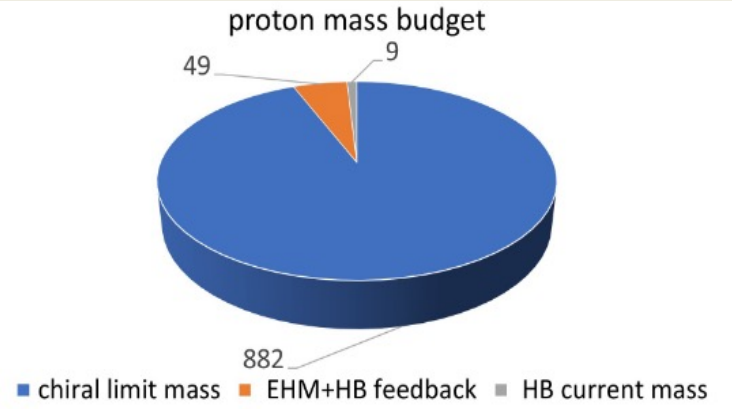
Parton distributions in Nambu-Goldstone Bosons

Lei Chang(常雷)
leichang@nankai.edu.cn

Nankai University

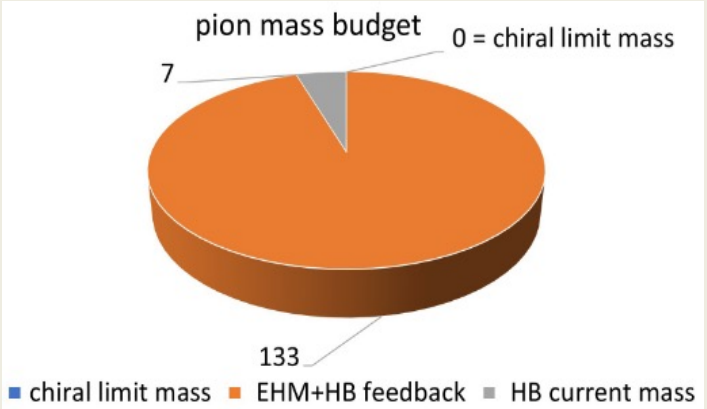
Revealing emergent mass through studies of hadron spectra and structure
2022/09/14, online(ECT*)

QCD: A modern goal

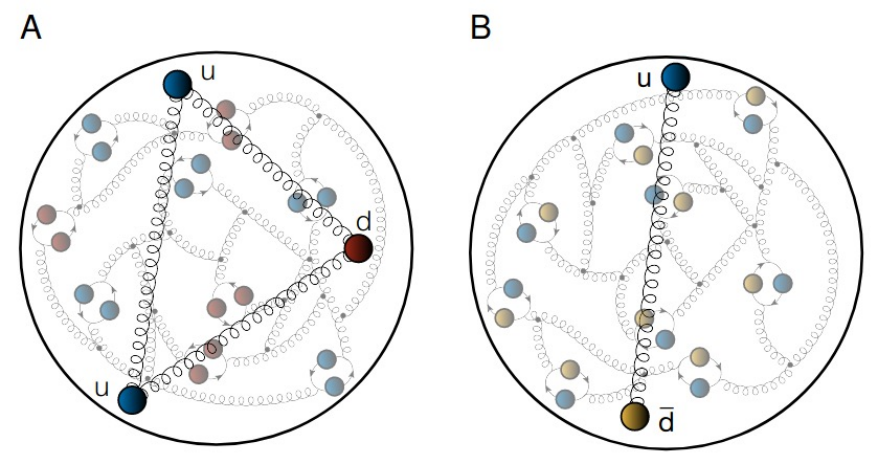


$r_p \approx 0.84fm$
$m_p \approx 0.94GeV$
Spin-1/2

$r_\pi \approx 0.64fm$
$m_\pi \approx 0.14GeV$
Spin-0



Proton **Pion**



Massive, regardless of **Higgs** mass generation

Massless in the absense of **Higgs** mass generation

- **QCD** should explain both the *massiveness* of the proton and the *masslessness* of the pion
- Quark and gluon structure(complex system…scale dependent)

QCD: the ways

Lattice QCD大有可为！

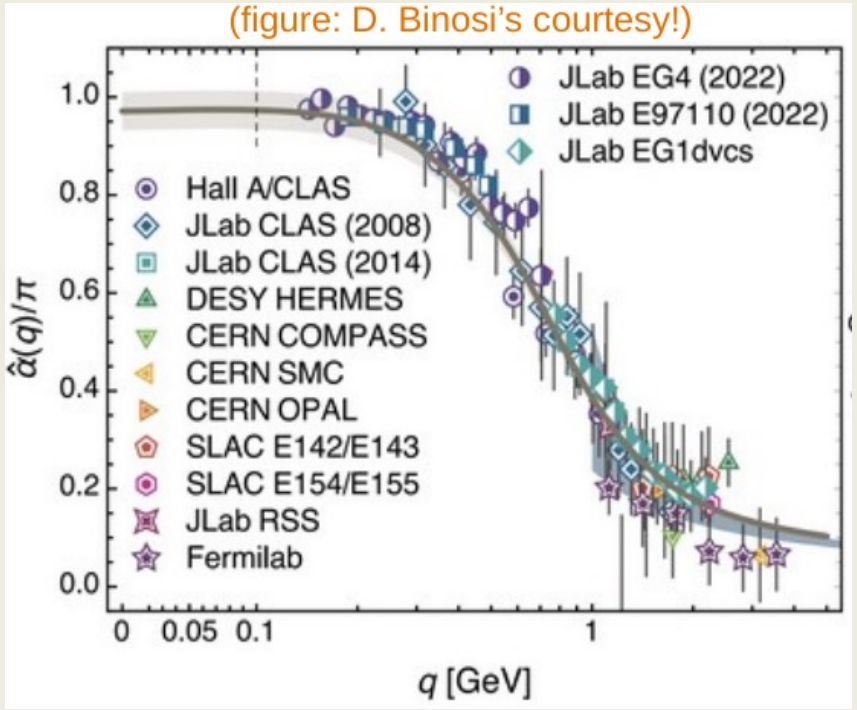
I. INTRODUCTION

Nowadays one sees relatively few papers on continuum non-perturbative QCD, compared to the numbers written a few years ago. Some of the reasons for this state of affairs: a general feeling that Monte Carlo simulations of lattice QCD are the best way to answer all questions, as well as an impression that no systematic non-perturbative treatment of continuum QCD is available or likely to become available.

While there is justification for this line of reasoning, it would certainly be wrong to abandon theoretical QCD in favor of the essentially experimental approach through simulations. At the same time it is hard to know what to make of models, like the bag model, which purport to mimic QCD yet have a prominent ad hoc component to them.

Cornwall, 1985

Effective coupling of QCD

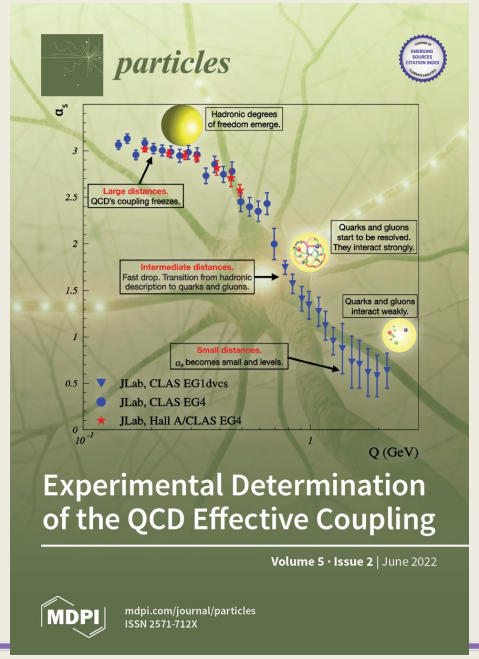


$$\hat{\alpha}(k^2) = \frac{\gamma_m \pi}{\ln\left[\frac{\mathcal{K}^2(k^2)}{\Lambda^2_{\rm QCD}}\right]}\,,\ \mathcal{K}^2(y) = \frac{a_0^2 + a_1 y + y^2}{b_0 + y}$$

Define a screening mass:

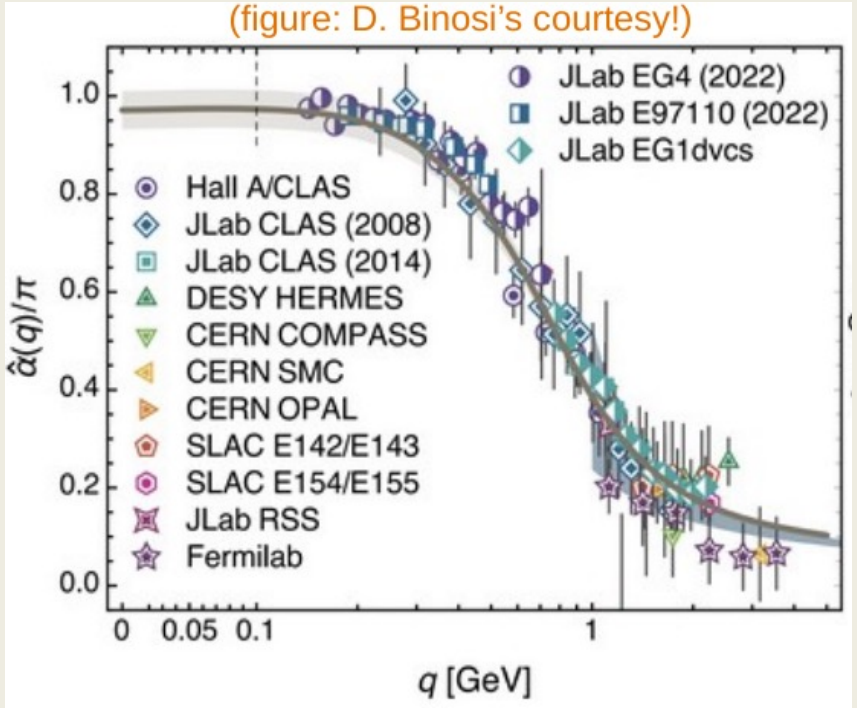
$$m_G := \mathcal{K}(k^2 = \Lambda^2_{QCD}) = 0.331\text{GeV}$$

The running coupling alters at m_G so that modes with $k^2<m^2$ are screened from interactions and theory enters a practically conformal domain.



A Hadronic Scale!

A rough picture

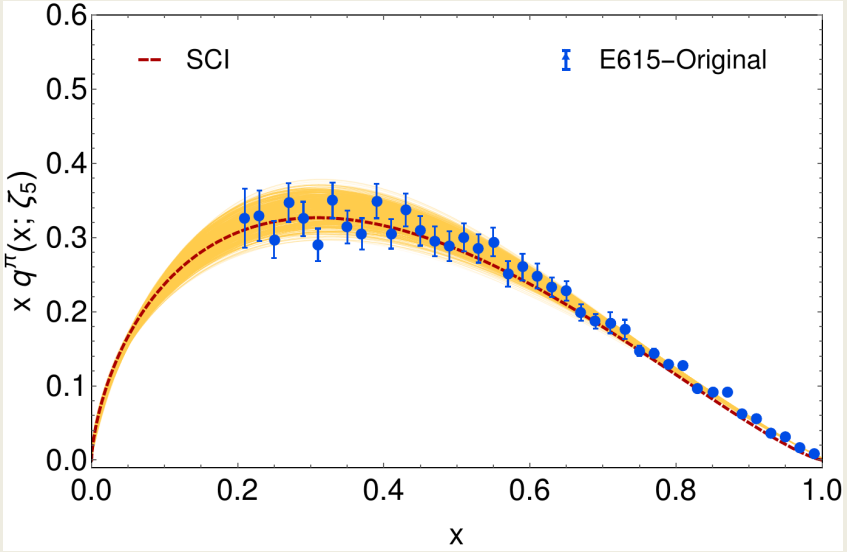


Suppose $\alpha(q) \to$ *constant*

- Gluon Momentum independent
- Quark Mass/Bound state Amplitude…

Pion DA=DF=1 at Hadronic scale in the chiral limit

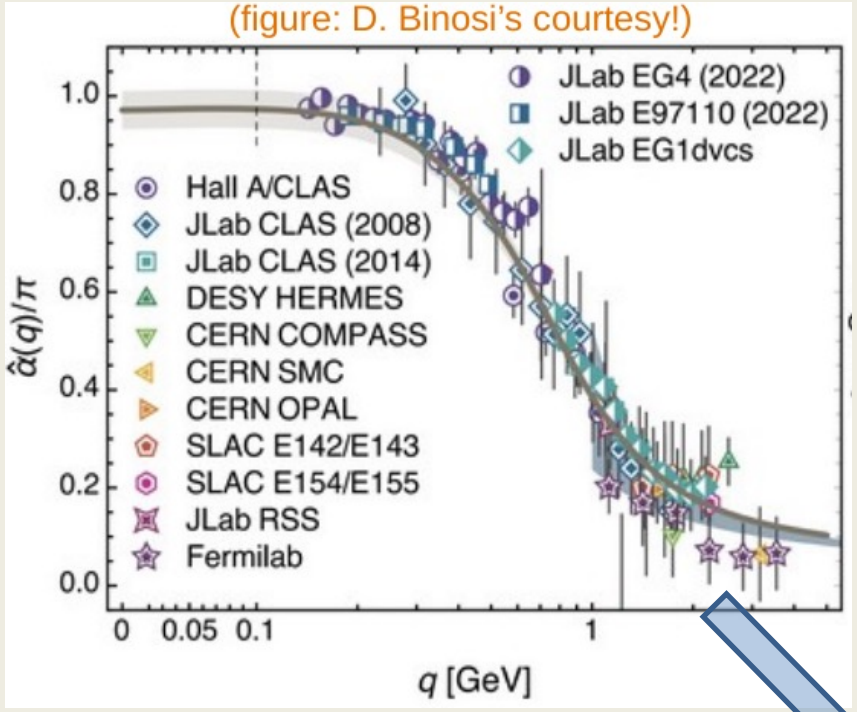
Evolution



E. R. Arriola, W. Broniowski, Phys. Rev. D 66 (2002) 094016.

……

Question?

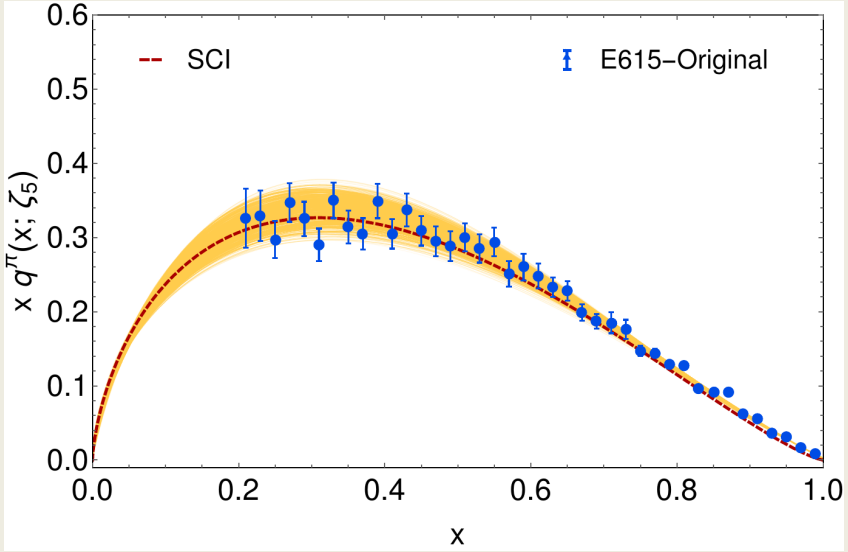


Suppose $\alpha(q) \to constant$

- Gluon Momentum independent
- Quark Mass/Bound state Amplitude…

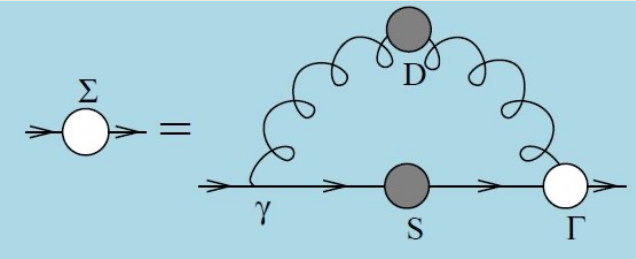
Pion DA=DF=1 at Hadronic scale in the chiral limit

Evolution

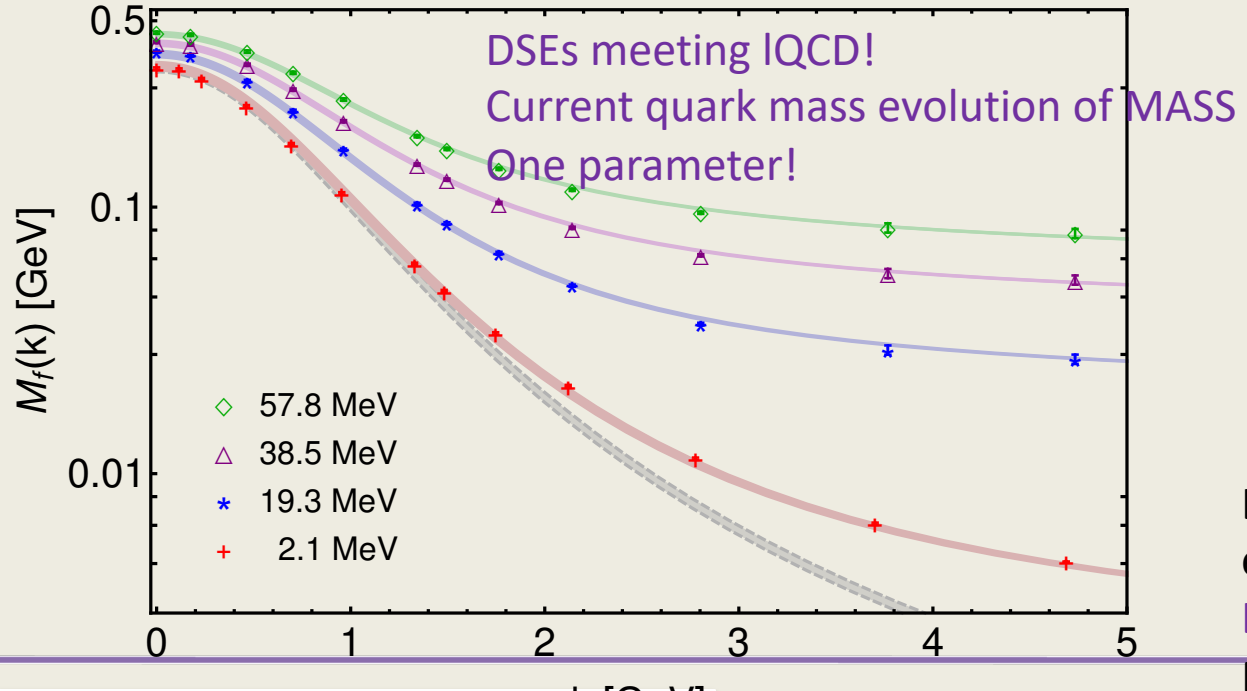


$\alpha(q) \to constant$(infrared)
$\alpha(q) \to$logrithm(ultraviolet)

Quark Mass----Minding the quark-gluon vertex



LC, and C.D.Roberts,PRL103(2009)081601, PRC85(2012)052201;
D. Binosi, et al., PLB742(20015) 183
Sixue Qin, C.D.Roberts, arXiv: 2009.13637



- Truncate quark-gluon vertex with DCSB-improvement ansatz;
- Performing the interaction from lattice QCD;
- Ward identity hold…guarantee proper current quark mass evolution;
- ACM generate quark mass and trigger DCSB.

Perspective of DCSB

Mass scale introduced by PI coupling

⬇

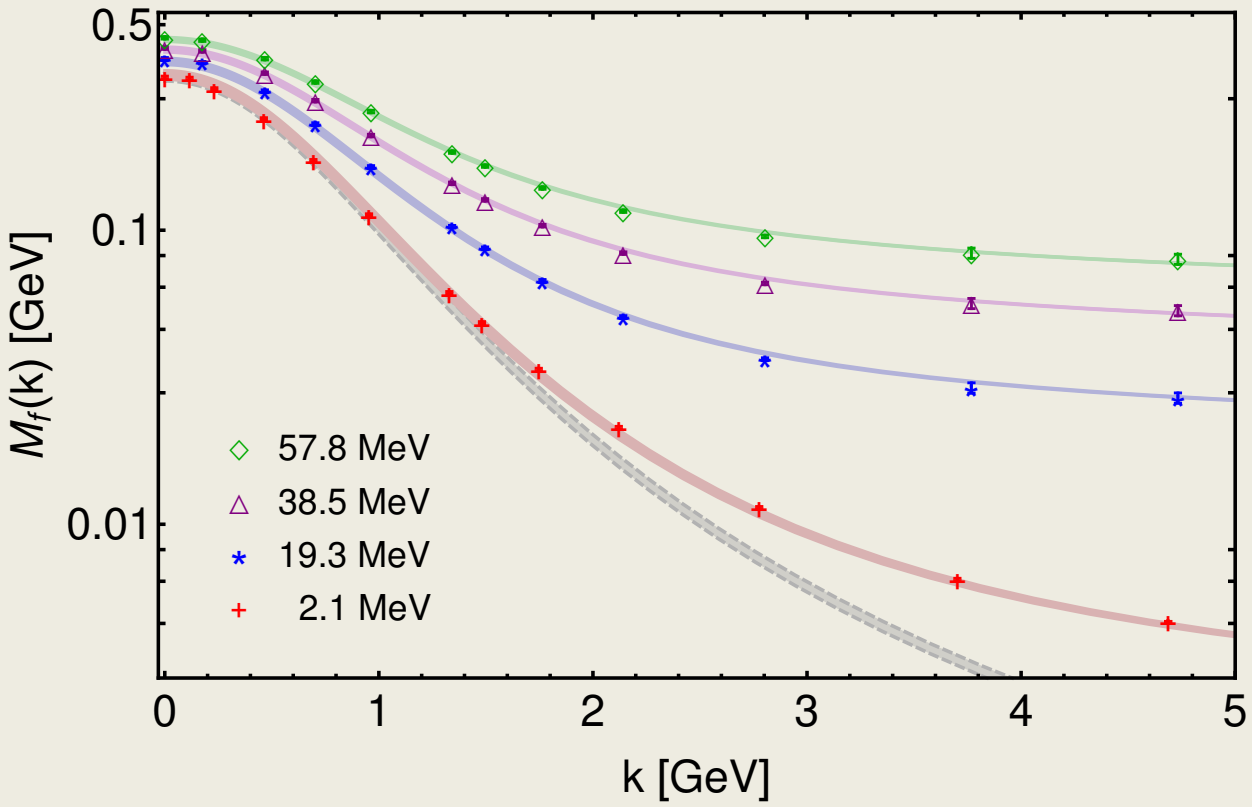
Trigger quark mass

⬇

DCSB enhance quark-gluon coupling

Linking continuum and lattice quark mass functions via an effective charge
LC, et al, Phys.ReV.D104(2021)094509
Bethe-Salpeter kernel and properties of strange-quark mesons
Zhen-Ni Xu, et al, arXiv:2208.13903(INP), See Zhen-Ni's talk

“Constituent” quarks



- The strong interaction of a quark with its (gluon) surrounding gives rise to a “constituent” quark with effective mass M_0;
- M_0 is about $m_p/3$ and runs as a logarithm-corrected $1/k^2$ power-law in the ultraviolet region;
- This consistuent quark has the finite size(B. Povh and J. Hufner, PLB245(1990)653);

Dressed-Quark Anomalous Magnetic Moments

Lei Chang, Yu-Xin Liu, and Craig D. Roberts

Phys. Rev. Lett. **106**, 072001 (2011) - Published 16 February 2011

Pion: Goldstone and Bound state

Maris, Roberts and Tandy, **Phys. Lett. B420**(1998) 267-273

- Pion's Bethe-Salpeter amplitude Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

- Dressed-quark propagator

$$S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$

- Axial-vector Ward-Takahashi identity entails(chiral limit)

$$\boxed{f_\pi E(k;P|P^2=0) = B(k^2)} + (k\cdot P)^2 \frac{d^2B(k^2)}{d^2k^2} + \ldots$$

2M+U=0

$r_M \approx r_\pi$

Valence parton distribution of pion at the hadronic scale

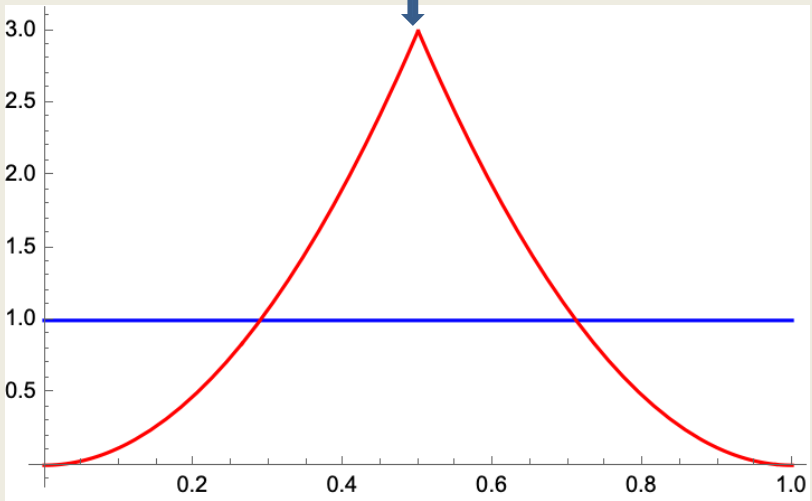
$$f_\pi E(k;P|P^2=0) = B(k^2) + (k\cdot P)^2 \frac{d^2B(k^2)}{d^2k^2} + \ldots$$

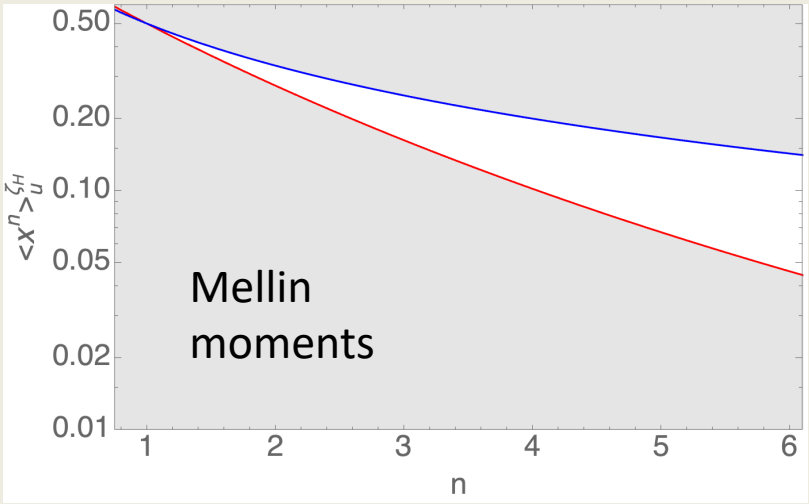
Nakanishi representation

$$E(k;P) \sim \int_{-1}^{1} dz\rho(z) \frac{M^2}{k^2 + z\,k\cdot P + M^2}$$

$\rho(z) = \delta(z)$

$\rho(z) = \delta(1+z) + \delta(1-z)$





Boundaries of Mellin moments at the hadronic scale

Valence parton distribution of pion at the hadronic scale

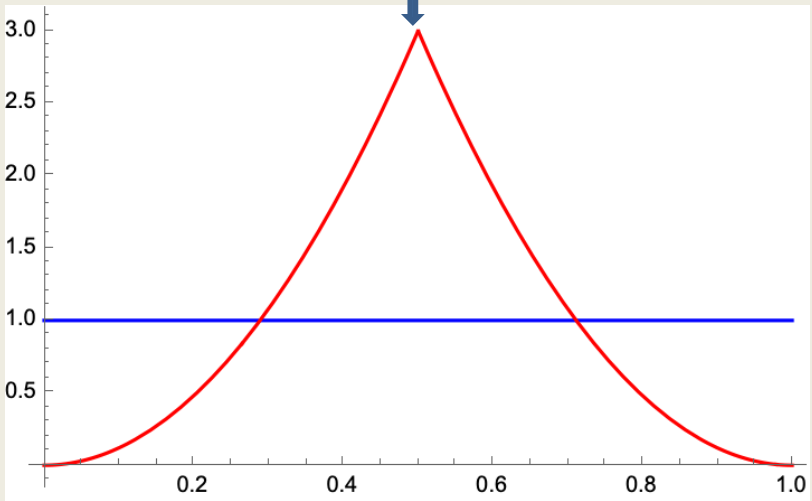
$$f_\pi E(k;P|P^2=0) = B(k^2) + (k\cdot P)^2 \frac{d^2B(k^2)}{d^2k^2} + \ldots$$

Nakanishi representation

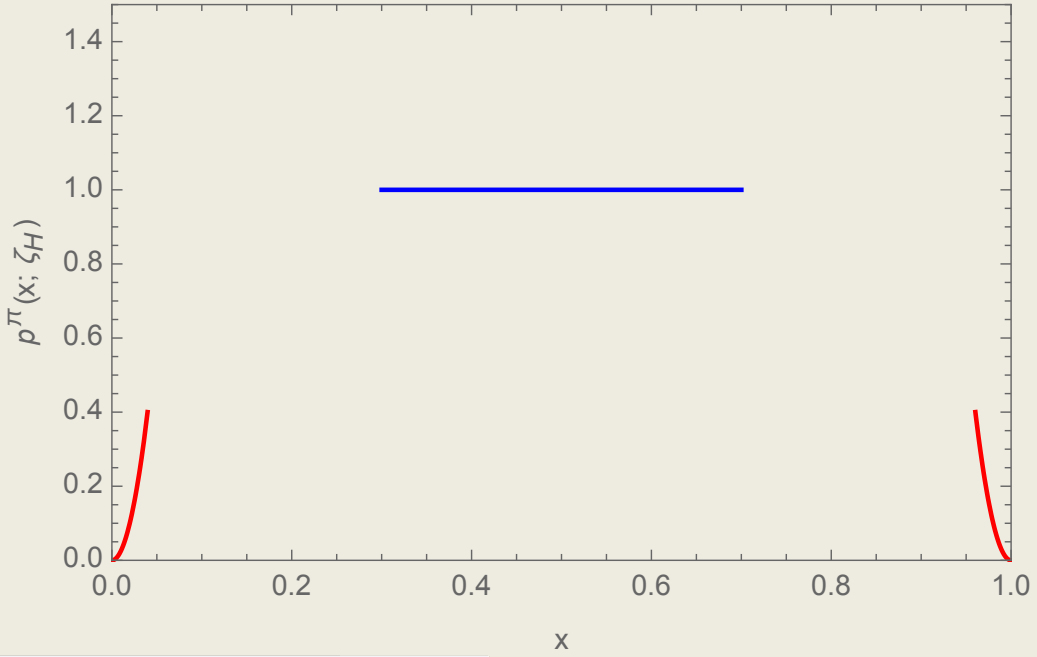
$$E(k;P) \sim \int_{-1}^{1} dz\rho(z) \frac{M^2}{k^2 + z\,k\cdot P + M^2}$$

$\rho(z) = \delta(z)$

$\rho(z) = \delta(1+z) + \delta(1-z)$



- QCD's asymptotic freedom…the spectrum of pion wave function is well defined at the endpoint of z…guarantee $(1-x)^{\beta\geq 2}$ behavior of DF near the endpoints.(in preparation)
- The screening of interaction below the hadronic scale indicates the DF is flat on the middle of x domain.



Pion PDF: Lattice and CSM

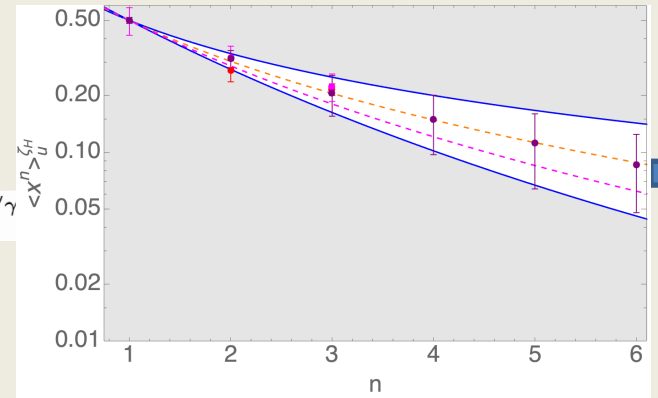
➢ lattice QCD:

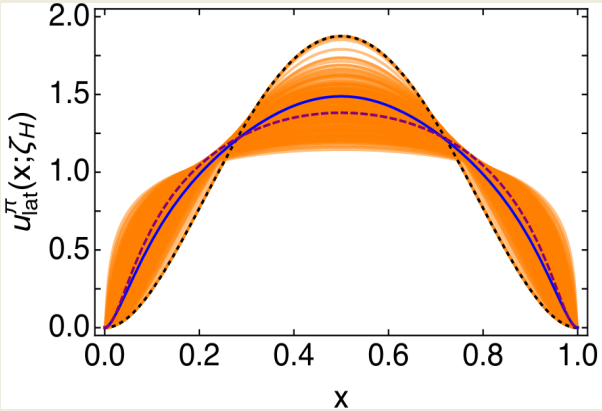
PRD100(2019)114512,99(2019)074507,104(2021)054504

TABLE I. Lattice-QCD results for Mellin moments of the pion valence-quark DF at $\zeta = \zeta_2 = 2\,\mathrm{GeV}$ [65] and $\zeta_5 = 5.2\,\mathrm{GeV}$ [66, 67]

n	[65]	[66]	[67]
1	0.254(03)	0.18(3)	0.23(3)(7)
2	0.094(12)	0.064(10)	0.087(05)(08)
3	0.057(04)	0.030(05)	0.041(05)(09)
4			0.023(05)(06)
5			0.014(04)(05)
6			0.009(03)(03)

$$\langle x^n \rangle^{\zeta}_{u_\pi} (\langle 2x \rangle^{\zeta}_{u_\pi})^{-\gamma_0^n/\gamma}$$

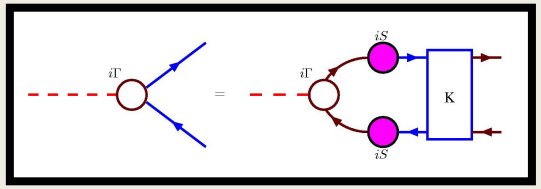


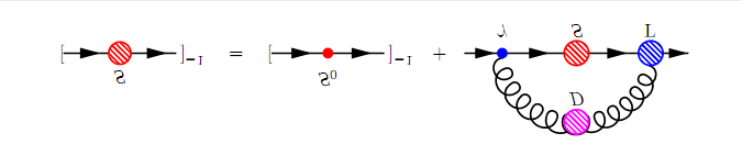


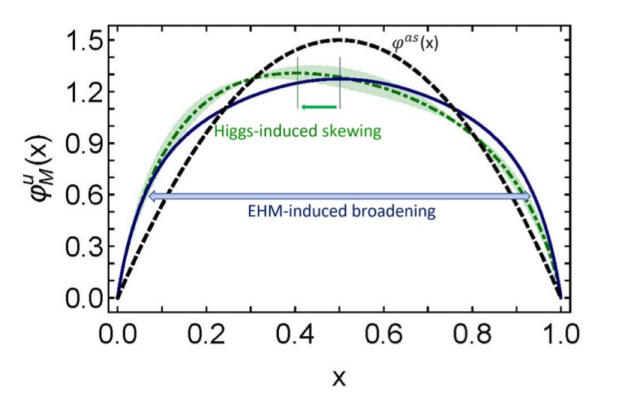
➢ CSM:

Minghui Ding, *et al.*, arXiv:1912.07529

Zhu-Fang Cui, *et al.*, arXiv:2006.14075

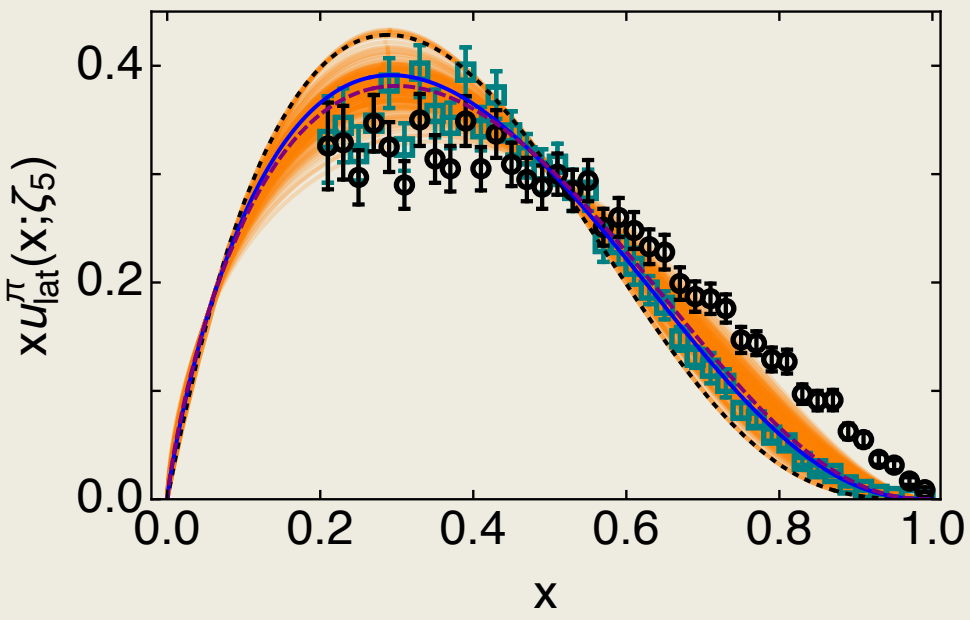


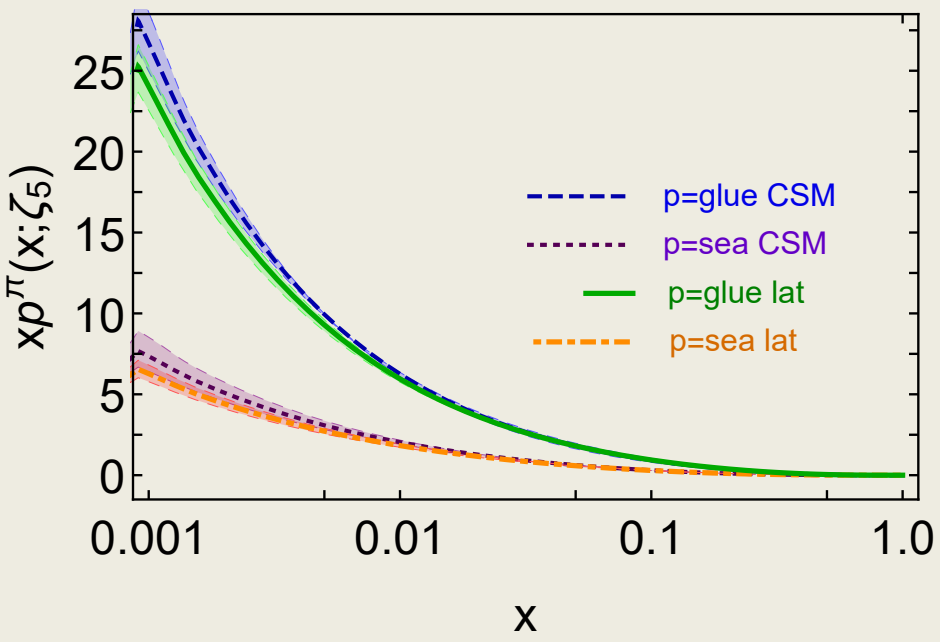




$$u^M(x;\zeta_H) \propto |\varphi_M^u(x;\zeta_H)|^2,$$

Pion PDF: experimental scale





- ✓ Evolved to $\zeta_5 = 5.2GeV$(central curve and associated 1σ-band)

$$u^{\pi}(x;\zeta_5) = n_0^{\zeta_5} x^{\alpha}(1-x)^{\beta}(1+\gamma x^2)\,,$$

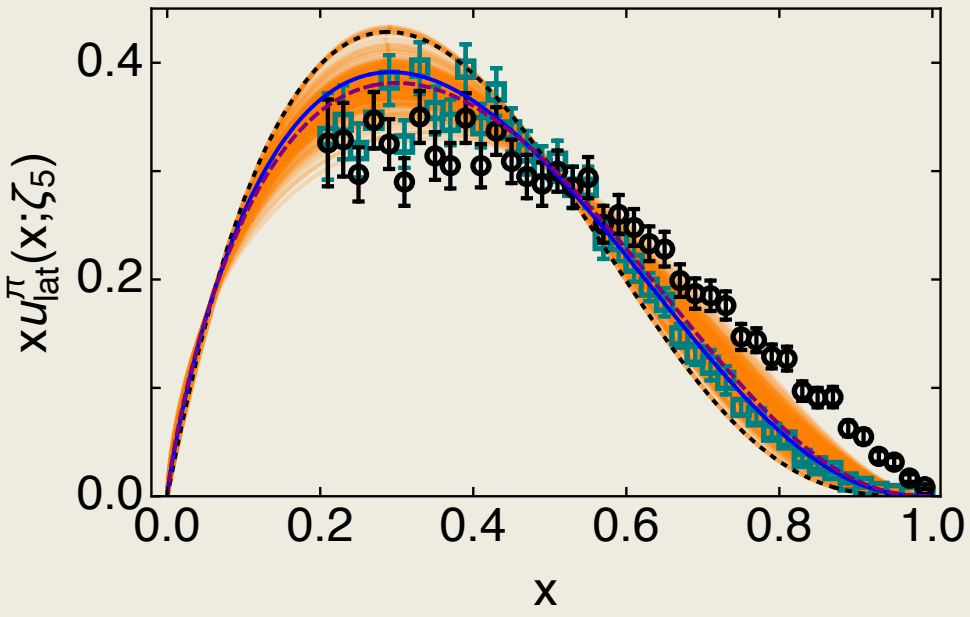
$$\alpha = -0.168(79),\ \beta = 2.49(40),\ \gamma = 1.51(74),$$

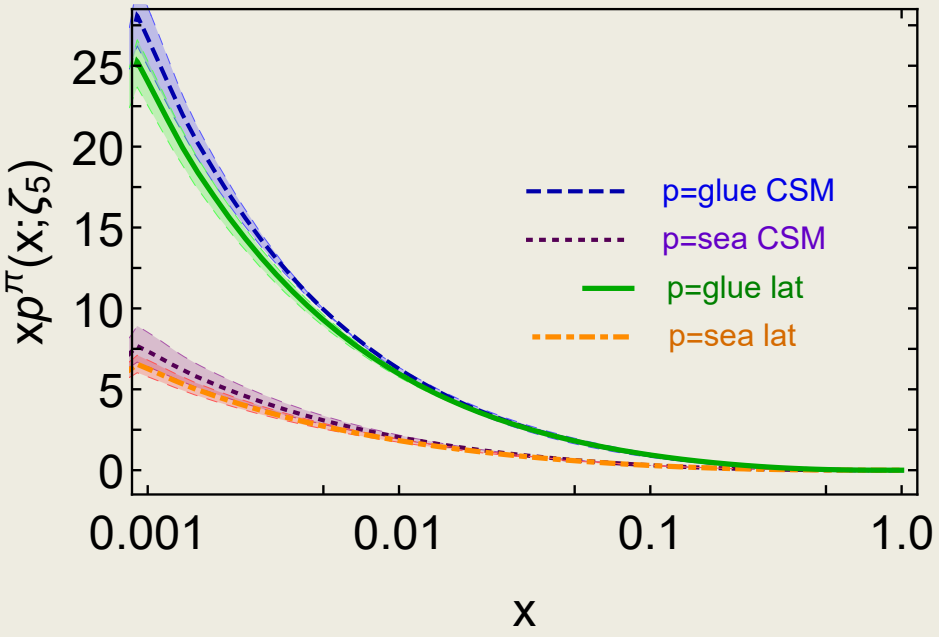
Lattice: $= 0.435(12), 0.435(06), 0.125(05)$

CSM: $= 0.40(2),\quad 0.45(1),\quad 0.14(1)$

- ✓ For the glue and sea quarks, within uncertainties, the lattice QCD based results agree with the CSM predictions.
- ✓ We expect a simulation of DFs(valence and gluon simultaneously) from lattice QCD!

Pion PDF





Thanks for your attention