3D STRUCTURE: BEAUTY AND THE BEAST

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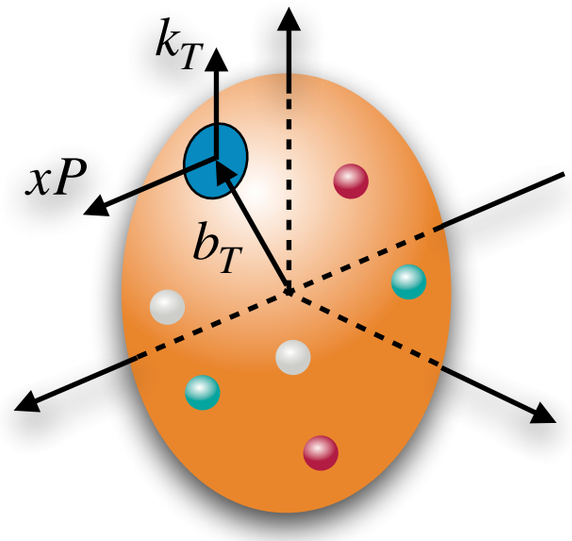
SOME INTRODUCTION

Wigner distributions (Fourier transform of GTMDs = Generalized Transverse Momentum Distributions)



see, e.g., C. Lorcé, B. Pasquini, M. Vanderhaeghen, JHEP 1105 (11)

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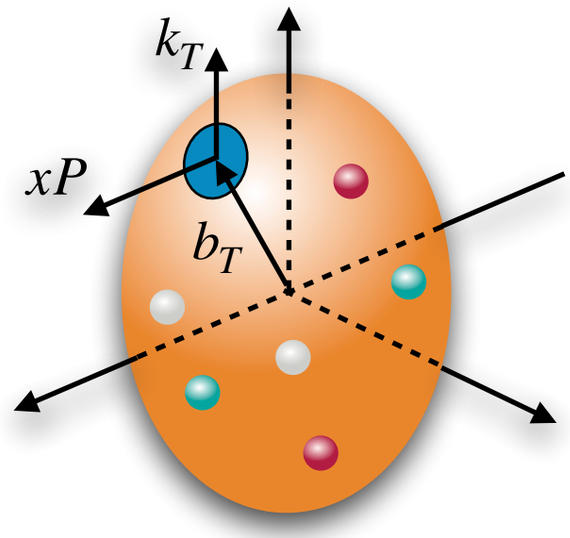
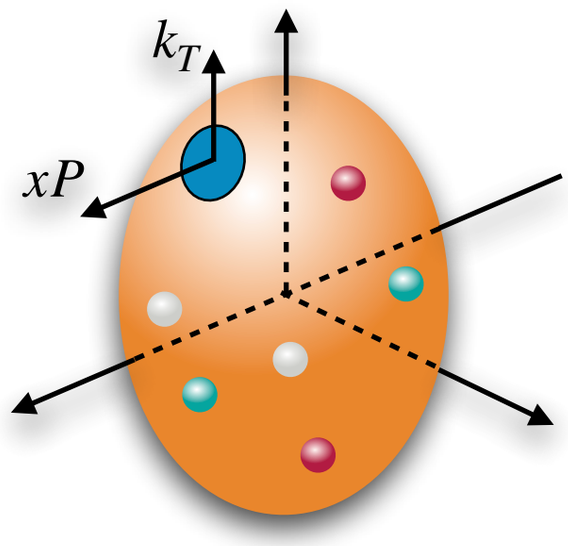


TMDs



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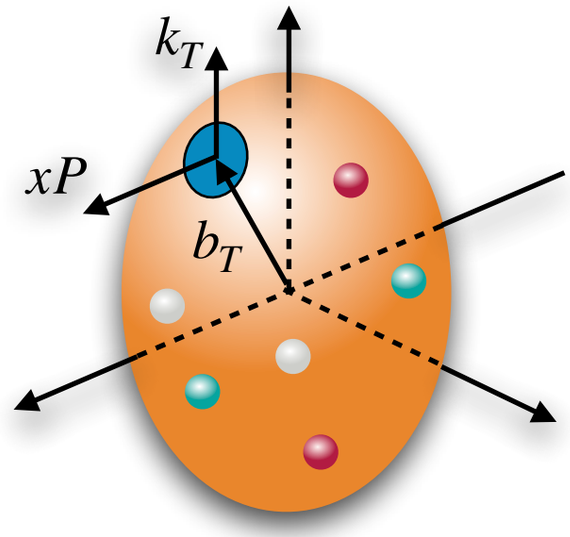

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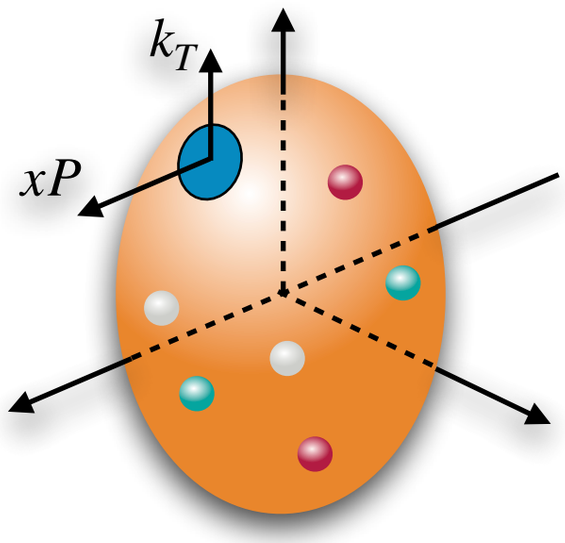
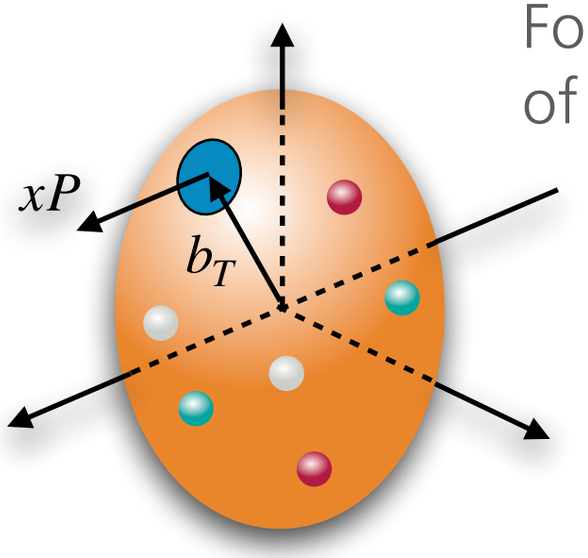


Fourier transform of GPDs



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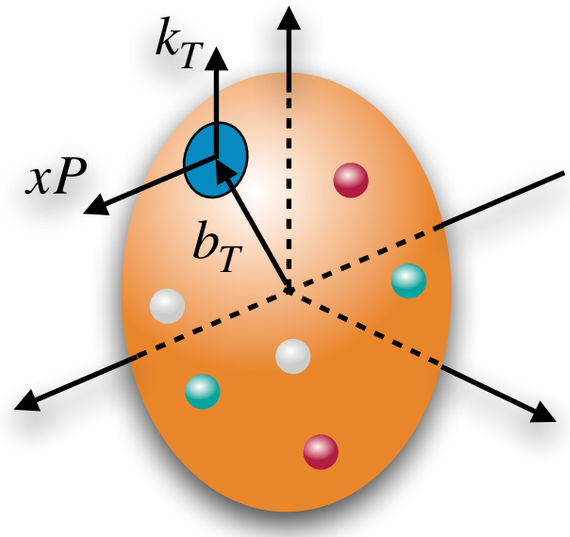

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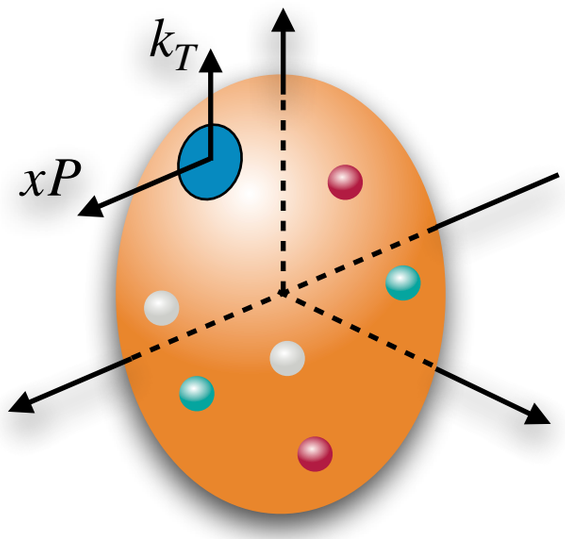

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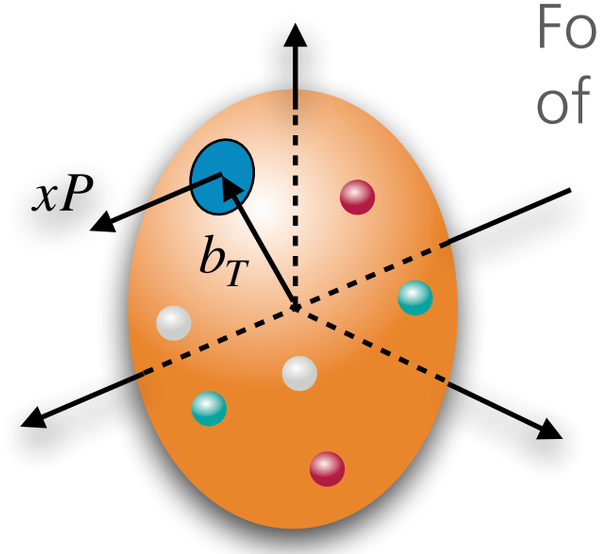

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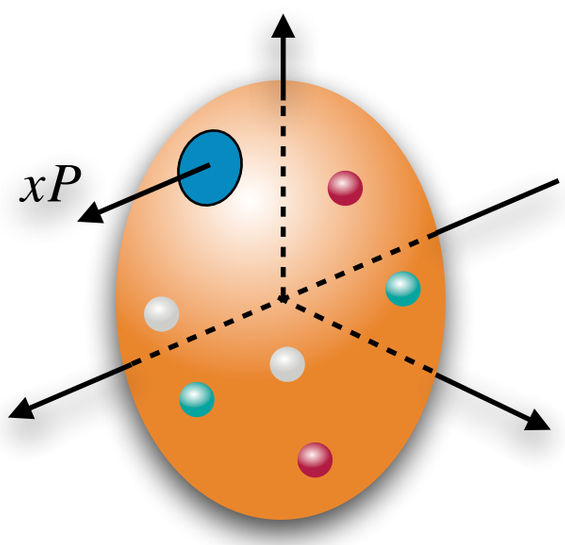
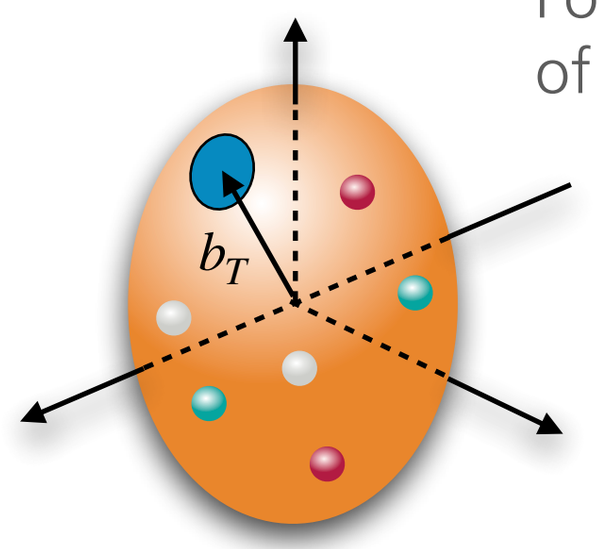


Fourier transform of Form Factors



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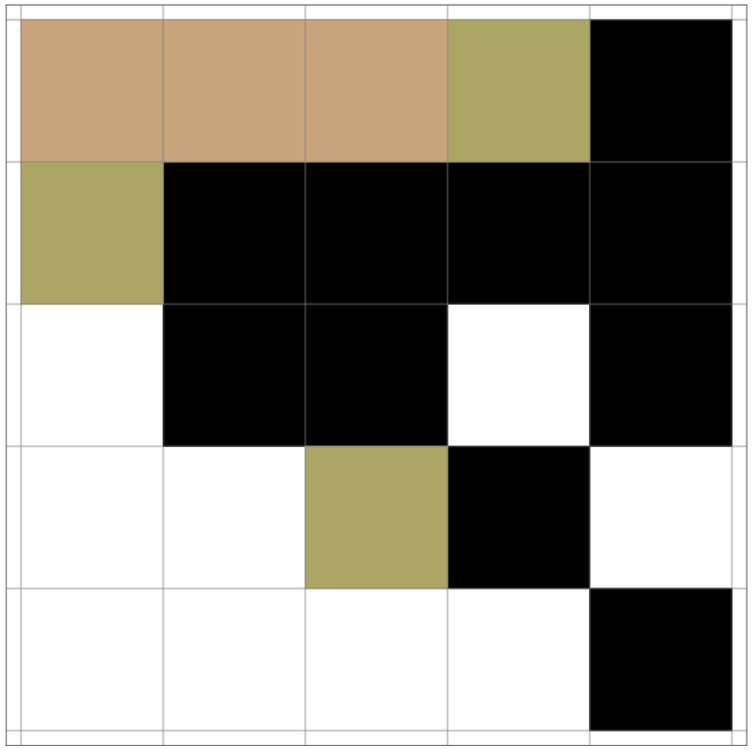
IMPROVING OUR 3D MAPS



With present data

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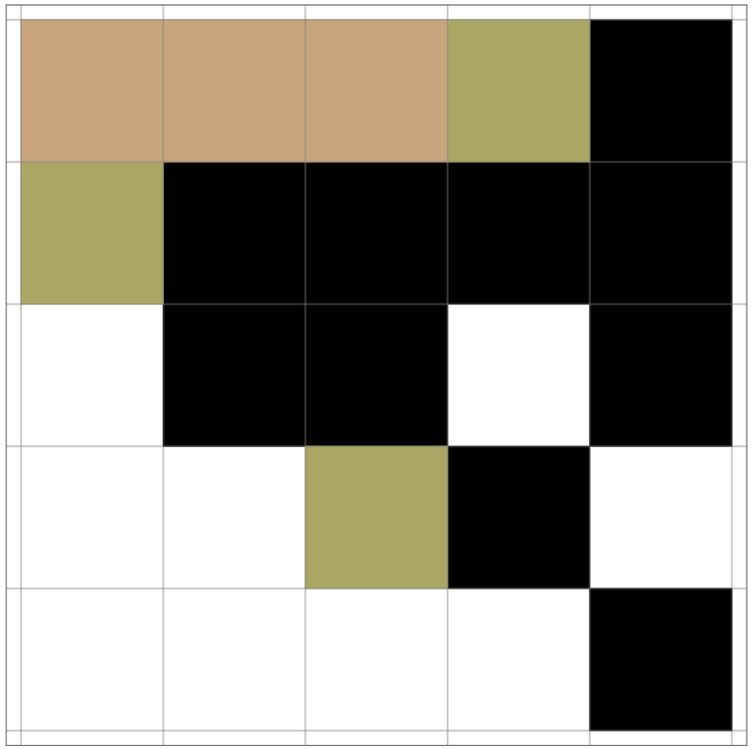

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With 10x data

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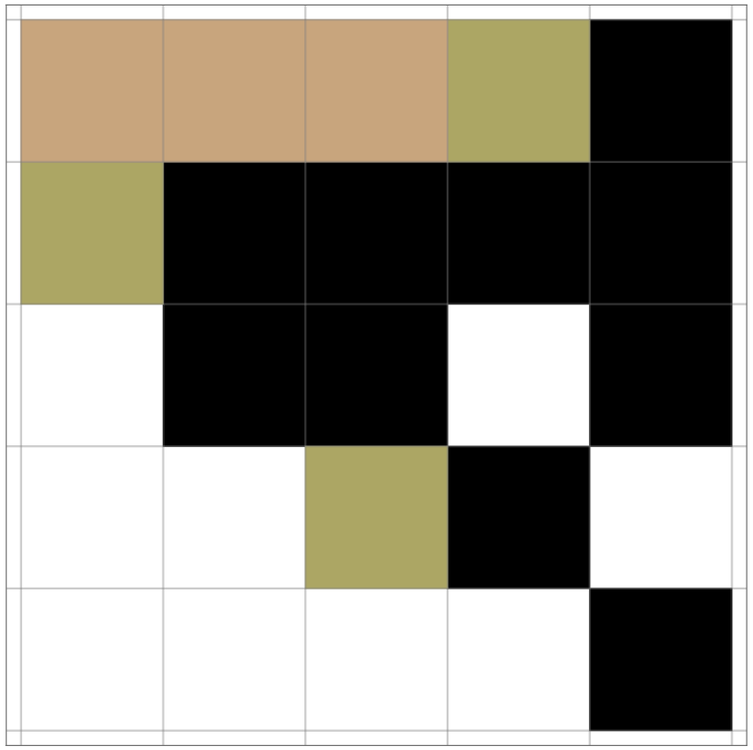

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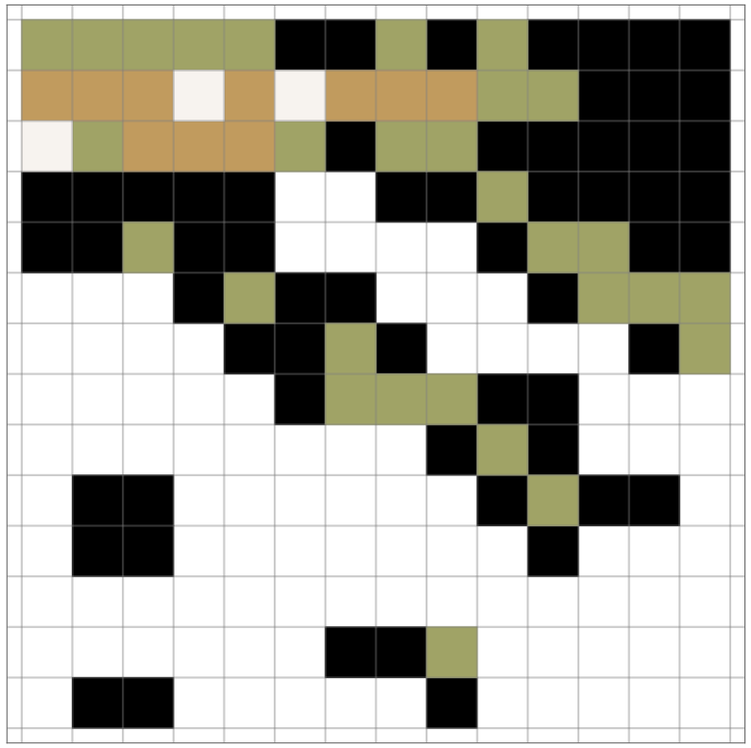

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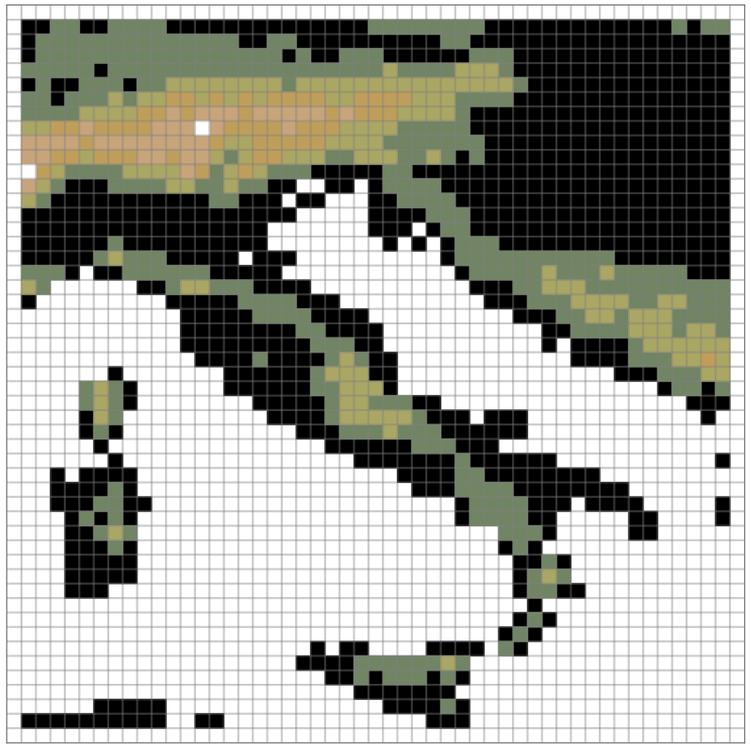

With 100x data

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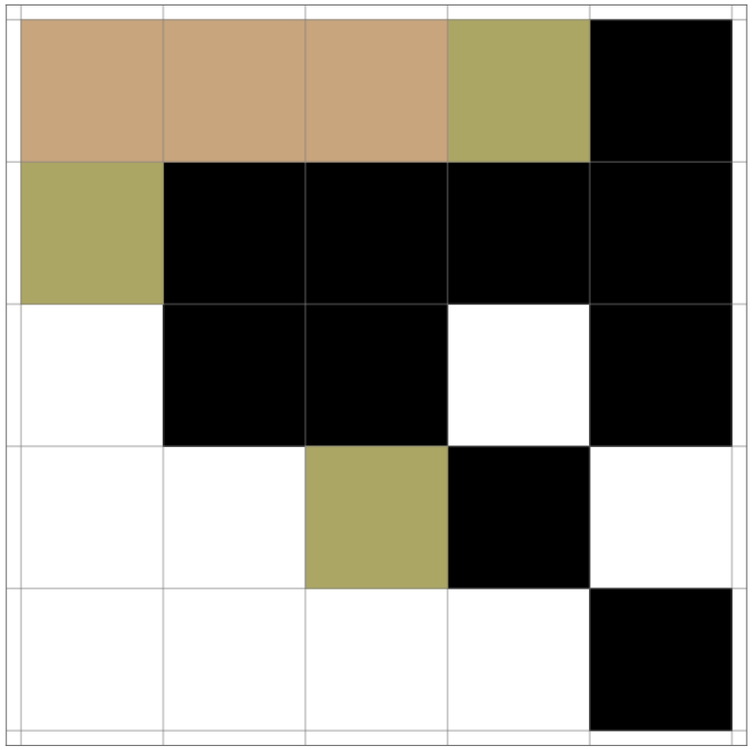

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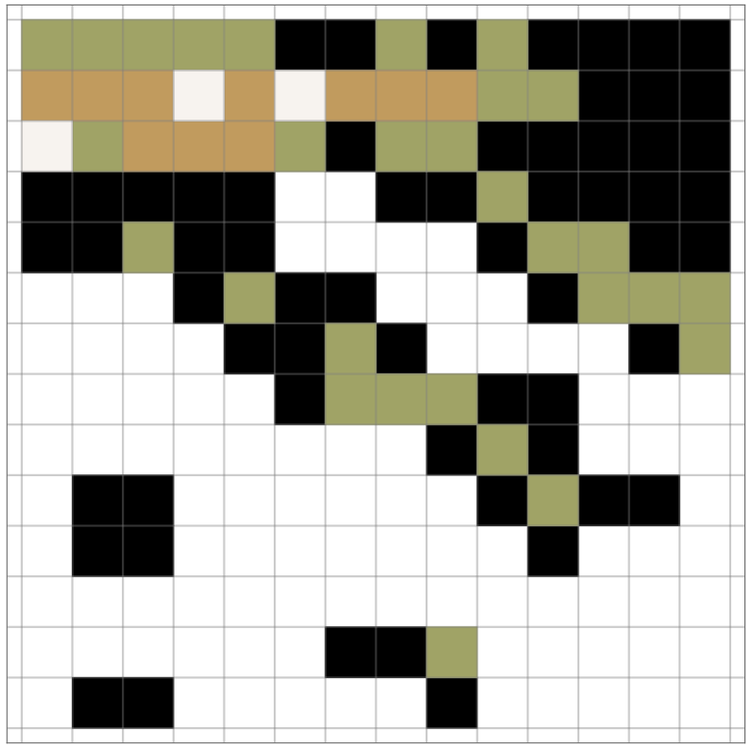

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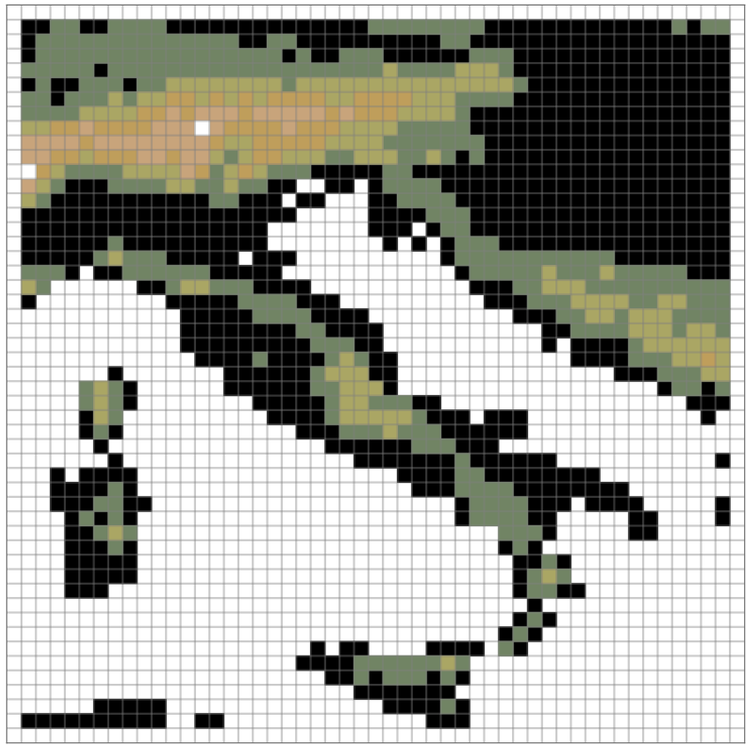

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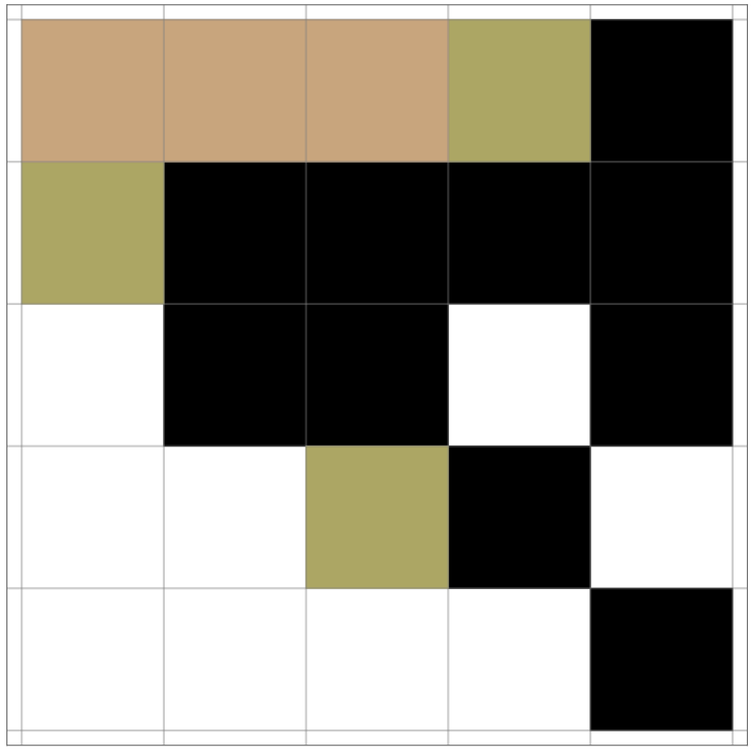

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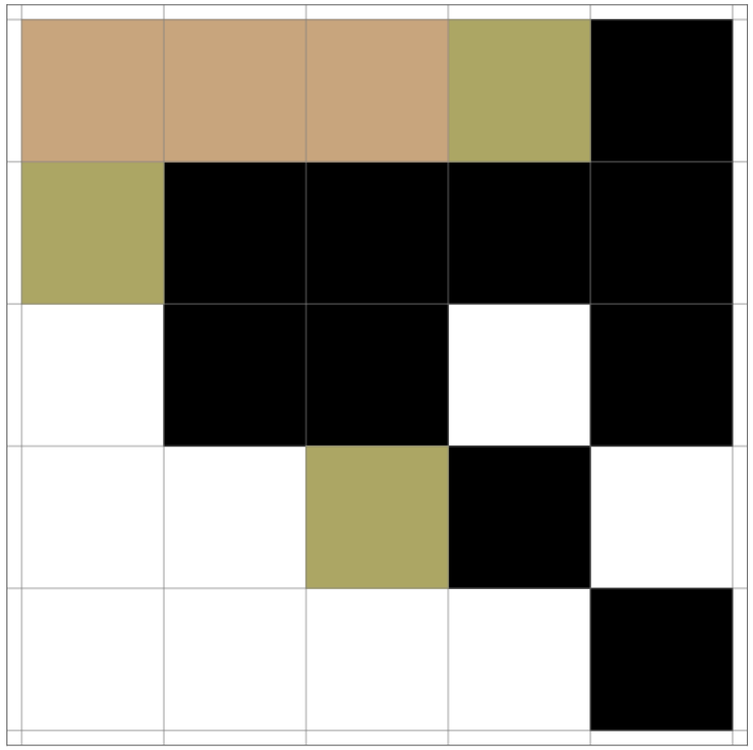
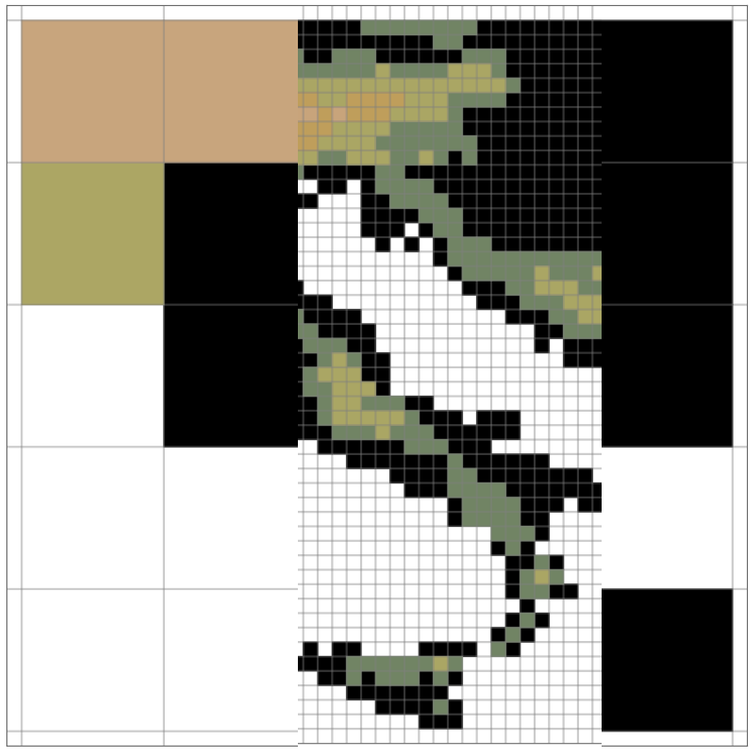


+ JLab 20



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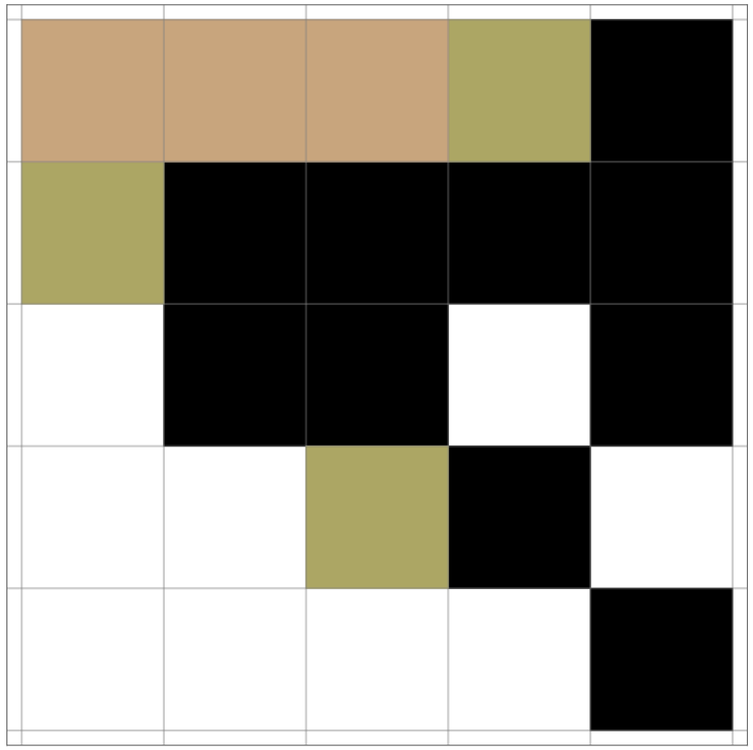

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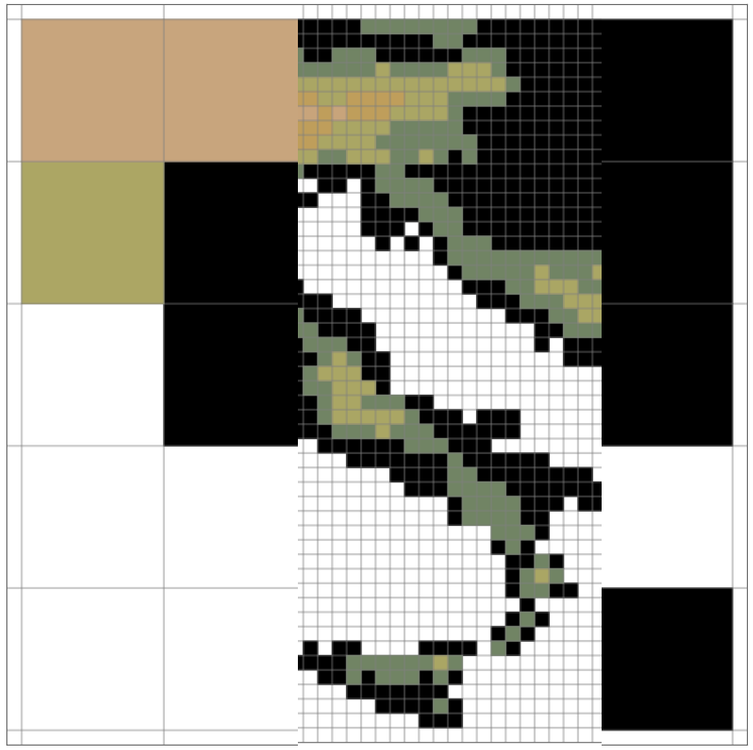

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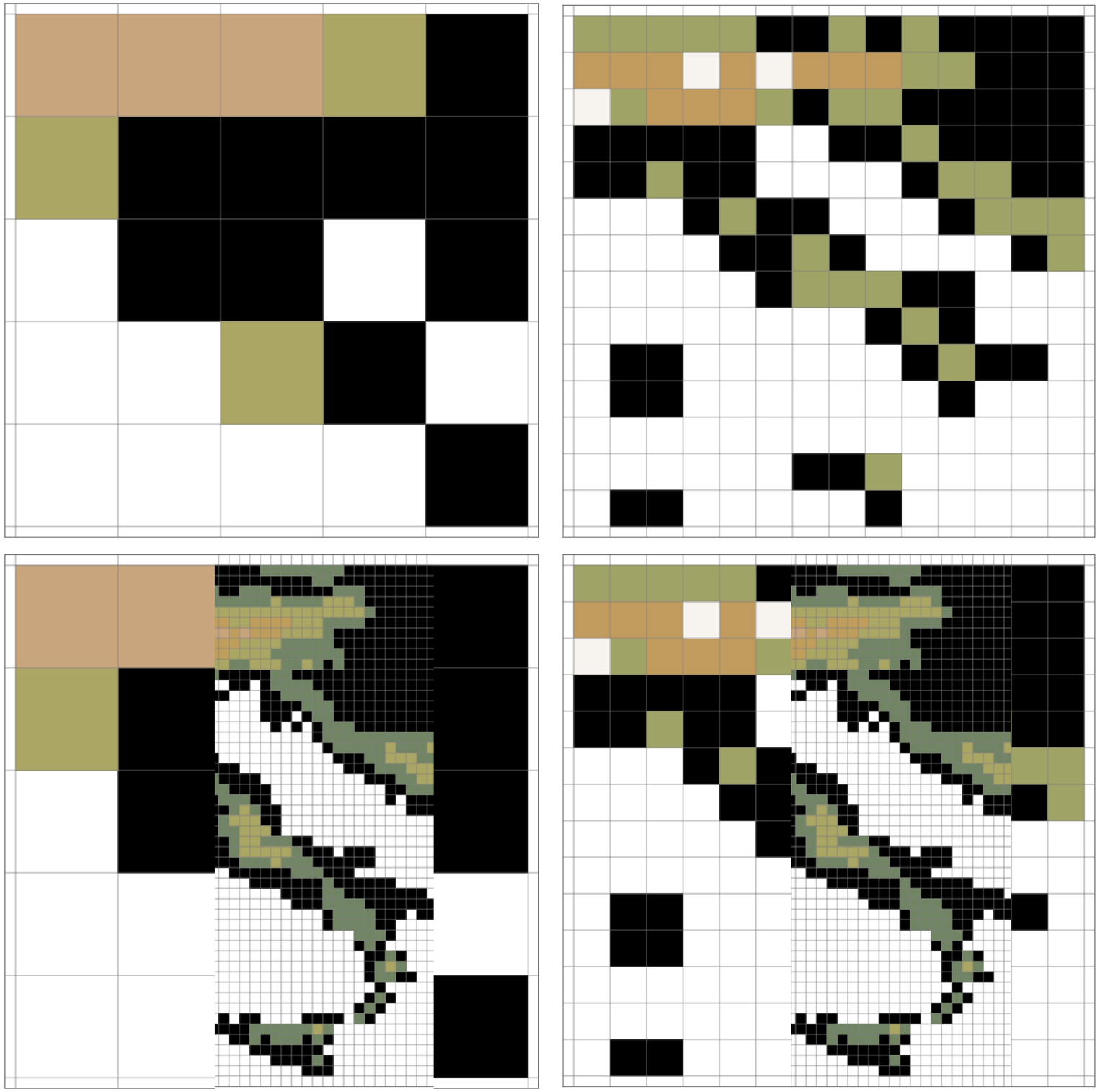

+ EIC

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TMD TABLE

quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Twist-2 TMDs

TMDs in black survive integration over transverse momentum
TMDs in red are time-reversal odd

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)

TMD TABLE

helicity

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nucleon pol.

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Sivers

Twist-2 TMDs

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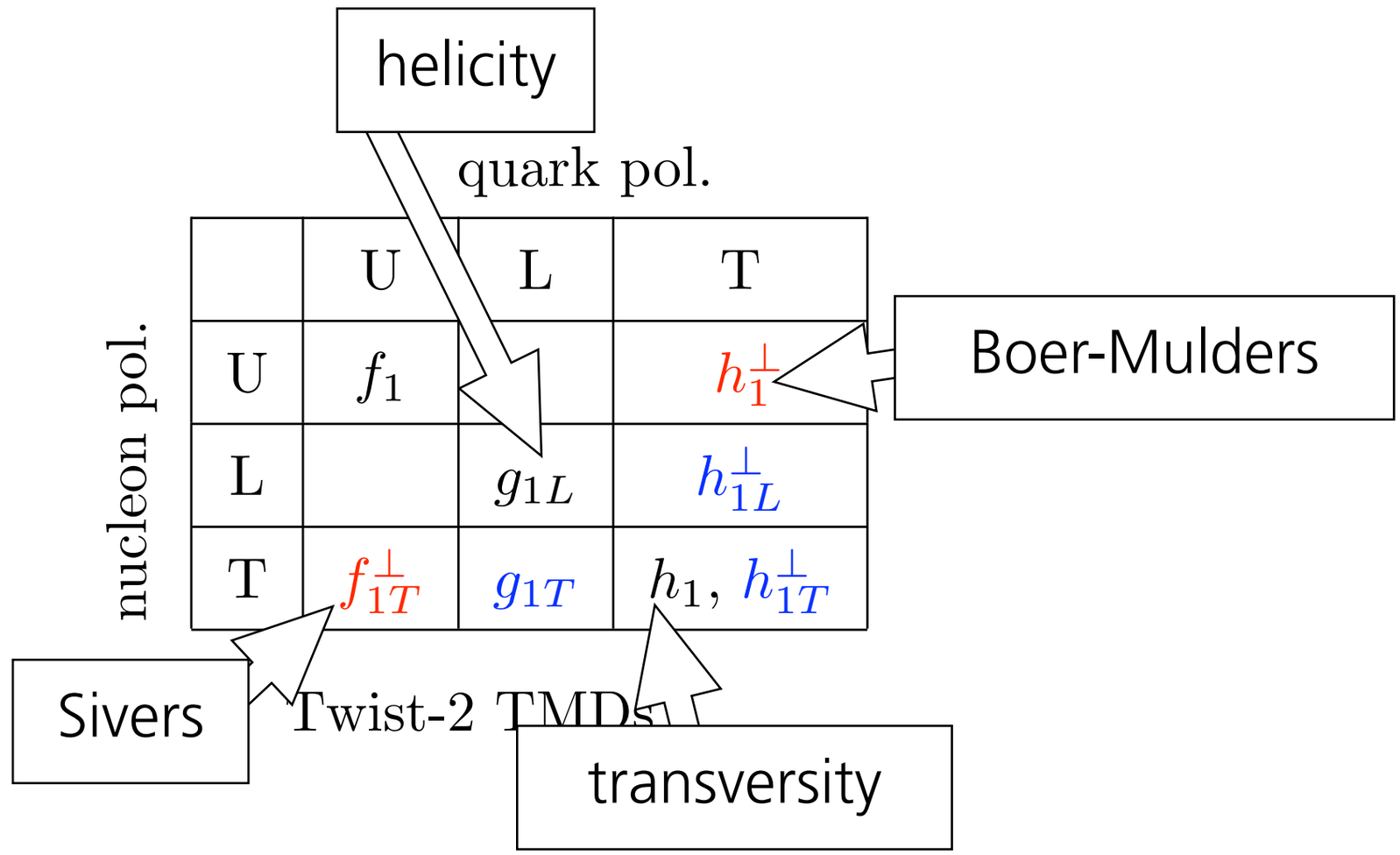

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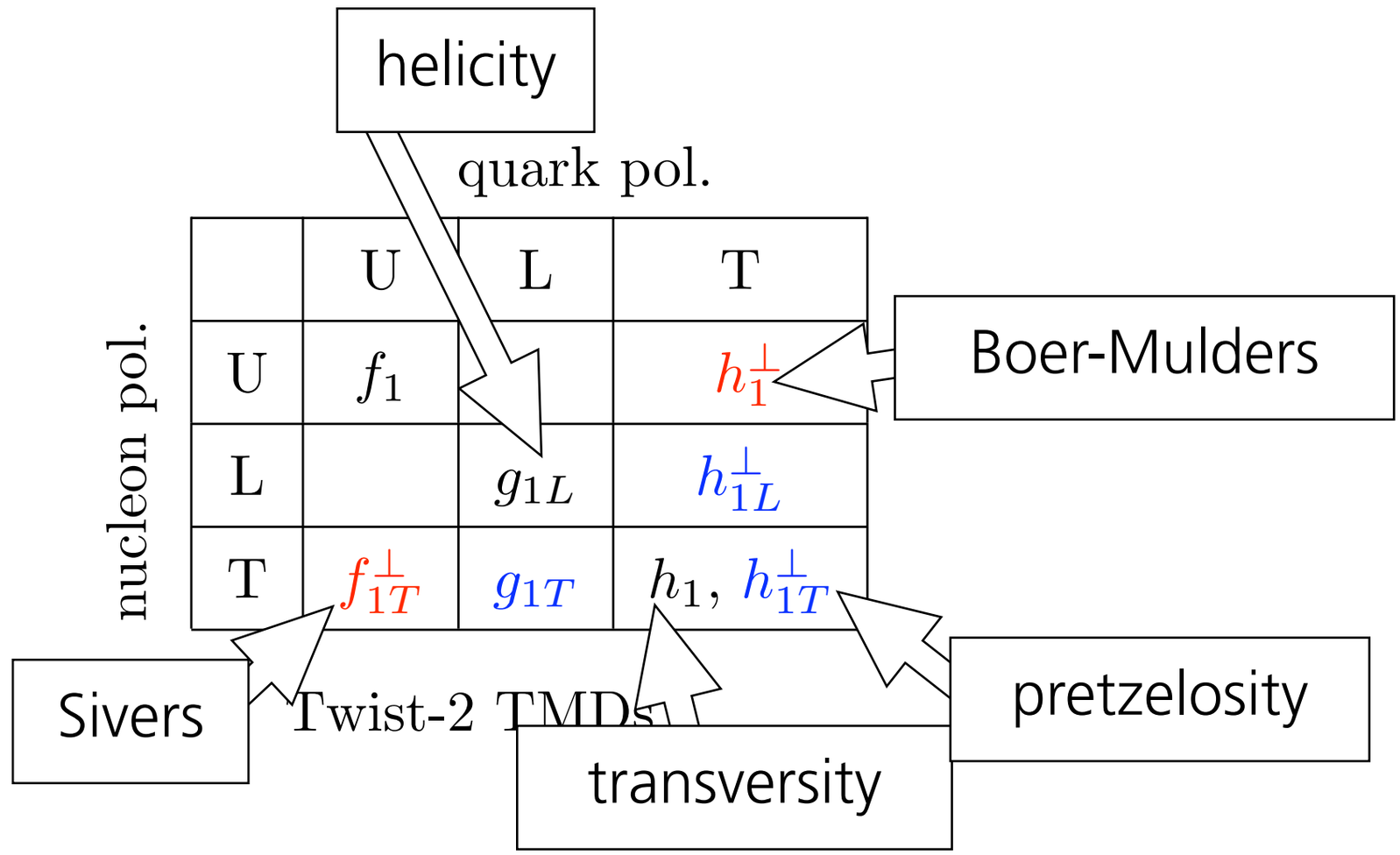

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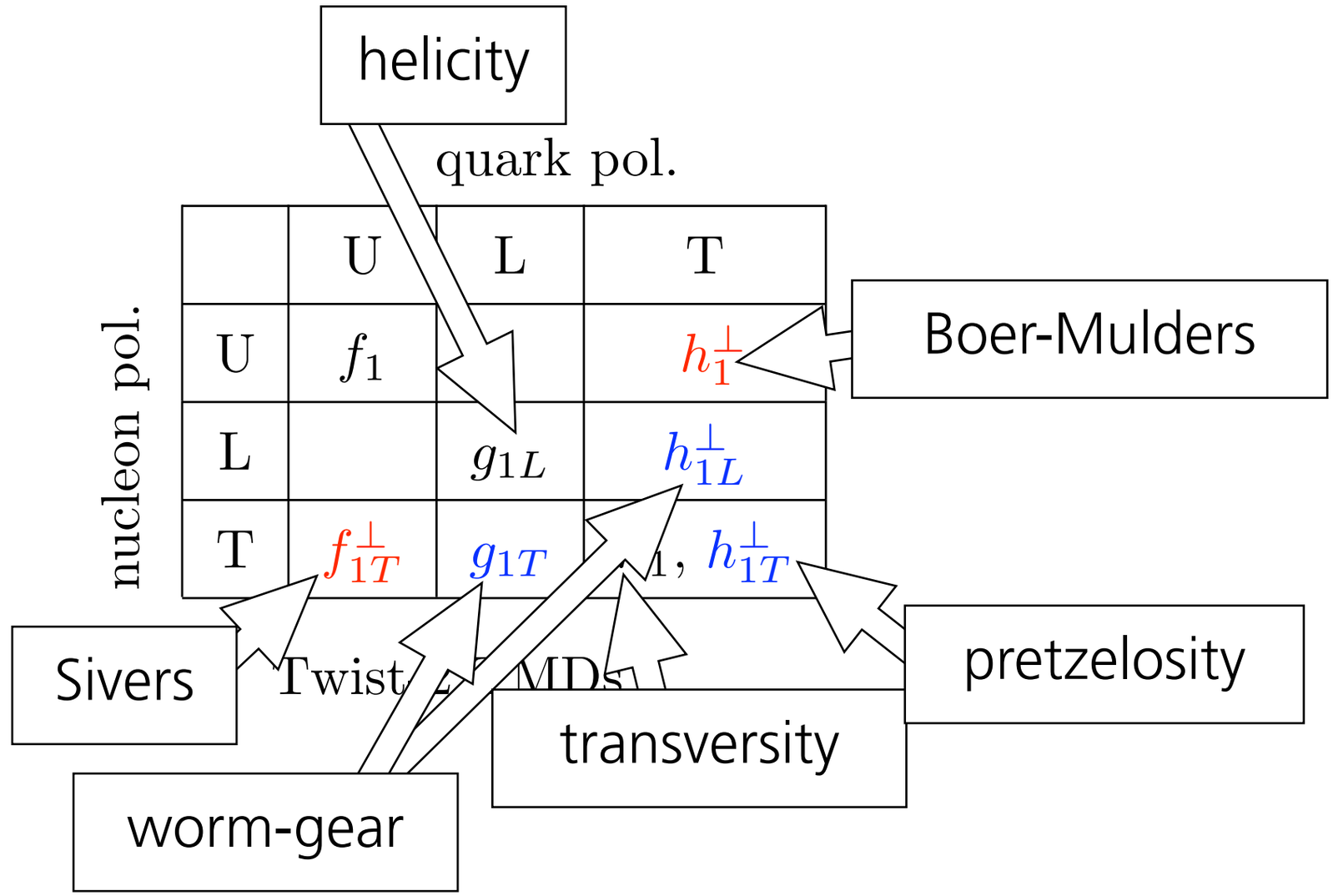

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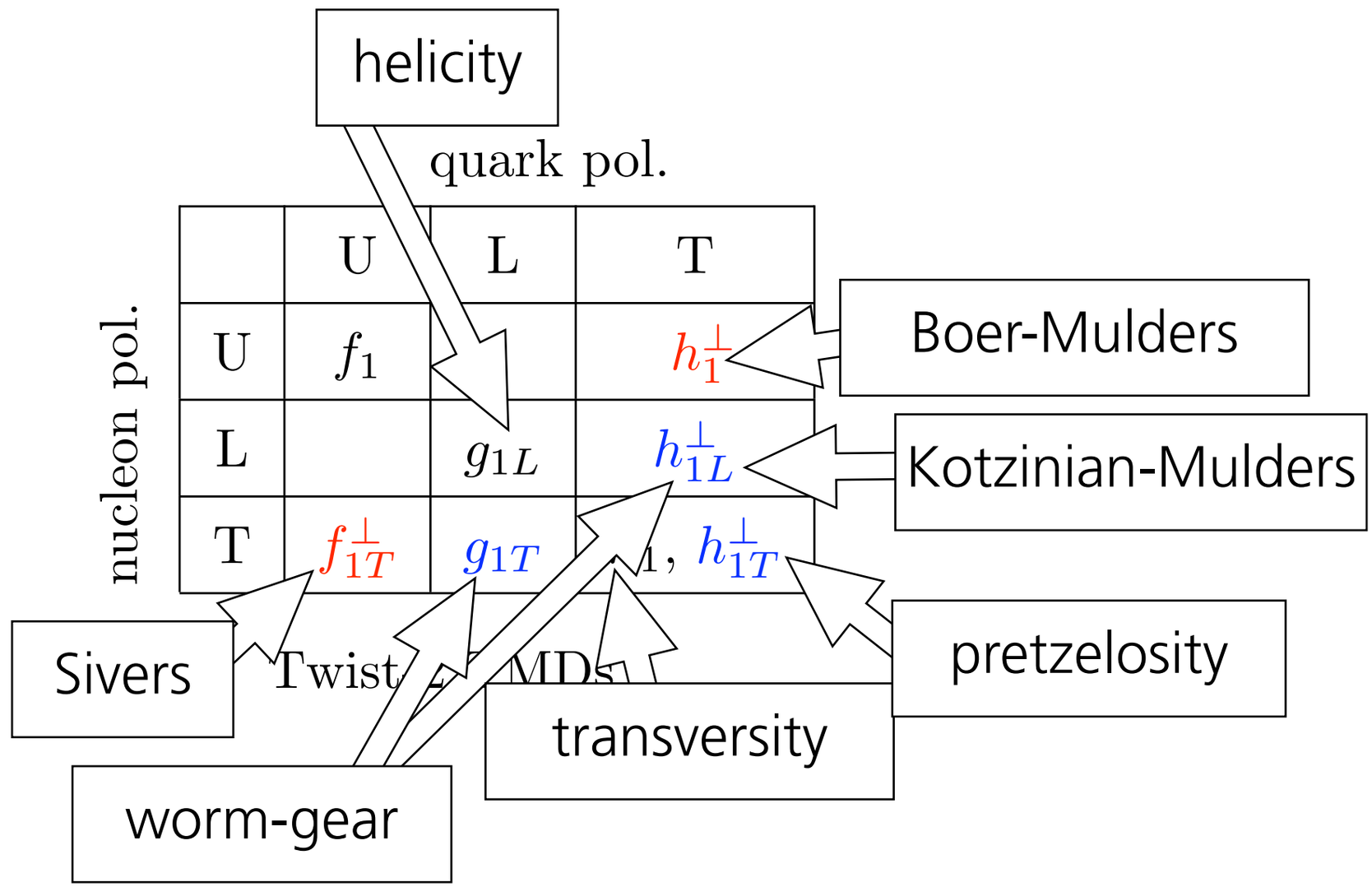

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TWIST-3 TMD TABLE

quark pol.

nucleon pol.	U	L	T	
U	$f^{\perp}$	$g^{\perp}$	e	h
L	$f_L^{\perp}$	$g_L^{\perp}$	h_L	e_L
T	$f_T,\ f_T^{\perp}$	$g_T,\ g_T^{\perp}$	$h_T,\ h_T^{\perp}$	$e_T,\ e_T^{\perp}$

TMDs in black survive integration over transverse momentum
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Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)
Bacchetta, Mulders, Pijlman, hep-ph/0405154
Goeke, Metz, Schlegel, hep-ph/0504130

TWIST-3 TMD TABLE

quark pol.

nucleon pol.	U	L	T	
U	$f^{\perp}$	$g^{\perp}$	e	h
L	$f_L^{\perp}$	$g_L^{\perp}$	h_L	e_L
T	f_T, $f_T^{\perp}$	g_T, $g_T^{\perp}$	h_T, $h_T^{\perp}$	e_T, $e_T^{\perp}$

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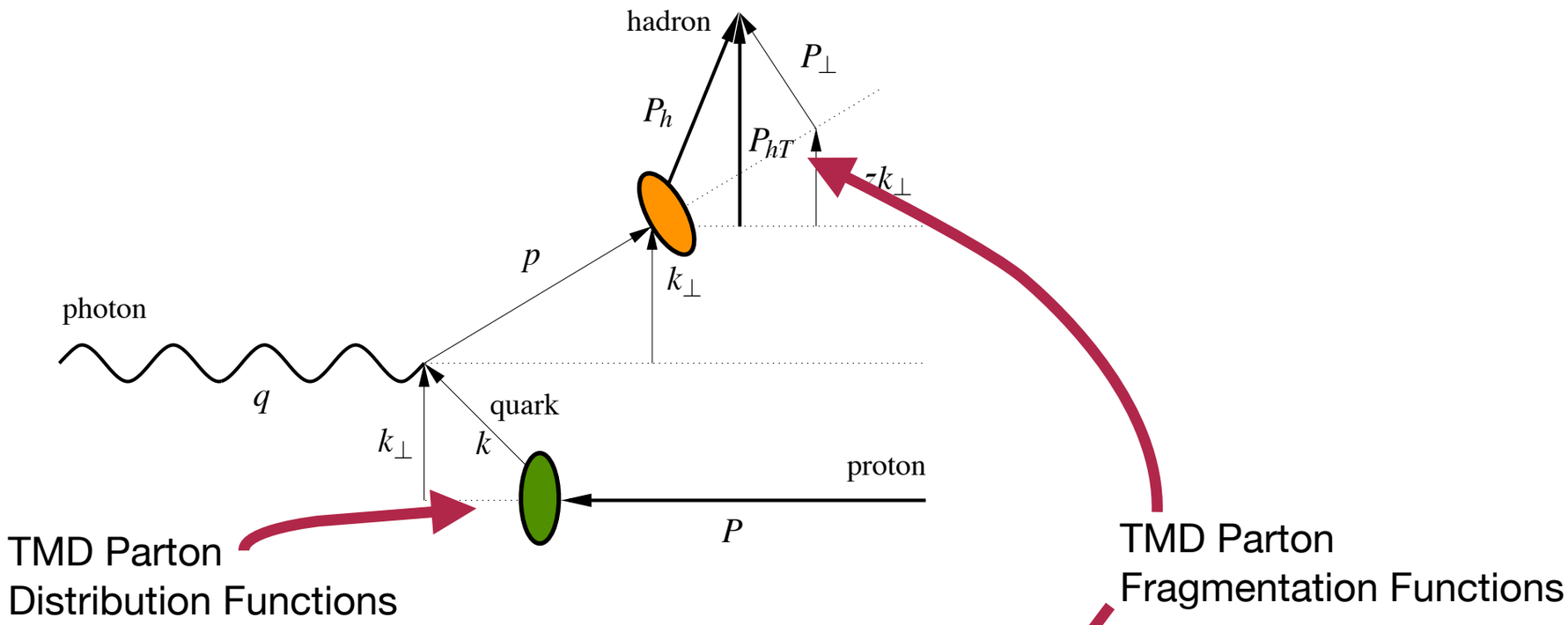
TMDS IN SEMI-INCLUSIVE DIS



$$F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2)$$
$$= x \sum_q \boxed{\mathcal{H}^q_{UU,T}(Q^2, \mu^2)} \int d^2\boldsymbol{k}_\perp \, d^2\boldsymbol{P}_\perp \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu^2) \, D_1^{a\to h}(z, \boldsymbol{P}_\perp^2; \mu^2) \, \delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp)$$
$$+ Y_{UU,T}(Q^2, \boldsymbol{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)$$

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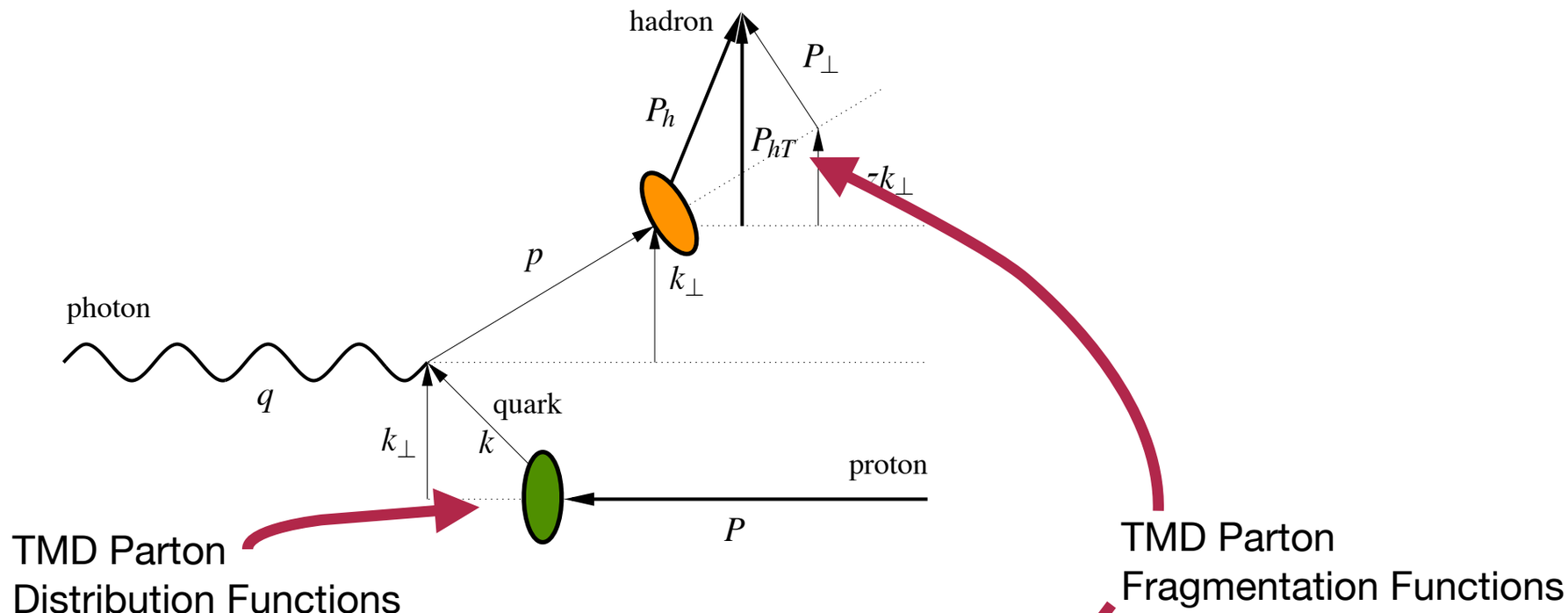

$$F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2)$$
$$= x \sum_q \boxed{\mathcal{H}^q_{UU,T}(Q^2, \mu^2)} \int d^2\boldsymbol{k}_\perp \, d^2\boldsymbol{P}_\perp \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu^2) \, D_1^{a\to h}(z, \boldsymbol{P}_\perp^2; \mu^2) \, \delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp)$$
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hard factor

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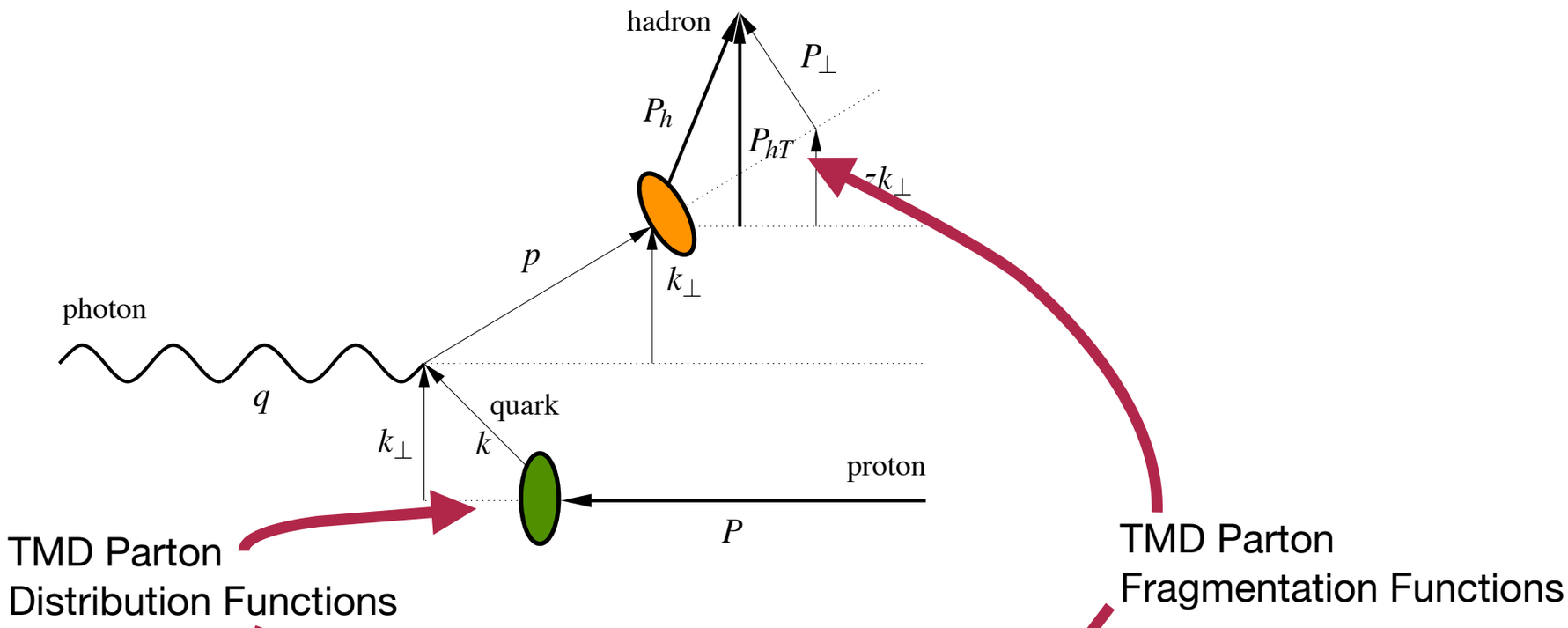
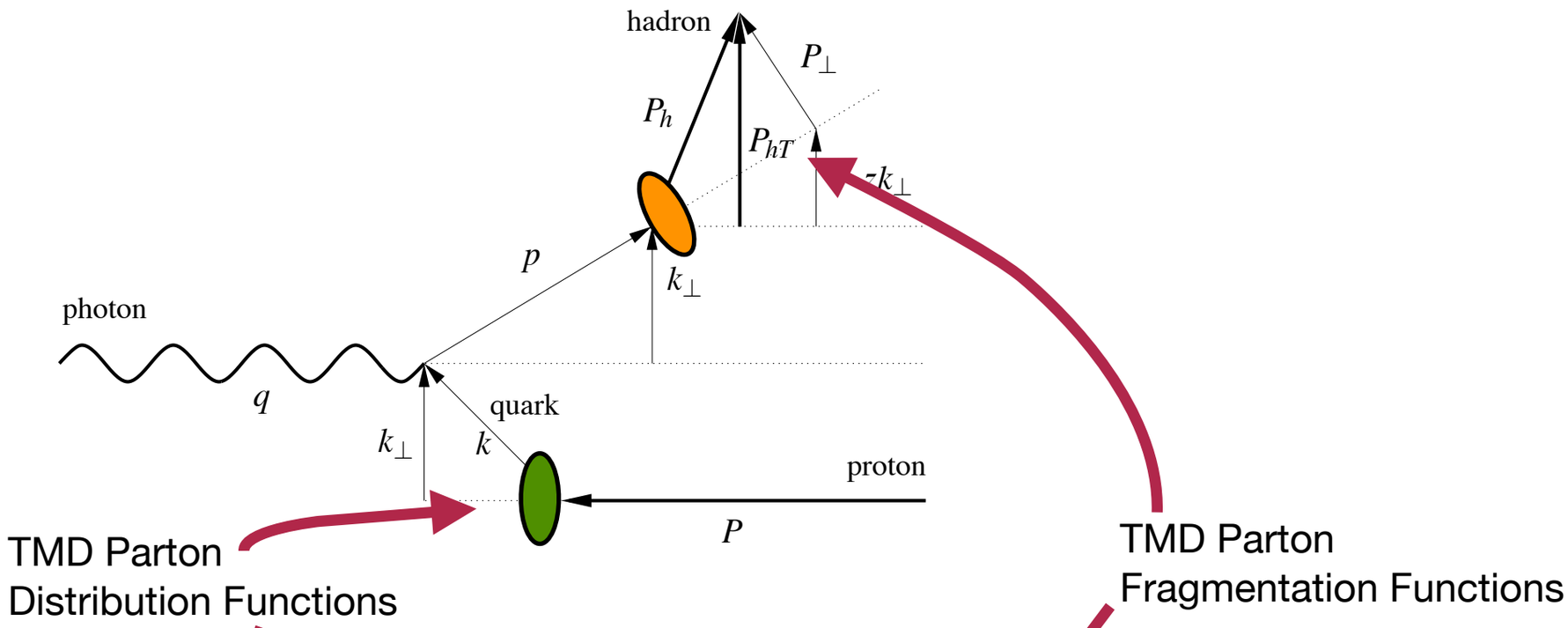


$$F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2) = x \sum_q \mathcal{H}_{UU,T}^q(Q^2, \mu^2) \int d^2\boldsymbol{k}_\perp \, d^2\boldsymbol{P}_\perp \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu^2) \, D_1^{a\to h}(z, \boldsymbol{P}_\perp^2; \mu^2) \, \delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp) + Y_{UU,T}(Q^2, \boldsymbol{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)$$

hard factor

W term

TMDS IN SEMI-INCLUSIVE DIS



$$F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2)$$
$$= x \sum_q \mathcal{H}^q_{UU,T}(Q^2, \mu^2) \int d^2\boldsymbol{k}_\perp \, d^2\boldsymbol{P}_\perp \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu^2) \, D_1^{a\to h}(z, \boldsymbol{P}_\perp^2; \mu^2) \, \delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp)$$
$$+ Y_{UU,T}(Q^2, \boldsymbol{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)$$

hard factor

W term

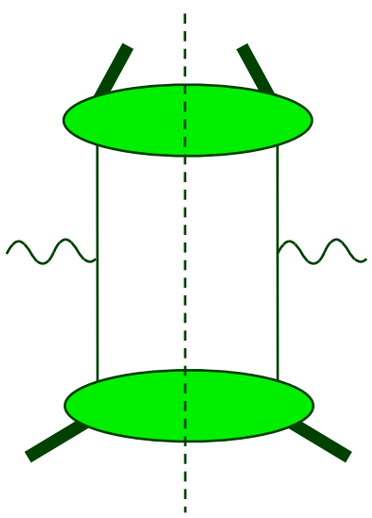
The W term, dominates at low transverse momentum $q_T = P_{hT}/z \ll Q$

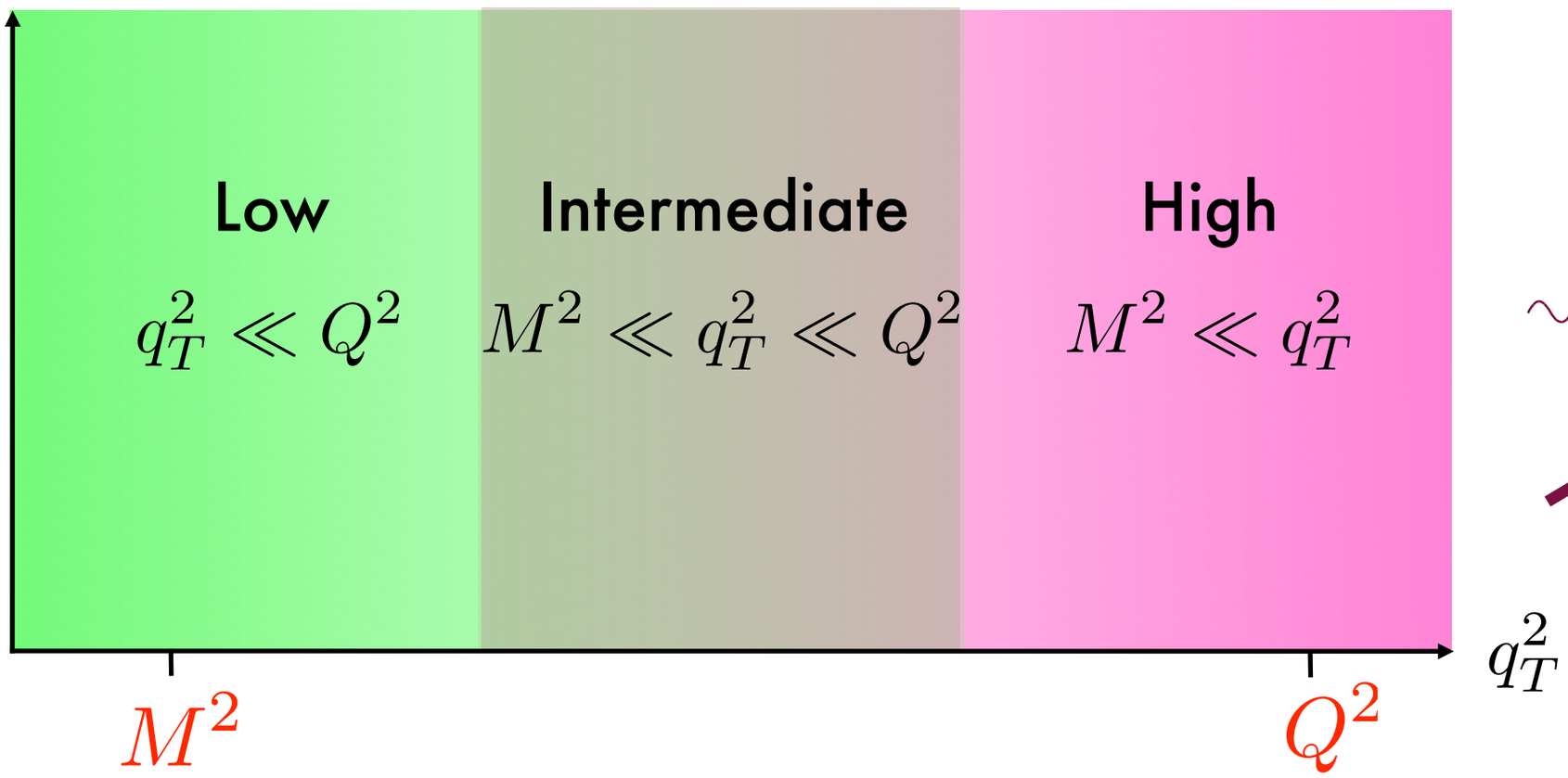
So far, the Y term has been neglected in TMD extractions

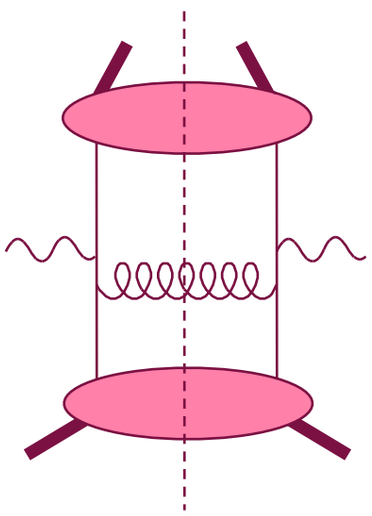
THE "MATCHING" PROCEDURE

TMDS

PDFS







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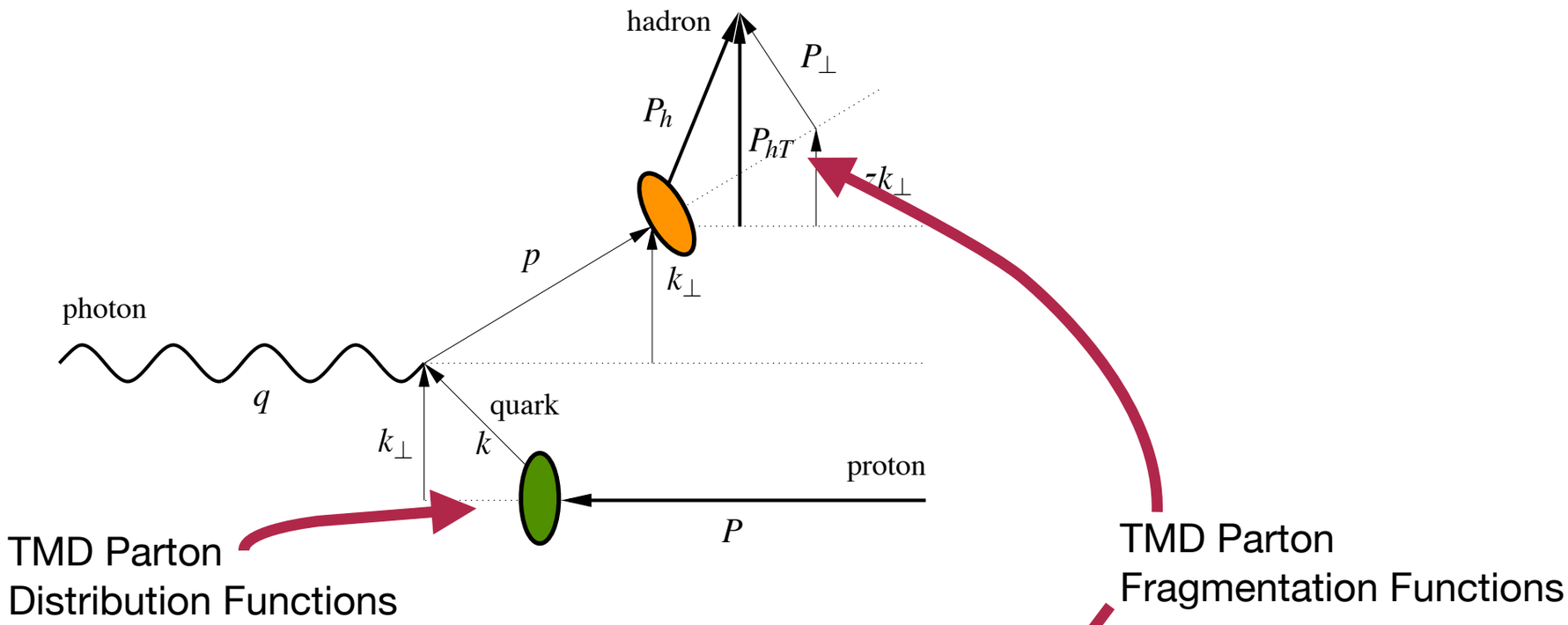

$$F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2)$$
$$= x \sum_q \mathcal{H}^q_{UU,T}(Q^2, \mu^2) \int d^2\boldsymbol{k}_\perp \, d^2\boldsymbol{P}_\perp \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu^2) \, D_1^{a\to h}(z, \boldsymbol{P}_\perp^2; \mu^2) \, \delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp)$$
$$= x \sum_a \mathcal{H}^q_{UU,T}(Q^2, \mu^2) \int db_T b_T J_0(b_T|\boldsymbol{P}_{h\perp}|) \hat{f}_1^q(x, z^2 b_\perp^2; \mu^2) \, \hat{D}_1^{a\to h}(z, b_\perp^2; \mu^2)$$

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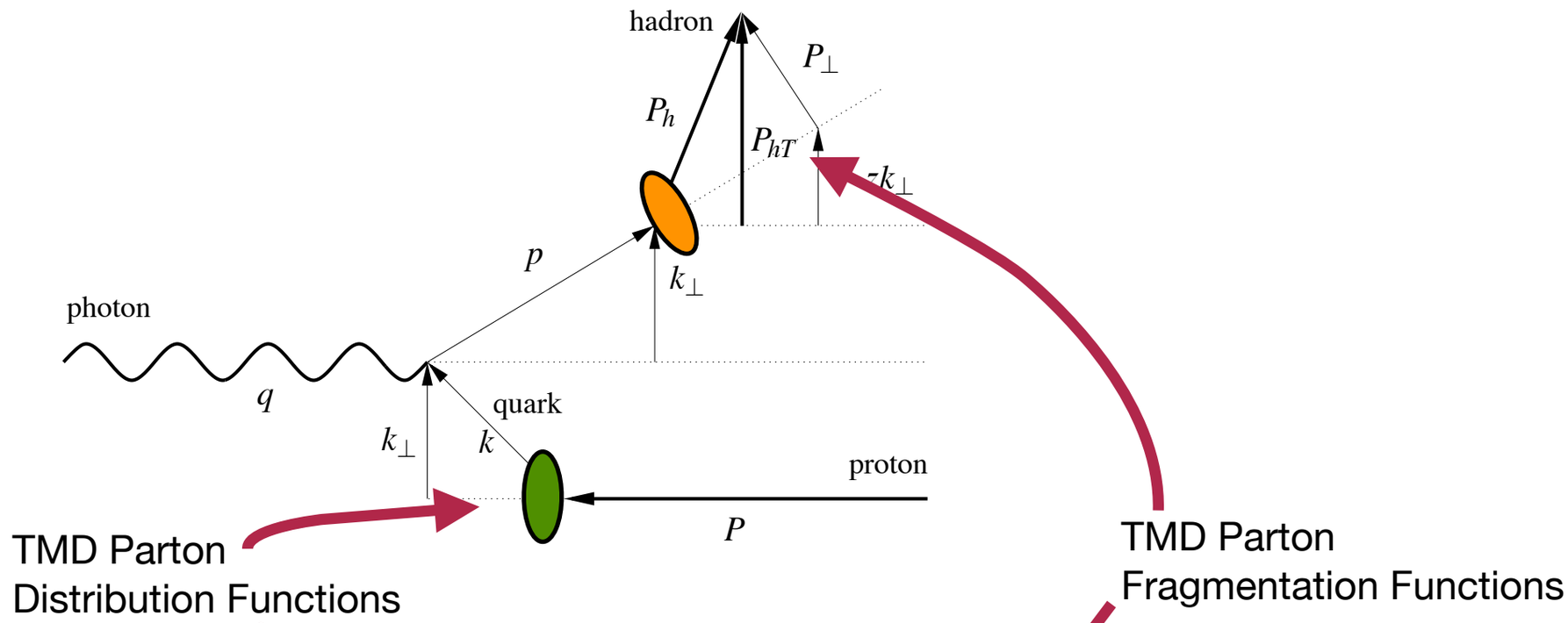

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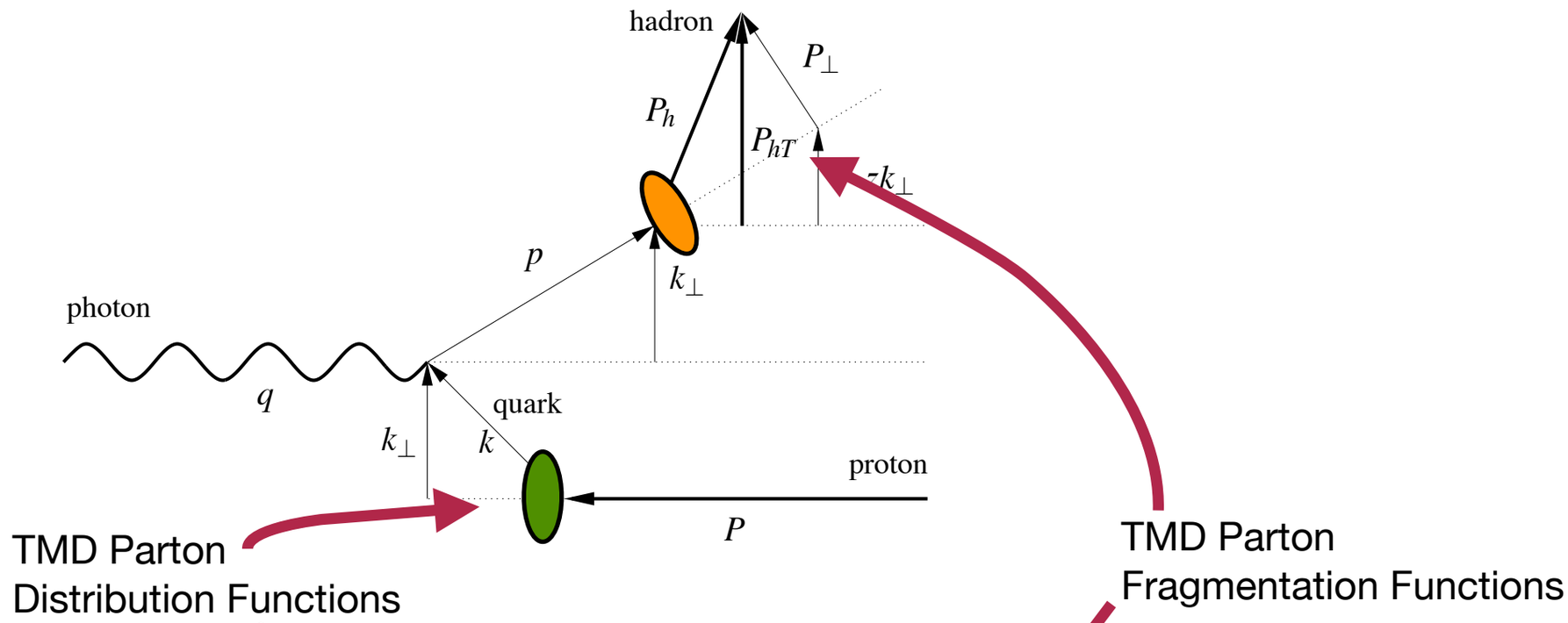
The analysis is usually done in Fourier-transformed space

TMDS IN SEMI-INCLUSIVE DIS



$$F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2)$$
$$= x \sum_q \mathcal{H}^q_{UU,T}(Q^2, \mu^2) \int d^2\boldsymbol{k}_\perp \, d^2\boldsymbol{P}_\perp \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu^2) \, D_1^{a\to h}(z, \boldsymbol{P}_\perp^2; \mu^2) \, \delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp)$$
$$= x \sum_a \mathcal{H}^q_{UU,T}(Q^2, \mu^2) \int db_T b_T J_0(b_T|\boldsymbol{P}_{h\perp}|) \hat{f}_1^q(x, z^2 b_\perp^2; \mu^2) \, \hat{D}_1^{a\to h}(z, b_\perp^2; \mu^2)$$

The analysis is usually done in Fourier-transformed space

TMDs depend on two scales, but they are set to be equal for convenience.

TMD STRUCTURE

$$\hat{f}_1^a(x, |\boldsymbol{b}_T|; \mu, \zeta) = \int d^2\boldsymbol{k}_\perp \, e^{i\boldsymbol{b}_T \cdot \boldsymbol{k}_\perp} \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu, \zeta)$$

$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = [C \otimes f_1](x, \mu_{b_*}) \, e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu} \left(\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_f}}{\mu}\right)} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}}\right)^{K_{\text{resum}} + g_K} f_{1\,NP}(x, b_T^2; \zeta_f, Q_0)$$

$$\mu_{b_*} = \frac{2e^{-\gamma_E}}{b_*}$$

see, e.g.,
Collins, "Foundations of Perturbative QCD" (11)

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perturbative Sudakov form factor

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collinear PDF

matching coefficients (perturbative)

Collins-Soper kernel (perturbative and nonperturbative)

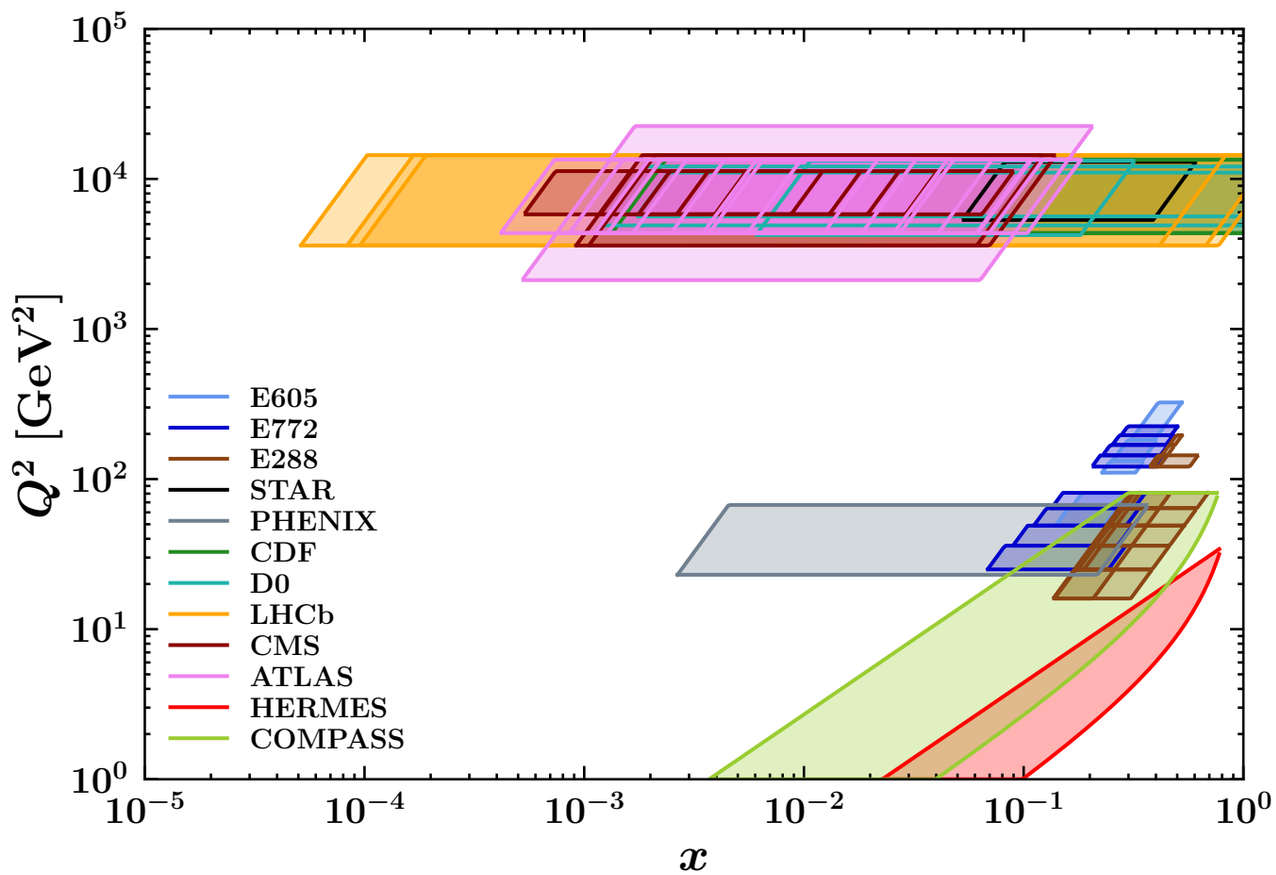
nonperturbative part of TMD

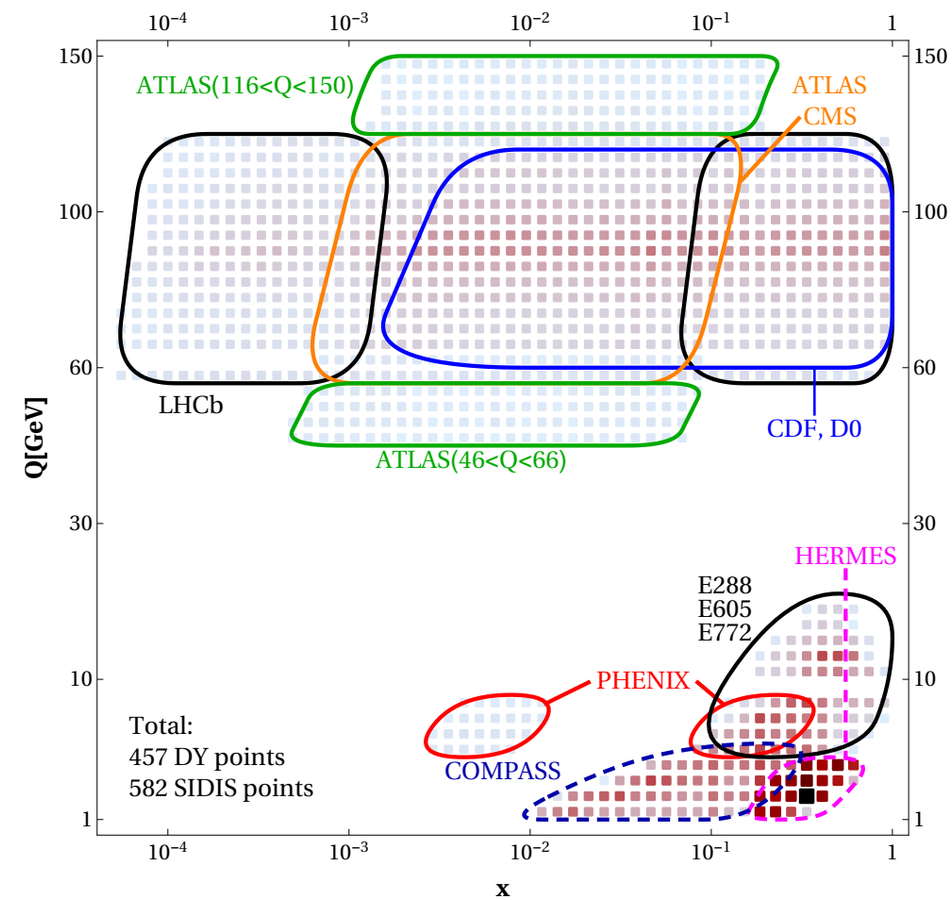
see, e.g.,
Collins, "Foundations of Perturbative QCD" (11)

TMD GLOBAL FITS

	Accuracy	HERMES	COMPASS	DY fixed target	DY collider	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✔	✔	✔	✔	8059	1.55
SV 2019 arXiv:1912.06532	N^3LL^-	✔	✔	✔	✔	1039	1.06
MAP22 arXiv:2206.07598	N^3LL^-	✔	✔	✔	✔	2031	1.06

x–Q² COVERAGE





MAP Collaboration
Bacchetta, Bertone, Bissolotti, Bozzi, Cerutti, Piacenza, Radici, Signori, arXiv:2206.07598

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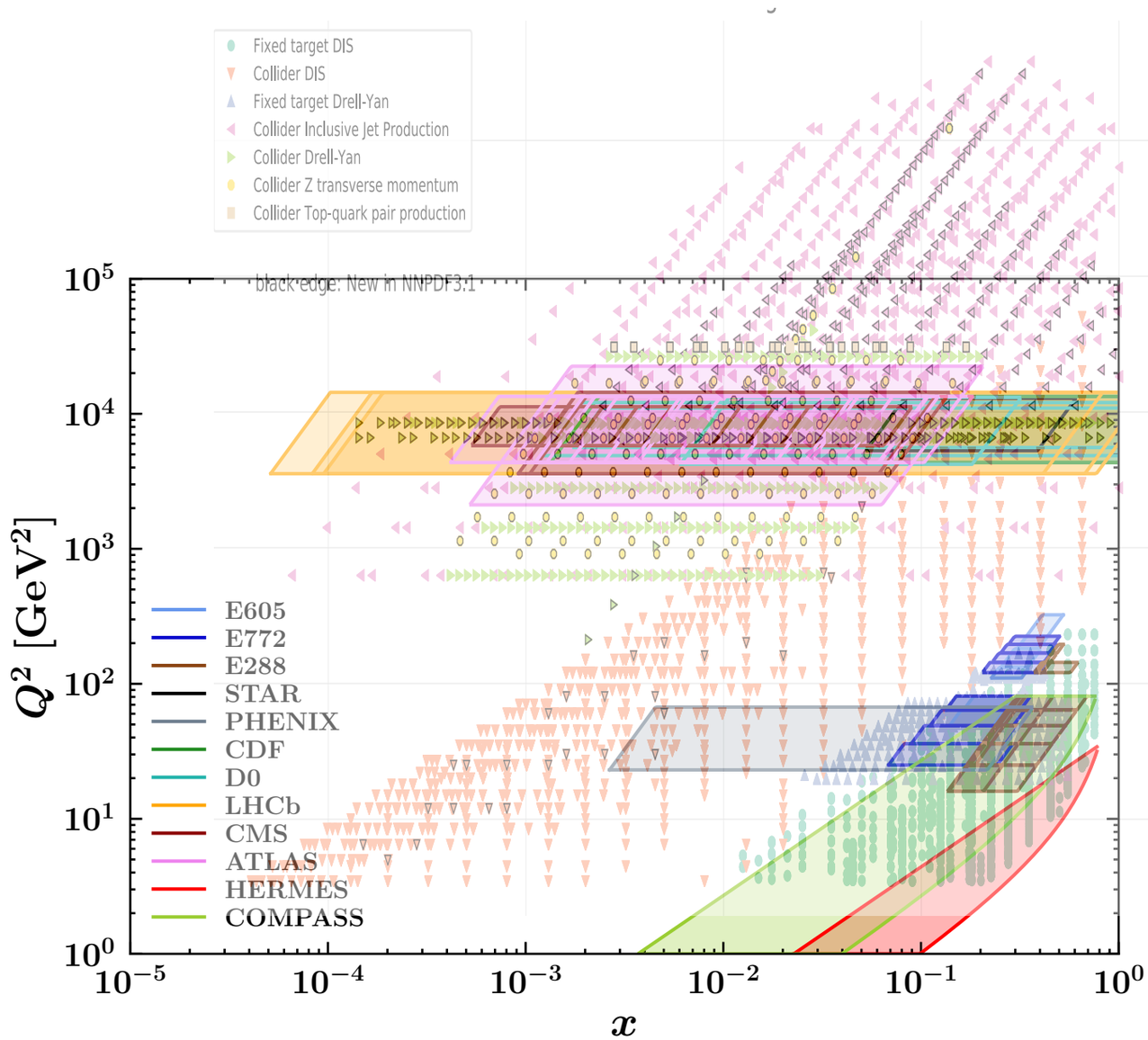

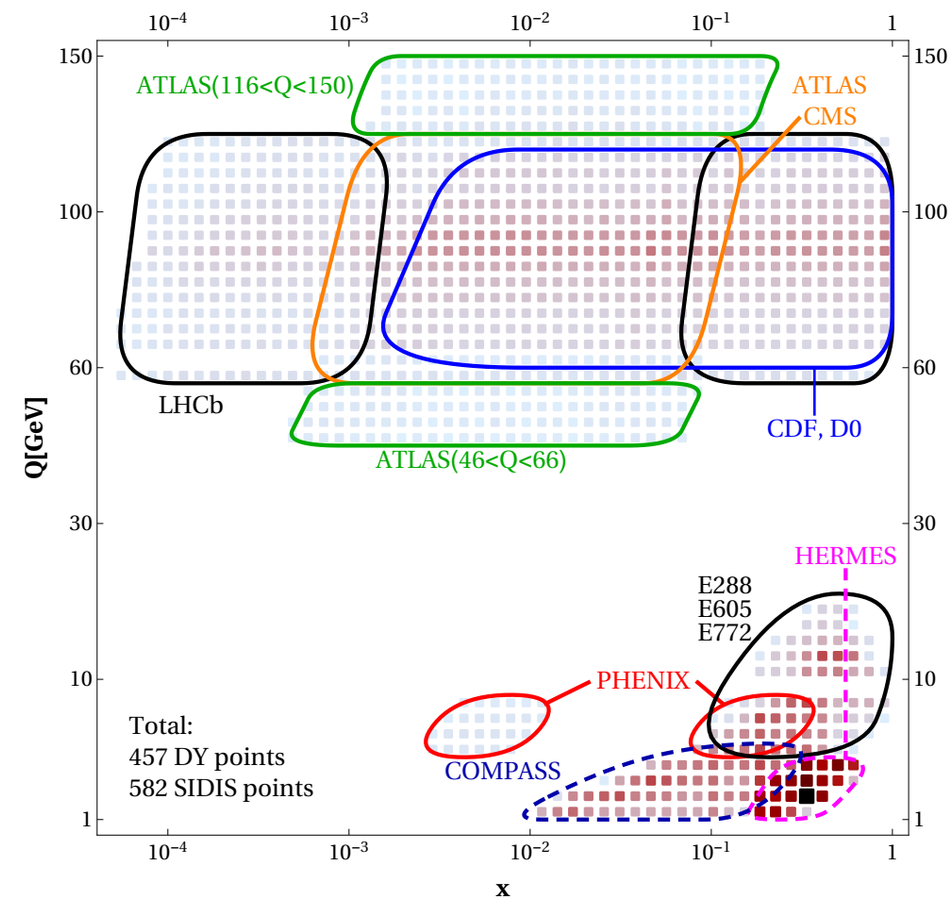

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MAP22 FUNCTIONAL FORM

$$f_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{k_T^2}{g1}} + \lambda^2 k_T^2 e^{-\frac{k_T^2}{g1B}} + \lambda_2^2 e^{-\frac{k_T^2}{g1C}} \right)$$

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11 parameters for TMD PDF
+ 1 for NP evolution +9 for FF
= 21 free parameters

EXAMPLE OF RESULTING TMDS

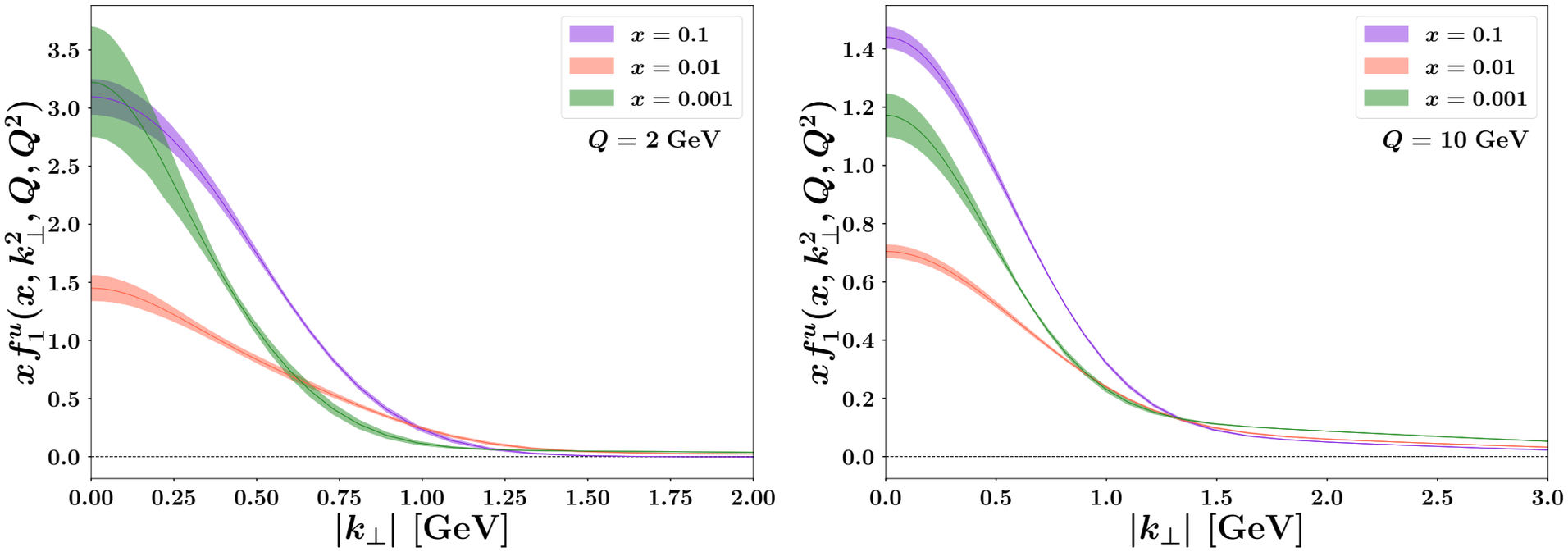

FIG. 13: The TMD PDF of the up quark in a proton at $\mu = \sqrt{\zeta} = Q = 2$ GeV (left panel) and 10 GeV (right panel) as a function of the partonic transverse momentum $|\boldsymbol{k}_\perp|$ for $x = 0.001$, 0.01 and 0.1. The uncertainty bands represent the 68% CL.

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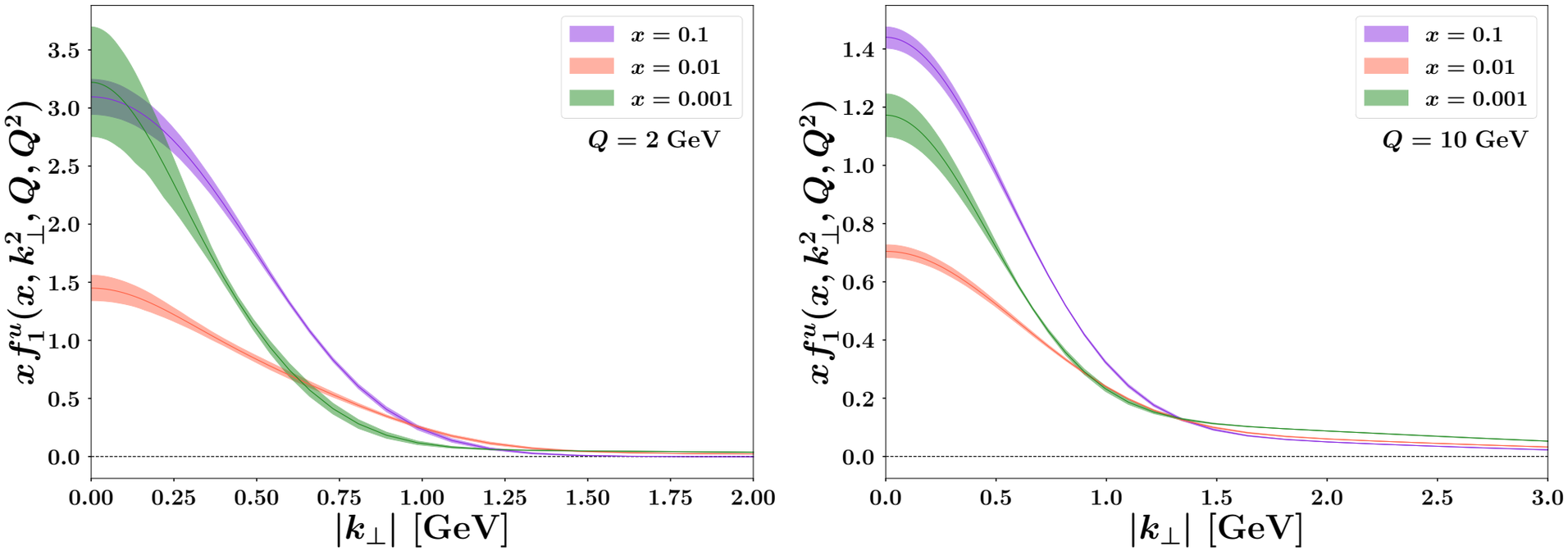

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As usual, the rigidity of the functional form plays a role and probably leads to underestimated bands

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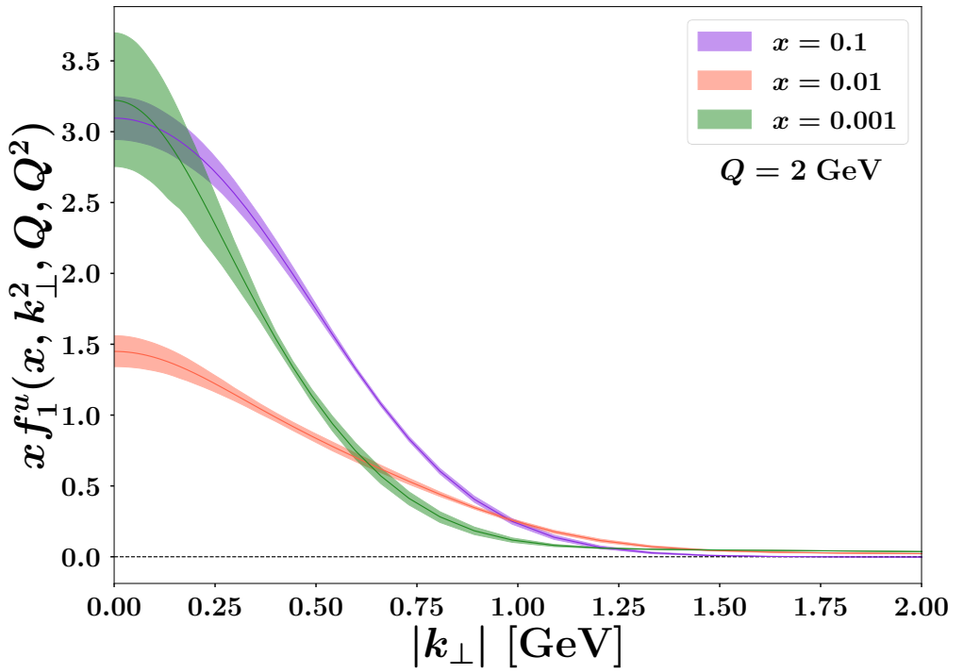

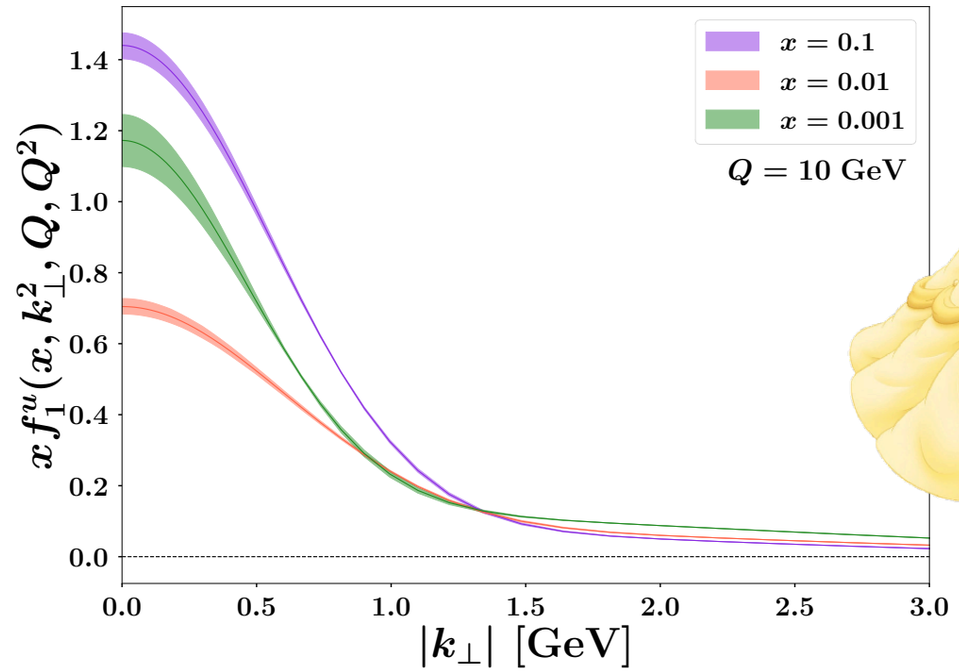

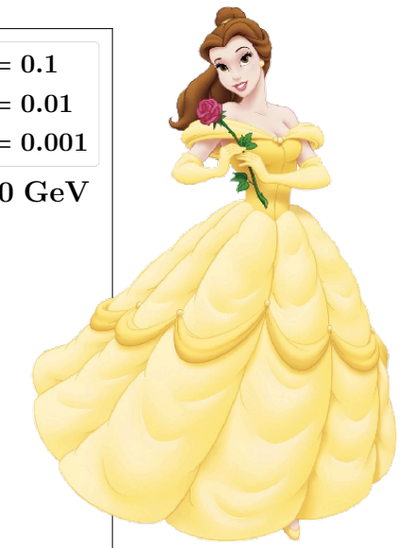

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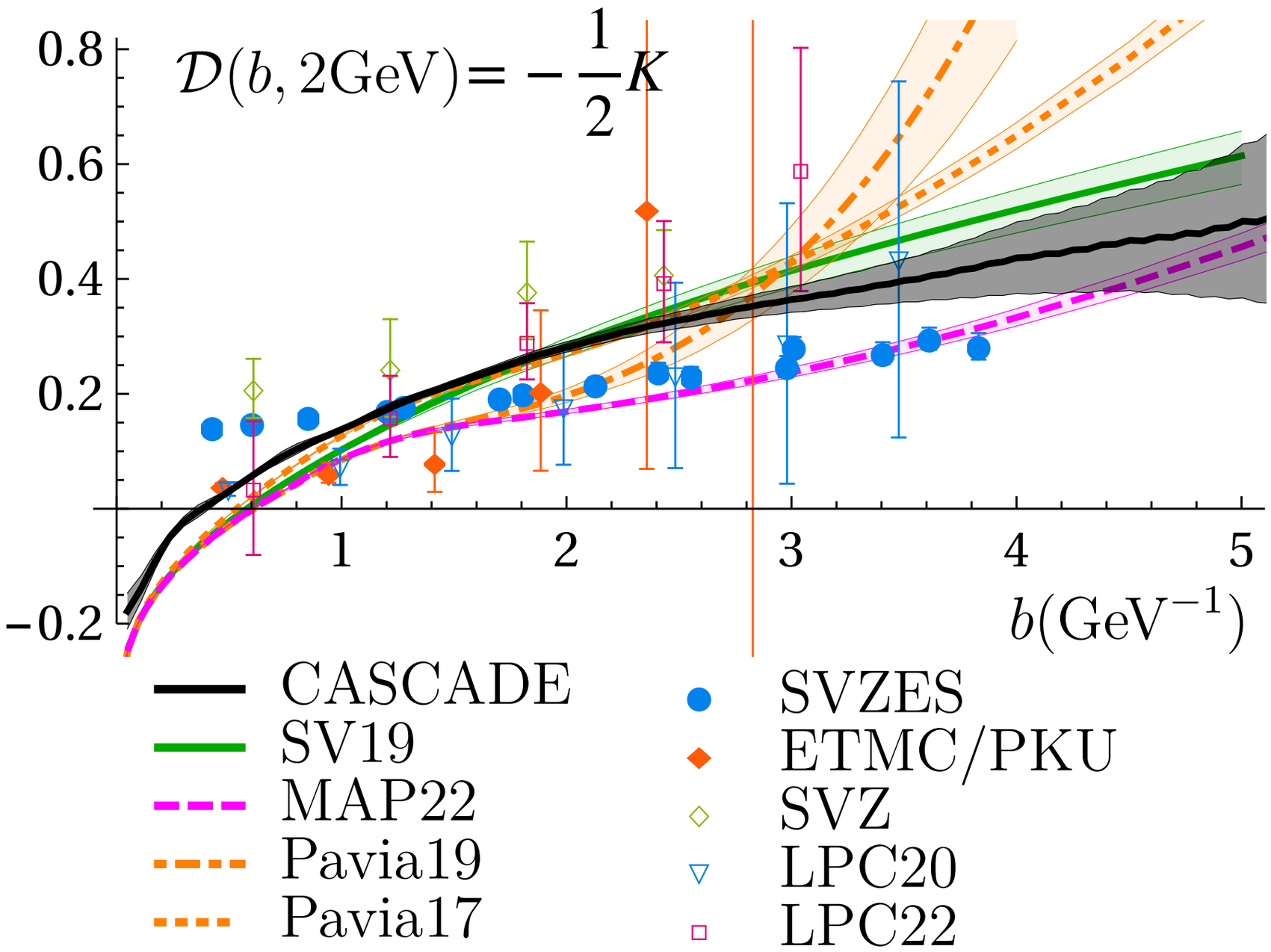
RESULTING COLLINS-SOPER KERNEL

Bermudez Martinez, Vladimirov, arXiv:2206.01105



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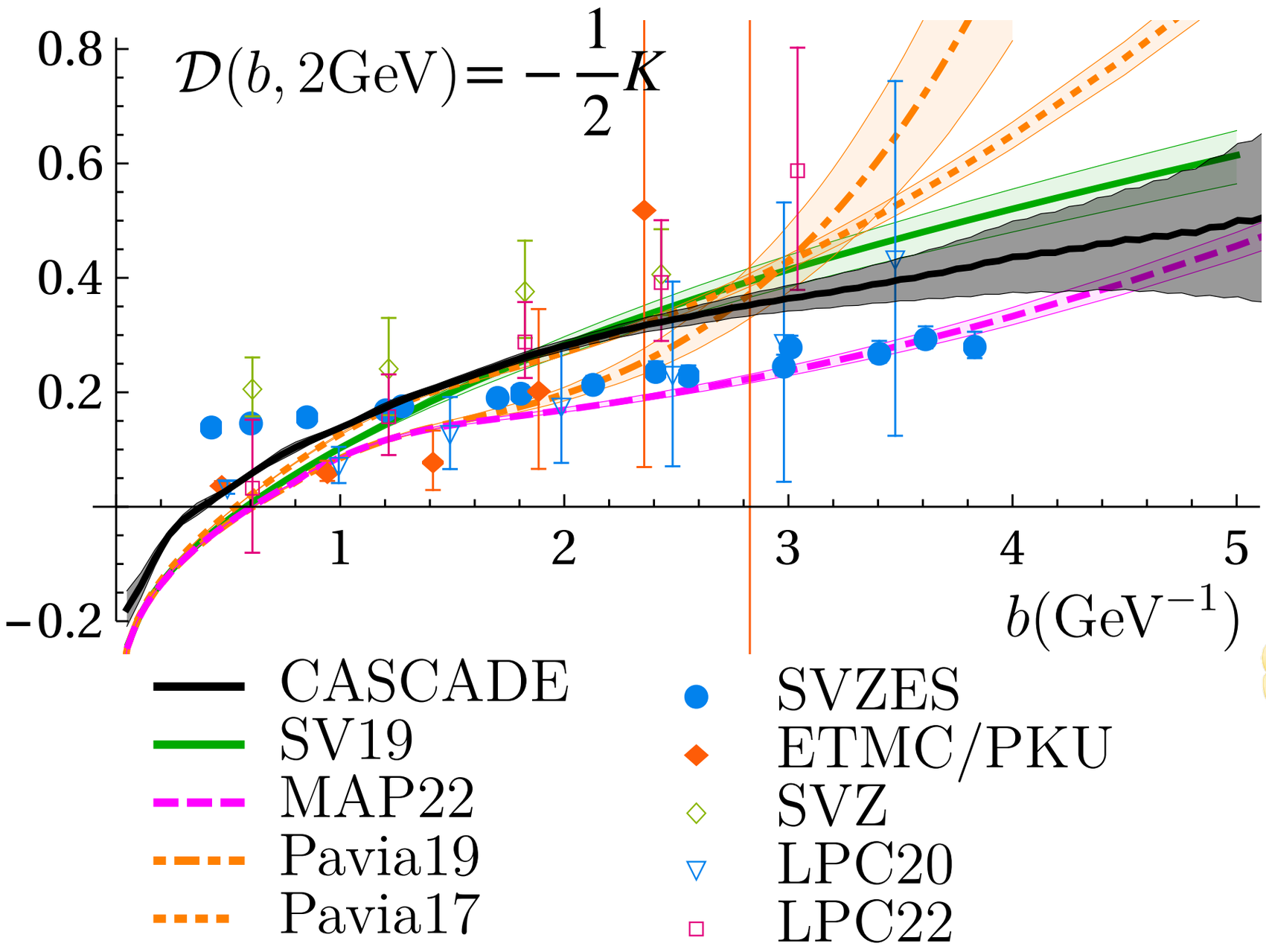
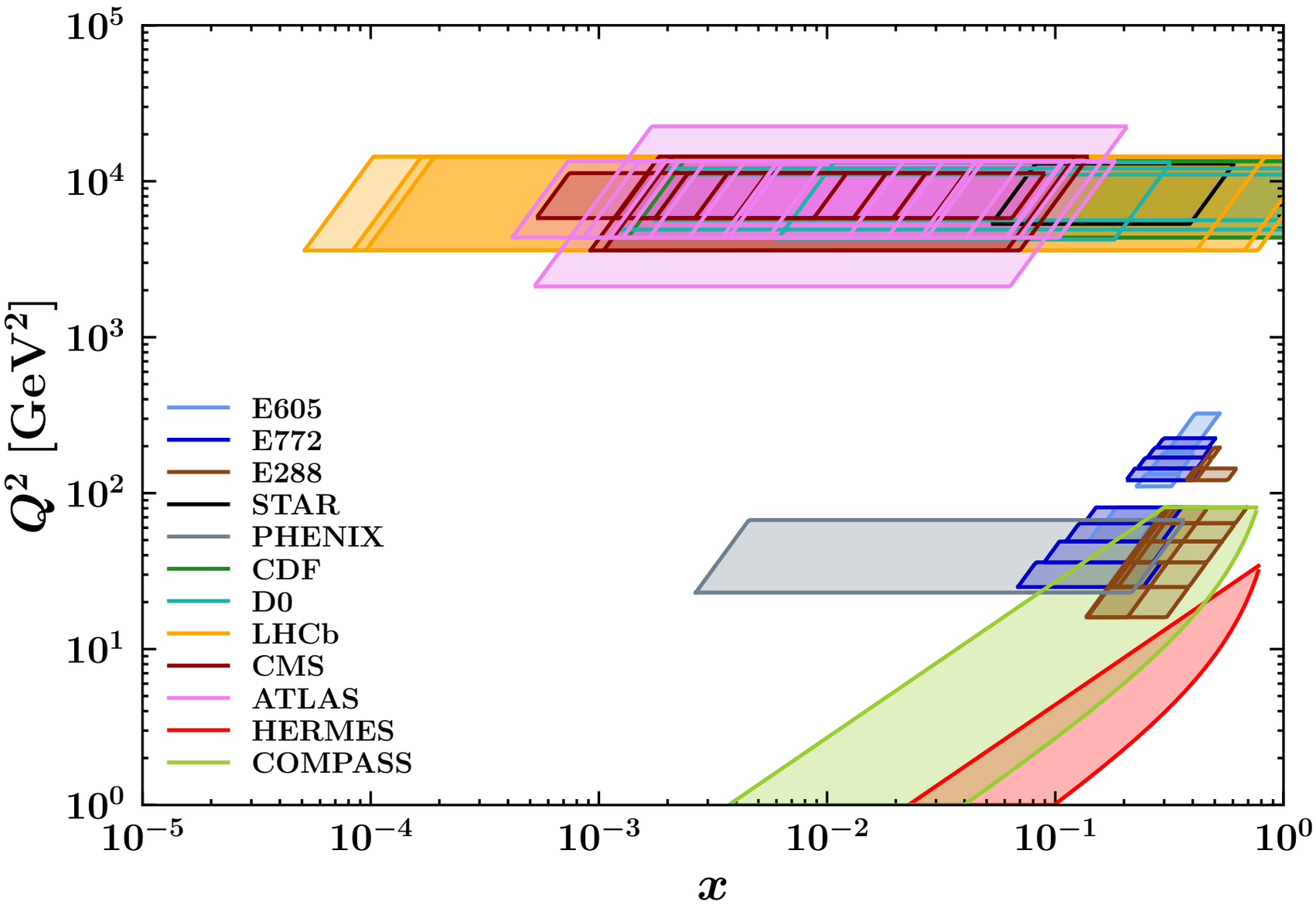


EXPECTED EXTENSIONS OF DATA RANGE



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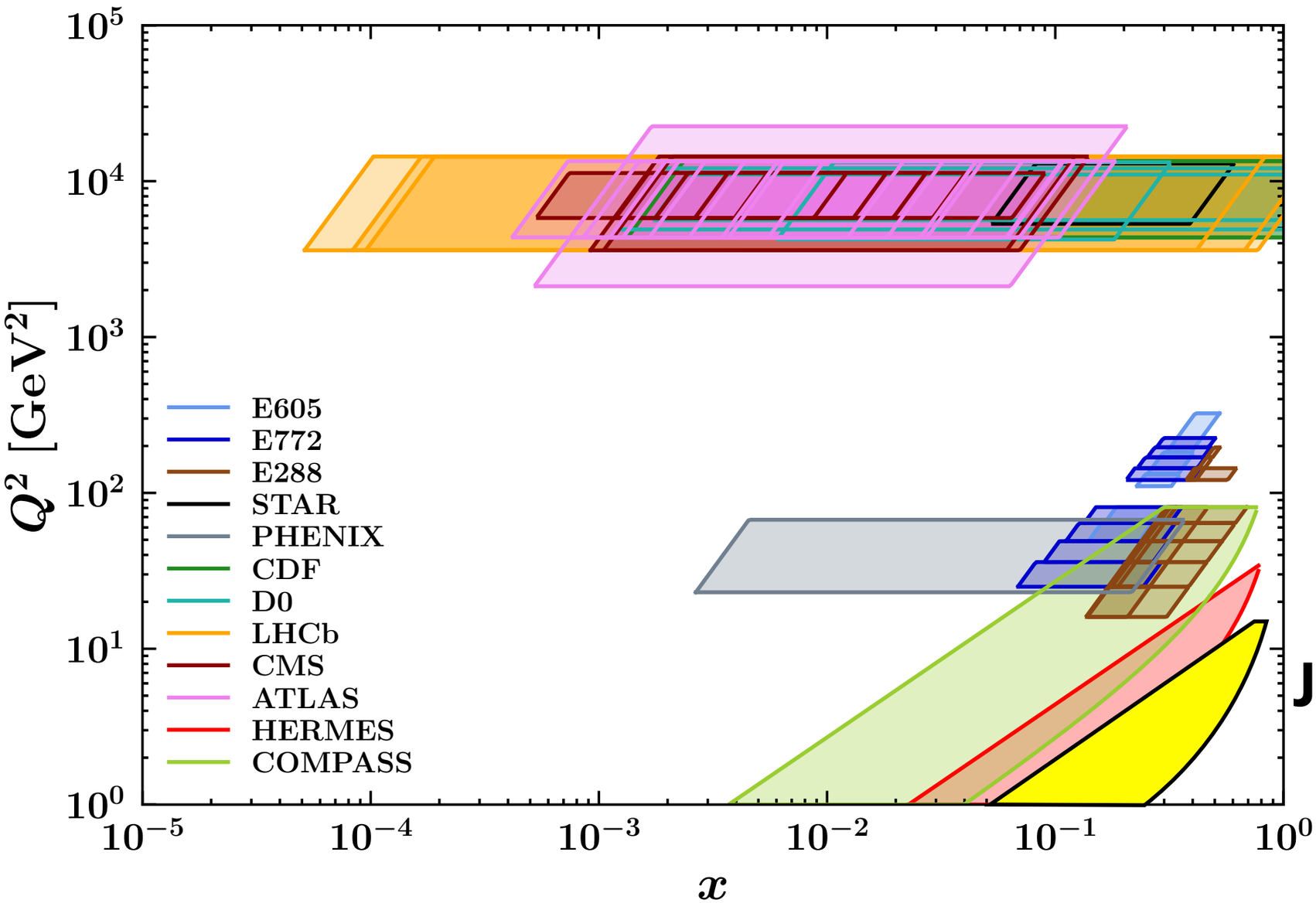
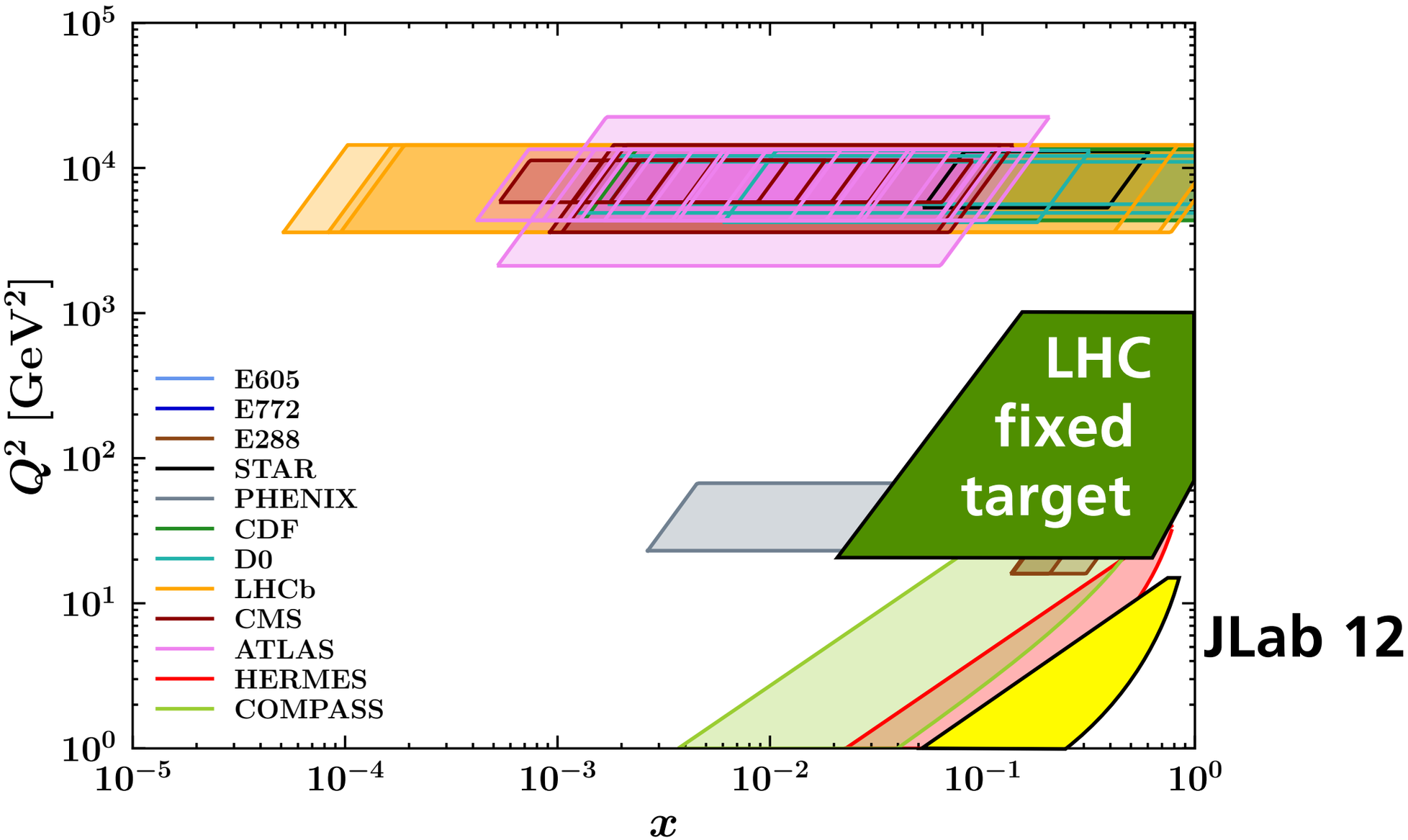


JLab 12

EXPECTED EXTENSIONS OF DATA RANGE



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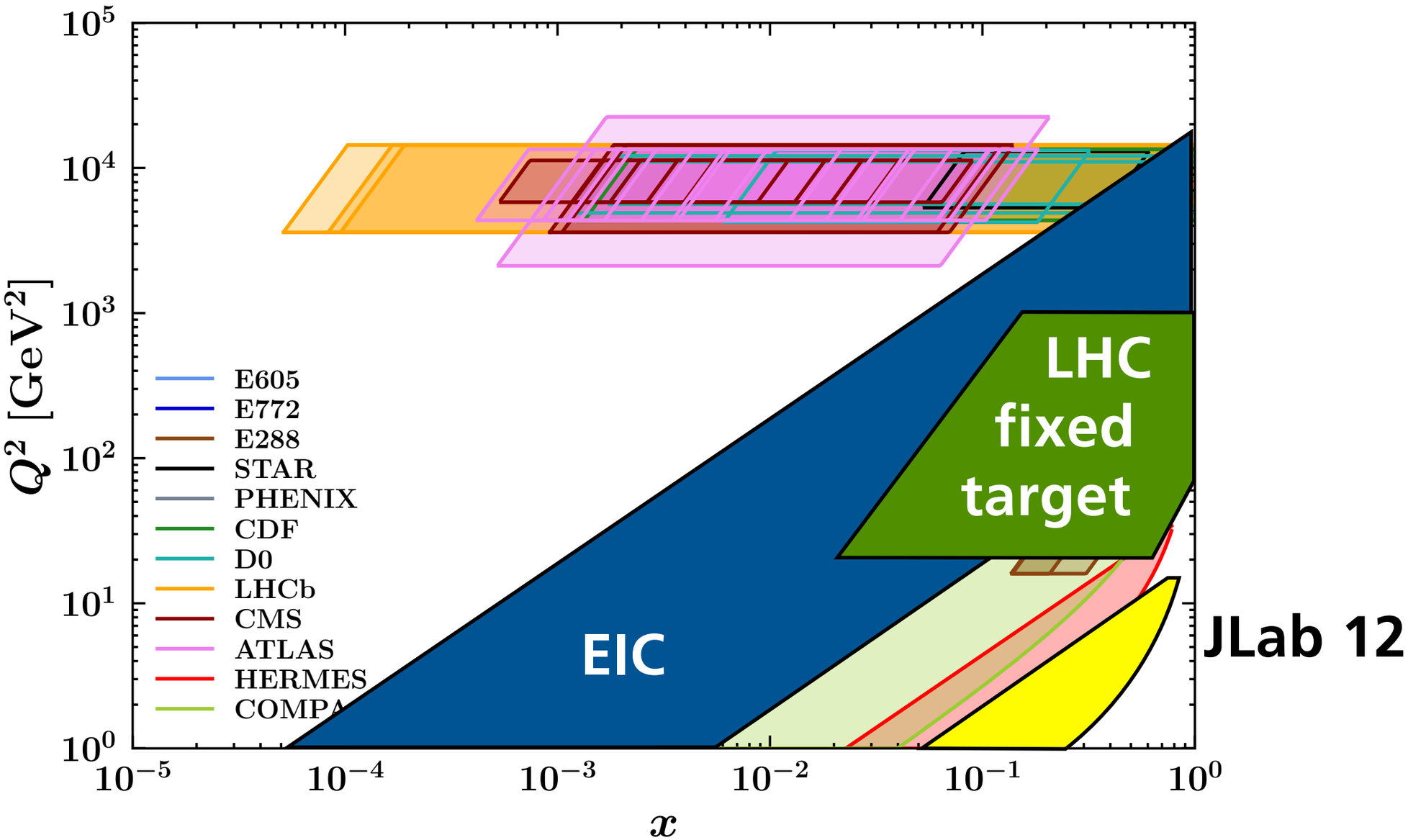

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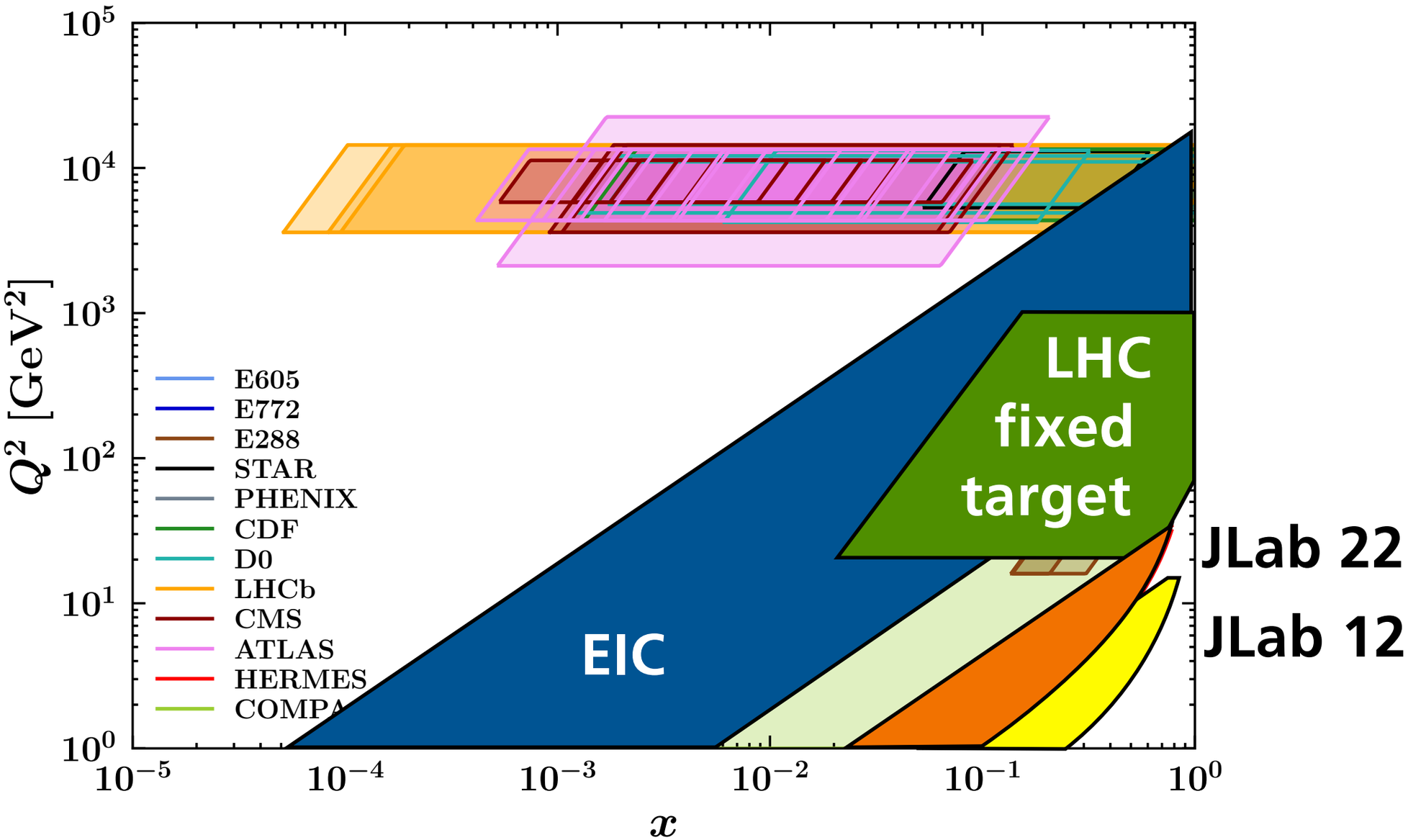
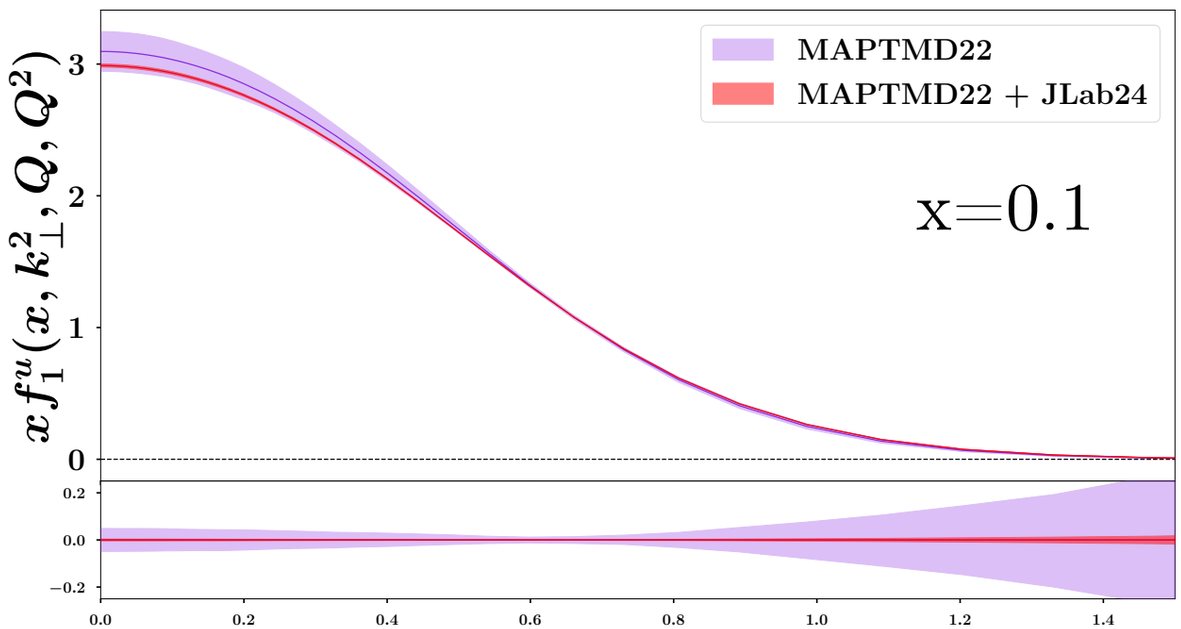


JLAB 20 IMPACT

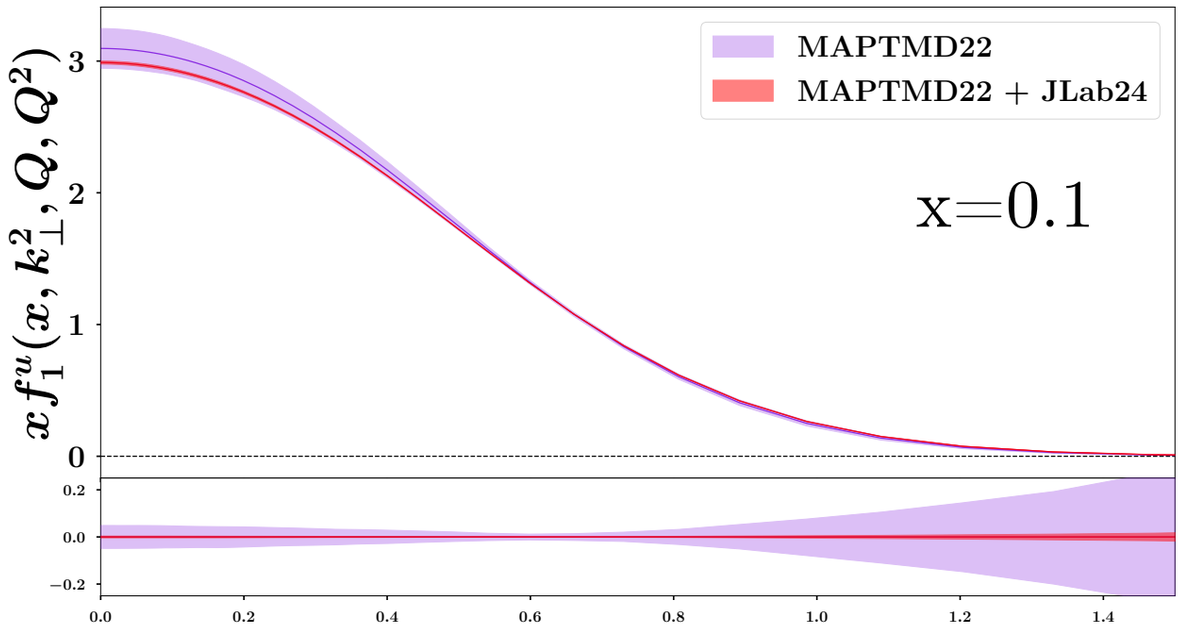
see talk on Wednesday



TMD at large x

JLAB 20 IMPACT

see talk on Wednesday



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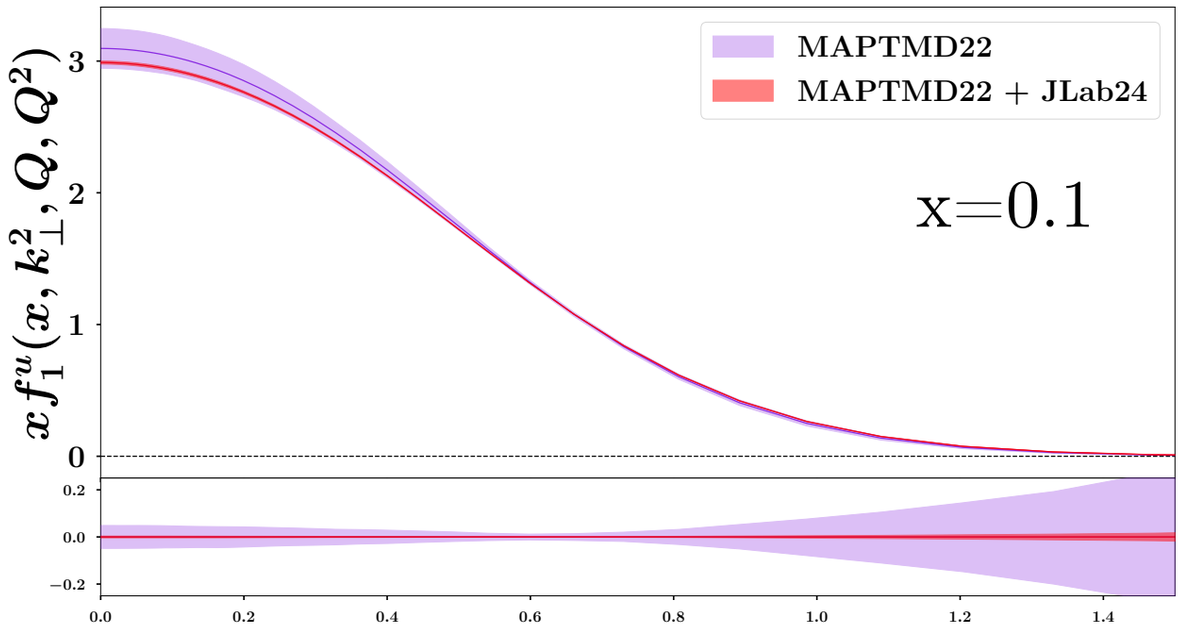
see talk on Wednesday



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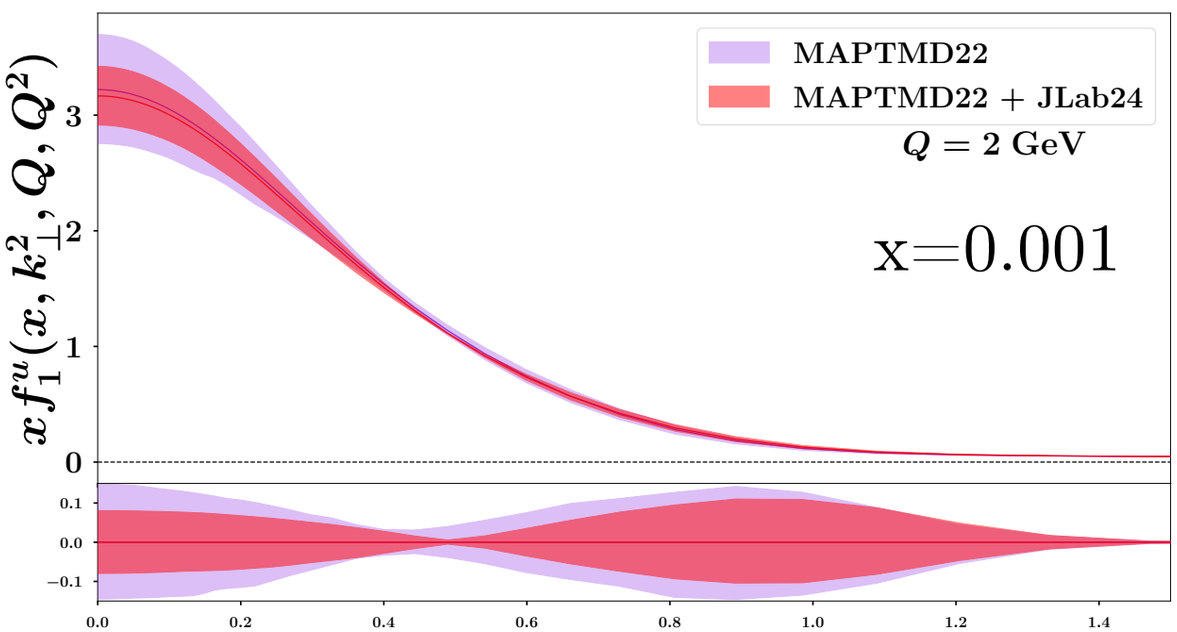
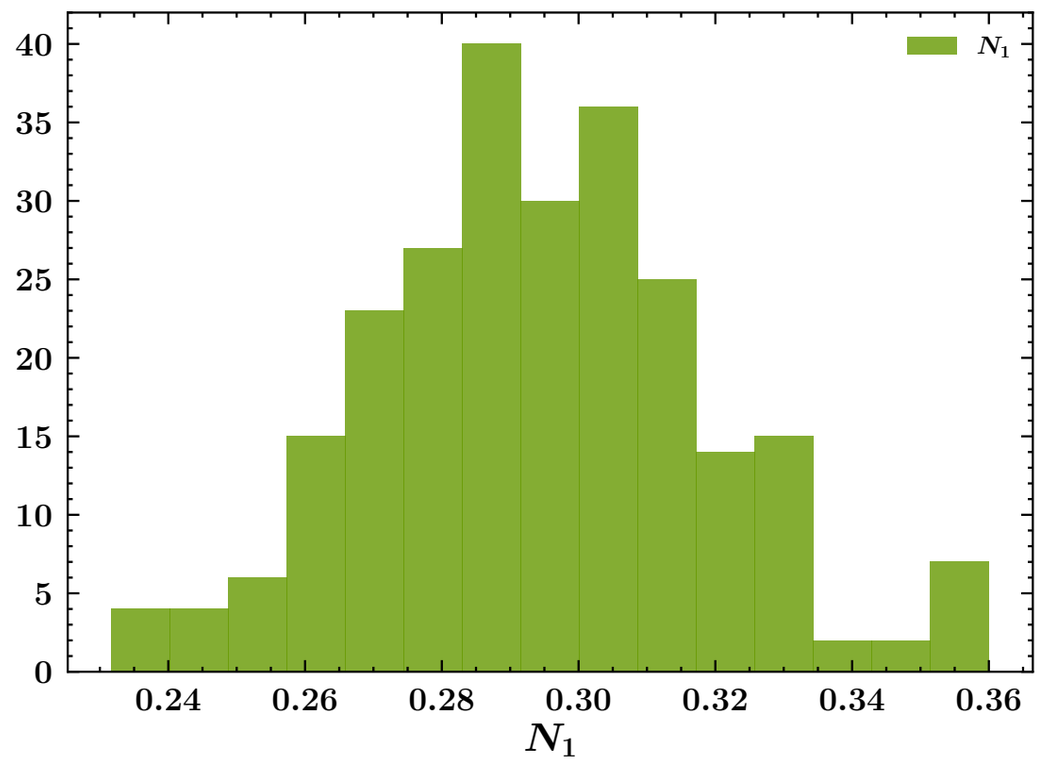


TMD at low x

JLAB 20 IMPACT

see talk on Wednesday



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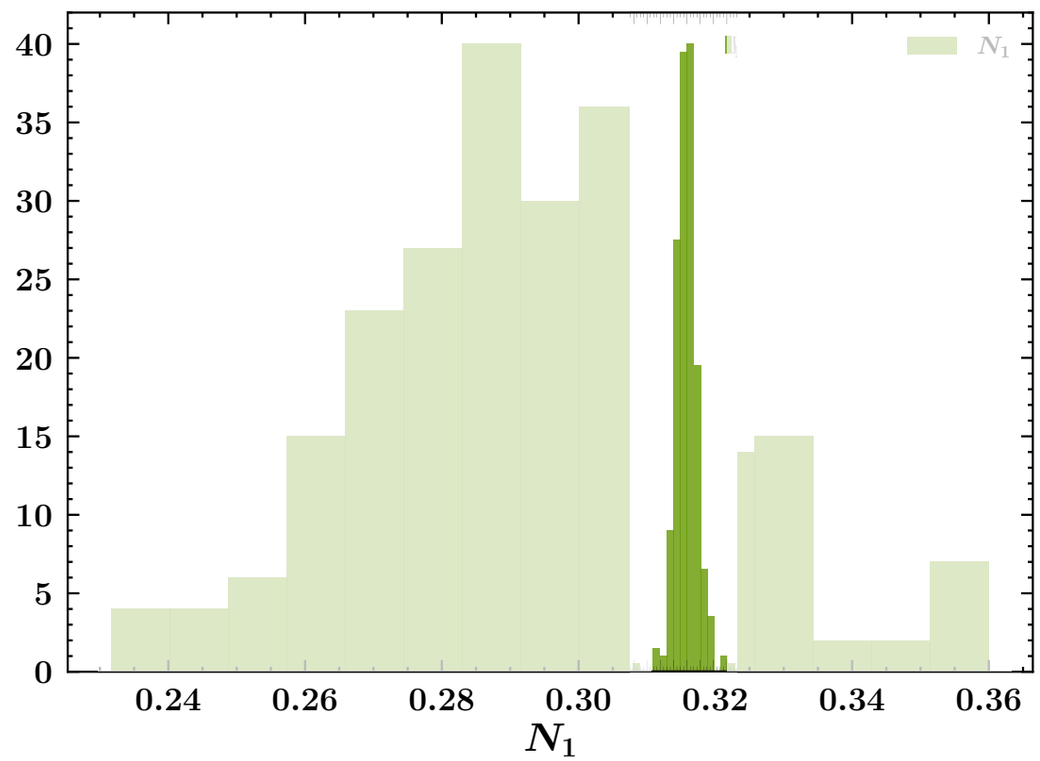

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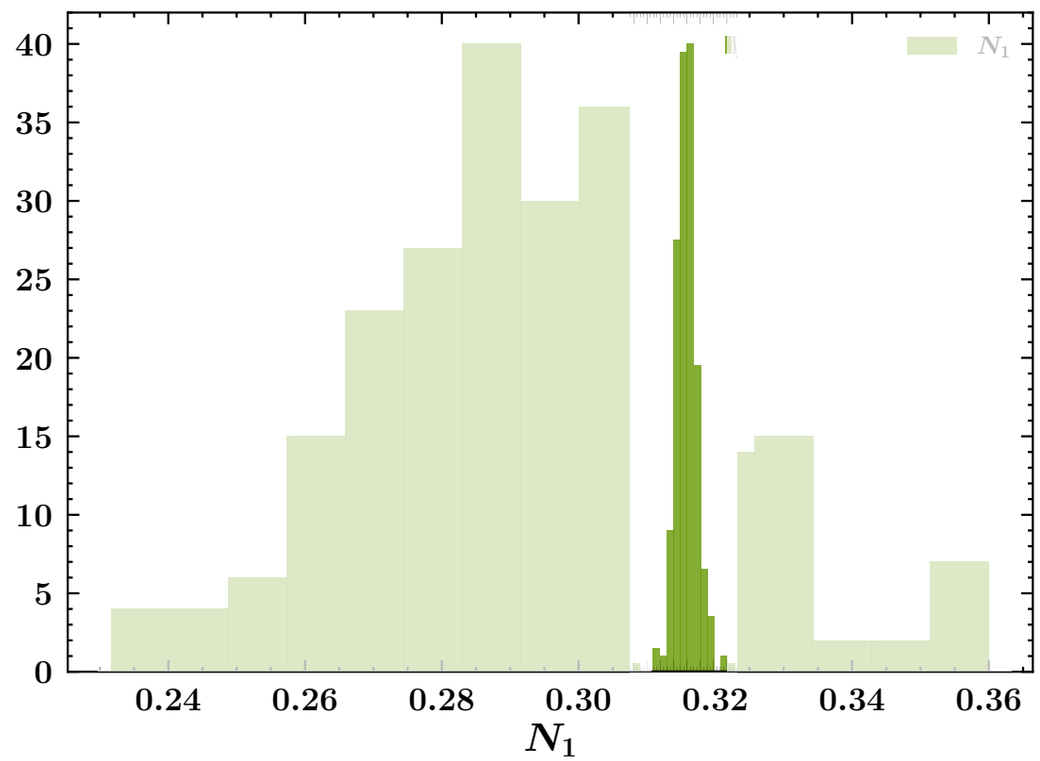

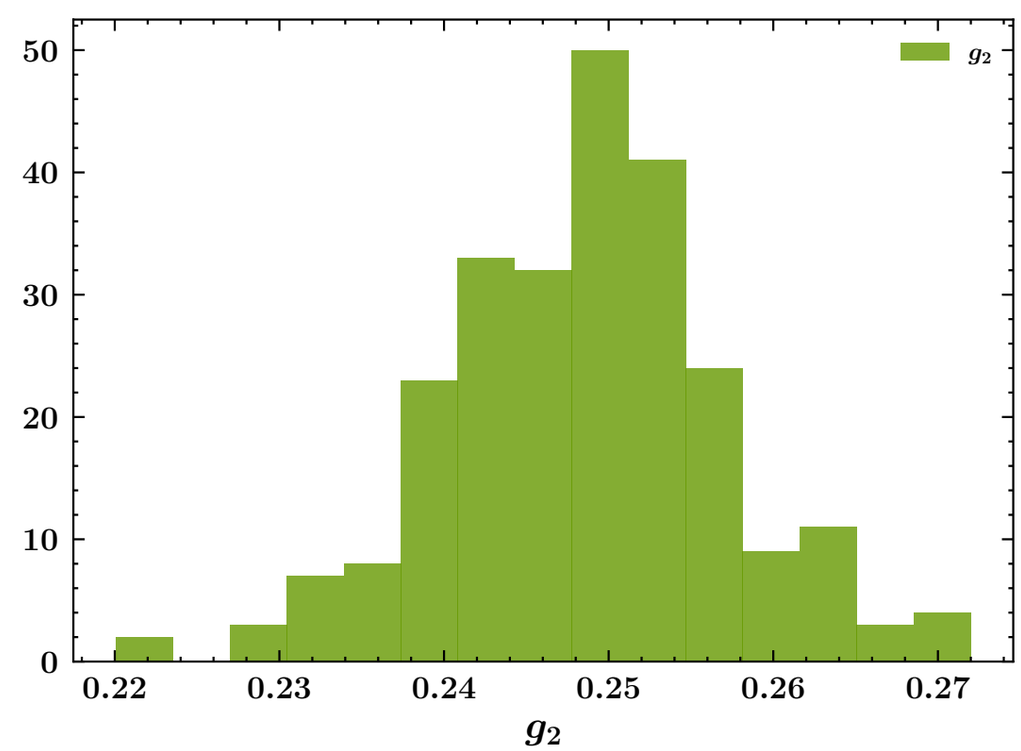

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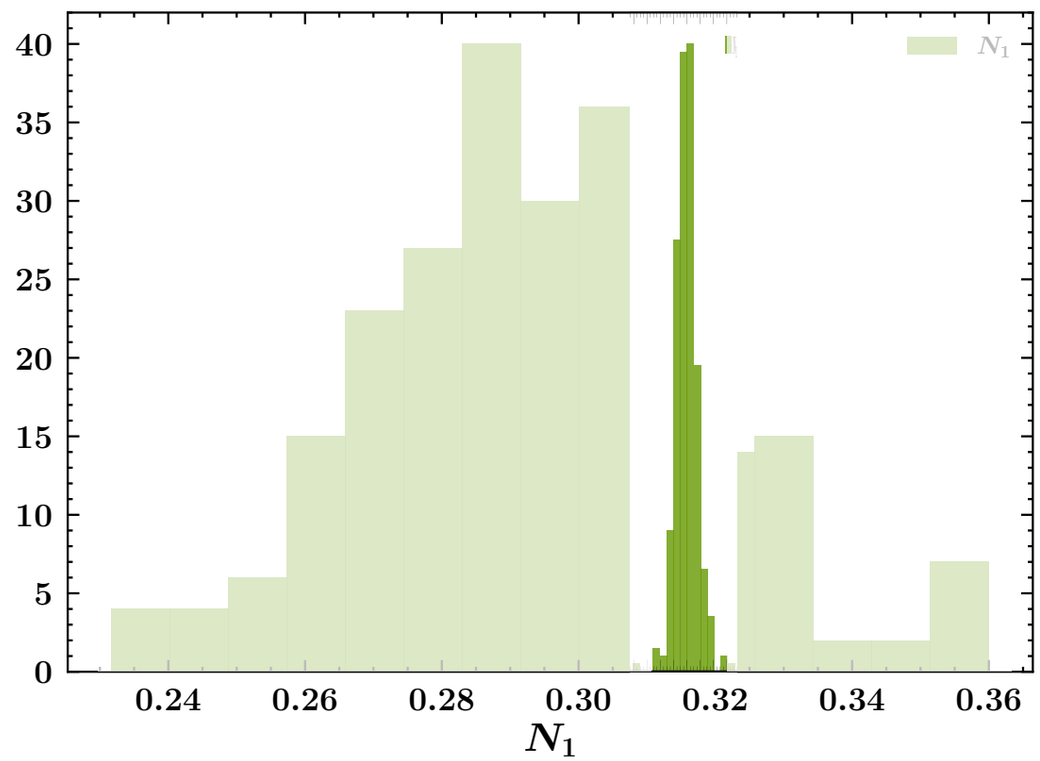

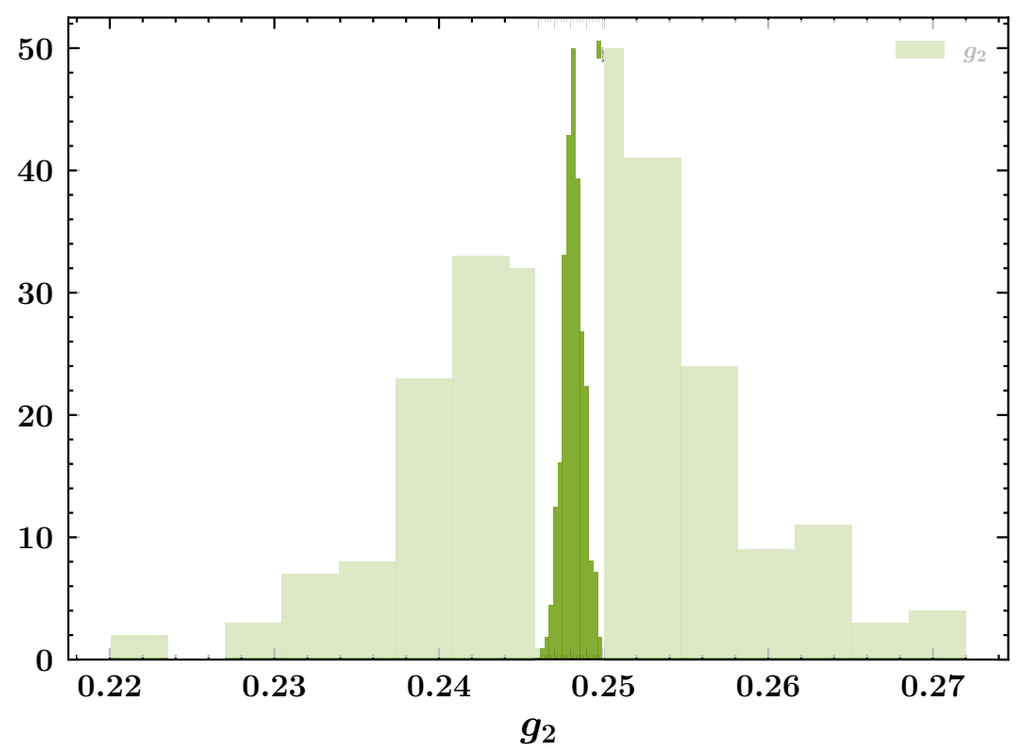

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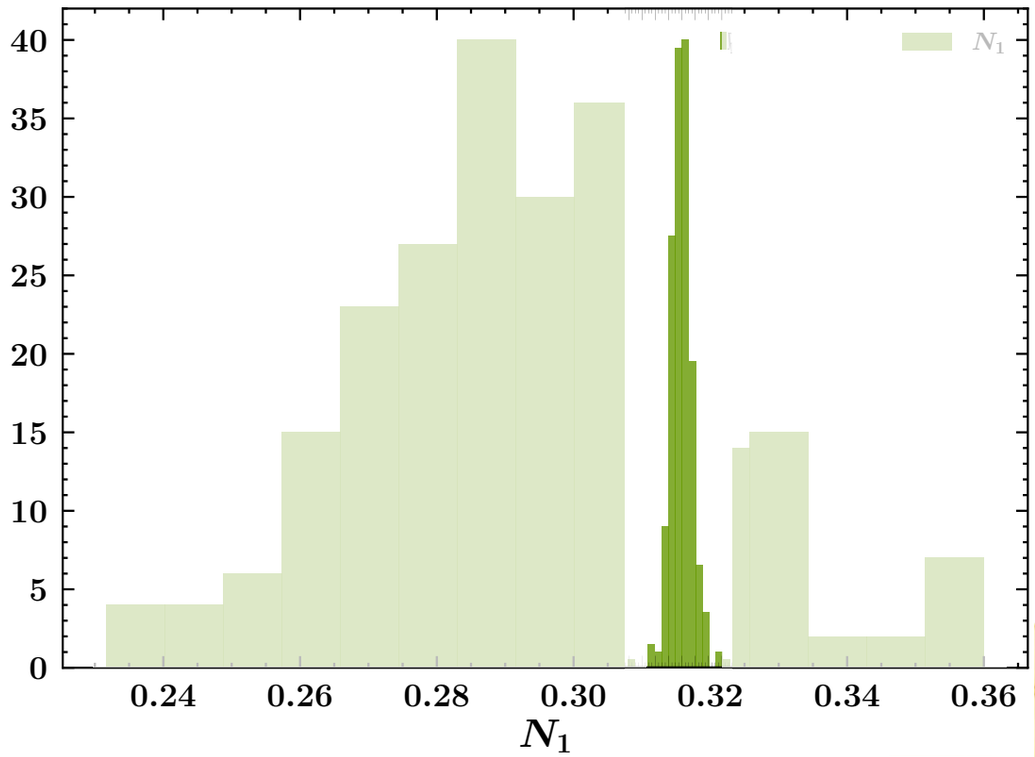

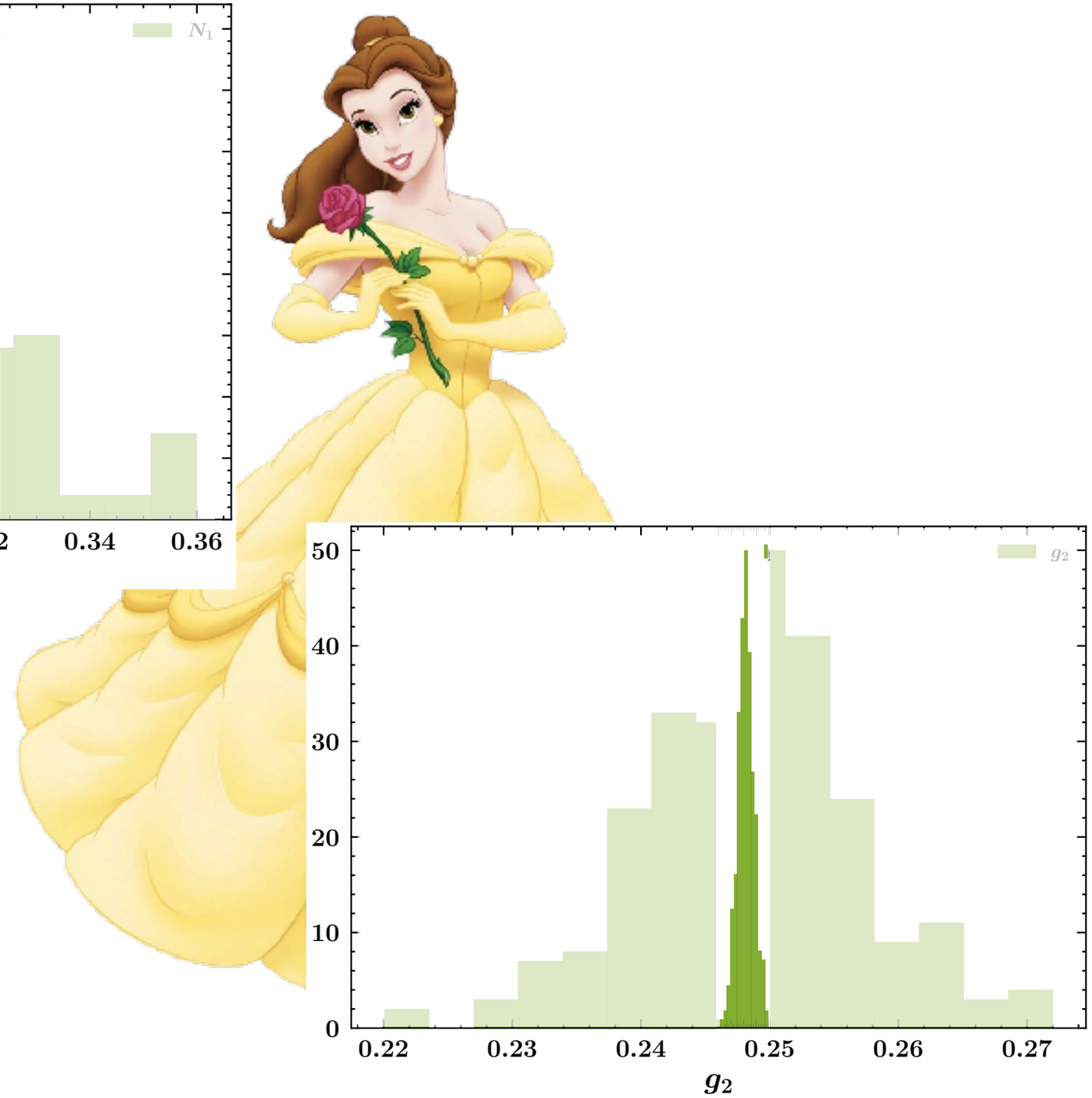

FROM KT TO BT SPACE?

MSc thesis C. Bissolotti, 2016

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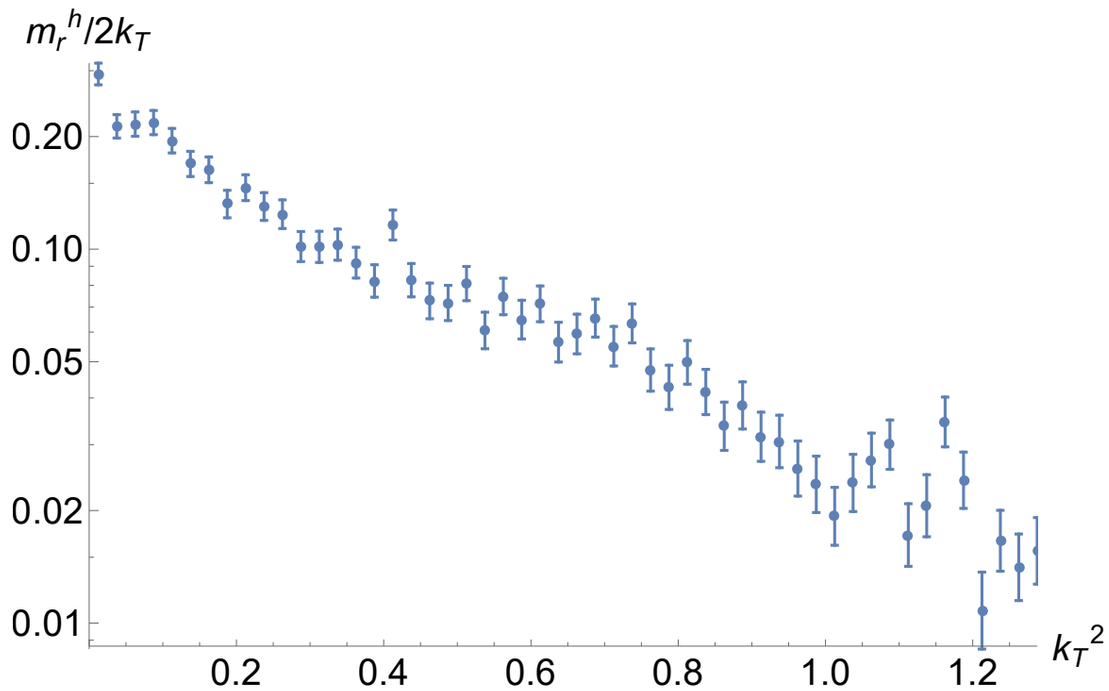

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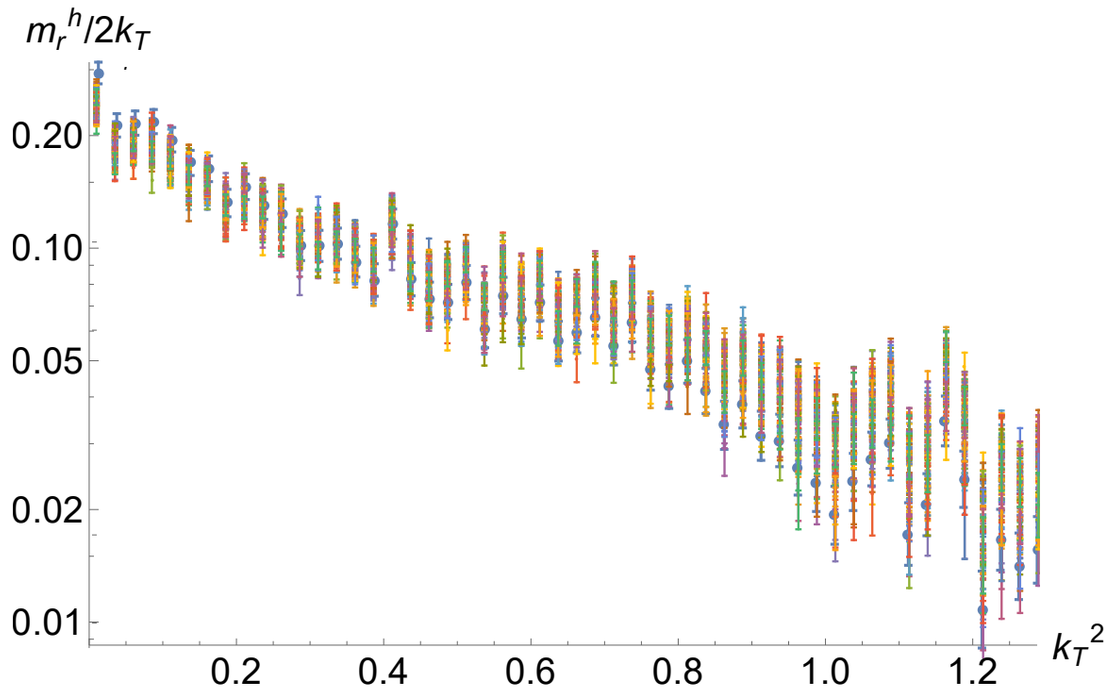

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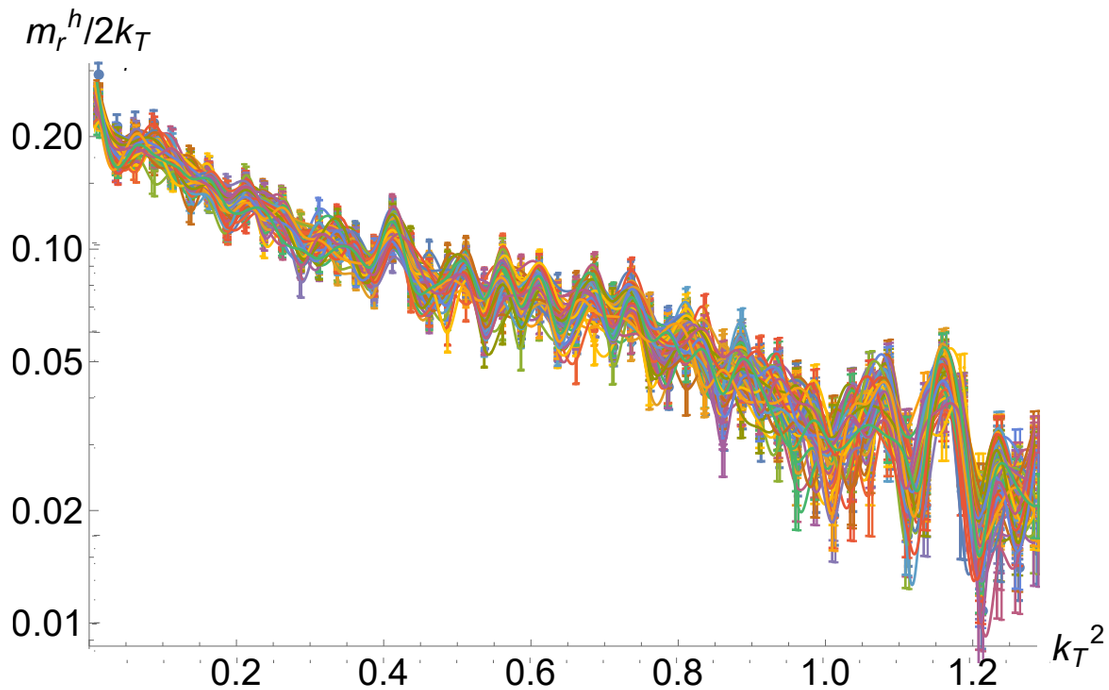

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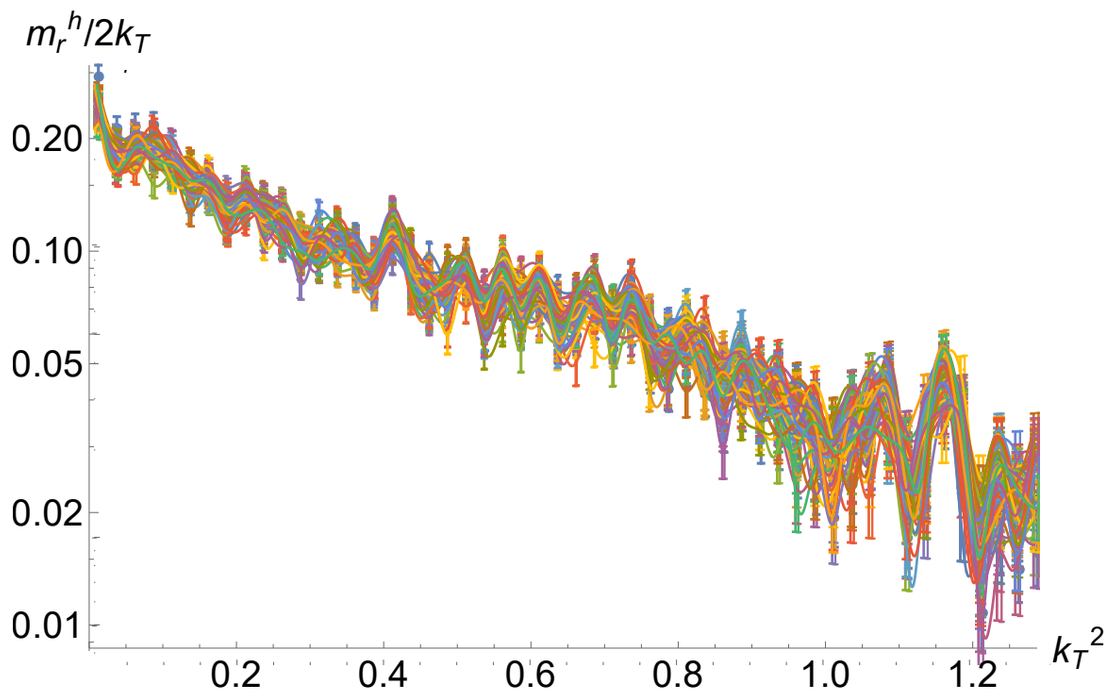

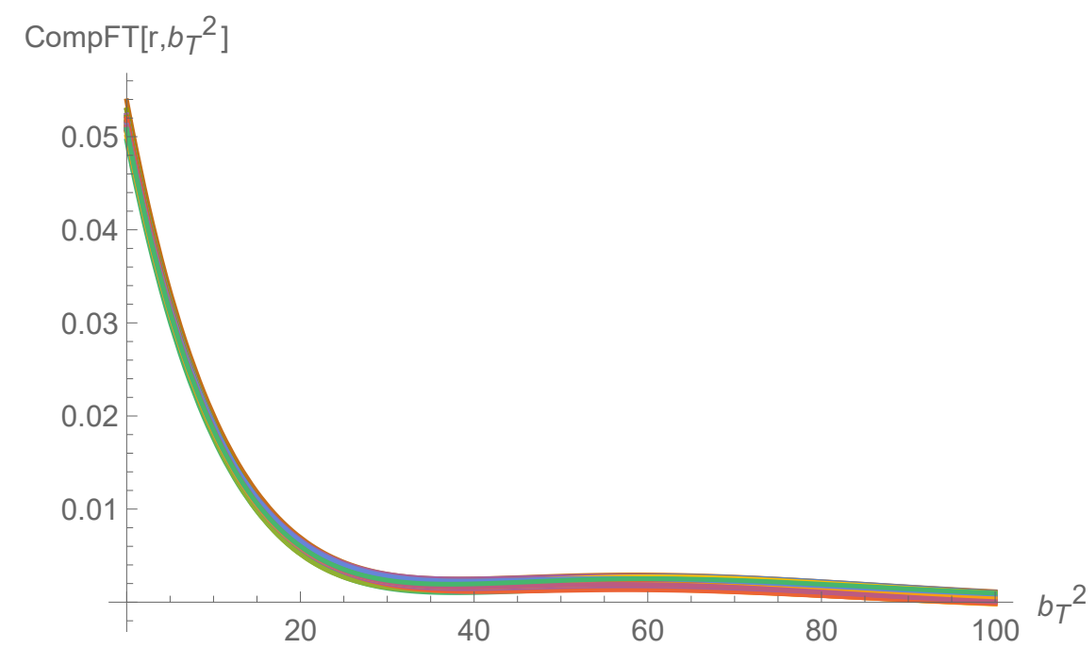

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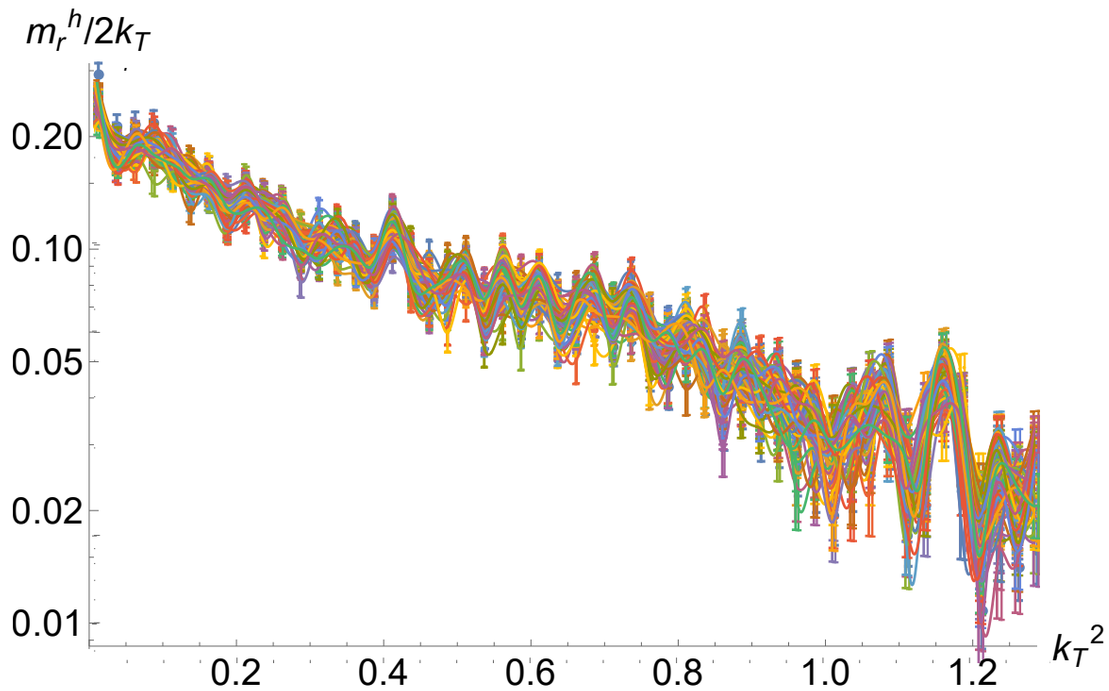
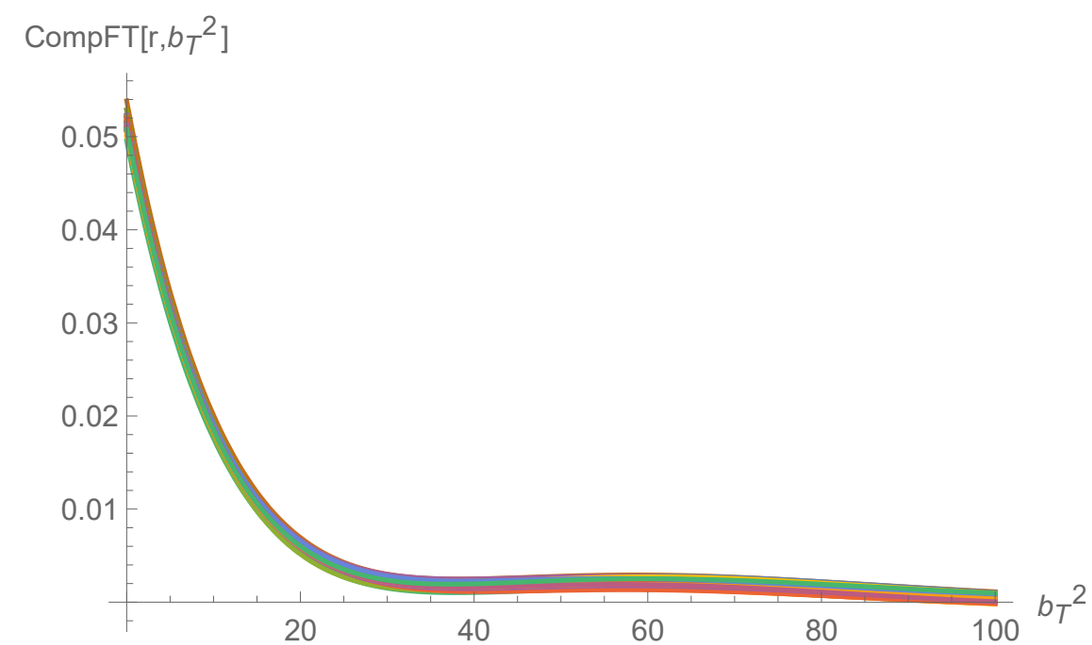


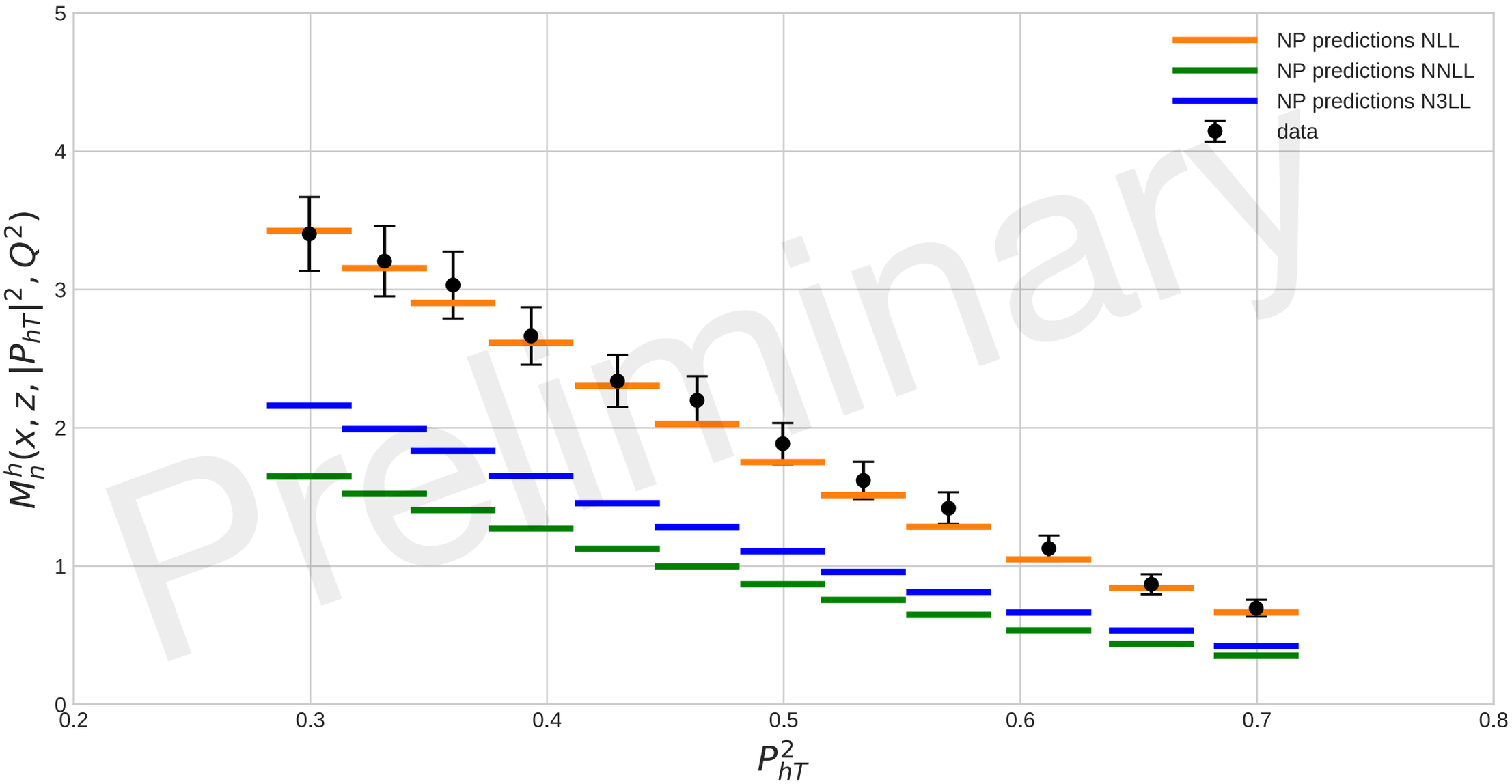
Fourier transform



BEAST 1: NORMALIZATION

PROBLEMS WITH LOW TRANSVERSE MOMENTUM

COMPASS multiplicities (one of many bins)



The description considerably worsens at higher orders

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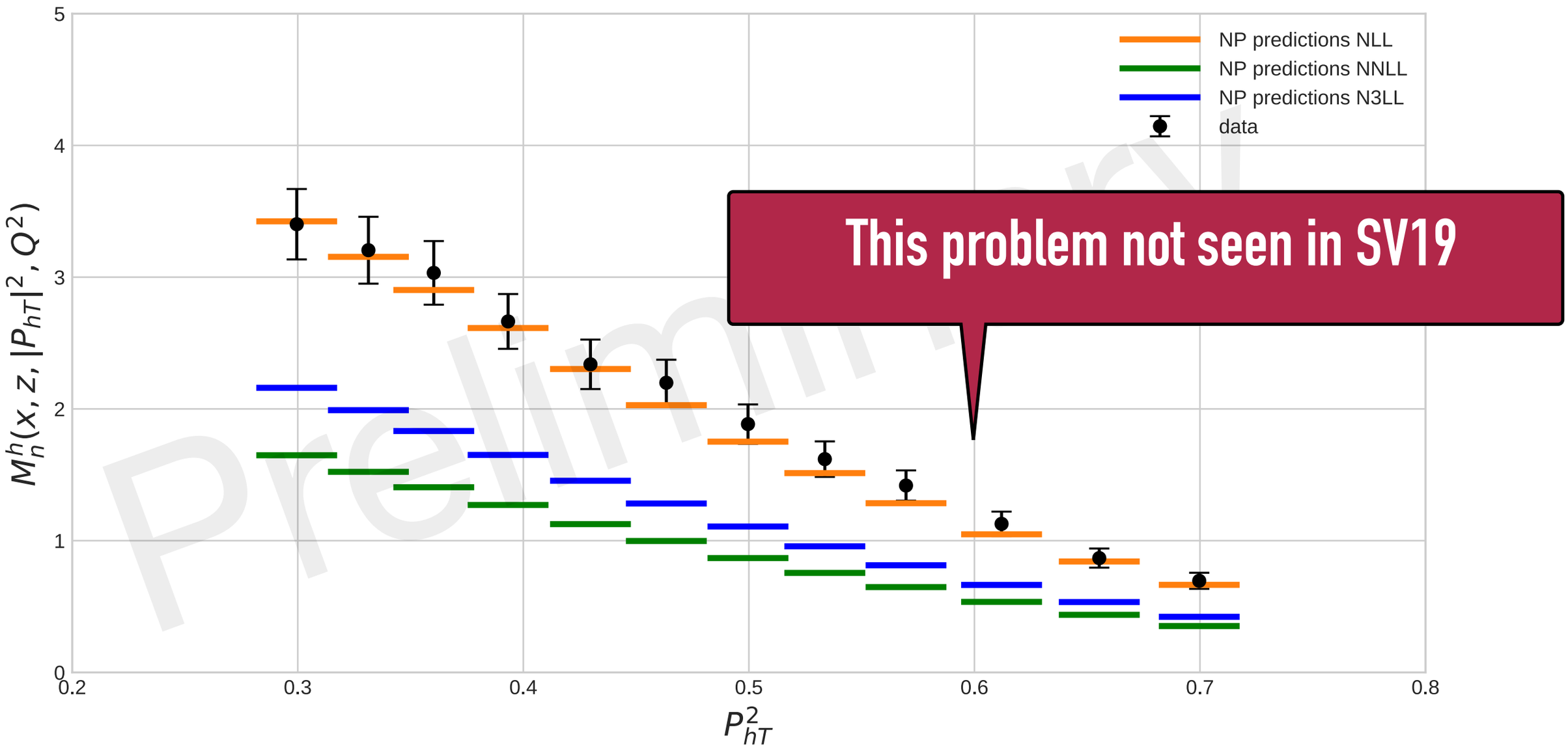

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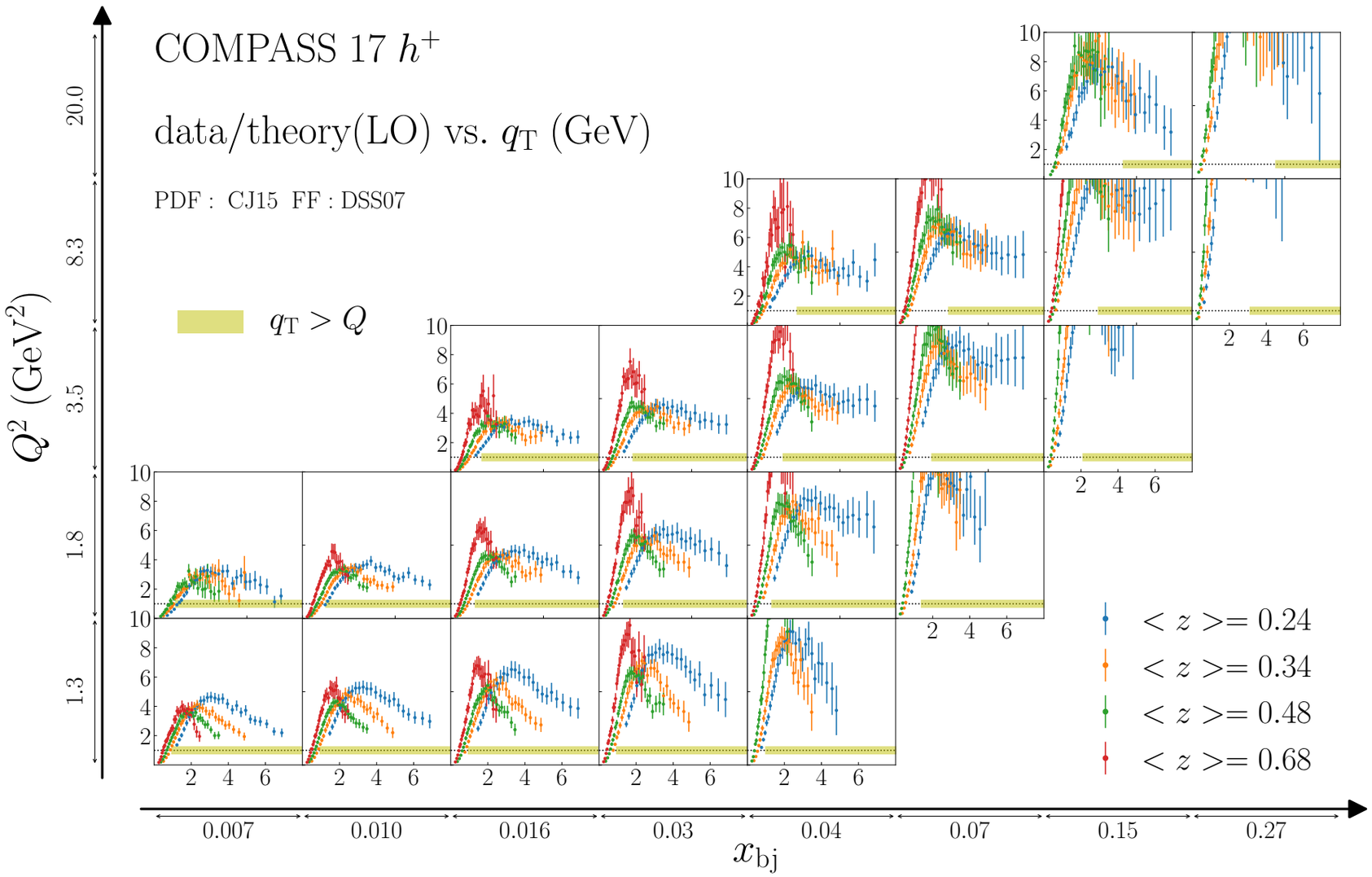
PROBLEMS WITH HIGH TRANSVERSE MOMENTUM

Gonzalez-Hernandez, Rogers, Sato, Wang arXiv:1808.04396



At high q_T, the collinear formalism should be valid, but large discrepancies are observed

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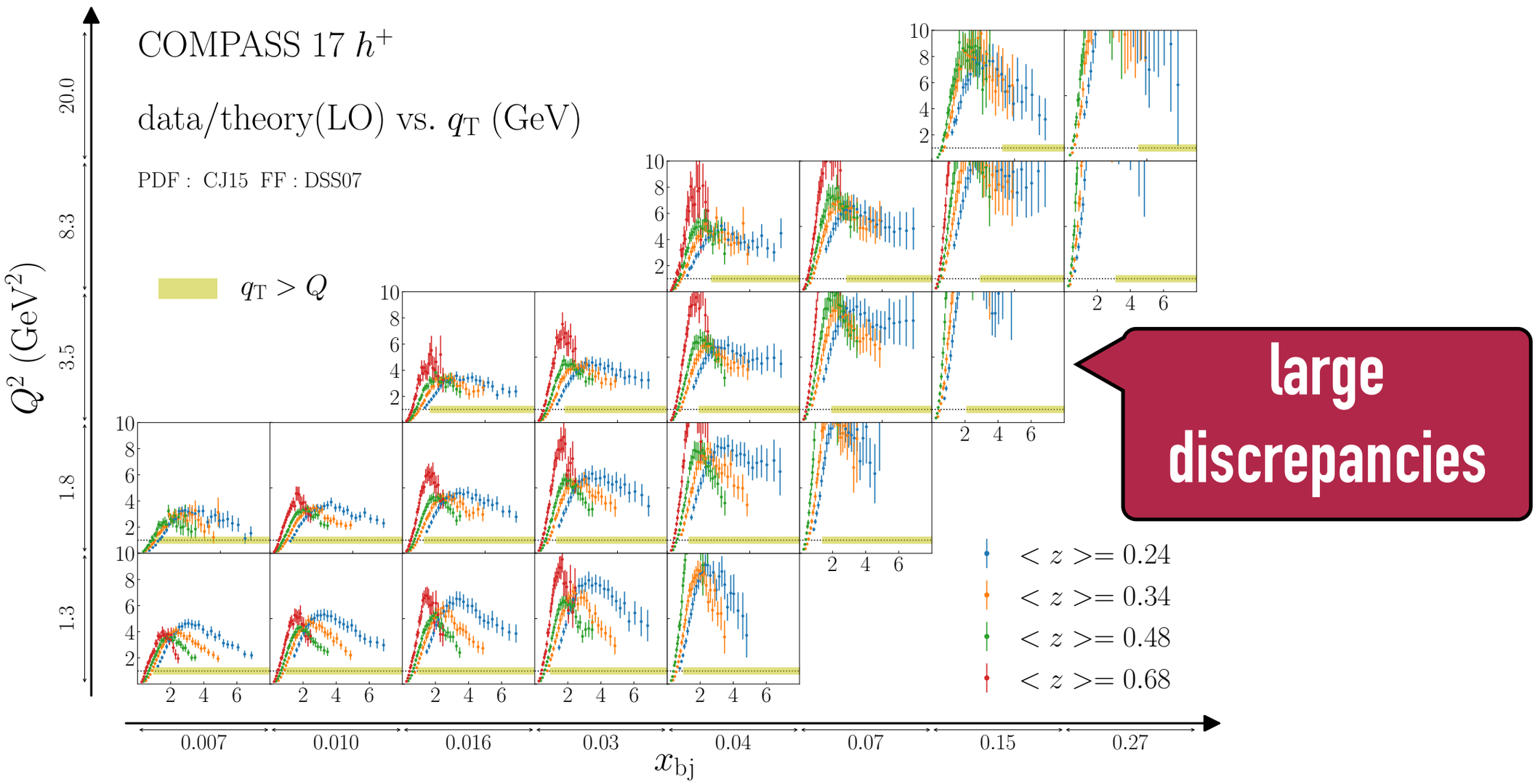

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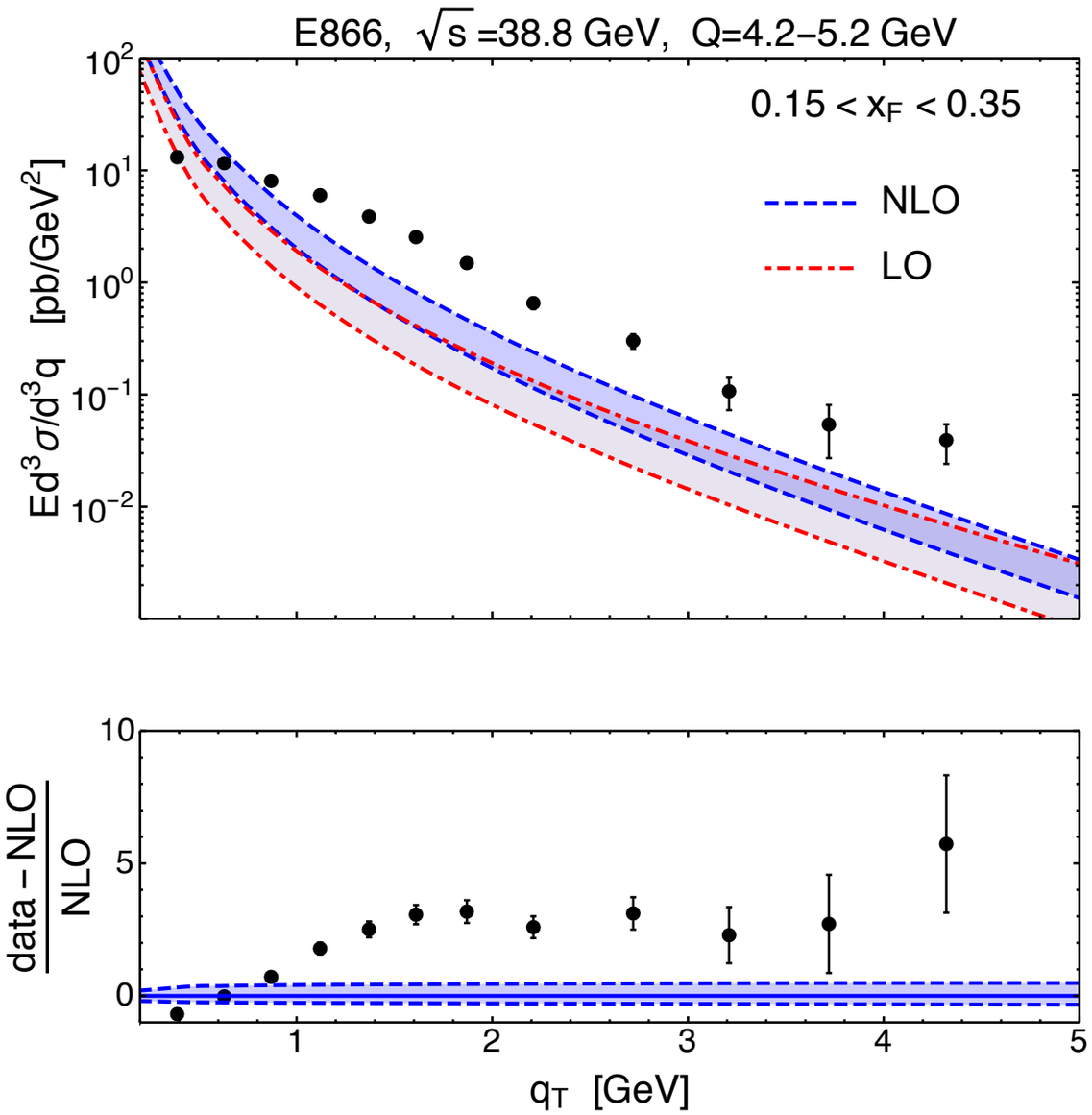
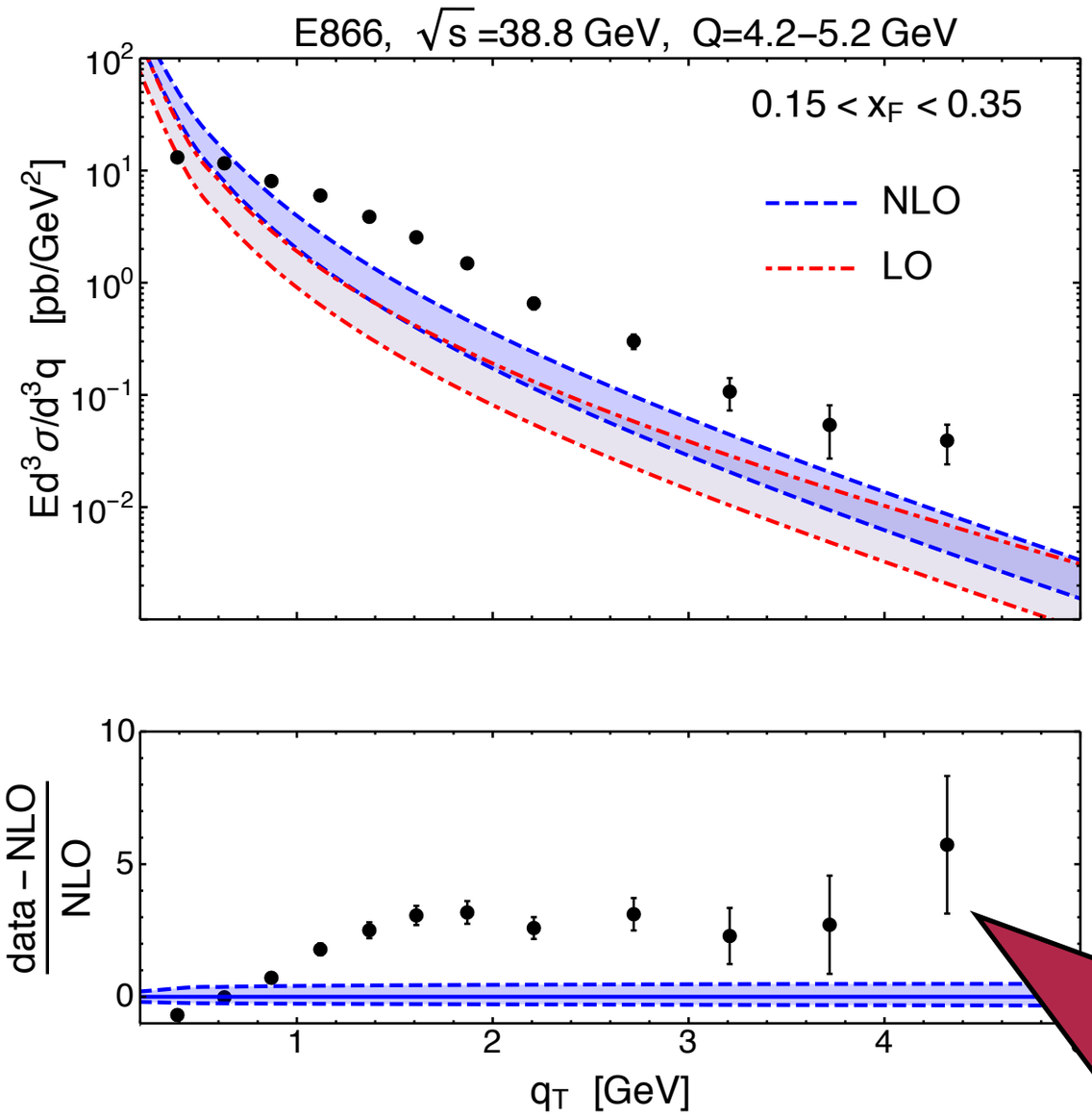
MATCHING PROBLEMS (DRELL-YAN EXAMPLE)



Bacchetta, Bozzi, Lambertsen, Piacenza, Steinglechner, Vogelsang arXiv:1901.06916

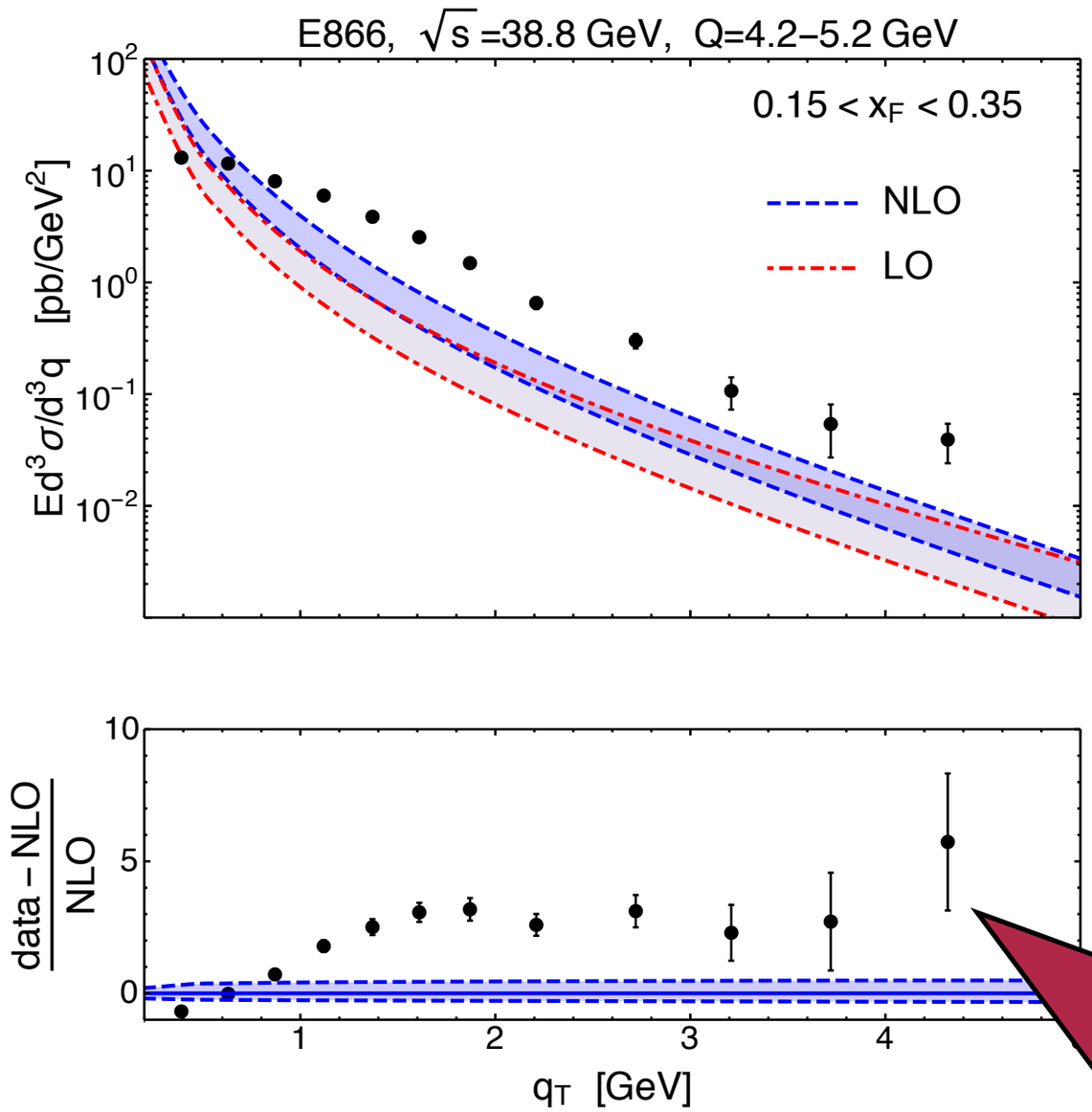
MATCHING PROBLEMS (DRELL-YAN EXAMPLE)

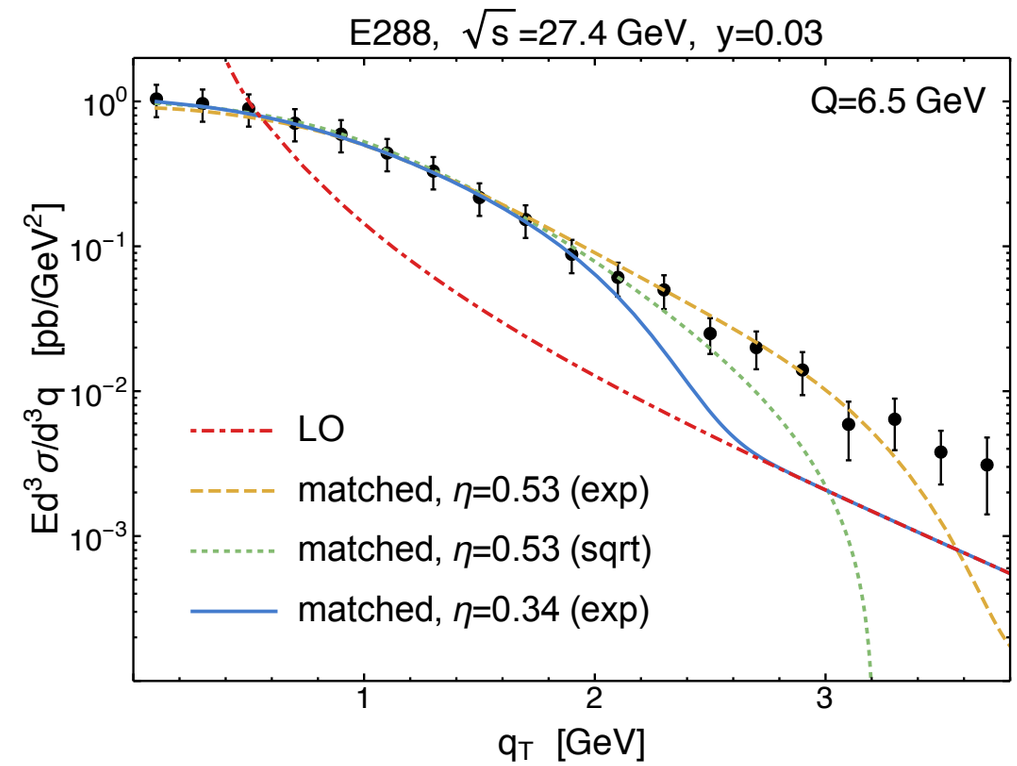


large discrepancies

Bacchetta, Bozzi, Lambertsen, Piacenza, Steinglechner, Vogelsang arXiv:1901.06916

MATCHING PROBLEMS (DRELL-YAN EXAMPLE)

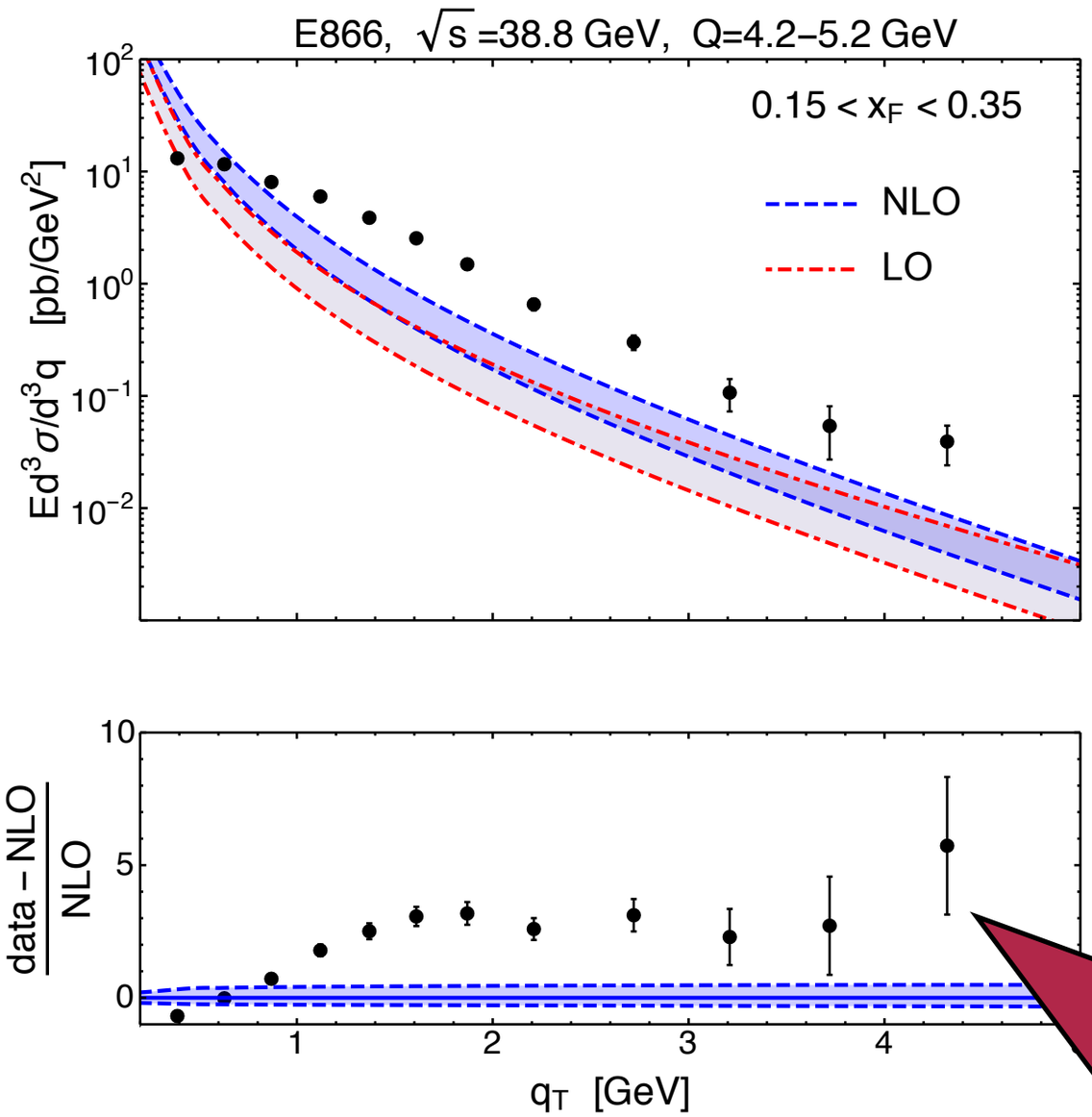


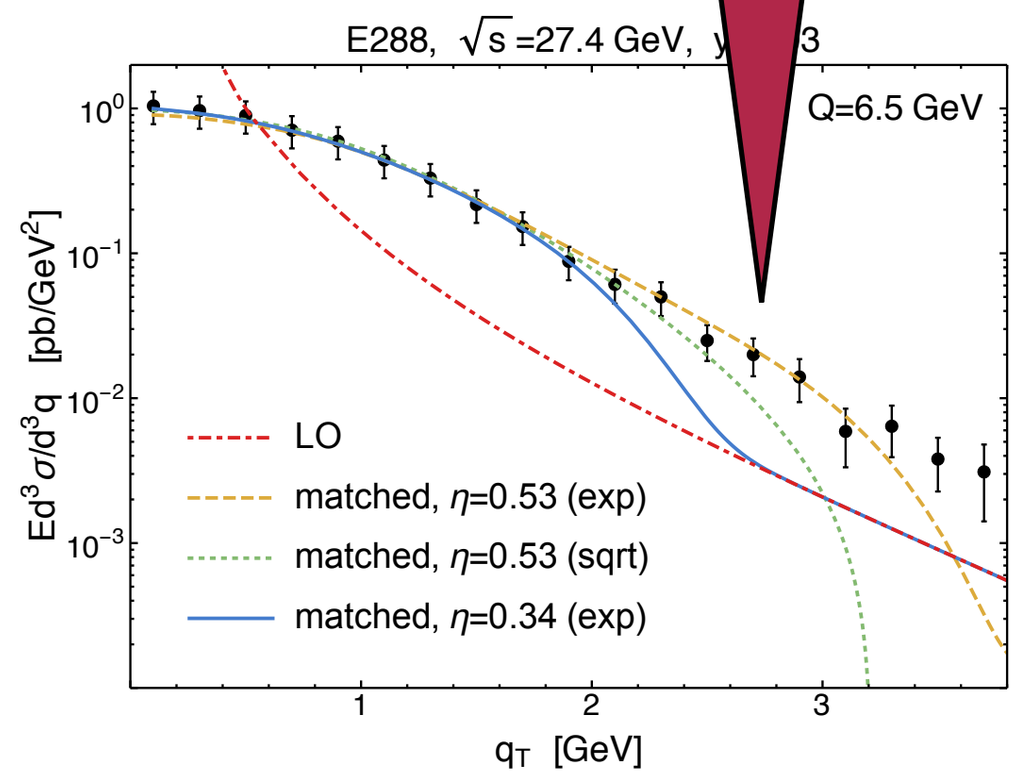


large discrepancies

Bacchetta, Bozzi, Lambertsen, Piacenza, Steinglechner, Vogelsang arXiv:1901.06916

MATCHING PROBLEMS (DRELL-YAN EXAMPLE)





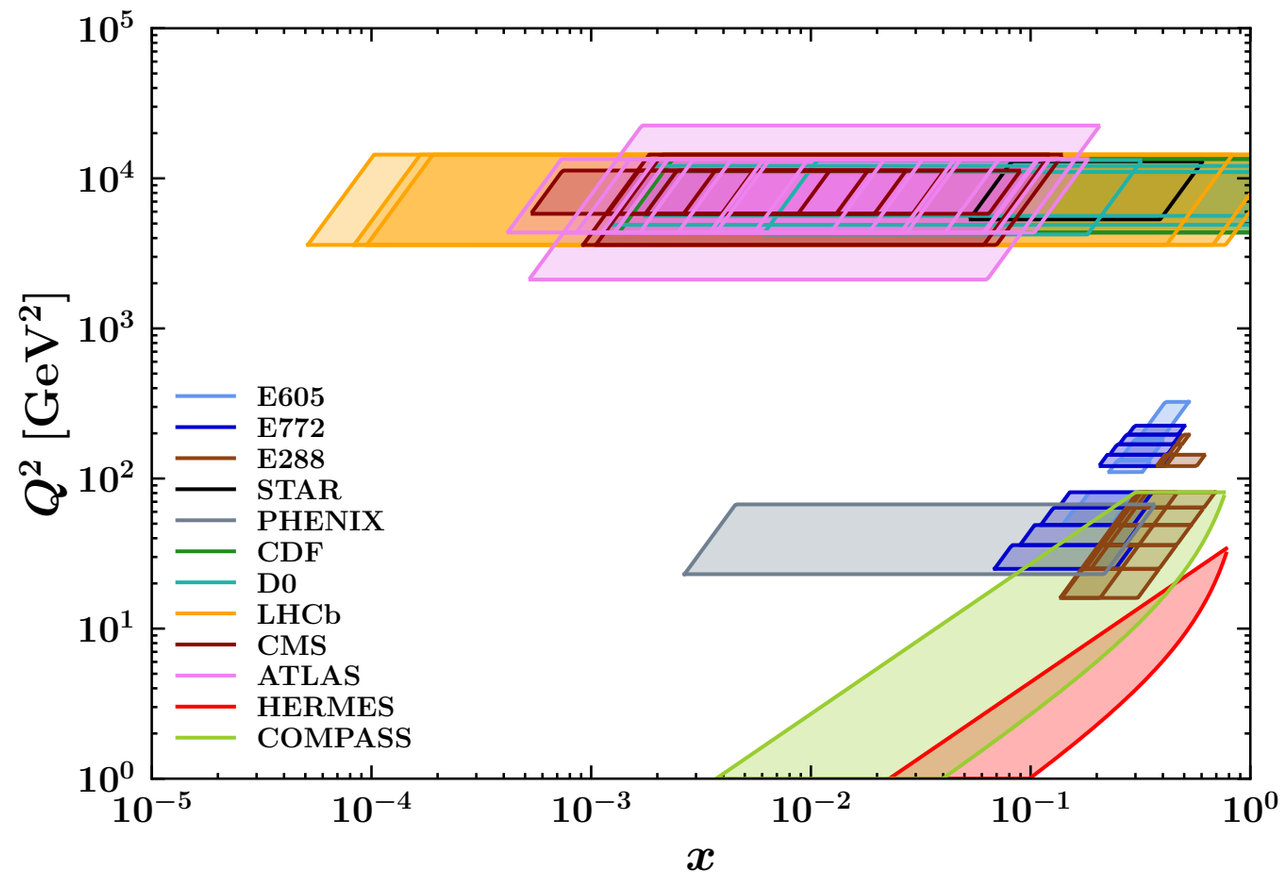
matching problem

large discrepancies

Bacchetta, Bozzi, Lambertsen, Piacenza, Steinglechner, Vogelsang arXiv:1901.06916

BEAST 2: TMD REGION

MAP22 TMD DATA SELECTION



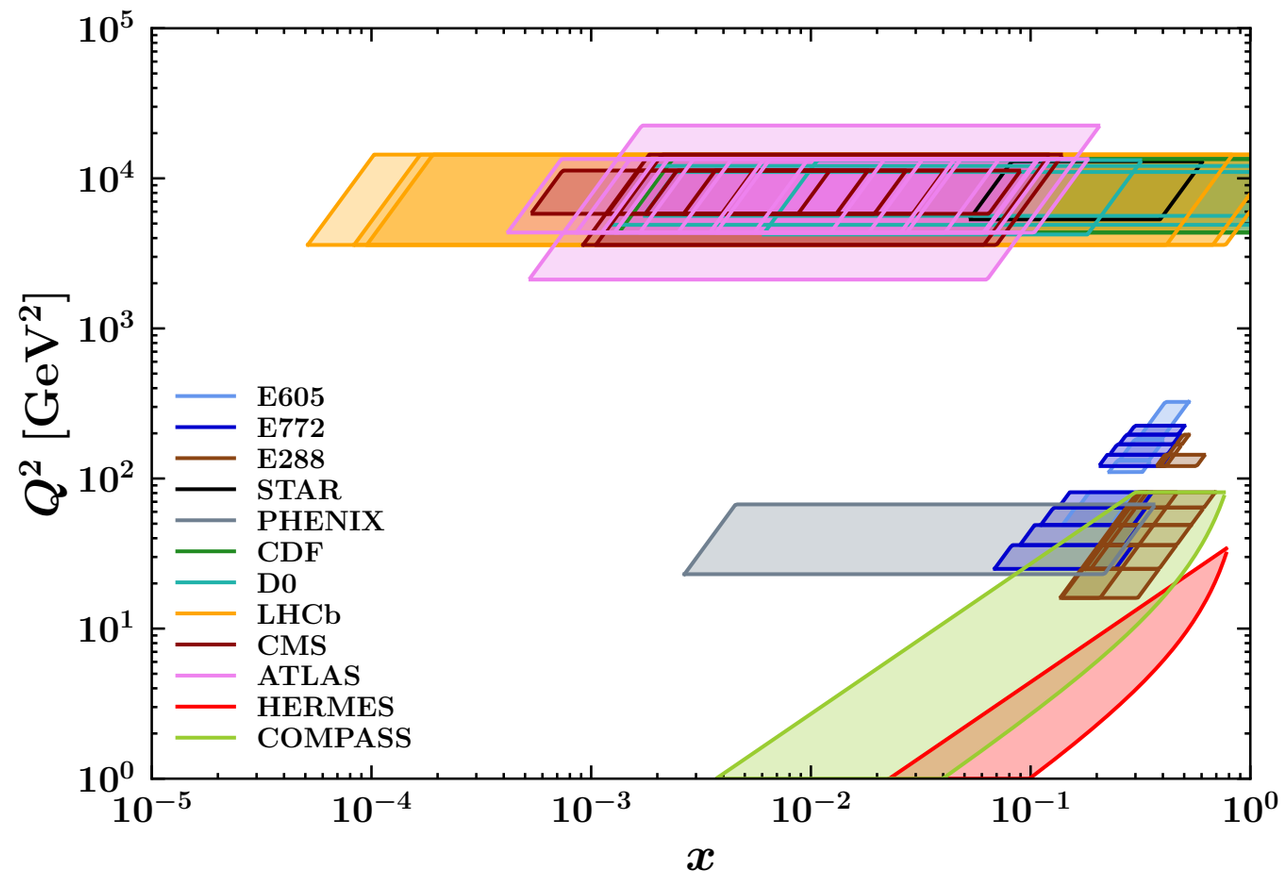
$$Q^2 > 1.4\ \mathrm{GeV}^2$$

$$0.2 < z < 0.7$$

$$q_T < 0.2\,Q \quad \text{(DY)}$$

$$P_{hT} < \min\left[\min\left[0.2\,Q, 0.5zQ\right] + 0.3\,\mathrm{GeV}, zQ\right] \quad \text{(SIDIS)}$$

MAP22 TMD DATA SELECTION



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$$q_T < 0.2\,Q \quad \text{(DY)}$$

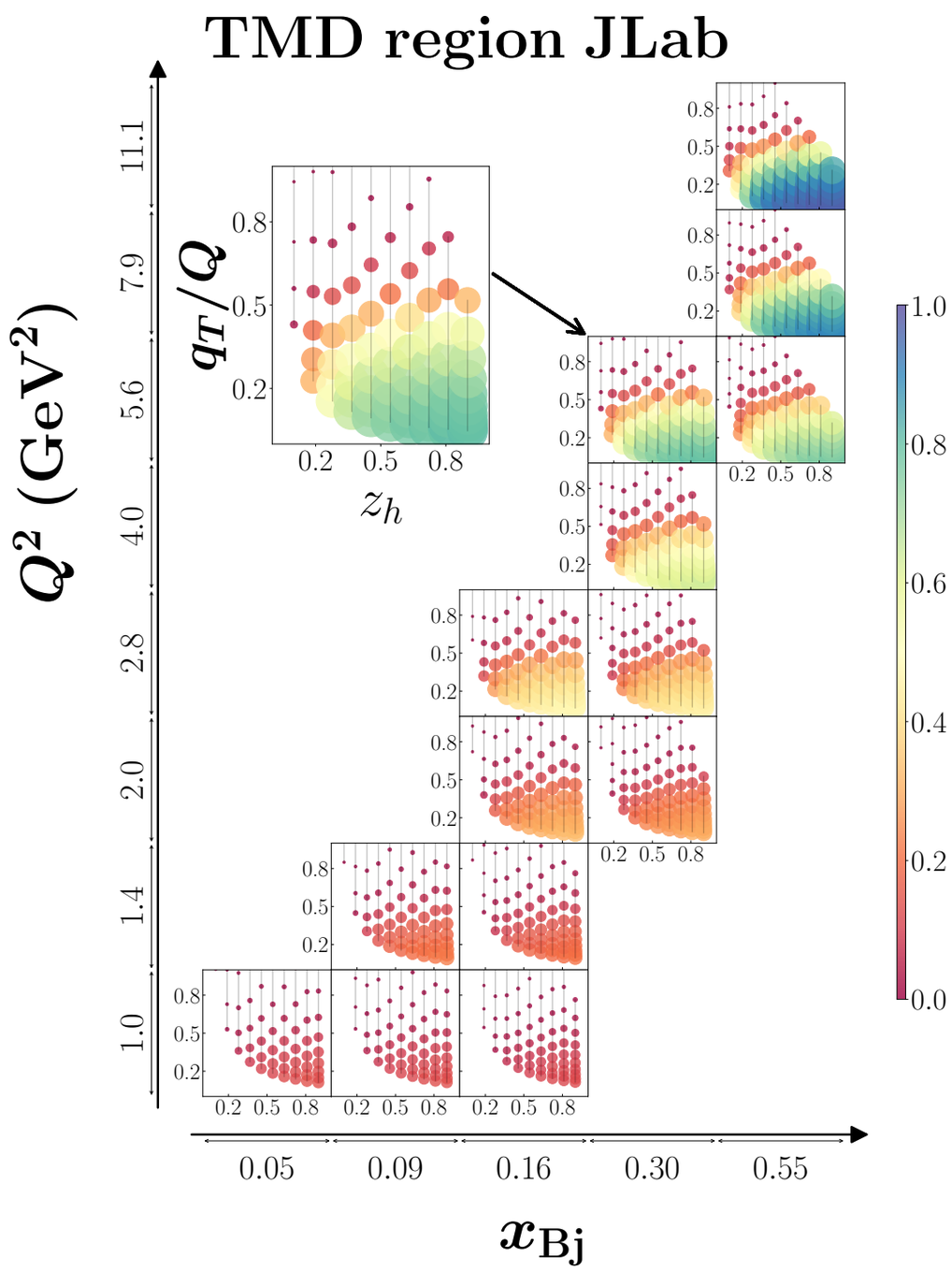
$$P_{hT} < \min\left[\min\left[0.2\,Q, 0.5zQ\right] + 0.3\,\mathrm{GeV}, zQ\right] \quad \text{(SIDIS)}$$

Number of points: 2031

REGION OF VALIDITY OF TMD FORMALISM

Boglione, Diefenthaler, Dolan, Gamberg, Melnitchouk, arXiv:2201.12197

$$|q_T| = |P_{hT}|/z \ll Q$$

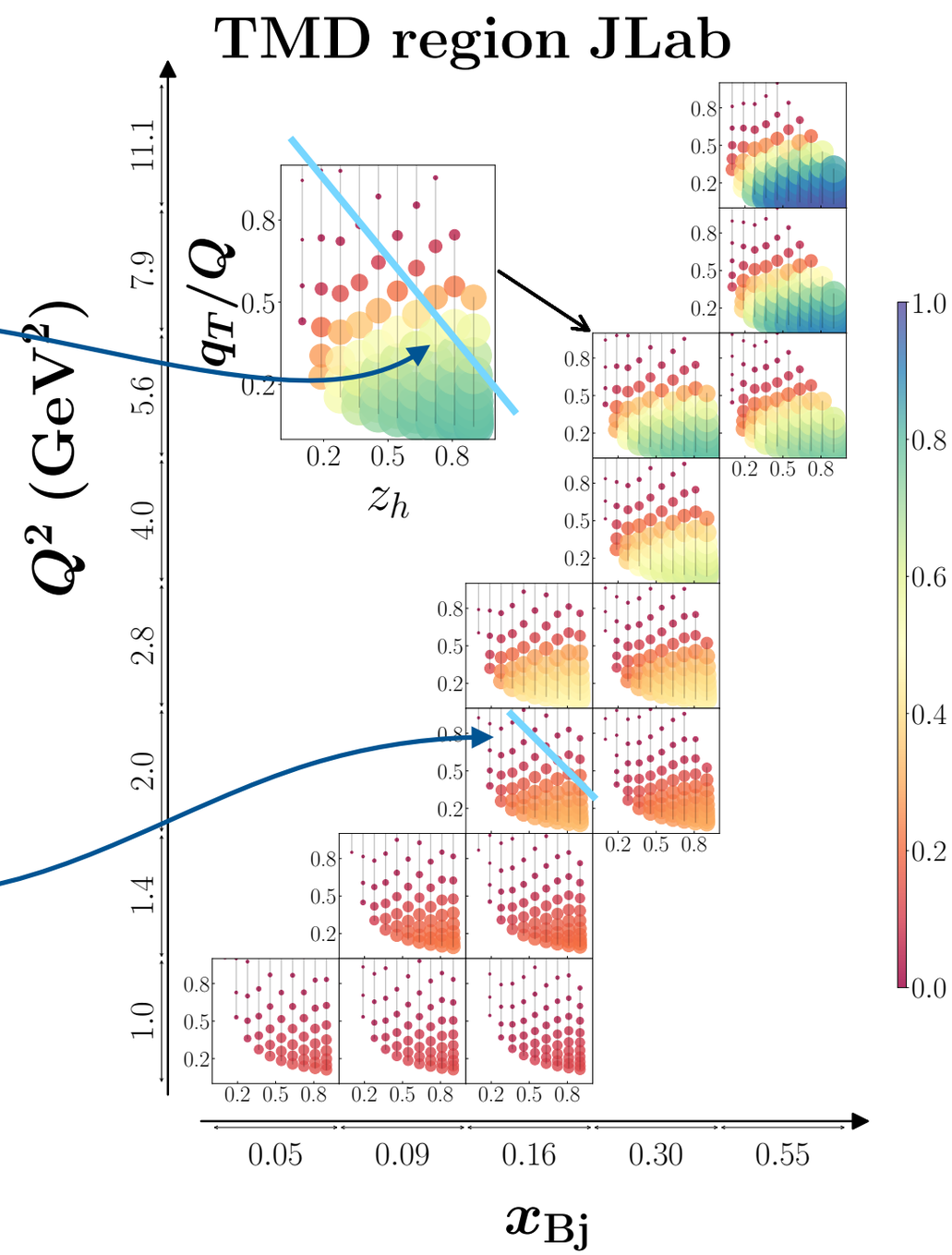


REGION OF VALIDITY OF TMD FORMALISM

Boglione, Diefenthaler, Dolan, Gamberg, Melnitchouk, arXiv:2201.12197

$$|q_T| = |P_{hT}|/z \ll Q$$

Approximate region corresponding to MAP22 cuts



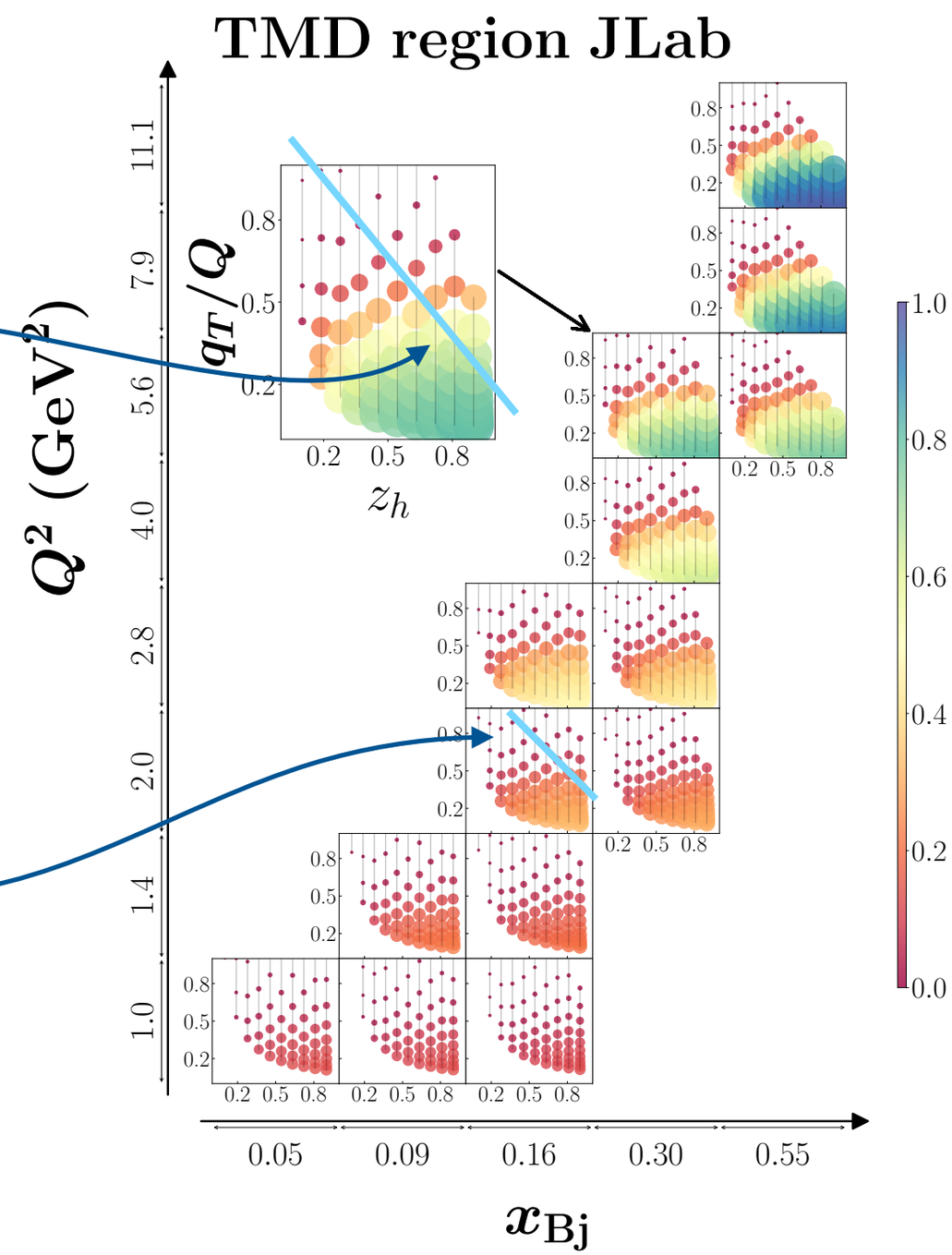
REGION OF VALIDITY OF TMD FORMALISM

Boglione, Diefenthaler, Dolan, Gamberg, Melnitchouk, arXiv:2201.12197

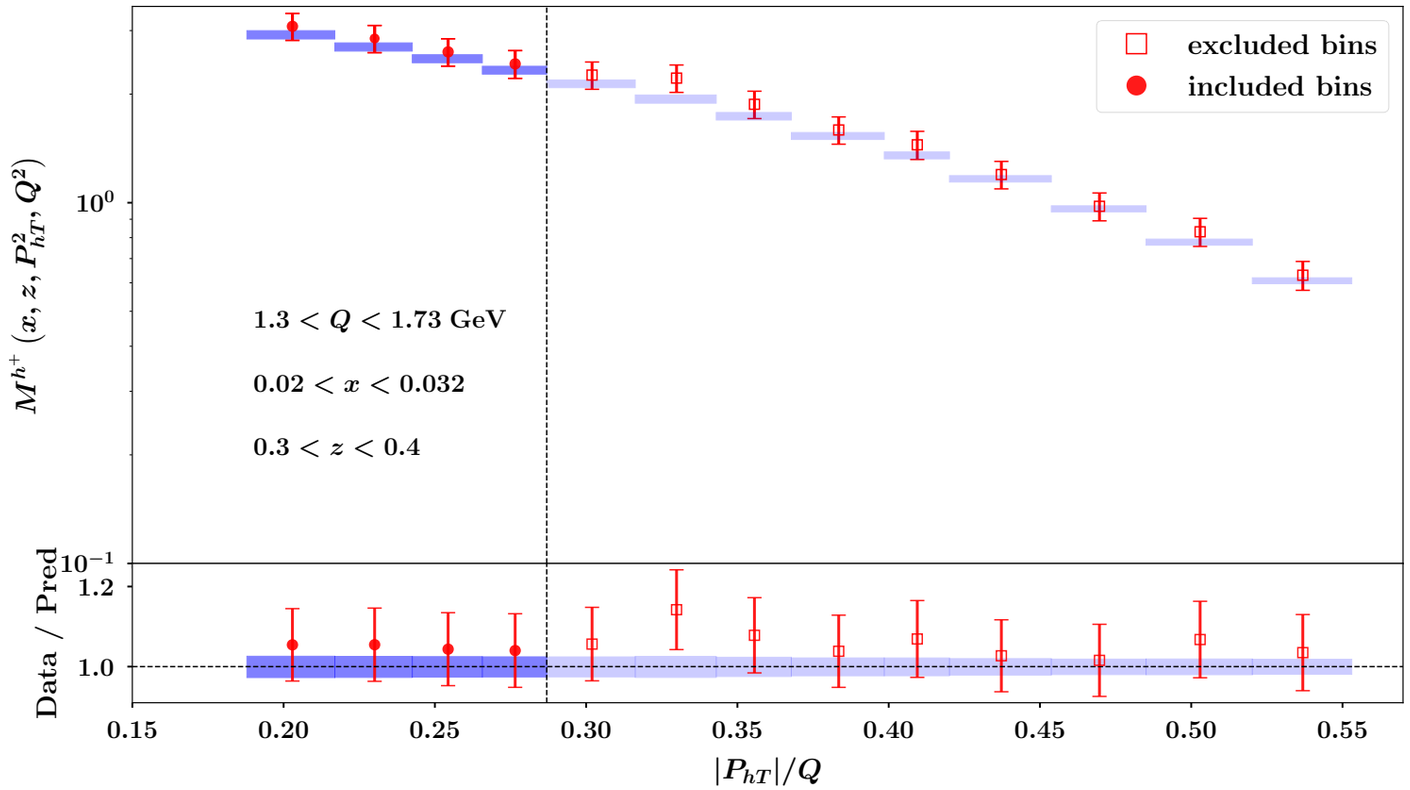
$$|q_T| = |P_{hT}|/z \ll Q$$

?

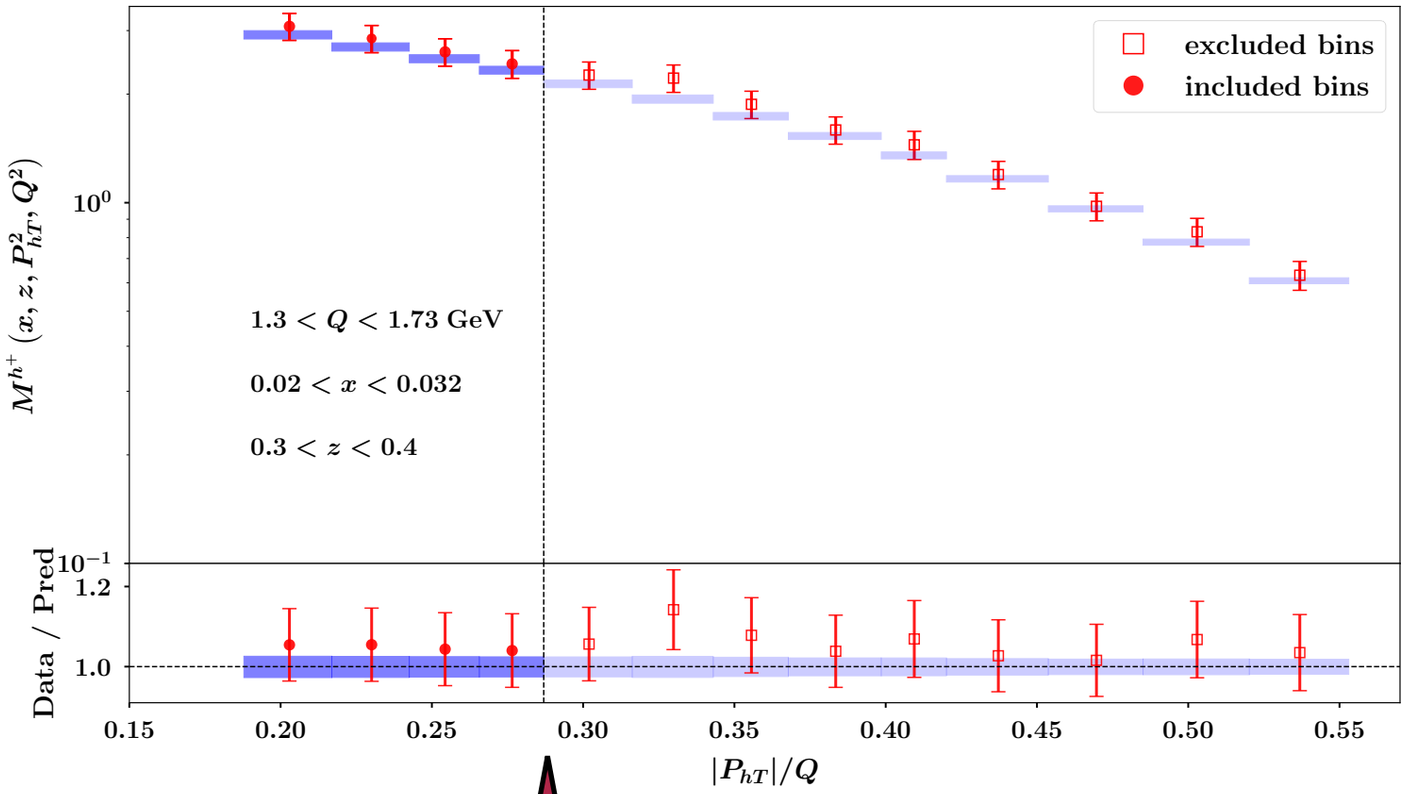
Approximate region corresponding to MAP22 cuts



REGION OF VALIDITY OF TMD FORMALISM

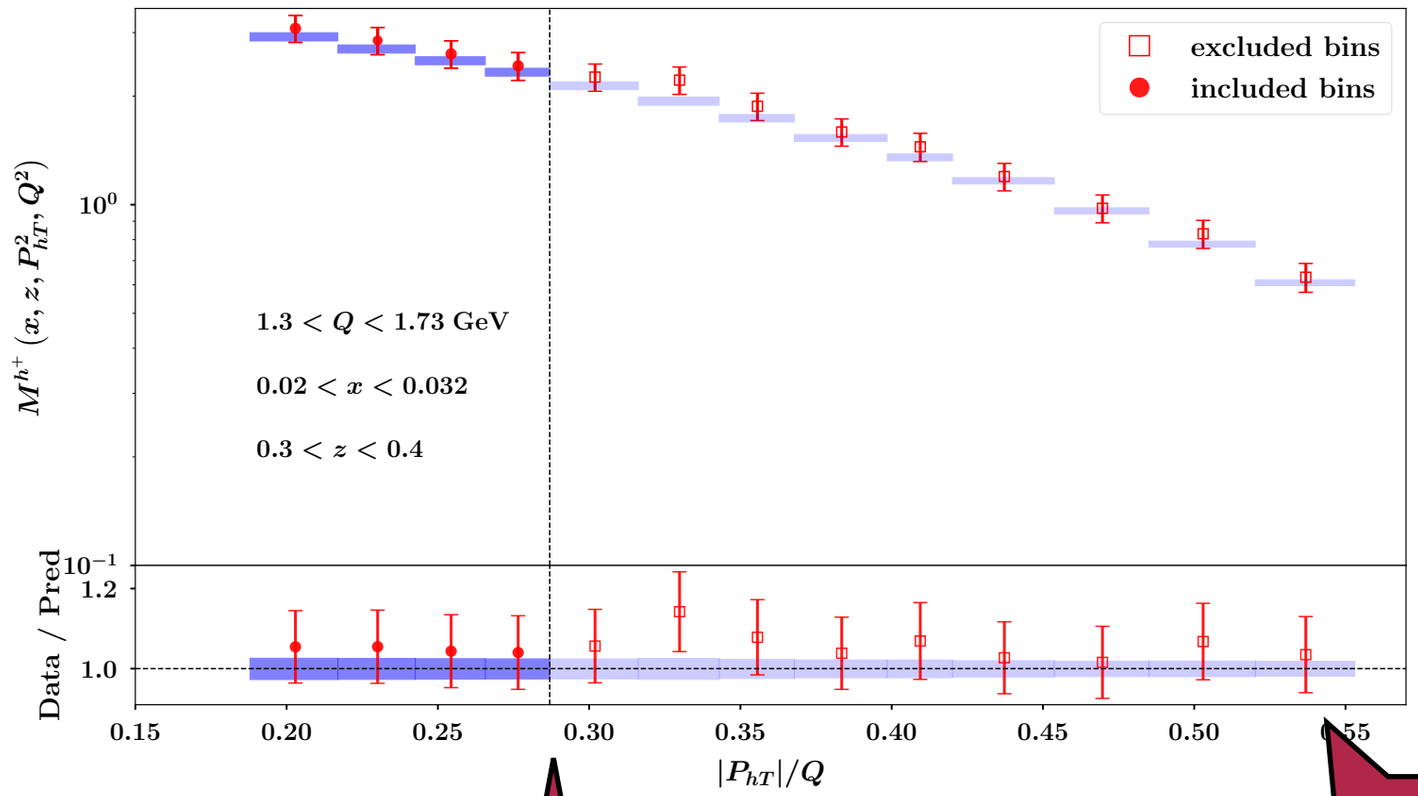


REGION OF VALIDITY OF TMD FORMALISM



MAP22 cut

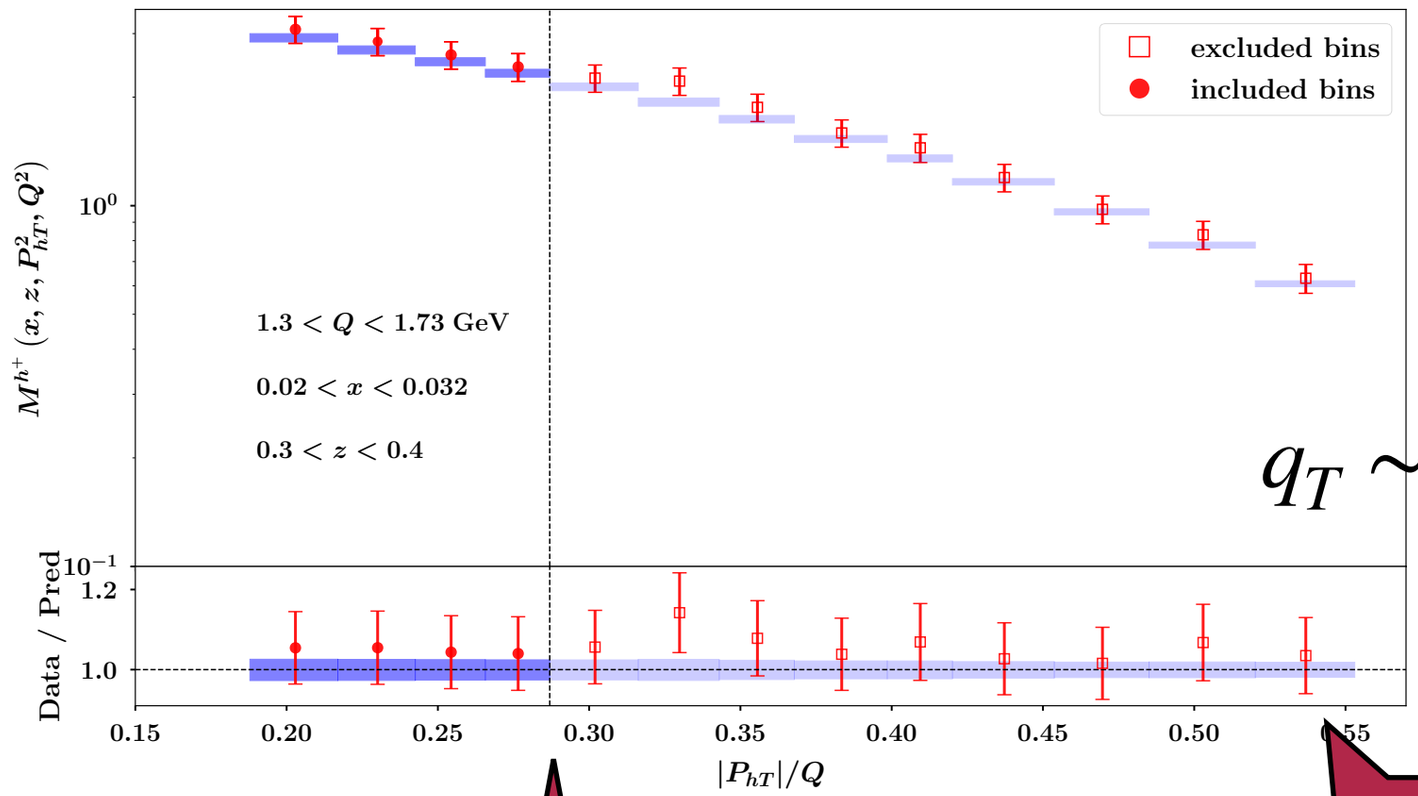
REGION OF VALIDITY OF TMD FORMALISM



MAP22 cut

MAP22 extrapolation

REGION OF VALIDITY OF TMD FORMALISM

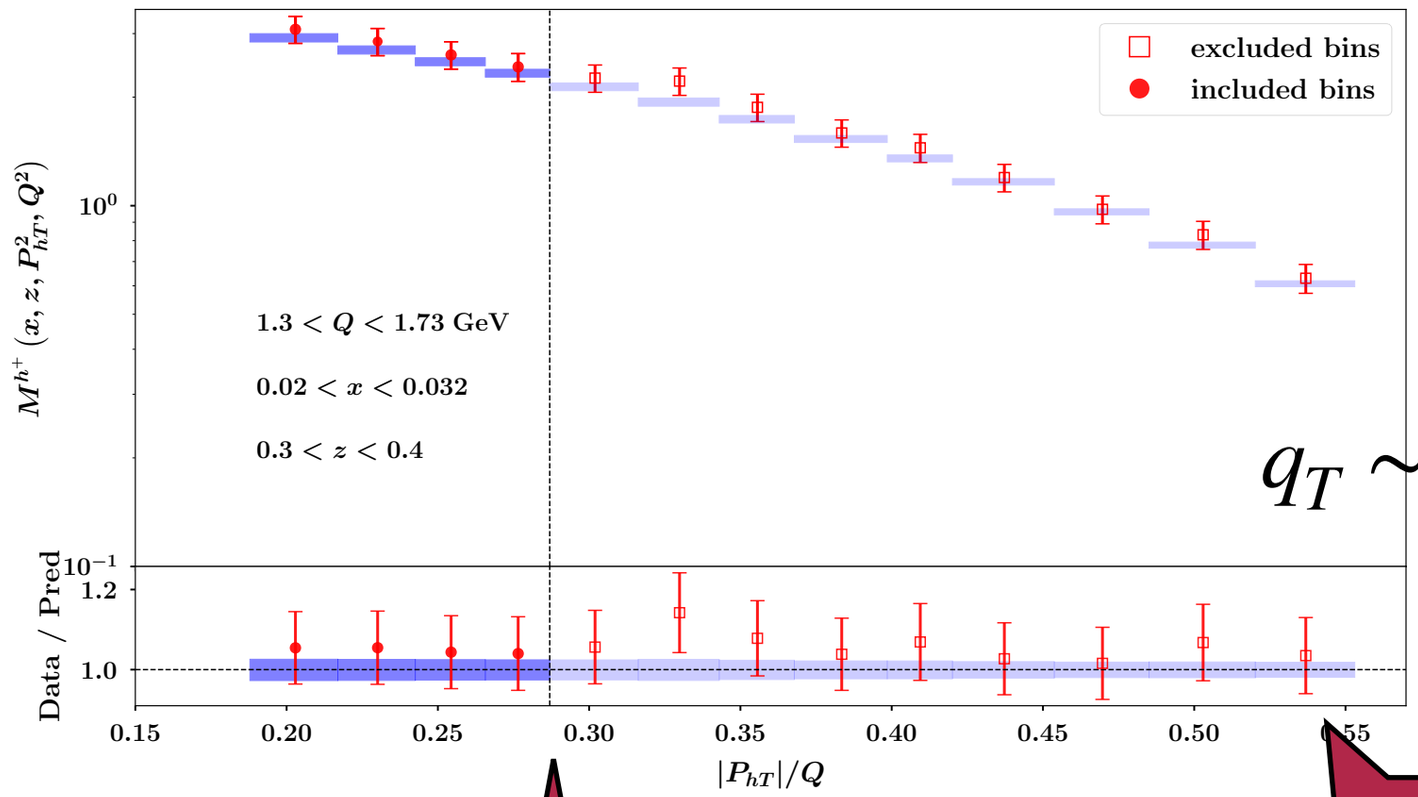


$q_T \sim 1.5\ Q$!

MAP22 cut

MAP22 extrapolation

REGION OF VALIDITY OF TMD FORMALISM



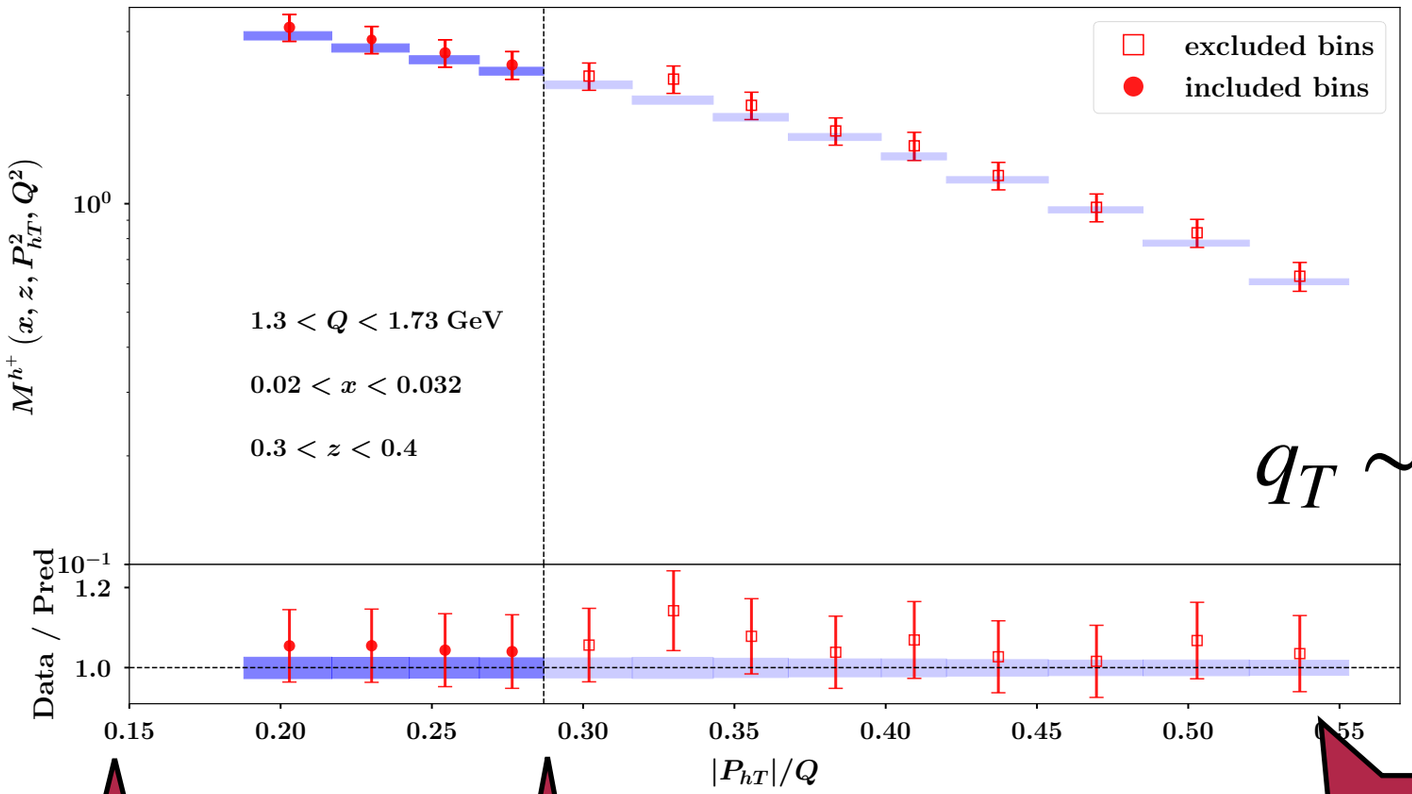
$q_T \sim 1.5\ Q$!

MAP22 cut

MAP22 extrapolation

The MAP22 cut is already considered to be "generous", but the physics seems to be the same for a much wider P_T

REGION OF VALIDITY OF TMD FORMALISM



$q_T \sim 1.5\, Q$!

$q_T < 0.2\, Q$

MAP22 cut

MAP22 extrapolation

The MAP22 cut is already considered to be "generous", but the physics seems to be the same for a much wider P_T

BEAST 3: HIGHER TWIST

TWIST-3 TMD TABLE

quark pol.

nucleon pol.	U	L	T	
U	$f^{\perp}$	$g^{\perp}$	e	h
L	$f_L^{\perp}$	$g_L^{\perp}$	h_L	e_L
T	$f_T,\ f_T^{\perp}$	$g_T,\ g_T^{\perp}$	$h_T,\ h_T^{\perp}$	$e_T,\ e_T^{\perp}$

TMDs in black survive integration over transverse momentum
TMDs in red are time-reversal odd

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)
Bacchetta, Mulders, Pijlman, hep-ph/0405154
Goeke, Metz, Schlegel, hep-ph/0504130

TWIST-3 TMD TABLE

quark pol.

nucleon pol.	U	L	T	
U	$f^{\perp}$	$g^{\perp}$	e	h
L	$f_L^{\perp}$	$g_L^{\perp}$	h_L	e_L
T	f_T, $f_T^{\perp}$	g_T, $g_T^{\perp}$	h_T, $h_T^{\perp}$	e_T, $e_T^{\perp}$

TMDs in black survive integration over transverse momentum
TMDs in red are time-reversal odd

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)
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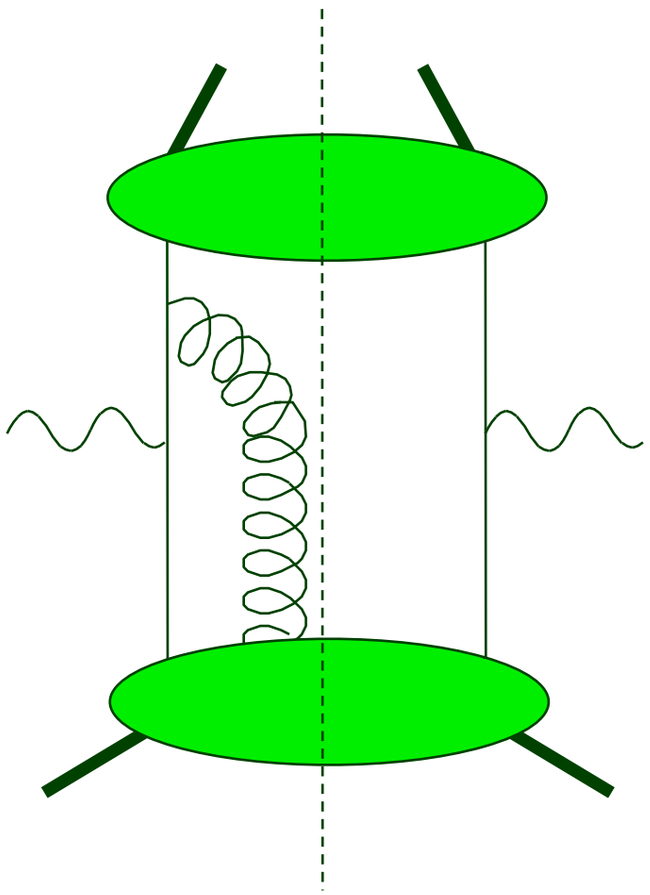
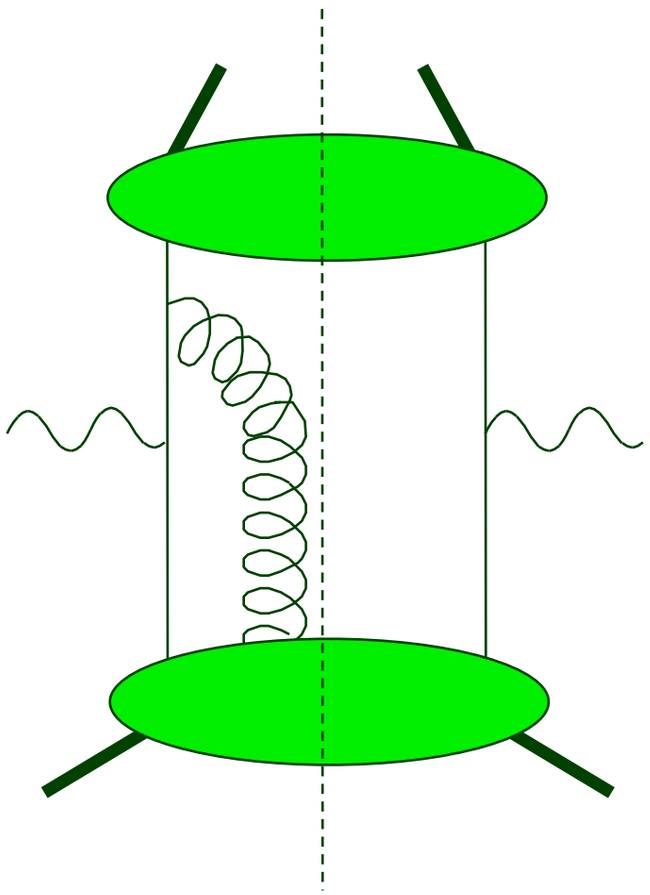
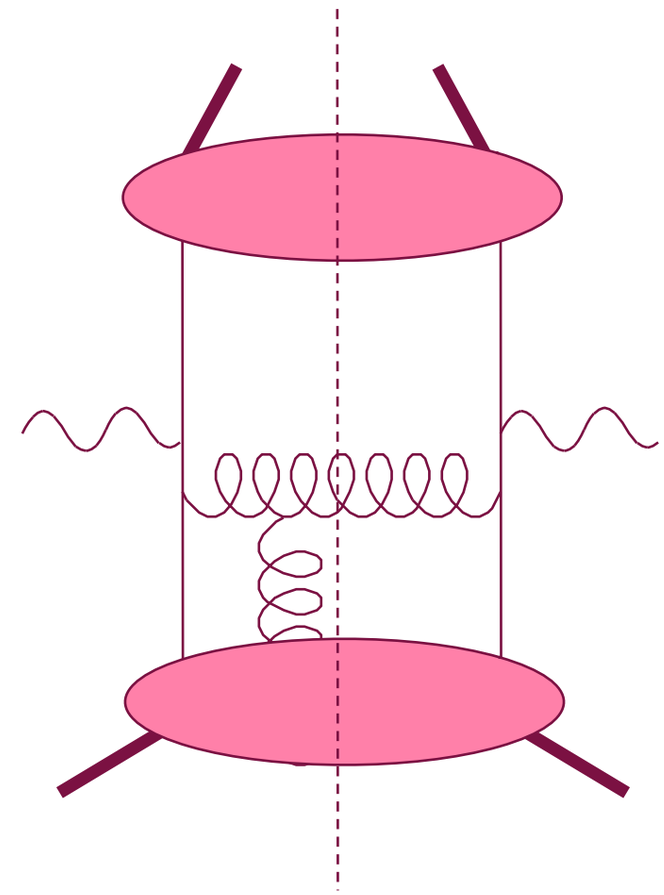
TWIST-3 CORRELATORS



TWIST-3 CORRELATORS





$$i\Phi_F^\alpha(x,x') = \int \frac{d\xi^- d\eta^-}{(2\pi)^2}\, e^{ik\cdot\xi}\, e^{i(k'-k)\cdot\eta}\, \delta_T^{\alpha\rho}$$
$$\times \langle P|\overline{\psi}(0)\, \mathcal{W}^v_{(0,\eta)}\, ig\, F^{+\alpha}(\eta)\, \mathcal{W}^v_{(\eta,\xi)}\, \psi(\xi)|P\rangle\Big|_{\substack{\xi^+=\xi_T=0\\ \eta^+=\eta_T=0}}$$

LIST OF STRUCTURE FUNCTIONS

Bacchetta, Boer, Diehl, Mulders, arXiv:0803.0227

	observable	low P_T twist	high P_T twist
"SIDIS F_T"	$F_{UU,T}$	2	2
"SIDIS F_L"	$F_{UU,L}$	4	2
"Cahn" - $f^\perp$	$F_{UU}^{\cos\phi_h}$	3	2
"Boer-Mulders"	$F_{UU}^{\cos 2\phi_h}$	2	2
e, $g^\perp$ and friends	$F_{LU}^{\sin\phi_h}$	3	2
	$F_{UL}^{\sin\phi_h}$	3	2
"Kotzinian-Mulders"	$F_{UL}^{\sin 2\phi_h}$	2	
"SIDIS g_1"	F_{LL}	2	2
	$F_{LL}^{\cos\phi_h}$	3	2
"Sivers"	$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	3
	$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4	3
"Collins"	$F_{UT}^{\sin(\phi_h+\phi_S)}$	2	3
"Pretzelosity"	$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	3
f_T and friends	$F_{UT}^{\sin\phi_S}$	3	3
	$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	3
"Worm gear"	$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	3
"SIDIS g_2" - g_T	$F_{LT}^{\cos\phi_S}$	3	
	$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	

LIST OF STRUCTURE FUNCTIONS

Bacchetta, Boer, Diehl, Mulders, arXiv:0803.0227

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"Pretzelosity"	$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	3
f_T and friends	$F_{UT}^{\sin\phi_S}$	3	3
	$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	3
"Worm gear"	$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	3
"SIDIS g_2" - g_T	$F_{LT}^{\cos\phi_S}$	3	
	$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	

Not all of them are easy to access at EIC due to: x-range, twist, evolution, prefactors

LIST OF STRUCTURE FUNCTIONS

	observable	low P_T twist	high P_T twist
"SIDIS F_T"	$F_{UU,T}$	2	2
"SIDIS F_L"	$F_{UU,L}$	4	2
"Cahn" - $f^{\perp}$	$F_{UU}^{\cos\phi_h}$	3	2
"Boer-Mulders"	$F_{UU}^{\cos 2\phi_h}$	2	2
e, $g^{\perp}$ and friends	$F_{LU}^{\sin\phi_h}$	3	2
	$F_{UL}^{\sin\phi_h}$	3	2
"Kotzinian-Mulders"	$F_{UL}^{\sin 2\phi_h}$	2	
"SIDIS g_1"	F_{LL}	2	2
	$F_{LL}^{\cos\phi_h}$	3	2
"Sivers"	$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	3
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"Pretzelosity"	$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	3
f_T and friends	$F_{UT}^{\sin\phi_S}$	3	3
	$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	3
"Worm gear"	$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	3
"SIDIS g_2" - g_T	$F_{LT}^{\cos\phi_S}$	3	
	$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	

LIST OF STRUCTURE FUNCTIONS

	observable	low P_T twist	high P_T twist
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"Boer-Mulders"	$F_{UU}^{\cos 2\phi_h}$	2	2
e, $g^{\perp}$ and friends	$F_{LU}^{\sin\phi_h}$	3	2
	$F_{UL}^{\sin\phi_h}$	3	2
"Kotzinian-Mulders"	$F_{UL}^{\sin 2\phi_h}$	2	
"SIDIS g_1"	F_{LL}	2	2
	$F_{LL}^{\cos\phi_h}$	3	2
"Sivers"	$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	3
	$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4	3
"Collins"	$F_{UT}^{\sin(\phi_h+\phi_S)}$	2	3
"Pretzelosity"	$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	3
f_T and friends	$F_{UT}^{\sin\phi_S}$	3	3
	$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	3
"Worm gear"	$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	3
"SIDIS g_2" - g_T	$F_{LT}^{\cos\phi_S}$	3	
	$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	

There are at least four possibilities:

LIST OF STRUCTURE FUNCTIONS

	observable	low P_T twist	high P_T twist
"SIDIS F_T"	$F_{UU,T}$	2	2
"SIDIS F_L"	$F_{UU,L}$	4	2
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"Boer-Mulders"	$F_{UU}^{\cos 2\phi_h}$	2	2
e, $g^\perp$ and friends	$F_{LU}^{\sin\phi_h}$	3	2
	$F_{UL}^{\sin\phi_h}$	3	2
"Kotzinian-Mulders"	$F_{UL}^{\sin 2\phi_h}$	2	
"SIDIS g_1"	F_{LL}	2	2
	$F_{LL}^{\cos\phi_h}$	3	2
"Sivers"	$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	3
	$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4	3
"Collins"	$F_{UT}^{\sin(\phi_h+\phi_S)}$	2	3
"Pretzelosity"	$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	3
f_T and friends	$F_{UT}^{\sin\phi_S}$	3	3
	$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	3
"Worm gear"	$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	3
"SIDIS g_2" - g_T	$F_{LT}^{\cos\phi_S}$	3	
	$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	

There are at least four possibilities:

- Twist 2 TMD matching twist 2 PDF

LIST OF STRUCTURE FUNCTIONS

	observable	low P_T twist	high P_T twist
"SIDIS F_T"	$F_{UU,T}$	2	2
"SIDIS F_L"	$F_{UU,L}$	4	2
"Cahn" - $f^\perp$	$F_{UU}^{\cos\phi_h}$	3	2
"Boer-Mulders"	$F_{UU}^{\cos 2\phi_h}$	2	2
e, $g^\perp$ and friends	$F_{LU}^{\sin\phi_h}$	3	2
	$F_{UL}^{\sin\phi_h}$	3	2
"Kotzinian-Mulders"	$F_{UL}^{\sin 2\phi_h}$	2	
"SIDIS g_1"	F_{LL}	2	2
	$F_{LL}^{\cos\phi_h}$	3	2
"Sivers"	$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	3
	$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4	3
"Collins"	$F_{UT}^{\sin(\phi_h+\phi_S)}$	2	3
"Pretzelosity"	$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	3
f_T and friends	$F_{UT}^{\sin\phi_S}$	3	3
	$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	3
"Worm gear"	$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	3
"SIDIS g_2" - g_T	$F_{LT}^{\cos\phi_S}$	3	
	$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	

There are at least four possibilities:

- Twist 2 TMD matching twist 2 PDF
- Twist 3 TMD matching twist 2 PDF

LIST OF STRUCTURE FUNCTIONS

	observable	low P_T twist	high P_T twist
"SIDIS F_T"	$F_{UU,T}$	2	2
"SIDIS F_L"	$F_{UU,L}$	4	2
"Cahn" - $f^\perp$	$F_{UU}^{\cos\phi_h}$	3	2
"Boer-Mulders"	$F_{UU}^{\cos 2\phi_h}$	2	2
e, $g^\perp$ and friends	$F_{LU}^{\sin\phi_h}$	3	2
	$F_{UL}^{\sin\phi_h}$	3	2
"Kotzinian-Mulders"	$F_{UL}^{\sin 2\phi_h}$	2	
"SIDIS g_1"	F_{LL}	2	2
	$F_{LL}^{\cos\phi_h}$	3	2
"Sivers"	$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	3
	$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4	3
"Collins"	$F_{UT}^{\sin(\phi_h+\phi_S)}$	2	3
"Pretzelosity"	$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	3
f_T and friends	$F_{UT}^{\sin\phi_S}$	3	3
	$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	3
"Worm gear"	$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	3
"SIDIS g_2" - g_T	$F_{LT}^{\cos\phi_S}$	3	
	$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	

There are at least four possibilities:

- Twist 2 TMD matching twist 2 PDF
- Twist 3 TMD matching twist 2 PDF
- Twist 2 TMD matching twist 3 PDF

LIST OF STRUCTURE FUNCTIONS

	observable	low P_T twist	high P_T twist
"SIDIS F_T"	$F_{UU,T}$	2	2
"SIDIS F_L"	$F_{UU,L}$	4	2
"Cahn" - $f^\perp$	$F_{UU}^{\cos\phi_h}$	3	2
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"Kotzinian-Mulders"	$F_{UL}^{\sin 2\phi_h}$	2	
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	$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4	3
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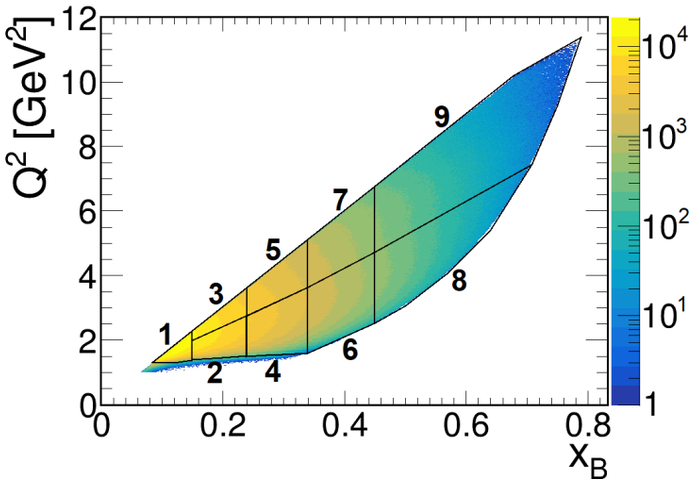
"Pretzelosity"	$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2	3
f_T and friends	$F_{UT}^{\sin\phi_S}$	3	3
	$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	3
"Worm gear"	$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	3
"SIDIS g_2" - g_T	$F_{LT}^{\cos\phi_S}$	3	
	$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	

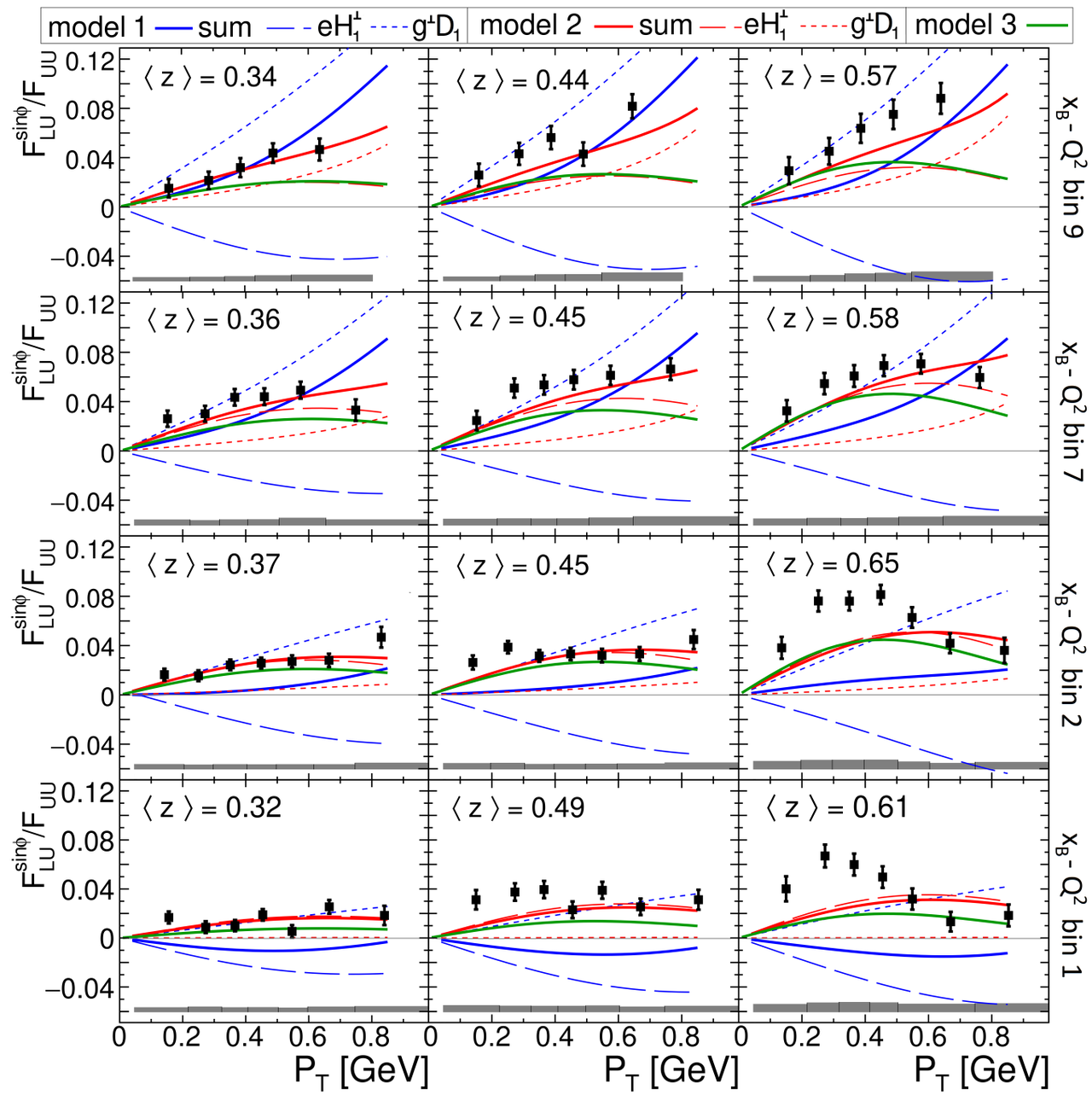
There are at least four possibilities:

- Twist 2 TMD matching twist 2 PDF
- Twist 3 TMD matching twist 2 PDF
- Twist 2 TMD matching twist 3 PDF
- Expected mismatch

BEAM-SPIN ASYMMETRY AT CLAS12

arXiv:2101.03544





THE SIVERS CASE

$$\hat{f}_1^a(x, b_T^2; Q, Q) = [C \otimes f_1](x, \mu_{b_*})\ S_{\rm pert}\ \ f_{1\,NP}(x, b_T^2; Q, Q_0)\,,$$

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Twist-3 collinear PDF: its evolution is not known exactly

BEAM-SPIN ASYMMETRY

start being known

$$F_{LU}^{\sin\phi} = \frac{2M}{Q}\,\mathcal{C}\Bigg[-\frac{\hat{\mathbf{h}}\cdot\mathbf{k}_T}{M_h}\left(x_B e H_1^{\perp} + \frac{M_h}{M} f_1 \frac{\tilde{G}^{\perp}}{z}\right) + \frac{\hat{\mathbf{h}}\cdot\mathbf{p}_T}{M}\left(x_B g^{\perp} D_1 + \frac{M_h}{M} h_1^{\perp}\frac{\tilde{E}}{z}\right)\Bigg],$$

accessible from other observables
and partially known

Bacchetta, Bozzi, Echevarria, Pisano, Prokudin, arXiv:1906.07037
Vladimirov, Moos, arXiv:21109.09771
Ebert, Gao, Stewart, arXiv:2112.07680

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**TMD interpretation at higher twist:
though theory work ongoing**

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$$W_{LU}^{\sin\phi_h} = \tilde{\mathcal{F}}\left\{\mathcal{H}^{(1)}\left[\frac{2xM_N}{Q}\left(\frac{k_{Tx}}{M_N}\tilde{g}^{\perp}D_1 - \frac{p_{Tx}}{M_h}\tilde{e}\,H_1^{\perp}\right) - \frac{2M_h}{zQ}\left(\frac{p_{Tx}}{M_h} f_1 \tilde{G}^{\perp} - \frac{k_{Tx}}{M_N} h_1^{\perp}\tilde{E}\right)\right]\right\}.$$

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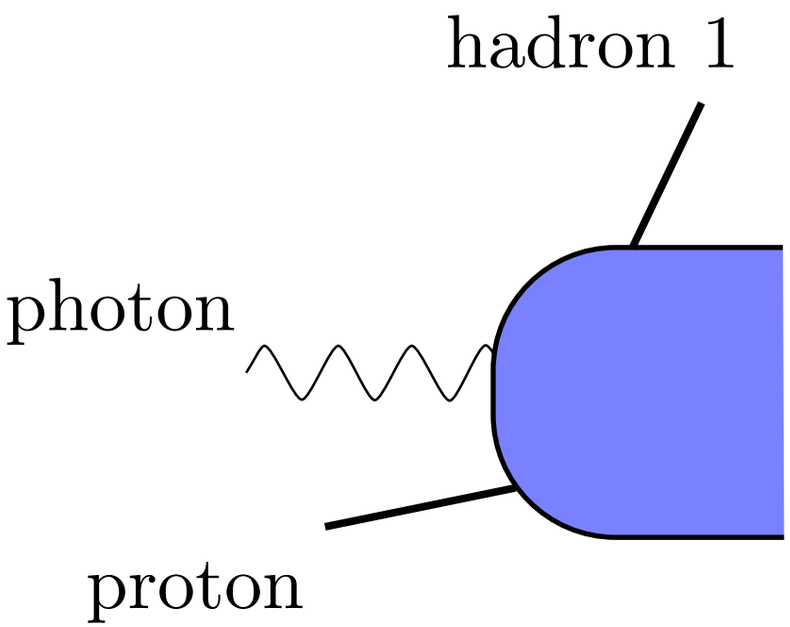
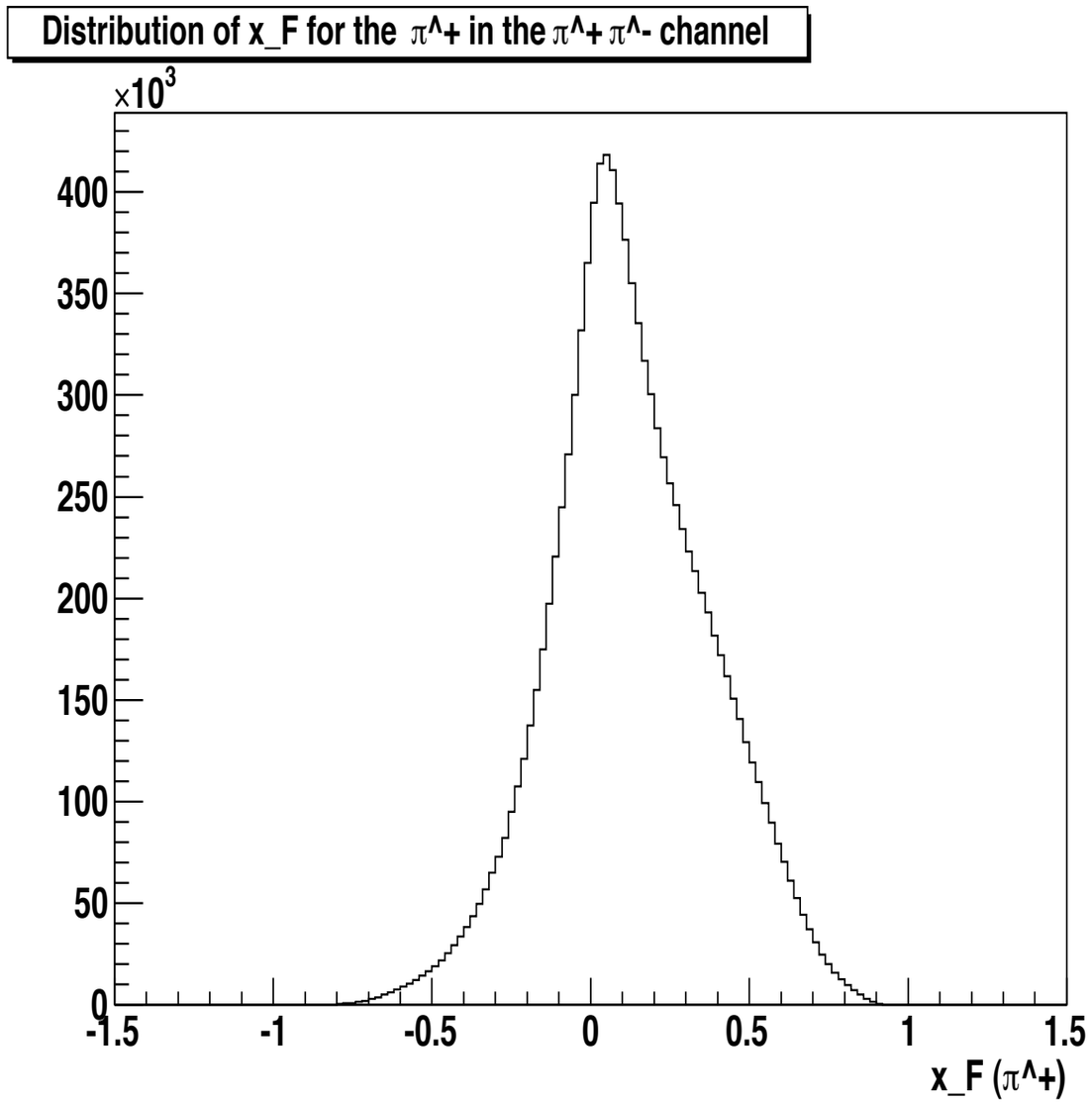
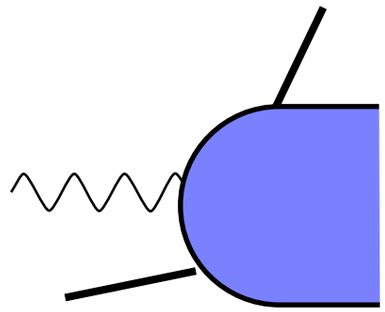
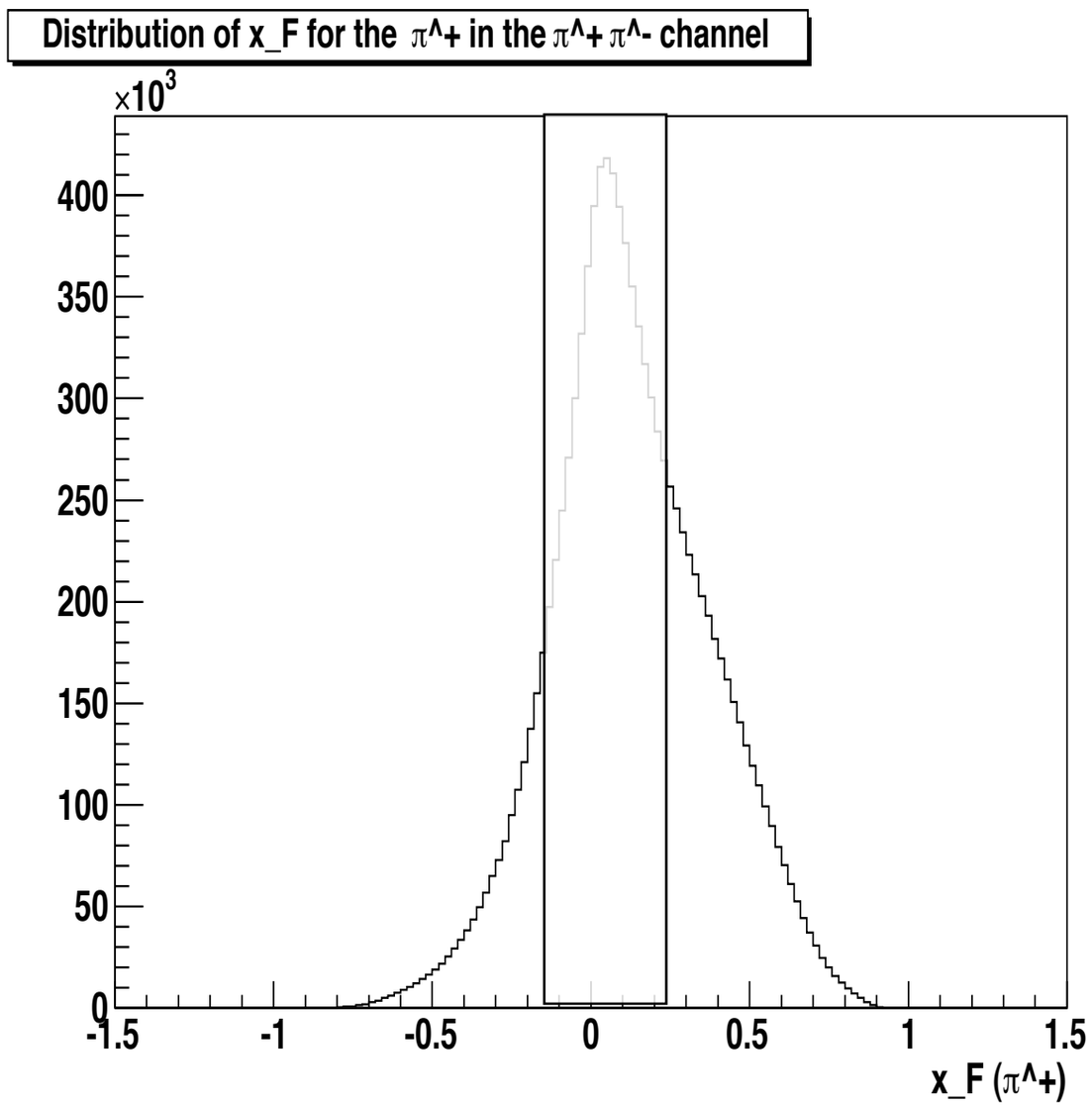
BEAST 4: TARGET FRAGMENTATION


hadron 1
photon
proton


Distribution of x_F for the π^+ in the π^+ π^- channel
×10^3
400
350
300
250
200
150
100
50
0
-1.5
-1
-0.5
0
0.5
1
1.5
x_F (π^+)

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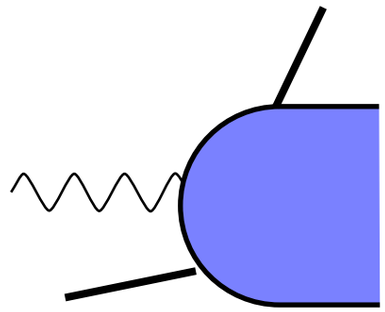
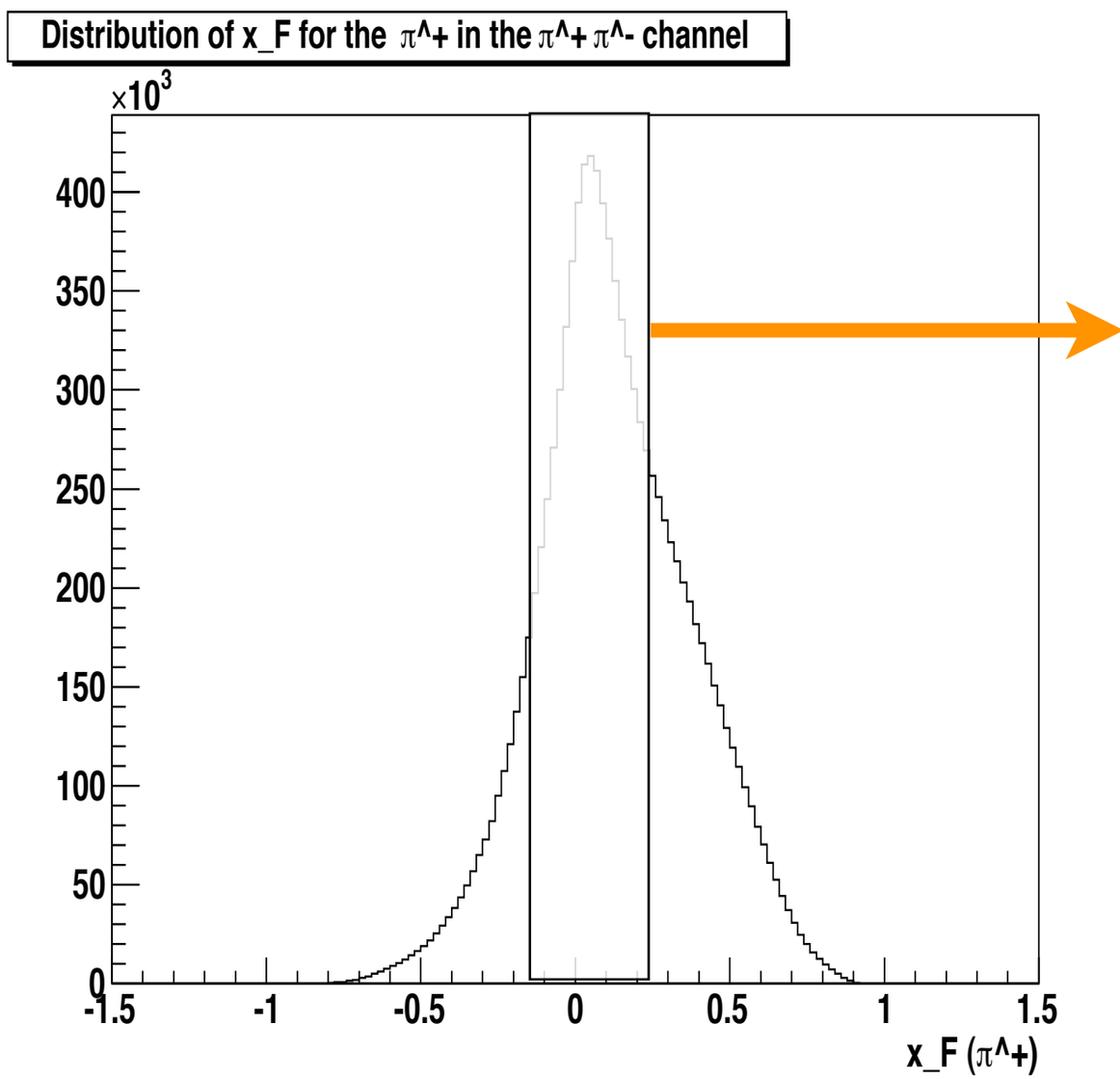
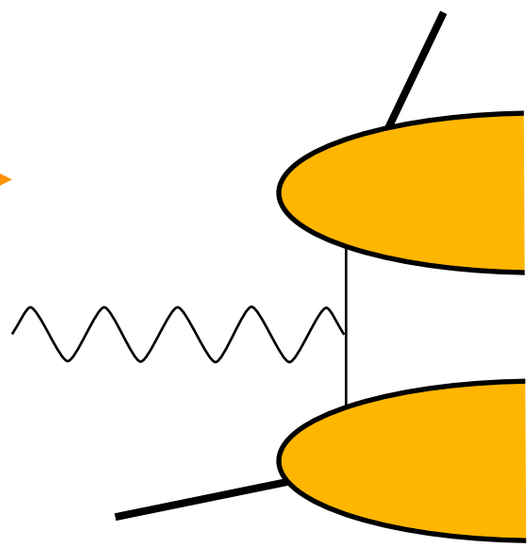
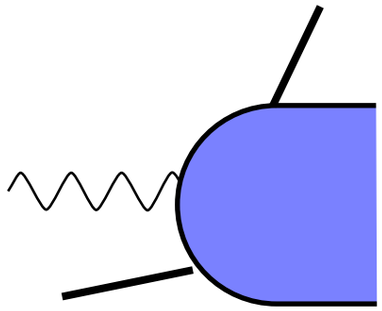




fragmentation function



distribution function



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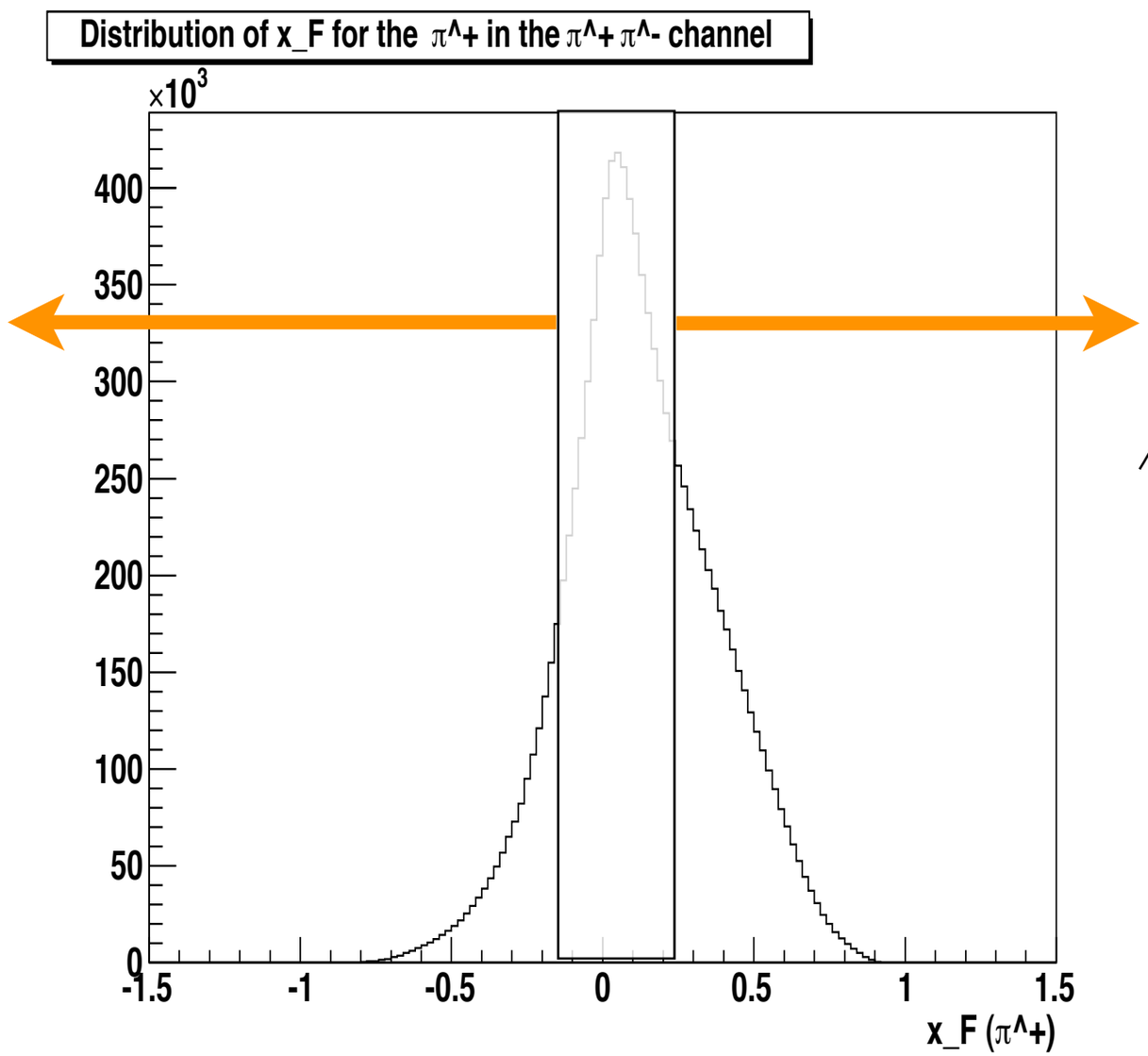
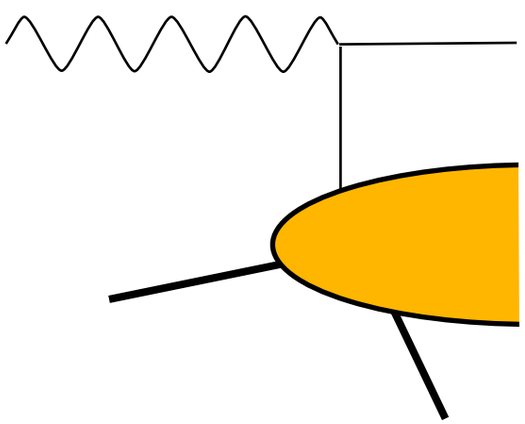
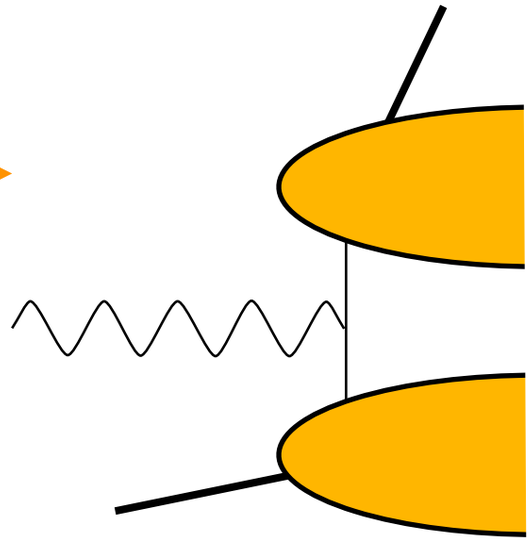




fracture function



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BACKUP SLIDES

LOW-b_T MODIFICATIONS

$$\log\left(Q^2 b_T^2\right) \to \log\left(Q^2 b_T^2 + 1\right)$$

see, e.g., Bozzi, Catani, De Florian, Grazzini
hep-ph/0302104

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$$b_*(b_c(b_{\rm T})) = \sqrt{\frac{b_{\rm T}^2 + b_0^2/(C_5^2 Q^2)}{1 + b_{\rm T}^2/b_{\rm max}^2 + b_0^2/(C_5^2 Q^2 b_{\rm max}^2)}}$$

$$b_{\rm min} \equiv b_*(b_c(0)) = \frac{b_0}{C_5 Q}\sqrt{\frac{1}{1 + b_0^2/(C_5^2 Q^2 b_{\rm max}^2)}}$$

Collins et al.
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- The justification is to recover the integrated result ("unitarity constraint")
- Modification at low b_T is allowed because resummed calculation is anyway unreliable there

PAVIA 2019 b_* PRESCRIPTION

$$b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}}$$

Collins, Soper, Sterman, NPB250 (85)

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These are all choices that should be at some point checked/challenged

PAVIA 2019 b_* PRESCRIPTION

$$\hat{f}_1^q(x, b_T; \mu^2) = \sum_i \left(C_{qi} \otimes f_1^i\right)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^q(x, b_T)$$

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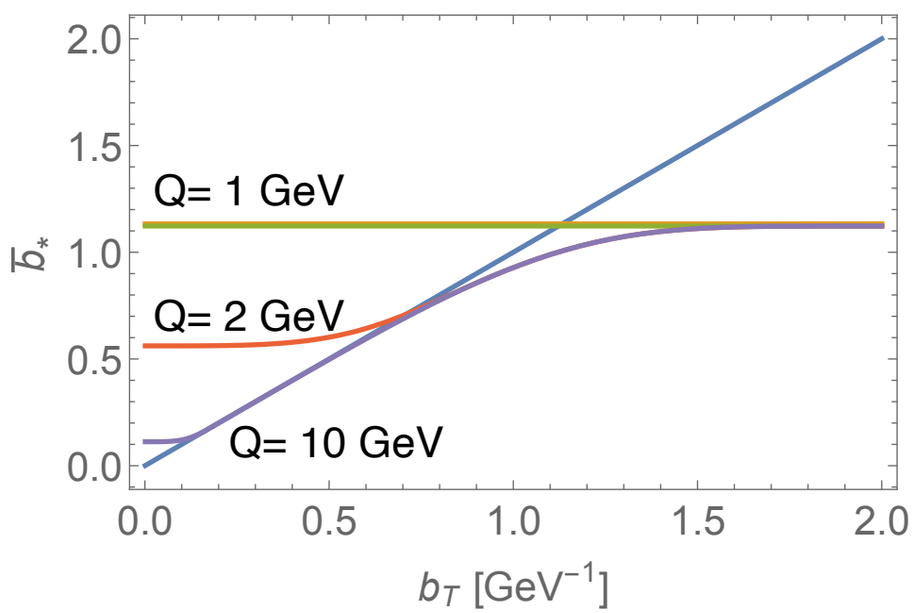

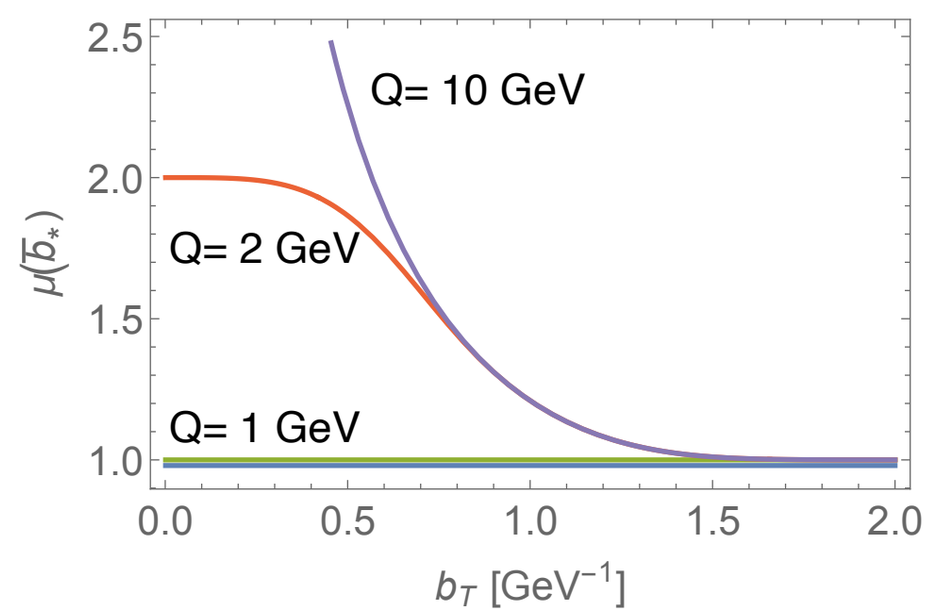

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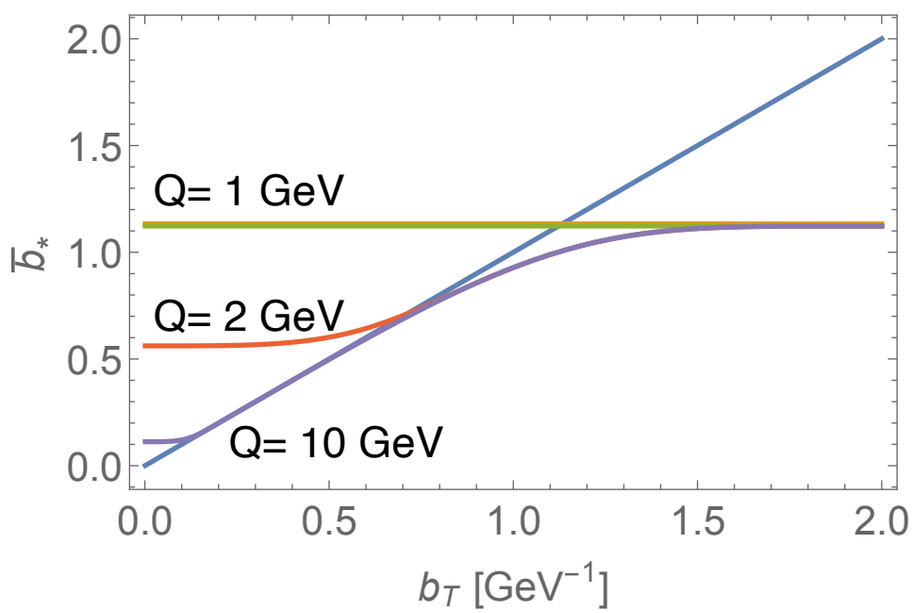
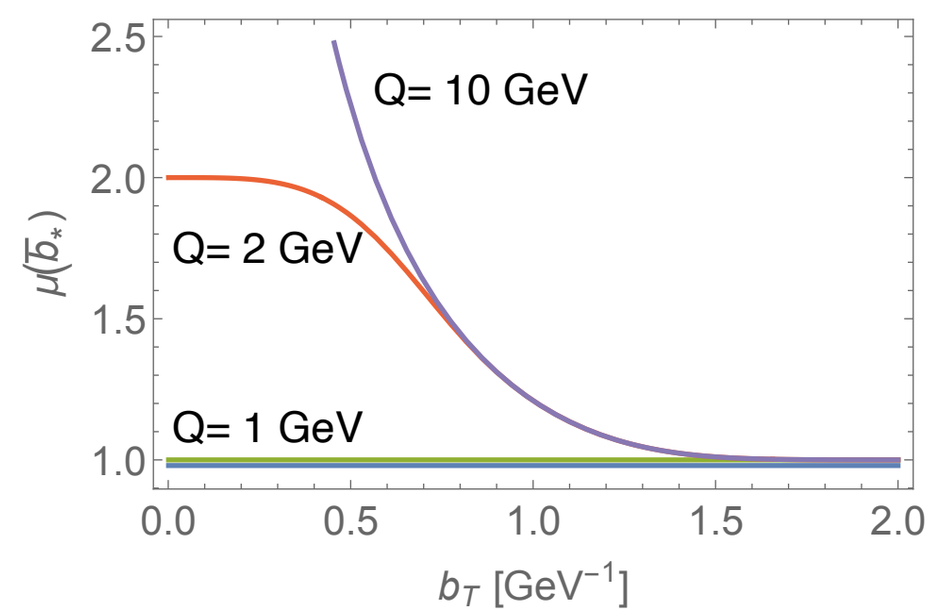




No significant effect at high Q, but large effect at low Q (inhibits perturbative contribution)

NONMIXED TERMS IN COLLINEAR SIDIS CROSS SECTION

$$\left.\frac{\mathrm{d}\sigma^h}{\mathrm{d}x\mathrm{d}Q^2\mathrm{d}z}\right|_{\mathcal{O}(\alpha_s^1)} = \sigma_0 \sum_{ff'} \frac{e_f^2}{z^2} \left(\delta_{f'f} + \delta_{f'g}\right) \frac{\alpha_s}{\pi} \Bigg\{ \left[D_1^{h/f'} \otimes C_1^{f'f} \otimes f_1^{f/N}\right](x,z,Q)$$

$$+ \frac{1-y}{1+(1-y)^2} \left[D_1^{h/f'} \otimes C_L^{f'f} \otimes f_1^{f/N}\right](x,z,Q) \Bigg\},$$

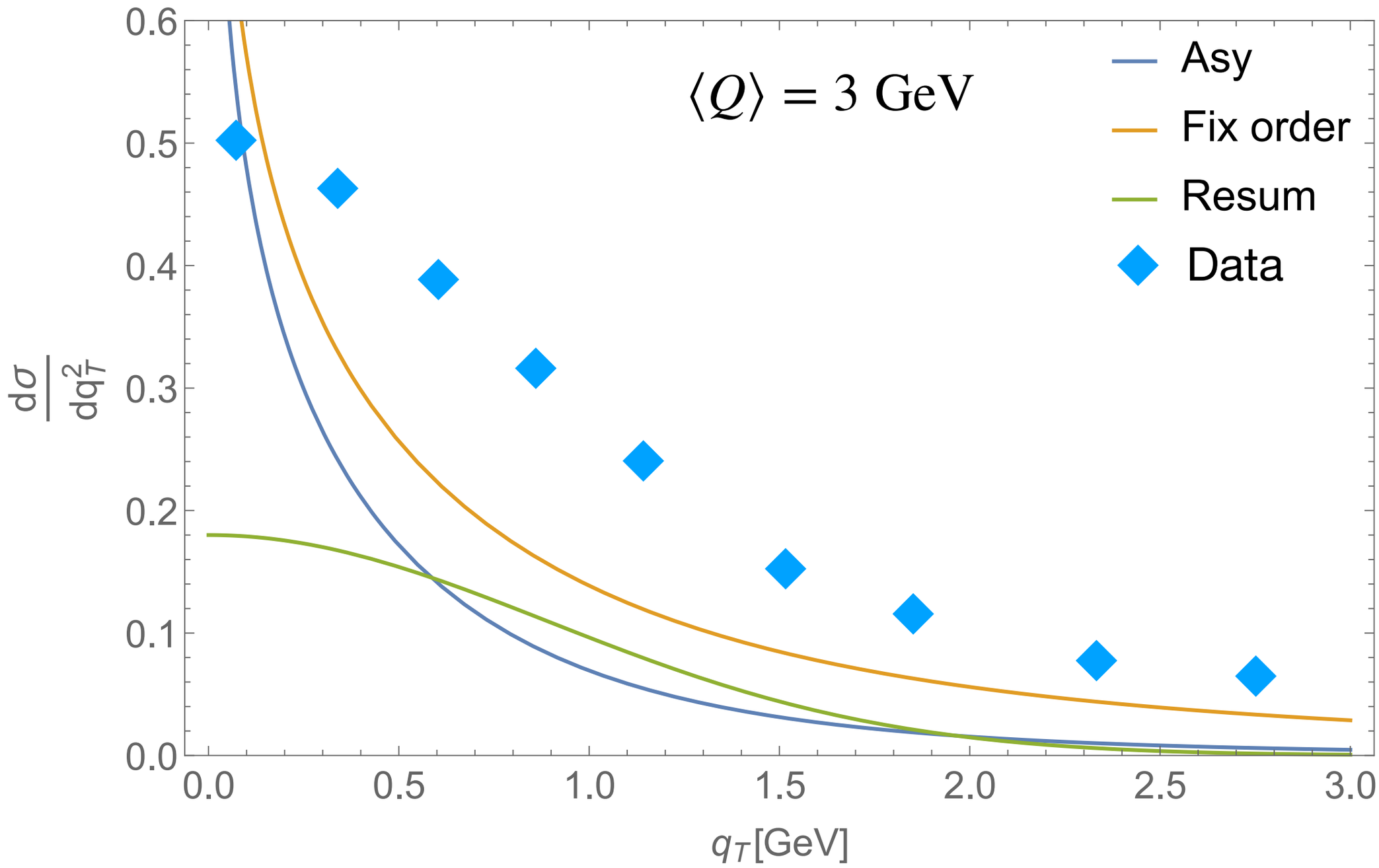
$$C_1^{qq} = \frac{C_F}{2} \Bigg\{ -8\delta(1-x)\delta(1-z)$$

$$+ \delta(1-x) \left[P_{qq}(z) \ln \frac{Q^2}{\mu_F^2} + L_1(z) + L_2(z) + (1-z)\right]$$

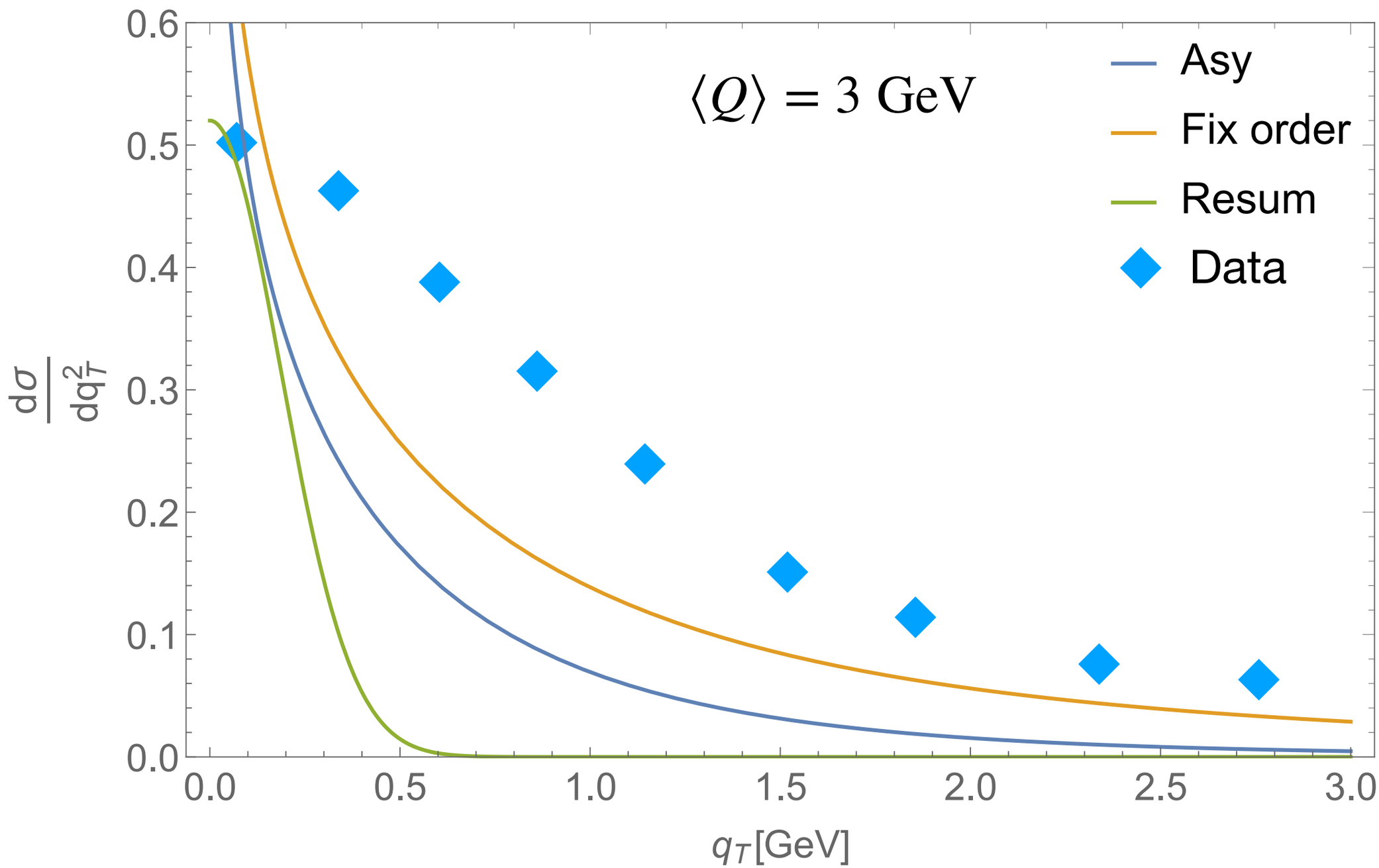
$$+ \delta(1-z) \left[P_{qq}(x) \ln \frac{Q^2}{\mu^2} + L_1(x) - L_2(x) + (1-x)\right]$$

$$+ 2\frac{1}{(1-x)_+}\frac{1}{(1-z)_+} - \frac{1+z}{(1-x)_+} - \frac{1+x}{(1-z)_+} + 2(1+xz) \Bigg\},$$

SOME JUSTIFICATION: INITIAL SITUATION



SOLUTION 1: RESTRICT TMD REGION



SOLUTION 2: ENHANCE TMD CONTRIBUTIONS