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Poincare-invariant analysis of strange quark mesons

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In collaboration with:
Prof. S.–X. Qin, Prof. C. D. Roberts, et.al.

September 13, 2022

Beginnings and Beliefs

- Hadron physics … began with the “prediction” of quarks
  - Gell-Mann and Zweig … circa 1964
- 20 years on … Godfrey & Isgur, *Mesons in a Relativized Quark Model with Chromodynamics*, Phys. Rev. D 32 (1985) 189–231

  “. . . All mesons – from the pion to the upsilon – can be described in a unified framework.”

  This statement was made with reference to a quark model potential with one-gluon- like exchange plus linear confinement.
- Remarkable overconfidence …

Hadron spectroscopy: quark model results

Owing to complications introduced by the need to preserve chiral symmetry and its pattern of breaking in QCD, i.e. to faithfully express DCSB, quark models are most reliable for baryons and mesons containing heavy quarks. In such systems, Higgs induced current-quark masses play the dominant role and differences between current- and constituent-quarks are least noticeable.

Aznauryan, A. Bashir, V. Braun, S. Brodsky, V. Burkert et al., Studies of nucleon resonance structure in exclusive meson electroproduction. Int. J. Mod. Phys. E **22**, 1330015 (2013)

J. Segovia, D.R. Entem, F. Fernandez, E. Hernandez, Constituentquark model description of charmonium phenomenology. Int. J. Mod. Phys. E 22, 1330026 (2013)

J. Segovia, D.R. Entem, F. Fernandez, E. Hernandez, Constituent quark model description of charmonium phenomenology. Int. J. Mod. Phys. E 22, 1330026 (2013)

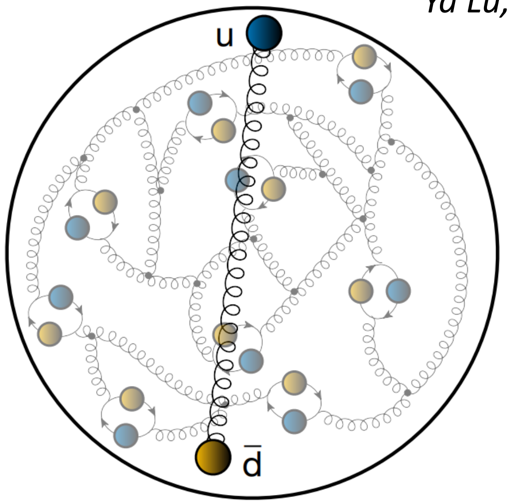
F. Fernández, J. Segovia, Historical introduction to chiral quark models. Symmetry 13(2), 252 (2021)

Strong interaction

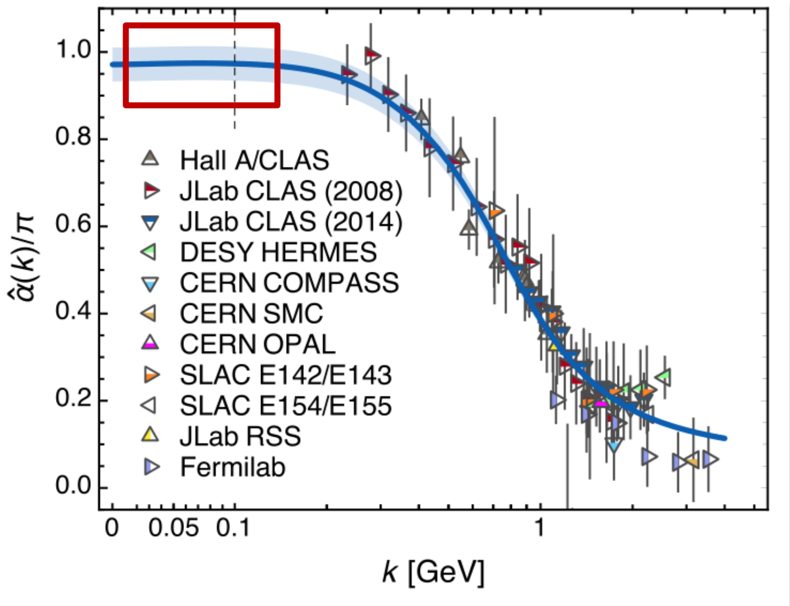
- Pion contains one valence u-quark, one valence $\bar{d}$-quark, and infinitely many gluons and sea quarks. These quarks are bound together by the strong interactions, described by quantum chromodynamics (QCD).

- These interactions are hard for people to make predictions with perturbative approaches due to the large coupling at the typical energy scales in low energy.

Ya Lu, *et al* Phys.Lett.B 830 (2022) 137130



Z.-F. Cui *et al* 2020 *Chinese Phys. C* **44** 083102



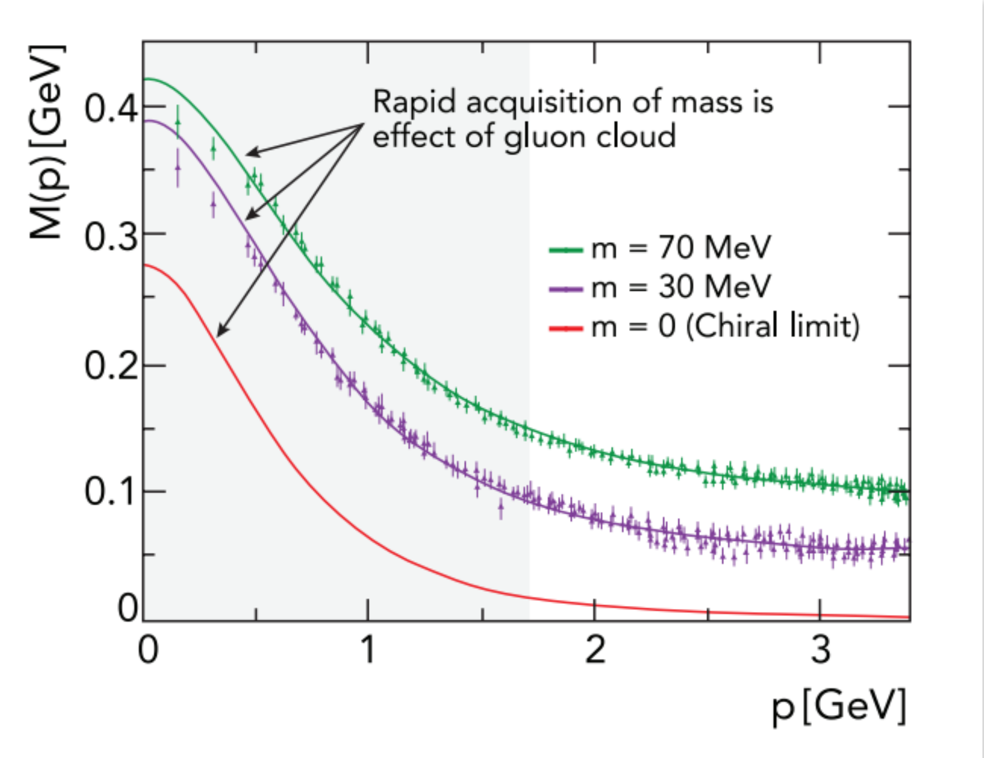
QCD's Running Coupling

Dynamical Chiral Symmetry Breaking

- Renormalisation-group-invariant dressed-quark mass function

$$S(p) = 1/[i\gamma \cdot pA(p^2) + B(p^2)] = Z(p^2)/[i\gamma \cdot p + M(p^2)]$$

M(p^2) : the dressed-quark mass-function



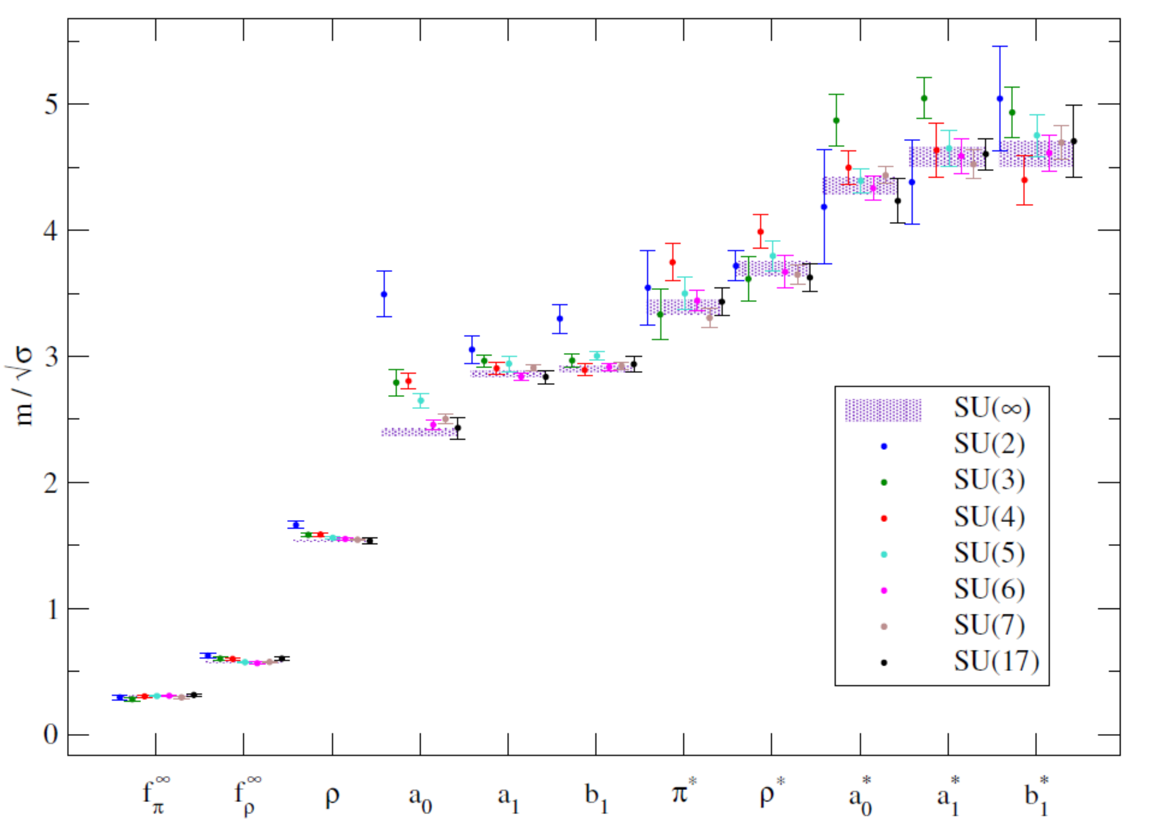
- Goldberger-Treiman relation

$$\boxed{f_\pi^0 E_\pi^0(k^2; P^2 = 0) = B_0(k^2)}$$

$$\Gamma_\pi(k;P) = \gamma_5 \left[iE_\pi(k;P) + \gamma \cdot PF_\pi(k;P) + \gamma \cdot kG_\pi(k;P) + \sigma_{kP} H_\pi(k;P) \right]$$

-- the most fundamental expression of Goldstone's theorem and DCSB

Hadron spectroscopy: lQCD results



G. S. Bali, F. Bursa, L. Castagnini, S. Collins, L. Del Debbio, B. Lucini, M. Panero, PoS (Confinement X) 278 (2013)

- numerical simulations of lattice-regularised quantum chromodynamics (lQCD) provide a direct connection with the Standard Model Lagrangian
  - Notably, few results are available on negative parity states and hadron radial excitations.

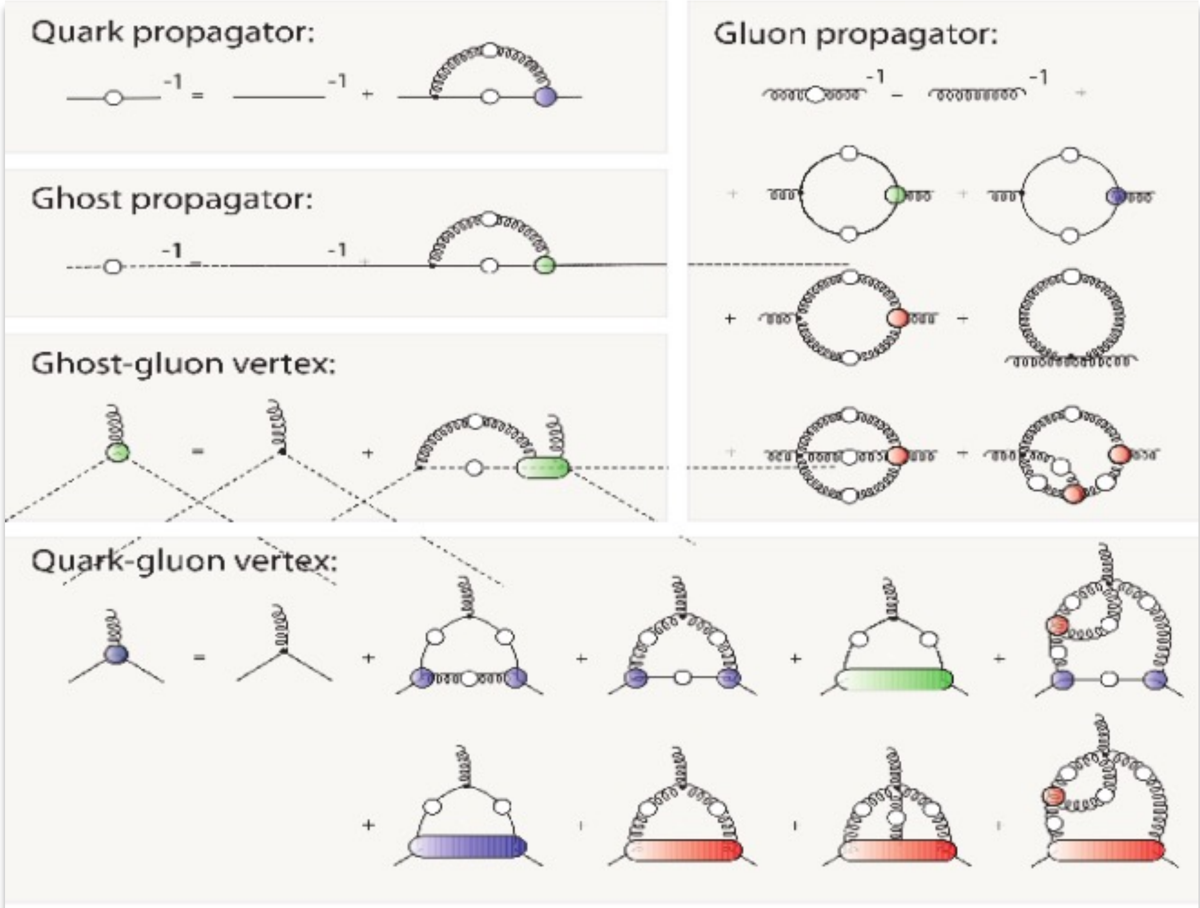
Hadron spectroscopy

- The spectrum of mesons with mass less-than 2 GeV, especially those with strangeness, is poorly understood - both experimentally and theoretically.

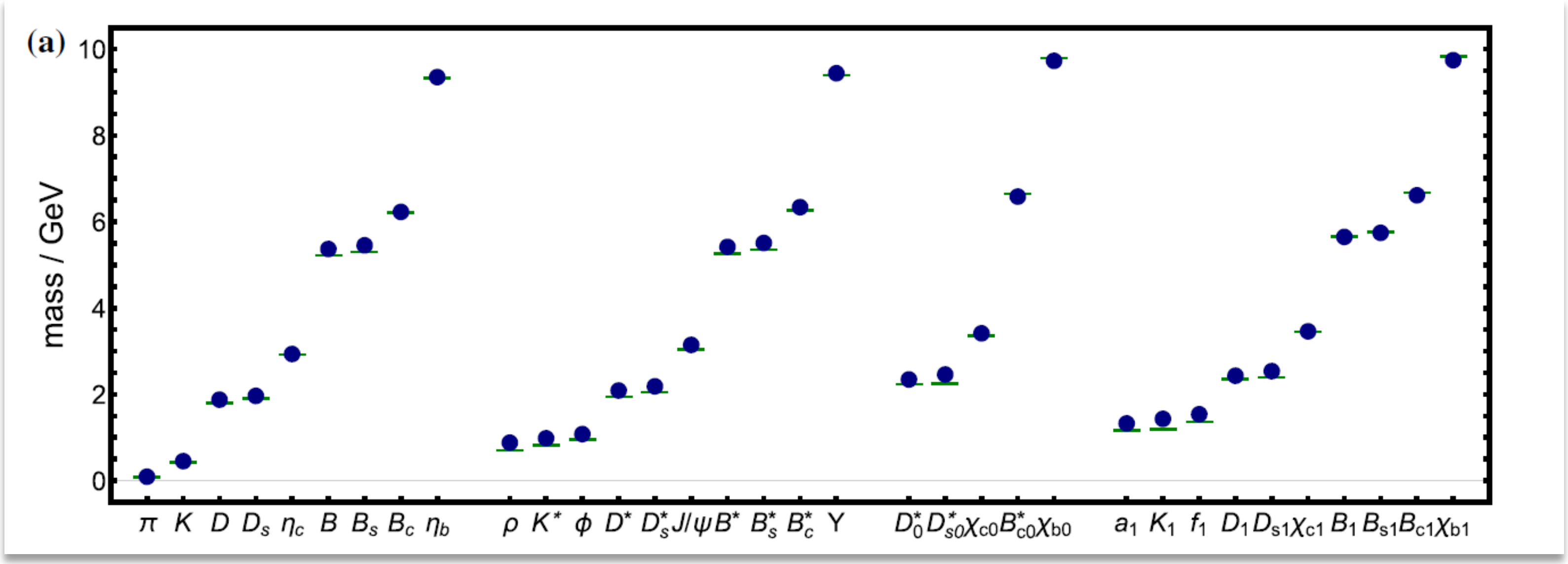
- We address this issue by employing a novel method for constructing a kernel for the meson bound-state problem.

➢ Dyson-Schwinger equations

- construction of a sound, symmetry preserving approximation to the quark + antiquark Bethe-Salpeter kernel



Hadron spectroscopy: contact interaction results



A confining, symmetry-preserving treatment of a vector × vector contact interaction (CI)

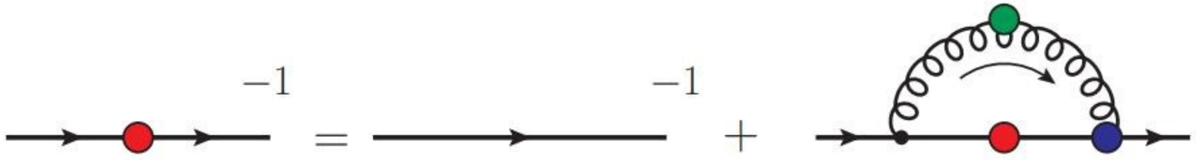
P.-L. Yin, Z.-F. Cui, C. D. Roberts, J. Segovia, Eur. Phys. J. C 81 (4) (2021) 327.

- A primary merit of the framework employed in the contact interaction model is its simplicity, enabling all analyses and calculations to be completed algebraically.

Dyson-Schwinger Equations

- Gap equation

$$S_g^{-1}(k) = i\gamma \cdot k + m_g + \Sigma_g(k)\,,$$
$$\Sigma_g(k) = \int_{dq} 4\pi\alpha\, D_{\mu\nu}(l)\gamma_\mu \frac{\lambda^a}{2} S(q)\Gamma_\nu^g(q,k)\frac{\lambda^a}{2}\,,$$



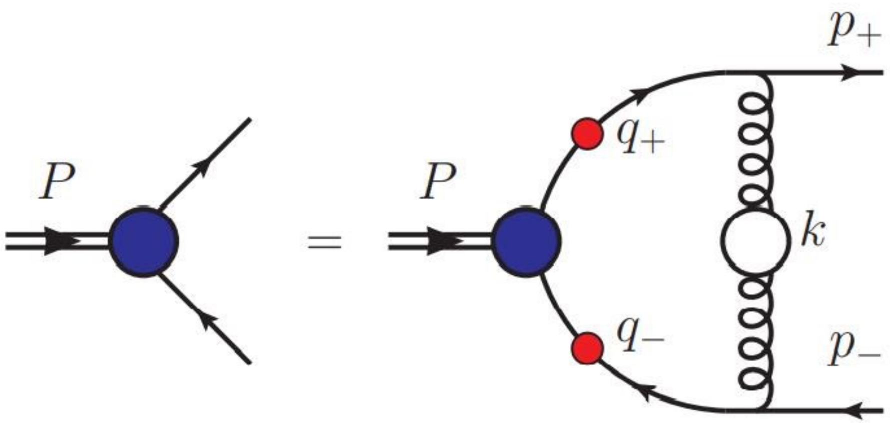
dressed propagator   bare propagator   self energy

$l = k - q$ m_g: Higgs-produced quark current mass $\{\frac{1}{2}\lambda^a | a = 1, \ldots, 8\}$ the generators of SU(3) colour

$D_{\mu\nu}$ the dressed-gluon propagator Γ_ν^g the dressed-gluon-quark vertex

➤ Bethe-Salpeter equation

$$\Gamma^{gh}_{H\alpha\beta}(k;P) = g_H + \int_{dq} K^{(2)}_{\alpha\alpha',\beta'\beta}\chi^{gh}_{H\alpha'\beta'}(q,P)$$



Γ^{gh}_H the bound-state amplitude $P = k_+ - k_-$ the total momentum

g_H Dirac matrix combination that species the $J^{P(c)}$

$K^{(2)}$ the two-particle irreducible quark + antiquark scattering kernel

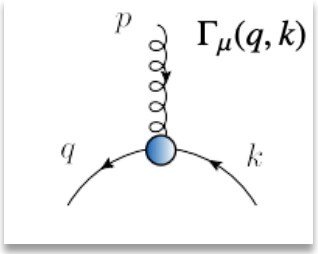
$\chi^{gh}_H(k,P) = S_f(k_+)\Gamma^{gh}_H(k,P)S_g(k_-)$ the unamputated vertex

$$\begin{aligned} K^{(2)}(q_\pm,k_\pm) &= \sum_n K^{(n)}_{L\,\alpha\alpha'}(q_\pm,k_\pm)K^{(n)}_{R\,\beta'\beta}(q_\pm,k_\pm) \\ &=: \sum_n K^{(n)}_L(q_\pm,k_\pm)\otimes K^{(n)}_R(q_\pm,k_\pm)\,. \end{aligned}$$

Truncation

- Rainbow truncation

$$4\pi\alpha D_{\mu\nu}(l)\Gamma^g_\nu(q,k) \to \mathcal{G}_{\mu\nu}(l)\gamma_\nu$$



- In the associated ladder truncation, the Bethe-Salpeter kernel is

$$\mathcal{K}^{rs}_{tu} = \mathcal{G}_{\mu\nu}(l)[i\gamma_\mu \frac{\lambda^a}{2}]_{ts}[i\gamma_\nu \frac{\lambda^a}{2}]_{ru}$$

$$\mathcal{G}_{\mu\nu}(l) = \tilde{\mathcal{G}}(y)T_{\mu\nu}(l)\,, \quad l^2 T_{\mu\nu}(l) \;=\; l^2\delta_{\mu\nu} - l_\mu l_\nu\,.$$

Munczek H J 1995 Phys. Rev. D 52 4736

Bender A, Roberts C D and von Smekal L
1996 Phys. Lett. B 380 7

- serves well for ground-state hadrons with little rest-frame orbital angular momentum between the dressed valence constituents.

- However, it fails for all other systems; its key weakness being omission of those structures which become large as a consequence of EHM.

Interaction

S.-X. Qin, L. Chang, Y.-X. Liu, C. D. Roberts, D. J. Wilson, Phys. Rev. C 85 (2012) 035202.

D. Binosi, L. Chang, J. Papavassiliou, C. D. Roberts, Phys. Lett. B 742 (2015) 183-188.

$$\tilde{\mathcal{G}}(y) = \boxed{\frac{8\pi^2 D}{\omega^4} e^{-y/\omega^2}} + \frac{8\pi^2 \gamma_m \mathcal{F}(y)}{\ln\left[\tau + (1 + y/\Lambda_{\rm QCD}^2)^2\right]}$$

where $\gamma_m = 4/\beta_0$, $\beta_0 = 25/3$, $\Lambda_{\rm QCD} = 0.234\,\mathrm{GeV}$, $\ln(\tau + 1) = 2$, and

$$\mathcal{F}(y) = \{1 - \exp(-y/[4m_t^2])\}/y$$

- $0 < \tilde{\mathcal{G}}(0) < \infty$ A nonzero gluon mass-scale appears as a consequence of EHM in QCD.

  Regarding masses, this is critical: even a symmetry-preserving treatment of a momentum-independent interaction can deliver good results.

- The large-y behaviour ensures that the one-loop renormalisation group flow of QCD is preserved.

EHM improved kernel

EHM is known to generate a large anomalous chromomagnetic moment (ACM) for the lighter quarks, and this ACM has a marked impact on the u, d meson spectrum.

D. Binosi, L. Chang, J. Papavassiliou, S.-X. Qin, C. D. Roberts, Phys. Rev. D 95 (2017) 031501(R).

L. Chang, Y.-X. Liu, C. D. Roberts, Phys. Rev. Lett. 106 (2011) 072001.

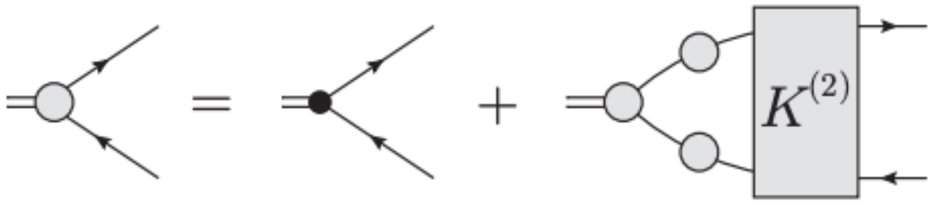
A. Bashir, R. Bermudez, L. Chang, C. D. Roberts, Phys. Rev. C 85 (2012) 045205.

S.-X. Qin, C. D. Roberts, Chin. Phys. Lett. Express 38 (7) (2021) 071201.

$$\Gamma^g_\nu(q,k) = \gamma_\nu + \tau_\nu(l)\,,\;\; \tau_\nu(l) = \eta\kappa(l^2)\sigma_{l\nu}$$

$$\sigma_{l\nu} = \sigma_{\rho\nu} l_\rho,\;\; \kappa(l^2) = (1/\omega)\exp(-l^2/\omega^2)$$

EHM improved kernel



$$K^{(2)}(q_\pm, k_\pm) = \sum_n K^{(n)}_{L\,\alpha\alpha'}(q_\pm, k_\pm) K^{(n)}_{R\,\beta'\beta}(q_\pm, k_\pm)$$
$$=: \sum_n K^{(n)}_L(q_\pm, k_\pm) \otimes K^{(n)}_R(q_\pm, k_\pm)\,.$$

$$\Gamma^{gh}_{H\alpha\beta}(k;P) = g_H + \int_{dq} K^{(2)}_{\alpha\alpha',\beta'\beta} \chi^{gh}_{H\alpha'\beta'}(q,P)$$

- Even under parity operations, forbidding the structures:

$$\mathbf{1} \otimes \gamma_5\,, \ \gamma_\mu \otimes \gamma_5\gamma_\mu\,, \ \text{etc.}$$

- C-parity even: with C the charge conjugation matrix, the following identity is required:

$$K^{(2)}(q_\pm, k_\pm) = \sum_n C[K^{(n)}_R(-k_\mp, -q_\mp)]^{\mathrm{T}} C^\dagger$$
$$\otimes\ C[K^{(n)}_L(-k_\mp, -q_\mp)]^{\mathrm{T}} C^\dagger\,.$$

- Continuous symmetries, prominent amongst which are those expressed in the vector and axial-vector Ward-Green-Takahashi (WGT) identities

$$P_\mu \chi^{gh}_\mu(k_+, k_-) = i\Delta^\pm_{S_{gh}}(k) + i(m_g - m_h)\chi^{gh}_0(k_+, k_-)\,,$$
$$P_\mu \chi^{gh}_{5\mu}(k_+, k_-) = i\Delta^\pm_{S5}(k) - i(m_g + m_h)\chi^{gh}_5(k_+, k_-)$$

Qin S X, Chang L, Liu Y X, Roberts C D and Schmidt S M 2013 Phys. Lett. B 722 384

Qin S X, Roberts C D and Schmidt S M 2014 Phys. Lett. B 733 202

EHM improved kernel

$$K^{(2)} = -\mathcal{G}_{\mu\nu}(l)\gamma_\mu \otimes \gamma_\nu - \mathcal{G}_{\mu\nu}(l)\gamma_\mu \otimes \tau_\nu(l) + \mathcal{G}_{\mu\nu}(l)\tau_\nu(l) \otimes \gamma_\mu + K_{\rm ad}.$$

$$K_{\rm ad} = [\mathbf{1} \otimes_+ \mathbf{1}] f_{p0}^{(+)} + [-\mathcal{G}_{\mu\nu}(l)\gamma_\mu \otimes_+ \gamma_\nu] f_{p1}^{(-)} + [\mathbf{1} \otimes_- \mathbf{1}] f_{n0}^{(+)} + [-\mathcal{G}_{\mu\nu}(l)\sigma_{l\mu} \otimes_- \sigma_{l\nu}] f_{n1}^{(+)}$$

one arrives at the following integral equations,

$$\int_{dq} \mathcal{G}_{\mu\nu}(l)\gamma_\mu s_A^g(q_+)\tau_\nu(l) = \int_{dq} [s_B^g(q_+) f_{p0}^{(+)} + \mathcal{G}_{\mu\nu}(l)\gamma_\mu s_B^h(q_-)\gamma_\nu f_{p1}^{(-)}],$$

$$\int_{dq} \mathcal{G}_{\mu\nu}(l)\gamma_\mu s_B^g(q_+)\tau_\nu(l) = \int_{dq} [s_A^g(q_+) f_{n0}^{(+)} - \mathcal{G}_{\mu\nu}(l)\sigma_{l\mu} s_A^h(q_+)\sigma_{l\nu} f_{n1}^{(+)}].$$

$$S_g(k) =: s_A^g(k) + s_B^g(k),\ \{s_A^g, \gamma_5\} = 0 = [s_B^g, \gamma_5].$$

- Minimal *Ansätze*
- deliver material improvements over leading-order results; in many cases, restoring crucial symmetries that would otherwise be broken.
- for an arbitrary vertex in the family specified by the following equations, one has symmetry-preserving EHM improved Bethe-Salpeter kernels for use in calculating u, d, s meson bound-state properties.

$$\Gamma_\nu^g(q,k) = \gamma_\nu + \tau_\nu(l),\ \tau_\nu(l) = \eta\kappa(l^2)\sigma_{l\nu}$$

GMOR and GT Relations

$$\tilde{\mathcal{G}}(y) = \frac{8\pi^2 D}{\omega^4} e^{-y/\omega^2}$$

$$\omega = 0.8\,\mathrm{GeV} \qquad \omega D_{\mathrm{RL}} = (1.01\,\mathrm{GeV})^3$$

$$m_u = m_d = 2.7\,\mathrm{MeV}, \; m_s = 72\,\mathrm{MeV}.$$

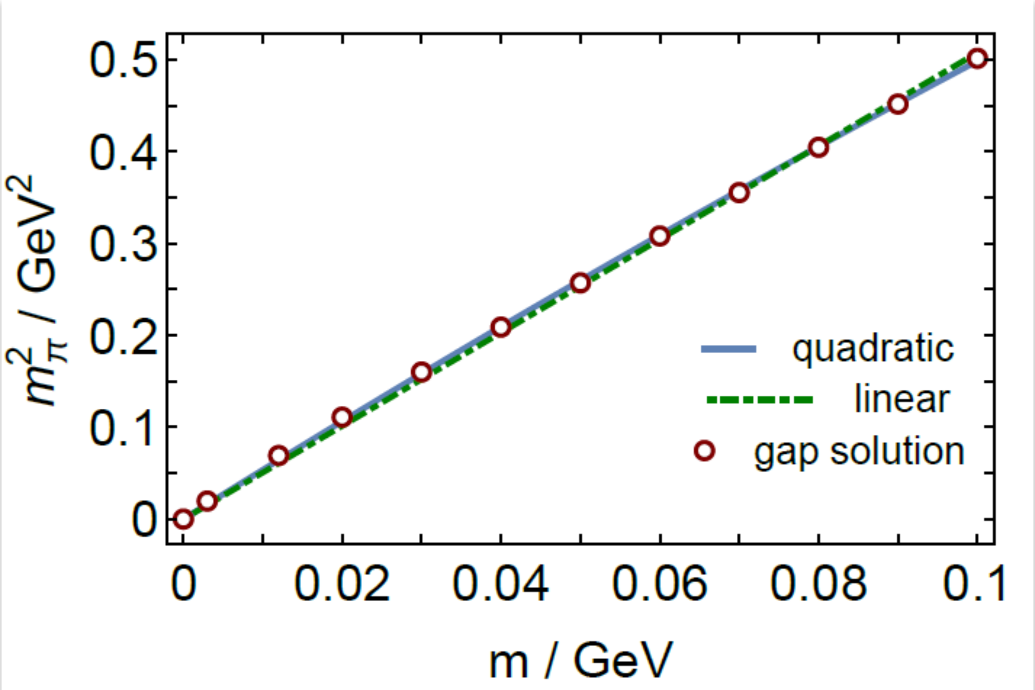
$2m_s/(m_u + m_d) = 26.7$ a good match, cf. $27.3^{+0.7}_{-0.8}$

$$\Gamma^g_\nu(q,k) = \gamma_\nu + \tau_\nu(l)\,, \; \tau_\nu(l) = \eta\kappa(l^2)\sigma_{l\nu}$$

Adjusting D so as to maintain $m_\rho = 0.77GeV$. Given that $\eta > 0$ adds EHM strength to the gap equation's kernel, then D must decrease with increasing η in order to achieve this outcome.

$$D(\eta) \stackrel{\eta\in[0,1.6]}{=} D_{\mathrm{RL}} \frac{1 + 0.27\eta}{1 + 1.47\eta}$$

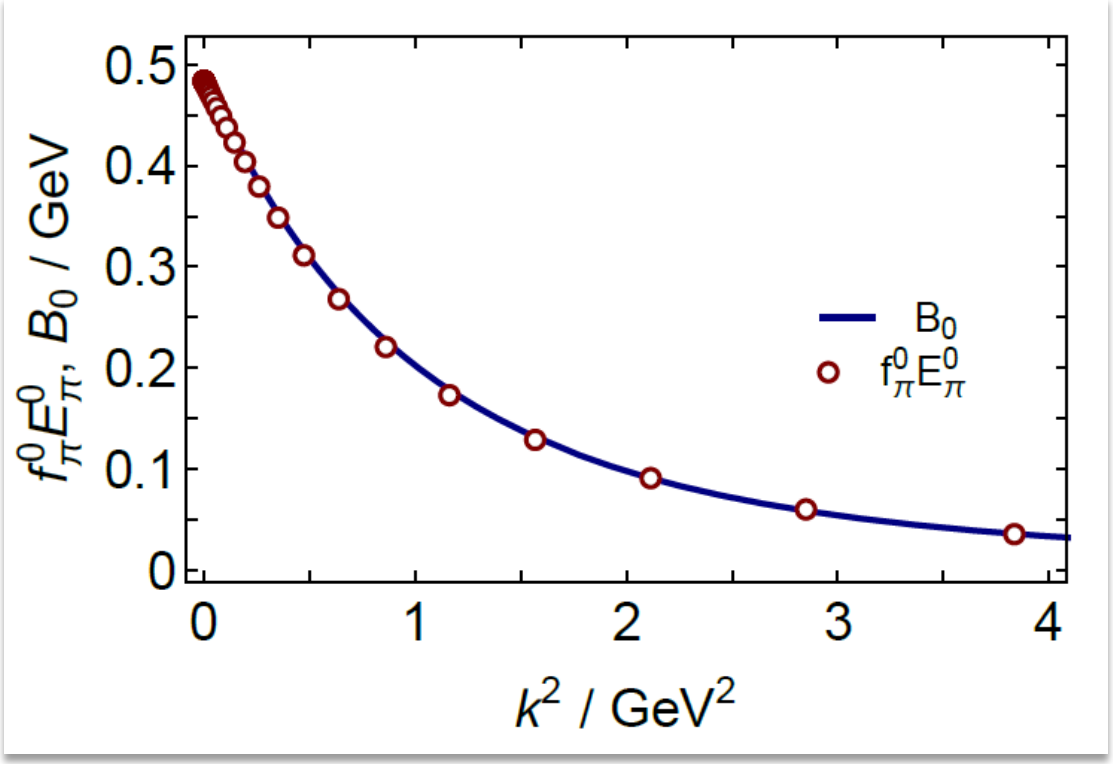
the Gell-Mann-Oakes-Renner relation



S.-X. Qin, C. D. Roberts, Chin. Phys. Lett. Express 38 (7) (2021) 071201.

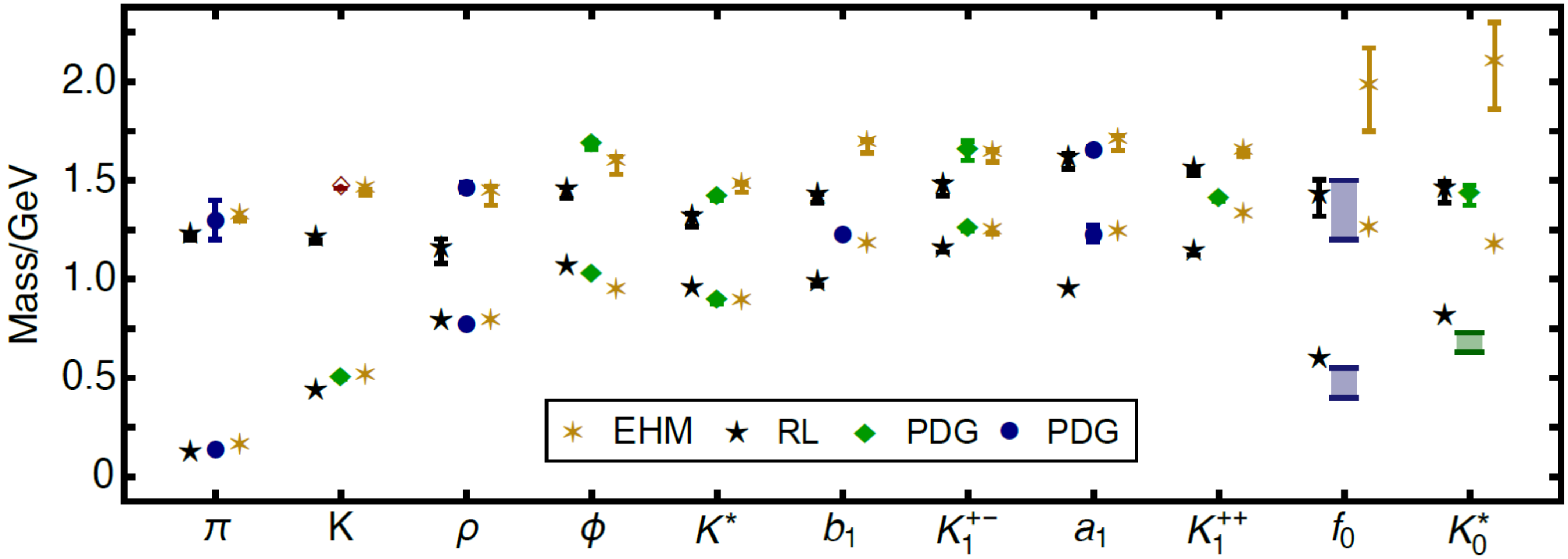
the chiral-limit Goldberger-Treiman relation:

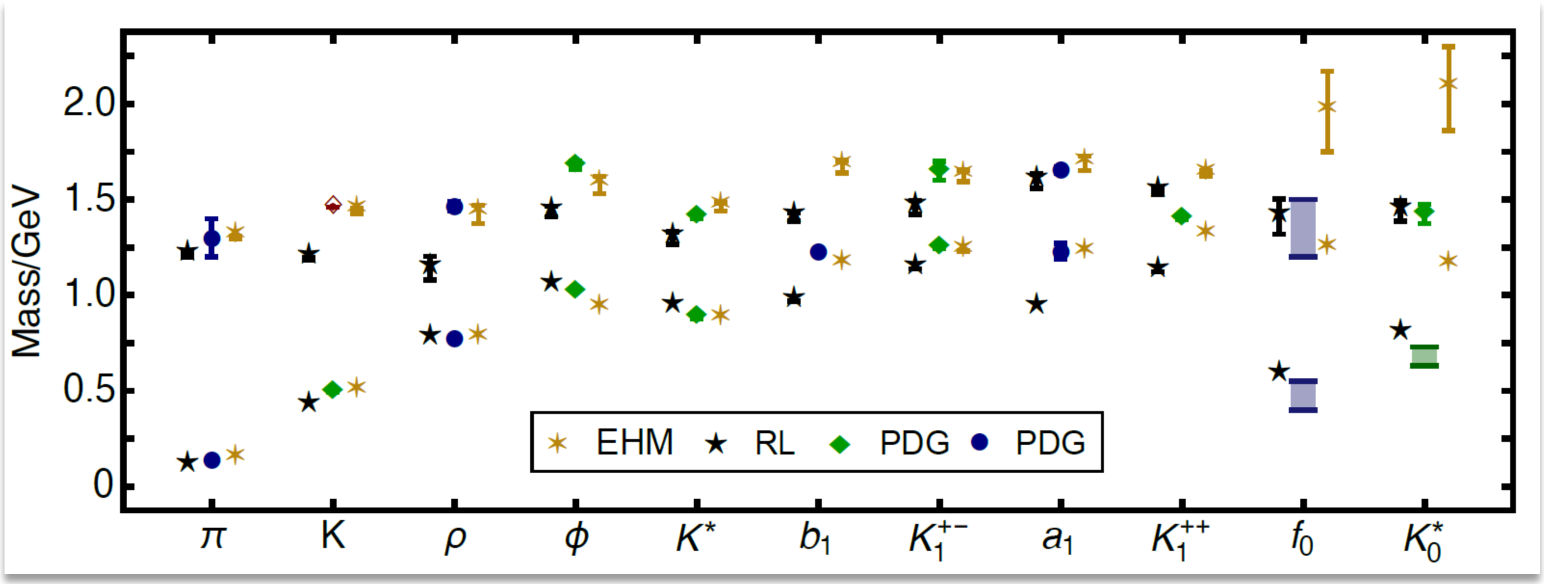
$$f_\pi^0 E_\pi^0(k^2; P^2 = 0) = B_0(k^2)$$



S.-X. Qin, C. D. Roberts, Chin. Phys. Lett. Express 38 (7) (2021) 071201.

Meson spectrum

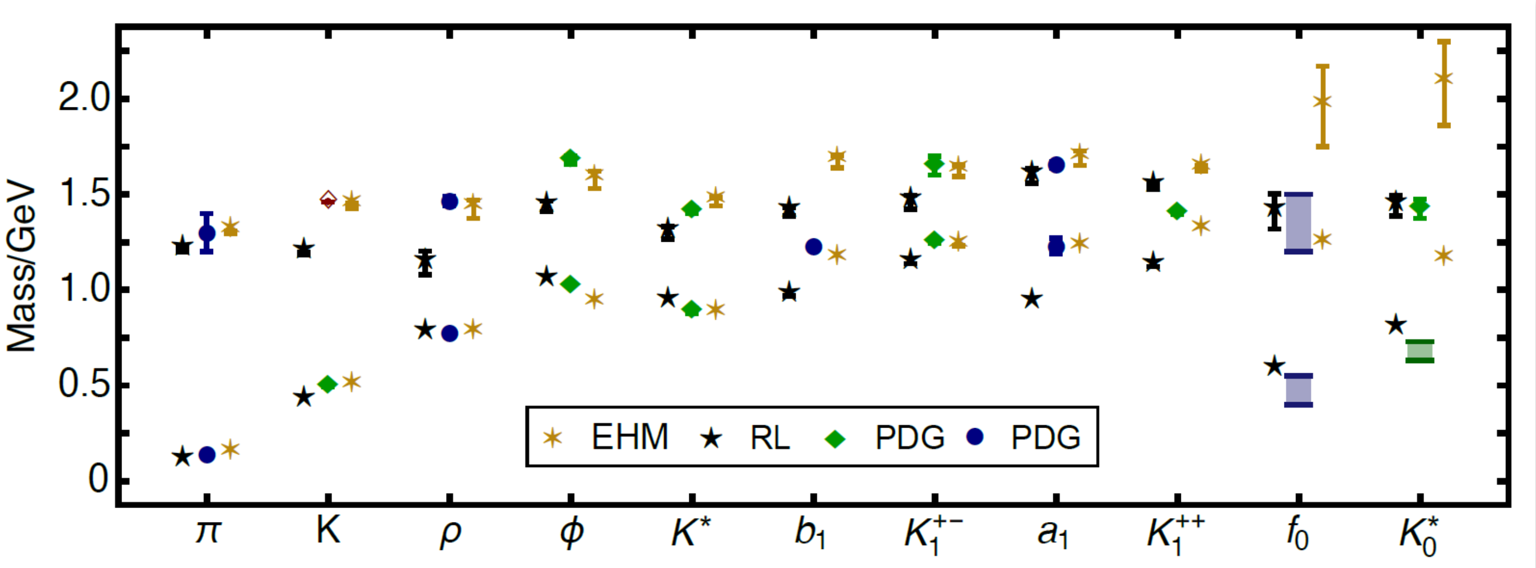




- RL truncation
  - overall, the mean absolute relative difference between RL masses and central experimental values is 13(8)%
  - many qualitative discrepancies:
    - Ordering of $(m_{\pi'}, m_{K'})$, $(m_{\rho'}, m_{\pi'})$, $(m_{\rho'}, m_{K^{*\prime}})$ is opposite to the empirical ordering.
    - $a_1 - \rho, b_1 - \rho$ mass splittings are one-third of the empirical values; $m_{\phi'} - m_\phi$ is half the experimental value.
    - the level ordering of the K_1^{+-}, K_1^{++} states is incorrect.

TABLE I. Masses calculated using RL and EHM-improved kernels compared with central values reported in Ref. [2, PDG – Summary Tables]: $n = 0$, ground states; and $n = 1$ first excited states. (All results in GeV. Where relevant, calculated results show Padé-fit extrapolation uncertainty.)

		RL	PDG	EHM-improved
π	$n=0$	0.103	0.138	0.140
	$n=1$	1.209(11)	1.3(1)	1.302(7)
K	$n=0$	0.415	0.494	0.494
	$n=1$	1.191(7)	1.460	1.440(14)
ρ	$n=0$	0.77	0.775	0.77
	$n=1$	1.140(62)	1.465(25)	1.423(48)
ϕ	$n=0$	1.049	1.020	0.926
	$n=1$	1.437(27)	1.680(20)	1.576(46)
K^*	$n=0$	0.936	0.890(14)	0.872
	$n=1$	1.298(32)	1.414(15)	1.461(22)
b_1	$n=0$	0.968	1.230(3)	1.159
	$n=1$	1.409(22)		1.671(32)
K_1^{+-}	$n=0$	1.138	1.253(7)	1.230
	$n=1$	1.458(37)	1.650(50)	1.623(30)
a_1	$n=0$	0.929	1.230(40)	1.218
	$n=1$	1.596(37)	1.655(16)	1.689(38)
K_1^{++}	$n=0$	1.123	1.403(7)	1.309
	$n=1$	1.541(12)		1.634(14)
f_0	$n=0$	0.577	0.4 – 0.55	1.237
	$n=1$	1.411(92)	1.2 – 1.5	1.96(21)
K_0^*	$n=0$	0.794	0.63 – 0.73	1.154
	$n=1$	1.439(53)	1.425(50)	2.08(22)



- **EHM improved kernel**
  - Compared with central experimental values, the overall mean absolute relative difference is 2.9(2.7)%,

    -- a factor of 4.6 improvement over the RL spectrum.
  - $m_{K'} > m_{\pi'}, m_{\rho'} > m_{\pi'}, m_{\rho'} \approx m_{K^{*\prime}}$, matching empirical results; the $a_1 - \rho, b_1 - \rho$ mass splittings are commensurate with empirical values; $m_{\phi'} - m_\phi$ matches experiment to within 2%; the level ordering of the K_1^{+-}, K_1^{++} states is correct.

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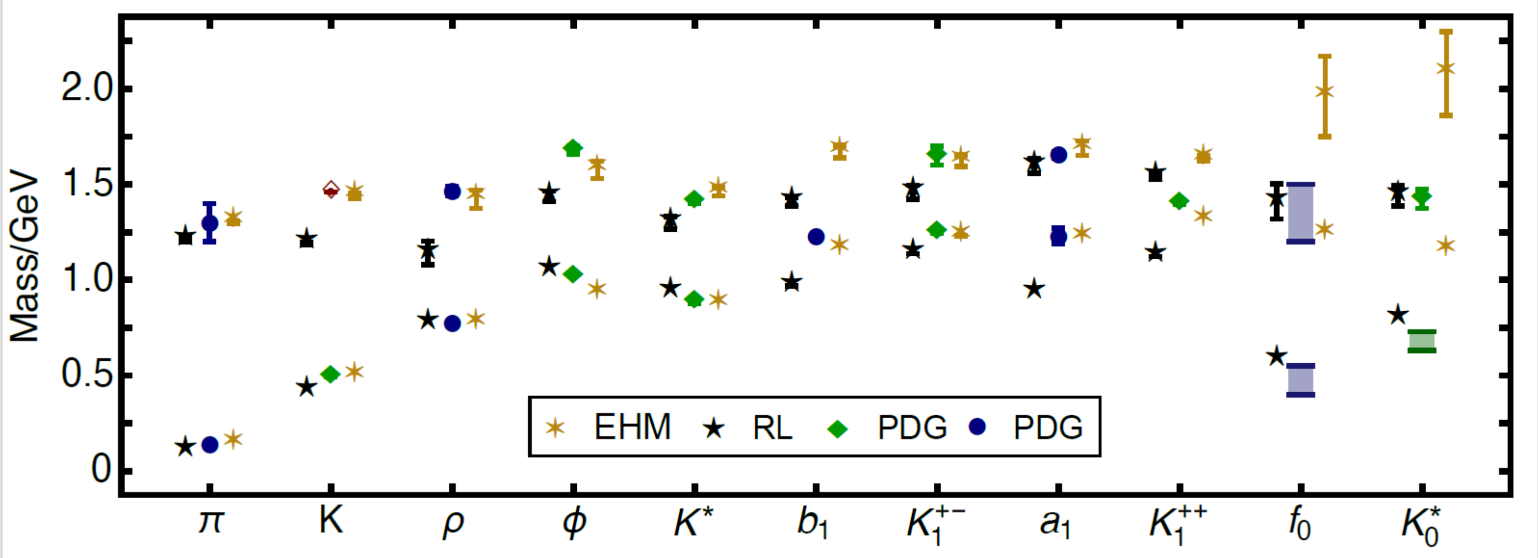

- EHM improved kernel

$$m_{b_1'} = 1.67(3)\,,\ m_{b_1'} - m_{b_1} = 0.51(3)\,,$$
$$m_{K_1^{++\prime}} = 1.63(1)\,,\ m_{K_1^{++\prime}} - m_{K_1^{++}} = 0.33(1)\,.$$

- The mass splittings in the partner channels are

$$m_{a_1'} - m_{a_1} = 0.47(4) \quad m_{K_1^{+-\prime}} - m_{K_1^{+-}} = 0.39(3)$$

the EHM-improved kernels predict $m_{b_1'} \approx m_{a_1'}\,,\ m_{K_1^{++\prime}} \approx m_{K_1^{+-\prime}}$.

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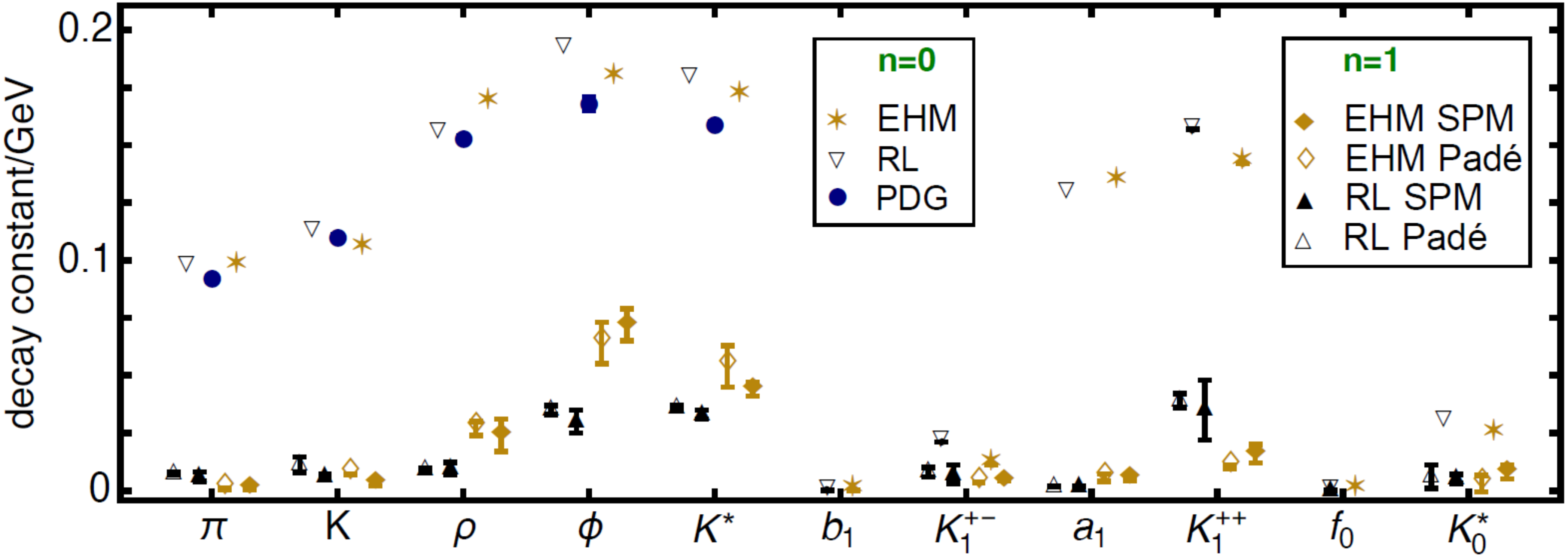
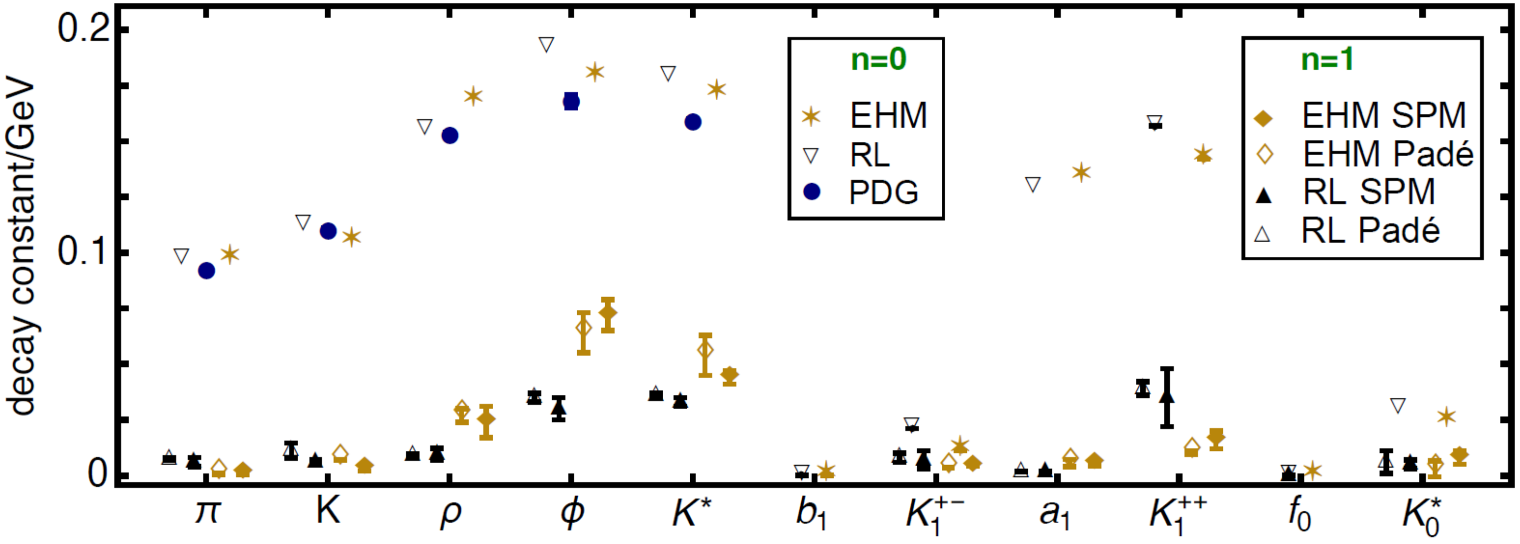
K_1^{++}	$n=0$	1.123	1.403(7)	1.309
	$n=1$	1.541(12)		1.634(14)
f_0	$n=0$	0.577	0.4 – 0.55	1.237
	$n=1$	1.411(92)	1.2 – 1.5	1.96(21)
K_0^*	$n=0$	0.794	0.63 – 0.73	1.154
	$n=1$	1.439(53)	1.425(50)	2.08(22)

Decay constants





meson $n=0$	RL	PDG	EHM-improved
π	0.097	0.092(1)	0.097
K	0.112	0.110(1)	0.105
ρ	0.155	0.153(1)	0.168
ϕ	0.192	0.168(3)	0.179
K^*	0.179	0.159(1)	0.171
b_1	0		0
K_1^{+-}	0.021		0.011
a_1	0.129		0.134
K_1^{++}	0.157		0.142
f_0	0		0
K_0^*	0.03		0.024

- For ground states, the leptonic decay constants can be calculated directly.
- the decay constants for excited states must be obtained by extrapolation.
- two extrapolation methods:
  - the Pade-approximant method
  - the Schlessinger point method (SPM)
    - the SPM provides reliable extrapolations with a rigorously quantified uncertainty.

meson	RL		EHM-improved	
$n=1$	Padé	SPM	Padé	SPM
π	0.0073(06)	0.006(2)	0.0008(3)	0.0010(06)
K	0.0112(34)	0.0061(11)	0.0071(2)	0.0033(14)
ρ	0.0088(11)	0.0095(26)	0.027(3)	0.024(7)
ϕ	0.035(2)	0.030(5)	0.064(9)	0.072(7)
K^*	0.036(1)	0.033(2)	0.054(9)	0.044(3)
b_1	0	0	0	0
K_1^{+-}	0.008(2)	0.007(4)	0.0034(1)	0.0042(1)
a_1	0.0018(2)	0.0017(2)	0.0056(17)	0.0056(12)
K_1^{++}	0.039(3)	0.035(13)	0.0101(7)	0.016(4)
f_0	0	0	0	0
K_0^*	0.006(5)	0.005(2)	0.0030(36)	0.008(3)

Conclusions

- We provide the first symmetry-preserving, Poincare covariant treatment of the spectrum of u, d, s mesons, both ground and first excited states.

- Our scheme produces a closed-form kernel that is symmetry-consistent (discrete and continuous) with the gap equation defined by any admissible gluon-quark vertex.

- The construction is applicable even when the diagrammatic content of that vertex is unknown, as would be the case if the vertex were obtained using lattice-QCD.

- It therefore establishes a route to new synergies between continuum and lattice approaches to strong interactions.

Conclusions

- The presence of a dressed-quark anomalous magnetic moment in the gluon-quark vertex, an emergent feature of strong interactions, can remedy many defects of widely used meson bound-state kernels
  - the mass splittings between vector and axial-vector mesons and the level ordering of pseudoscalar and vector meson radial excitations.

- Adding the ACM is sufficient to explain the light+light, light+strange, and strange+strange mesons.

Outlook

- repeat the work using a more realistic interaction.

- Extend this work to charm quark mesons

- Having benchmarked the method against known lighter-quark states, desirable to extend the approach to …
  - heavy + light mesons,
  - hybrid mesons and glueballs

  especially given world-wide investments in studies of and searches for such states.

Thank You!