

Dynamical mass generation in 'tHooft Model

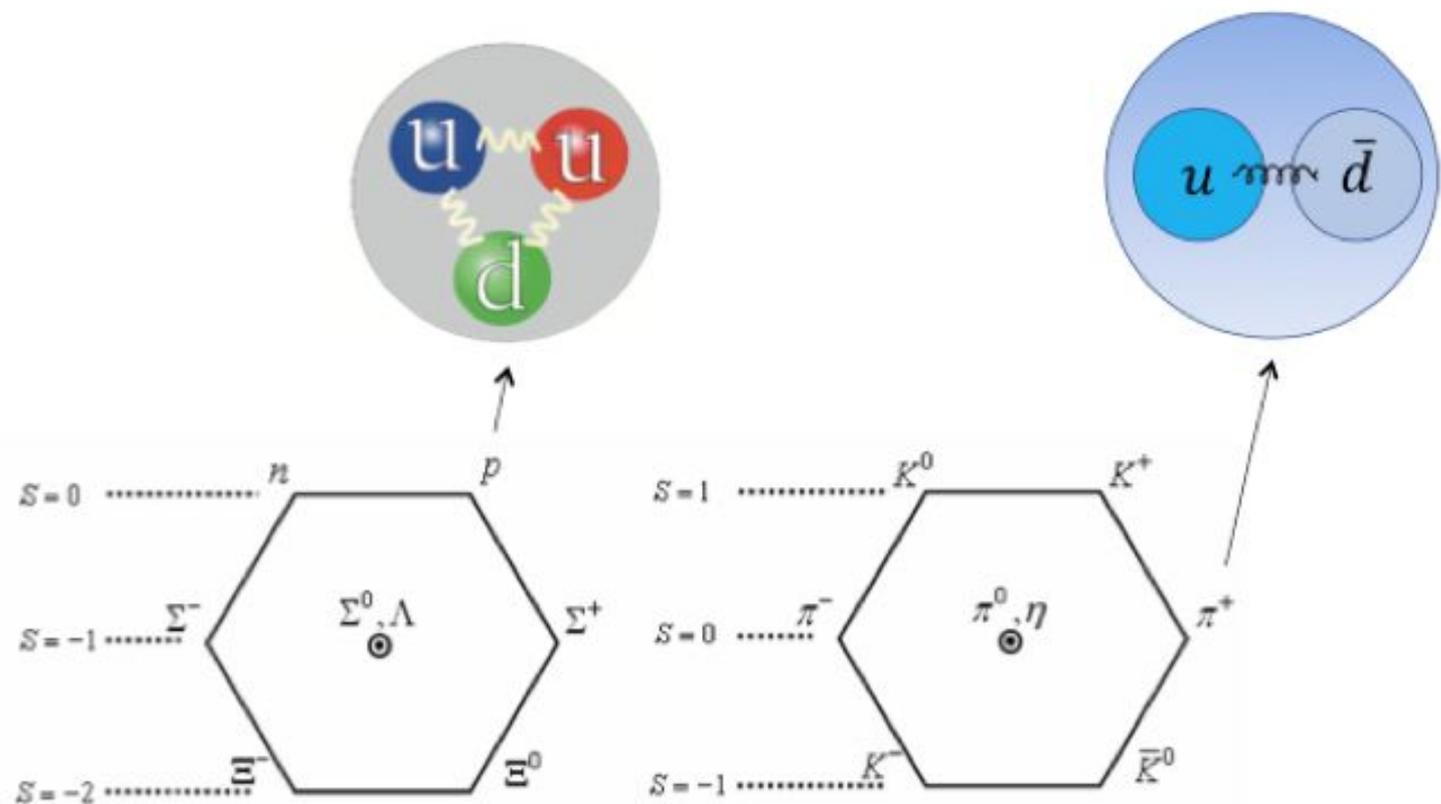
Chueng-Ryong Ji

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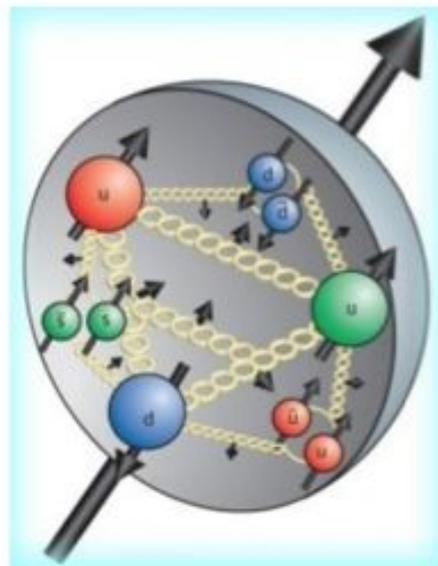
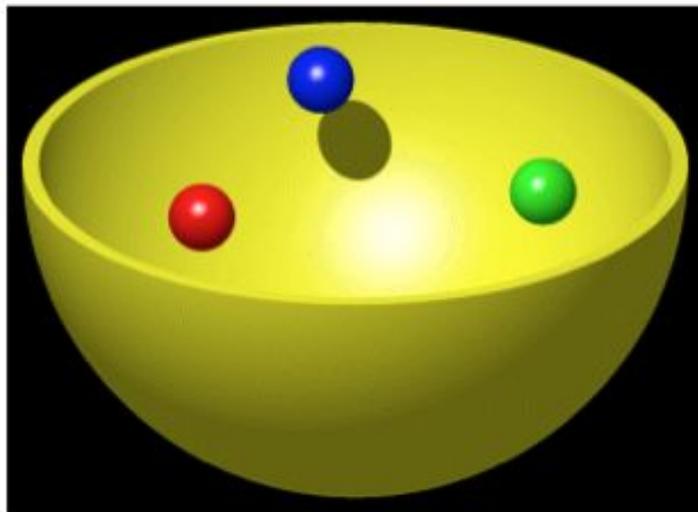
September 12, 2022, ECT*

How do we understand the Quark Model in Quantum Chromodynamics?

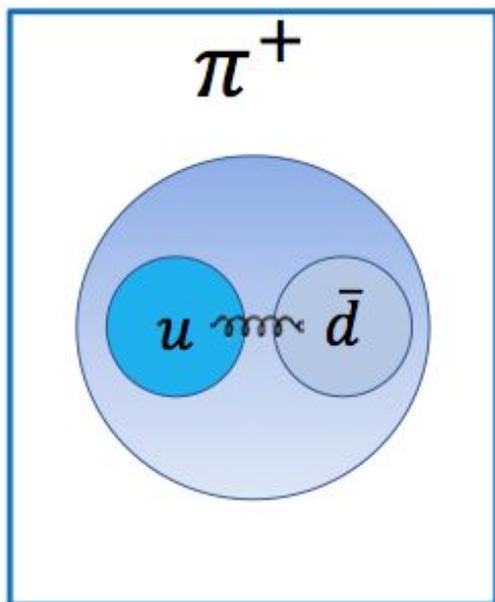


$$M_p = 938.272046 \pm 0.000021 \text{ MeV}$$

$$M_n = 939.565379 \pm 0.000021 \text{ MeV}$$



$$m_u = 2.3_{-0.5}^{+0.7} \text{ MeV} \quad ; \quad m_d = 4.8_{-0.3}^{+0.7} \text{ MeV}$$



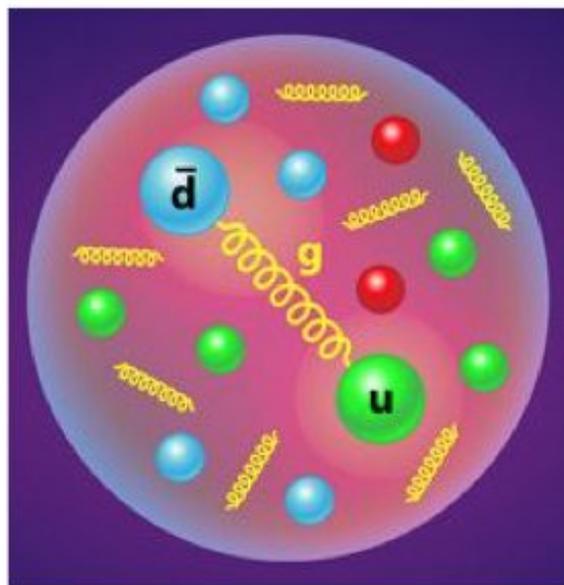
Constituent Quark Model

$$M = m_1 + m_2 + A \frac{\vec{s}_1 \cdot \vec{s}_2}{m_1 m_2}$$

$$m_u = m_d = 310 \text{ MeV} / c^2$$

$$A = \left(\frac{2m_u}{\hbar} \right)^2 160 \text{ MeV} / c^2$$

vs.



Quantum Chromodynamics

Isospin symmetry

Chiral symmetry

$SU(2)_R \times SU(2)_L$

Spontaneous symmetry breakdown

Goldstone Bosons

$$F_\pi^2 M_\pi^2 = -(m_u + m_d) \langle 0 | \bar{u}u | 0 \rangle$$

Effective field theory

Outline

- 'tHooft model and short list of references
- Dirac's proposition for relativistic dynamics
- Interpolation between IFD and LFD
- Mass gap solutions in 'tHooft model
- Spontaneous breaking of chiral symmetry
- Meson spectroscopy and wavefunctions
- Application to hadron physics

Large N_c QCD in 1+1 dim. ('tHooft Model)

$$\mathcal{L} = -\frac{1}{4} F_{\hat{\mu}\hat{\nu}}^a F^{\hat{\mu}\hat{\nu}a} + \bar{\psi}(i\gamma^{\hat{\mu}} D_{\hat{\mu}} - m)\psi$$

$$D_{\hat{\mu}} = \partial_{\hat{\mu}} - igA_{\hat{\mu}}^a t_a$$

$$F_{\hat{\mu}\hat{\nu}}^a = \partial_{\hat{\mu}} A_{\hat{\nu}}^a - \partial_{\hat{\nu}} A_{\hat{\mu}}^a + gf^{abc} A_{\hat{\mu}}^b A_{\hat{\nu}}^c$$

'tHooft Coupling $\lambda = \frac{g^2 (N_c - 1/N_c)}{4\pi}$ and mass m

$$g \rightarrow 0, N_c \rightarrow \infty; \lambda \rightarrow \text{finite}$$

Short List of References

- G.'tHooft, NPB75,461(74) - LFD
- Y.Frishman, et al., PRD15(75) - Interpol Gauges IFD&LFD
- I.Bars&M.Green, PRD17,537(78) - IFD(formulation)
- A.Zhitnitsky, PLB165,405(85) - LFD(chiral sym breaking)
- M.Li, et al., JPG13, 915(87) - IFD(rest frame)
- K.Hornbostel, Ph.D. Dissertation(88) - LFD(DLCQ)
- M.Burkardt, PRD53,933(96) - LFD(vacuum condensates)
- Y.Kalashnikov&A.Nefed'ev,Phys.-Usp.45,346('02) - IFD(rev)
- Y. Jia, et al., JHEP11, 151('17) - IFD(moving frame)
- Y. Jia, et al., PRD98, 054011('18) - IFD(quasi-PDFs)
- B.Ma&C.Ji, PRD104,036004('21) - [Link IFD&LFD](#)

Dirac's Proposition for Relativistic Dynamics



Equal t

1949

Equal τ

$$p^0 \leftrightarrow p^- = p^0 - p^3$$

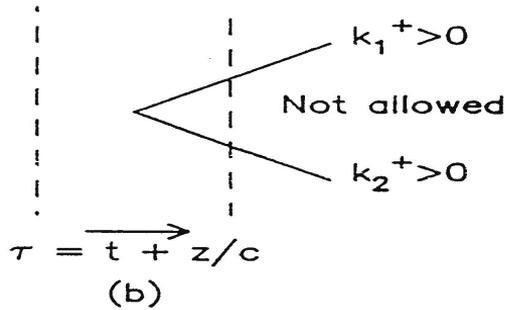
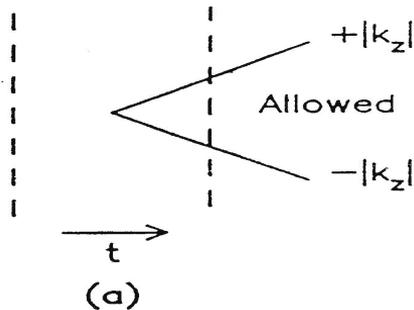
$$(p^1, p^2) \leftrightarrow \vec{p}_\perp$$

$$p^3 \leftrightarrow p^+ = p^0 + p^3$$

Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^2 + m^2}$$

$$p^- = \frac{\vec{p}_\perp^2 + m^2}{p^+}$$



Except zero-modes

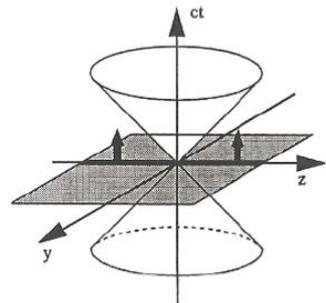
$$k_1^+ = k_2^+ = 0$$

IFD

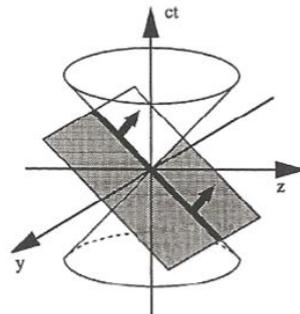
Instant Form Dynamics

LFD

Light-Front Dynamics

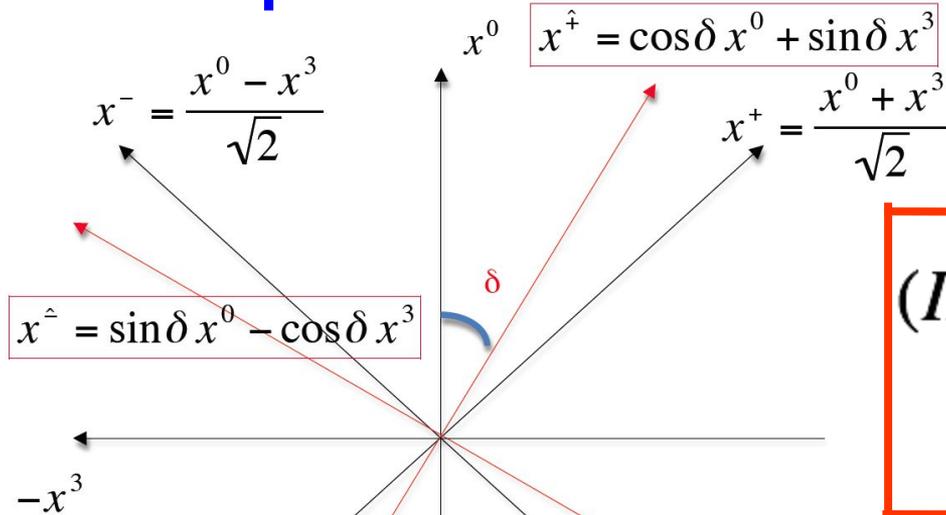


The instant form



The front form

Interpolation between IFD and LFD



$$(IFD) \quad 0 \leq \delta \leq \frac{\pi}{4} \quad (LFD)$$

$$1 \geq C \equiv \cos(2\delta) \geq 0$$

K. Hornbostel, PRD45, 3781 (1992) – RQFT

C.Ji and S.Rey, PRD53, 5815(1996) – Chiral Anomaly

C.Ji and C. Mitchell, PRD64, 085013 (2001) – Poincare Algebra

C.Ji and A. Suzuki, PRD87, 065015 (2013) – Scattering Amps

C.Ji, Z. Li and A. Suzuki, PRD91, 065020 (2015) – EM Gauges

Z.Li, M. An and C.Ji, PRD92, 105014 (2015) – Spinors

C.Ji, Z.Li, B.Ma and A.Suzuki, PRD98, 036017(2018) – QED

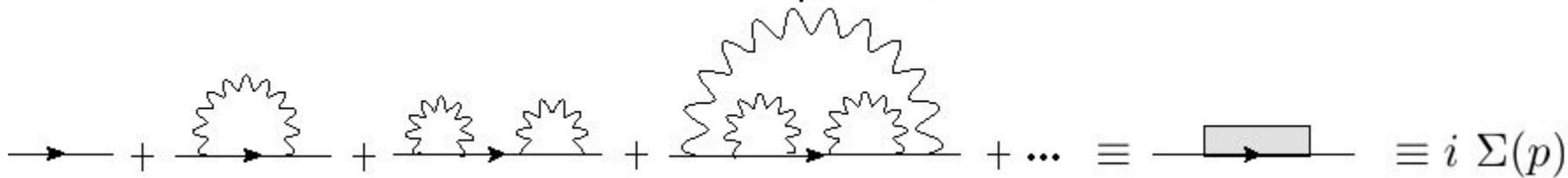
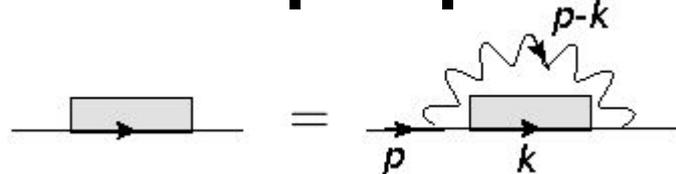
B.Ma and C.Ji, PRD194, 036004(2021) – QCD₁₊₁

Interpolating Axial Gauge

$$A_{\hat{\pm}}^a = 0$$

$$\mathcal{L} = \frac{1}{2} \left(\partial_{\hat{\pm}} A_{\hat{\mp}}^a \right)^2 + \bar{\psi} (i\gamma^{\hat{\mp}} D_{\hat{\mp}} + i\gamma^{\hat{\pm}} \partial_{\hat{\pm}} - m) \psi$$

Mass Gap Equation



$$\Sigma(p_{\hat{\pm}}) = i \frac{\lambda}{2\pi} \int \frac{dk_{\hat{\pm}} dk_{\hat{\mp}}}{(p_{\hat{\pm}} - k_{\hat{\pm}})^2} \gamma^{\hat{\mp}} \frac{1}{\not{k} - m - \Sigma(k_{\hat{\pm}}) + i\epsilon} \gamma^{\hat{\mp}}$$

Fermion Propagator

Free Propagator

$$S_f(p) = \frac{1}{\not{p} - m + i\epsilon}$$



Interacting Propagator

$$S(p) = \frac{1}{\not{p} - m - \Sigma(p) + i\epsilon}$$
$$= \frac{F(p)}{\not{p} - M(p) + i\epsilon}$$

$$\Sigma(p) = \Sigma_s(p) + \Sigma_v(p)\not{p}$$

$$F(p) = (1 - \Sigma_v(p))^{-1} \quad \text{“Wave function renormalization factor”}$$

$$M(p) = \frac{m + \Sigma_s(p)}{1 - \Sigma_v(p)} \quad \text{“Renormalized fermion mass function”}$$

Energy-Momentum Dispersion Relation

Free particle

Interacting particle

$$E = \sqrt{p_z^2 + m^2}$$

$$\frac{F(p'_\perp)E(p'_\perp)}{\sqrt{C}} = \sqrt{p'^2_\perp + M(p'_\perp)^2} \equiv \tilde{E}(p'_\perp)$$

$$\theta_f = \tan^{-1}(p_z / m)$$

$$\theta(p'_\perp) = \theta_f(p'_\perp) + 2\zeta(p'_\perp)$$

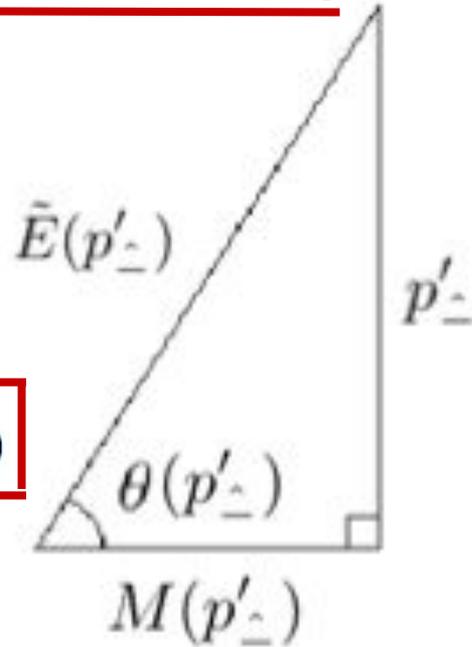
$$\beta = p_z / E$$

$$\begin{pmatrix} b^i(p'_\perp) \\ d^{+i}(p'_\perp) \end{pmatrix} = \begin{pmatrix} \cos\zeta(p'_\perp) & -\sin\zeta(p'_\perp) \\ \sin\zeta(p'_\perp) & \cos\zeta(p'_\perp) \end{pmatrix} \begin{pmatrix} b^i_f(p'_\perp) \\ d^{+i}_f(p'_\perp) \end{pmatrix}$$

$$= \sin\theta_f$$

$$= \tanh\eta$$

$$b^i_f |0\rangle = 0, d^i_f |0\rangle = 0 \quad \text{vs.} \quad b^i |\Omega\rangle = 0, d^i |\Omega\rangle = 0$$



Interpolation

$$(E, p_z) \Rightarrow (p^\dagger / \sqrt{C}, p_\perp / \sqrt{C} \equiv p'_\perp)$$

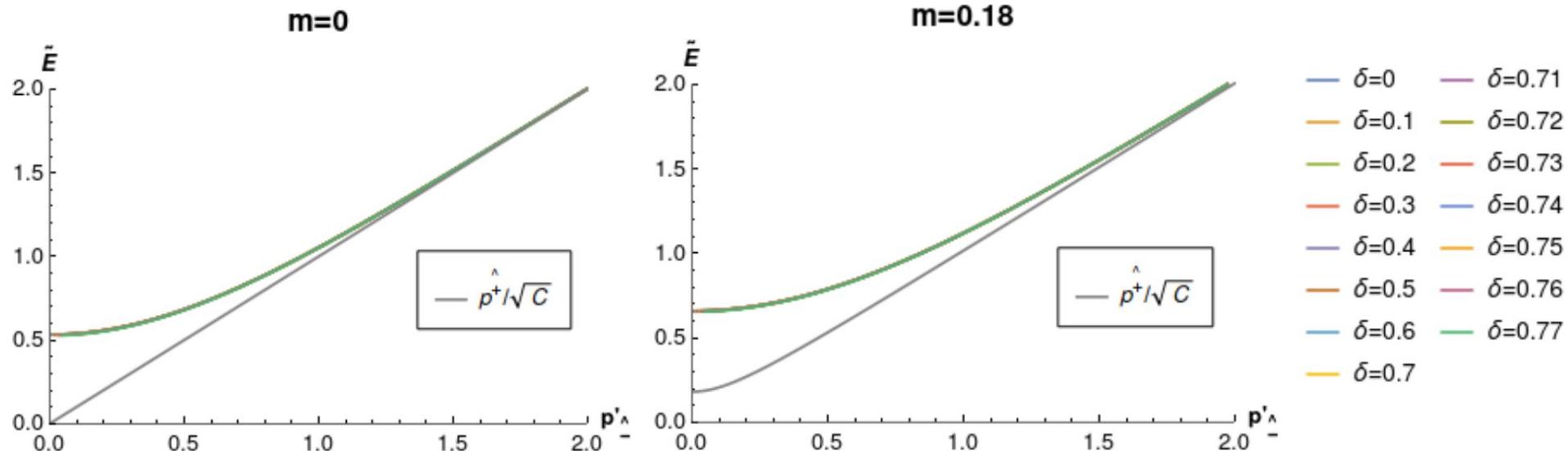
Mass Gap Equation in Scaled Variables

$$\bar{p}'_{\hat{_}} = \frac{\bar{p}_{\hat{_}}}{\sqrt{\mathbb{C}}}, \quad \bar{E}' = \frac{\bar{E}}{\sqrt{\mathbb{C}}}, \quad \bar{p}_{\hat{_}} = \frac{p_{\hat{_}}}{\sqrt{2\lambda}}, \quad \bar{E} = \frac{E}{\sqrt{2\lambda}}, \quad \bar{m} = \frac{m}{\sqrt{2\lambda}}$$

$$\begin{aligned} \bar{p}'_{\hat{_}} \cos \theta(\bar{p}'_{\hat{_}}) - \bar{m} \sin \theta(\bar{p}'_{\hat{_}}) &= \frac{1}{4} \int \frac{d\bar{k}'_{\hat{_}}}{(\bar{p}'_{\hat{_}} - \bar{k}'_{\hat{_}})^2} \sin \left(\theta(\bar{p}'_{\hat{_}}) - \theta(\bar{k}'_{\hat{_}}) \right) \\ \bar{E}'(\bar{p}'_{\hat{_}}) &= \bar{p}'_{\hat{_}} \sin \theta(\bar{p}'_{\hat{_}}) + \bar{m} \cos \theta(\bar{p}'_{\hat{_}}) + \frac{1}{4} \int \frac{d\bar{k}'_{\hat{_}}}{(\bar{p}'_{\hat{_}} - \bar{k}'_{\hat{_}})^2} \cos \left(\theta(\bar{p}'_{\hat{_}}) - \theta(\bar{k}'_{\hat{_}}) \right) \end{aligned}$$

$$\begin{aligned} \frac{p_{\hat{_}}}{\mathbb{C}} \cos \theta(p_{\hat{_}}) - \frac{m}{\sqrt{\mathbb{C}}} \sin \theta(p_{\hat{_}}) &= \frac{\lambda}{2} \int \frac{dk_{\hat{_}}}{(p_{\hat{_}} - k_{\hat{_}})^2} \sin \left(\theta(p_{\hat{_}}) - \theta(k_{\hat{_}}) \right) \\ E(p_{\hat{_}}) &= p_{\hat{_}} \sin \theta(p_{\hat{_}}) + \sqrt{\mathbb{C}} m \cos \theta(p_{\hat{_}}) + \frac{\mathbb{C}\lambda}{2} \int \frac{dk_{\hat{_}}}{(p_{\hat{_}} - k_{\hat{_}})^2} \cos \left(\theta(p_{\hat{_}}) - \theta(k_{\hat{_}}) \right) \end{aligned}$$

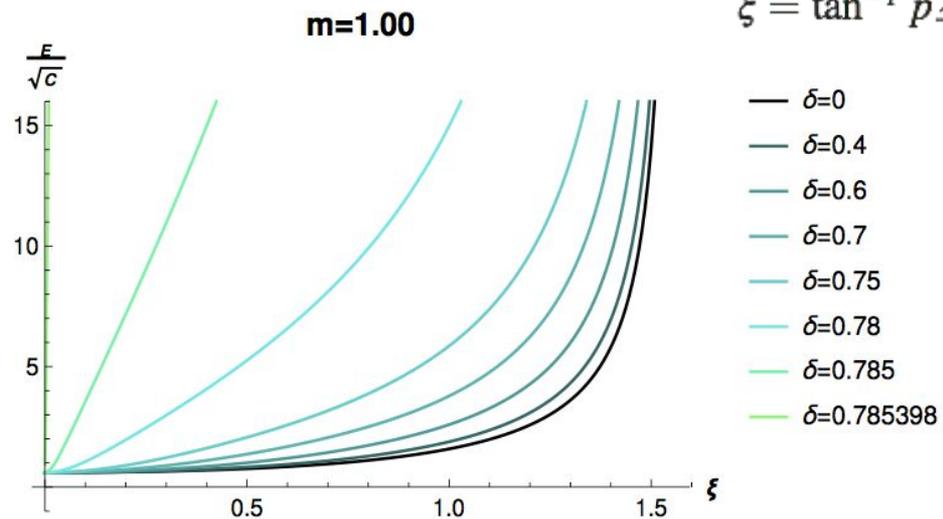
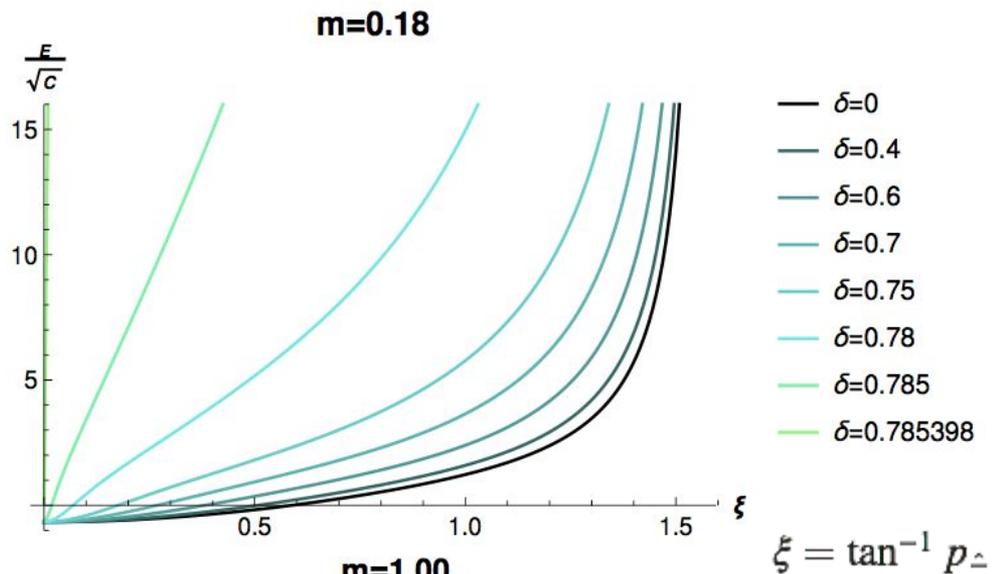
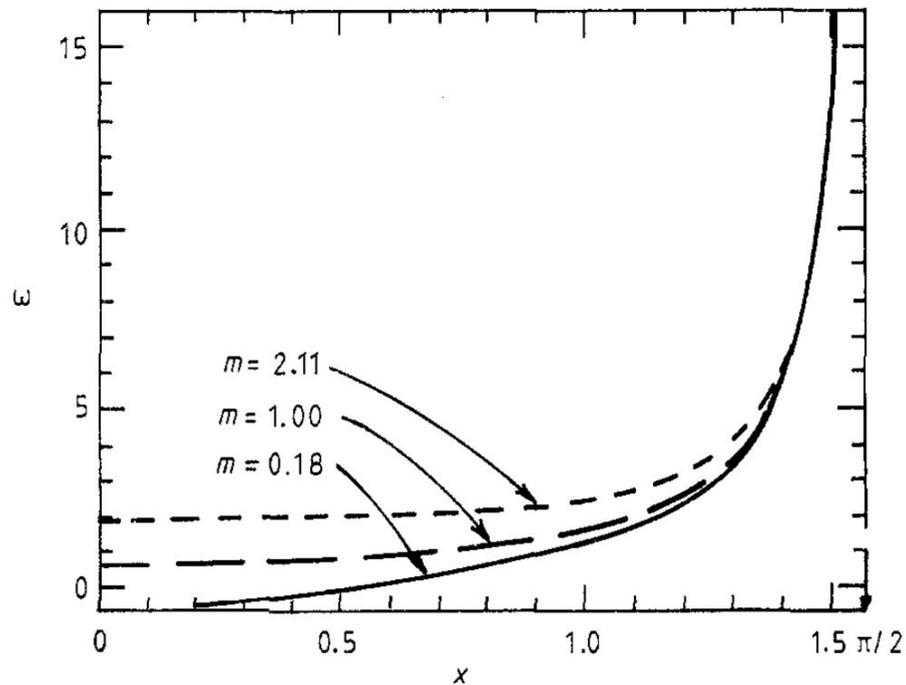
Mass Gap Solutions



$$\tilde{E}(0) = \frac{F(0)E(0)}{\sqrt{C}} = M(0)$$

m	0	0.045	0.18	0.749	1.00	2.11	4.23
$M(0)$	0.532778	0.563644	0.659112	1.10105	1.31167	2.30969	4.34358
$F(0)$	-0.495173	-0.584175	-0.987673	4.11079	2.17976	1.22134	1.05526

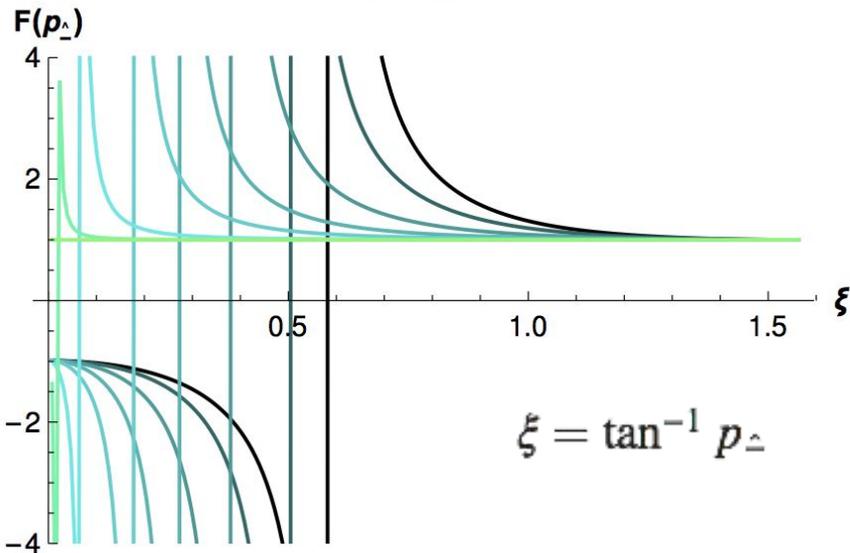
$$m \lesssim 0.56$$



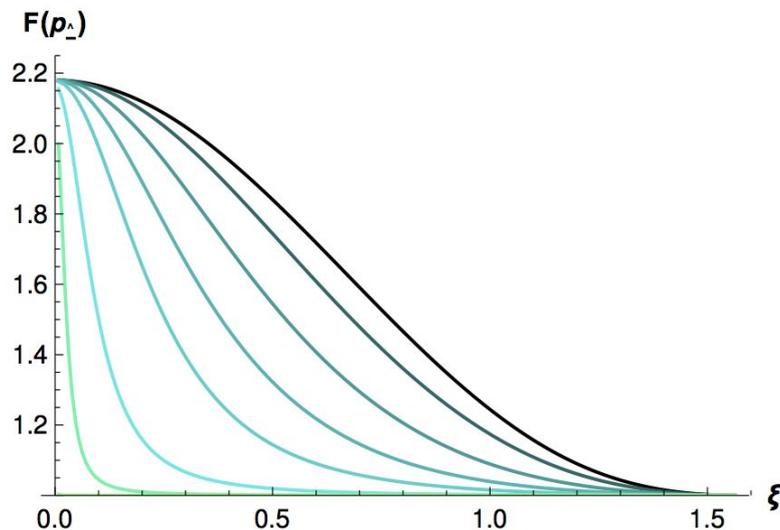
- M.Li, et al., JPG13, 915(87) - IFD(rest frame)

Wave Function Renormalization Factors

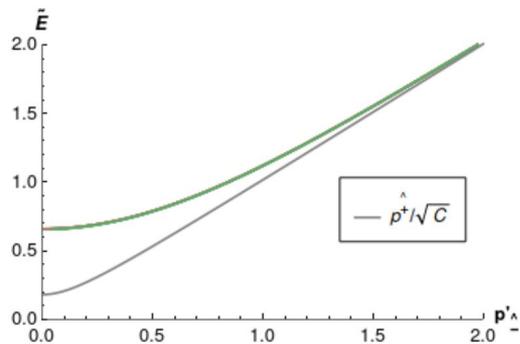
$m=0.18$



$m=1.00$

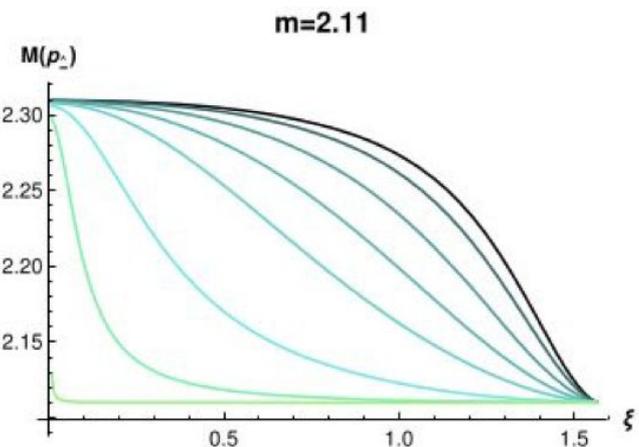
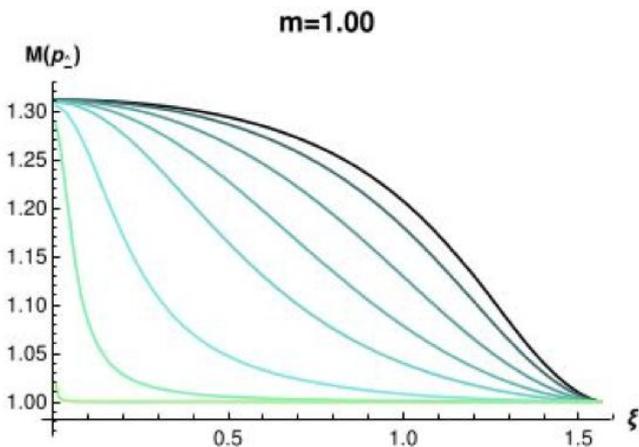
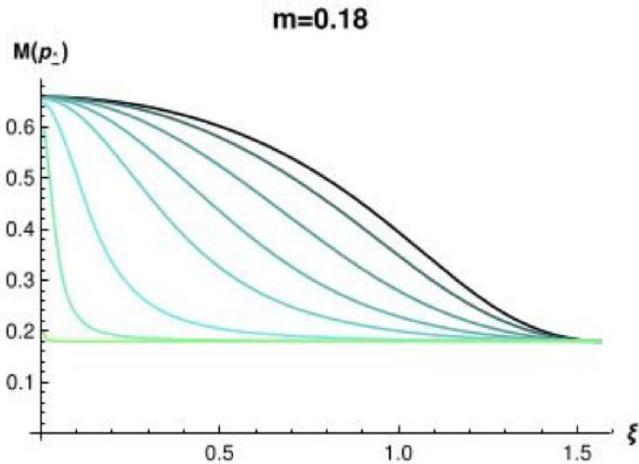
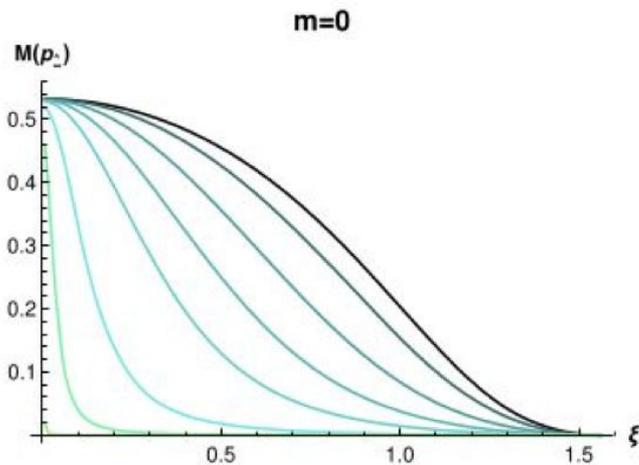


$m=0.18$



$$\frac{F(p'_{\perp})E(p'_{\perp})}{\sqrt{C}} = \sqrt{p'^2_{\perp} + M(p'_{\perp})^2} \equiv \tilde{E}(p'_{\perp})$$

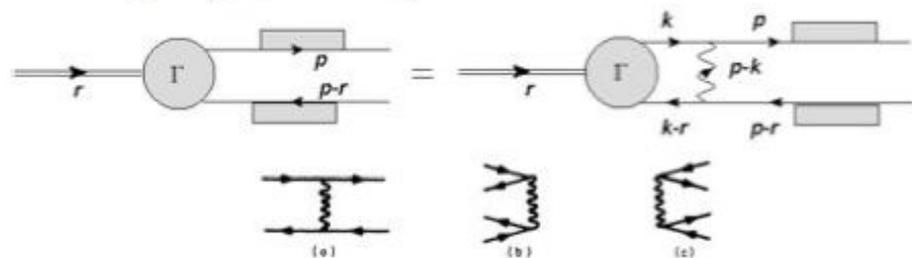
Running quark mass function



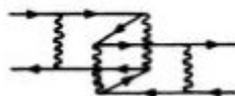
$$\xi = \tan^{-1} p_{\perp}$$

BOUND-STATE EQUATION

$$\Gamma(r, p) = \frac{i\lambda}{2\pi} \int \frac{dk_{\perp} dk_{\parallel}}{(p_{\perp} - k_{\perp})^2} S(p) \gamma^{\dagger} \Gamma(r, k) \gamma^{\dagger} S(p - r)$$



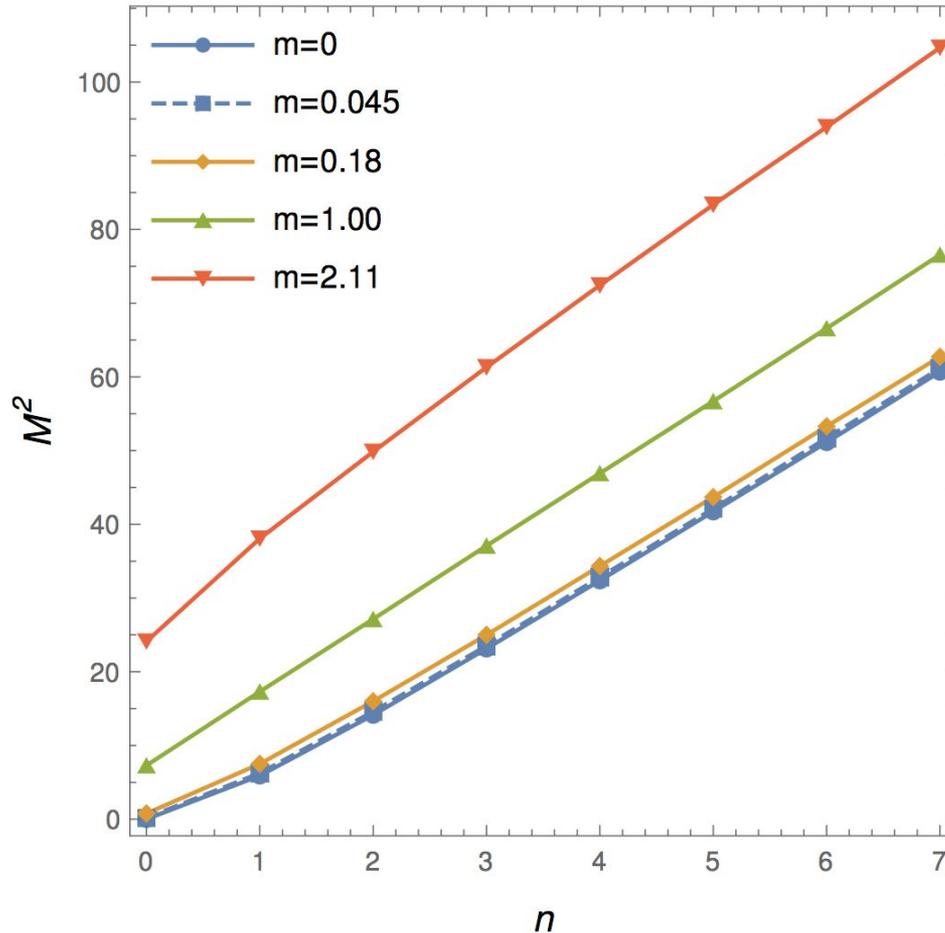
$$\begin{aligned} & \left[-r_{\parallel} + \frac{-S p_{\perp} + E(p_{\perp})}{C} + \frac{S(p_{\perp} - r_{\perp}) + E(p_{\perp} - r_{\perp})}{C} \right] \hat{\phi}_{+}(r_{\perp}, p_{\perp}) \\ &= \lambda \int \frac{dk_{\perp}}{(p_{\perp} - k_{\perp})^2} \left[C(p_{\perp}, k_{\perp}, r_{\perp}) \hat{\phi}_{+}(r_{\perp}, k_{\perp}) - S(p_{\perp}, k_{\perp}, r_{\perp}) \hat{\phi}_{-}(r_{\perp}, k_{\perp}) \right], \\ & \left[r_{\parallel} + \frac{-S(p_{\perp} - r_{\perp}) + E(p_{\perp} - r_{\perp})}{C} + \frac{S p_{\perp} + E(p_{\perp})}{C} \right] \hat{\phi}_{-}(r_{\perp}, p_{\perp}) \\ &= \lambda \int \frac{dk_{\perp}}{(p_{\perp} - k_{\perp})^2} \left[C(p_{\perp}, k_{\perp}, r_{\perp}) \hat{\phi}_{-}(r_{\perp}, k_{\perp}) - S(p_{\perp}, k_{\perp}, r_{\perp}) \hat{\phi}_{+}(r_{\perp}, k_{\perp}) \right]. \end{aligned}$$



LFD

$$\left[\mathcal{M}^2 - \frac{m^2 - 2\lambda}{x} - \frac{m^2 - 2\lambda}{1-x} \right] \phi(x) = -2\lambda \int_0^1 \frac{dy}{(x-y)^2} \phi(y)$$

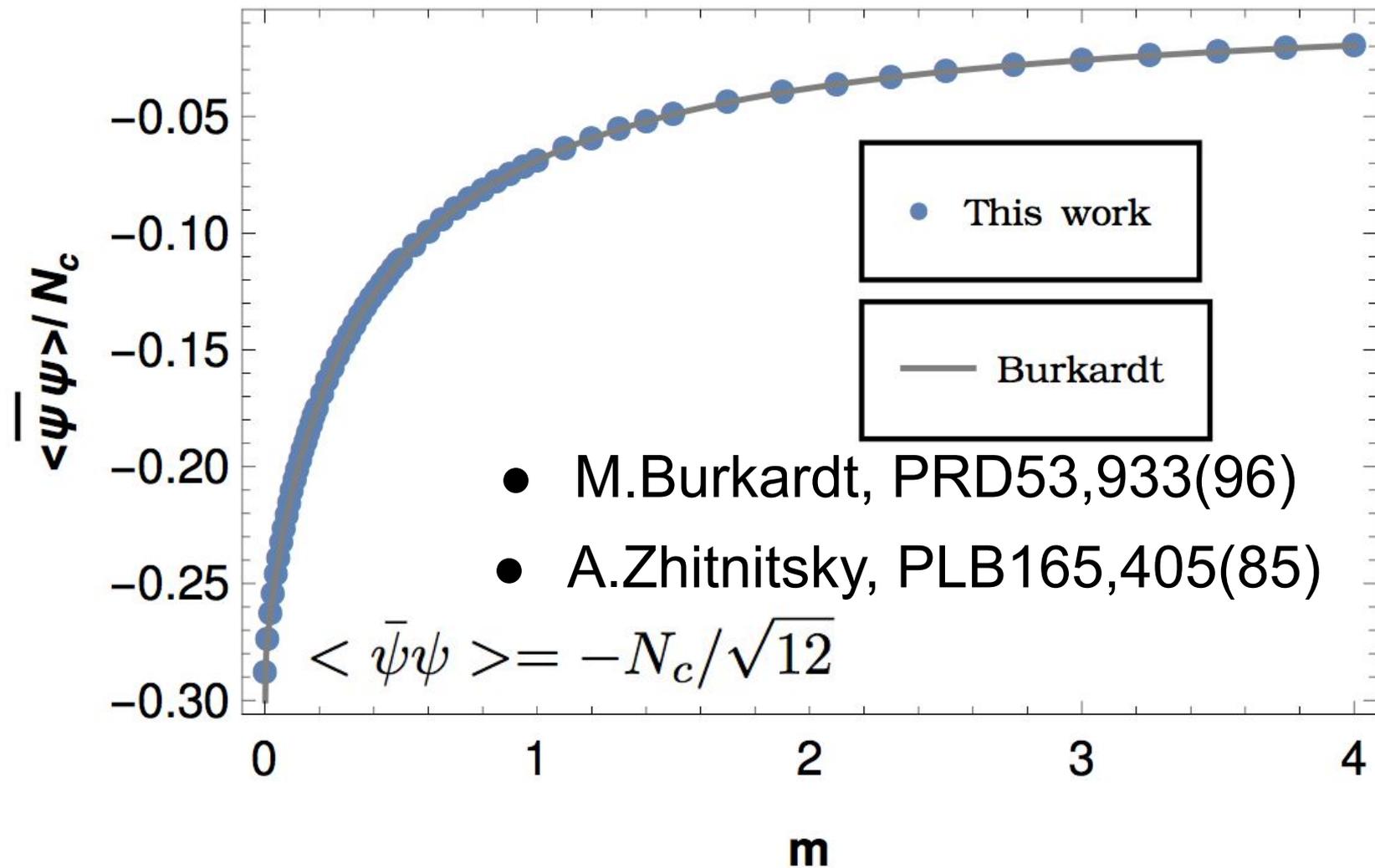
Meson Spectroscopy



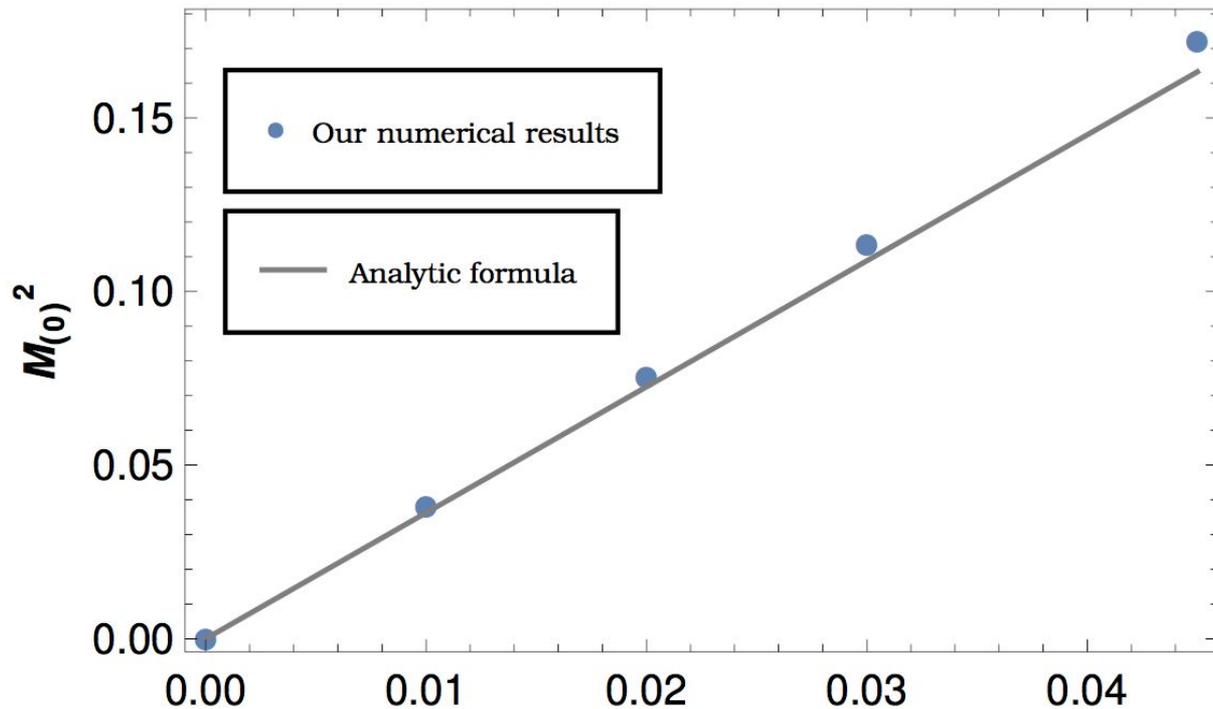
- G. 'tHooft, NPB75, 461(74) - LFD

- M. Li, et al., JPG13, 915(87) - IFD (rest frame)

- Y. Jia, et al., JHEP11, 151('17) - IFD (moving frame)



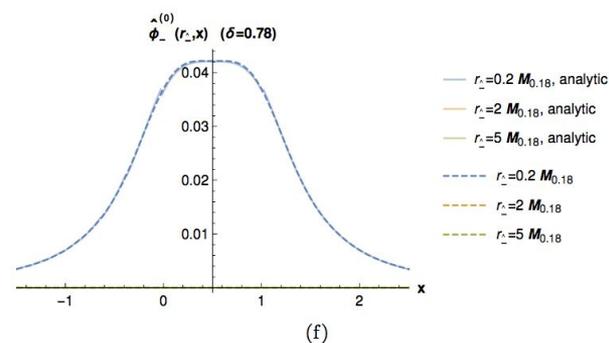
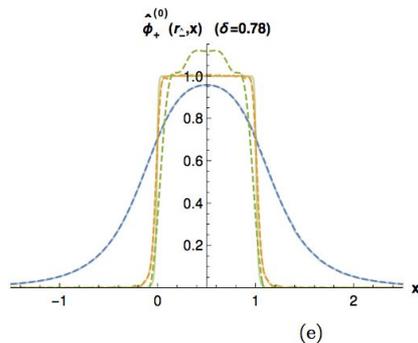
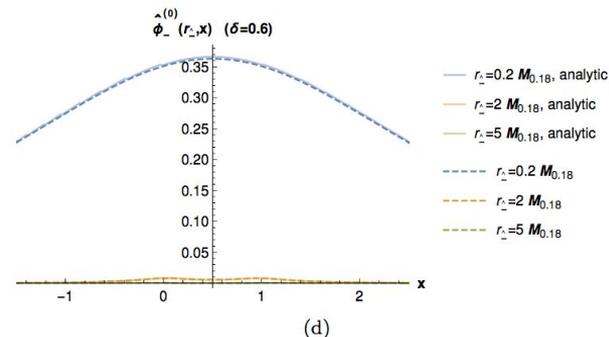
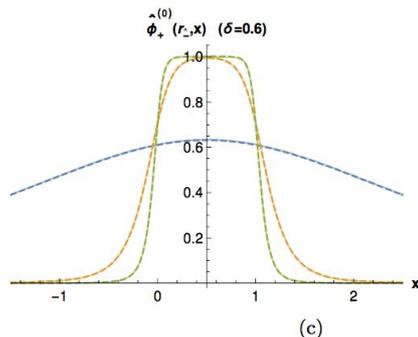
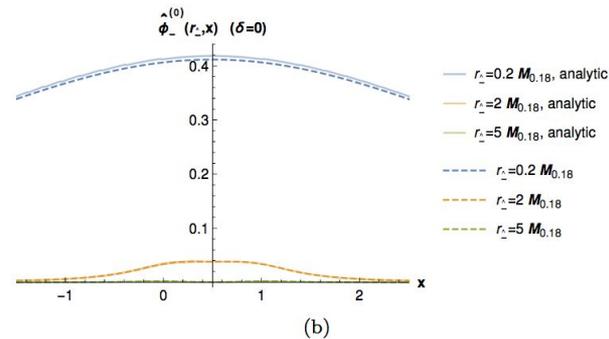
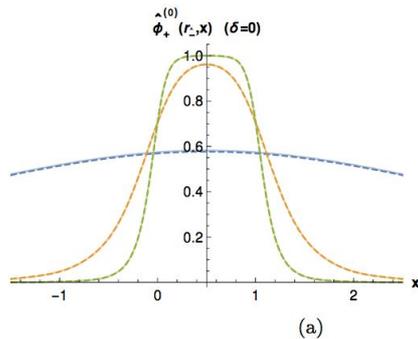
Gell-Mann - Oaks - Renner Relation



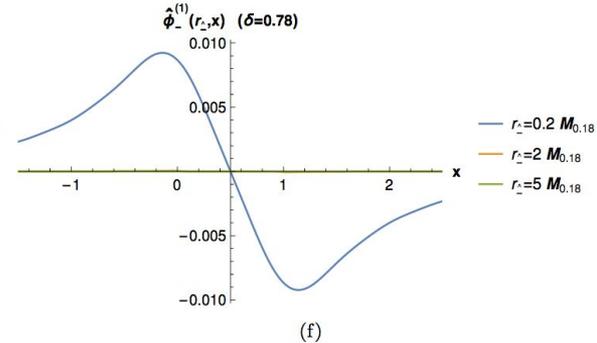
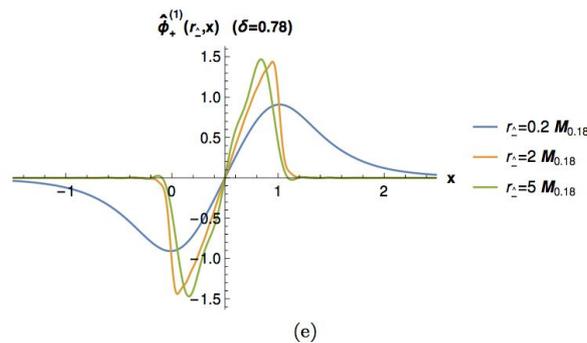
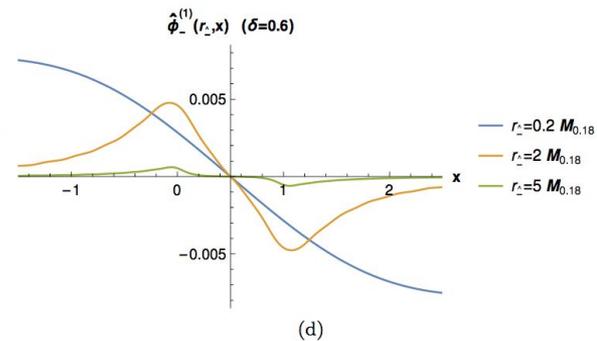
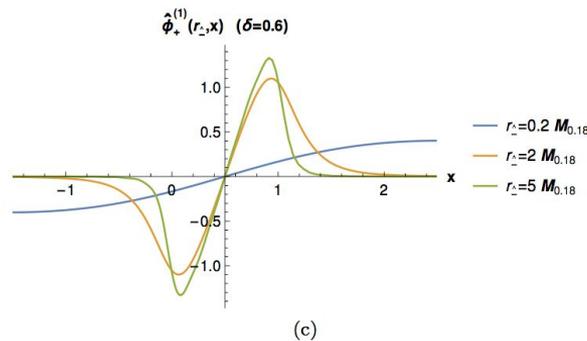
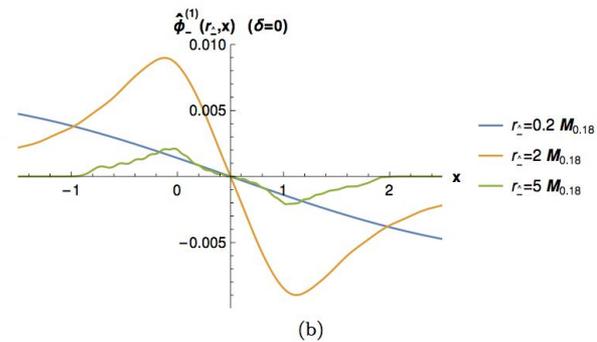
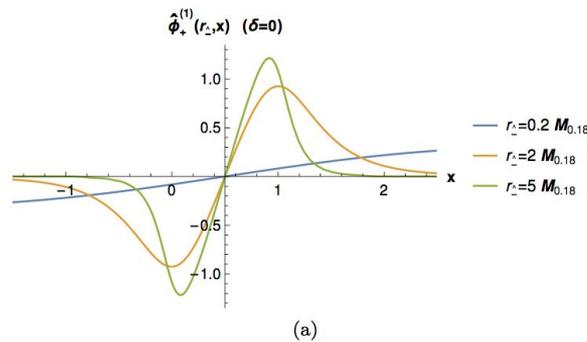
$$\mathcal{M}_{\pi}^2 = -\frac{4m \langle \bar{\psi}\psi \rangle}{f_{\pi}^2} = \sqrt{\frac{8\pi^2 m^2 \lambda}{3}} \quad m \quad f_{\pi} = \sqrt{N_c/\pi}$$

Meson Ground-state Wave-function for $m=0$ case

$$\hat{\phi}_{\pm}^{(0)}(r_{\pm}, p_{\pm}) = \frac{1}{2} \left(\cos \frac{\theta(r_{\pm} - p_{\pm}) - \theta(p_{\pm})}{2} \pm \sin \frac{\theta(r_{\pm} - p_{\pm}) + \theta(p_{\pm})}{2} \right)$$



First Excited-state Meson Wave-functions for $m=0$ case



Parton Distribution Functions (PDFs)

$$q_n(x) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \\ \times \langle P_n^-, P^+ | \bar{\psi}(\xi^-) \gamma^+ \mathcal{W}[\xi^-, 0] \psi(0) | P_n^-, P^+ \rangle_C,$$

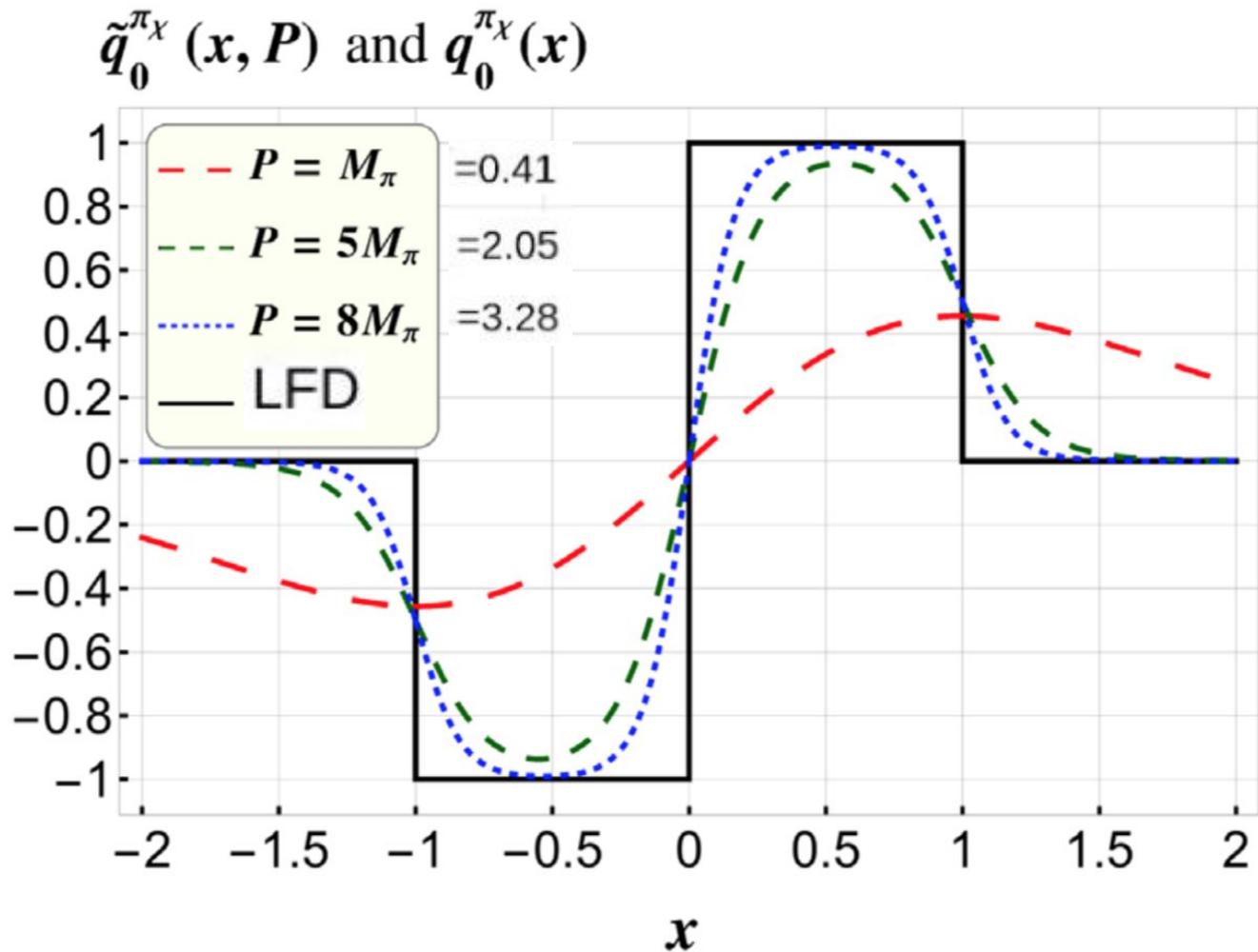
$$\mathcal{W}[\xi^-, 0] = \mathcal{P} \left[\exp \left(-ig_s \int_0^{\xi^-} d\eta^- A^+(\eta^-) \right) \right] \mathbf{A^+=0 Gauge} \\ \mathbf{in LFD}$$

Quasi-PDFs

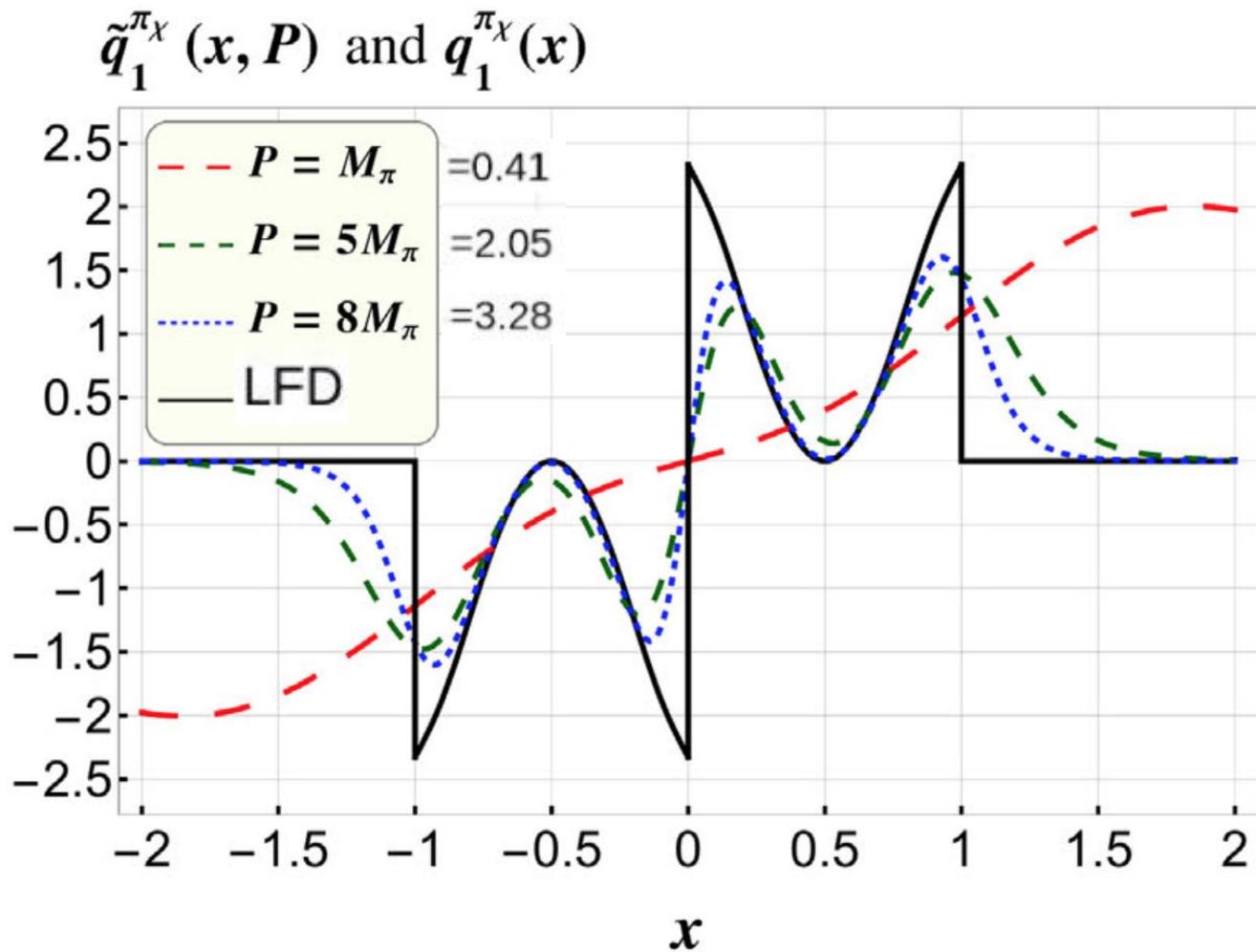
$$\tilde{q}_{(n)}(\hat{r}_\perp, x) = \int_{-\infty}^{+\infty} \frac{dx^\hat{-}}{4\pi} e^{ix^\hat{-} r_\perp} \\ \times \langle r_{(n)}^\hat{+}, r_\perp | \bar{\psi}(x^\hat{-}) \gamma_\perp \mathcal{W}[x^\hat{-}, 0] \psi(0) | r_{(n)}^\hat{+}, r_\perp \rangle_C,$$

$$\mathcal{W}[x^\hat{-}, 0] = \mathcal{P} \left[\exp \left(-ig \int_0^{x^\hat{-}} dx'^\hat{-} A_\perp(x'^\hat{-}) \right) \right] \mathbf{Interpolating} \\ \mathbf{dynamics}$$

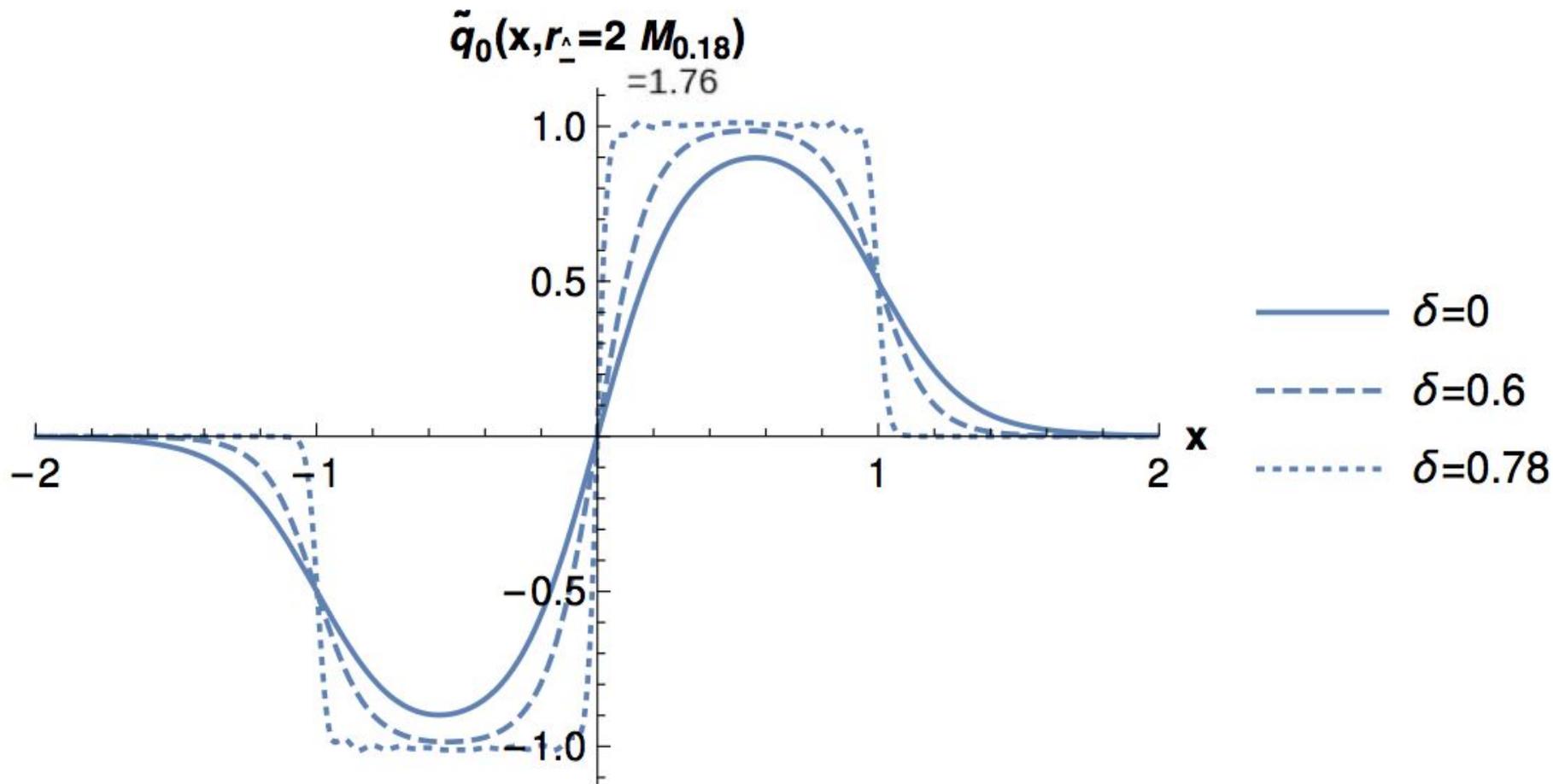
- Y. Jia, et al.,
PRD98,
054011('18)
- IFD
(quasi-PDFs)



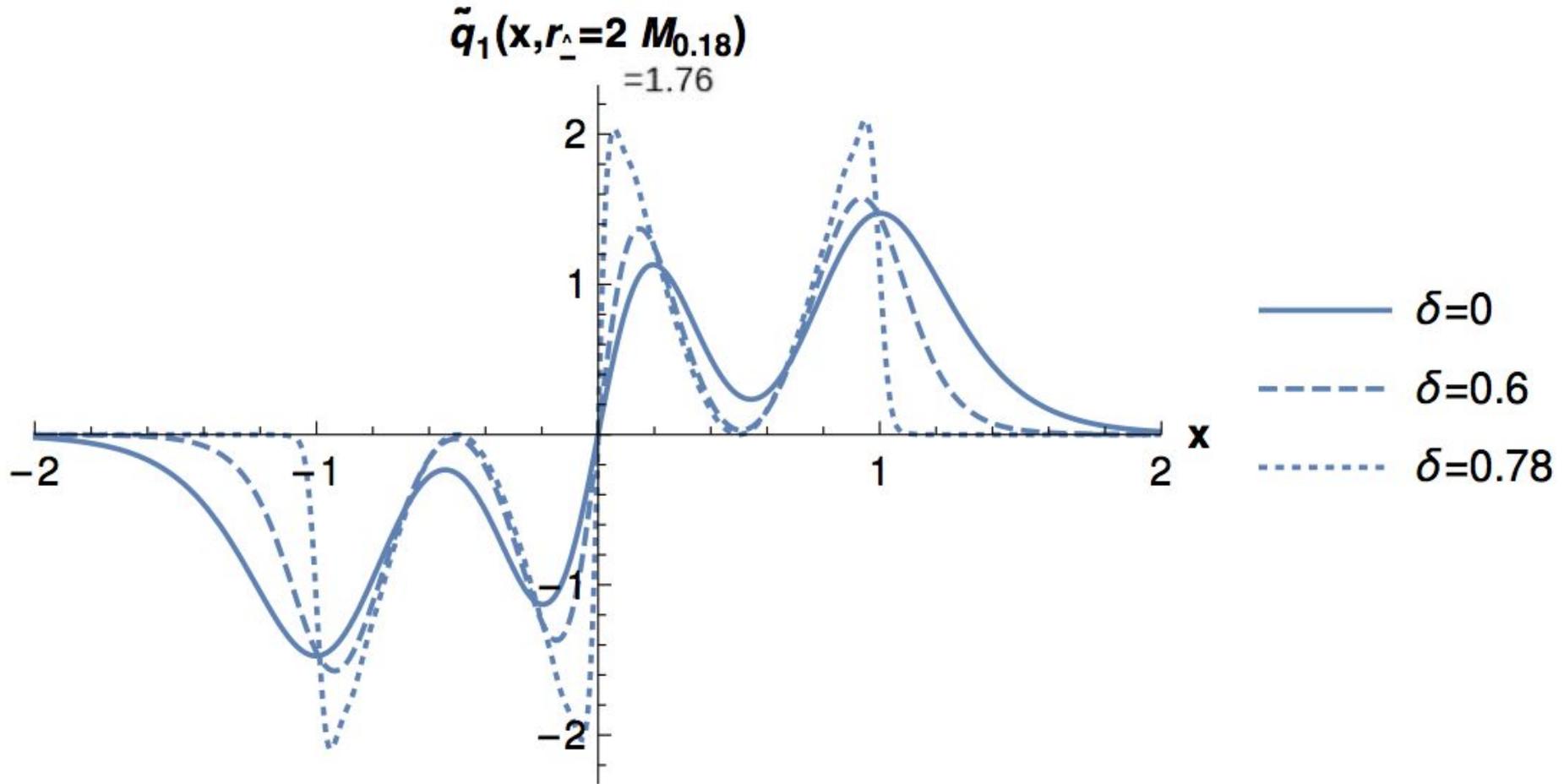
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- B.Ma&C.Ji, PRD194,036004(2021)



- B.Ma&C.Ji, PRD194,036004(2021)



Extended Wick Rotation

$$p^0 \rightarrow \tilde{P}^0 = ip^0 \quad (\delta = 0)$$

For $0 < \delta < \pi / 4$,

$$p^{\hat{\dagger}} / \sqrt{C} \rightarrow \tilde{P}^{\hat{\dagger}} / \sqrt{C} = ip^{\hat{\dagger}} / \sqrt{C} .$$

Correspondence to Euclidean Space

$$p_{\hat{_}}'^2 = p_{\hat{_}}^2 / C \leftrightarrow -\tilde{P}^2$$

Conclusions and Outlook

- QCD(1+1) in large N_c ‘tHooft model’ is interpolated between IFD and LFD and solved for its mass gap to find interpolation angle independent energy function including the wavefunction renormalization.
- Chiral condensate is found independent of interpolation angle indicating the persistence of nontrivial vacuum even in LFD.

- Mass spectra of mesons bearing the feature of Regge trajectories are found and GOR relation for the pionic ground-state in the zero fermion mass limit.
- Applying to quasi-PDFs in the interpolating formulation, we note a possibility of utilizing not only the reference frame dependence but also the interpolation angle dependence to get an alternative effective approach to the LFD's PDFs.