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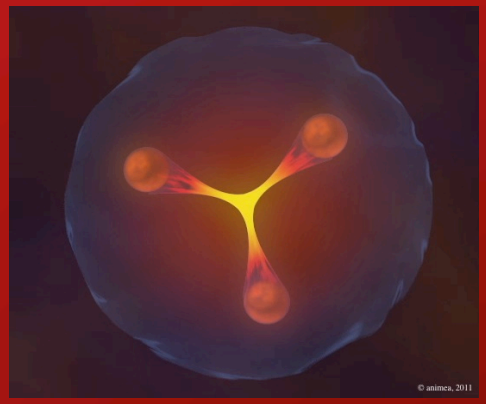


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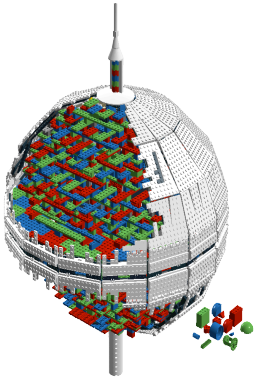
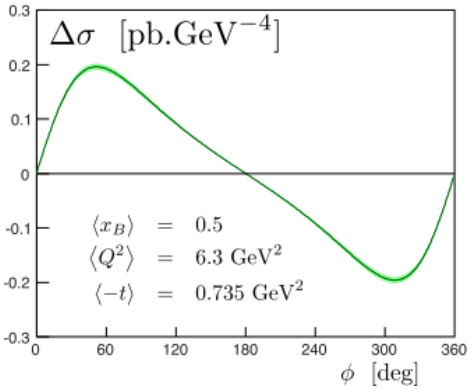
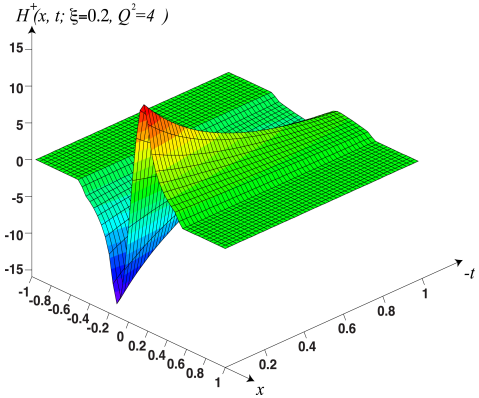
# Phenomenology of the nucleon internal pressure






**Revealing emergent mass** | **Hervé MOUTARDE**

université PARIS-SACLAY

**Sep. 16, 2022**

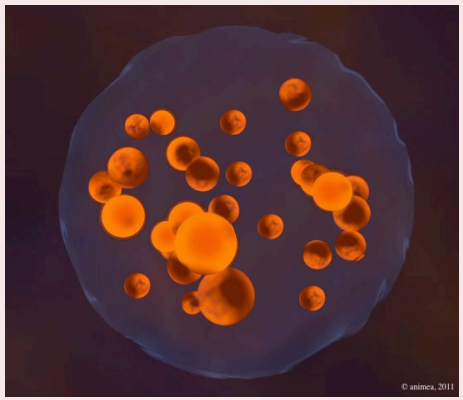
This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 824093.

# Perturbative and nonperturbative QCD.

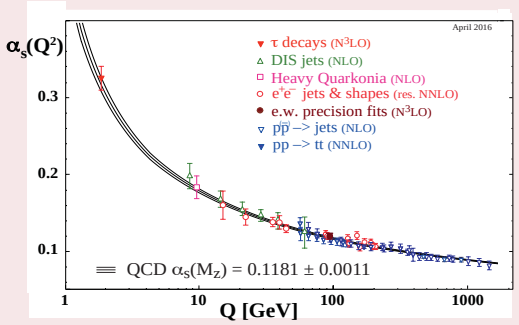
Study hadron structure to shed new light on nonperturbative QCD.



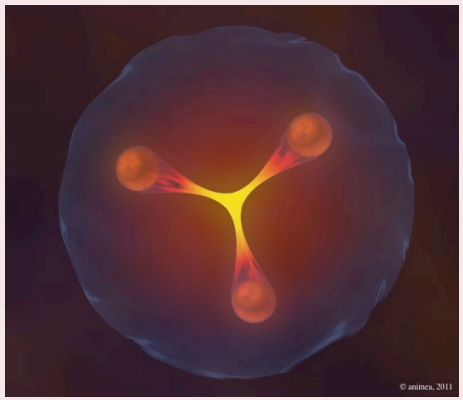
## Perturbative QCD



## Asymptotic freedom




## Nonperturbative QCD



## Interface between perturbative and nonperturbative regimes

- Define **universal** objects describing 3D hadron structure: **Generalized Parton Distributions** (GPD).
- Relate GPDs to measurements using **factorization**: **Virtual Compton Scattering** (DVCS, TCS), **Deeply Virtual Meson production** (DVMP).
- Get **experimental knowledge** of hadron structure.

# Energy-momentum tensor

# Energy-momentum tensor.
## Quark and gluon contributions.



- EMT defined from the invariance under space and time translations.
- **Quark** and **gluon** contributions

$$T_q^{\mu\nu} = \bar{q}\gamma^\mu \frac{i}{2} \overleftrightarrow{\mathrm{D}}\, q$$

$$T_g^{\mu\nu} = -F^{\mu\lambda} F^{\nu}{}_{\lambda} + \frac{1}{4}\eta^{\mu\nu} F^2$$

with $\overleftrightarrow{\mathrm{D}}$ the symmetric covariant derivative and $F^{\mu\nu}$ the field strength tensor.

- $T^{\mu\nu} = \sum_a T_a^{\mu\nu}$ $(a = q, g)$.

# Parameterization: massive spin-1/2 target.

## Introduction of 5 GFFs.

- Local, gauge-invariant, asymmetric EMT:

$$\begin{aligned}\langle p', s' | T_a^{\mu\nu}(0) | p, s\rangle = \bar{u}(p', s') \Bigg\{ & \frac{P^\mu P^\nu}{M} A_a(t) + M\eta^{\mu\nu} \bar{C}_a(t) \\ & + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C_a(t) \\ & + \frac{P^{\{\mu} i\sigma^{\nu\}\Delta}}{4M} [A_a(t) + B_a(t)] \\ & + \frac{P^{[\mu} i\sigma^{\nu]\Delta}}{4M} D_a(t) \Bigg\} u(p, s)\end{aligned}$$

with $P = (p' + p)/2$, $\Delta = p' - p$, $t = \Delta^2$ and polarizations $s, s'$. Shorthand notations: $a^{\{\mu} b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$, $a^{[\mu} b^{\nu]} = a^\mu b^\nu - a^\nu b^\mu$, and $i\sigma^{\mu\Delta} = i\sigma^{\mu\lambda}\Delta_\lambda$

✍ Lorcé et al. (2019)

# 3D profile of GFFs.

## Localization in the Wigner sense.





- Define distribution of a physical quantity inside a system, by first **localizing the system in both position and momentum** space.
- Breit frame where $P^\mu = (P^0, \vec{0})$ and $\Delta^\mu = (0, \vec{\Delta})$

$$\langle T_a^{\mu\nu} \rangle_{\mathrm{BF}(\vec{r})} = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3} e^{-i\vec{\Delta}\vec{r}} \left[ \frac{\langle p', s | T_a^{\mu\nu}(0) | p, s \rangle}{2P^0} \right]_{\vec{P}=\vec{0}}$$

- Specific role of 3D Fourier transform of GFFs.

✍ Lorcé *et al.* (2019)

# Gravitational form factors.
## Definition of pressure.



- **Matrix element** in the Breit frame ($a = q, g$):

$$\left\langle \frac{\Delta}{2} \left| T_a^{\mu\nu}(0) \right| -\frac{\Delta}{2} \right\rangle = M \left\{ \eta^{\mu 0}\eta^{\nu 0} \left[ A_a(t) + \frac{t}{4M^2} B_a(t) \right] + \eta^{\mu\nu} \left[ \bar{C}_a(t) - \frac{t}{M^2} C_a(t) \right] + \frac{\Delta^\mu \Delta^\nu}{M^2} C_a(t) \right\}$$

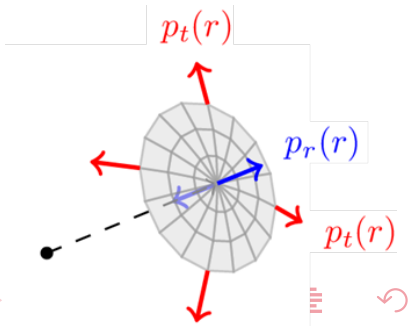
- Anisotropic fluid in **relativistic hydrodynamics**:

$$\Theta^{\mu\nu}(\vec{r}) = [\varepsilon(r) + p_t(r)]\, u^\mu u^\nu - p_t(r)\eta^{\mu\nu} + [p_r(r) - p_t(r)]\, \chi^\mu \chi^\nu$$

where $u^\mu$ and $\chi^\mu = x^\mu / r$.

- Define **isotropic pressure** and **pressure anisotropy**:

$$p(r) = \frac{p_r(r) + 2\, p_t(r)}{3}$$
$$s(r) = p_r(r) - p_t(r)$$




✍ Lorcé *et al.* (2019)

- Write dictionary between quantum and fluid pictures:

$$\frac{\varepsilon_a(r)}{M} = \int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ A_a(t) + \bar{C}_a(t) + \frac{t}{4M^2}\left[B_a(t) - 4C_a(t)\right] \right\}$$

$$\frac{p_{r,a}(r)}{M} = \int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ -\bar{C}_a(t) - \frac{4}{r^2}\frac{t^{-1/2}}{M^2}\frac{\mathrm{d}}{\mathrm{d}t}\left(t^{3/2}\, C_a(t)\right) \right\}$$

$$\frac{p_{t,a}(r)}{M} = \int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ -\bar{C}_a(t) + \frac{4}{r^2}\frac{t^{-1/2}}{M^2}\frac{\mathrm{d}}{\mathrm{d}t}\left[t\frac{\mathrm{d}}{\mathrm{d}t}\left(t^{3/2}\, C_a(t)\right)\right] \right\}$$

$$\frac{p_a(r)}{M} = \int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ -\bar{C}_a(t) + \frac{2}{3}\frac{t}{M^2}\, C_a(t) \right\}$$

$$\frac{s_a(r)}{M} = \int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ -\frac{4}{r^2}\frac{t^{-1/2}}{M^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(t^{5/2}\, C_a(t)\right) \right\}$$

✍ Lorcé *et al.* (2019)

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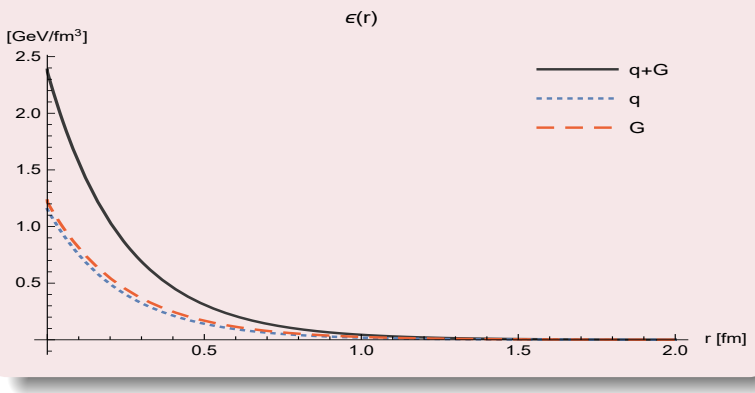
# Equation of state.

## Elaborating on the relation between energy and pressure.

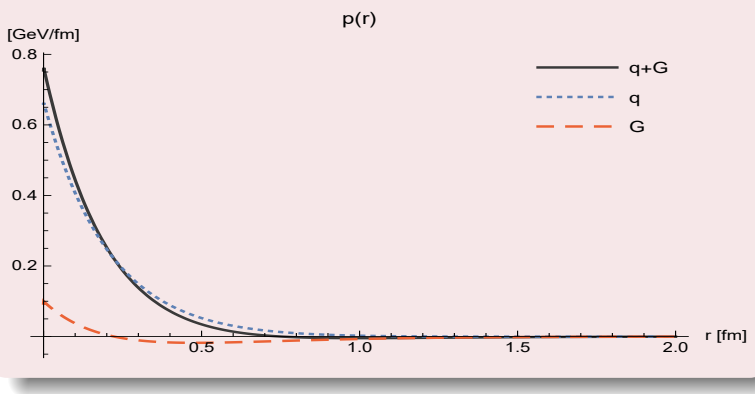


- Simple multiple models: dipole for GFFs $A$ and $\bar{C}$, tripole for GFFs $B$ and $C$.

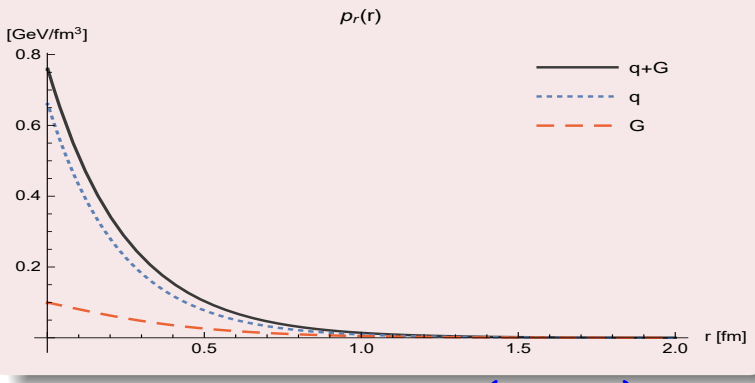
### quark



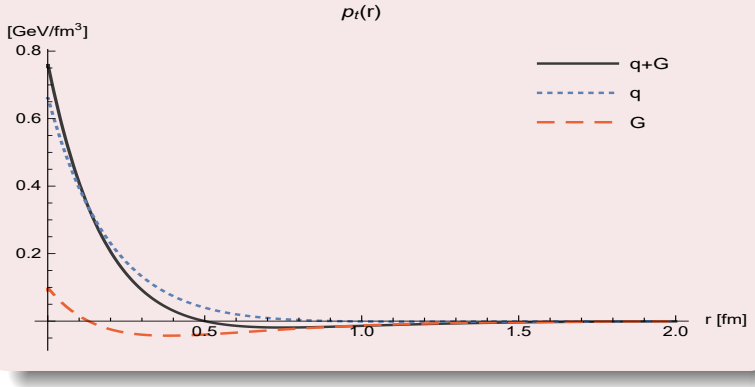

### glue




### quark + glue




### Neutron stars




✍ Lorcé *et al.* (2019)

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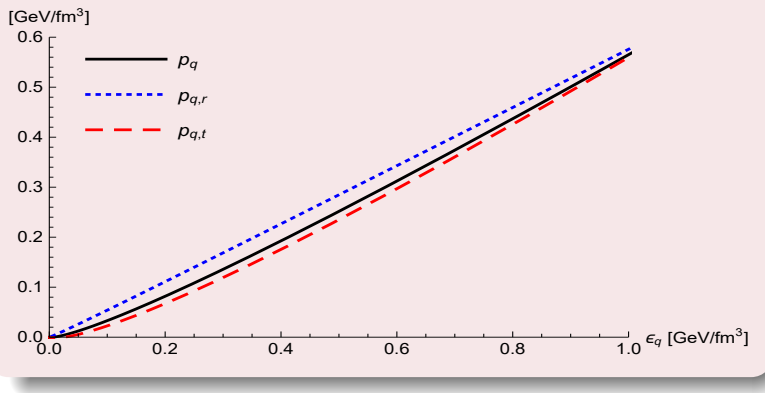
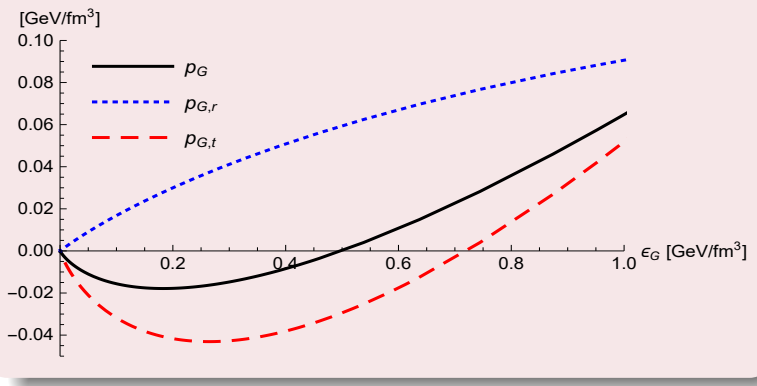
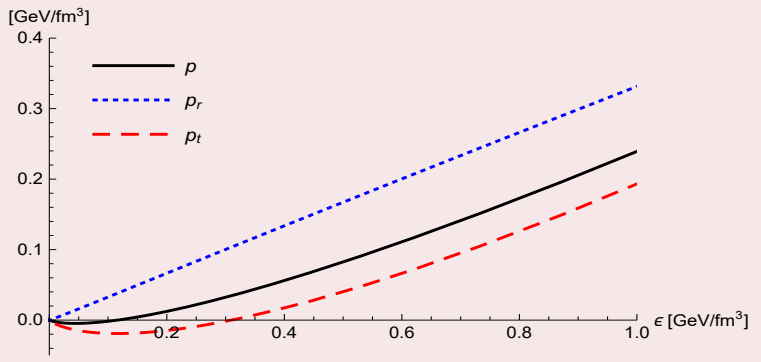



**glue**




**quark + glue**




✍ Lorcé *et al.* (2019)

- Parametric plots of EOS
  - $(\epsilon(r), p_r(r))$
  - $(\epsilon(r), p_t(r))$
  - $(\epsilon(r), p(r))$
- Quark and gluon contributions

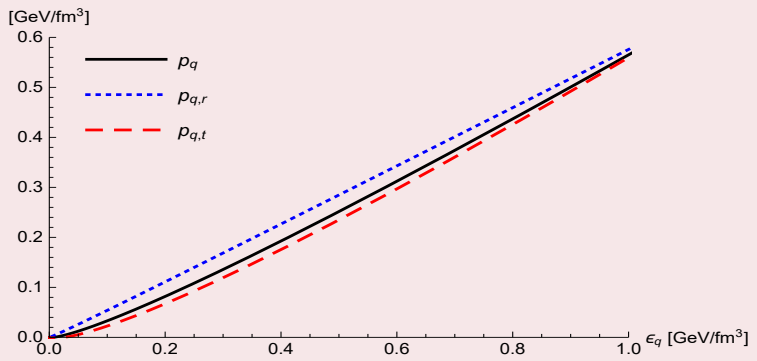
# Equation of state.

## Elaborating on the relation between energy and pressure.



- From the nucleon to compact stars?

### quark

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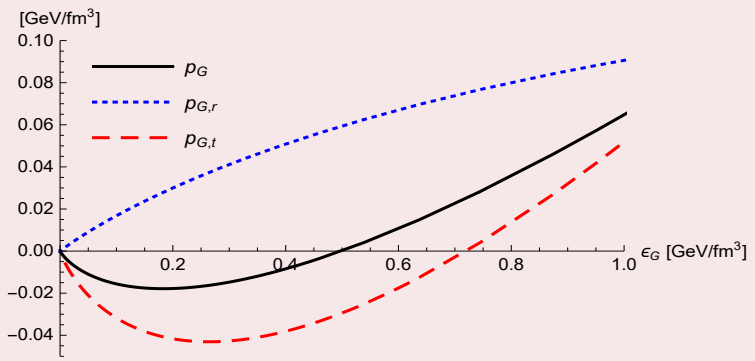


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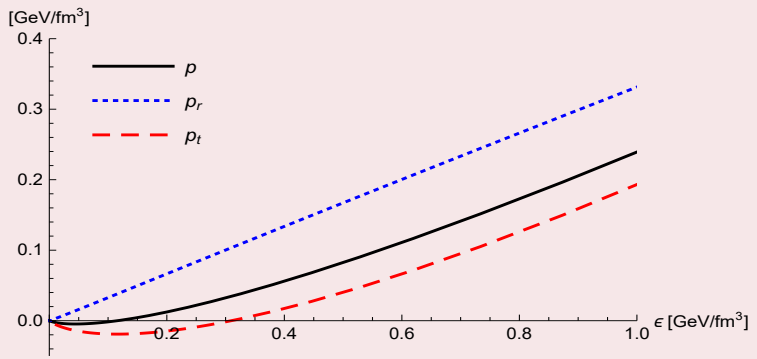


✍ Lorcé *et al.* (2019)

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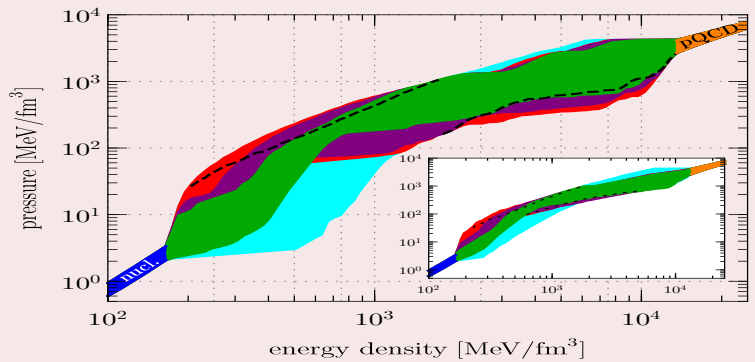



✍ Annala *et al.* (2018)

# Connection with experimental data.

## Gravitational form factors from generalized parton distributions.



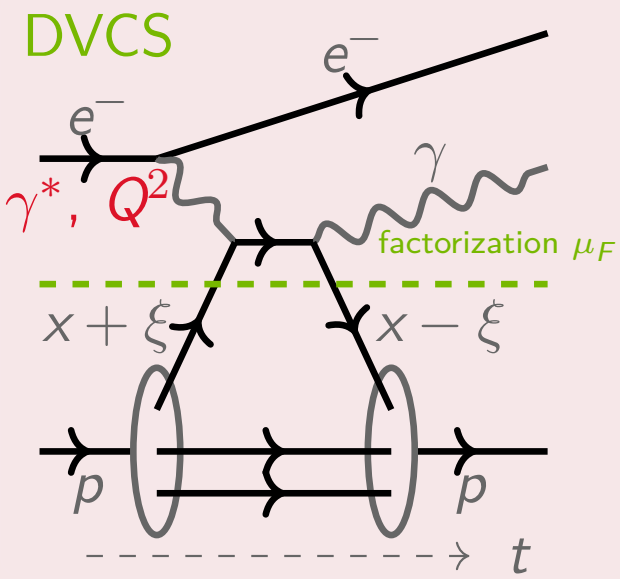
### Link between GPDs and GFFs

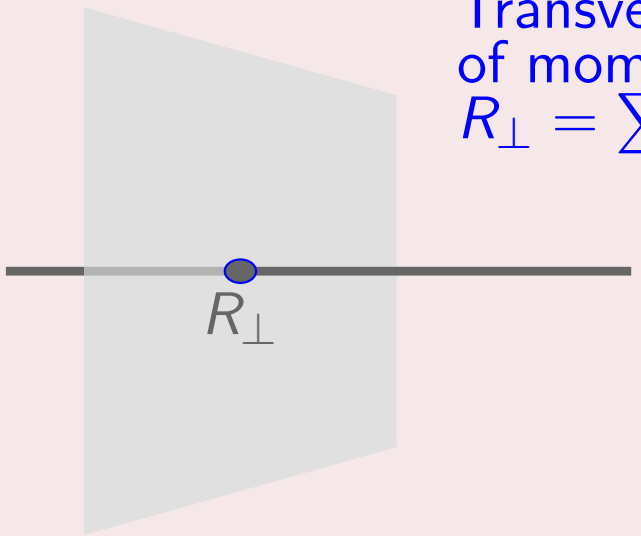
$$\int \mathrm{d}x\, x H^q(x,\xi,t) = A^q(t) + 4\xi^2 C^q(t)$$

$$\int \mathrm{d}x\, x E^q(x,\xi,t) = B^q(t) - 4\xi^2 C^q(t)$$

✍ Ji (1997), ✍ Goeke (2001)

### Deeply Virtual Compton Scattering (DVCS)




Transverse center of momentum $R_\perp$

$R_\perp = \sum_i x_i r_{\perp i}$

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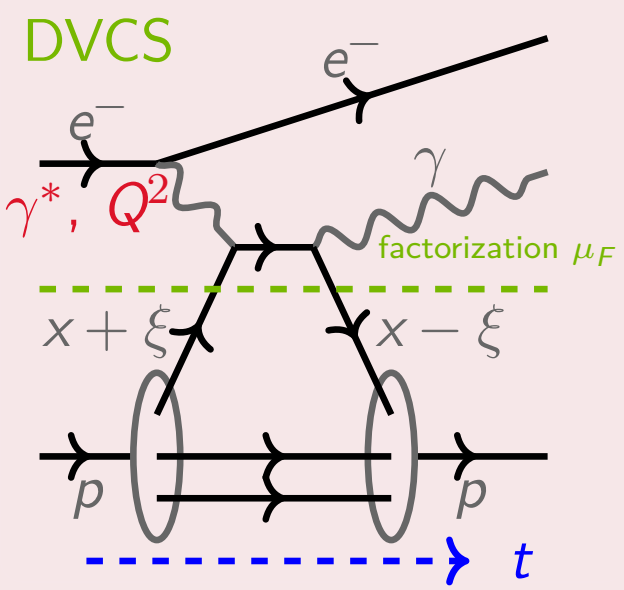


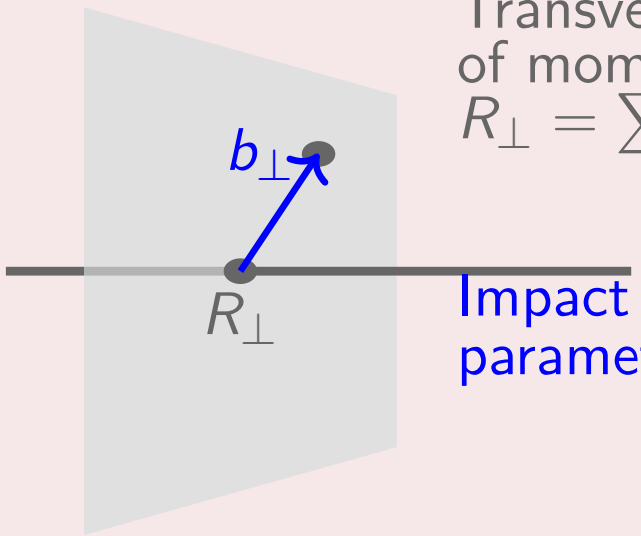


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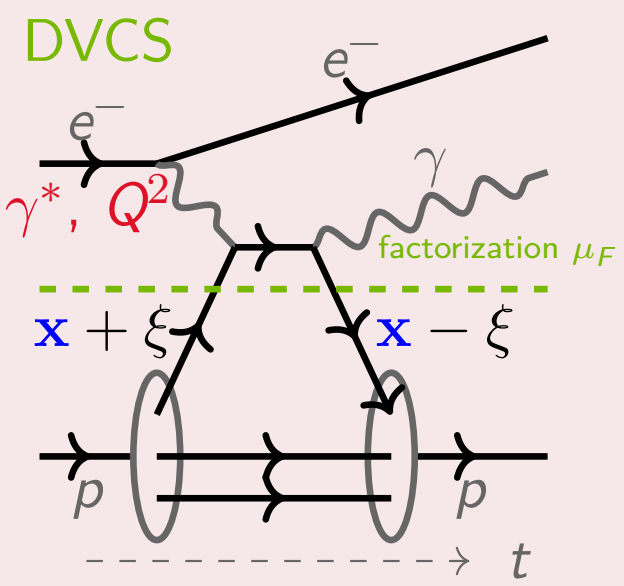


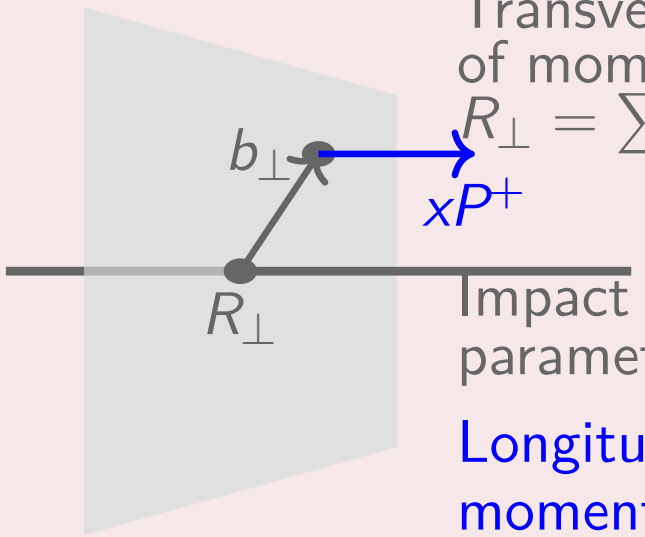


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Impact parameter $b_\perp$

Longitudinal momentum $xP^+$

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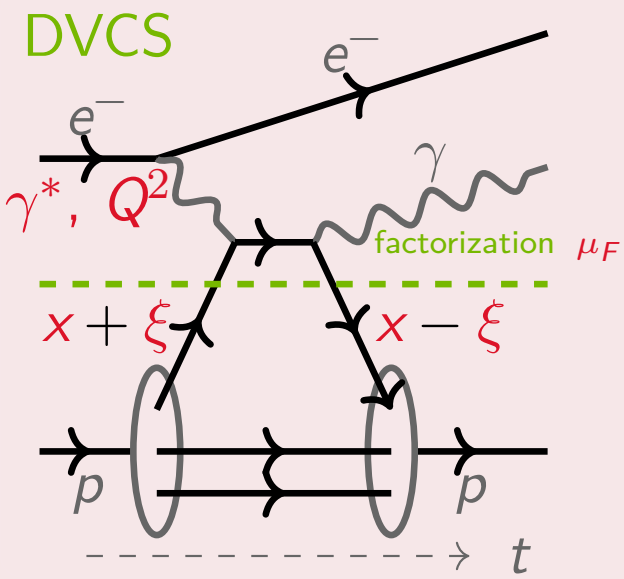


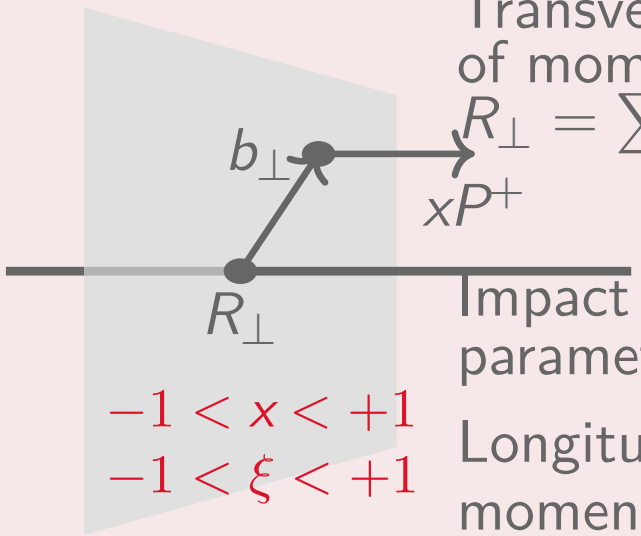


Transverse center of momentum $R_\perp$
$R_\perp = \sum_i x_i r_{\perp i}$

Impact parameter $b_\perp$

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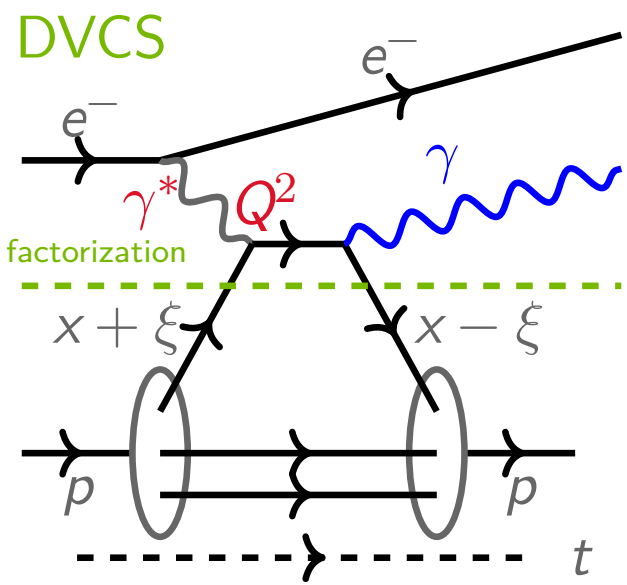
# Exclusive processes of current interest.

## Factorization, universality and need for high luminosity.





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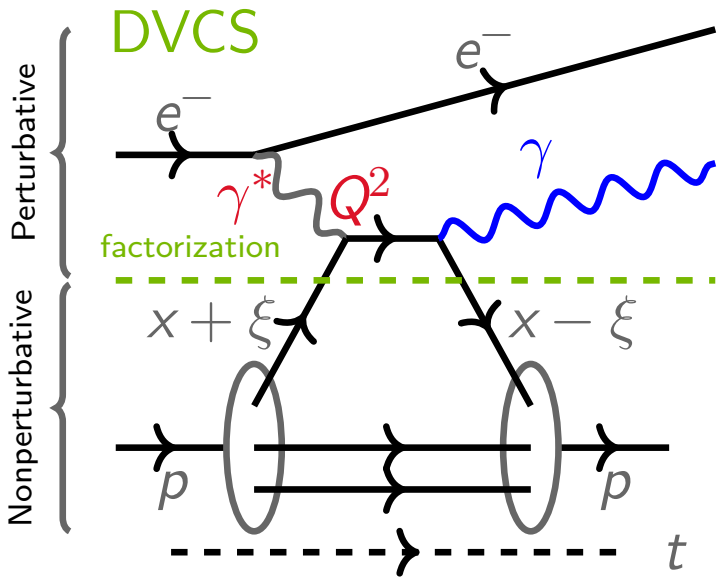

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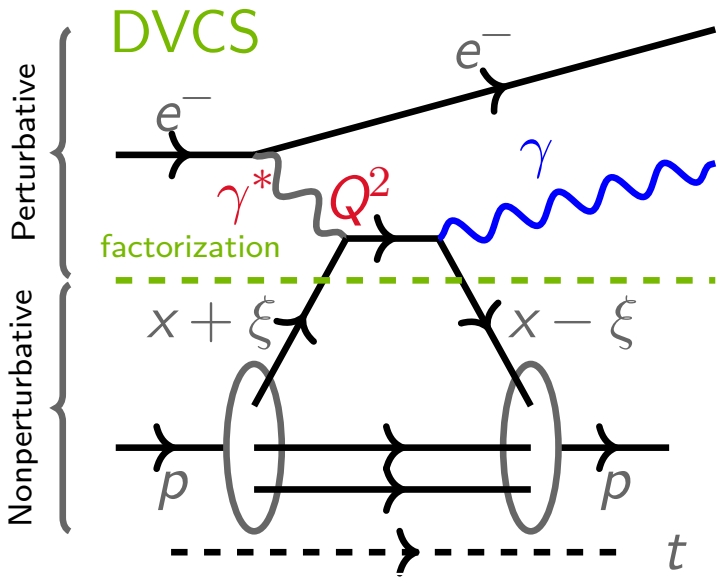


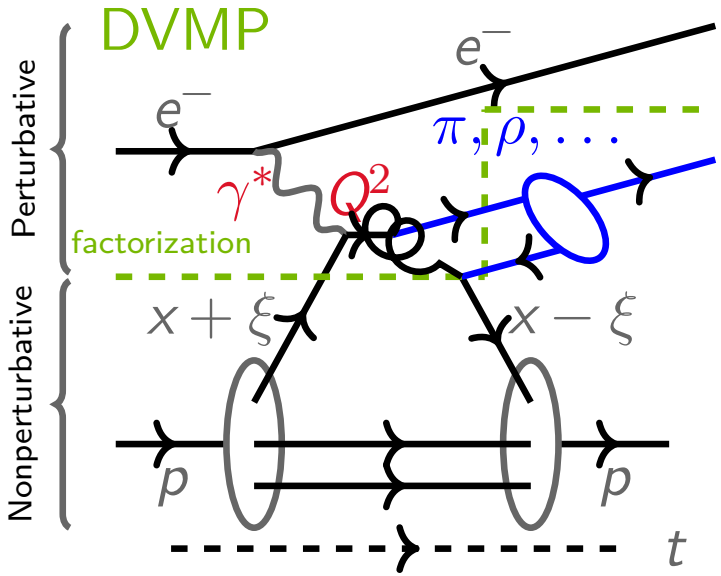

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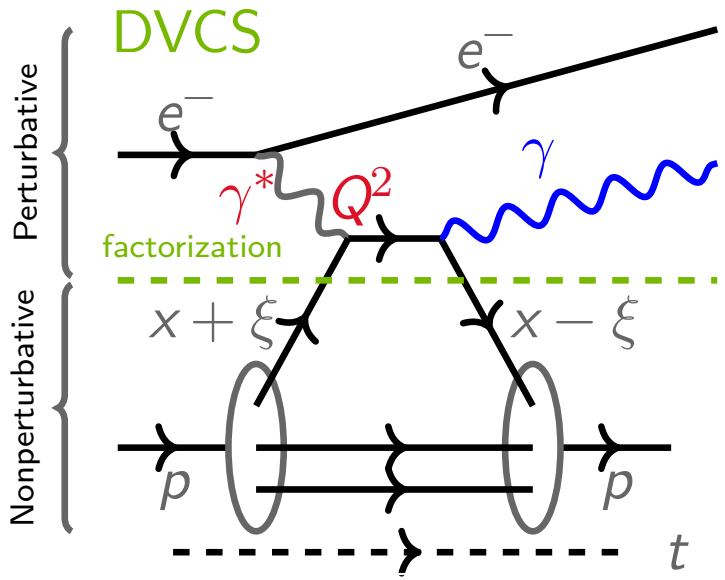


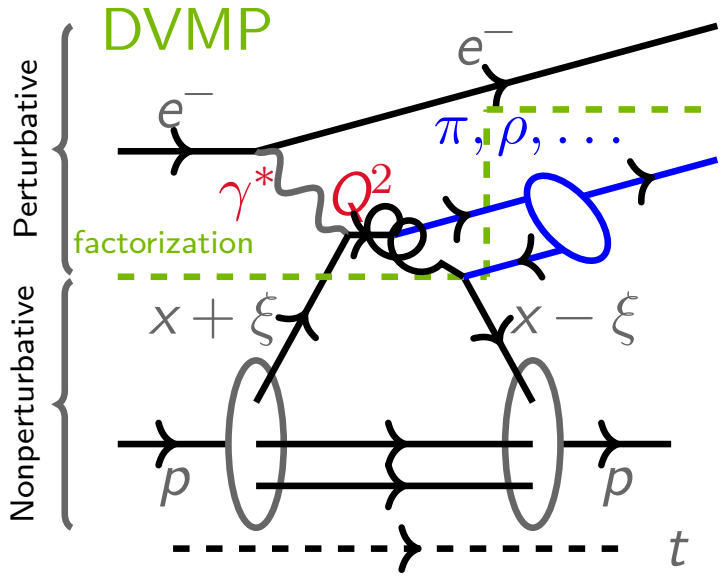


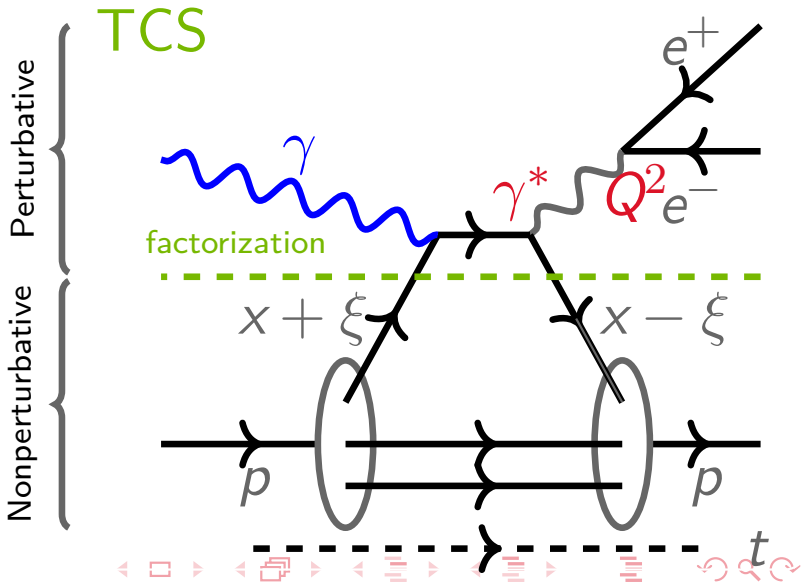

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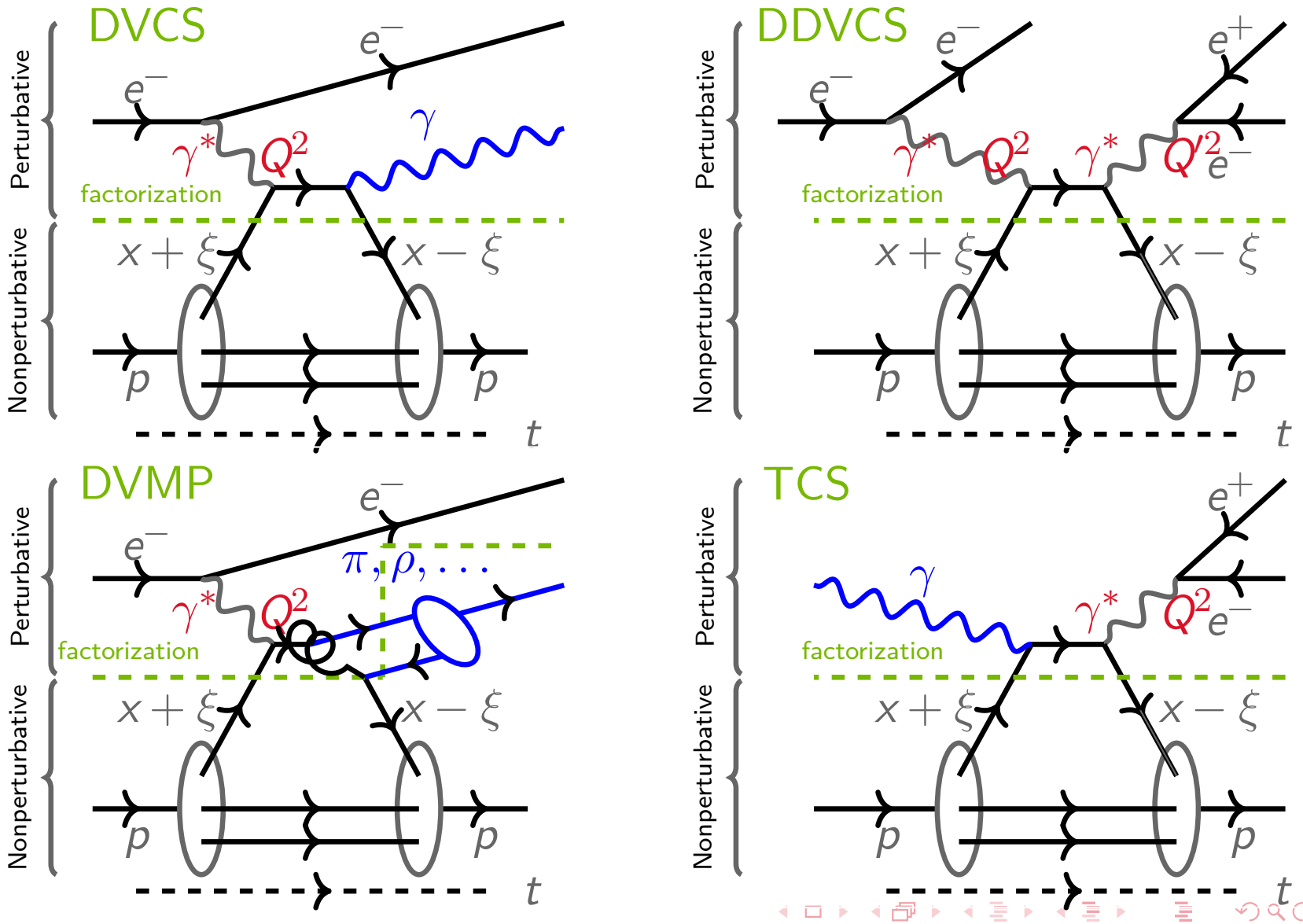
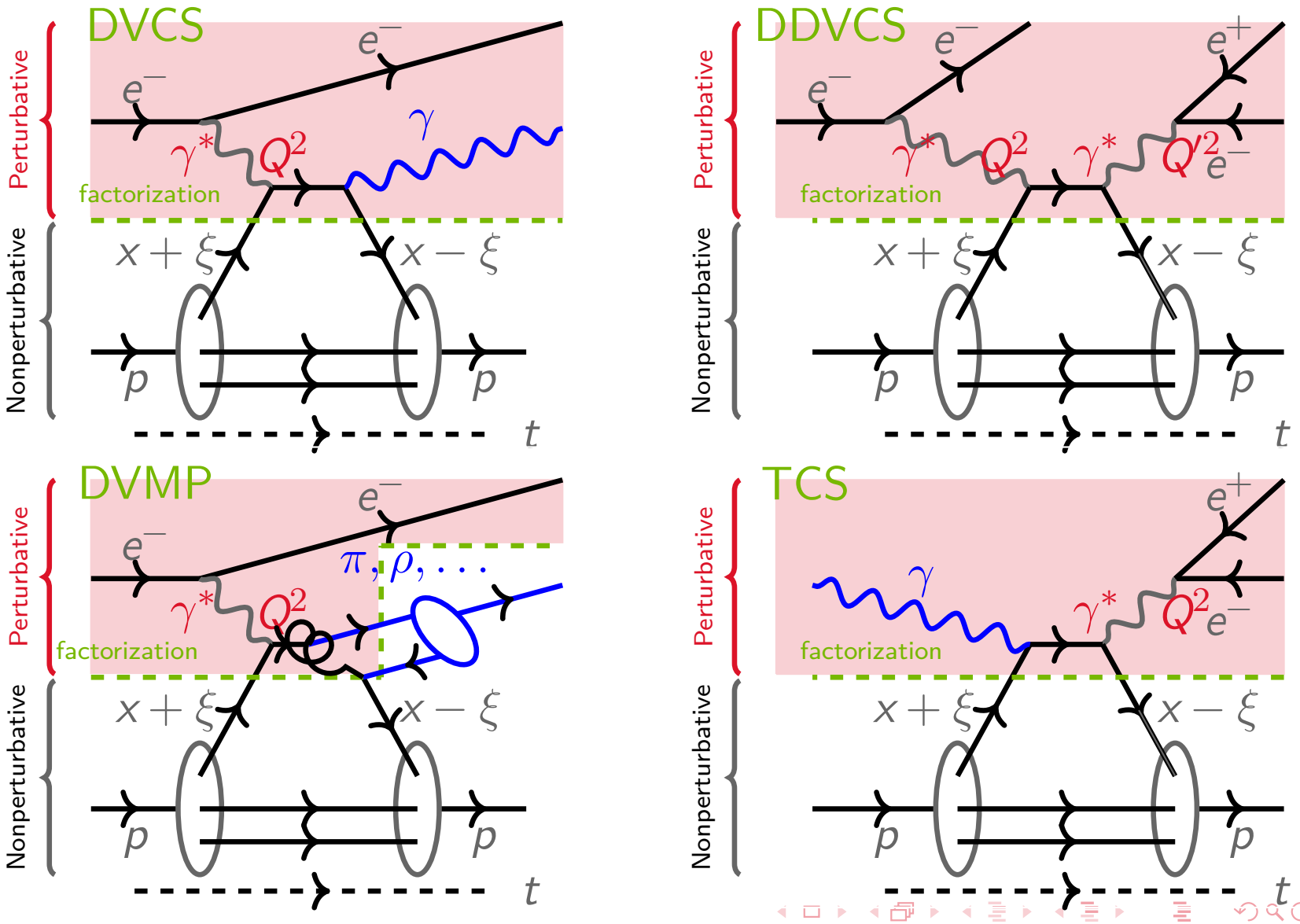


# Exclusive processes of current interest.

## Factorization, universality and need for high luminosity.





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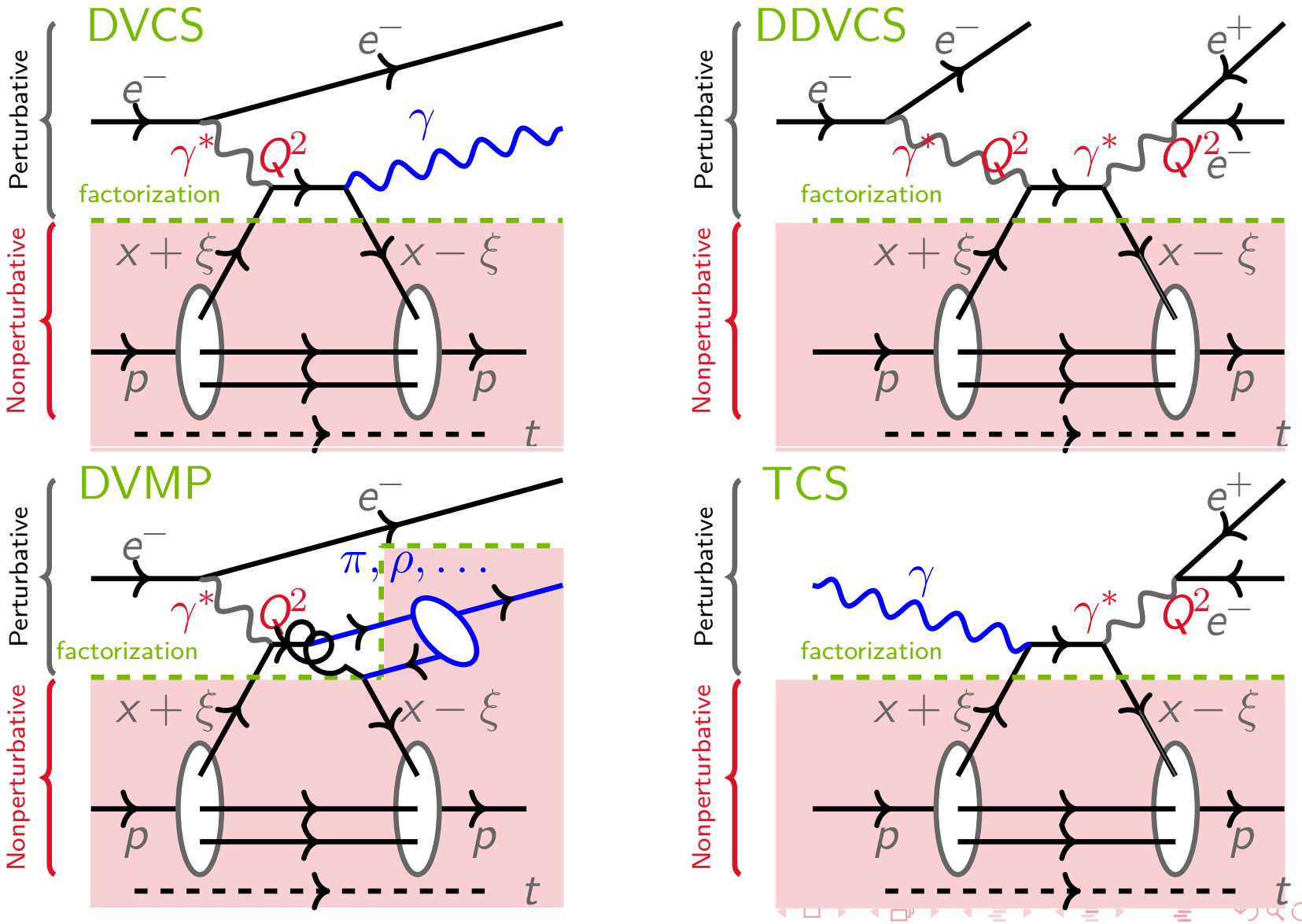


## Bjorken regime : large $Q^2$ and fixed $xB \simeq 2\xi/(1+\xi)$

- Partonic interpretation relies on **factorization theorems**.
- All-order proofs for DVCS.
- GPDs depend on a (arbitrary) factorization scale $\mu_F$.
- **Consistency** requires the study of **different channels**.

- GPDs enter DVCS through **Compton Form Factors** :

$$\mathcal{F}(\xi, t, Q^2) = \int_{-1}^{1} \mathrm{d}x\, T\left(x, \xi, \alpha_S(\mu_F), \frac{Q}{\mu_F}\right) F(x, \xi, t, \mu_F)$$

for a given GPD $F$.

- Kernels $T$ derived at NLO and (partially) NNLO.

✍ Belitsky and Müller (1998)

✍ Braun *et al.* (2022)

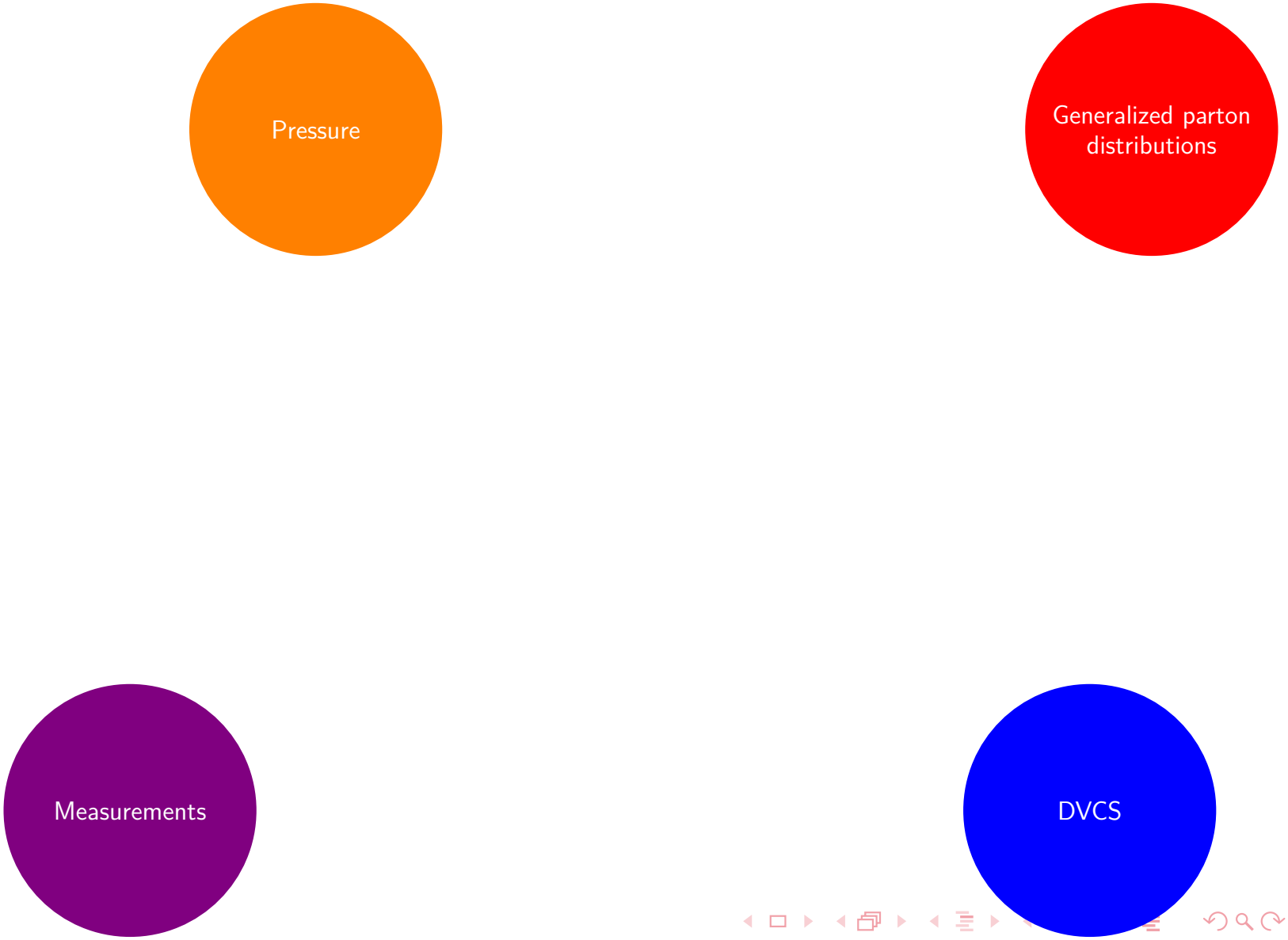
- CFF $\mathcal{F}$ is a **complex function**.

# Phenomenology

# What is the proton internal pressure?

## Identifying the concepts.





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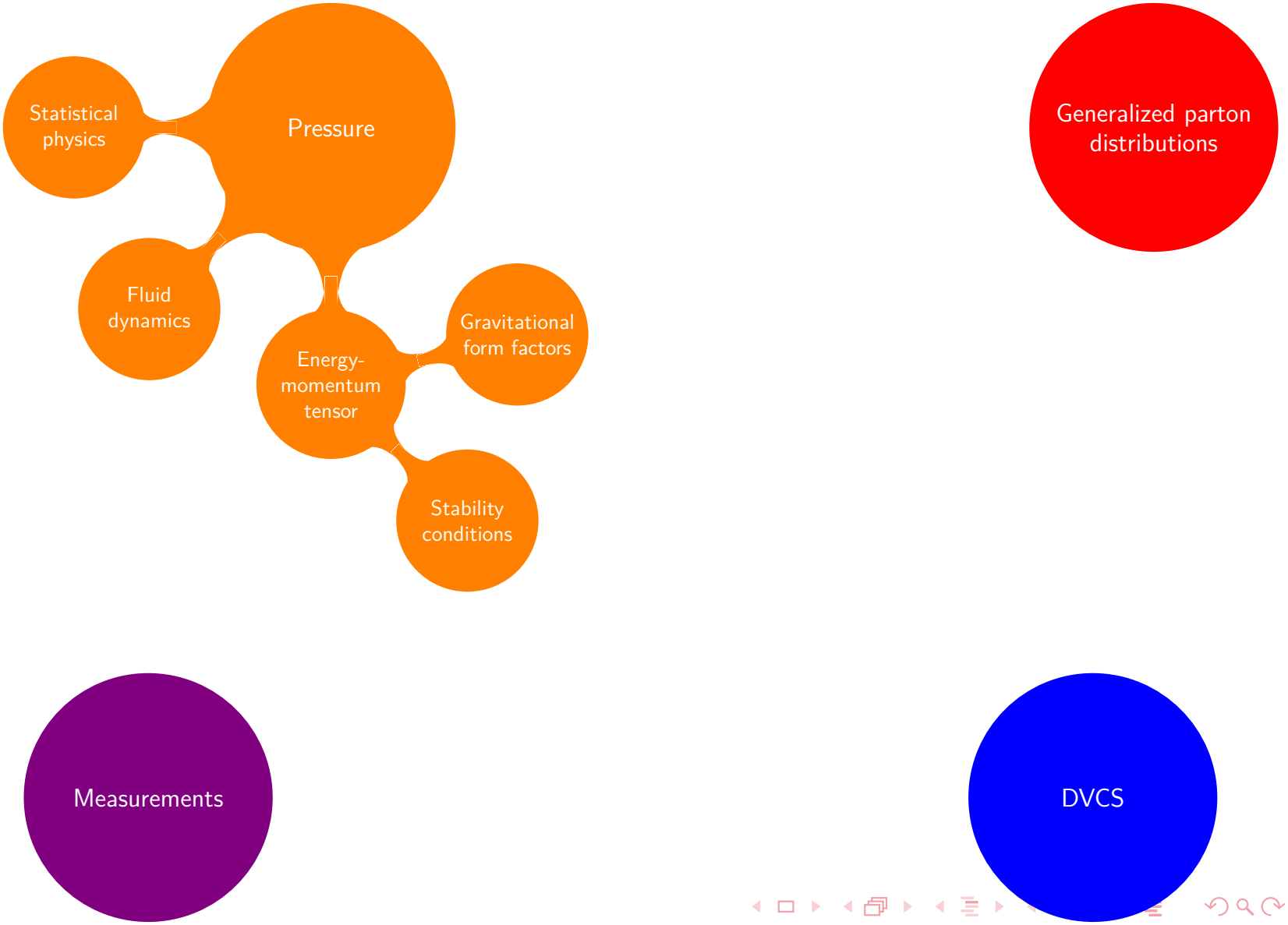

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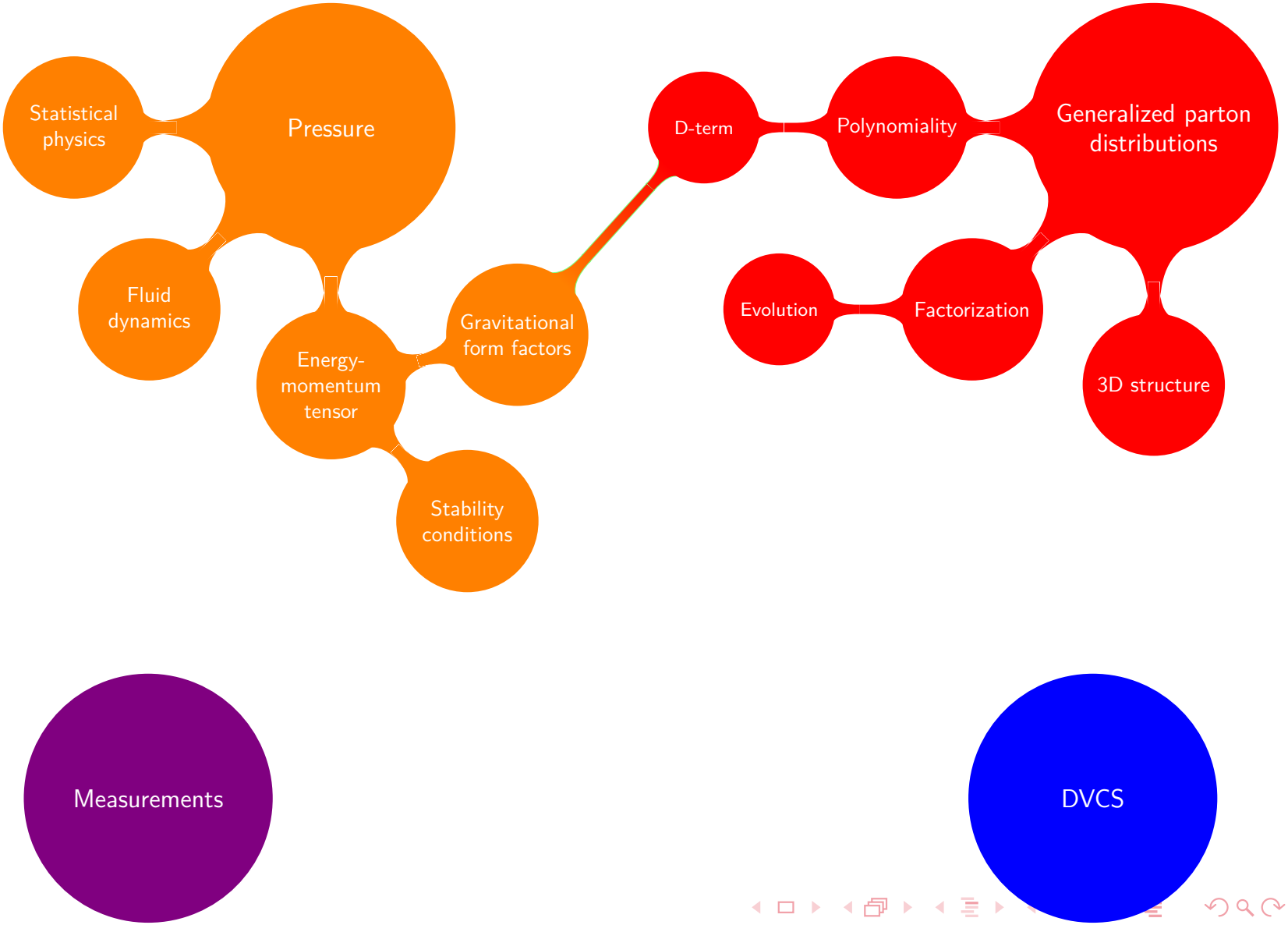

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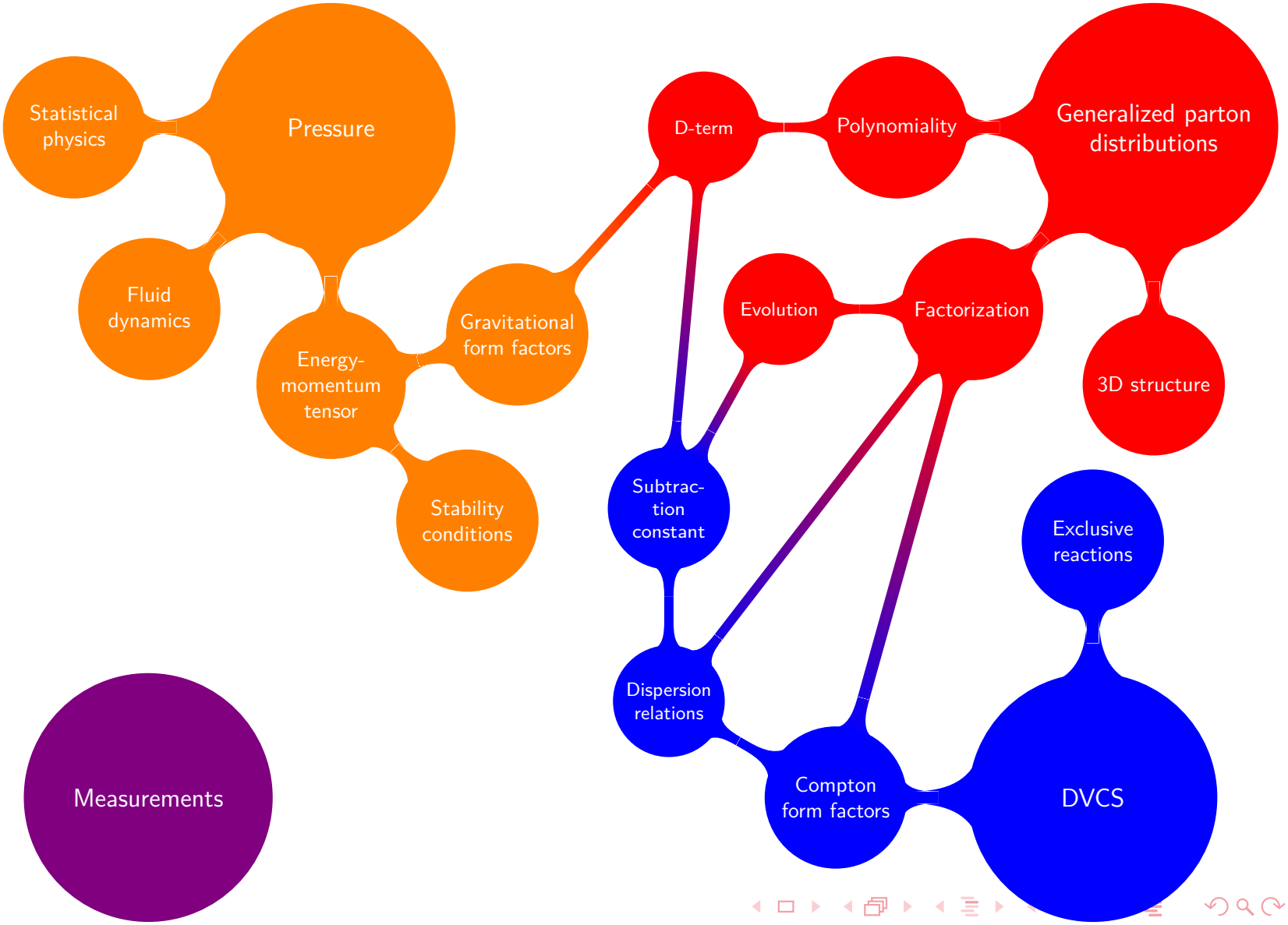

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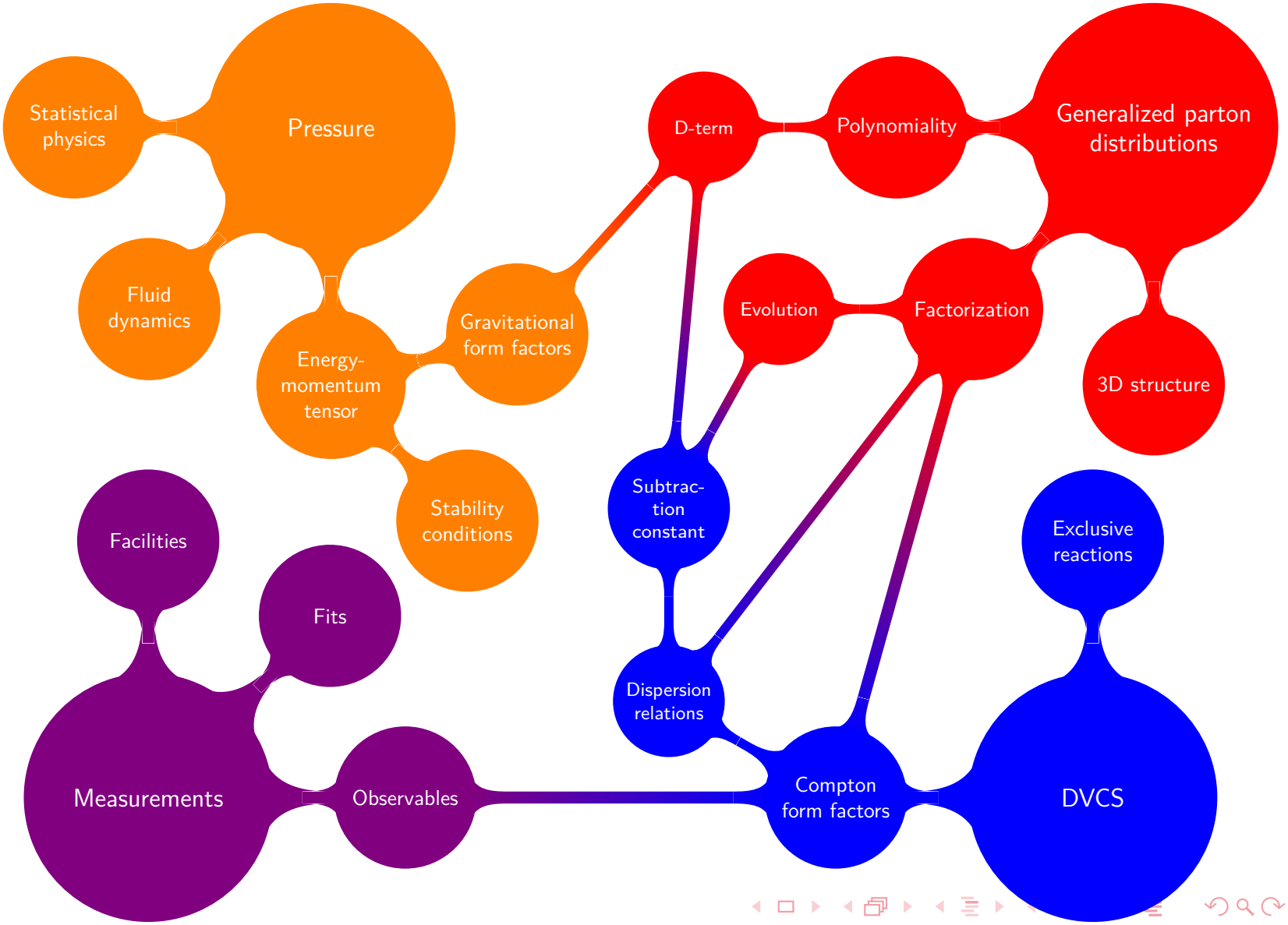

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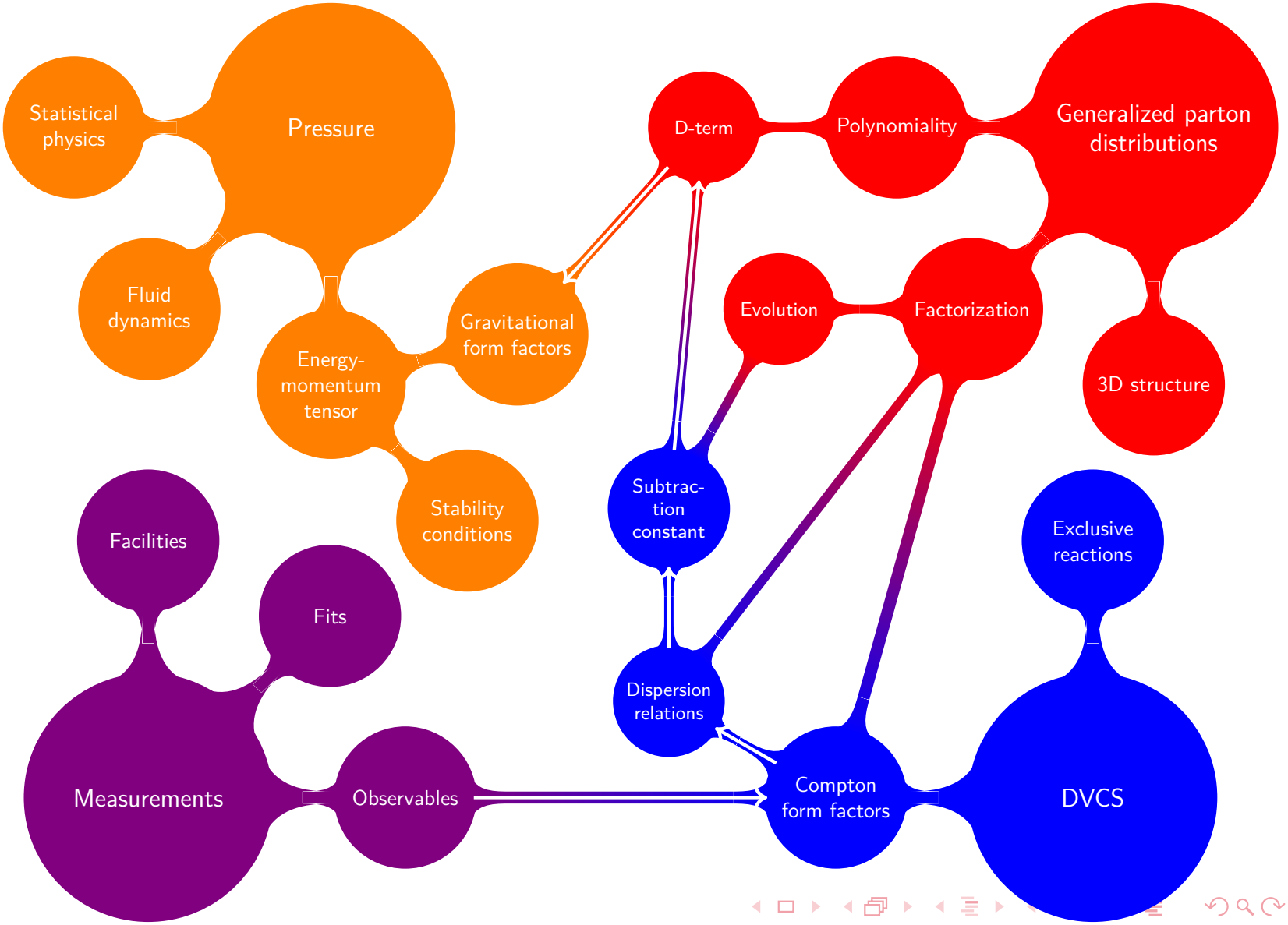

# Almost all existing DVCS data sets.

## 2600+ measurements of 30 observables published during 2001-17.



| No. | Collab. | Year | Ref. | Observable | | Kinematic dependence | No. of points used / all |
|---|---|---|---|---|---|---|---|
| 1 | HERMES | 2001 | 40 | $A_{LU}^{+}$ | | $\phi$ | 10 / 10 |
| 2 | | 2006 | 41 | $A_{C}^{\cos i\phi}$ | $i=1$ | $t$ | 4 / 4 |
| 3 | | 2008 | 42 | $A_{C}^{\cos i\phi}$ | $i=0,1$ | $x_{\mathrm{Bj}}$ | 18 / 24 |
| | | | | $A_{UT,\mathrm{DVCS}}^{\sin(\phi-\phi_S)\cos i\phi}$ | $i=0$ | | |
| | | | | $A_{UT,\mathrm{I}}^{\sin(\phi-\phi_S)\cos i\phi}$ | $i=0,1$ | | |
| | | | | $A_{UT,\mathrm{I}}^{\cos(\phi-\phi_S)\sin i\phi}$ | $i=1$ | | |
| 4 | | 2009 | 43 | $A_{LU,\mathrm{I}}^{\sin i\phi}$ | $i=1,2$ | $x_{\mathrm{Bj}}$ | 35 / 42 |
| | | | | $A_{LU,\mathrm{DVCS}}^{\sin i\phi}$ | $i=1$ | | |
| | | | | $A_{C}^{\cos i\phi}$ | $i=0,1,2,3$ | | |
| 5 | | 2010 | 44 | $A_{UL}^{+,\sin i\phi}$ | $i=1,2,3$ | $x_{\mathrm{Bj}}$ | 18 / 24 |
| | | | | $A_{LL}^{+,\cos i\phi}$ | $i=0,1,2$ | | |
| 6 | | 2011 | 45 | $A_{LT,\mathrm{DVCS}}^{\cos(\phi-\phi_S)\cos i\phi}$ | $i=0,1$ | $x_{\mathrm{Bj}}$ | 24 / 32 |
| | | | | $A_{LT,\mathrm{DVCS}}^{\sin(\phi-\phi_S)\sin i\phi}$ | $i=1$ | | |
| | | | | $A_{LT,\mathrm{I}}^{\cos(\phi-\phi_S)\cos i\phi}$ | $i=0,1,2$ | | |
| | | | | $A_{LT,\mathrm{I}}^{\sin(\phi-\phi_S)\sin i\phi}$ | $i=1,2$ | | |
| 7 | | 2012 | 46 | $A_{LU,\mathrm{I}}^{\sin i\phi}$ | $i=1,2$ | $x_{\mathrm{Bj}}$ | 35 / 42 |
| | | | | $A_{LU,\mathrm{DVCS}}^{\sin i\phi}$ | $i=1$ | | |
| | | | | $A_{C}^{\cos i\phi}$ | $i=0,1,2,3$ | | |
| 8 | CLAS | 2001 | 47 | $A_{LU}^{-,\sin i\phi}$ | $i=1,2$ | — | 0 / 2 |
| 9 | | 2006 | 48 | $A_{UL}^{-,\sin i\phi}$ | $i=1,2$ | — | 2 / 2 |
| 10 | | 2008 | 49 | $A_{LU}^{-}$ | | $\phi$ | 283 / 737 |
| 11 | | 2009 | 50 | $A_{LU}^{-}$ | | $\phi$ | 22 / 33 |
| 12 | | 2015 | 51 | $A_{LU}^{-}$, $A_{UL}^{-}$, $A_{LL}^{-}$ | | $\phi$ | 311 / 497 |
| 13 | | 2015 | 52 | $d^4\sigma_{UU}^{-}$ | | $\phi$ | 1333 / 1933 |
| 14 | Hall A | 2015 | 34 | $\Delta d^4\sigma_{LU}^{-}$ | | $\phi$ | 228 / 228 |
| 15 | | 2017 | 35 | $\Delta d^4\sigma_{LU}^{-}$ | | $\phi$ | 276 / 358 |
| 16 | COMPASS | 2018 | 36 | $d^3\sigma_{UU}^{\pm}$ | | t | 2 / 4 |
| 17 | ZEUS | 2009 | 37 | $d^3\sigma_{UU}^{+}$ | | t | 4 / 4 |
| 18 | H1 | 2005 | 38 | $d^3\sigma_{UU}^{+}$ | | t | 7 / 8 |
| 19 | | 2009 | 39 | $d^3\sigma_{UU}^{\pm}$ | | t | 12 / 12 |
| | | | | | | SUM: | 2624 / 3996 |

✍ Moutarde *et al.* (2019)

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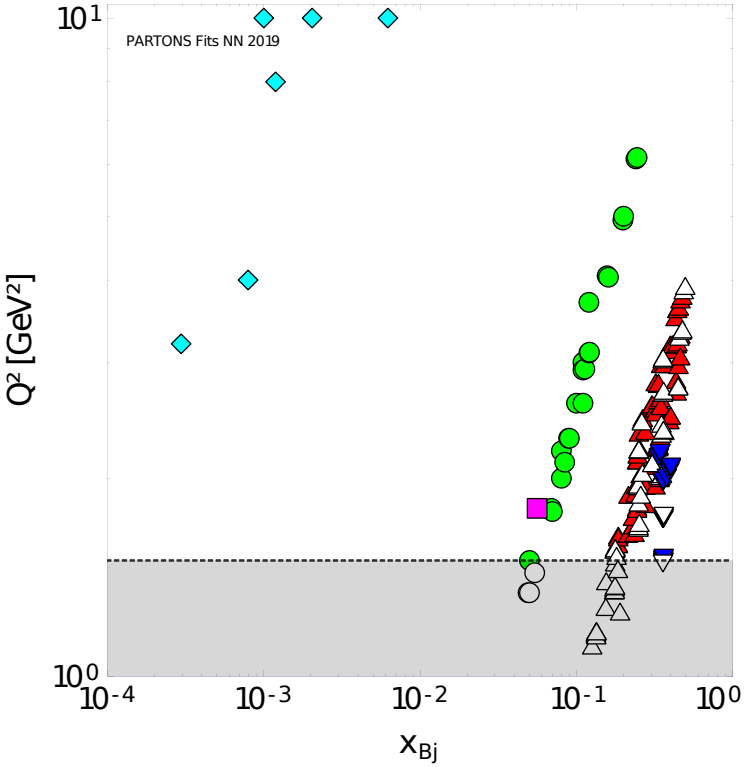
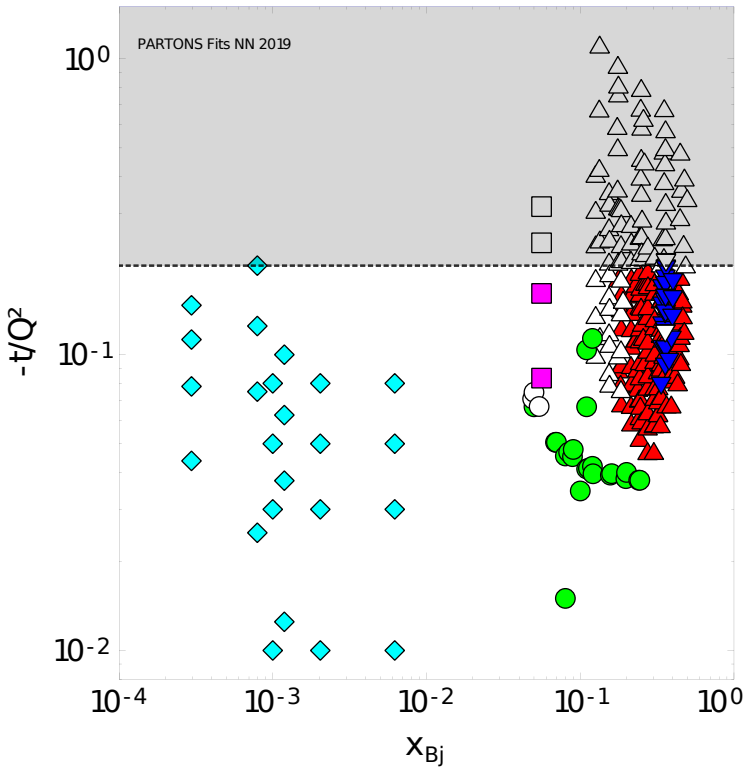



▼ Hall A ● HERMES ■ COMPASS

▲ CLAS ♦ H1 and ZEUS

✍ Moutarde *et al.* (2019)

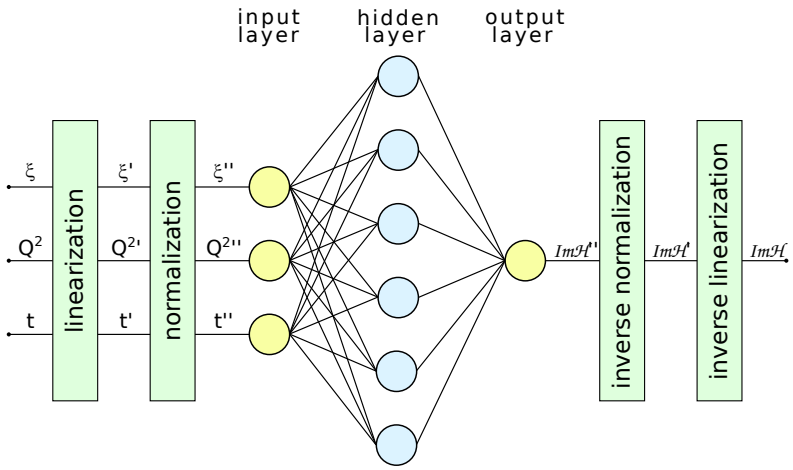
**Need many more precise data at small $x_B$, large $Q^2$ and also as close as possible to the valence region.**

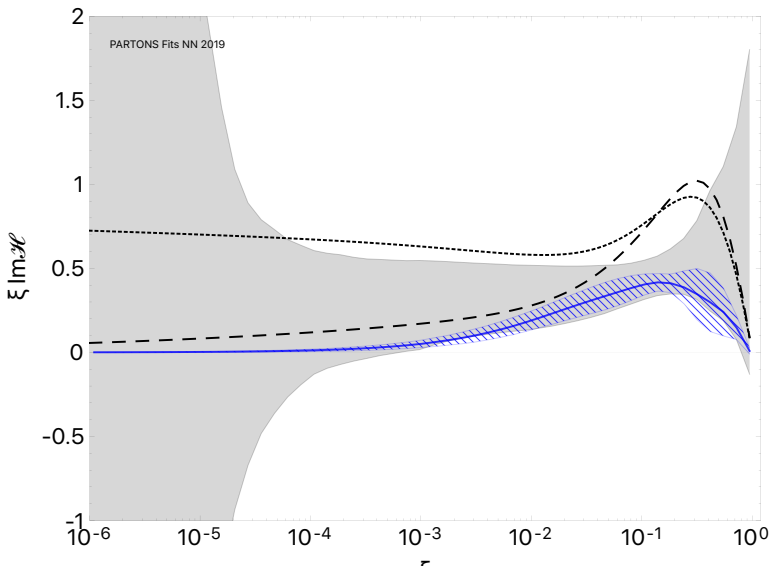
# Modeling of $\mathcal{H}$, $\widetilde{\mathcal{H}}$, $\mathcal{E}$ and $\widetilde{\mathcal{E}}$.

## Independent descriptions of real and imaginary parts.



- Real and imaginary parts of CFFs parameterized by **neural networks**.
- Propagation of uncertainties through **replica method** and evaluation of 68 % **confidence levels**.




✍ Moutarde *et al.* (2019)

# Pressure forces from DVCS measurements.

## A first-principle connection.



1. Expand D-term on Gegenbauer polynomials

$$D^q_{\text{term}}(z,t,\mu_F^2) = (1-z^2)\sum_{\text{odd } n} d_n^q(t,\mu_F^2)\, C_n^{3/2}(z)$$

2. Write dispersion relation for CFF (true at all pQCD orders)

$$\mathcal{C}_H(t,Q^2) = \mathrm{Re}\mathcal{H}(\xi) - \frac{1}{\pi}\int_0^1 \mathrm{d}\xi'\, \mathrm{Im}\mathcal{H}(\xi')\left(\frac{1}{\xi-\xi'} - \frac{1}{\xi+\xi'}\right)$$

3. Compute subtraction constant

$$\mathcal{C}_H^{q,g}(t,Q^2) = \frac{2}{\pi}\int_1^{+\infty} \mathrm{d}\omega\, \mathrm{Im}\, T^{q,g}(\omega)\int_{-1}^{1}\mathrm{d}z\, \frac{D^{q,g}(z)}{\omega - z}$$

✍ Diehl & Ivanov (2007)

4. Retrieve GFF

$$d_1^q(t,\mu_F^2) = 5\, C_q(t,\mu_F^2)$$

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3. Compute subtraction constant at LO

$$\mathcal{C}_H(t,Q^2) = 4\sum_q e_q^2 \sum_{\text{odd } n} d_n^q(t,\mu_F^2 \equiv Q^2)$$

✍ Diehl & Ivanov (2007)

4. Retrieve GFF

$$d_1^q(t,\mu_F^2) = 5\,C_q(t,\mu_F^2)$$

# GFF extracting chain.

## Implementing this first-principle connection.



Experimental data

GFF $C$
Internal pressure

# GFF extracting chain.

## Implementing this first-principle connection.



Experimental data

GPD $H$

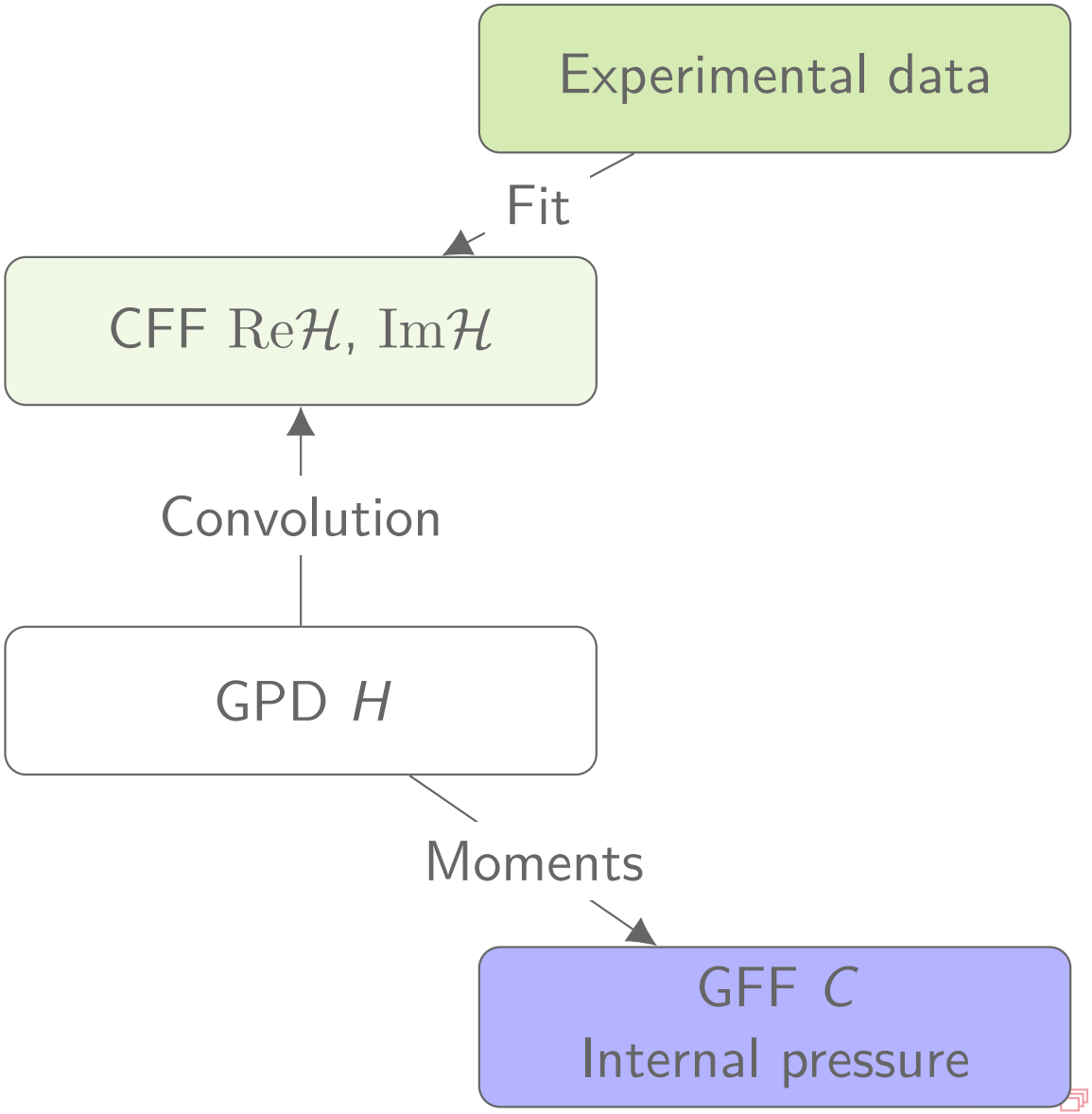
Moments

GFF $C$
Internal pressure

# GFF extracting chain.

## Implementing this first-principle connection.





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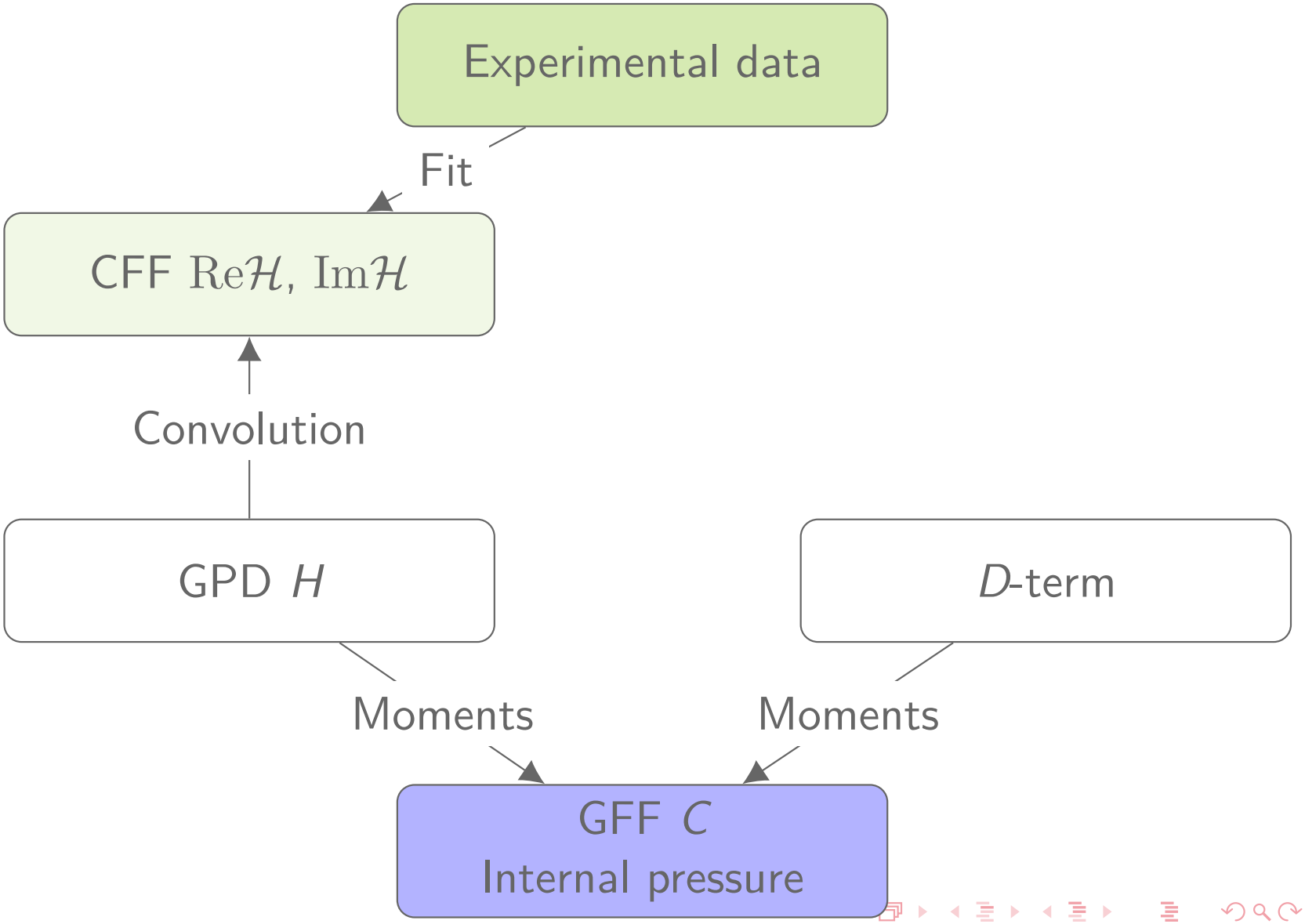

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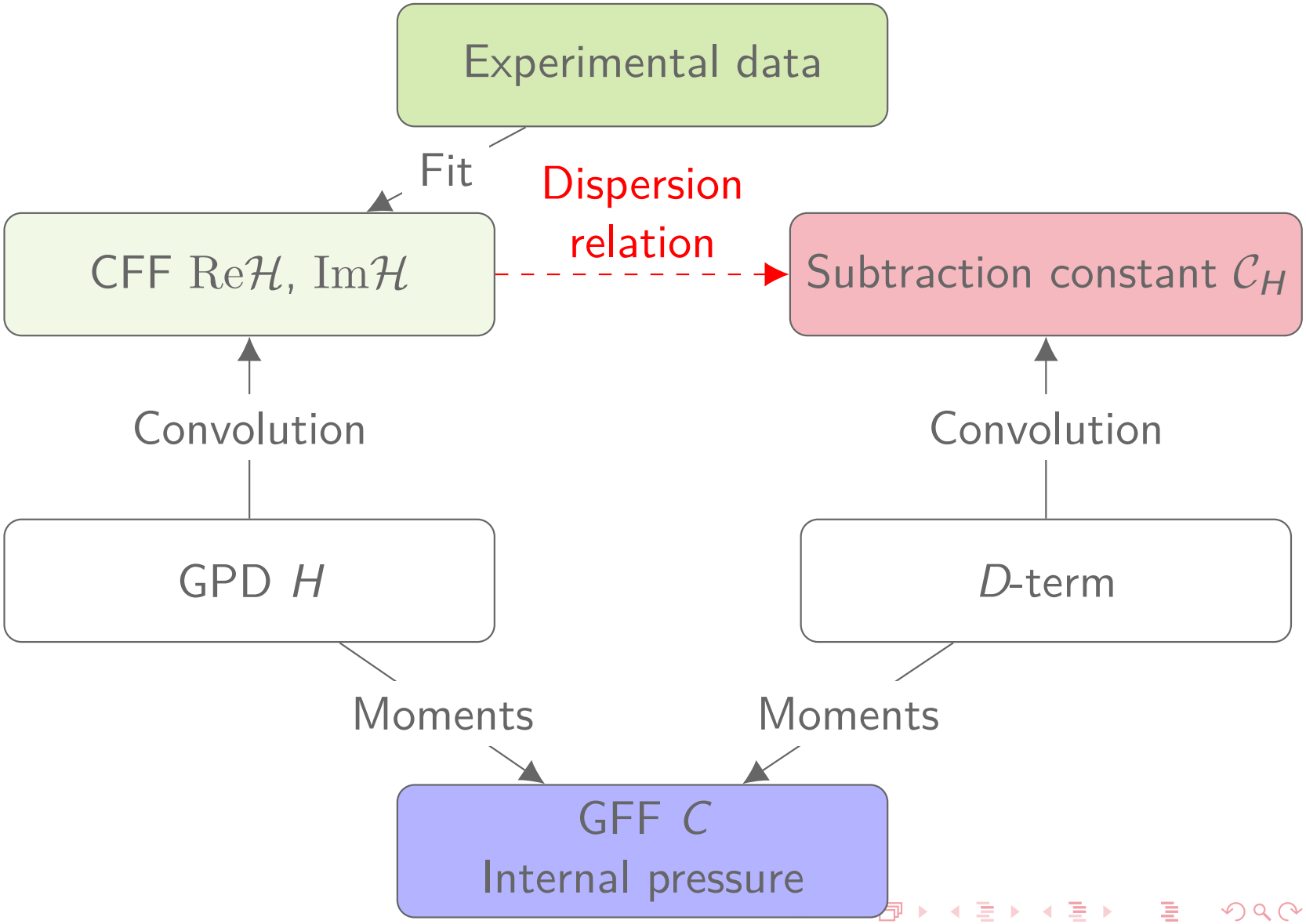

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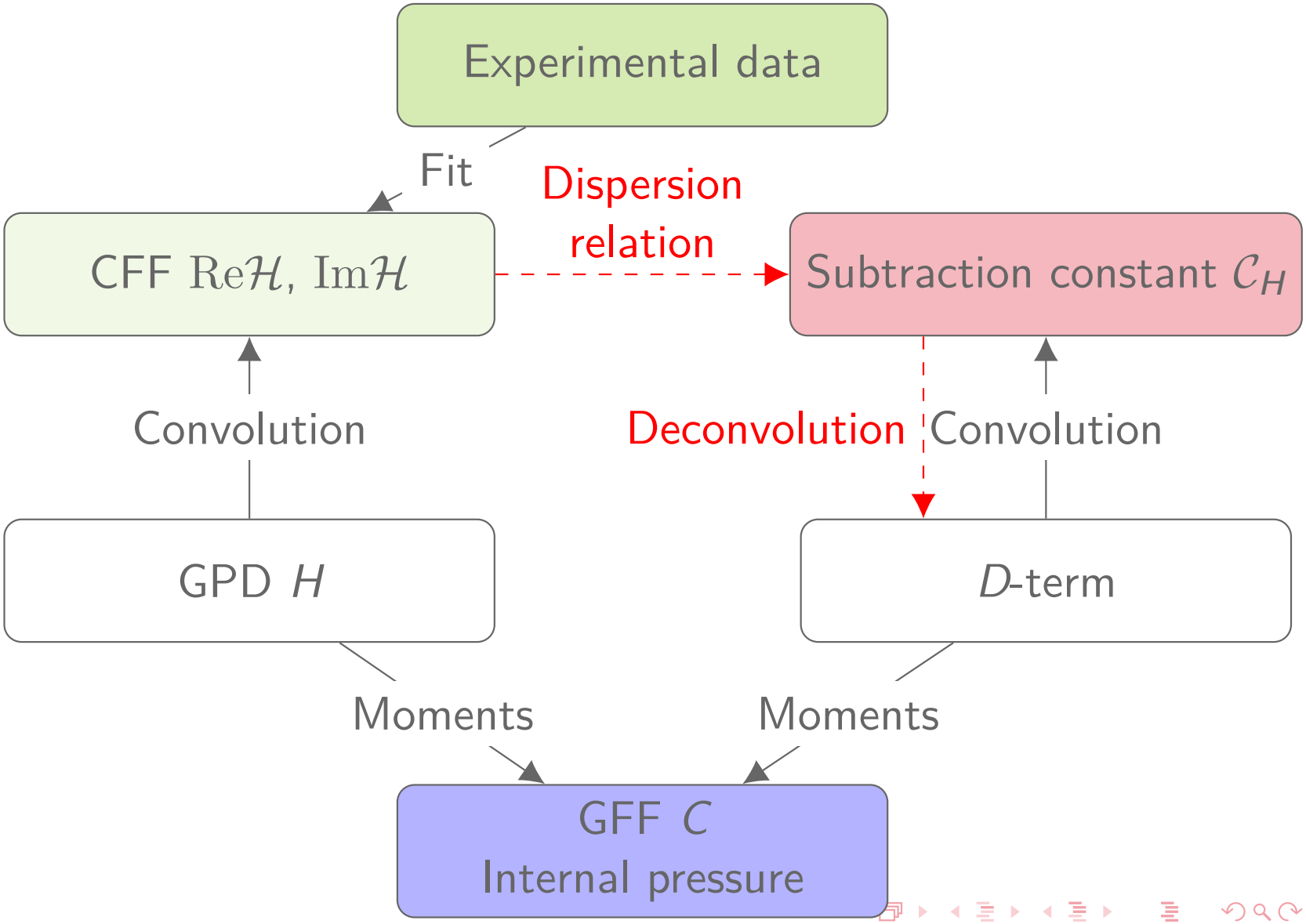


# Subtraction constant from measurements.

## EIC prospect: determination over a wide kinematic domain.



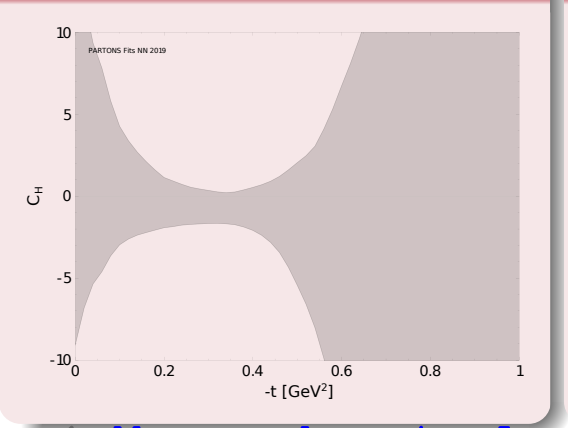
- Range of kinematic variables in neural networks

$$\begin{array}{ccccc} 10^{-6} & < & \xi & < & 1 \\ 0 & < & -t & < & 1\ \text{GeV}^2 \\ 1 & < & Q^2 & < & 100\ \text{GeV}^2 \end{array}$$

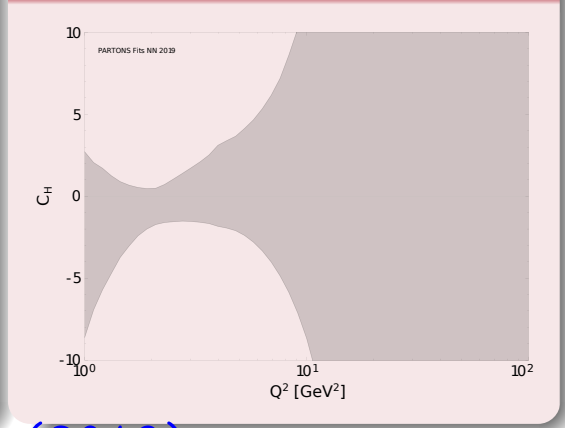
- Implement DVCS dispersion relation

$$\mathcal{C}_H(t, Q^2) = \mathrm{Re}\mathcal{H}(\xi) - \frac{1}{\pi}\int_{10^{-6}}^{1} \mathrm{d}\xi'\, \mathrm{Im}\mathcal{H}(\xi) \left( \frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right)$$

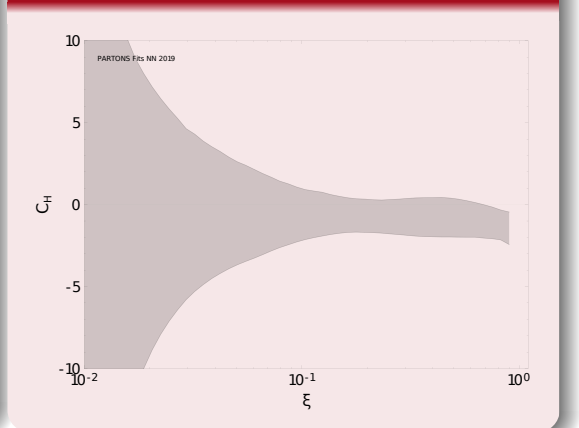
$\xi = 0.2$, $Q^2 = 2\ \text{GeV}^2$




$\xi = 0.2$, $t = -0.3\ \text{GeV}^2$




$t = -0.3\ \text{GeV}^2$, $Q^2 = 2\ \text{GeV}^2$




✍ Moutarde *et al.* (2019)

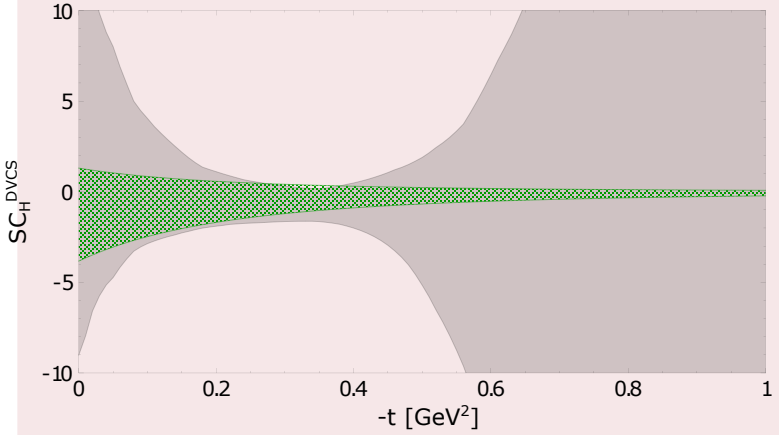
# Pressure forces from DVCS measurements.

## Working assumptions.



1. Subtraction constant assumed equal to $d_1$.
2. Equal values for light quark contributions $d_1^{uds}$.
3. Radiative generation of gluon $d_1^g$ and charm $d_1^c$ contributions.
4. Tripole Ansatz for the $t$-dependence of $d_1$.

### Tripole Ansatz




| Parameter | Value |
|---|---|
| $d_1^{uds}(\mu_F^2)$ | $-0.45 \pm 0.92$ |
| $d_1^c(\mu_F^2)$ | $-0.0020 \pm 0.0041$ |
| $d_1^g(\mu_F^2)$ | $-0.6 \pm 1.3$ |

✍ Dutrieux *et al.* (2021)

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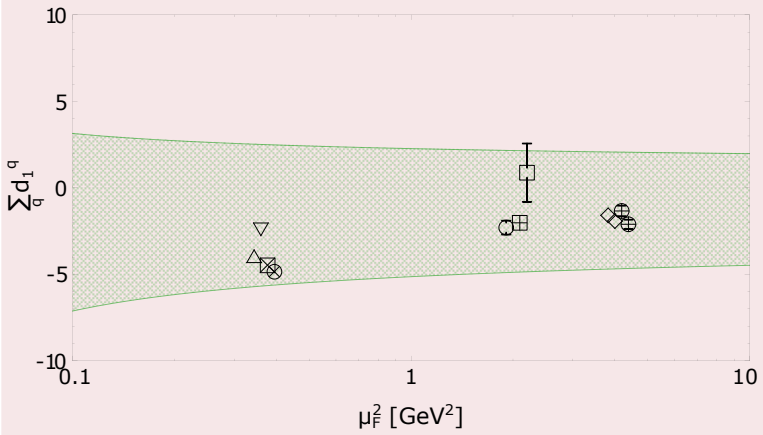
### $d_1$ from DVCS data

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### Summary of existing determinations

| No. | Marker in Fig. 3 | $\sum_q d_1^q(\mu_F^2)$ | $\mu_F^2$ in GeV$^2$ | # of flavours | Type | Ref. |
|---|---|---|---|---|---|---|
| 1 | ○ | $-2.30 \pm 0.16 \pm 0.37$ | 2.0 | 3 | from experimental data | [13] |
| 2 | □ | $0.88 \pm 1.69$ | 2.2 | 2 | from experimental data | [14] |
| 3 | ◇ | $-1.59$ | 4 | 2 | $t$-channel saturated model | [55] |
| | | $-1.92$ | 4 | 2 | $t$-channel saturated model | [55] |
| 4 | △ | $-4$ | 0.36 | 3 | $\chi$QSM | [30] |
| 5 | ▽ | $-2.35$ | 0.36 | 2 | $\chi$QSM | [10] |
| 6 | ⊠ | $-4.48$ | 0.36 | 2 | Skyrme model | [56] |
| 7 | ⊞ | $-2.02$ | 2 | 3 | LFWF model | [57] |
| 8 | ⊗ | $-4.85$ | 0.36 | 2 | $\chi$QSM | [58] |
| 9 | ⊕ | $-1.34 \pm 0.31$ | 4 | 2 | lattice QCD ($\overline{\text{MS}}$) | [59] |
| | | $-2.11 \pm 0.27$ | 4 | 2 | lattice QCD ($\overline{\text{MS}}$) | [59] |

✍ Dutrieux *et al.* (2021)

# Pressure forces from DVCS measurements.

## Relaxing some modelling assumptions.



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- 3 active quark flavors ($uds$), and radiative generation of $c$ contribution.
- 4 parameters

$$d_1^{uds}(\mu_0) \qquad d_3^{uds}(\mu_0) \qquad d_1^{g}(\mu_0) \qquad d_3^{g}(\mu_0)$$

## Investigate 3 fitting scenarios

**4 parameters**

$d_1^{uds}(0.1\mathrm{GeV}^2)$

$d_1^{g}(0.1\mathrm{GeV}^2)$

$d_3^{uds}(0.1\mathrm{GeV}^2)$

$d_3^{g}(0.1\mathrm{GeV}^2)$

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**4 parameters**

$d_1^{uds}(0.1\text{GeV}^2)$
$d_1^{g}(0.1\text{GeV}^2)$
$d_3^{uds}(0.1\text{GeV}^2)$
$d_3^{g}(0.1\text{GeV}^2)$

Free
Fixed

1. Nominal fit:
   - 1 free parameter for light quarks.
   - Gluon and charm radiatively generated.

| | LO | NLO |
|---|---|---|
| $d_1^{uds}(0.1\text{GeV}^2)$ | $-0.7 \pm 1.7$ | $-0.8 \pm 2.0$ |
| $d_1^{uds}(2\text{GeV})$ | $-0.5 \pm 1.2$ | $-0.5 \pm 1.4$ |
| $d_1^{g}(2\text{GeV})$ | $-0.6 \pm 1.6$ | $-0.7 \pm 1.9$ |
| $d_1^{c}(2\text{GeV})$ | $-0.002 \pm 0.0005$ | $-0.002 \pm 0.006$ |

Dutrieux *et al.*, *in preparation*

## Investigate 3 fitting scenarios

**4 parameters**

$d_1^{uds}(0.1\text{GeV}^2)$
$d_1^{g}(0.1\text{GeV}^2)$
$d_3^{uds}(0.1\text{GeV}^2)$
$d_3^{g}(0.1\text{GeV}^2)$

Free
Fixed

2. Alternative fit:
   - 2 free parameters: light quarks and gluons.
   - Charm radiatively generated.

| | LO | NLO |
|---|---|---|
| $d_1^{uds}(0.1\text{GeV}^2)$ | $-6.2 \pm 14$ | $-0.4 \pm 2.3$ |
| $d_1^{g}(0.1\text{GeV}^2)$ | $68 \pm 152$ | $6.3 \pm 22$ |
| $d_1^{uds}(2\text{GeV})$ | $-0.7 \pm 1.2$ | $0.4 \pm 2.8$ |
| $d_1^{g}(2\text{GeV})$ | $51 \pm 111$ | $5.3 \pm 19$ |
| $d_1^{c}(2\text{GeV})$ | $0.2 \pm 0.4$ | $0.02 \pm 0.06$ |

Dutrieux *et al.*, *in preparation*

## Investigate 3 fitting scenarios

**4 parameters**

$d_1^{uds}(0.1\text{GeV}^2)$
$d_1^{g}(0.1\text{GeV}^2)$
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Free
Fixed

2. Alternative fit:
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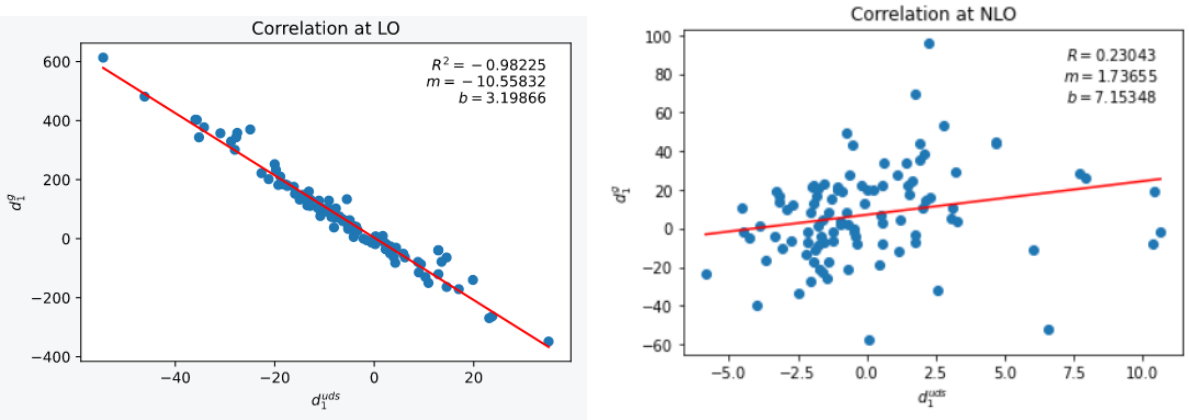



**Decorrelation of $d_1^g$ and $d_1^{uds}$ at NLO.**

Dutrieux *et al.*, *in preparation*

## Investigate 3 fitting scenarios

3. Alternative fit:
   - 2 free parameters: $d_1$ and $d_3$ for light quarks.
   - Gluon and charm radiatively generated.

**4 parameters**

$d_1^{uds}(0.1\text{GeV}^2)$
$d_1^{g}(0.1\text{GeV}^2)$
$d_3^{uds}(0.1\text{GeV}^2)$
$d_3^{g}(0.1\text{GeV}^2)$

Free
Fixed

| | LO | NLO |
|---|---|---|
| $d_1^{uds}(0.1\text{GeV}^2)$ | $16 \pm 37$ | $15 \pm 34$ |
| $d_3^{uds}(0.1\text{GeV}^2)$ | $-26 \pm 59$ | $-18 \pm 39$ |
| $d_1^{uds}(2\text{GeV})$ | $11 \pm 25$ | $11 \pm 23$ |
| $d_1^{g}(2\text{GeV})$ | $15 \pm 34$ | $15 \pm 32$ |
| $d_1^{c}(2\text{GeV})$ | $-0.05 \pm 0.1$ | $-0.05 \pm 0.1$ |
| $d_3^{uds}(2\text{GeV})$ | $-11 \pm 26$ | $-7.7 \pm 17$ |
| $d_3^{g}(2\text{GeV})$ | $-1.8 \pm 3.9$ | $-1.2 \pm 2.6$ |
| $d_3^{c}(2\text{GeV})$ | $-0.04 \pm 0.01$ | $-0.003 \pm 0.007$ |

Dutrieux *et al.*, *in preparation*

## Investigate 3 fitting scenarios

**4 parameters**

$d_1^{uds}(0.1\text{GeV}^2)$

$d_1^{g}(0.1\text{GeV}^2)$

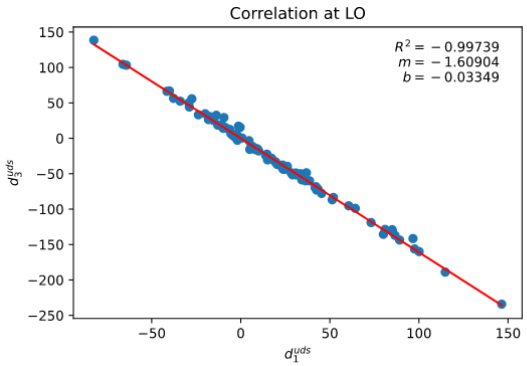
$d_3^{uds}(0.1\text{GeV}^2)$

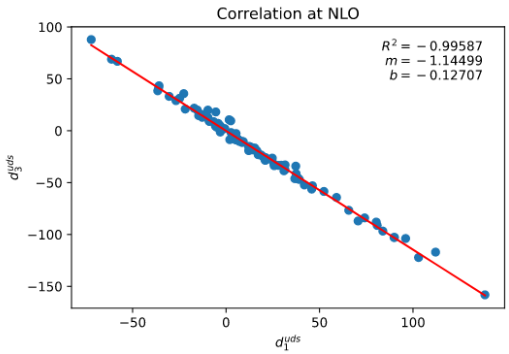
$d_3^{g}(0.1\text{GeV}^2)$

Free

Fixed

3. Alternative fit:
   - 2 free parameters: $d_1$ and $d_3$ for light quarks.
   - Gluon and charm radiatively generated.




**Strong correlation of $d_1^{uds}$ and $d_3^{uds}$ both at LO and NLO.**

Dutrieux *et al.*, *in preparation*

# Areas for improvement

# Increase the physics input in the global fit.

## An example of the bias-variance trade-off.



- So far the CFF fit gathering most of the world DVCS measurements relies on an **independent modeling** of the CFF real and imaginary parts by **neural networks**.
- Convenient because of the **dimensionality** of the problem but yields **large statistical uncertainties**.

✍ Moutarde *et al.* (2019)

- Conversaly a fit to the same data with a **physically motivated** parameterization still required ***ad hoc* assumptions**.

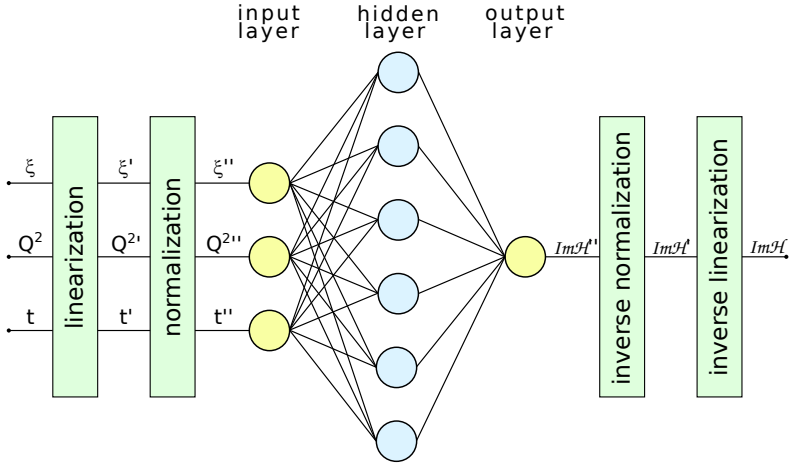
✍ Moutarde *et al.* (2018)

- Many **first-principle constraints** expressed at the GPD level are not implemented at the CFF level.

# Increase the physics input in the global fit.

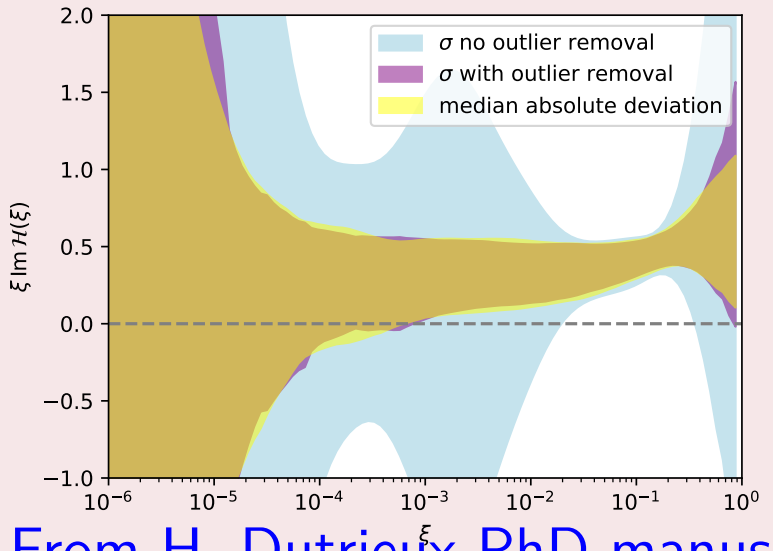
## An example of the bias-variance trade-off.



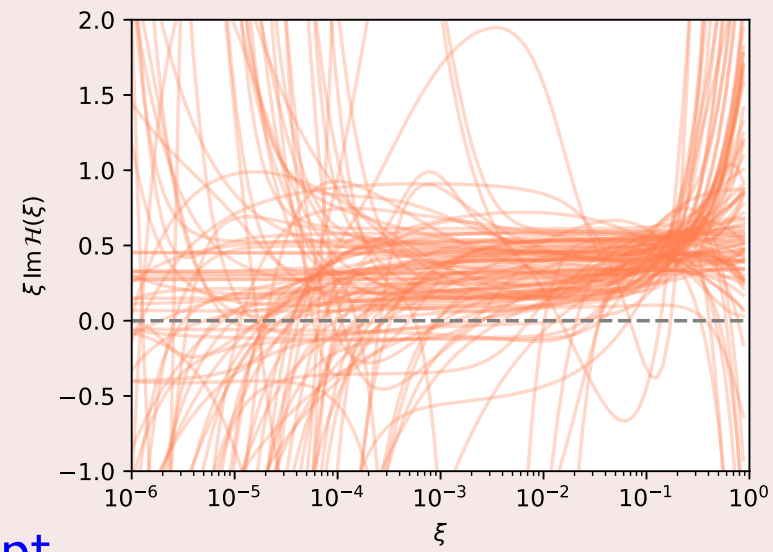



- Polynomiality?
- Positivity?
- Reduction to PDF or EFF?
- Evolution?

### Outliers




### Replicas



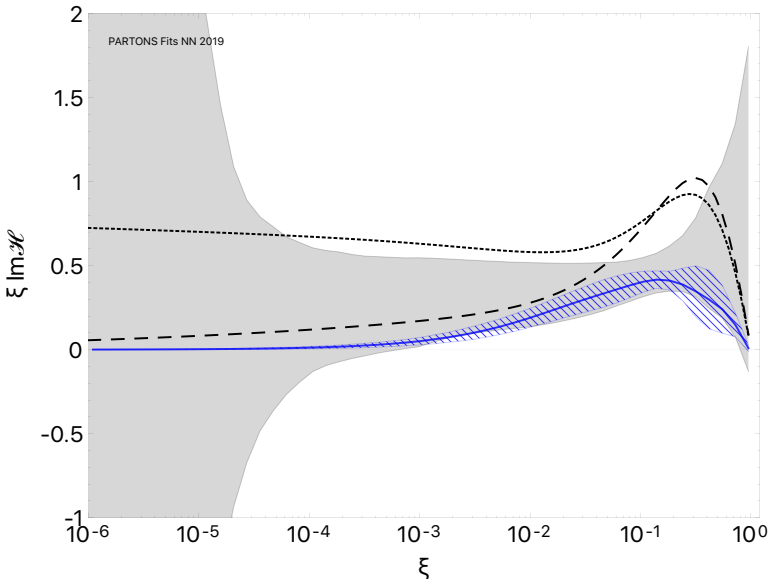

From H. Dutrieux PhD manuscript

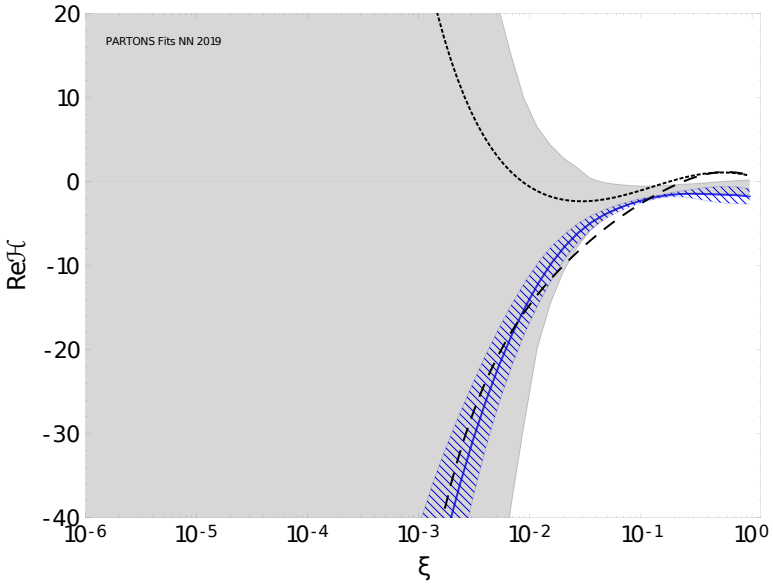
# Increase the physics input in the global fit.

## An example of the bias-variance trade-off.



- Next step requires a (challenging) **GPD global fit** to world data.
- On the long run, need more experimental data to
  - Increase the $Q^2$-**lever arm**.
  - Provide a better handle on the real part of $\mathcal{H}$.
  - Improve the **accuracy** of existing measurements.
  - Probe the kinematic regions insufficiently constrained.




✍ Moutarde *et al.* (2019)

# Relax modeling assumptions on $d_1(t)$.

## Shape of pressure distribution not set by the current fit...



- Use **multipole Ansatz**

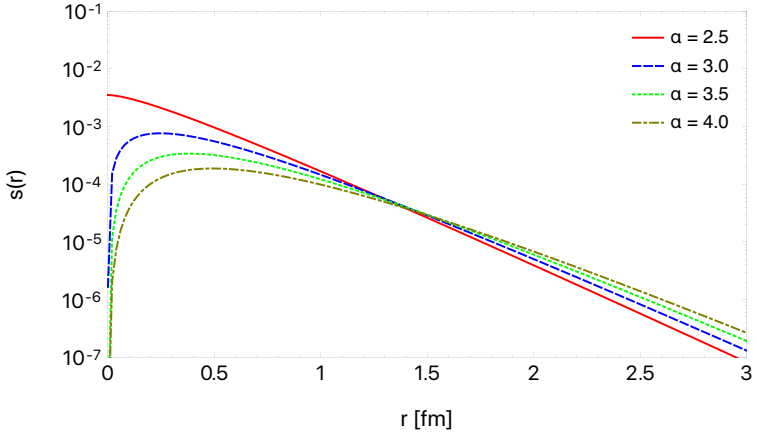
$$d_1(t,\mu_F) = \frac{d_1(\mu_F)}{\left(1-\frac{t}{\Lambda^2}\right)^\alpha}$$

- Remind $d_1^q(t,\mu_F^2) = 5C_q(t,\mu_F^2)$.
- Plug in pressure anisotropy

$$\frac{s(r)}{M} \propto \int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ -\frac{4}{r^2}\frac{t^{-1/2}}{M^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(t^{5/2}\, d_1(t)\right)\right\}$$

- Normalization $d_1(\mu_F)$ set by fit.
- Position of node in $r$ depends on $\Lambda$.

✍ Dutrieux *et al.* (2021)



# Relax modeling assumptions on $d_1(t)$.

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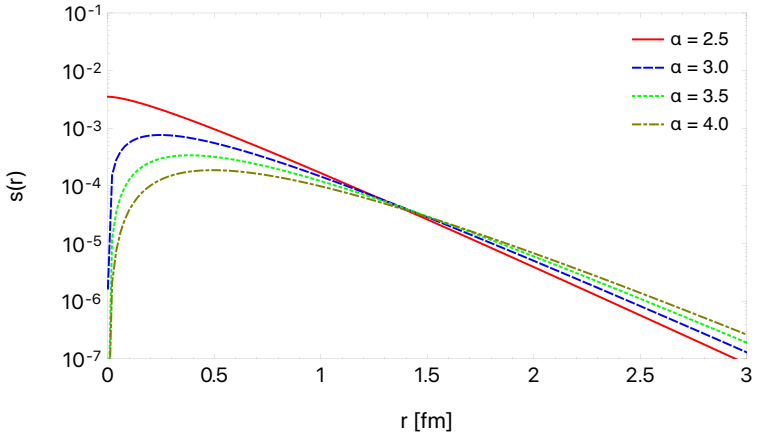



✍ Dutrieux *et al.* (2021)

- **Asymptotic** information on $|t|$-dependence from perturbative QCD. *But how large is "asymptotic"?*
- **Factorization** constraint: $Q^2 \gg |t|$. *Most of the experimental data used as fit input has low $|t|$.*
- Need for more experimental data points.

# Increase $Q^2$-lever arm.

## Evolution equations bring a slow $\log Q^2$ dependence.



- Remind computation of subtraction constant at LO

$$\mathcal{C}_H(t, Q^2) = 4 \sum_q e_q^2 \sum_{\text{odd } n} d_n^q(t, \mu_F^2 \equiv Q^2)$$

✍ Diehl & Ivanov (2007)

- Plug LO evolution of D-term to obtain the following **pattern**

$$\mathcal{C}_H(t, Q^2) \propto \sum_{\text{odd } n} d_n(t, \mu_F) \left( \frac{\alpha_s(Q^2)}{\alpha_s(\mu_F^2)} \right)^{\gamma_n}$$

with $\gamma_n$ computed in perturbative QCD.

- Since $\alpha_s(Q^2) \propto 1/\log Q^2$, an exact knowledge of $\mathcal{C}_H(t, Q^2)$ on an $Q^2$-interval allows to exactly retrieve $d_n$.

# Increase $Q^2$-lever arm.

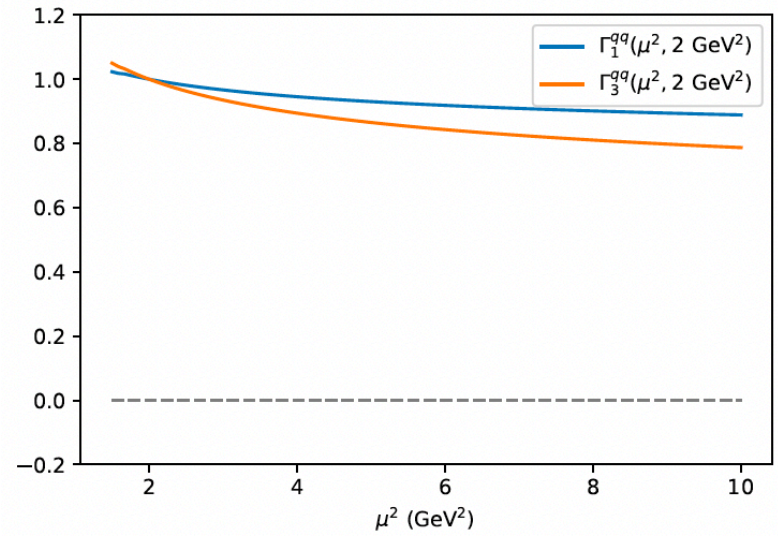
Anomalous dimensions $\gamma_n$ are small and take comparable values.

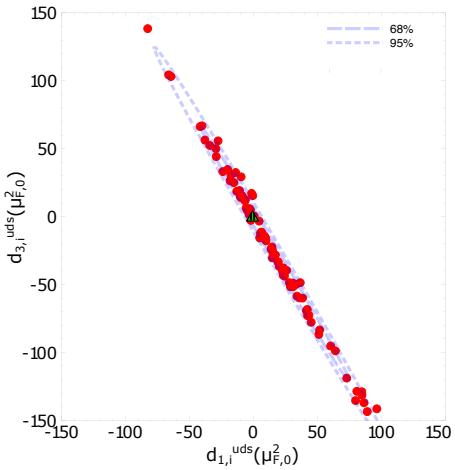


- Introduce evolution operator $\Gamma$ so that

$$d_n(\mu_1) = \Gamma_n(\mu_1, \mu_2) d_n(\mu_2)$$

- Probed $Q^2$-range in CFF fit: $[1.5, 4]$ GeV$^2$.
- $\Gamma_1$ and $\Gamma_3$ are **numerically very close**.




- Evolution **"too slow"** to separate $d_1$ and $d_3$ for $Q^2 \in [1.5, 4]$ GeV$^2$.
- Experimental data mostly constrain $d_1 + d_3 + \ldots$

✍ Dutrieux *et al.* (2021)

# Anomalous dimensions $\gamma_n$ and $Q^2$-lever arm.

## Inverse problem and regularization.



- Remind **pattern** of the problem

$$\mathcal{C}_H(t, Q^2) \propto \sum_{\text{odd } n} d_n(t, \mu_F) \left( \frac{\alpha_s(Q^2)}{\alpha_s(\mu_F^2)} \right)^{\gamma_n}$$

- If $Q^2$-range is too small, a solution with $d_1(t, \mu_F) + d_3(t, \mu_F) + d_5(t, \mu_F) + \ldots = 0$ can remain **hidden within experimental uncertainties** over the whole range $Q^2 \in [Q^2_{\text{min}}, Q^2_{\text{max}}]$.
- In practice: act as if the problem of retrieving $d_1$, $d_3, \ldots$ from measurements has infinitely many solutions.
- Add extra **regularization** to select one solution **robust with respect to statistical uncertainties**.
- Today **cannot reliably estimate** the uncertainty associated to the neglect of $d_3, \ldots$

# Conclusion

# Conclusion and prospects.

## The phenomenological quest towards proton internal pressure.



- Concept **well-defined** and suitable for phenomenology.
- Strong **first-principle connection** between concept and experimental data.
- Need for **multi-channel** analysis **beyond LO** on a **wide kinematic coverage**. EIC and EIcC much needed!
- The GPD deconvolution problem is **ill-posed**. **Huge sensitivity** to numerical noise or experimental uncertainties.

gg75478317 GoGraph.com

- Development of the **software ecosystem** PARTONS for 3D hadron structure studies.
- Need for **coordinated effort** involving fits, computing chains, continuum and lattice QCD to make the best from experiments.

# Abbreviations

# Abbreviations used in this presentation.



| | |
|---|---|
| ANN | artificial neural network |
| CFF | Compton form factor |
| DDVCS | double deeply virtual Compton scattering |
| DVCS | deeply virtual Compton scattering |
| DVMP | deeply virtual meson production |
| DR | dispersion relation |
| EIC | electron-ion collider |
| EIcC | electron-ion collider in China |
| EFF | elastic form factor |
| GFF | gravitational form factor |
| GPD | generalized parton distribution |
| LO | leading order |
| NLO | next-to-leading order |
| PDF | parton distribution function |
| TCS | timelike Compton scattering |

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