Modelling and Imaging the nucleon LFWFs

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Definitions and Classification of LFWFs

Hadrons seen as Fock States

- Lightfront quantization allows to expand hadrons on a Fock basis:

$$|P,\pi\rangle \propto \sum_\beta \Psi_\beta^{q\bar{q}}|q\bar{q}\rangle + \sum_\beta \Psi_\beta^{q\bar{q},q\bar{q}}|q\bar{q},q\bar{q}\rangle + \dots$$

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- Non-perturbative physics is contained in the N-particles Lightfront-Wave Functions (LFWF) Ψ^N
- Schematically a distribution amplitude φ is related to the LFWF through:

$$\varphi(x) \propto \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} \Psi(x,k_\perp)$$

S. Brodsky and G. Lepage, PRD 22, (1980)

$$\langle 0|O^{\alpha,\cdots}(\left\{z_1^-, z_{\perp 1}\right\}, \ldots, \left\{z_n^-, z_{\perp n}\right\})|P,\lambda\rangle\Big|_{z_i^+=0}$$

- Lightfront operator O of given number of quark and gluon fields

LFWFs: definitions

$$\langle 0|O^{\alpha,\cdots}(\left\{z_1^-, z_{\perp 1}\right\}, \ldots, \left\{z_n^-, z_{\perp n}\right\})|P,\lambda\rangle\Big|_{z_i^+=0} = \sum_j^n \tau_j^{\alpha,\cdots} N(P,\lambda) F_j(z_i)$$

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Both mesons and baryons can (in principle) have multiple independent leading-twist LFWFs

We introduce to lightlike vector p and n such that:

$$P^\mu = p^\mu + n^\mu \frac{M^2}{2p\cdot n} \quad \text{and} \quad P^+ = p^+$$

see for instance V. Braun *et al.*, Nucl Phys B589 381 (2000)

From these four-vectors we define the projectors:

$$N(p) = \underbrace{\frac{p\cdot\gamma n\cdot\gamma}{2p\cdot n} N(P)}_{\substack{\text{Dominant contribution} \\ \text{when } P^+ \to \infty}} + \frac{n\cdot\gamma p\cdot\gamma}{2p\cdot n} N(p) = N^+(P) + N^-(P)$$

Spinor decomposition and twist selection

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The same procedure is applied to all quark fields (and a similar one to gluon fields), selecting the leading twist contributions

Sorting LFWFs

$$\langle 0|\tilde{O}^{\alpha,\cdots}(\left\{z_1^-, z_{\perp 1}\right\}, \ldots, \left\{z_n^-, z_{\perp n}\right\})|P,\lambda\rangle\Big|_{z_i^+=0} = \sum_{\substack{j \\ \mathrm{LT}}}^{n} \tilde{\tau}_j^{\alpha,\cdots} N^+(P,\lambda)\tilde{F}_j(z_i)$$

- With the previous procedure we can select the leading-twist combinations scalar functions $\tilde{F}$

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- But an additional classification can be performed, by selecting the helicity projection of the quark fields involved through:

$$\psi = \frac{1+\gamma_5}{2}\psi + \frac{1-\gamma_5}{2}\psi = \psi^\uparrow + \psi^\downarrow$$

Nucleon LFWFs classification

- In the nucleon case, the procedure applies with three quarks at leading Fock state:

$$\langle 0|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|P,\uparrow\rangle$$

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X. Ji, *et al.*, Nucl Phys B652 383 (2003)

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- The LFWFs carry different amount of OAM projections:

states	$\langle\downarrow\downarrow\downarrow\|P,\uparrow\rangle$	$\langle\downarrow\downarrow\uparrow\|P,\uparrow\rangle$	$\langle\uparrow\downarrow\uparrow\|P,\uparrow\rangle$	$\langle\uparrow\uparrow\uparrow\|P,\uparrow\rangle$
OAM	2	1	0	-1
LFWFs	ψ^6	ψ^3, ψ^4	ψ^1, ψ^2	ψ^5

Relation with the Faddeev Wave function

- Since the Faddeev wave function χ is given as:

$$\langle 0| T\{q(z_1)q(z_2)q(z_3)\}|P,\lambda\rangle = \frac{1}{4} f_N N_\sigma(P,\lambda)$$

$$\times \int \prod_{j=1}^{3} \mathrm{d}^{(4)} k_j e^{-ik_j z_j} \delta^{(4)}(P - \sum_j k_j)\chi_\sigma(k_1,k_2,k_3),$$

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$$\psi^i = \int \prod_{j=1}^{3} [\mathrm{d}k_j^-] \mathcal{P}_i \chi$$

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Important

The FWF allows a **consistent** derivation of the 6 leading-fock states LFWFs of the nucleon

Modelling the Faddeev wave Function

Baryon and Diquarks

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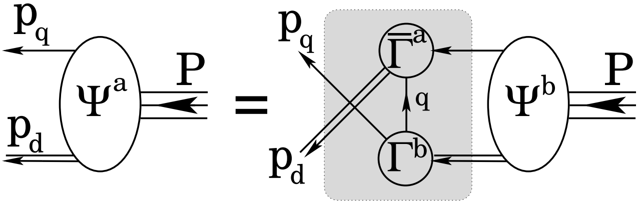
- It predicts the existence of strong diquarks correlations inside the nucleon.



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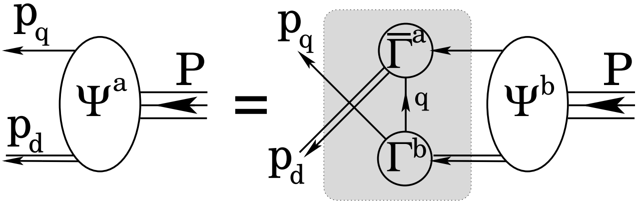

- Mostly two types of diquark are dynamically generated by the Faddeev equation:
  - Scalar diquarks,
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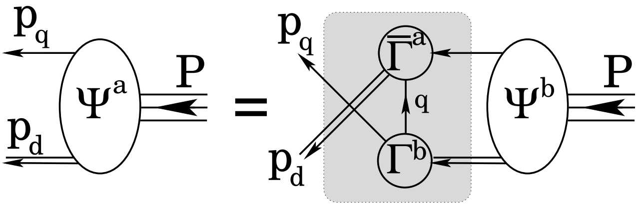

- Mostly two types of diquark are dynamically generated by the Faddeev equation:
  - Scalar diquarks,
  - Axial-Vector (AV) diquarks.
- In the following we build a model inspired by numerical solutions of the Faddeev equations

- DA is obtained by integrating the transverse momentum degrees of feedom

$$\langle 0|\epsilon^{ijk}\left(u_{\uparrow}^{i}(z_1^-,0_\perp)C\not{n}u_{\downarrow}^{j}(z_2^-,0_\perp)\right)\not{n}d_{\uparrow}^{k}(z_3^-,0_\perp)|P,\lambda\rangle \rightarrow \varphi(x_1,x_2,x_3),$$

Braun *et al.*, Nucl.Phys. B589 (2000)

Example on the Nucleon Distribution Amplitude

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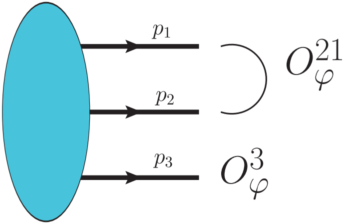
- We can apply it on the wave function:



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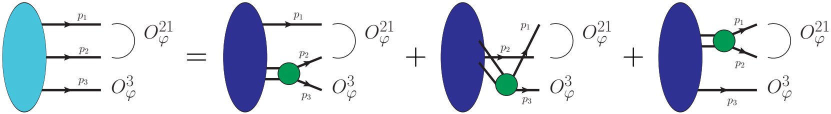

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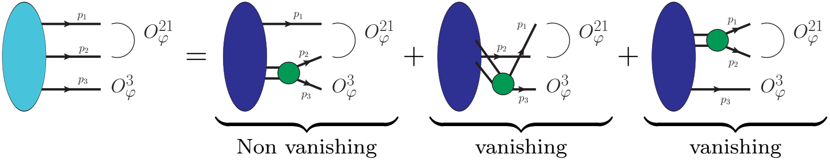

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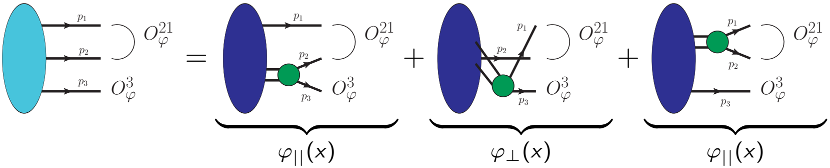

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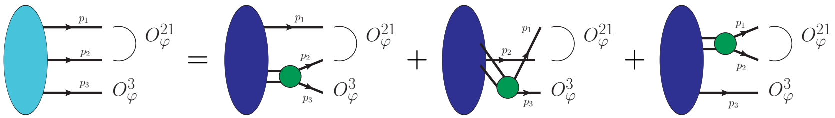

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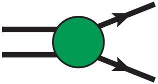
- Our ingredients are:
  - Perturbative-like quark and diquark propagator
  - Nakanishi based diquark Bethe-Salpeter-like amplitude (green disks)
  - Nakanishi based quark-diquark amplitude (dark blue ellipses)

Scalar Diquark part of the nucleon

Modelling the Scalar Diquark DA

- We need to obtain the structure of the scalar diquark itself



$$= \mathcal{N} \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{(\Lambda_q^2 + (q + \frac{z}{2}K)^2)}$$

  - q is the relative momentum between the quarks and K the total diquark momentum
  - Λ_q is a free parameter to be fit on DSE computations
  - $\rho(z,\gamma) = \rho(z) = 1 - z^2 \rightarrow$ we keep only the 0th degree coefficient in a Gegenbauer expansion of the Nakanishi weight

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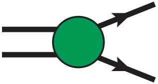

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- We couple this with a simple massive fermion propagator:

$$S(p) = \frac{-ip\cdot\gamma + M}{p^2 + M^2}$$

Adjusting the parameters

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  - Sum of the frozen mass bigger than the nucleon mass for stability (binding energy)
  - Avoid singularities in the complex plane

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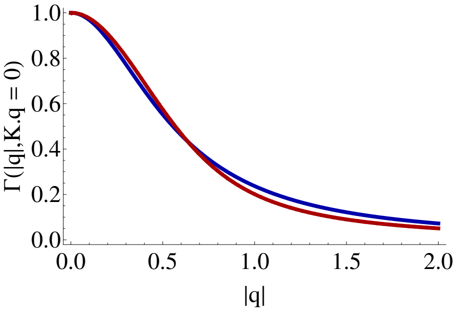
- Width of the diquark BSA $\Lambda_q = 3/5M_N$ fitted on previous computations:



red curve from Segovia *et al.*,Few Body Syst. 55 (2014) 1185-1222

- From that we can compute the scalar diquark DA as:

$$\phi(x) \propto \int d^4q\delta\left(q\cdot n - xK\cdot n\right)\mathrm{Tr}\left[S\Gamma^{0\,T}S^T L^{\downarrow}C^{\dagger}n\cdot\gamma L^{\uparrow}\right]$$

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- Nakanishi representation $\rightarrow$ analytic treatments of singularities and analytic reconstruction of the function from the moment

$$\phi(x) = \int_x^1 \mathrm{d}u \int_0^x \mathrm{d}v \frac{F(u,v,x)}{M^2_{eff}(u,v,x,M^2,\Lambda^2)+K^2}$$

F and M_{eff} are computed analytically

Analytic results

- In the specific case $M^2 = \Lambda_q^2$, the PDA can be analytically obtained:

$$\phi(x) \propto \frac{M^2}{K^2}\left[1 - \frac{M^2}{K^2}\frac{\ln\left[1+\frac{K^2}{M^2}x(1-x)\right]}{x(1-x)}\right]$$

C. Mezrag *et al.*, Springer Proc.Phys. 238 (2020) 773-781

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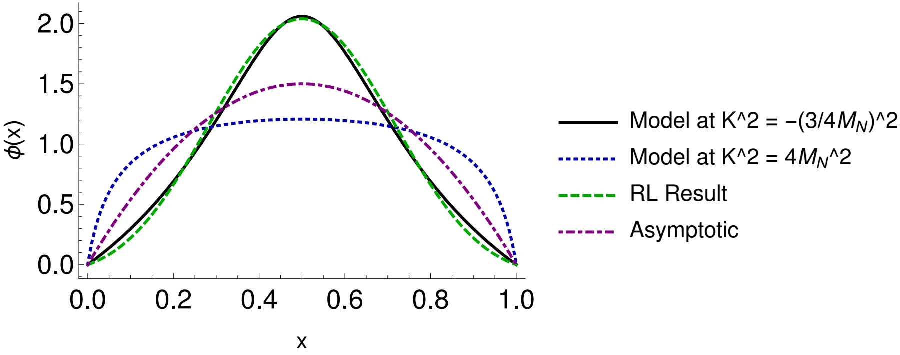
- Note that expanding the log, one get:

$$\phi(x) \propto \frac{1}{2}x(1-x) - \frac{1}{3}K^2/M^2x^2(1-x)^2 + \ldots$$

so that:

  - at the end point the DA remains linearly decreasing (important impact on observable)
  - at vanishing diquark virtuality, one recovers the asymptotic DA



RL results from Y. Lu *et al.*, Eur.Phys.J.A 57 (2021) 4, 115

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  - Need of spectral representation with running mass to bypass this?

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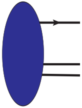
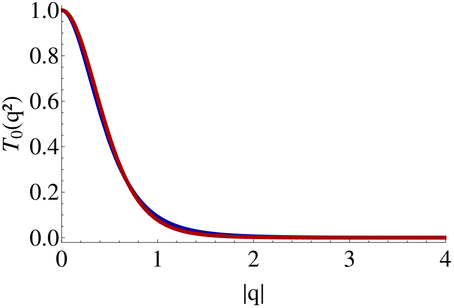
But overall, we expect to gain insights from this simple model

Quark-diquark amplitude



$$= \mathcal{N}\int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)\tilde{\rho}(z)}{(\Lambda^2 + (\ell - \frac{1+3z}{6}P)^2)^3}, \quad \tilde{\rho}(z) = \prod_j (z-a_j)(z-\bar{a}_j)$$

Fits of the parameters through comparison to Chebychev moments:



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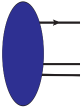

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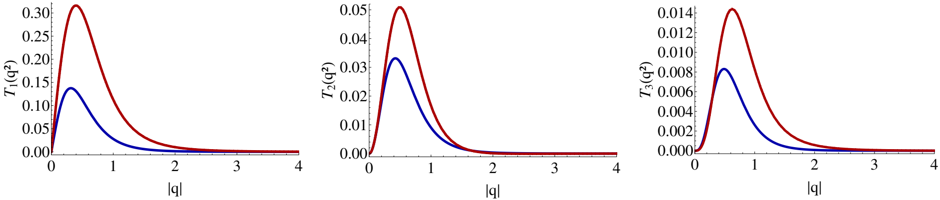

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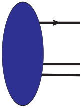
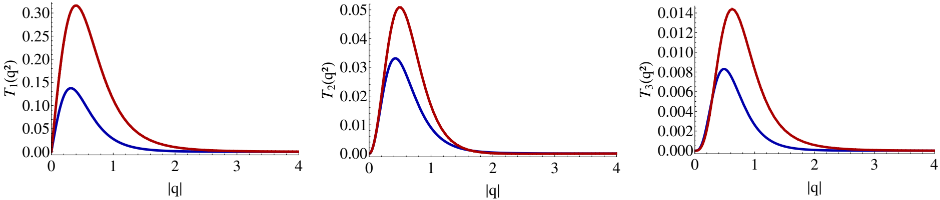


$$= \mathcal{N}\int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)\tilde{\rho}(z)}{(\Lambda^2 + (\ell - \frac{1+3z}{6}P)^2)^3}, \quad \tilde{\rho}(z) = \prod_j (z-a_j)(z-\bar{a}_j)$$

Fits of the parameters through comparison to Chebychev moments:



red curves from Segovia *et al.*,

Modification of the $\tilde{\rho}$ Ansatz ? $\tilde{\rho}(z) \to \tilde{\rho}(\gamma, z)$?

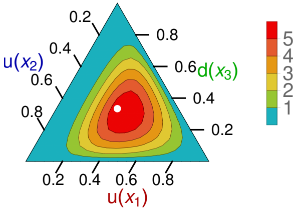
- We do not compute the PDA directly but Mellin moments of it:

$$\langle x_1^m x_2^n \rangle = \int_0^1 \mathrm{d}x_1 \int_0^{1-x_1} \mathrm{d}x_2 \; x_1^m x_2^n \varphi(x_1, x_2, 1 - x_1 - x_2)$$

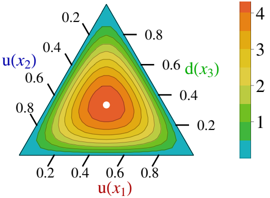
- For a general moment $\langle x_1^m x_2^n \rangle$, we change the variable in such a way to write down our moments as:

$$\langle x_1^m x_2^n \rangle = \int_0^1 \mathrm{d}\alpha \int_0^{1-\alpha} \mathrm{d}\beta \; \alpha^m \beta^n f(\alpha, \beta)$$

- f is a complicated function involving the integration on 6 parameters
- Uniqueness of the Mellin moments of continuous functions allows us to identify f and φ



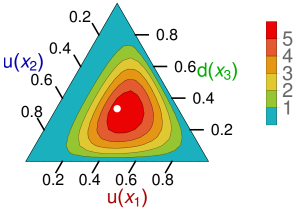
Scalar diquark Only



Asymptotic DA

- Typical symmetry in the pure scalar case

C. Mezrag *et al.*, Phys.Lett. B783 (2018)



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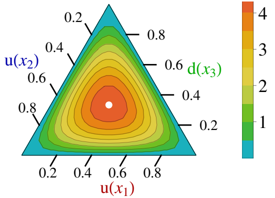
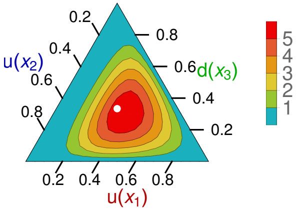


Asymptotic DA

- Typical symmetry in the pure scalar case
- Nucleon DA is skewed compared to the asymptotic one

C. Mezrag *et al.*, Phys.Lett. B783 (2018)

Preliminary Results



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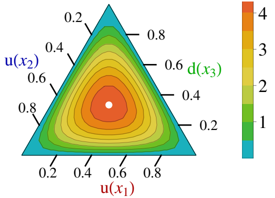

Asymptotic DA

- Typical symmetry in the pure scalar case
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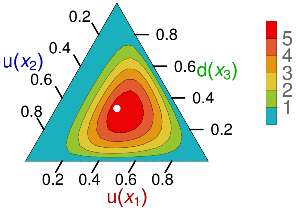
- Deformation along the symmetry axis and orthogonally to it
  - Impact of the virtuality dependence of the diquark WF

C. Mezrag *et al.*, Phys.Lett. B783 (2018)



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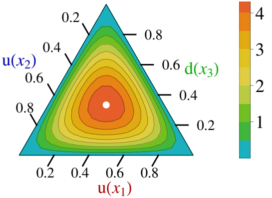

Asymptotic DA

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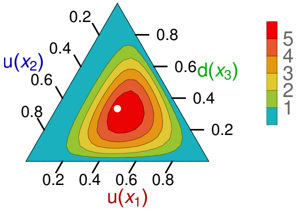
C. Mezrag *et al.*, Phys.Lett. B783 (2018)



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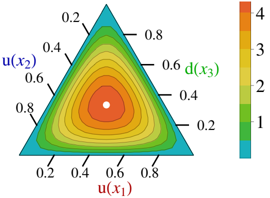

Asymptotic DA

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- Deformation along the symmetry axis and orthogonally to it
  - Impact of the virtuality dependence of the diquark WF
- These properties are consequences of our quark-diquark picture
- Improvement in the modelling with respect to our previous work

C. Mezrag *et al.*, Phys.Lett. B783 (2018)

LFWFs and images of the nucleon with GPDs

- Generalised parton distributions are defined as off-forward matrix elements:

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}\gamma^+u + E^q(x,\xi,t)\bar{u}\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u\right].$$

D. Müller *et al.*, Fortsch. Phy. 42 101 (1994)
X. Ji, Phys. Rev. Lett. **78**, 610 (1997)
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see also Jianwei Qiu talk

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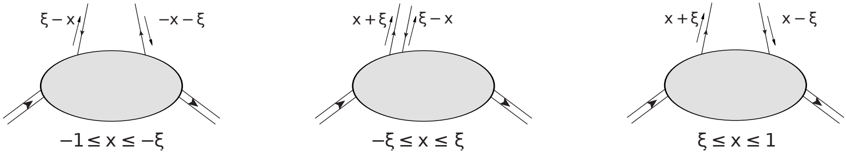

  - x: average momentum fraction carried by the active parton
  - ξ: skewness parameter $\xi \simeq \frac{x_B}{2-x_B}$
  - t: the Mandelstam variable

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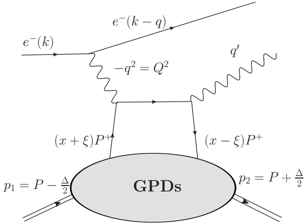
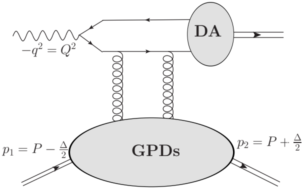
- They can be accessed in exclusive processes





3D picture of the nucleon

- In the limit $\xi \to 0$, one recovers a density interpretation:
  - 1D in momentum space (x)
  - 2D in coordinate space $\vec{b}_\perp$ (related to t)

M. Burkardt, Phys. Rev. **D62**, 071503 (2000)

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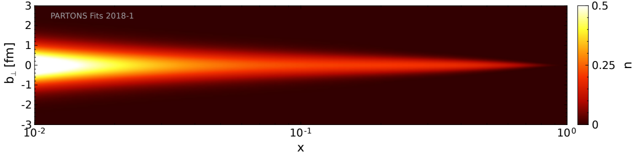

figure from H. Moutarde *et al.*, EPJC 78 (2018) 890

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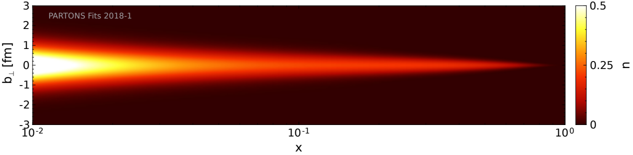

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- Correlation between x and $b_\perp \to$ going beyond PDF and FF.

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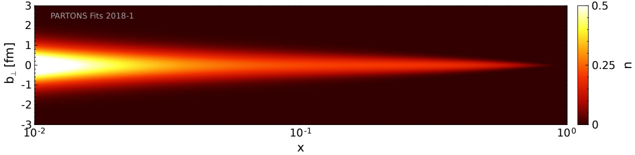

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- Correlation between x and $b_\perp \to$ going beyond PDF and FF.
- Caveat: no experimental data at $\xi = 0$
  $\to$ extrapolations (and thus model-dependence) are necessary

- In the limit $\xi \to 0$ the connection between GPDs and LFWFs can be computed:

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-\Big|_{\substack{z^+=0,\mathbf{z}=0\\ \xi=0}}$$

$$\underset{N=3}{=}\sum_{\beta,\beta'}\int\left[\prod_{i=1}^{3}\mathrm{d}x_i\mathrm{d}k_\perp^i\right]K_{\beta,\beta'}(x,\xi,t,k_\perp^i)\psi^*_{\beta'}\psi_\beta$$

GPDs and LFWFs

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- The impact of LFWFs with definite OAM projection can be followed up to the GPD expressions for $|x| \geq |\xi|$:

$$H(x,\xi,t) = F_H(x,\xi,t,\psi_1,\ldots,\psi_6)$$
$$E(x,\xi,t) = F_E(x,\xi,t,\psi_5,\psi_6)$$

M. Riberdy *et al.*, in preparation

Consequences I: OAM deformation of the nucleon

- OAM projection dependence on the 3D probability density:

$$F_H(x, \xi, t, \psi_1, \ldots, \psi_6) \to \rho(x, b_\perp, \psi_1, \ldots, \psi_6)$$

Visualisation of the impact of OAM

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Visualisation of the impact of OAM

- NR charged proton radius

$$F_1(t) = \int \mathrm{d}x \; H(x,0,t,\psi_1,\dots,\psi_6)$$

$$F_2(t) = \int \mathrm{d}x \; E(x,0,t,\psi_5,\psi_6)$$

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- However, no input on pressure or energy distributions (no access to D-term)

Consequences II: Experimental sensibility to OAM

- In principle we could assess the impact of OAM pojection on GPD-sensitive observables

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- In principle we could assess the impact of OAM pojection on GPD-sensitive observables
- but two main difficulties:
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  - amplitude convolution filters the information accessible experimentally

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- but two main difficulties:
  - Evolution mixes OAM projections $\rightarrow$ one can draw conclusion only for OAM at the original scale
  - amplitude convolution filters the information accessible experimentally
- Observables sensitive to GPD E remain the best ones able to tell us something on OAM projection within the nucleon, but they are hard to measure (requires polarised proton targets or the ability to measure the polarisation of the recoil proton)

O. Bessidskaia Bylund *et al.*, arXiv:2209.04313

Summary

Summary and conclusion

Achievements

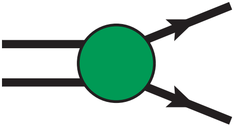
- **CSM compatible** framework for Nucleon LFWFs computations
- Based on the Nakanishi representation
- Improved models from the first exploratory work
- Relation between LFWFs and GPDs has been worked out

Work in progress/future work

- Tackling the AV-diquark contributions
- Improvement of the Nakanishi Ansätze
- Computations of GPDs

Thank you for your attention

Back up slides



At all order of perturbation theory, one can write (Euclidean space):

$$\Gamma(k,P) = \mathcal{N}\int_0^\infty \mathrm{d}\gamma \int_{-1}^{1} \mathrm{d}z \frac{\rho_n(\gamma,z)}{(\gamma + (k+\frac{z}{2}P)^2)^n}$$

We use a "simpler" version of the latter as follow:

$$\tilde{\Gamma}(q,P) = \mathcal{N}\int_{-1}^{1} \mathrm{d}z \frac{\rho_n(z)}{(\Lambda^2 + (q+\frac{z}{2}P)^2)^n}$$