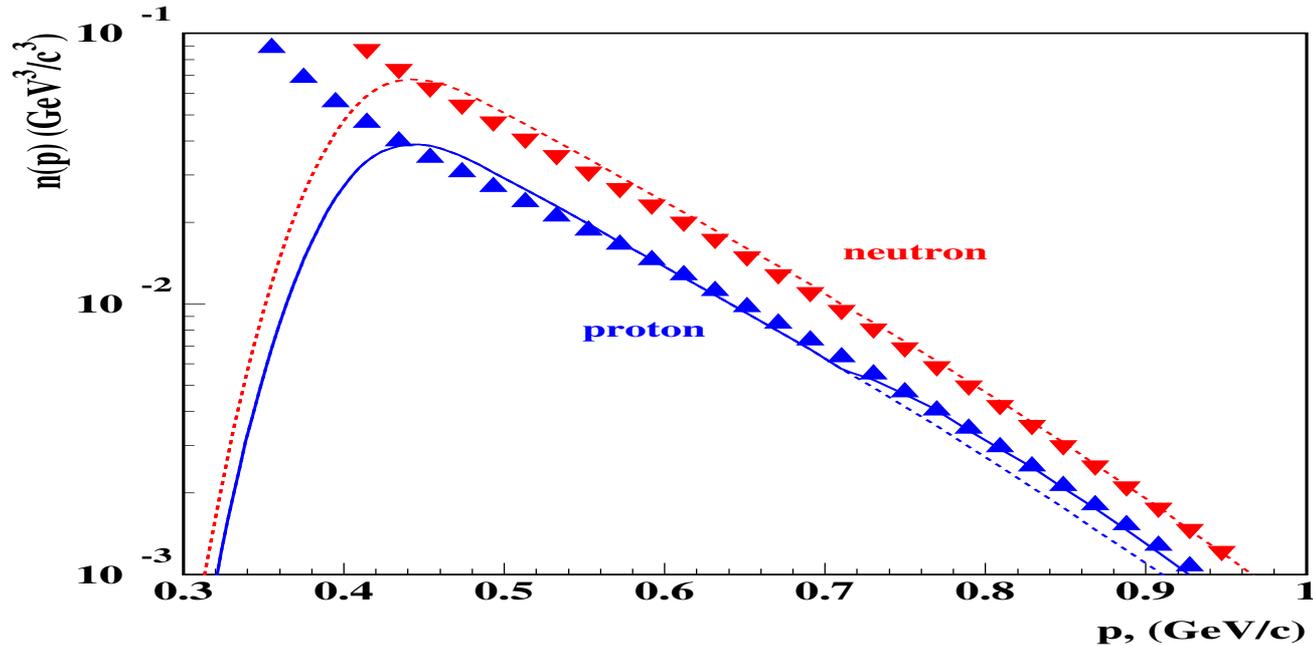
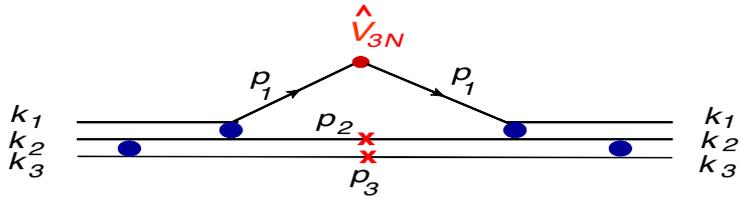
$$\begin{aligned}
 \rho_{3N}(\alpha_1) = & \int \frac{1}{4} \left[\frac{3 - \alpha_3}{(2 - \alpha_3)^3} \rho_{pn}(\alpha_3, p_{3\perp}) \rho_{pn} \left(\frac{2\alpha_2}{3 - \alpha_3}, p_{2\perp} + \frac{\alpha_1}{3 - \alpha_3} p_{3\perp} \right) + \right. \\
 & \left. \frac{3 - \alpha_2}{(2 - \alpha_2)^3} \rho_{pn}(\alpha_2, p_{2\perp}) \rho_{pn} \left(\frac{2\alpha_3}{3 - \alpha_2}, p_{3\perp} + \frac{\alpha_1}{3 - \alpha_2} p_{2\perp} \right) \right] \delta \left(\sum_{i=1}^3 \alpha_i - 3 \right) \\
 & d\alpha_2 d^2 p_{2\perp} d\alpha_3 d^2 p_{3\perp}, \tag{1}
 \end{aligned}$$

$$\rho_{pn}(\alpha, p_{\perp}) \approx a_2(A) \rho_d(\alpha, p_{\perp})$$

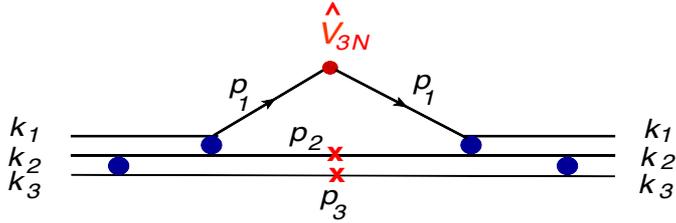
3N SRC:

Light-Cone Momentum Fraction Distribution

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3N SRC: Light-Cone Momentum Fraction Distribution



$$-\rho_{3N} \sim a_2(A, z)^2$$

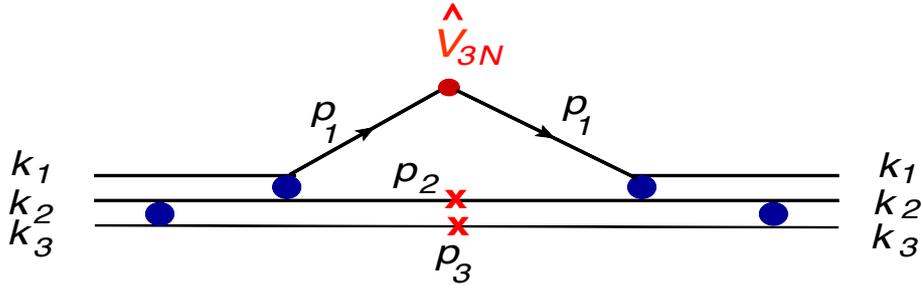
- For $A(e, e')$ X reactions: $\sigma_{eA} = \sum_N \sigma_{eN} \rho_{3N}(\alpha_{3N})$

- Defining: $R_3(A, Z) = \frac{3\sigma_{eA}}{A\sigma_{e^3He}} \Big|_{\alpha_{3N} \geq \alpha_{3N}^0}$

- We predict: $R_3(A, Z) = \frac{9}{8} \frac{(\sigma_{ep} + \sigma_{en})/2}{(2\sigma_{ep} + \sigma_{en})/3} \left(\frac{a_2(A, Z)}{a_2(^3He)} \right)^2 = \frac{9}{8} \frac{(\sigma_{ep} + \sigma_{en})/2}{(2\sigma_{ep} + \sigma_{en})/3} R_2^2(A, Z)$

- Where: $R_2(A, Z) = \frac{3\sigma_{eA}}{A\sigma_{e^3He}} \Big|_{1.3 \leq \alpha_{3N} \leq 1.5}$ where: $\alpha_{3N} \approx \alpha_{2N}$

3N SRC: Predictions



$$-\rho_{3N} \sim a_2(A, z)^2$$

- ppp and nnn strongly suppressed compared with ppn or pnn
- pp/nn recoil state is suppressed compared with pn

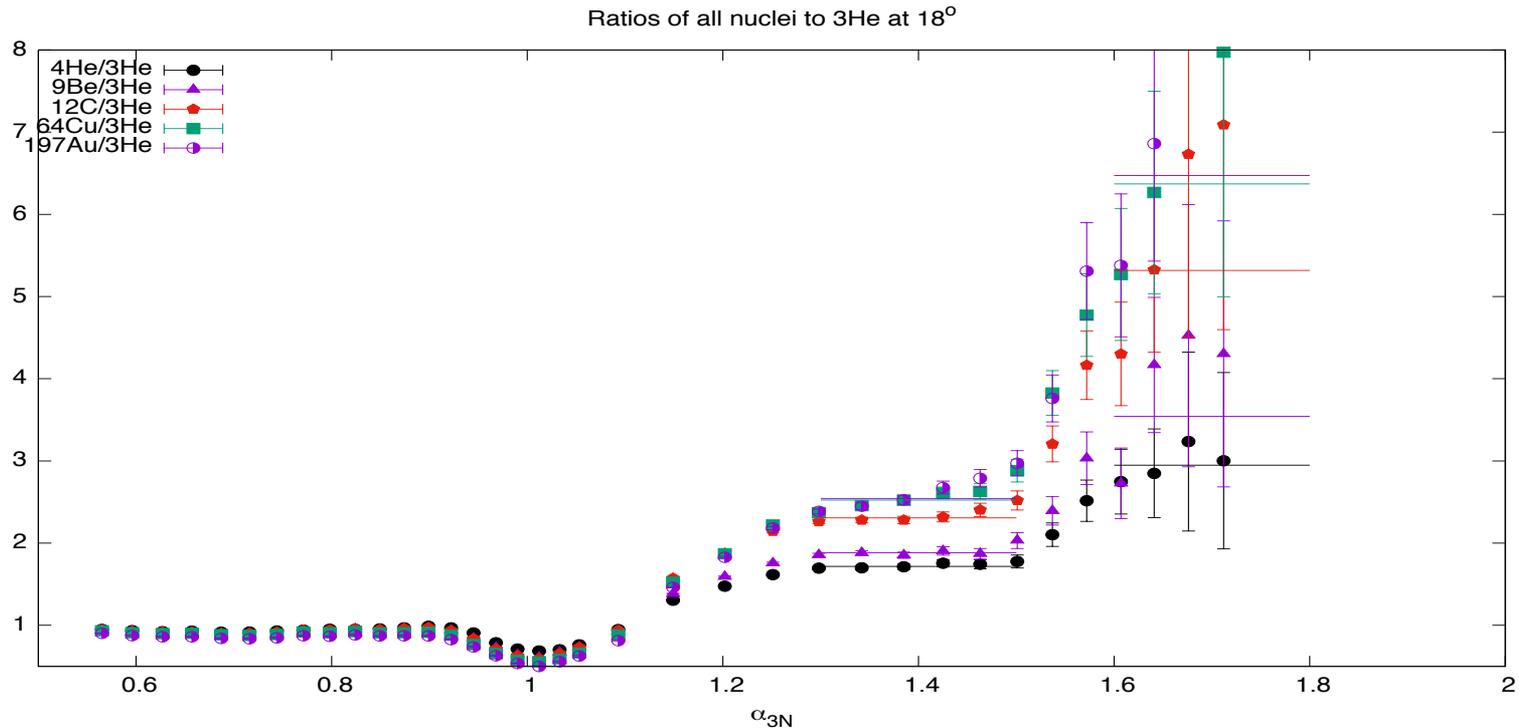
$$R_3(A, Z) = \frac{9}{8} \frac{(\sigma_{ep} + \sigma_{en})/2}{(2\sigma_{ep} + \sigma_{en})/3} \left(\frac{a_2(A, Z)}{a_2(^3He)} \right)^2 = \frac{9}{8} \frac{(\sigma_{ep} + \sigma_{en})/2}{(2\sigma_{ep} + \sigma_{en})/3} R_2^2(A, Z),$$

3N SRC model

$$R_2 = \frac{3\sigma_{eA}(\alpha_{3N})}{A\sigma_{3A}(\alpha_{3N})} \quad 1.3 \leq \alpha_{3N} \leq 1.5 \quad 1.6 \leq \alpha_{3N} < 3$$

$$R_3 = \frac{3\sigma_{eA}(\alpha_{3N})}{A\sigma_{3A}(\alpha_{3N})} \quad 1.6 \leq \alpha_{3N} \leq 1.8$$

$$R_3(A, Z) \approx R_2(A, Z)^2$$



3N SRC model: Prediction

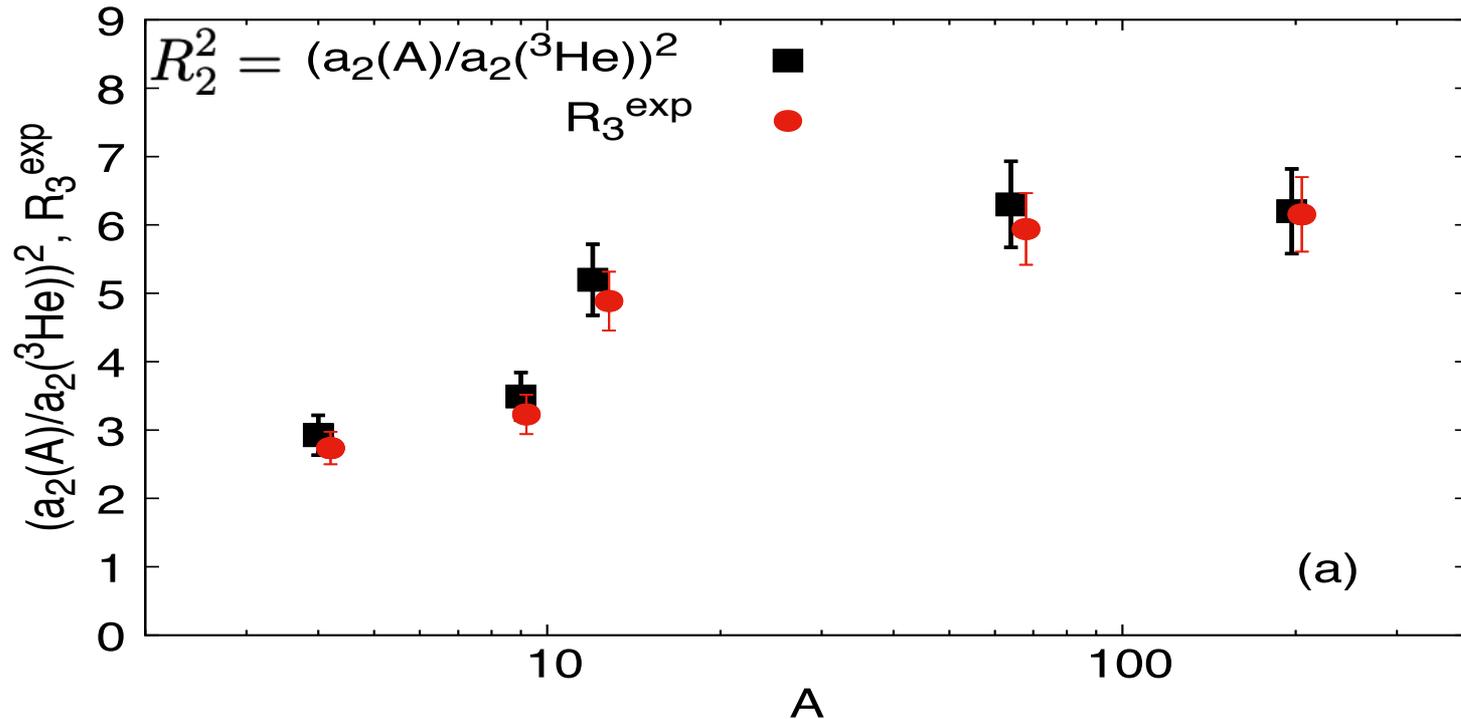
$$R_2 = \frac{3\sigma_{eA}(\alpha_{3N})}{A\sigma_{3A}(\alpha_{3N})} \quad 1.3 \leq \alpha_{3N} \leq 1.5$$

$$1.6 \leq \alpha_{3N} < 3$$

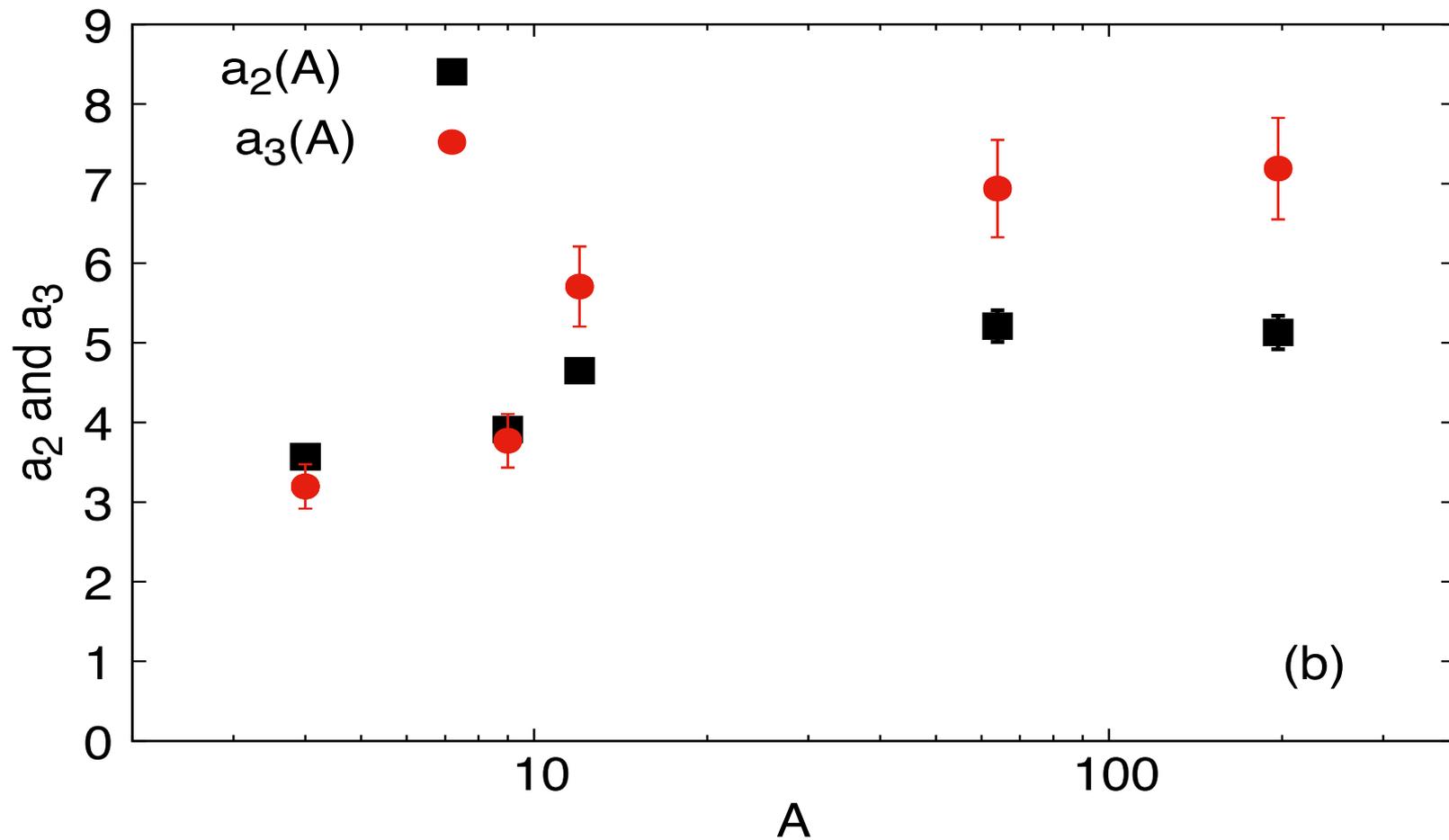
$$R_3 = \frac{3\sigma_{eA}(\alpha_{3N})}{A\sigma_{3A}(\alpha_{3N})} \quad 1.6 \leq \alpha_{3N} \leq 1.8$$

$$R_3(A) = R_2(A)^2$$

M.S. D.Day, L.Frankfurt, M.S, M.Strikman,
PRC 2019



One of the goals: Extrapolating to infinite nuclear matter

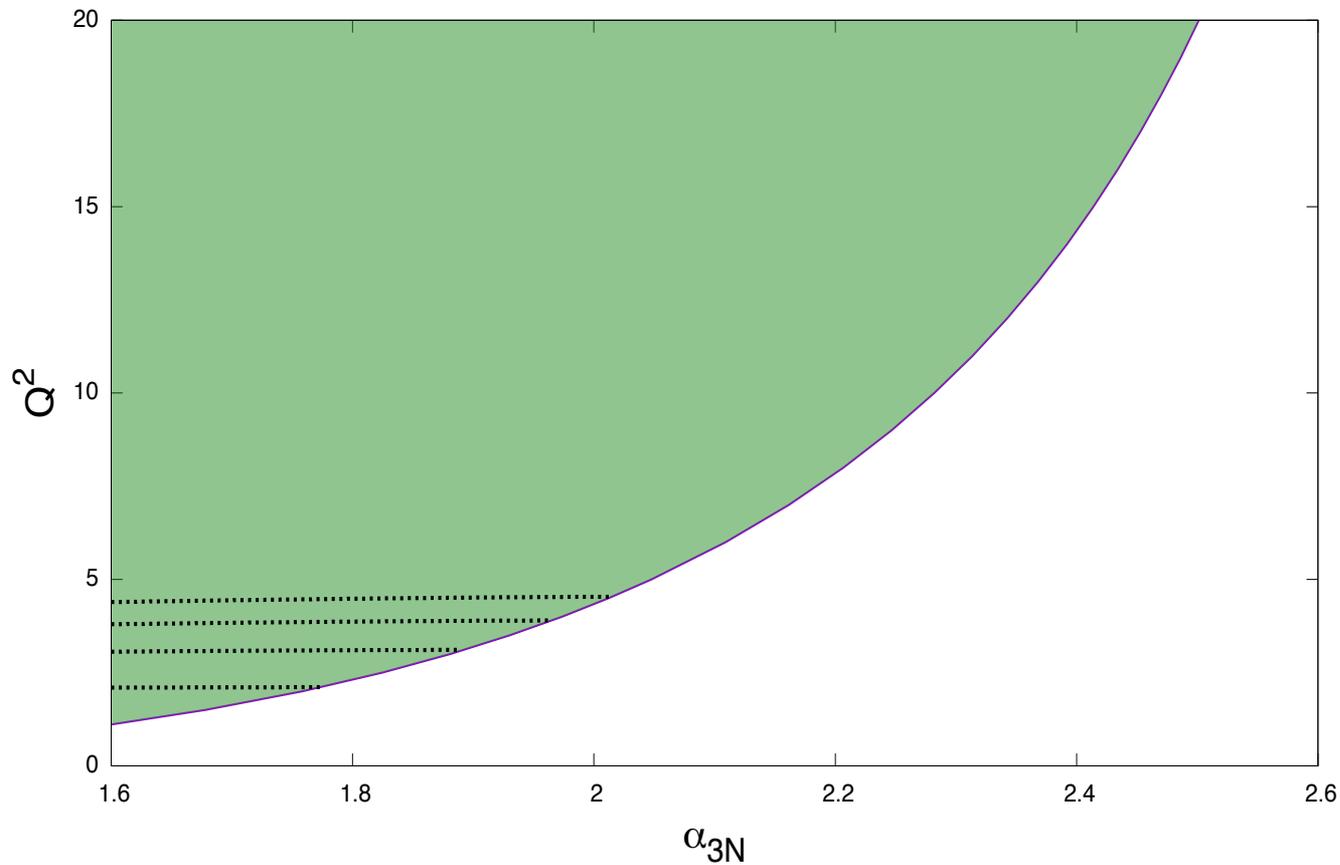


3N SRC Summary & Outlook

- Proper variable for studies of 2N and 3N SRS are Light-Cone momentum fractions: α_{2N}, α_{3N}
- It seems we observed first signatures of 3N SRCs in the form of the “scaling”
- Existing data in agreement with the prediction of: $R_3(A, Z) \approx R_2(A, Z)^2$
- Unambiguous verification will require larger Q² data to cover larger α_{3N} region
- Reaching Q² > 5 GeV² will allow to reach: $\alpha_{3N} > 2$

3N SRC Outlook

${}^3\text{He}$ at $Q^2 = 5 \text{ GeV}^2$ $\alpha_{3N} = 2$



$E = 12 \text{ GeV}$, 30 days

$E = 24 \text{ GeV}$, 6 days

Probing 3N SRCs in Inclusive Scattering:

$$\frac{2\sigma(eA \rightarrow e'X)}{A\sigma(ed \rightarrow e'X)} = \frac{\rho_A(\alpha_{2N})}{\rho_d(\alpha_{2N})} = a_2(A) \quad \text{For } 1 < \alpha_{2N} < 2$$

$$q + 2m = p_f + p_s$$

$$\alpha_{2N} = 2 - \frac{q_- + 2m_N}{2m_N} \left(1 + \frac{\sqrt{W_{2N}^2 - 4m_N^2}}{W_{2N}^2} \right)$$

$$\frac{3\sigma(eA \rightarrow e'X)}{A\sigma(e^3He \rightarrow e'X)} = \frac{\rho_A(\alpha_{3N})}{\rho_{^3He}(\alpha_{3N})} = a_3(A) \quad \text{For } 2 < \alpha_{3N} < 3$$

$$q + 3m = p_f + p_s$$

$$\alpha_{3N} = 3 - \frac{q_- + 3m_N}{2m_N} \left[1 + \frac{m_S^2 - m_N^2}{W_{3N}^2} + \sqrt{\left(1 - \frac{(m_S + m_n)^2}{W_{3N}^2} \right) \left(1 - \frac{(m_S - m_n)^2}{W_{3N}^2} \right)} \right]$$

Probing Deuteron at Core Distances at large Q^2

$$\Psi_d = \Psi_{pn} + \Psi_{\Delta\Delta} + \Psi_{NN^*} + \Psi_{hc} \dots$$

$$e + d \rightarrow e' + \Delta_{backward} + X$$

$$e + d \rightarrow e' + N_{backward}^* + X$$

