

Structure of pseudo-scalar mesons from lattice QCD

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What is the internal structure of the pseudo-scalar mesons, and how it is related to their Goldstone boson nature ? Very well suited for lQCD

- Valence pion PDF

X. Gao, L. Jing, C. Kallidonis, S. Mukherjee, PP, C. Shugert, S. Syritsyn, Y. Zhao, PRD104 ('21) 114515

X. Gao, A.D. Hanlon, S. Mukherjee, PP, P. Scior, S. Syritsyn, Y. Zhao, PRD 103 ('21) 094510

X. Gao, A.D. Hanlon, S. Mukherjee, PP, P. Scior, S. Syritsyn, Y. Zhao, PRL 128 ('22) 142003

X. Gao, A.D. Hanlon, N. Karthik, S. Mukherjee, PP, S. Shi, P. Scior, S. Syritsyn, Y. Zhao, K. Zhou, arXiv:2208.02297 [hep-lat]

- Pion distribution amplitude (DA)

X. Gao, A.D. Hanlon, N. Karthik, S. Mukherjee, PP, P. Scior, S. Syritsyn, Y. Zhao, arXiv:2207.04084

- Pion and kaon form-factors

X. Gao, N. Karthik, S. Mukherjee, PP, S. Syritsyn, Y. Zhao, PRD104 (2021) 114515, and work in progress

Lattice QCD setup

2+1 flavor HISQ (HotQCD) lattices, pions boosted along the z-direction
 HYP smeared clover action for valence quarks, $m_\pi^{val} \simeq 300$ MeV
 $48^3 \times 64$, $a = 0.06$ fm, $P_z^{max} = 2.15$ GeV, 64^4 , $a = 0.04$ fm, $P_z^{max} = 2.40$ GeV
 64^4 , $a = 0.076$ fm, $m_\pi = 140$ MeV, $P_z^{max} = 2$ GeV

$$P_z = \frac{2\pi}{L} n_z, \quad n_z = 0, 1, 2, 3, 4, 5 \quad \text{Boosted sources}$$

Bali et al, PRD 93 ('16) 094515

$$C_{2\text{pt}}^{ss'}(t; \mathbf{P}) = \langle \pi_s(\mathbf{P}, t) \pi_{s'}^\dagger(\mathbf{P}, 0) \rangle \quad \mathbf{P}^f = \mathbf{P} = \mathbf{P}^i + \mathbf{q}$$

$$C_{3\text{pt}}(\mathbf{P}^f, \mathbf{P}^i, \tau, t_s) = \langle \pi_s(\mathbf{P}^f, t_s) O_{\gamma_t}(\tau) \pi_s^\dagger(\mathbf{P}^i, 0) \rangle \quad \text{PDF, Form factor}$$

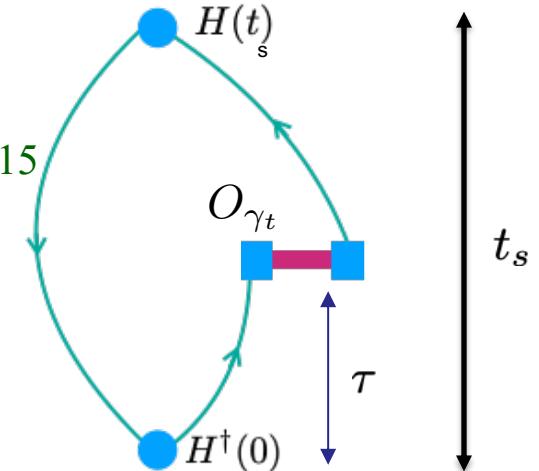
$$C_{\text{pt-split}}(\mathbf{P}^i, t_s) = \langle O_{\gamma_5 \gamma_3}(t_s) \pi_s^\dagger(\mathbf{P}^i, 0) \rangle \quad \text{Pion DA}$$

$$O_\Gamma(z) = \left[\bar{u}(z) \Gamma W(z, 0) u(0) - \bar{d}(z) \Gamma W(z, 0) d(0) \right]$$

Iso-vector,
 no disconnected
 diagrams

$$ME = \lim_{t_s \rightarrow \infty, \tau \rightarrow \infty} R(\tau, t_s), \quad R^{fi} \sim \frac{3\text{pt}}{2\text{pt}}$$

Extrapolation is done using 2 and 3 exp. Fits
 With energy levels from 2pt functions



LaMET and pion PDF

1) Quasi-PDF approach (x -space matching):

$$\tilde{f}_v(x, P^z, \mu) = \int \frac{dz}{2\pi} e^{ixP^z z} \tilde{h}(z, P^z, \mu)$$

calculable in perturbation theory,
 known to NNLO

$$C = \delta\left(\frac{x}{y} - 1\right) + \alpha_s C^{(1)}\left(\frac{x}{y}, \frac{\mu}{P_z y}\right) + \dots$$

$$f_v(x, \mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C^{-1}\left(\frac{x}{y}, \frac{\mu}{y P^z}\right) \tilde{f}_v(y, P^z, \mu) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{(x P^z)^2}, \frac{\Lambda_{\text{QCD}}^2}{((1-x) P^z)^2}\right)$$

Light cone PDF
Access the x -dependence for PDF, except for $x \rightarrow 0$ and $x \rightarrow 1$

2) Short distance factorization:

Wilson coefficients $C_n = 1 + \alpha_s C_n^{(1)}(\mu^2 z^2) + \dots$

$$\tilde{h}(z, P_z, \mu) = \sum_n C_n (\mu^2 z^2) \langle x^n \rangle_\mu \frac{(-iz P_z)^n}{n!} + z^2 \Lambda_{\text{QCD}}^2, \quad \langle x^n \rangle_\mu = \int_{-1}^{+1} dx x^n f_v(x, \mu)$$

$\lambda = z P_z$ - Ioffe time; one needs large λ to probe higher moments of PDF

1) And 2) are equivalent in the infinite momentum limit $P_z \rightarrow \infty$

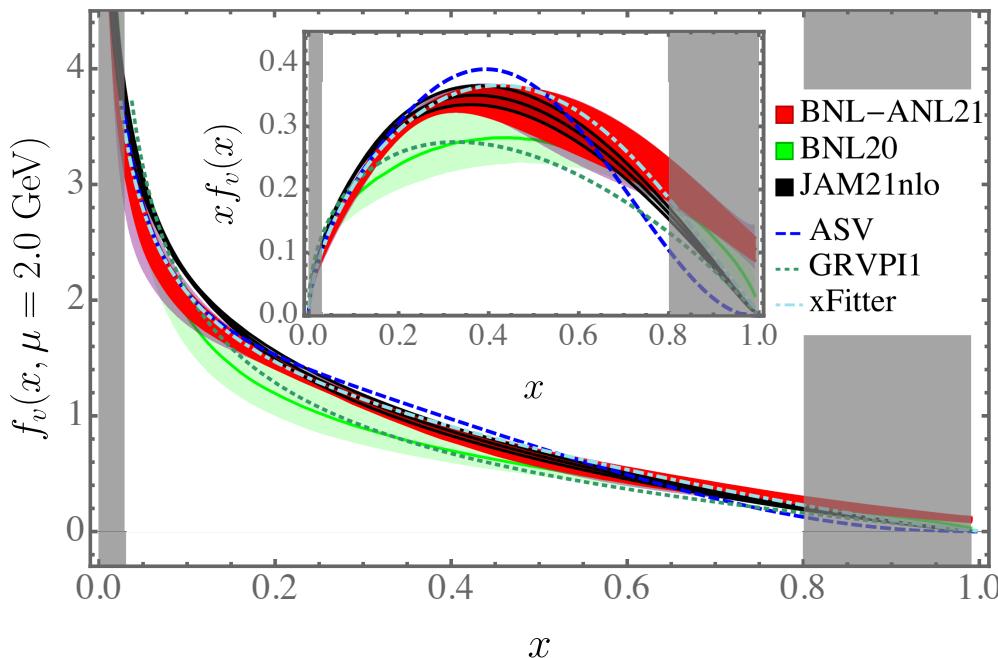
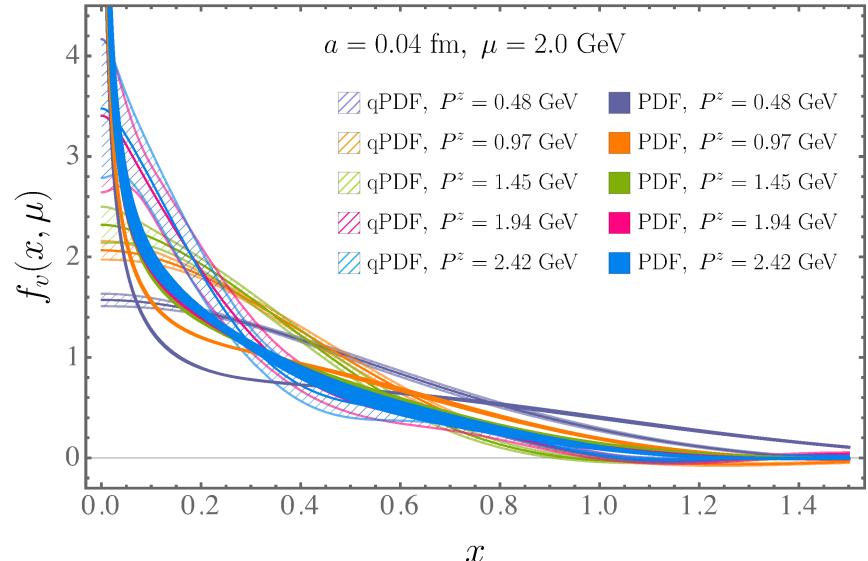
Renormalization: RI-MOM, ratio scheme, hybrid scheme

Valence pion PDF from x -space matching at NNLO

Hybrid renormalization:

$$\tilde{h}(z, P_z, a) \rightarrow h^R(z, P_z) =$$

$$\begin{cases} \frac{\tilde{h}(z, P_z, a)}{\tilde{h}(z, 0, a)}, & z \leq z_S \\ \frac{\tilde{h}(z, P_z, a)}{\tilde{h}(z_S, 0, a)} e^{\delta m(a)|z - z_S|}, & z > z_S. \end{cases}$$



P_z -dependence: $f_v(x) + a_v/P_z^2$
 \Rightarrow results are reliable at x ,
where $a_v/P_z^2|f_v(x)| < 0.1$

P_z -dependence of the result
is small for $P_z \geq 1.45 \text{ GeV}$

Good agreement between the lattice
results and the results from global
analysis

Valence pion PDF from short distance factorization: moments

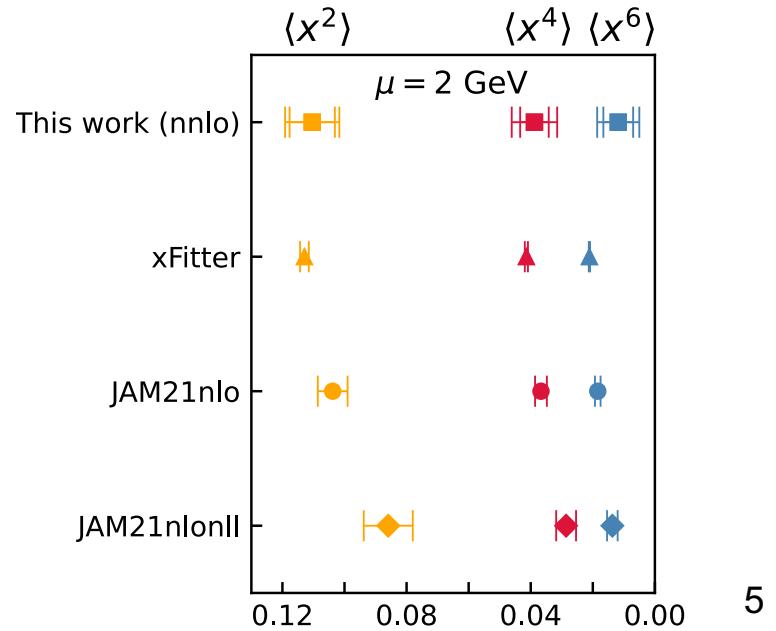
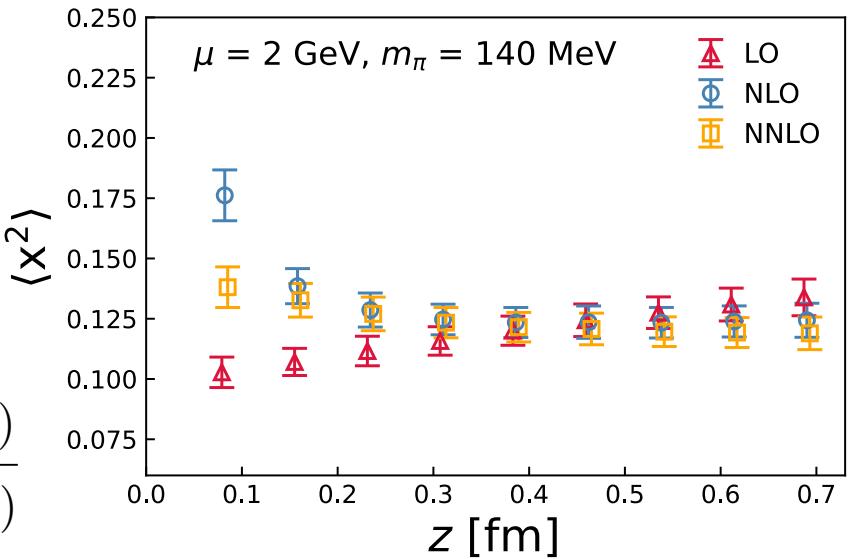
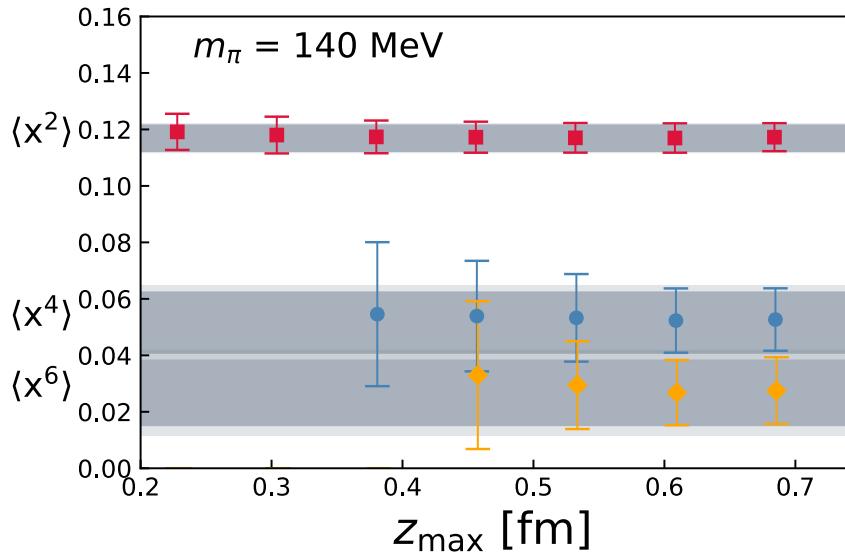
Generalized ratio scheme:

$$\mathcal{M}(z, P_z, P_z^0) = \frac{h^B(z, P_z, a)}{h^B(z, P_z^0, a)} = \frac{h^R(z, P_z, \mu)}{h^R(z, P_z^0, \mu)}$$

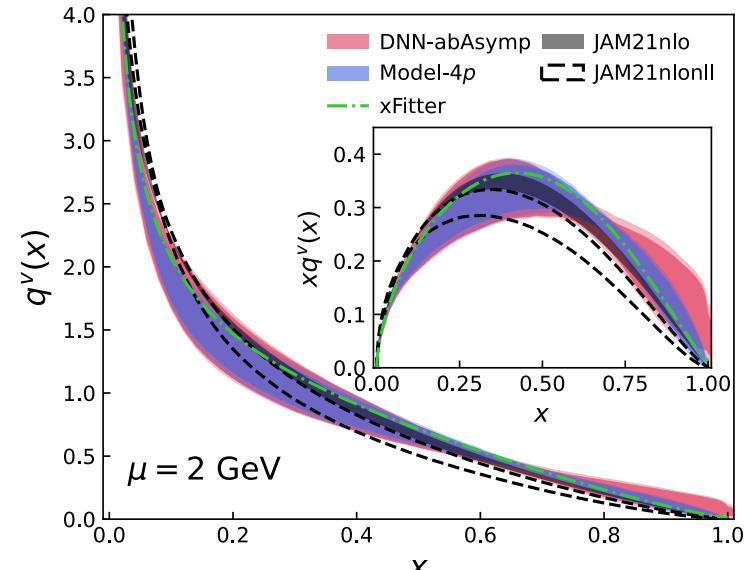
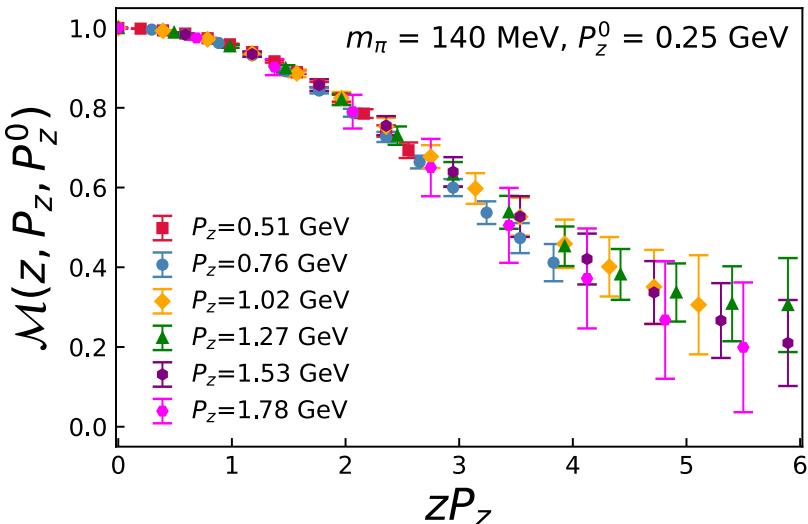
$$\mathcal{M}(z, P_z, P_z^0) =$$

$$\frac{\sum_{n=0} c_n (\mu^2 z^2)^n \frac{(-izP_z)^n}{n!} \langle x^n \rangle(\mu) + \mathcal{O}(\Lambda_{\text{QCD}}^2 z^2)}{\sum_{n=0} c_n (\mu^2 z^2)^n \frac{(-izP_z^0)^n}{n!} \langle x^n \rangle(\mu) + \mathcal{O}(\Lambda_{\text{QCD}}^2 z^2)}$$


 $\langle x^n \rangle_\mu$



Valence pion PDF short distance factorization: fits of PDF



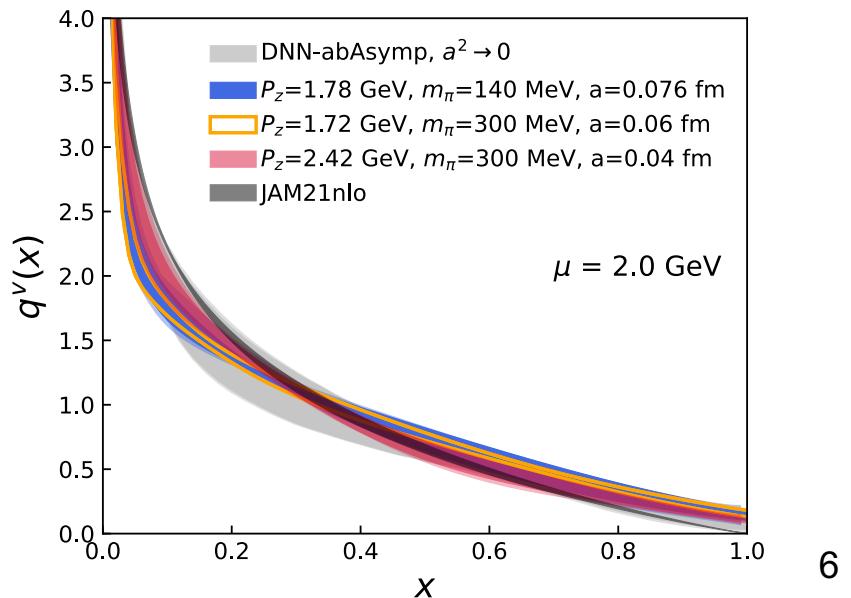
2 and 4 parameter forms of valence pion PDF:

$$f_v(x; \alpha, \beta) = \mathcal{N} x^\alpha (1-x)^\beta,$$

$$f_v(x; \alpha, \beta, s, t) = \mathcal{N}' x^\alpha (1-x)^\beta (1 + s\sqrt{x} + tx),$$

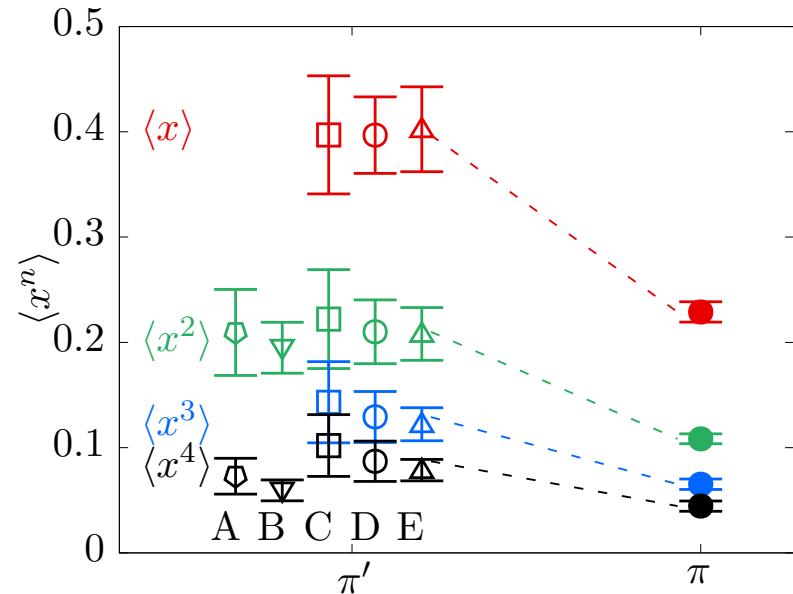
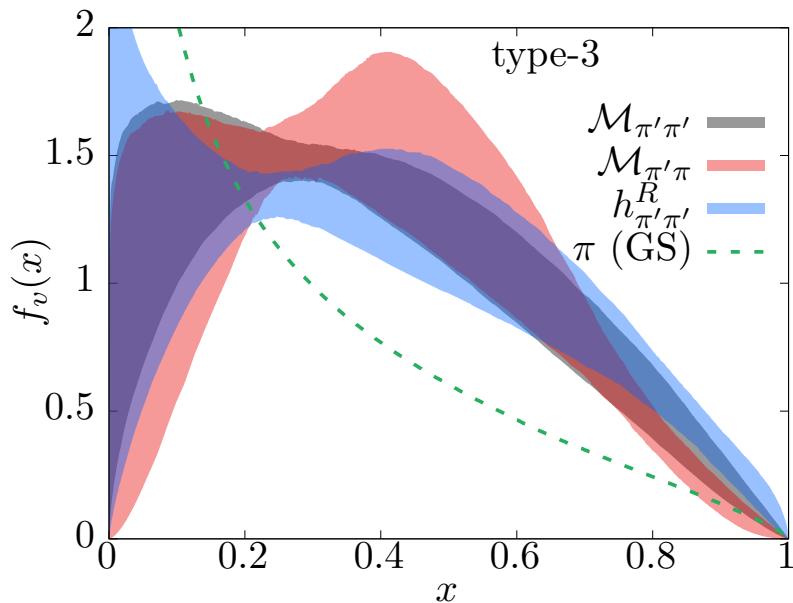
Deep Neural Network (DNN)

agreement with x -space matching



Valence PDF of excited pion

Valence PDF of first radial excitation of the pion, $\pi(1300)$ on $a = 0.04$ fm with $m_\pi = 300$ MeV



The lance PDF is much larger than for the ground state in the intermediate x region \Rightarrow all moments up to the 4th one are larger

Pion distribution amplitude

$$if_\pi P^+ \mathcal{I}(\lambda, \mu) = \langle 0 | \bar{d}(-z^-/2) \gamma^+ \gamma_5 W_+ u(z^-/2) | \pi^+; P \rangle \quad \phi(x, \mu) = \int \frac{d\lambda}{2\pi} e^{-i\frac{x}{2}\lambda} \mathcal{I}(\lambda, \mu)$$

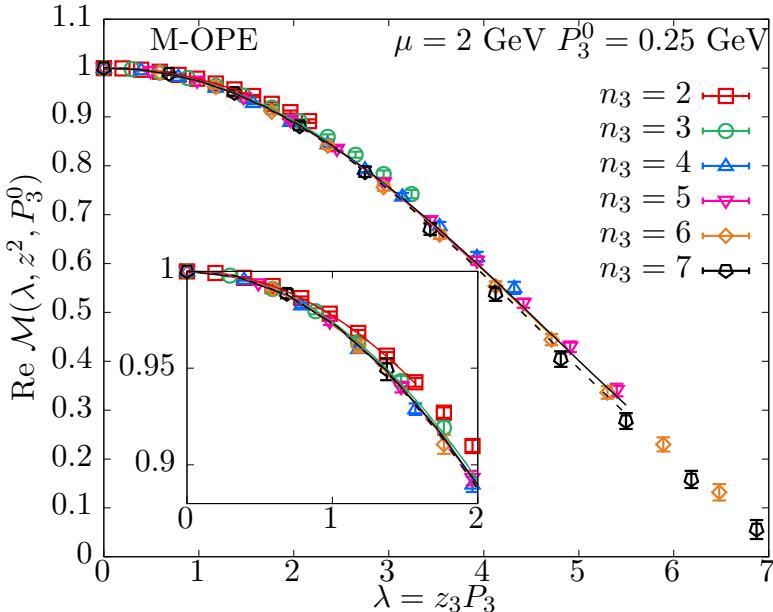
$\lambda = P^+ z^-$

Start with quasi-DA $iP_z h^R(z \cdot P_z, z^2, \mu) \equiv \langle 0 | O_{\gamma_5 \gamma_3}(z, \mu) | \pi^+; P \rangle$ and use short distance factorization: **Mellin-OPE**

$$h^{\text{tw2}}(\lambda, z^2, \mu) = \sum_{n=0} \frac{(-i\lambda/2)^n}{n!} \sum_{m=0}^n C_{n,m}(z^2 \mu^2) \langle x^m \rangle, \quad \lambda = zP_z, \quad \langle x^n \rangle = \int_{-1}^1 \phi(x, \mu) x^n dx$$

Ratio scheme:

$$\mathcal{M}(\lambda, z^2, P^0) \equiv \frac{h^B(\lambda, z^2)}{h^B(\lambda_0, z^2)} = \frac{h^R(\lambda, z^2, \mu)}{h^R(\lambda_0, z^2, \mu)}, \leftrightarrow \frac{h^{\text{tw2}}(\lambda, z^2, \mu)}{h^{\text{tw2}}(\lambda_0, z^2, \mu)}, \quad \lambda_0 = P_z^0 z \Rightarrow \langle x^n \rangle_\mu$$



$$\langle x^2 \rangle = 0.2848(52)(71), \\ \langle x^4 \rangle = 0.124(11)(20).$$

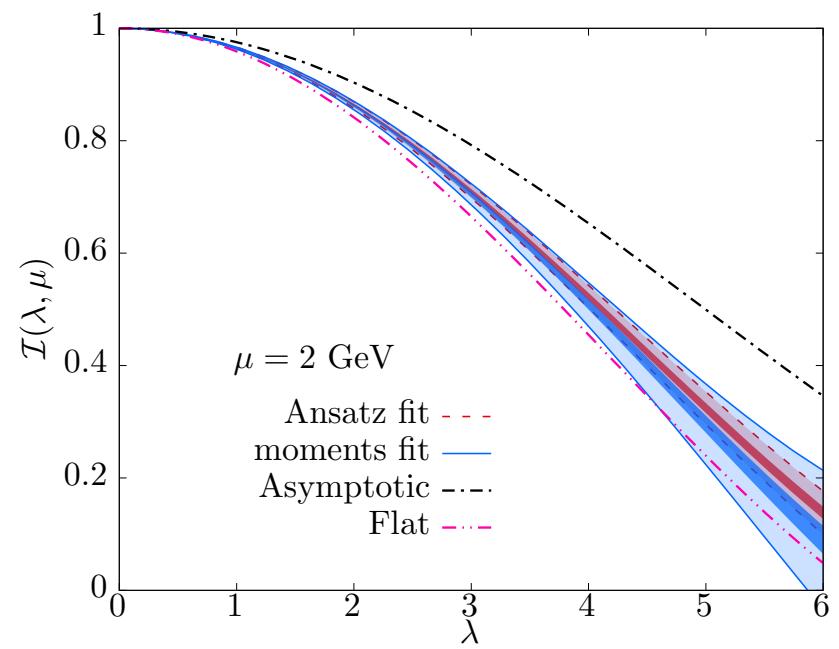
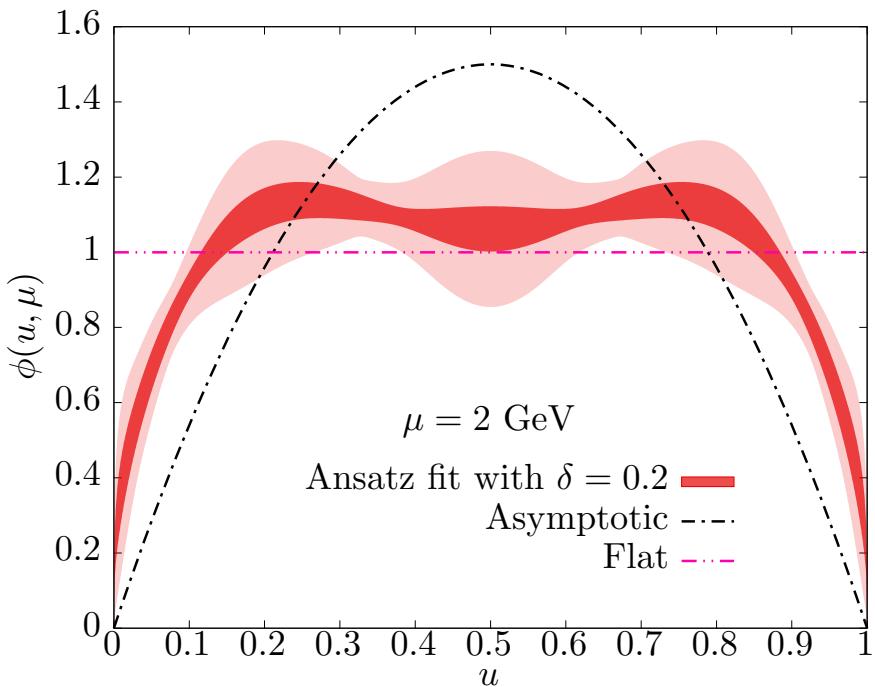
Very different from asymptotic values:
 $\langle x^2 \rangle = 0.2$ and $\langle x^4 \rangle = 0.0857$
or flat DA ($\phi(x) = 1/2$):
 $\langle x^2 \rangle = 1/3$ and $\langle x^4 \rangle = 0.2$

Results are cross-checked with conformal OPE
 $a_2 = 0.227(18)(23), a_4 = -0.16(13)(30) \Rightarrow$
same Mellin moments

Pion distribution amplitude

Conformal OPE inspired fit form $x = 2u - 1$

$$\phi(u) = \mathcal{N} u^\alpha (1-u)^\alpha \sum_{n=0}^{N_G+1} s_n C_{2n}^{\frac{1}{2}+\alpha} (1-2u), \quad s_0 = 1 \quad |s_n| < \delta, \quad n > 0$$



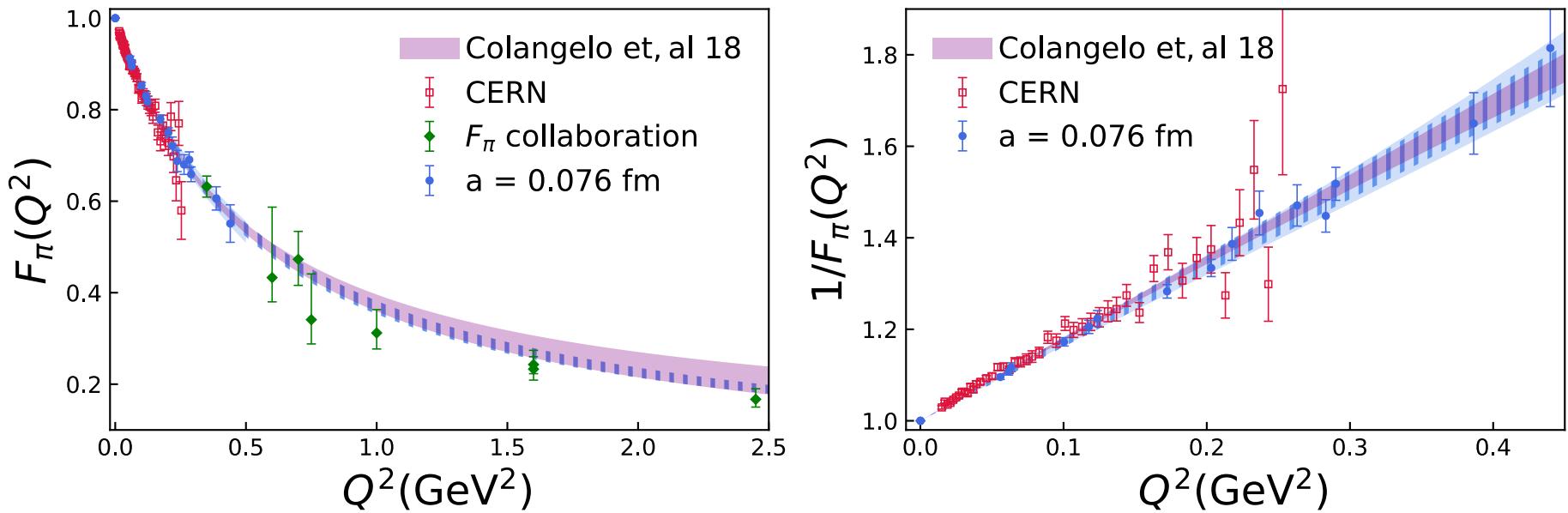
$$\phi(x) \Rightarrow \langle x^2 \rangle = 0.2845(44)(58), \quad \langle x^4 \rangle = 0.1497(50)(38)$$

Agreement with direct fits of moments

Pion form factor

Form factors are sensitive to the light quark masses

$$64^4, a = 0.076 \text{ fm}, m_\pi = 140 \text{ MeV}, P_z \sim 2 \text{ GeV}$$



Lattice and experimental results on the pion form factor agree

Lattice results also agree with the results of the dispersive analysis of the time-like pion form factor

Colangelo, Hofferichter, Stoffer, JHEP 02 (2019) 006

The monopole Ansatz $F_\pi(Q^2) = (1 + Q^2/M^2)^{-1}$, $M \simeq 0.8 \text{ GeV}$ works well

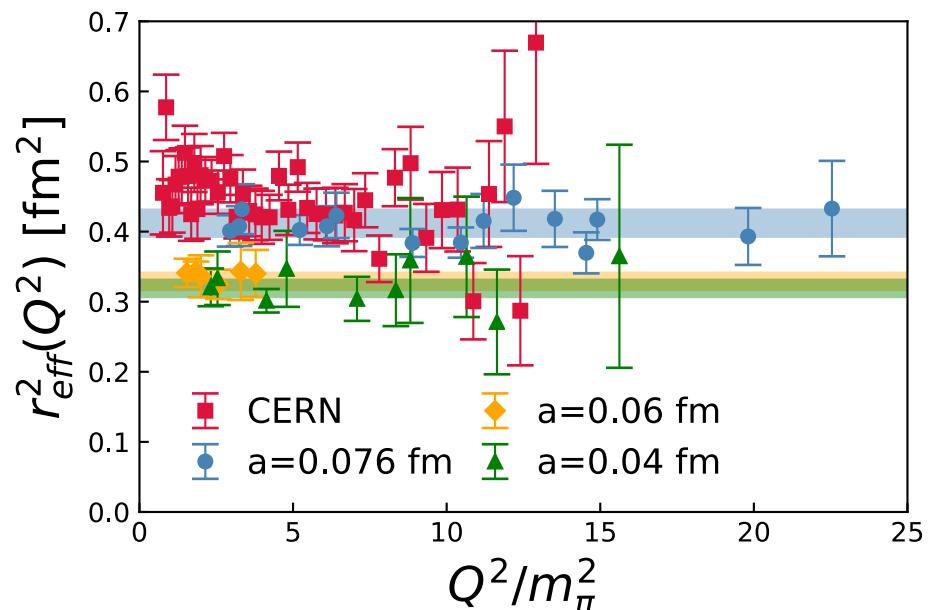
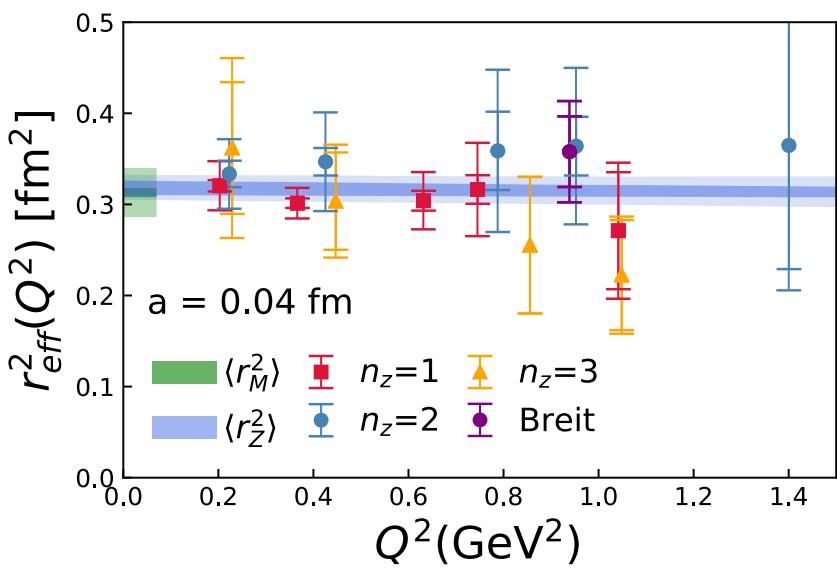
Pion form factor (con't)

Pion charge radius: $\langle r_\pi^2 \rangle = -6 \frac{dF_\pi(Q^2)}{dQ^2} |_{Q^2=0}$ $\langle r_\pi^2 \rangle = 6/M^2$ for monopole fit

The effective radius $r_{eff}^2(Q^2) = \frac{6(1/F_\pi(Q^2) - 1)}{Q^2}$ is constant for all Q^2



monopole Ansatz works for extended Q^2 range



Pion form factor is very sensitive to the quark mass

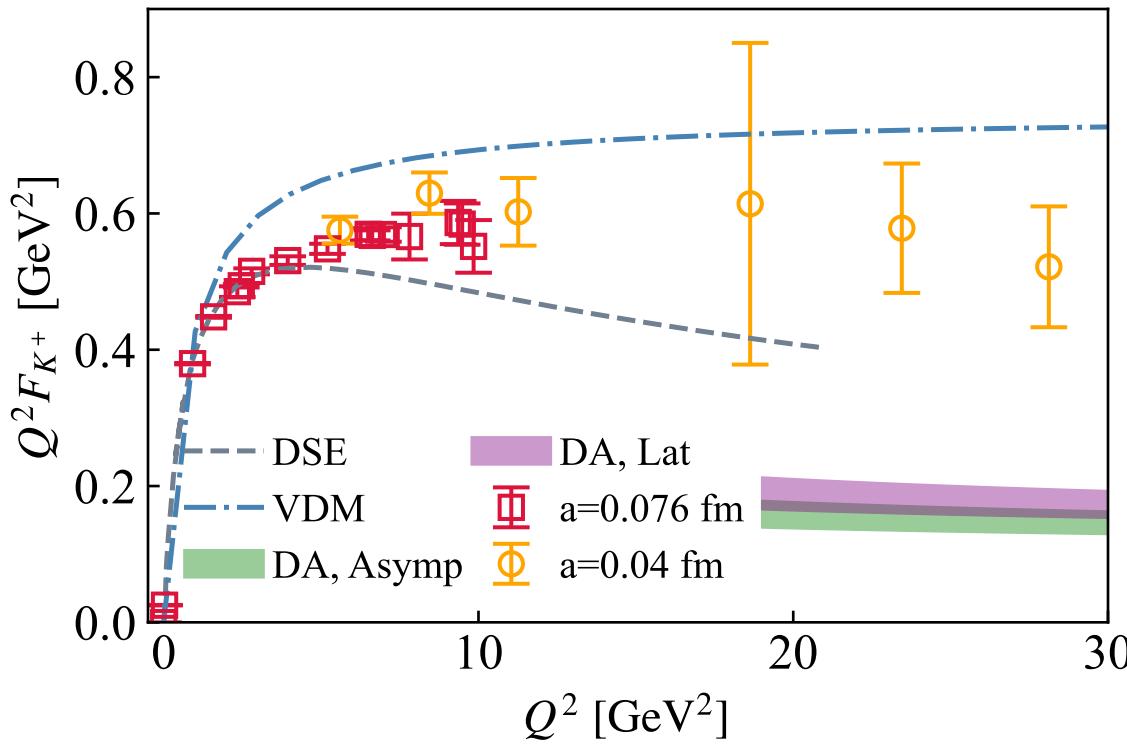
$$\langle r_\pi^2 \rangle = 0.42(2) \text{ fm}^2 \quad (\text{monopole fit, } z\text{-expansion})$$

$$\langle r_\pi^2 \rangle_{PDG} = 0.434(5) \text{ fm}^2$$

Kaon form factor

H.-T. Ding, X. Gao, A.D. Hanlon, N. Karthik, S. Mukherjee, PP, P. Scior, Qi Shi, S. Syritsyn, Y. Zhao,

Use Breit-Frame and boosted source to reach large momentum transfer



Summary

- Valence pion PDF is very suitable for LaMET based lattice studies and current lattice results agree with and complement pion PDF obtained with global analysis
- There is good consistency between different lattice approaches for pion PDF
- First result for excited pion valence PDF show that it is much harder than for the ground state pion
- Pion DA can be estimated from the lattice and it is very different from the asymptotic regime or flat form
- Pion and kaon form factor can be studied at large momentum transfer using lattice QCD and the first result indicate very different behavior from naïve perturbative behavior

Back-up: perturbative convergence at small z

