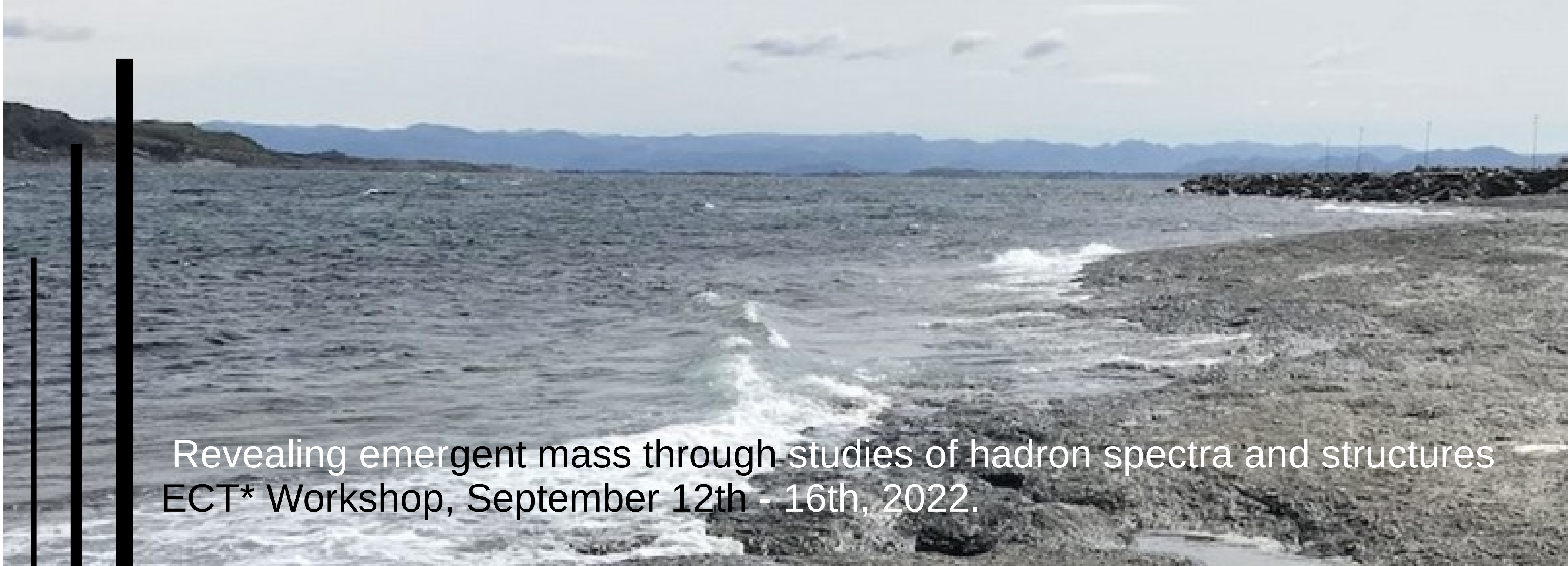
Gravitational Form Factors of Nambu-Godstone Bosons

J. Rodríguez-Quintero

FÍSICA MATEMÁTICAS COMPUTACIÓN

In collaboration with Yin-Zhen Xu, Khépani Raya, C.D. Roberts, ...



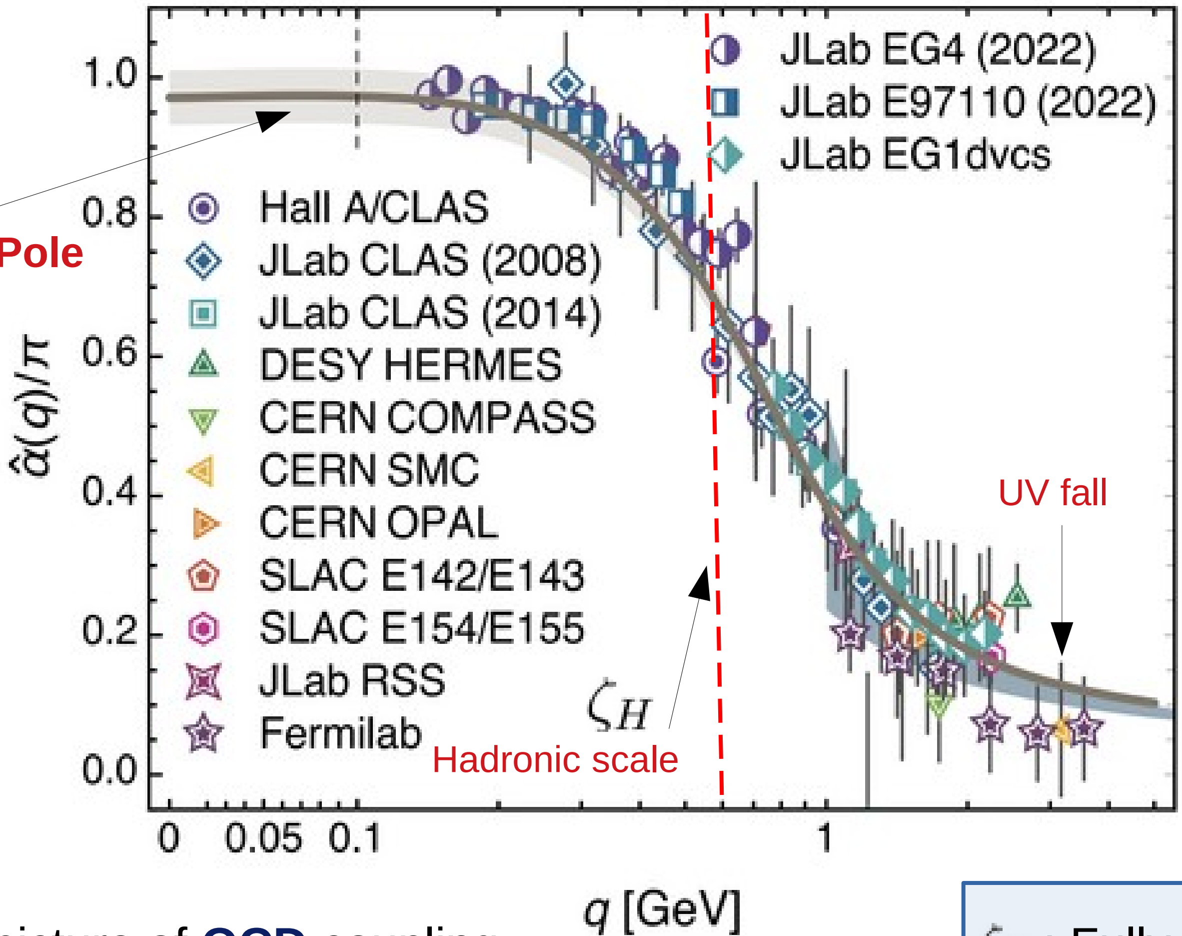
Revealing emergent mass through studies of hadron spectra and structures
ECT* Workshop, September 12th - 16th, 2022.

QCD: Basic Facts

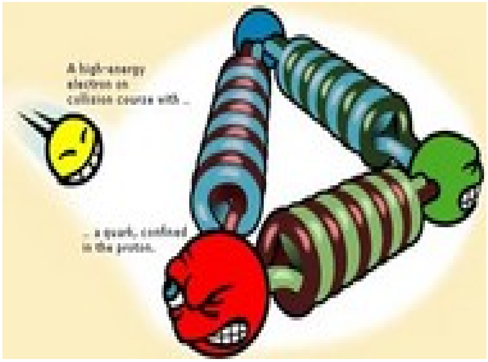
- **Confinement** and the **EHM** are tightly connected with **QCD's running coupling**.

'Effective Charge' (figure: D. Binosi's courtesy!)

No Landau Pole
Enhancement



$$\mathcal{L}_{QCD} = \sum_{j=u,d,s,\ldots} \bar{q}_j [\gamma_\mu D_\mu + m_j] q_j + \tfrac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu},$$
$$D_\mu = \partial_\mu + i g \tfrac{1}{2} \lambda^a A^a_\mu ,$$
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu + \partial_\nu A^a_\mu - \underline{g f^{abc} A^b_\mu A^c_\nu},$$



Modern picture of **QCD** coupling.

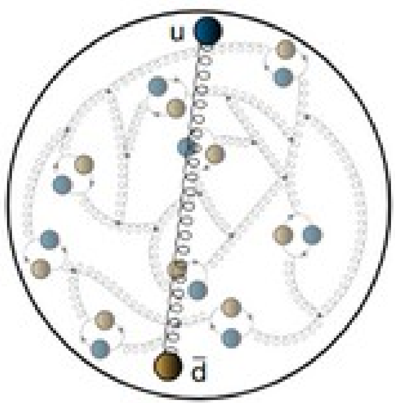
Combined continuum + QCD lattice analysis

ζ_H : Fully **dressed valence** quarks express all hadron's properties

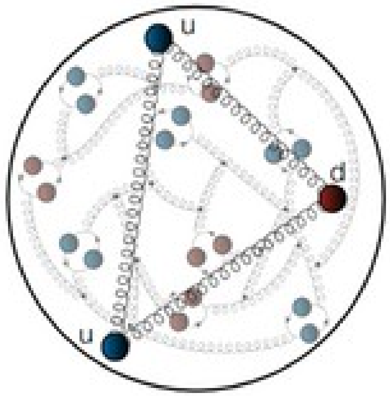
Why pions and kaons?: understanding EHM

➢ **Pions** and **kaons** emerge as (pseudo)-**Goldstone** bosons of **DCSB**.

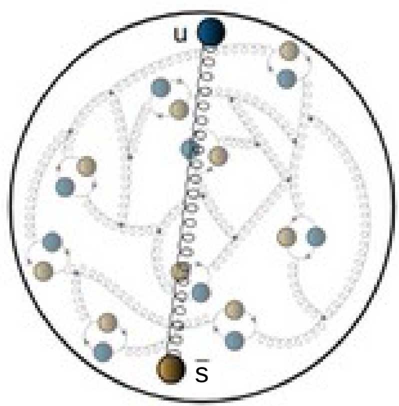
(besides being 'simple' bound states)



$m_\pi \approx 0.140$ GeV



$m_p \approx 0.940$ GeV



$m_K \approx 0.490$ GeV

'Higgs' masses

$m_{u/d} \approx 0.004$ GeV

$m_s \approx 0.095$ GeV

→ Their study is crucial to understand the **EHM** and the ***hadron structure***:

- Dominated by **QCD** dynamics

  Simultaneously explains the mass of the proton and the *masslessness* of the pion

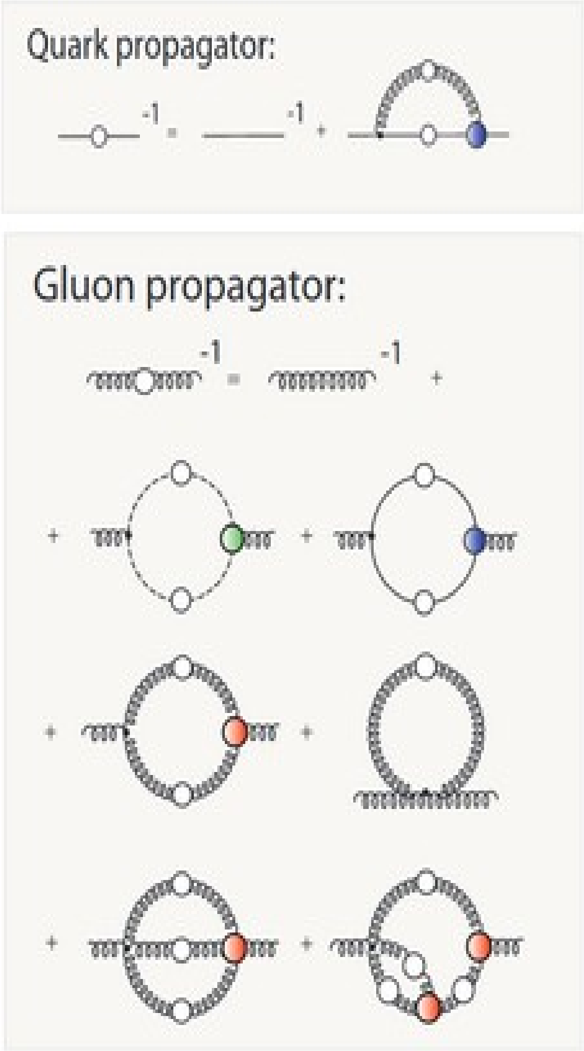
- Interplay between **Higgs** and **strong** mass generating mechanisms.

CSM: the DSE approach

- Equations of motion of a **quantum field theory**
- Relate Green functions with higher-order Green functions
  - **Infinite** tower of coupled equations.
  - Systematic **truncation** required
- ✓ **No assumptions** on the **coupling** for their derivation.
  - ✓ Capture both **perturbative** and **non-perturbative** facets of **QCD**
- ✓ **Not limited** to a certain domain of current **quark masses**
- ✓ Maintain a **traceable connection** to QCD.

C.D. Roberts and a A.G. Williams,
Prog.Part.Nucl.Phys. 33 (1994) 477-575

Example DSEs



Eichmann:2009zx

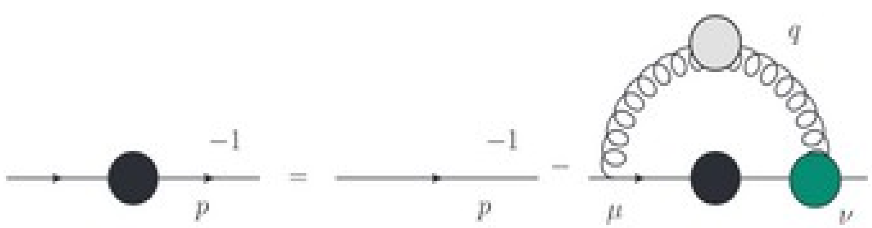
CSM: the DSE approach

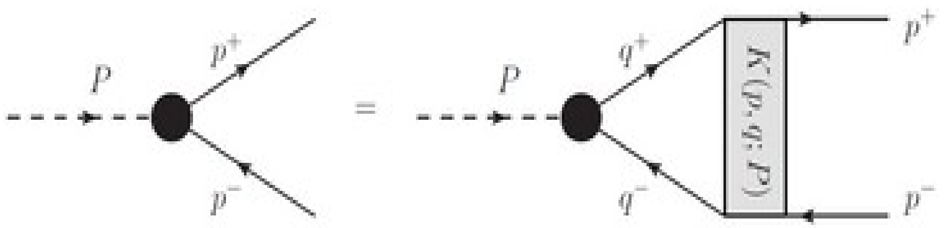
- **BSWF**: sandwich of the Bethe-Salpeter amplitude and quark propagators:

$$\chi_H(k^H;P_H) = S_q(k)\Gamma_H(k^H;P_H)S_{\bar{q}}(k-P_H)\ ,\ k^H = k - P_H/2\ .$$

$P^2 = -m_H^2$ **: meson's mass**; Γ_H **BS amplitude**; $S_{q(\bar{q})}$ **quark (antiquark) propagator**

- Quark **propagator** and **BSA** should come from solutions of:





Quark DSE

Meson BSE

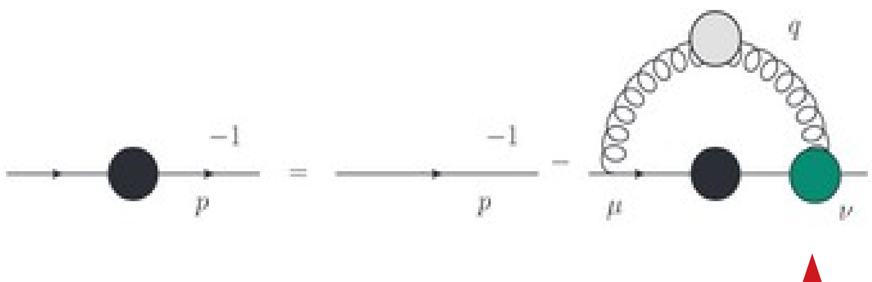
- Relates the quark propagator with **QGV** and **gluon propagator**.
- Contains **all interactions** between the quark and antiquark

CSM: the DSE approach

- For the ground-state pseudoscalar and vector mesons, it is typical to employ the so called Rainbow-Ladder (**RL**) truncation:

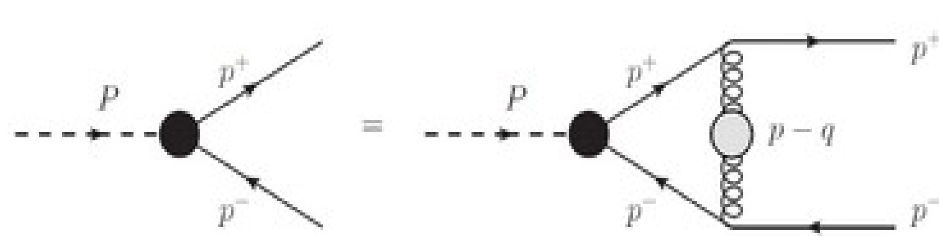
Y-Z Xu et al., PRD 100 (2019) 11, 114038.

K. Raya et al., PRD 101 (2020) 7, 074021.



bare

Quark DSE



Meson BSE

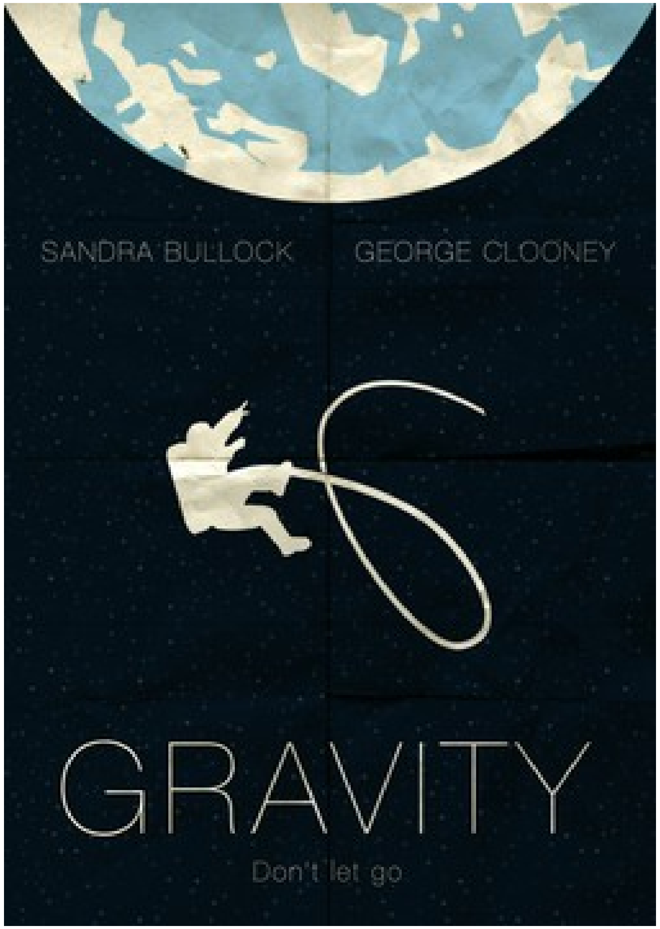
- It preserves the **Goldstone's Theorem**, whose most fundamental expression is captured in:

$$f_\pi E_\pi(k; P=0) = B(k^2)$$

Leading BSA　　“***Mass*** Function”

> “**Pions** exists, if and only if, **DCSB** occurs.”

Gravitational form factors



Gravitational **form factors**

- For a given parton class, the **spin-0** energy-momentum tensor (**EMT**) can take the following form:

$$\Lambda^{a}_{\mu\nu}(P,Q) = 2P_\mu P_\nu \theta^{a}_2(Q^2) + \frac{1}{2}\left(Q^2 g_{\mu\nu} - Q_\mu Q_\nu\right)\theta^{a}_1(Q^2) + 2m_\pi^2 g_{\mu\nu}\bar{c}^{a}(Q^2)$$

$$\underbrace{\Lambda^{a}_{\mu\nu}(P,Q)}_{\langle P_f | T_{\mu\nu}(0) | P_i \rangle}$$

With: $P = [P_f + P_i]/2 \text{ and } Q = P_f - P_i$

- Such that $\theta_{1,2}(Q^2),\ \bar{c}(Q^2)$ define the so called **gravitational form factors** (**GFFs**).

They can be extracted with the appropriate projections.
Particularly:

$$\theta_{1,2}(Q^2) = P^{\mu\nu}_{1,2}\Lambda_{\mu\nu}(P,Q)$$

With:

$$P_2^{\mu\nu} = \frac{3P^\mu P^\nu}{4P^4} + \frac{Q^\mu Q^\nu - Q^2 g^{\mu\nu}}{4P^2Q^2}$$

$$P_1^{\mu\nu} = -\frac{P^\mu P^\nu}{P^2Q^2} - \frac{3\left(Q^\mu Q^\nu - Q^2 g^{\mu\nu}\right)}{Q^4} - \frac{2g^{\mu\nu}}{Q^2}$$

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p(r) : pressure
s(r) : shear forces

$$T^{ij}_q(\vec{r}) = p_q(r)\,\delta_{ij} + s_q(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) \Rightarrow \theta_1(Q^2)$$

Is connected with the **mechanical** properties of the hadron

$$\int d^3r\, T^{00}_q(\vec{r}) = m_\pi \Theta_{2,q}(0) \Rightarrow \theta_2(Q^2)$$

connected with the **mass** distribution inside the hadron

M. Polyakov, Phys.Lett. B555 (2003) 56-62
M. Polyakov, P. Schweitzer, Int. J. Mod. Phys. A 33 (2018) 1830025

Gravitational form factors

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- Such that $\theta_{1,2}(Q^2)$, $\bar{c}(Q^2)$ define the so called **gravitational form factors** (**GFFs**).

- Energy-momentum **conservation** entails the following **sum rules**:

$$\sum_{q,g} \theta_2(0) = 1 \qquad \sum_{q,g} \bar{c}(t) = 0$$

- While, in the **chiral limit**, the **soft-pion theorem** constraints:

$$\sum_{q,g} \theta_1(0) = 1$$

Gravitational form factors: CSM

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In pion's case, both u- and d-in-π valence-quark contributions are the same

$$\Lambda_{\mu\nu}(P,Q) = N_c \int_{dk} \mathrm{Tr}\left[\Gamma_\pi\left(k-\frac{Q}{4}, P-\frac{Q}{2}\right) S\left(k-\frac{P}{2}\right)\Gamma_\pi\left(k+\frac{Q}{4}, P+\frac{Q}{2}\right)\right.$$
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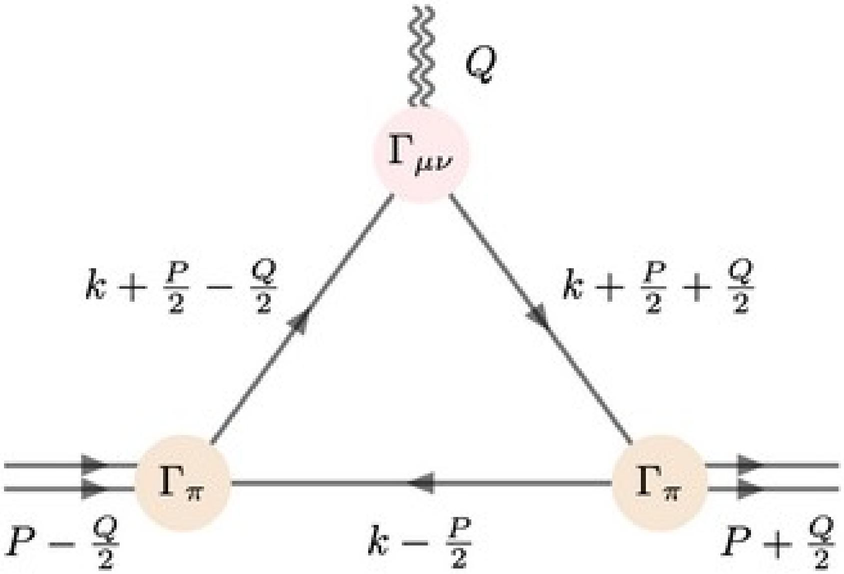

+ beyond **I.A.**

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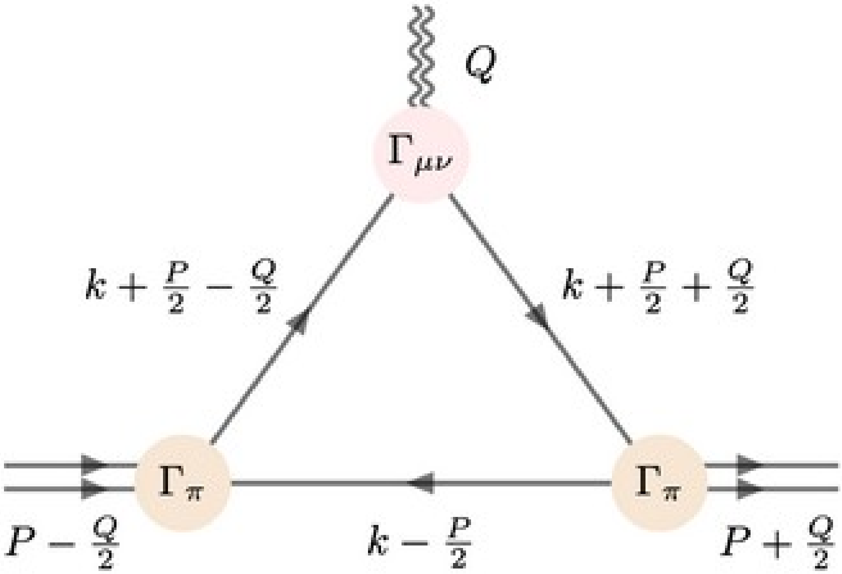

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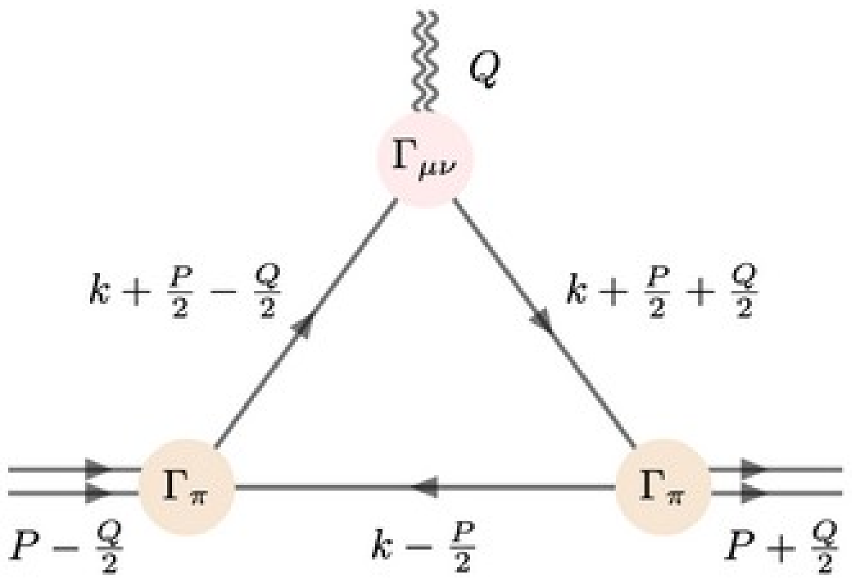

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Also note:

$$Q_\mu \Lambda_{\mu\nu}(P,Q) \sim \bar{c}(Q^2)$$

Thus restricting the structure of contributions beyond I.A.

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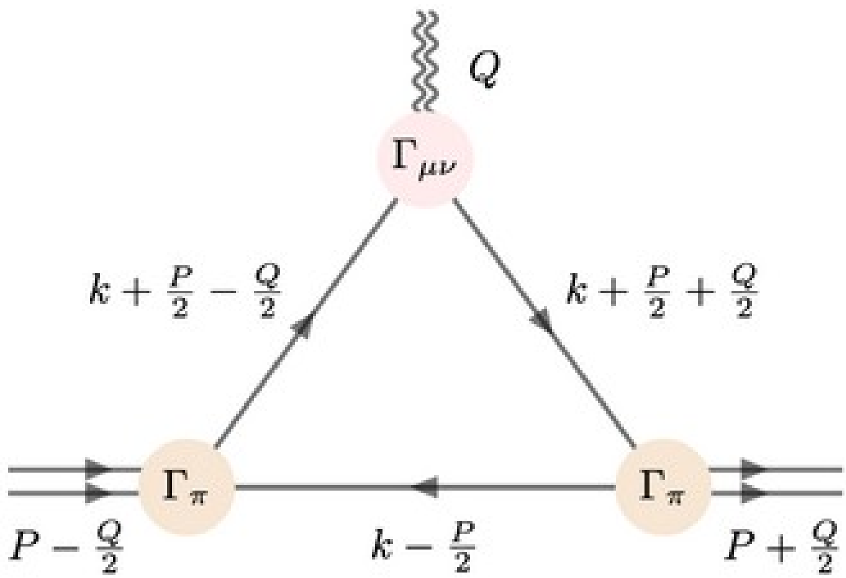

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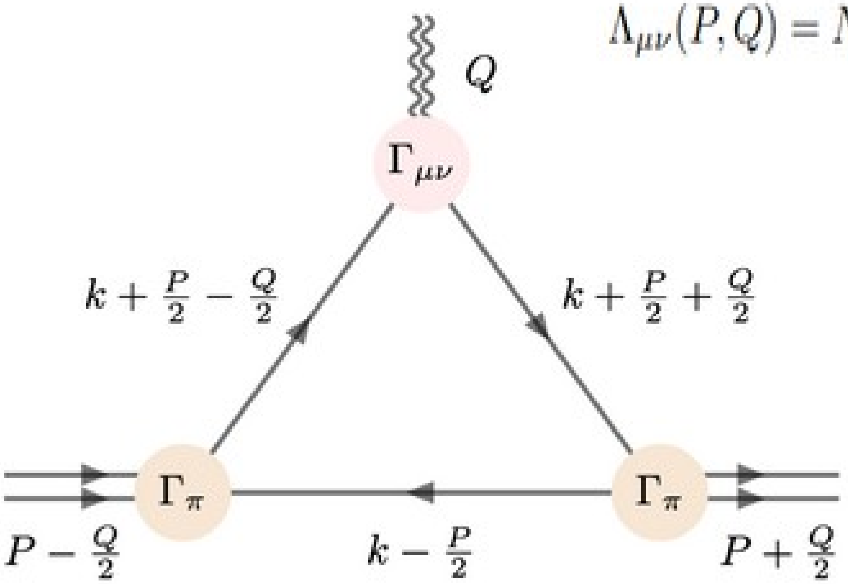
Thus restricting the structure of contributions beyond I.A.

Remark: the graviton-quark vertex obeys a tensor WGTI making it to rely on the quark propagators and such that $\bar{c}(Q^2)$ is irrespective of it.

$$iQ_\mu \Gamma^{\mu\nu}(P,Q) = P_i^\nu S^{-1}(P_f) - P_f^\nu S^{-1}(P_i)$$

Gravitational form factors: CSM ingredients



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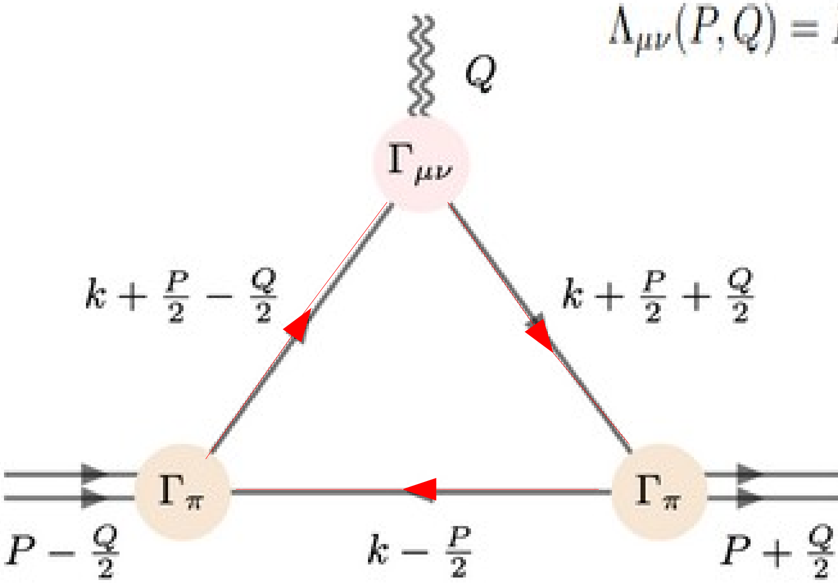
Gravitational form factors: CSM ingredients



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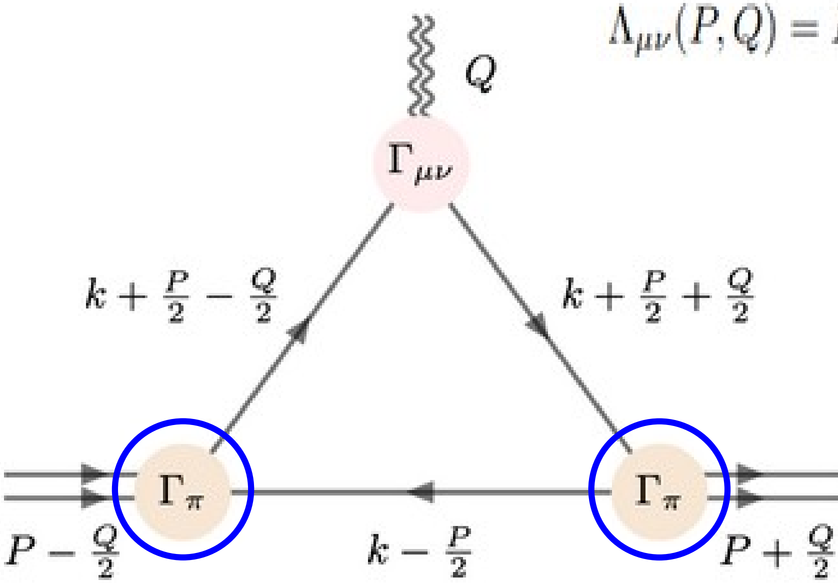
+ beyond **I.A.**

Solutions of the quark gap equation in RL with a realistic quark-gluon interaction.

Gravitational form factors: CSM ingredients



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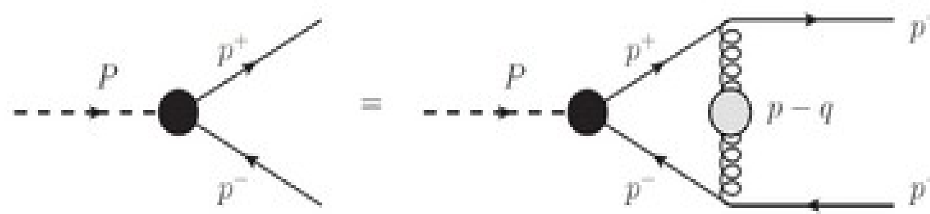
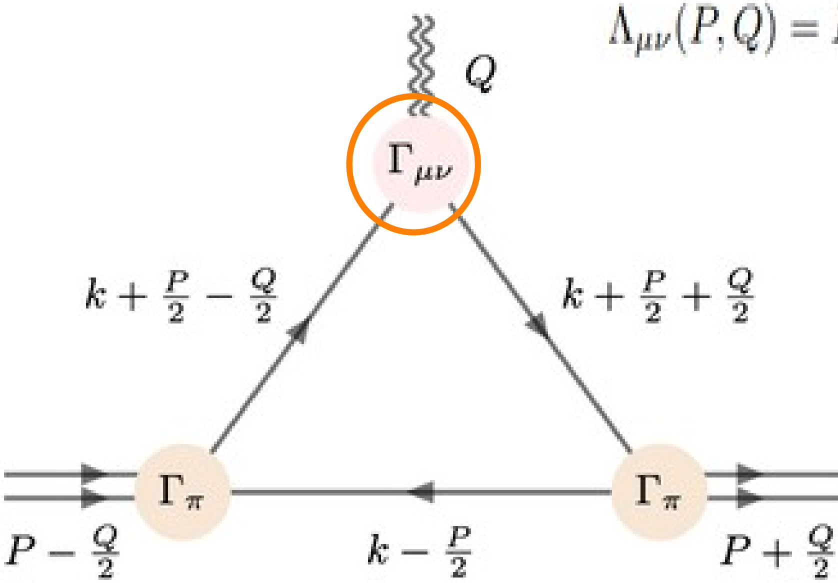
Solutions of the quark gap equation in RL with a realistic quark-gluon interaction.

Solutions of the Bethe-Salpeter equation with the corresponding RL kernel, derived from the realistic quark-gluon interaction.



The interaction parameters are properly fixed such that: $m_\pi = 0.14, m_K = 0.49, f_\pi = 0.095, f_K = 0.116 [GeV]$
but one can also consistently compute with an effective interaction relying on the PI effective charge.

Quark-tensor vertex



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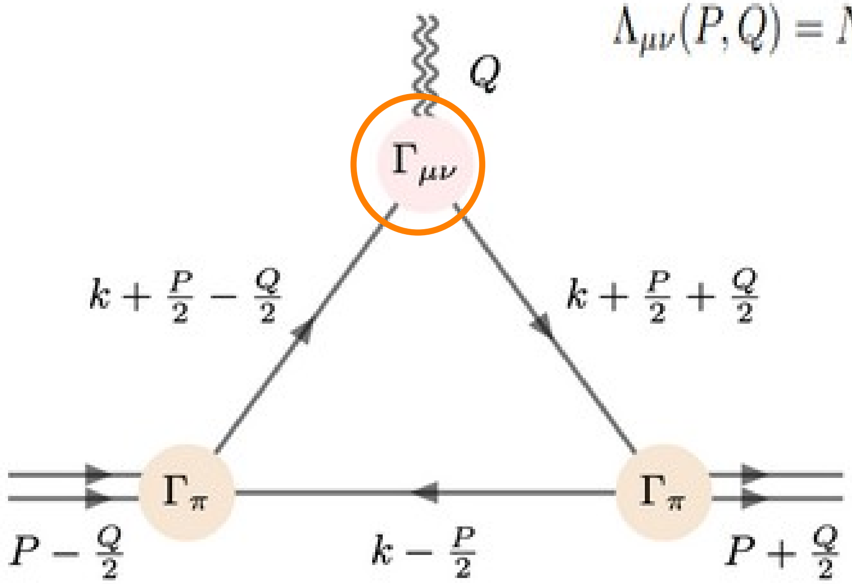
Quark-tensor vertex (QTV): The brand-new ingredient encoding the interaction of a quark and a spin-2 probe. To be determined by its **DSE**, but also its Ward-Green-Takahashi identity (**WGTI**) and other **symmetry properties**.

As the quark-photon vertex (QPV), QTV obeys its own DSE:

$$i\Gamma^{\mu\nu}(P,Q) = \underbrace{i\Gamma_0^{\mu\nu}(P,Q)}_{\text{Tree-level}} + \int \underbrace{K^{(2)}(P,Q|P',Q')}_{\text{IA Kernel}} i\Gamma^{\mu\nu}(P',Q') + \underbrace{\Delta^{\mu\nu}(P,Q)}_{\text{Symmetry-restoring term}}$$

$$i\Gamma_0^{\mu\nu}(P,Q) = i\gamma^\mu P_i^\nu - g^{\mu\nu} S_0^{-1}(P_i)$$

Quark-tensor vertex



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$$i\Gamma_0^{\mu\nu}(P,Q) = i\gamma^\mu P_i^\nu - g^{\mu\nu}S_0^{-1}(P_i)$$

...and its own WGTI, constraining its structure from symmetry principles:

$$iQ_\mu\Gamma^{\mu\nu}(P,Q) = P_i^\nu S^{-1}(P_f) - P_f^\nu S^{-1}(P_i)$$

Quark-tensor vertex

A symmetry preserving quark-tensor vertex can be minimally built as:

$$i\Gamma^{\mu\nu}(P,Q) = i\Gamma^{\mu}_{L}(P,Q)P_i^{\nu} - g^{\mu\nu}S^{-1}(P_i) + i\Gamma^{\mu}_{T}(P,Q)P_i^{\nu} + i\Gamma^{\mu\nu}_{T}(P,Q)$$

$$Q_\mu \Gamma^{\mu\nu}_{T} = 0$$

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$$Q_\mu \Gamma^{\mu\nu}_{T} = 0$$

This part being fully determined by the **quark-propagator** and **QPV**,

$$\Gamma^{\mu} = \Gamma^{\mu}_{L} + \Gamma^{\mu}_{T}\,, \qquad Q_\mu \Gamma^{\mu}_{T} = 0$$

Obeying its vector WGTI: (the transverse part resulting from the QV DSE.)

$$iQ_\mu \Gamma^{\mu}_{L}(P,Q) = S^{-1}(P_f) - S^{-1}(P_i)$$

$$i\Gamma^{\mu\nu}_{L}(P,Q) = \sum_{i=1}^{14} F_i(P^2, Q^2, P\cdot Q)\, \tau^{\mu\nu}_{i}(P,Q)$$

Quark-tensor vertex

A symmetry preserving quark-tensor vertex can be minimally built as:

$$i\Gamma^{\mu\nu}(P,Q) = \underbrace{i\Gamma^{\mu}_{L}(P,Q)P^{\nu}_{i} - g^{\mu\nu}S^{-1}(P_i) + i\Gamma^{\mu}_{T}(P,Q)P^{\nu}_{i}}_{i\Gamma^{\mu\nu}_{L}(P,Q)} + i\Gamma^{\mu\nu}_{T}(P,Q)$$

$$Q_\mu \Gamma^{\mu\nu}_{T} = 0$$

This, a priori unknown, can be obtained from solving the QTV SDE.

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$$i\Gamma^{\mu\nu}_{L}(P,Q) = \sum_{i=1}^{14} F_i(P^2,Q^2,P\cdot Q)\,\tau^{\mu\nu}_{i}(P,Q)$$

Capitalizing on the latter, we propose the following **minimal representation**:

$$i\Gamma^{\mu\nu}_{T}(P,Q) \;=\; F_{15}(P^2,Q^2,P\cdot Q)\,\tau^{\mu\nu}_{15}(P,Q) = \boxed{i\mathbb{1}\left(Q^2 g^{\mu\nu} - Q^\mu Q^\nu\right)} F_{15}(P^2,Q^2,P\cdot Q)$$

Quark-tensor **vertex**

A symmetry preserving quark-tensor vertex can be minimally built as:

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Obeying its vector WGTI: (the transverse part resulting from the QV DSE.)

$$iQ_\mu \Gamma^{\mu}_{L}(P,Q) = S^{-1}(P_f) - S^{-1}(P_i)$$

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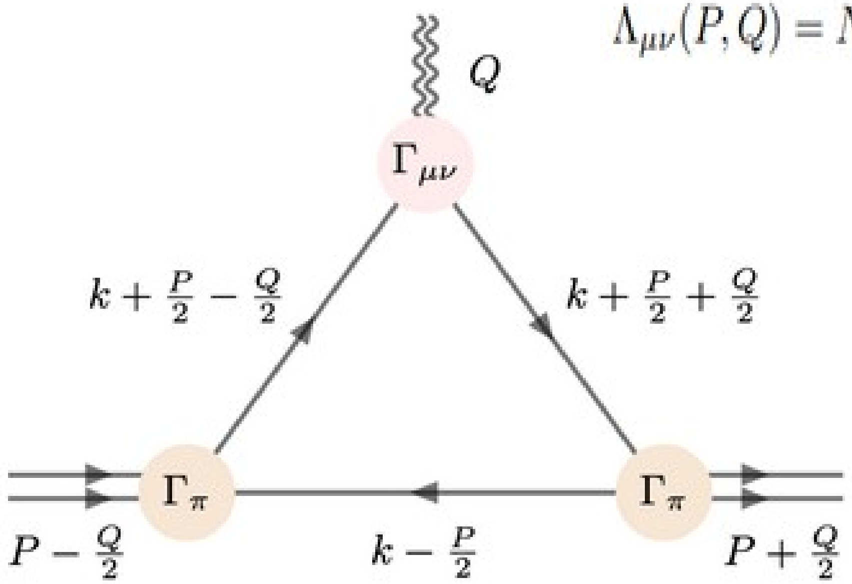
$$i\Gamma^{\mu\nu}_{T}(P,Q) = F_{15}(P^2,Q^2,P\cdot Q)\,\tau^{\mu\nu}_{15}(P,Q) = i\mathbb{1}\left(Q^2 g^{\mu\nu} - Q^{\mu}Q^{\nu}\right)F_{15}(P^2,Q^2,P\cdot Q)$$

Then we proceed to solve the **QTV DSE**.

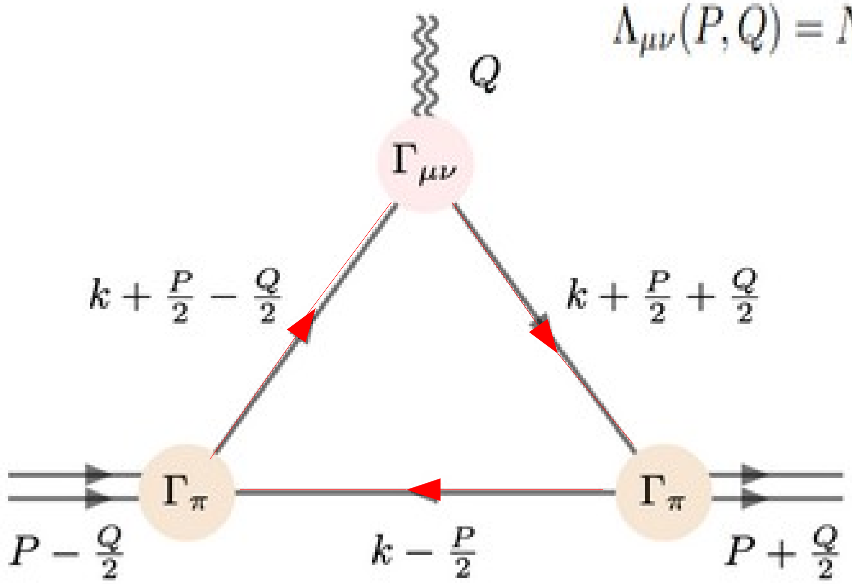
Gravitational form factors: Algebraic Model



$$\Lambda_{\mu\nu}(P,Q) = N_c \int_{dk} \mathrm{Tr}\left[\Gamma_\pi\left(k-\frac{Q}{4},P-\frac{Q}{2}\right)S\left(k-\frac{P}{2}\right)\Gamma_\pi\left(k+\frac{Q}{4},P+\frac{Q}{2}\right)\right.$$
$$\left.S\left(k+\frac{P}{2}+\frac{Q}{2}\right)\Gamma_{\mu\nu}\left(k+\frac{P}{2},Q\right)S\left(k+\frac{P}{2}-\frac{Q}{2}\right)\right]$$

+ beyond **I.A.**

Gravitational form factors: Algebraic Model



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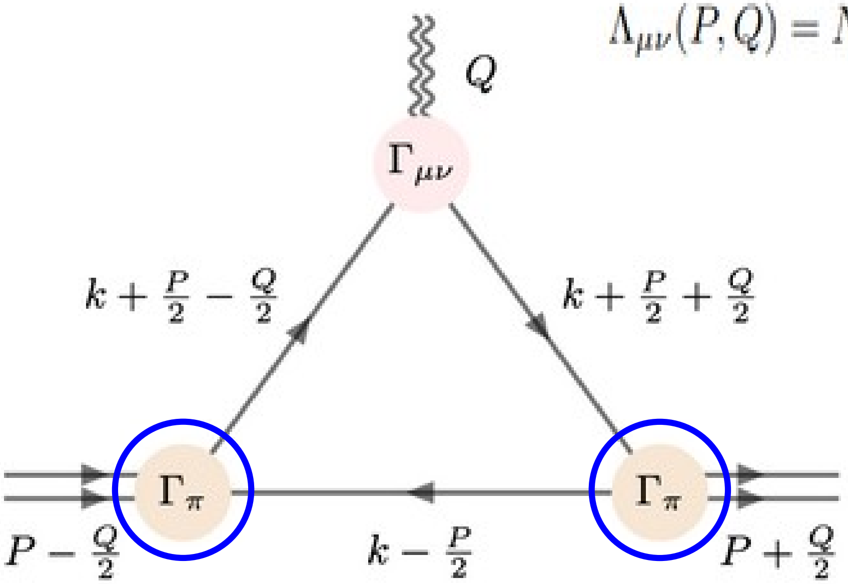
$$S(p) = (-i\gamma\cdot p + M)\Delta_M(p^2)\,,\ \Delta_M(p^2) = (p^2+M^2)^{-1}$$

Gravitational form factors: Algebraic Model



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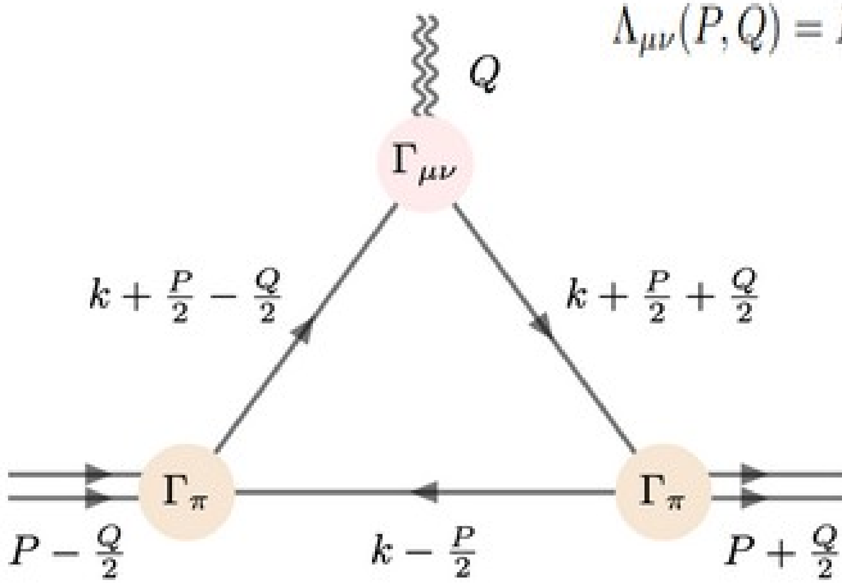
$$\Gamma_\pi(k;P) = i\gamma_5 \int_{-1}^{1} d\omega\, \rho(\omega)\hat{\Delta}_M(k_\omega^2)\,,\ \begin{cases}\hat{\Delta}_M(s) = M^2\Delta_M(s)\\ k_\omega = k + (\omega/2)P\end{cases}$$

Gravitational form factors: Algebraic Model



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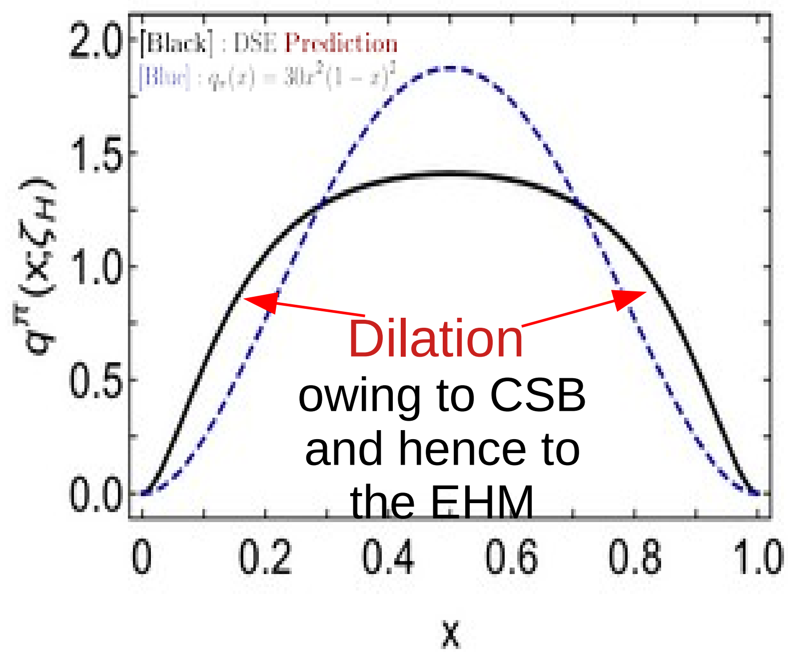
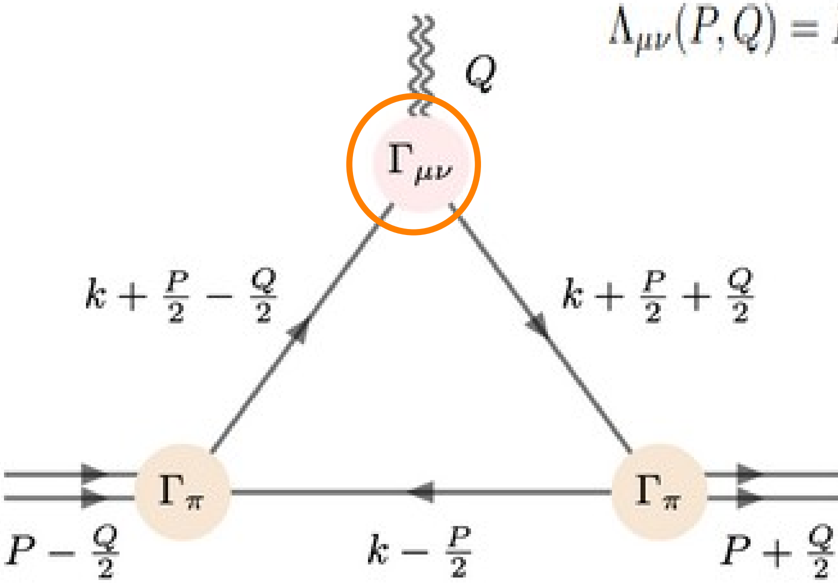


Gravitational form factors: Algebraic Model



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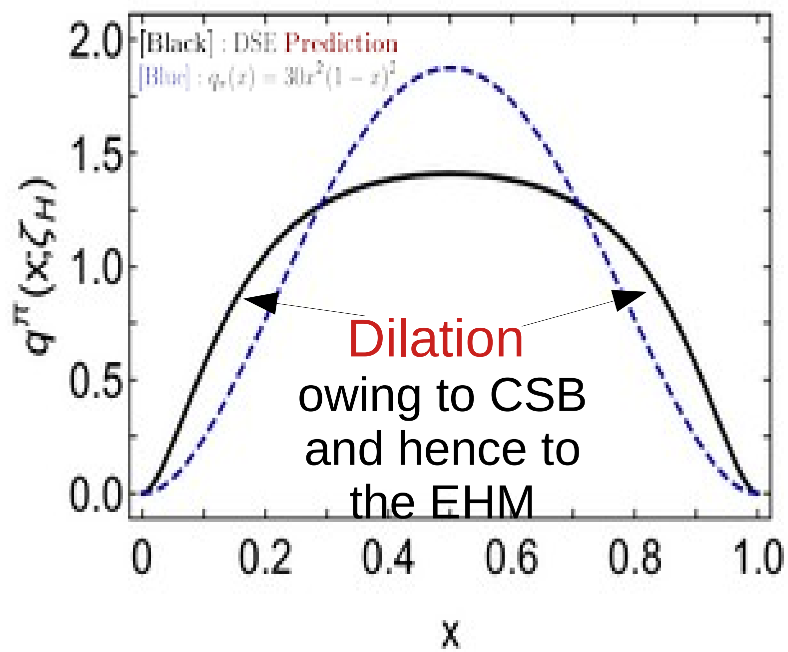
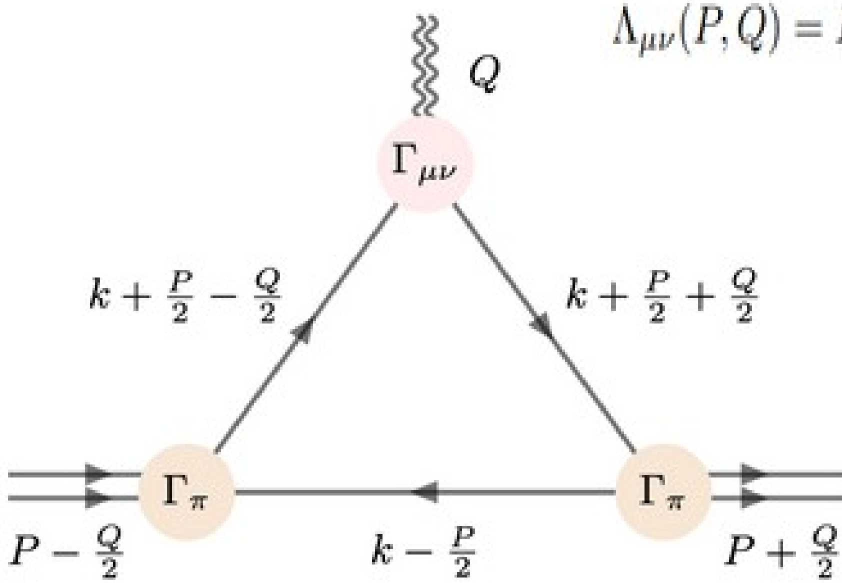
$$i\Gamma^{\mu\nu} = [i\gamma^\mu p^\nu - g^{\mu\nu}S^{-1}(p)] + i(Q^\mu Q^\nu - Q^2 g^{\mu\nu})F_{15}(k,p)$$



Gravitational form factors: Algebraic Model



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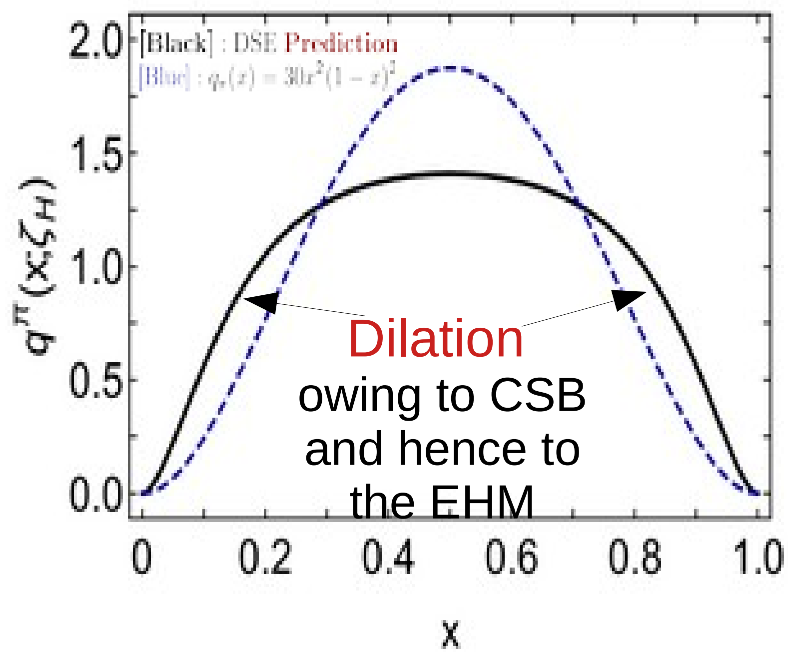
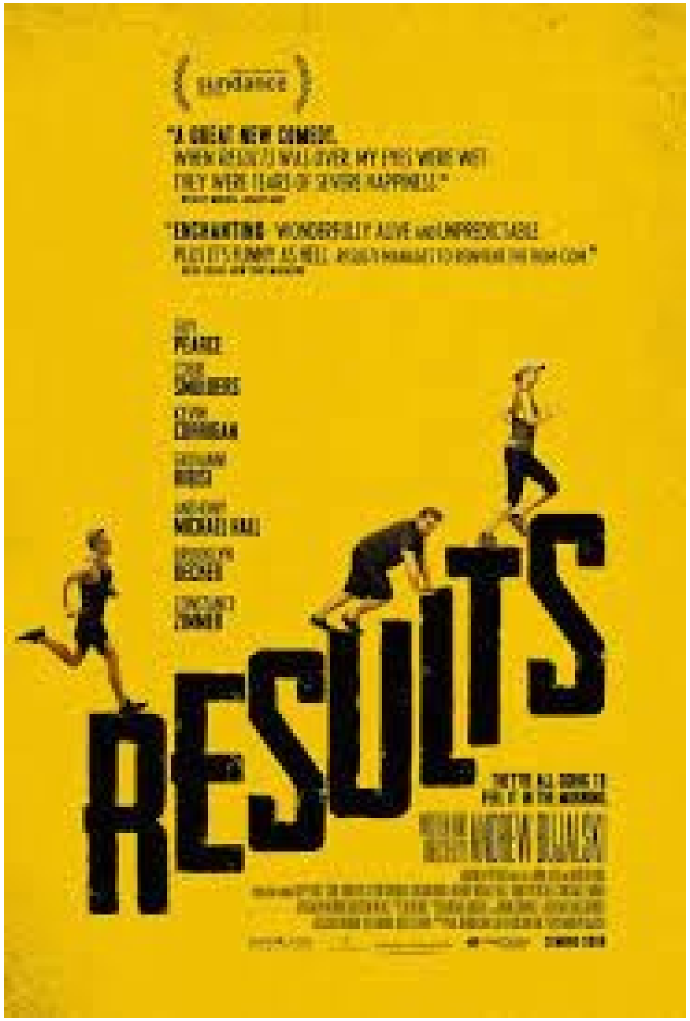
$$i\Gamma^{\mu\nu} = [i\gamma^\mu p^\nu - g^{\mu\nu}S^{-1}(p)] + i(Q^\mu Q^\nu - Q^2 g^{\mu\nu})F_{15}(k,p)$$

$$F_{15}(k,p) \to F_{15}(Q^2) := \frac{\eta_{15}}{1+Q^2/m_\sigma^2}$$

Inspired by a SCI analysis




sundance
"A GREAT NEW COMEDY.
"ENCHANTING
WONDERFULLY ALIVE AND UNPREDICTABLE
PEARCE
SMULDERS
CORRIGAN
RESULTS
ANDREW BUJALSKI

Results: Pion's GFFs

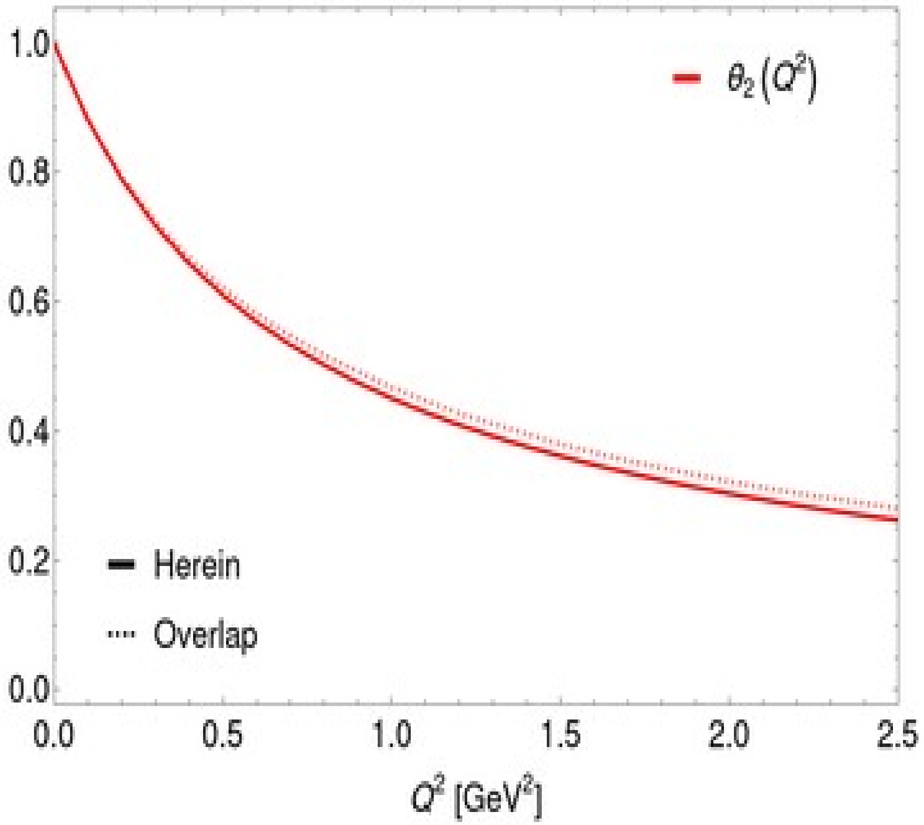
- Recall the **GFFs** are extracted from: $\Lambda_{\mu\nu}(P,Q) = 2P_\mu P_\nu \theta_2(Q^2) + \frac{1}{2}(Q^2 g_{\mu\nu} - Q_\mu Q_\nu)\theta_1(Q^2) + 2m_\pi^2 g_{\mu\nu}\bar{c}(Q^2)$

0

- $\theta_2(Q^2)$ Is well described by the part of the **QTV** that satisfies its **WGTI** alone:

$$iQ_\mu \Gamma^{\mu\nu}(P,Q) = P_i^\nu S^{-1}(P_f) - P_f^\nu S^{-1}(P_i)$$

Which is fully determined by the **QPV** and the **quark propagator**



$$i\Gamma^{\mu\nu}(P,Q) = \underbrace{i\Gamma_L^\mu(P,Q)P_i^\nu - g^{\mu\nu}S^{-1}(P_i)}_{i\Gamma_L^{\mu\nu}(P,Q)} + i\Gamma_T^\mu(P,Q)P_i^\nu$$

Overlap: Result obtained via the computation of the pion **LFWF** and **GPD**

$$\int_{-1}^{1} dx\, x\, H_{\mathsf{P}}^q(x,\xi,-\Delta^2;\zeta_{\mathcal{H}}) = \theta_2^{\mathsf{P}}(\Delta^2) - \xi^2\theta_1^{\mathsf{P}}(\Delta^2)$$

Raya:2021zrz

Results: Pion's GFFs

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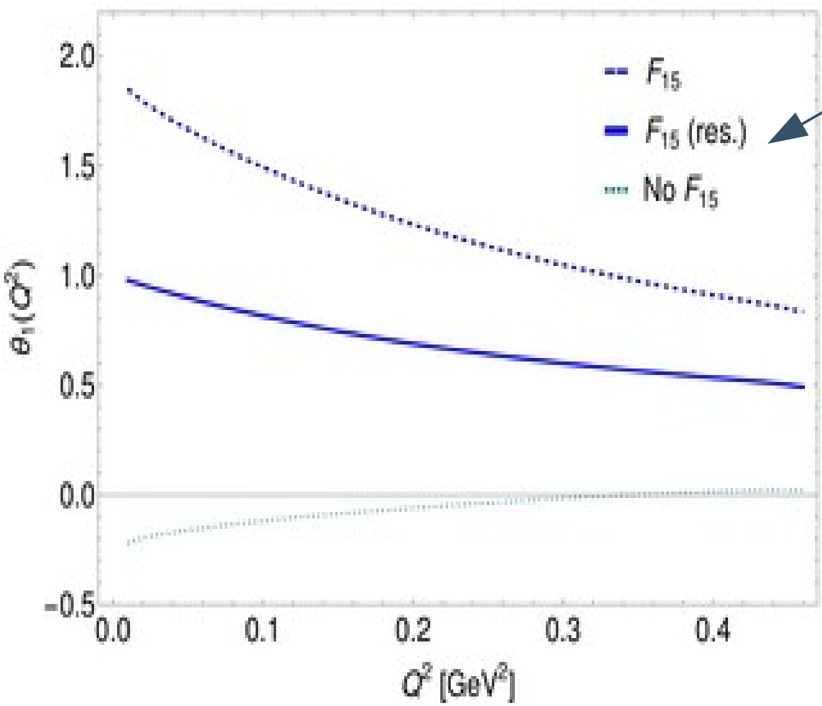
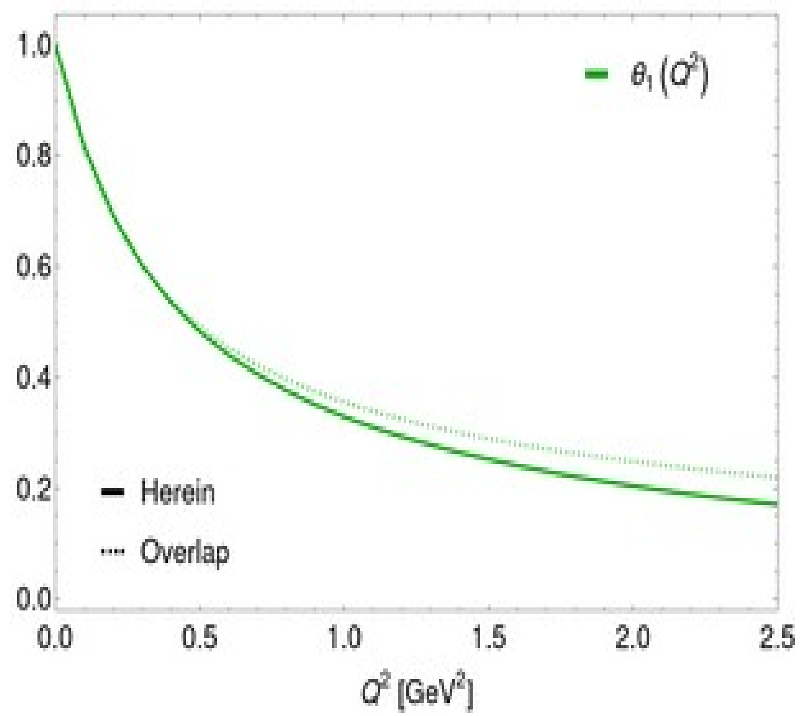
- $\theta_1(Q^2)$ Requires the inclusion of fully transverse pieces in the **QTV;** our *minimal* extension:

$$i\Gamma_T^{\mu\nu}(P,Q) \;=\; F_{15}(P^2,Q^2,P\cdot Q)\,\tau_{15}^{\mu\nu}(P,Q) = i\mathbb{1}\left(Q^2 g^{\mu\nu} - Q^\mu Q^\nu\right)F_{15}(P^2,Q^2,P\cdot Q)$$



Rescaled to account for soft-pion theorem: $\sum_{q,g}\theta_1(0) = 1$

- The **complete** result:



Results: Pion's GFFs

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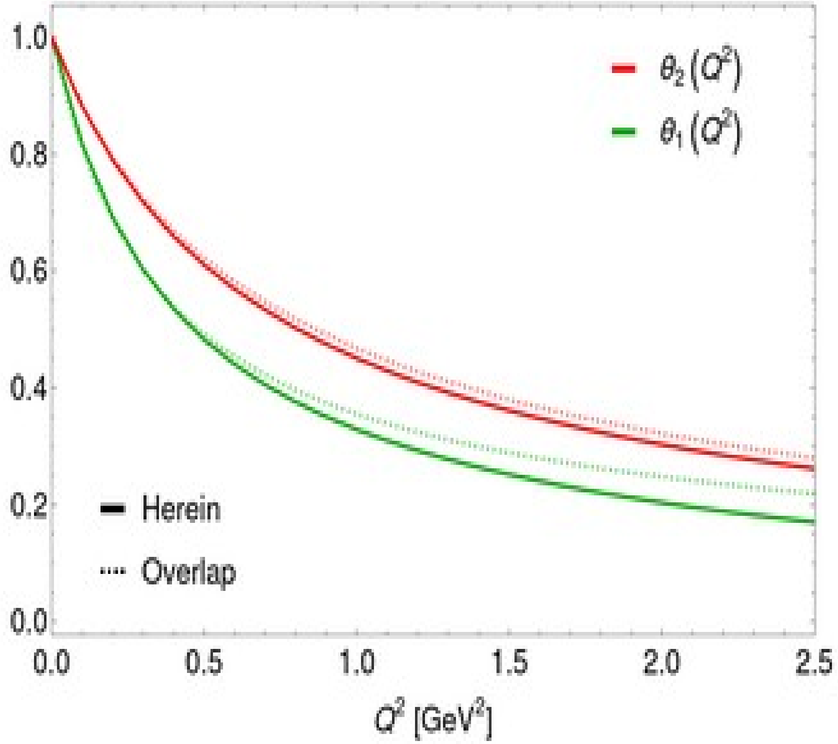
- $\theta_2(Q^2)$ Is harder than $\theta_1(Q^2)$ (and than the pion electromagnetic form factor):

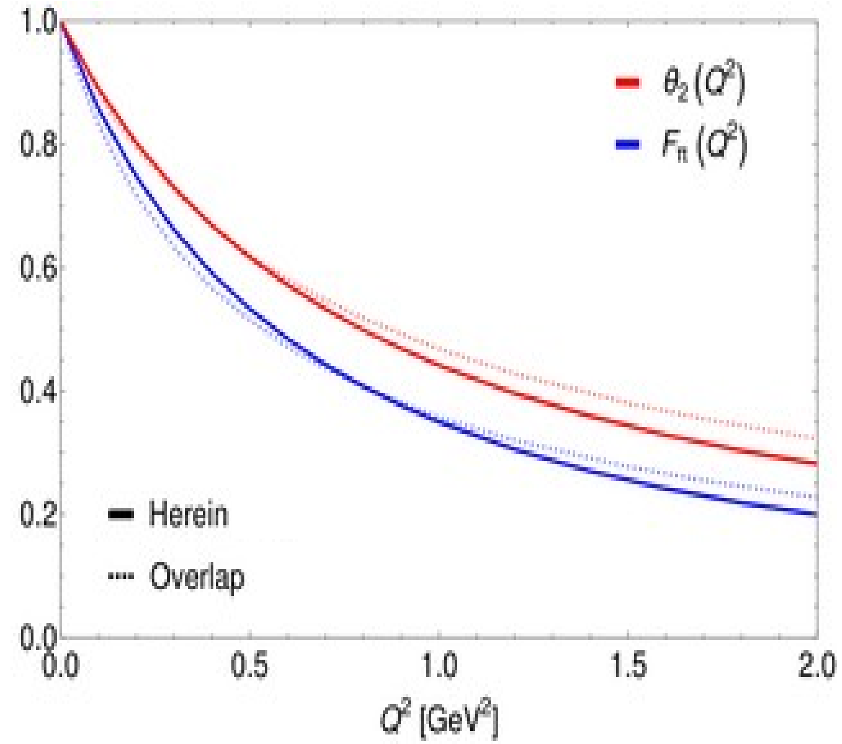


Overlap: Result obtained via the computation of the pion **LFWF** and **GPD**

- In fact, one finds: $r_{\theta_2} \approx 0.8 r_\pi$, $r_{\theta_2} < r_\pi < r_{\theta_1}$

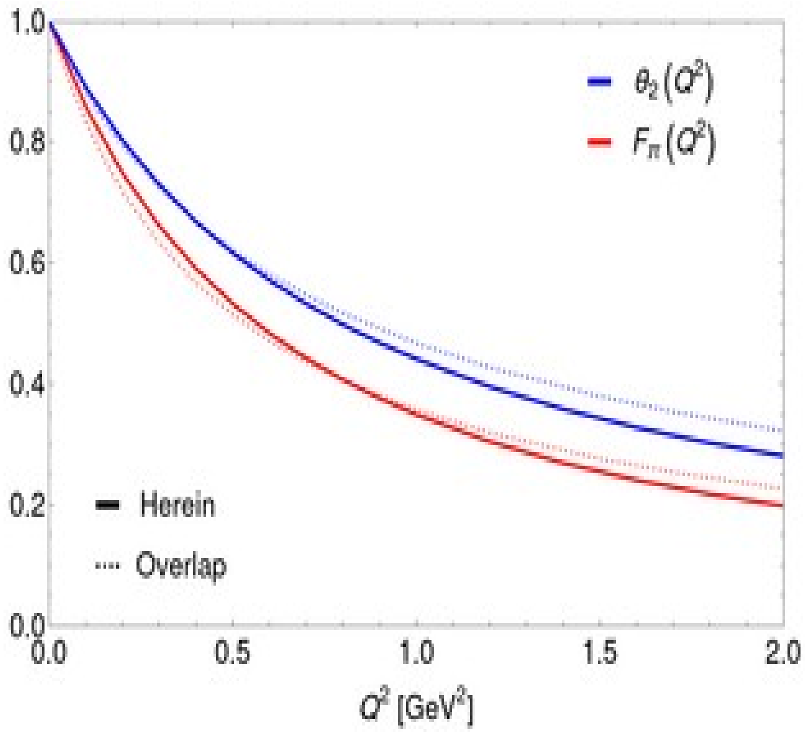
Not an accident! Can be proven via GPD

Raya:2021zrz



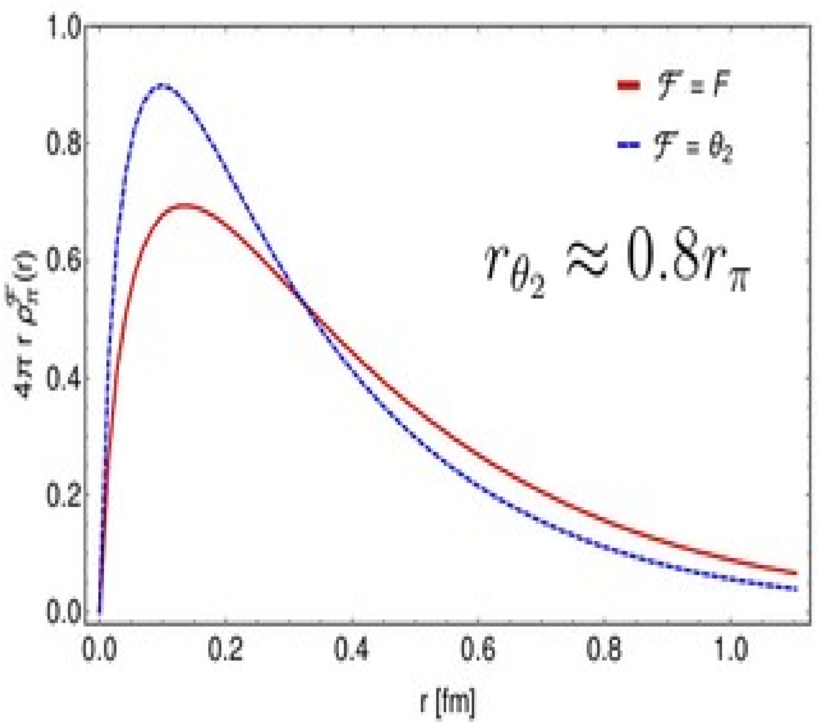
Results: Mass distribution

- Recall the **GFFs** are extracted from: $\Lambda_{\mu\nu}(P,Q) = 2P_\mu P_\nu \theta_2(Q^2) + \frac{1}{2}(Q^2 g_{\mu\nu} - Q_\mu Q_\nu)\theta_1(Q^2) + 2m_\pi^2 g_{\mu\nu}\bar{c}(Q^2)$ ← 0
- $\theta_2(Q^2)$ Is harder than $\theta_1(Q^2)$ (and than the pion electromagnetic form factor):



- The **charge** and **mass** distributions:

$$\rho_{\mathbf{P}}^{\mathfrak{F}}(r) = \frac{1}{2\pi}\int_0^\infty d\Delta\Delta J_0(\Delta r)\mathfrak{F}_{\mathbf{P}}(\Delta^2)$$



Overlap: Result obtained via the computation of the pion **LFWF** and **GPD**

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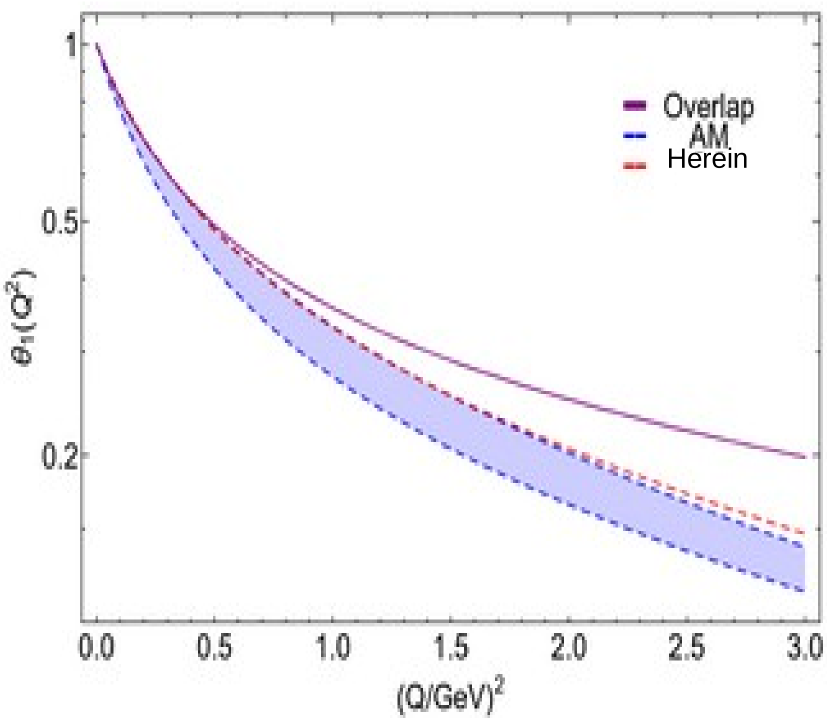
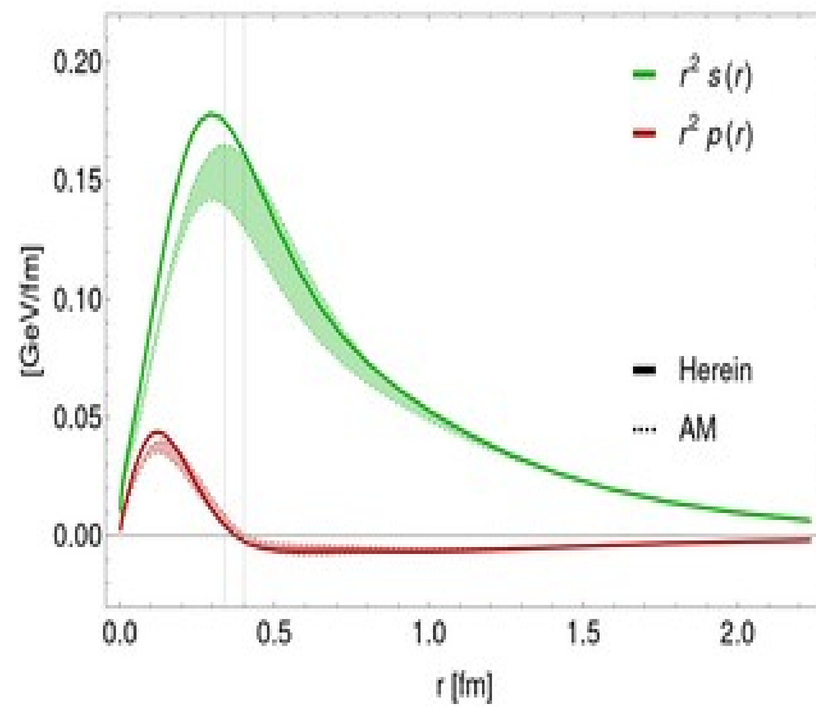
Results: Pressure profiles

- **Pressure** and **shear** forces are obtained from $\theta_1(Q^2)$:

$$p_{\mathbf{P}}(r) = \frac{1}{6\pi^2 r}\int_0^\infty d\Delta \, \frac{\Delta}{2E(\Delta)} \sin(\Delta r)[\Delta^2 \theta_1^{\mathbf{P}}(\Delta^2)]$$

$$s_{\mathbf{P}}(r) = \frac{3}{8\pi^2}\int_0^\infty d\Delta \, \frac{\Delta^2}{2E(\Delta)} j_2(\Delta r)\,[\Delta^2 \theta_1^{\mathbf{P}}(\Delta^2)]$$



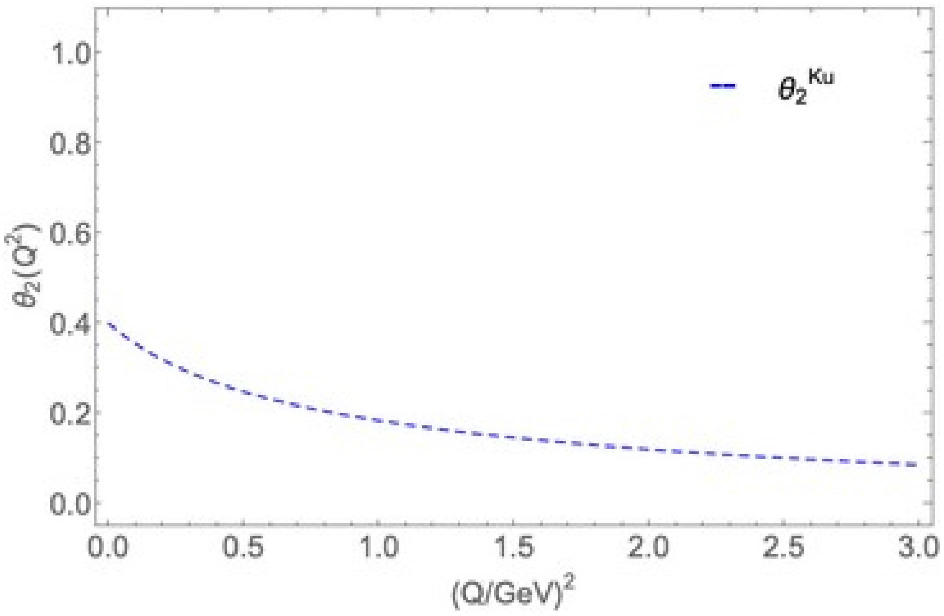


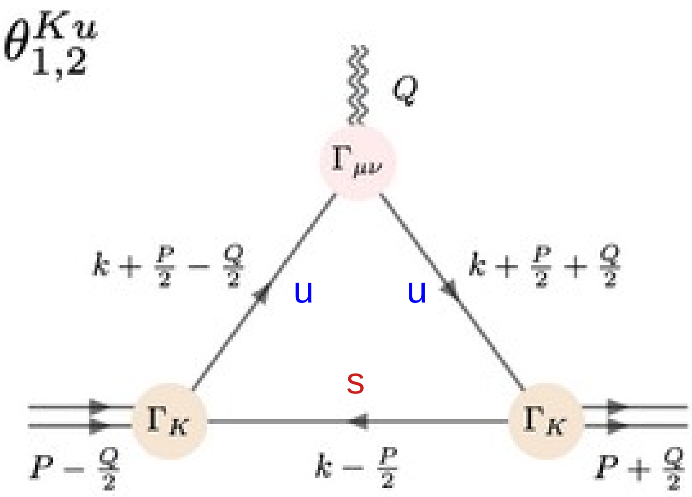
AM: Ingredients from **Overlap** but using the diagrammatic approach discussed herein.

Raya:2021zrz

Shear forces are maximal where the **pressure** shifts sign, i.e. where **confinement** forces become dominant.

Results: Kaon’s GFFs and profiles





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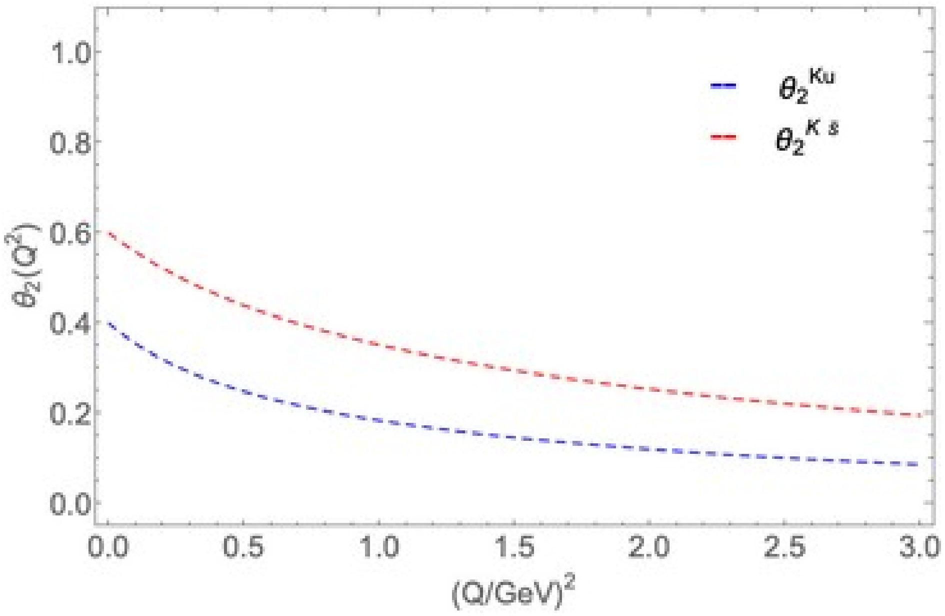

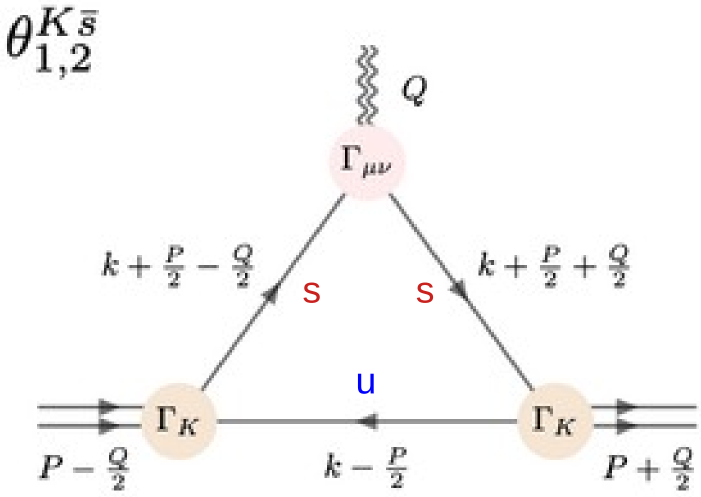

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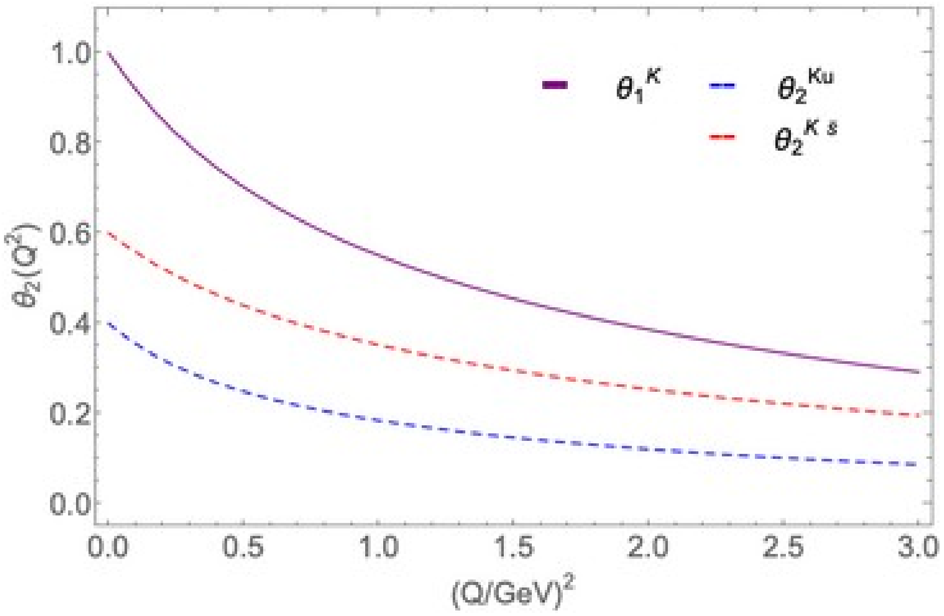

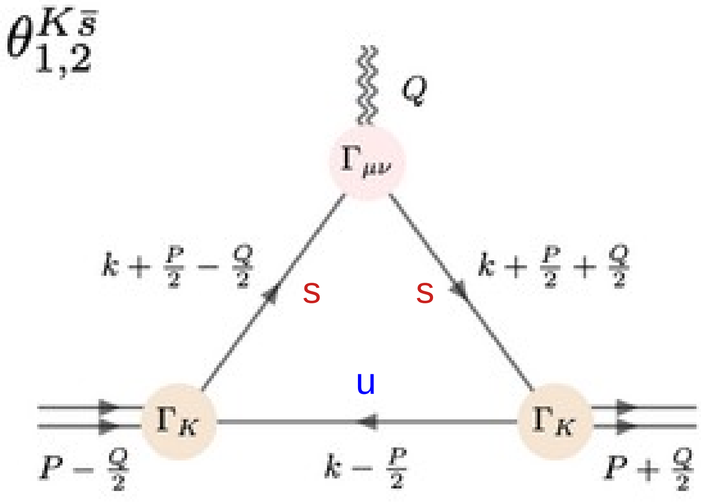

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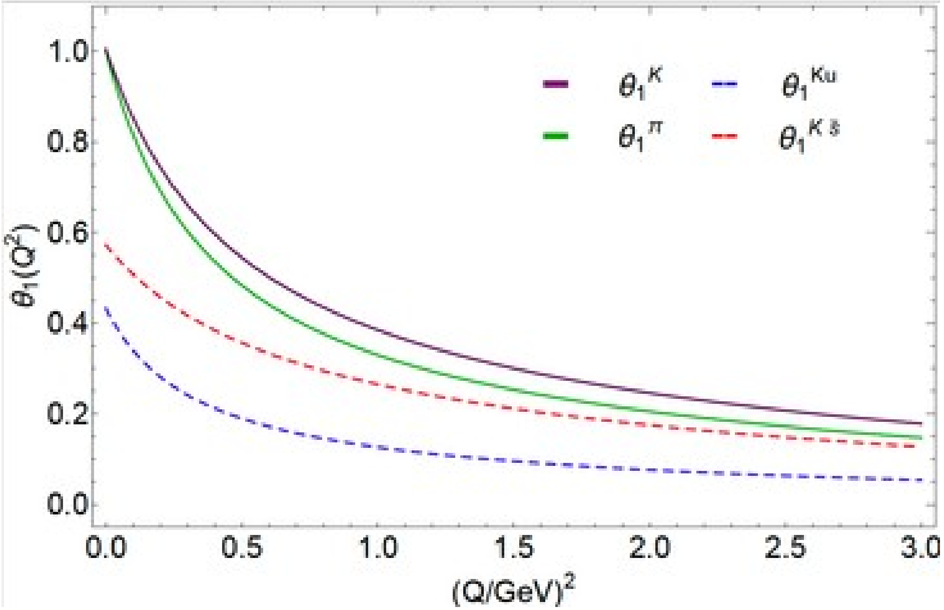
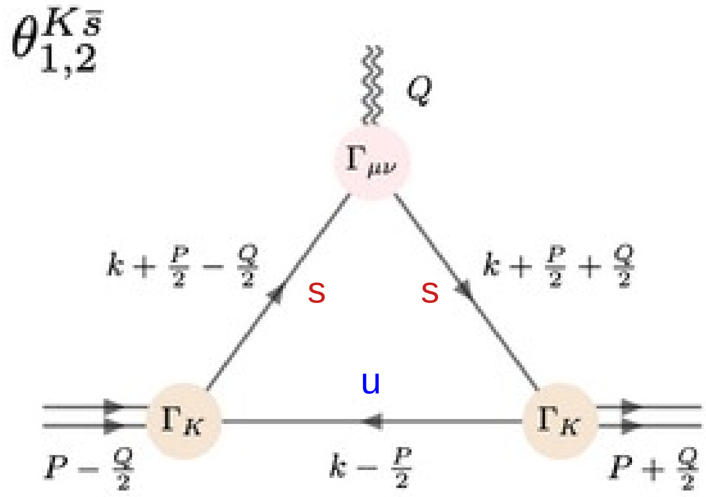
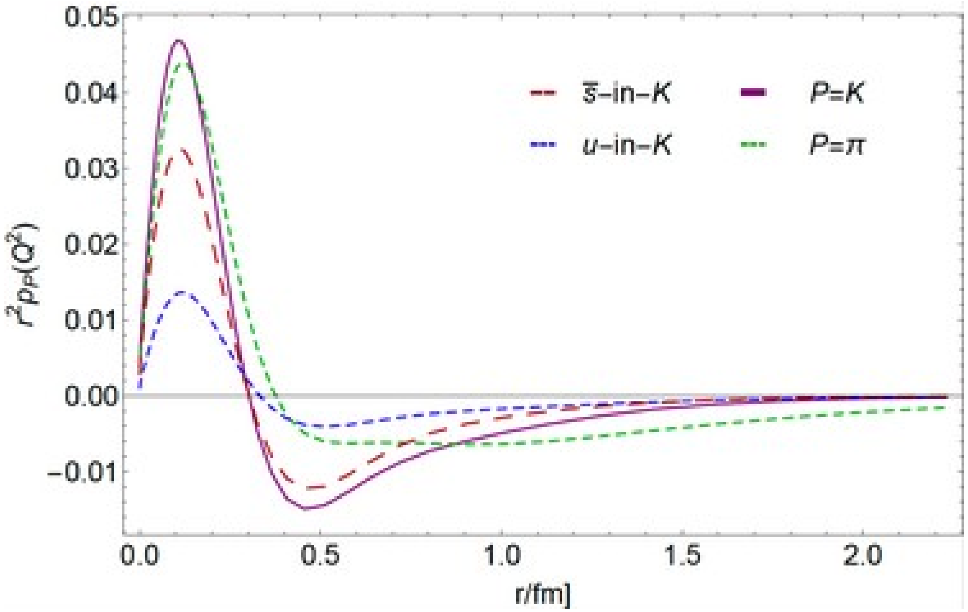
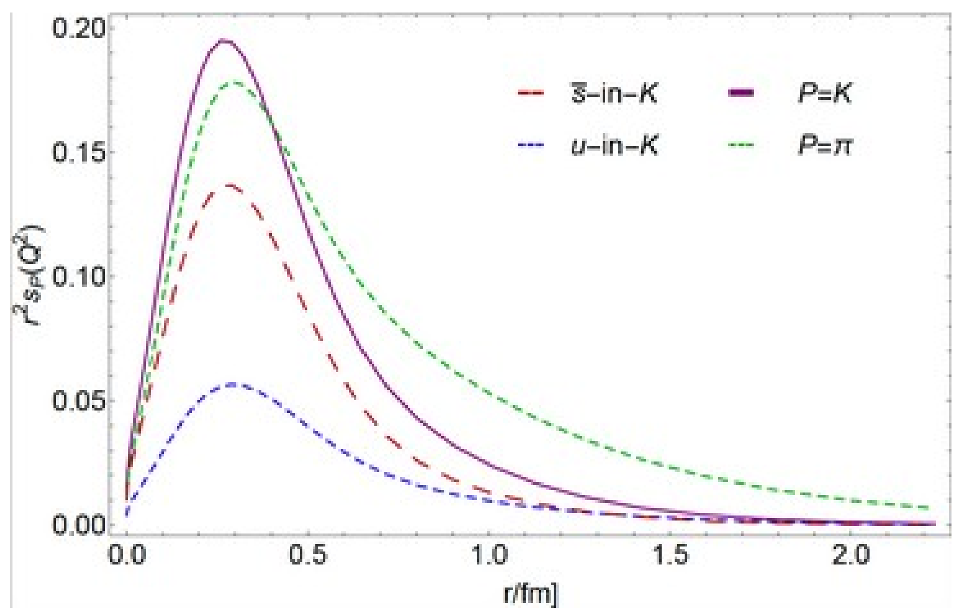








Summary and scopes

Summary and scopes

- We have described a **CSM based** computation of the pion **GFFs,** the new-brand ingredient for which is the **QTV** entering the **game**.

- The obtained results expose the **robustness** of the framework and the importance of **symmetries**:
  - Both **QPV** and **QTV** obey their own **WGTI**
  - This is sufficient to produce a sensible result for $\theta_2(Q^2)$
  - The **QTV** is completed by accounting for the soft-pion theorem, fixing the normalization of $\theta_1(Q^2)$
  - **Beyond I.A.**, additional diagrams are crucial to ensure $\sum_{q,g} \bar{c}(t) = 0$, but not needed for the two other form factors.

- **Physically** meaningful pictures are drawn:
  - **Charge** effects span over a larger domain than **mass** effects
  - **Shear** forces are maximal where **confinement** forces become dominant

- Other hadrons are **within reach:**
  - we can **analogously** proceed with **heavy quarkonia**
  - and, capitalizing on **Faddeev amplitudes**, compute **proton GFFs**

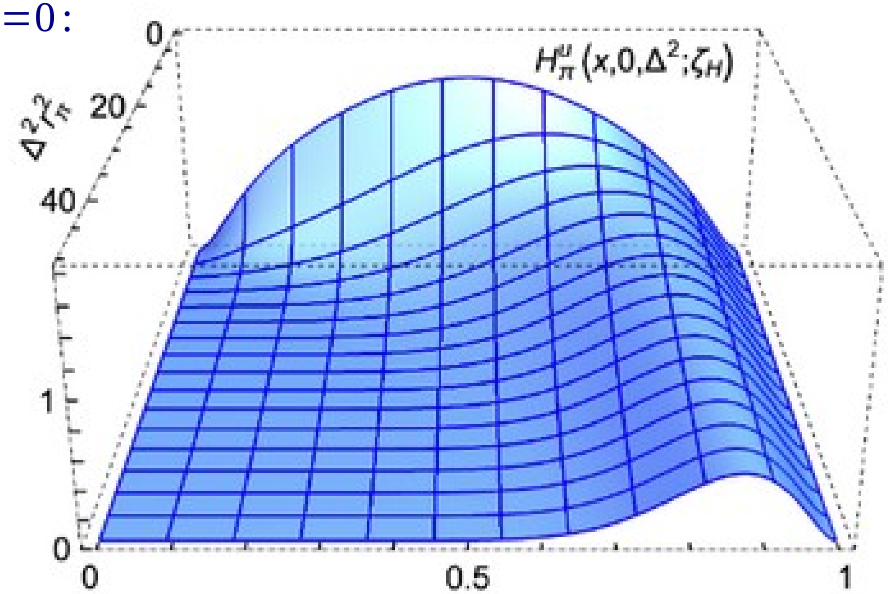
To be continued...

Backslides

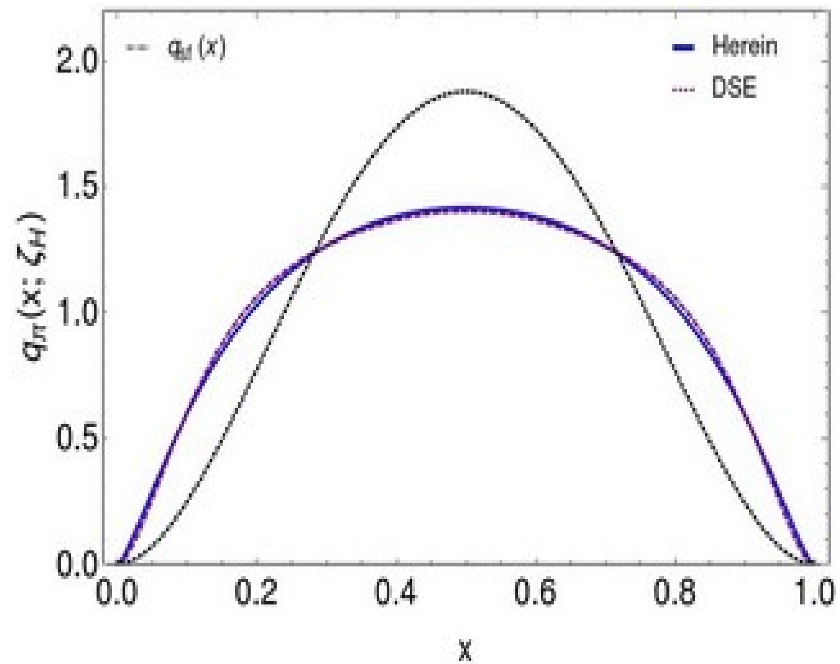
GPDs from LFWFs

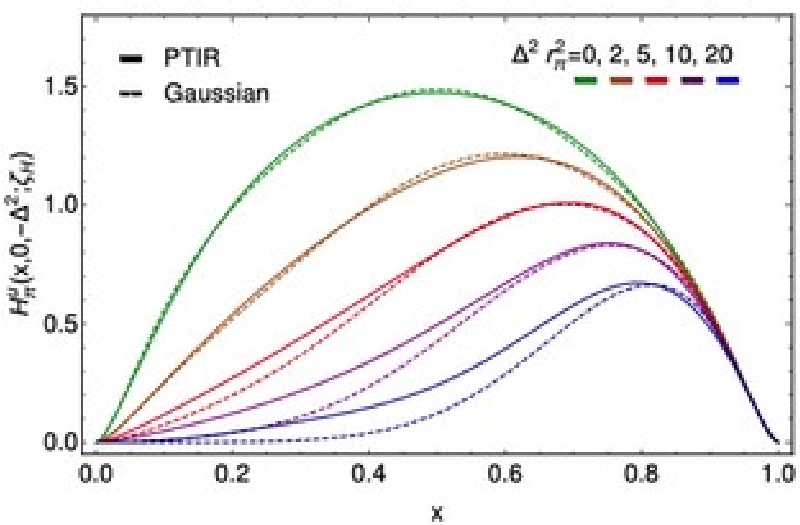
Pion GPD: $H_\pi^u(x,\xi,t;\zeta_H) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\psi^{\uparrow\downarrow *}_{\pi\,u}\left(\frac{x-\xi}{1-\xi},\left(\mathbf{k}_\perp + \frac{\mathbf{1-x}}{\mathbf{1-\xi}}\frac{\mathbf{\Delta}_\perp}{\mathbf{2}}\right)^2;\zeta_H\right)\psi^{\uparrow\downarrow}_{\pi\,u}\left(\frac{x+\xi}{1+\xi},\left(\mathbf{k}_\perp - \frac{\mathbf{1-x}}{\mathbf{1+\xi}}\frac{\mathbf{\Delta}_\perp}{\mathbf{2}}\right)^2;\zeta_H\right)$

$\xi=0:$



Valence-quark overlap GPD and forward PDF limit





Factorized gaussian ansatz:

$$H_\pi^u(x,\xi,t;\zeta_H) = \theta(x-\xi)\sqrt{u^\pi\left(\frac{x-\xi}{1-\xi}\right)u^\pi\left(\frac{x+\xi}{1+\xi}\right)}\,\exp\left(-\frac{-t\,r_\pi^2(1-x)^2}{6\langle x^2\rangle_u^{\zeta_H}(1-\xi^2)}\right)$$

The only (additional) input needed to fix an approximated compact result is the pion charge radius

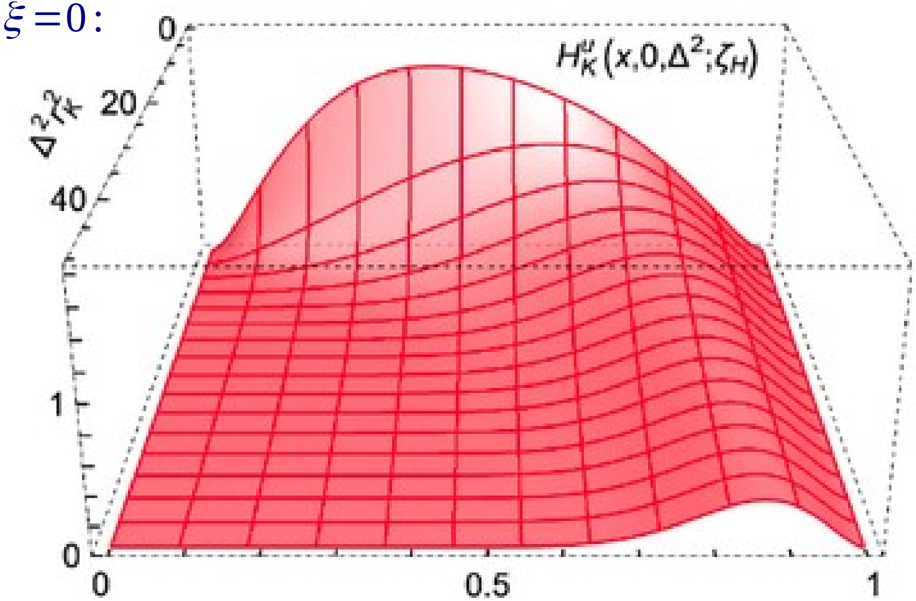
$PDG:\ r_\pi = 0.659(8)\,fm \qquad DSE:\ r_\pi = 0.69\,fm\,[PTIR]$

GPDs from LFWFs

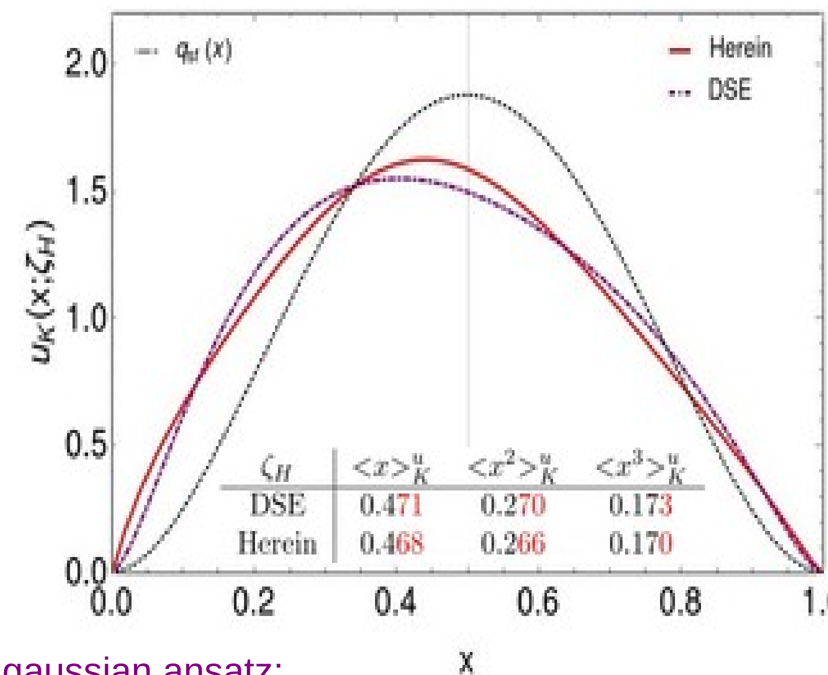
Kaon GPD:

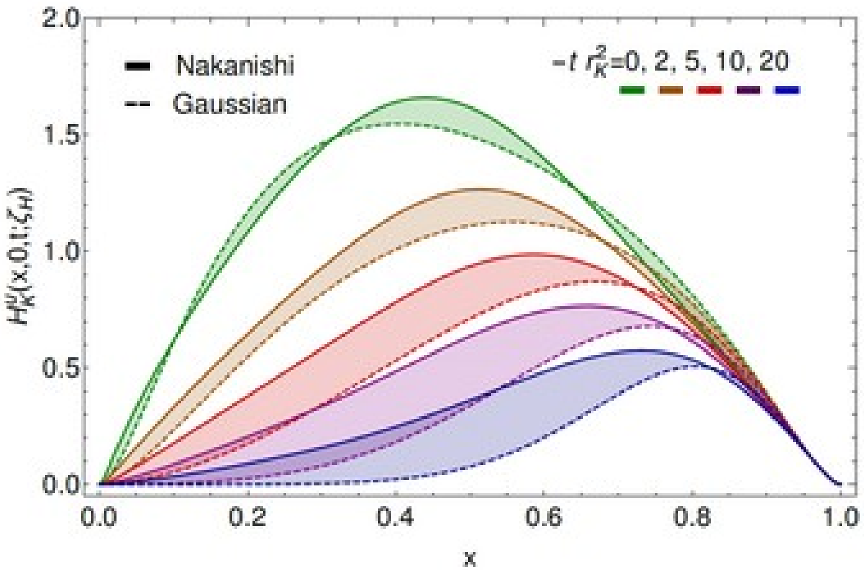
$$H_K^u(x,\xi,t;\zeta_H) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \psi_{K^u}^{\uparrow\downarrow *}\left(\frac{x-\xi}{1-\xi}, \left(\mathbf{k}_\perp + \frac{\mathbf{1-x}}{\mathbf{1-\xi}}\frac{\mathbf{\Delta}_\perp}{\mathbf{2}}\right)^2; \zeta_H\right) \psi_{K^u}^{\uparrow\downarrow}\left(\frac{x+\xi}{1+\xi}, \left(\mathbf{k}_\perp - \frac{\mathbf{1-x}}{\mathbf{1+\xi}}\frac{\mathbf{\Delta}_\perp}{\mathbf{2}}\right)^2; \zeta_H\right)$$

$\xi = 0:$



Valence-quark overlap GPD and forward PDF limit





Factorized gaussian ansatz:

$$H_K^u(x,\xi,t;\zeta_H) = \theta(x-\xi)\sqrt{u^K\left(\frac{x-\xi}{1-\xi}\right)u^K\left(\frac{x+\xi}{1+\xi}\right)} \times \exp\left(-\frac{-t\, r_K^2(1-x)^2}{\left(4\langle x^2\rangle_{\bar{s}}^{\zeta_H} + 2(1+\delta)\langle x^2\rangle_u^{\zeta_H}\right)(1-\xi^2)}\right)$$

The only (additional) input needed to fix an approximated compact result is the pion charge radius

$PDG:\ r_K = 0.560(31)\, fm \qquad DSE:\ r_K = 0.56\, fm\, [PTIR]$

Meson gravitational Form Factors

- **Gravitational** form factors connect with Energy-momentum tensor and are obtained from the **t-dependence** of the **GPD's 1-st Mellin moment**:

$$\theta^{\mathrm{M}}_{1,2}(-t) = \theta^{\mathrm{M}_u}_{1,2}(-t) + \theta^{\mathrm{M}_{\bar{h}}}_{1,2}(-t)$$

$$\int_{-1}^{1} dx\, x H^q_{\mathrm{M}}(x,\xi,t;\zeta_H) = \theta_2^{\mathrm{M}_q}(-t) - \xi^2 \theta_1^{\mathrm{M}_q}(-t)$$

Owing to GPD's polynomiality:

mass distribution

pressure distribution

$$\int_{-1}^{1} dx\, x H^q_{\mathrm{M}}(x,0,t;\zeta_H) = \theta_2^{\mathrm{M}_q}(-t)$$

One needs both DGLAP ($|x| \geq \xi$) and ERBL ($|x| \leq \xi$) GPD to derive the pressure distribution.

ERBL completion → Radon transform inversion

J-L. Zhang et al., arXiv:2101.12286