The flavour structure of baryons from lattice QCD

Gunnar S. Bali, Sara Collins, Piotr Korcyl, Andreas Schäfer, Enno E. Scholz, Jakob Simeth, Wolfgang Söldner, Simon Weishäupl, Daniel Jenkins . . .

RQCD Collaboration

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Revealing emergent mass through studies of hadron spectra and structure, ECT*, Sept. 12th 2022

Overview

Investigate flavour structure of the octet baryons (N, Λ, Σ, Ξ): how the properties of the baryon change with the flavour content.

Properties of the baryons are related in SU(3) flavour limit, e.g. $m_N = m_\Lambda = m_\Sigma = m_\Xi$.

Determine the pattern of symmetry breaking $m_u = m_d \neq m_s$.

Test assumptions made on the size of the symmetry breaking in some phenomenology studies.

- ⋆ Preliminaries: interest in the baryon octet (predominantly the nucleon) sigma terms and the current status.
- ⋆ Determination via the Feynmann-Hellmann theorem: fitting the baryon octet masses.
- ⋆ Axial and scalar charges of the baryon octet.
- ⋆ Summary.

Baryon sigma terms

$$\sigma_{q,B} = m_q \left[\frac{\langle B \,|\bar{q}\mathbb{1}q|\, B\rangle}{\langle B|B\rangle} - \langle \Omega \,|\bar{q}\mathbb{1}q|\, \Omega\rangle \right],$$

where $|\Omega\rangle$ denotes the vacuum. Consider $\sigma_{\pi B} = \sigma_{uB} + \sigma_{dB}$ and σ_{sB}.

Mass decomposition of a hadron

$$M_B = \underbrace{\sum_q m_q \langle \bar{q}q\rangle_B}_{\langle \mathbb{H}_m\rangle_B} \underbrace{+\frac{1}{2}\langle \mathbf{B}^2 - \mathbf{E}^2\rangle_B + \sum_q \langle \bar{q}\mathbf{D}\cdot\gamma q\rangle_B}_{\langle \mathbb{H}_{\mathrm{kin}}\rangle_B = 3\langle \mathbb{H}_a\rangle_B}$$

$$\underbrace{-\frac{1}{4}\left[\gamma_m(\alpha_s)\sum_q m_q\langle \bar{q}q\rangle_B + \frac{\beta(\alpha_s)}{4\alpha_s}\langle \mathbf{E}^2 + \mathbf{B}^2\rangle_B\right]}_{\langle \mathbb{H}_a\rangle_B = \frac{1}{4}(M_B - \langle \mathbb{H}_m\rangle_B)}$$

Knowledge of $\langle \mathbb{H}_m\rangle_B$ and M_B gives $\langle \mathbb{H}_a\rangle_B$ and $\langle \mathbb{H}_{\mathrm{kin}}\rangle_B$.

Nucleon: relevant for computing the spin-independent WIMP-nucleon scattering cross-section for direct dark matter experiments.

Phenomenological determinations

These include

- Mass decompositions of the baryon octet with $\langle \overline{q}q \rangle_H$ related via $SU_F(3)$ flavour breaking [Cheng,(1989)].

  $\sigma_{\pi N} = \frac{1}{2}(m_u + m_d)\langle \bar{u}u + \bar{d}d \rangle_N = 26$ MeV assuming $\langle \bar{s}s \rangle_N = 0$.

- Pion-nucleon scattering (Cheng-Dashen theorem): e.g. [Gasser et al.,(1991)] $\sigma_{\pi N} = 45(8)$ MeV, [Pavan et al.,hep-ph/0111066] 64(7) MeV, [Alarcon et al.,1110.3797] 59(7) MeV, [Chen et al.,1212.1893] 52(7) MeV and [Hoferichter et al.,1510.06039] 59.1(3.5) MeV.

- σ_s harder to estimate, not well constrained:

$$\sigma_0 = \frac{1}{2}(m_u + m_d)\langle \bar{u}u + \bar{d}d - 2\bar{s}s \rangle_N, \qquad \sigma_s = \frac{m_s}{m_u + m_d}(\sigma_{\pi N} - \sigma_0)$$

  Chiral perturbation theory+experimental input (masses+...) including $\sigma_{\pi N}$, e.g. [Gasser,(1981)], [Borasoy and Meißner,(1997)], [Alarcon et al.,(1209.2870)].

  (Unnaturely) large $\sigma_s \sim 200$ MeV was not ruled out.

Lattice determinations of the sigma terms

Via direct evaluation of the scalar matrix element.

Indirect evaluation via the Feynman-Hellmann theorem:

$$\sigma_{q,B} = m_q \left[\frac{\langle B \,|\bar{q}\mathbb{1}q|\, B\rangle}{\langle B|B\rangle} - \langle \Omega \,|\bar{q}\mathbb{1}q|\, \Omega\rangle \right] = m_q \frac{\partial m_B}{\partial m_q},$$

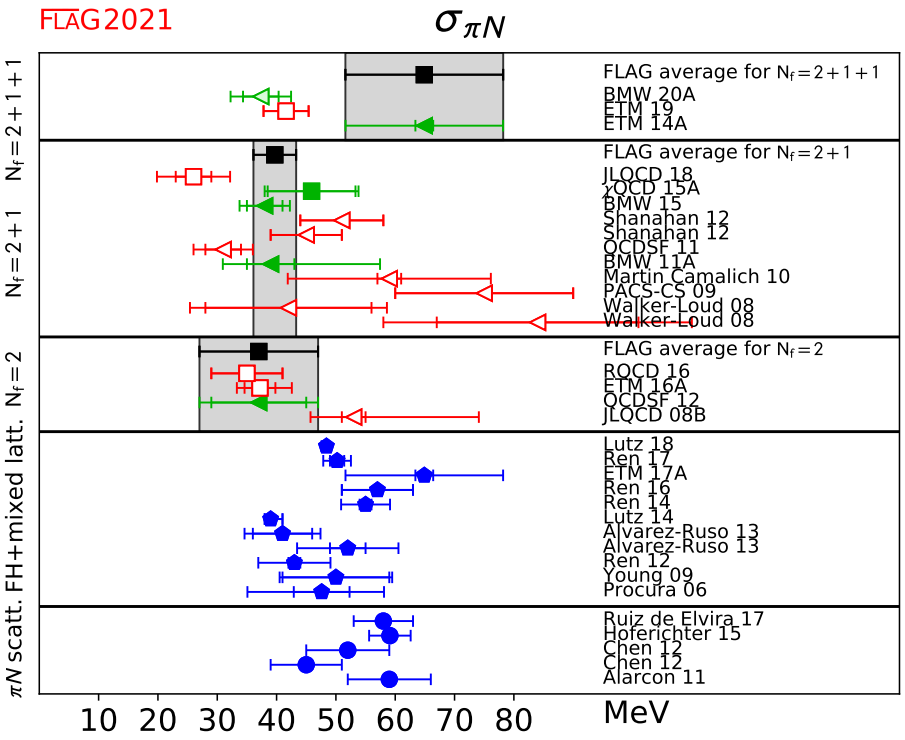
Using the Gell-Mann-Oakes-Renner (GMOR) relation: $M_{PS}^2 \approx B_0(m_{q1} + m_{q2})$

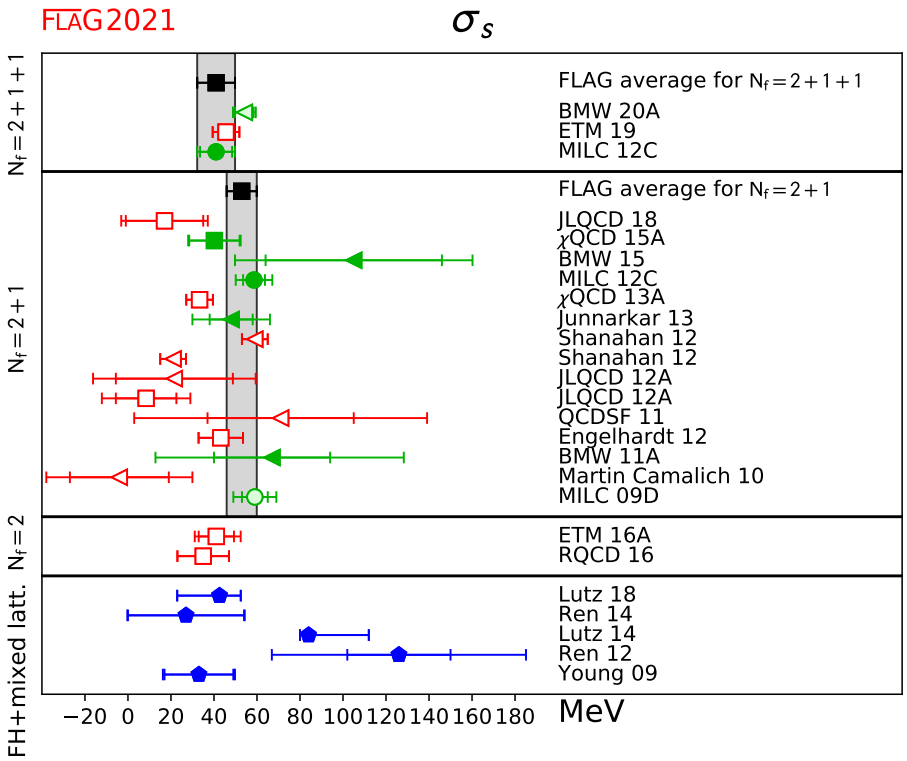
$$\sigma_{\pi B} \approx \tilde{\sigma}_{\pi B} = M_\pi^2 \frac{\partial m_B}{\partial M_\pi^2} \qquad \sigma_{sB} \approx \tilde{\sigma}_{sB} = M_{s\bar{s}}^2 \frac{\partial m_B}{\partial M_{s\bar{s}}^2}$$

where $M_{s\bar{s}}^2 = 2M_K^2 - M_\pi^2$.

Difficult to determine $\sigma_{s,B}$ via indirect (FH) approach (in particular if m_s is kept roughly constant).

Current status: nucleon sigma terms

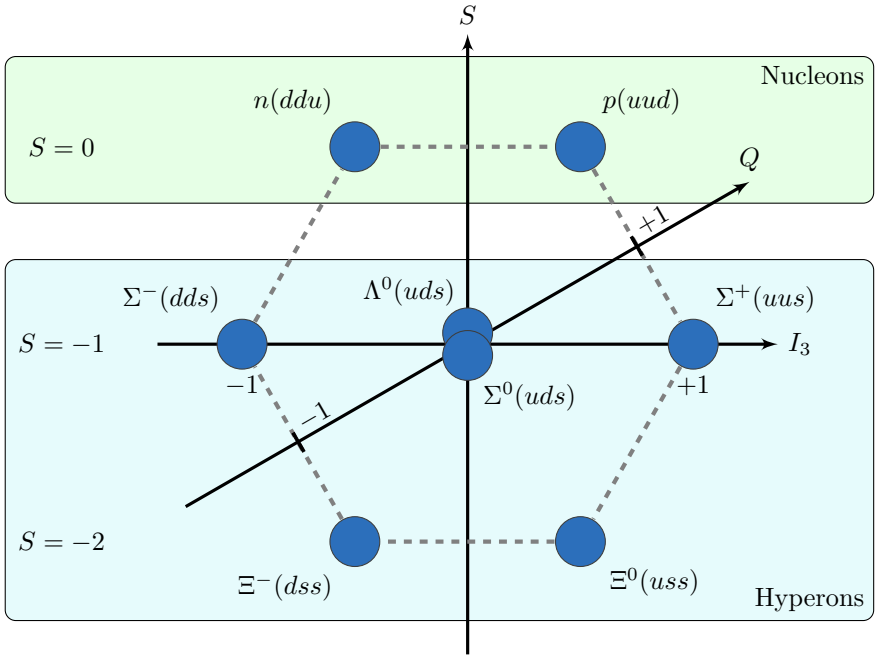




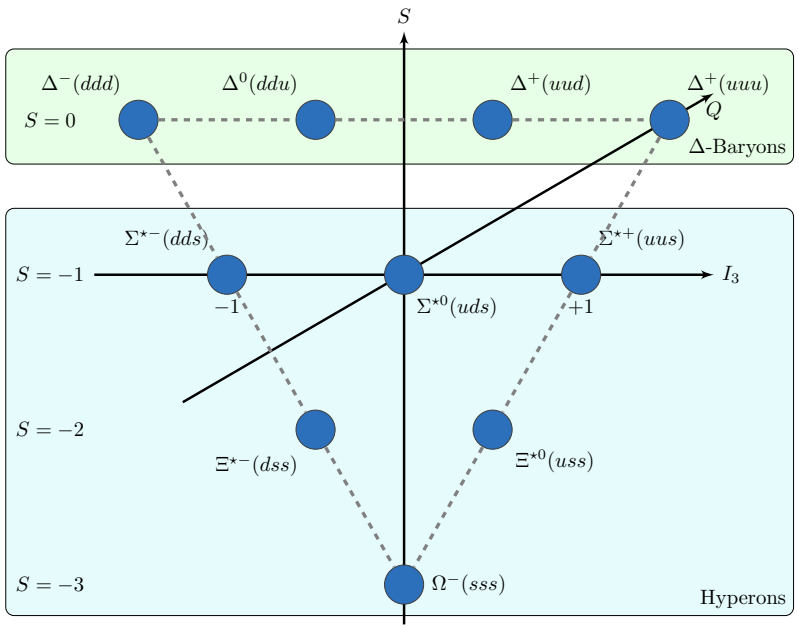
Sigma terms for the rest of the baryon octet are less well known.

Baryon spectrum

Octet: $J^P = \frac{1}{2}^+$



decuplet $J^P = \frac{3}{2}^+$



Simulating QCD (isospin-symmetric, electrically neutral): N, Σ, Λ, Ξ and Δ, Σ^*, Ξ^*, Ω.

Unstable under strong decay: $\Delta \to N\pi$, $\Sigma^* \to \Lambda\pi, \Sigma\pi$ and $\Xi^* \to \Xi\pi$.

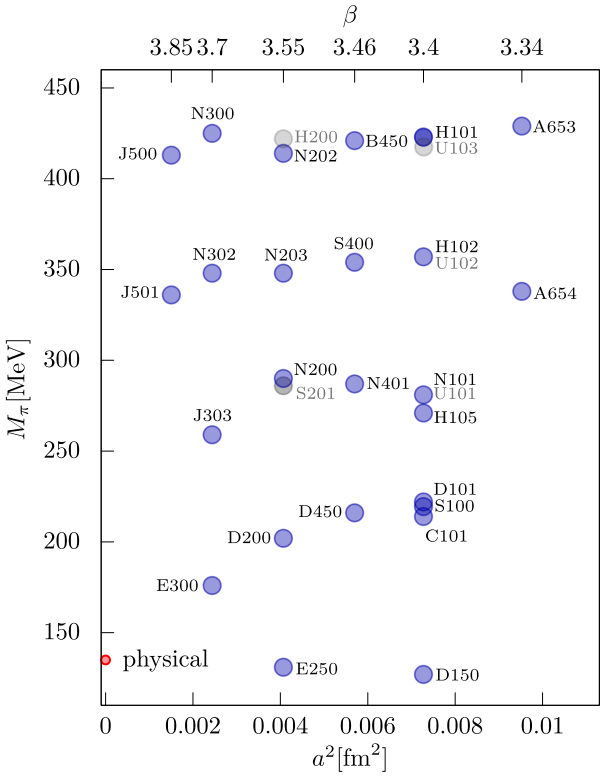
Omit data for which, e.g. $m_\Delta > E_N(\vec{p}) + E_\pi(-\vec{p})$. Consider finite volume, $\vec{p} = 2\pi/L$ and infinite volume ($\vec{p} \to 0$) cases. Ideally, a scattering analysis via the Lüscher formalism is needed.

Coordinated Lattice Simulations (CLS) ensembles

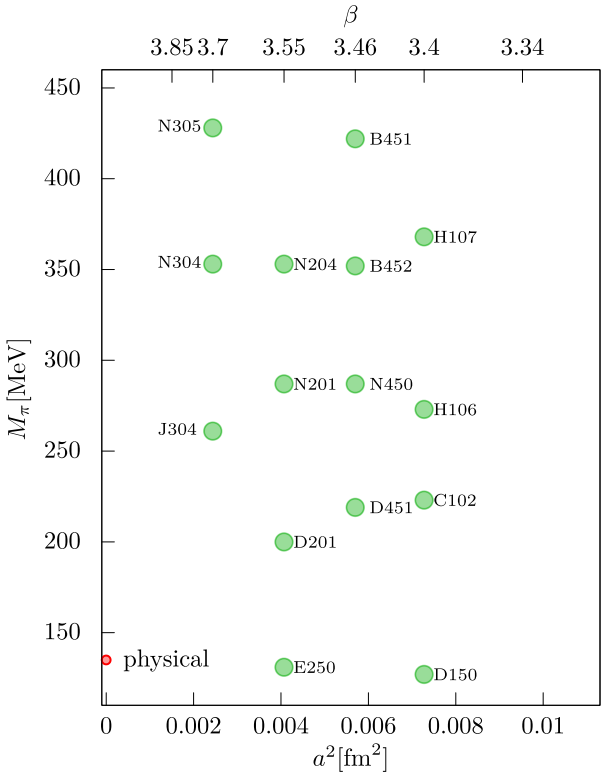
- ⋆ **High statistics**: typically 6000-8000 MDUs, 1000-2000 configurations.

Aim to control all main sources of systematics (a, m_q and V).

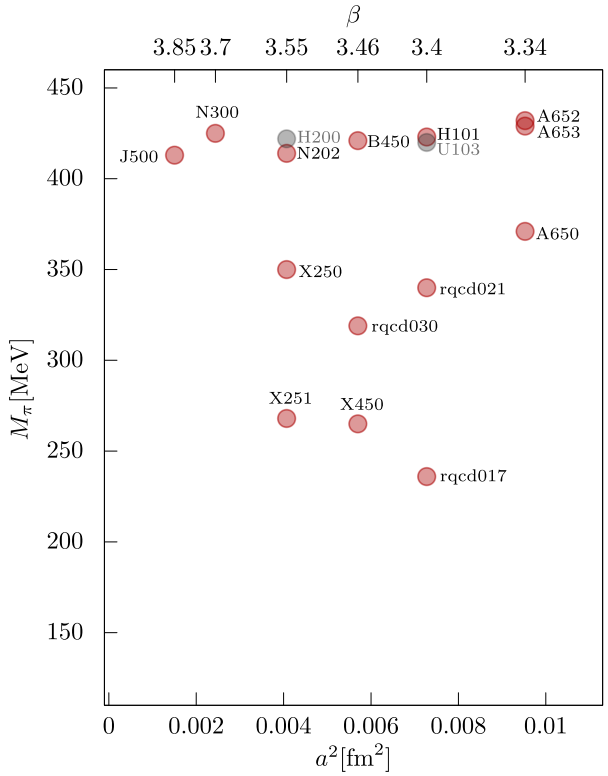
- ⋆ **Discretisation**: Six lattice spacings: $a = 0.1 - 0.04$ fm.
- ⋆ **Finite volume**: $Lm_\pi \gtrsim 4$ with additional smaller volumes.
- ⋆ **Quark mass**: $m_\pi = 410$ MeV down to m_π^{phys}.



$2m_\ell + m_s = \text{const.}$

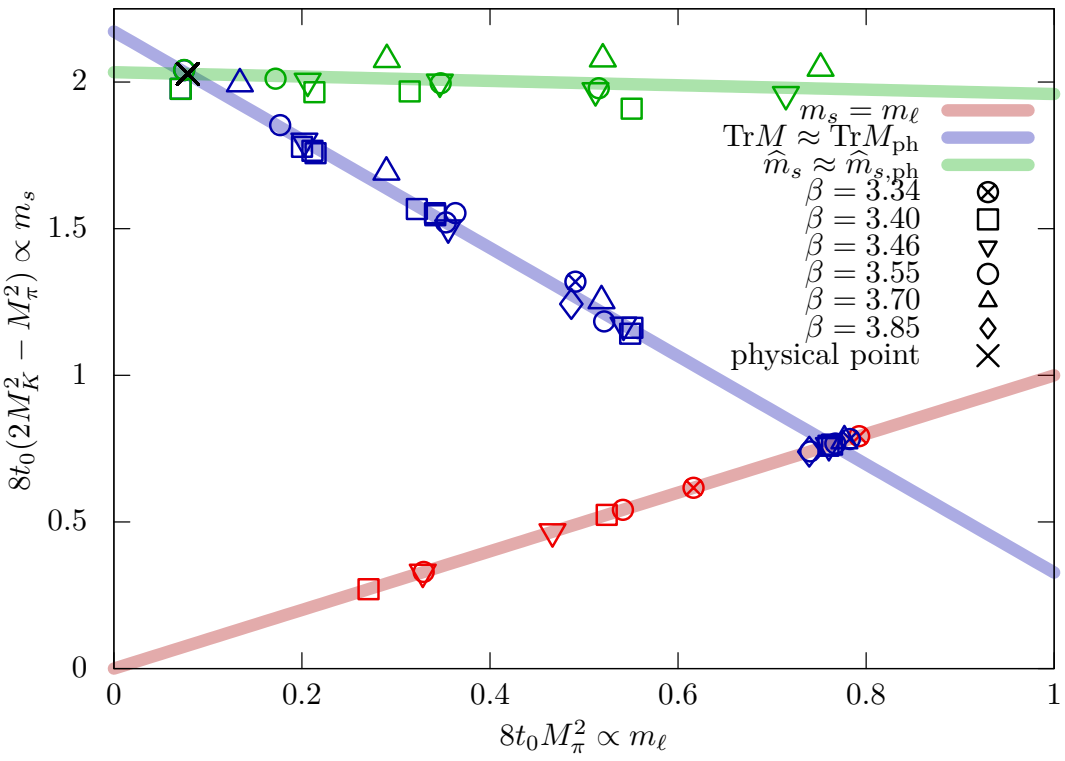


$m_s = \text{const.}$



$m_\ell = m_s$

CLS $N_f = 2 + 1$ ensembles: m_ℓ-m_s plane



Three trajectories: good control over the quark mass dependence. Can correct for mis-tuning of the trajectories. Observables sensitive to m_s are tightly constrained.

$2m_\ell + m_s = \mathrm{const.}$: investigate SU(3) flavour breaking (flavour average quantities roughly constant), approach to physical point involves $m_\pi \downarrow$ and $m_K \uparrow$.

$m_\ell = m_s$: important for determination of SU(3) ChPT low energy constants (and renormalisation factors).

Extrapolation of baryon multiplets

Simultaneous fit to the baryon multiplets using the fit form

$$\mathrm{m}_B(\mathbb{M}_\pi, \mathbb{M}_K, L, \mathbb{a}) = \left[\mathrm{m}_B(\mathbb{M}_\pi, \mathbb{M}_K, \infty, 0) + \delta m_B^{FV}(\mathbb{M}_\pi, \mathbb{M}_K, L)\right] \times \left[1 + \mathbb{a}^2\left(c + \bar{c}\,\overline{\mathbb{M}}^2 + \delta c_B\,\delta\mathbb{M}^2\right)\right].$$

where $\overline{M}^2 = \frac{1}{3}(2M_K^2 + M_\pi^2) \propto \overline{m} = \frac{1}{3}(2m_\ell + m_s)$, $\delta M^2 = 2(M_K^2 - M_\pi^2) \propto \delta m = m_s - m_\ell$

Natural choice for $\mathrm{m}_B(\mathbb{M}_\pi, \mathbb{M}_K, \infty, 0)$ is to use SU(3) baryon ChPT (and in a finite volume for δm_B^{FV} with no additional low energy constants).

BChPT: $O(p^3)$ baryon ChPT with EOMS regularisation [Ellis et al.,nucl-th/9904017] 6 low energy constants (LECs) including m_0 and F and D, also appearing in ChPT expressions for g_A^B.

$$m_O(M_\pi, M_K, \infty, 0) = m_0 + \bar{b}\,\overline{M}^2 + \delta b_O\,\delta M^2 + \frac{m_0^3}{(4\pi F_0)^2}\left[g_{O,\pi} f_O\left(\frac{M_\pi}{m_0}\right) + g_{O,K} f_O\left(\frac{M_K}{m_0}\right) + g_{O,\eta_8} f_O\left(\frac{M_{\eta_8}}{m_0}\right)\right],$$

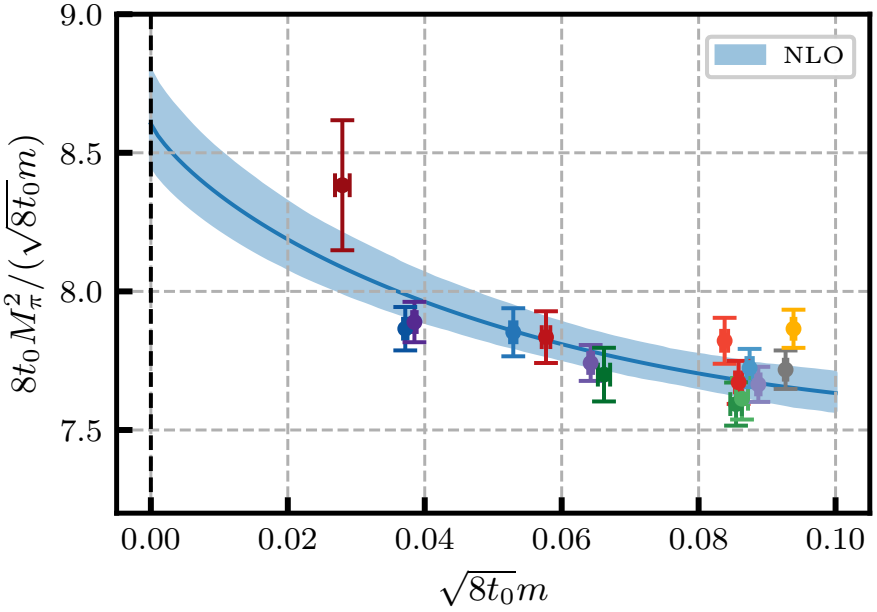
Heavy limit (HBChPT) [Jenkins and Manohar,Phys. Lett. B 255 (1991) 558.]

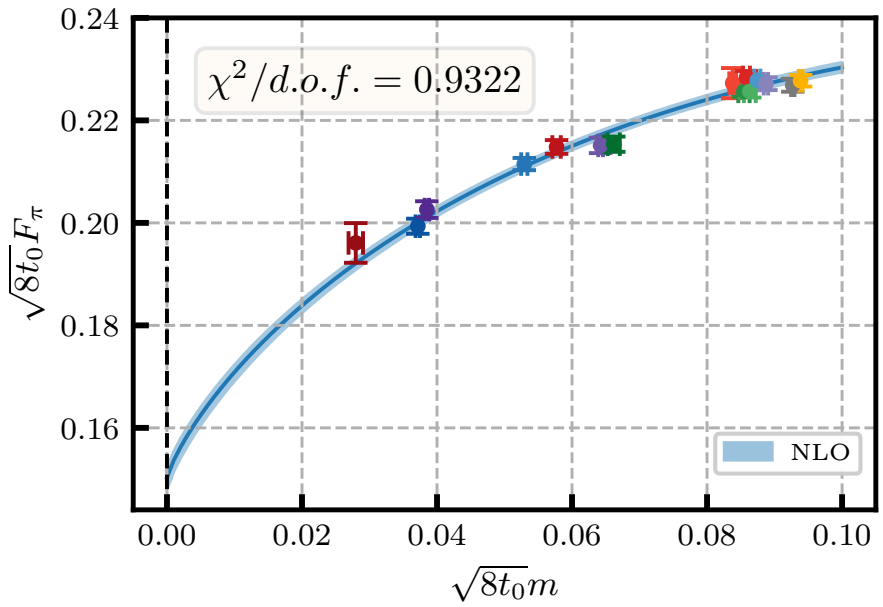
Joint baryon octet and decuplet fits via the small scale expansion, see e.g. [Martin Camalich et al.,1003.1929] **Two new LECs:** $\mathcal{C}$ and $\delta = m_{D0} - m_0$.

Future: simultaneous fit to several observables

[RQCD,2201.05591]: CLS $N_f = 3$ ($m_s = m_\ell$) ensembles, $a = 0.04 - 0.1$ fm, $M_\pi = 430 - 240$ MeV.

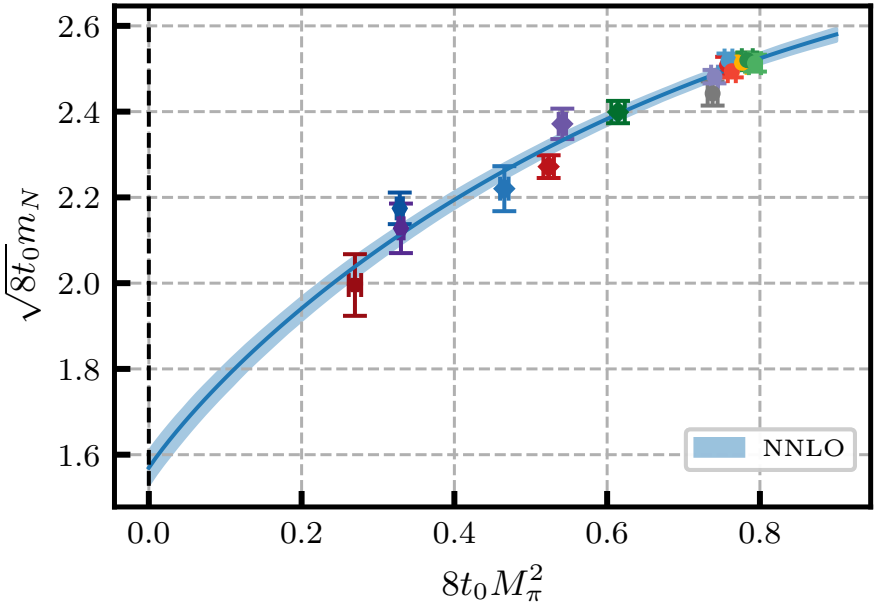
Fit M_π^2 and F_π as a function of the renormalised quark mass $m = m_\ell$ to extract the ChPT low energy constants B_0 and F_0.

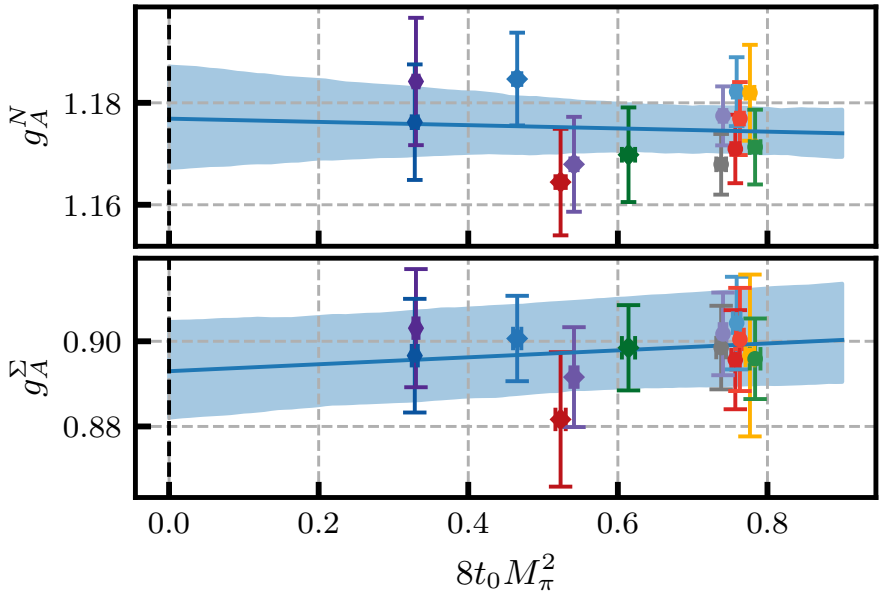


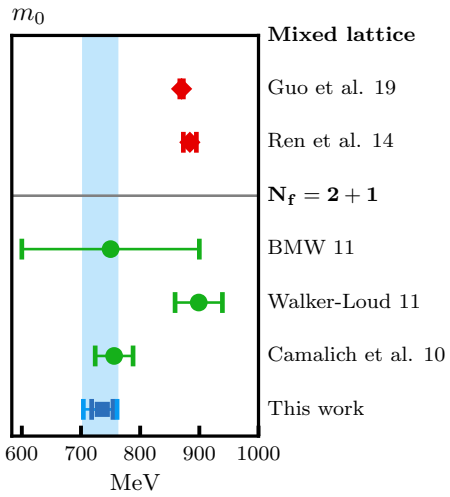


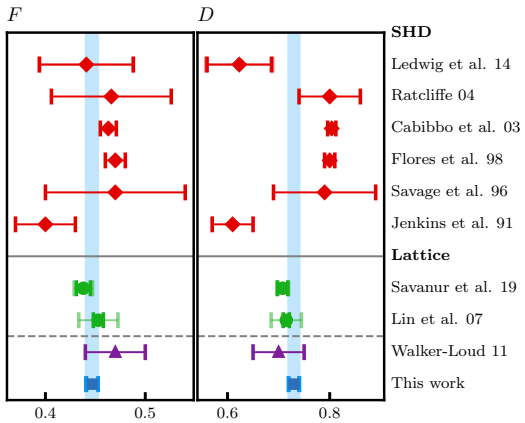
Future: simultaneous fit to several observables

Fit m_N and $g_A^{N,\Sigma}$ as a function of M_π^2 to extract the ChPT low energy constants m_0, F and D.





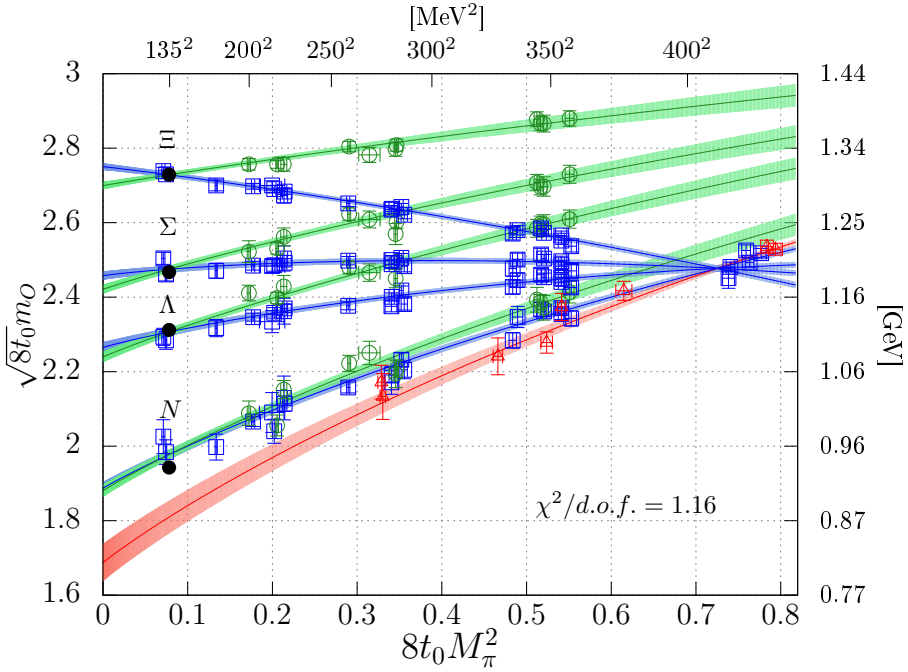


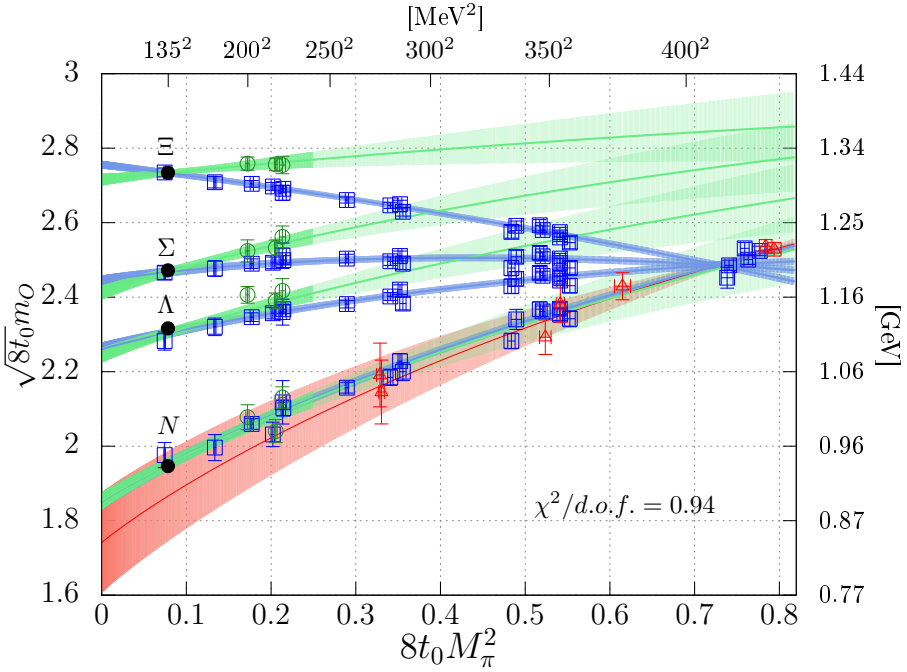


SHD: semi-leptonic hyperon decay.

NNLO BChPT fit to the baryon octet: m_q dependence

FV terms included in the fit. 12 parameters to fit the 4 octet baryon masses.

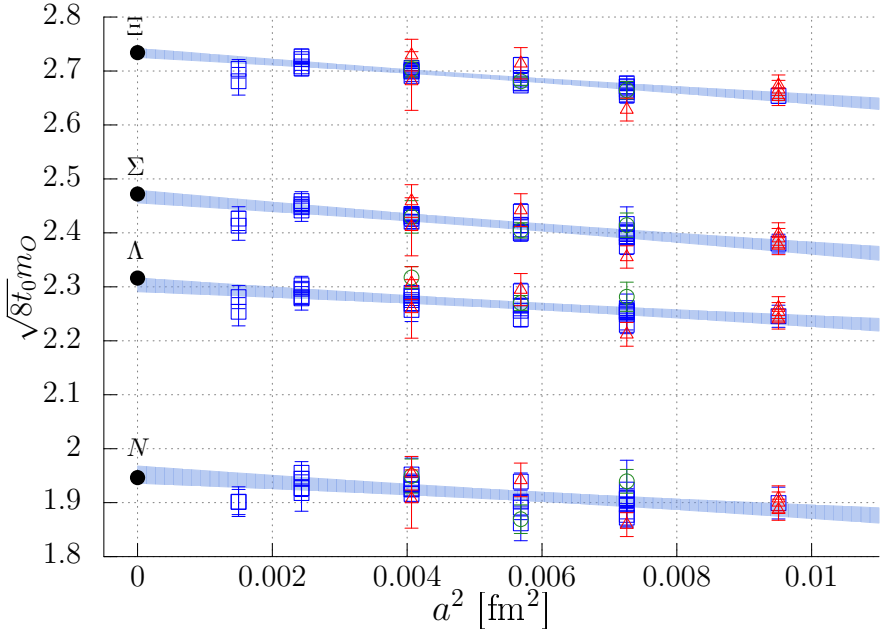


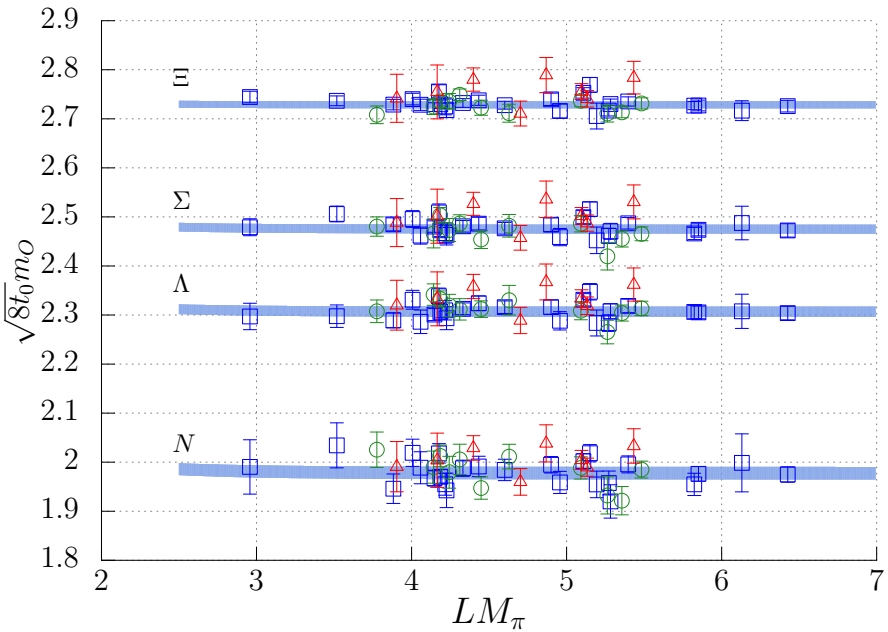


Curves show $\mathrm{m}_O(\mathbb{M}_\pi, \mathbb{M}_K, \infty, 0)$, while the data points are shifted to correct for finite a, finite V and mis-tuning of the trajectory.

NNLO BChPT fit to the baryon octet: a and V effects

The data points are shifted to the physical point ($M_{\pi,ph}$, $M_{K,ph}$).





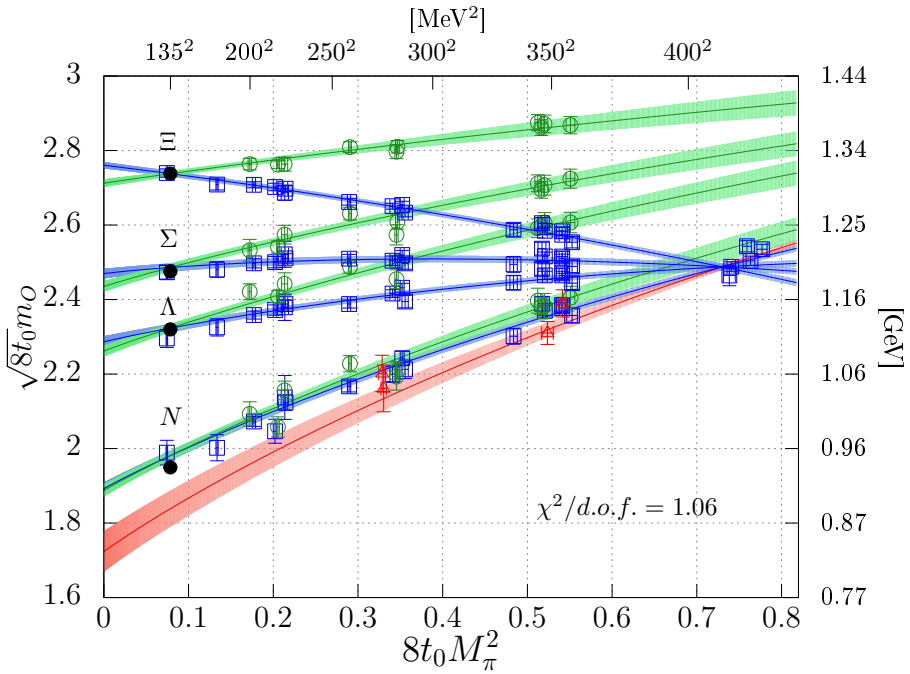
Discretisation effects are mild.

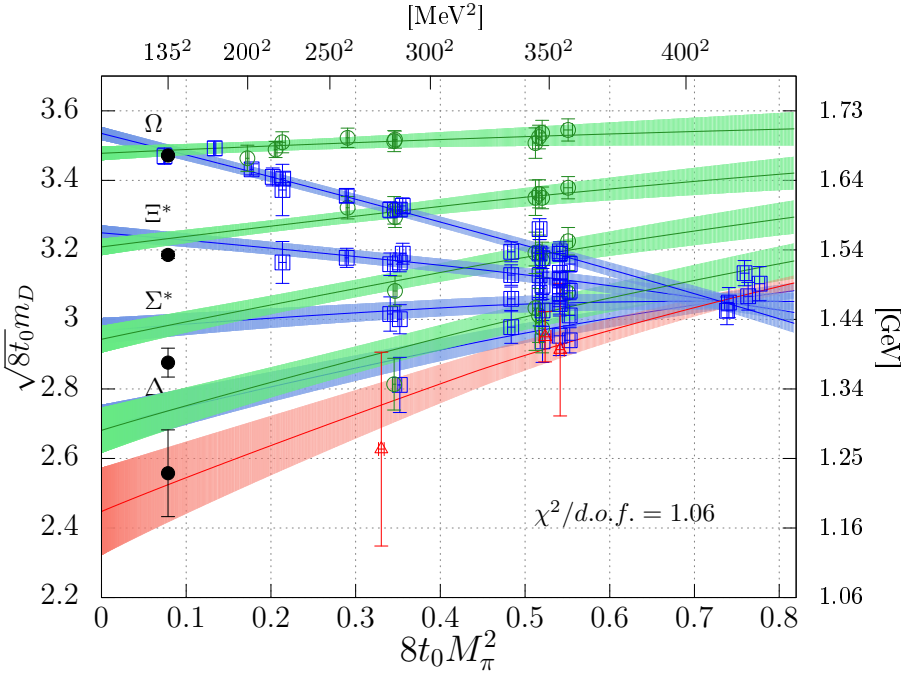
Finite volume effects are small.

Variation with the continuum fit form

Fit the octet and decuplet masses simultaneously using NNLO BChPT + SSE + FV terms (23 parameters).

Δ, Σ^*, Ξ^* baryons are unstable: cut out data where $D \to O\pi$ in the infinite volume limit.



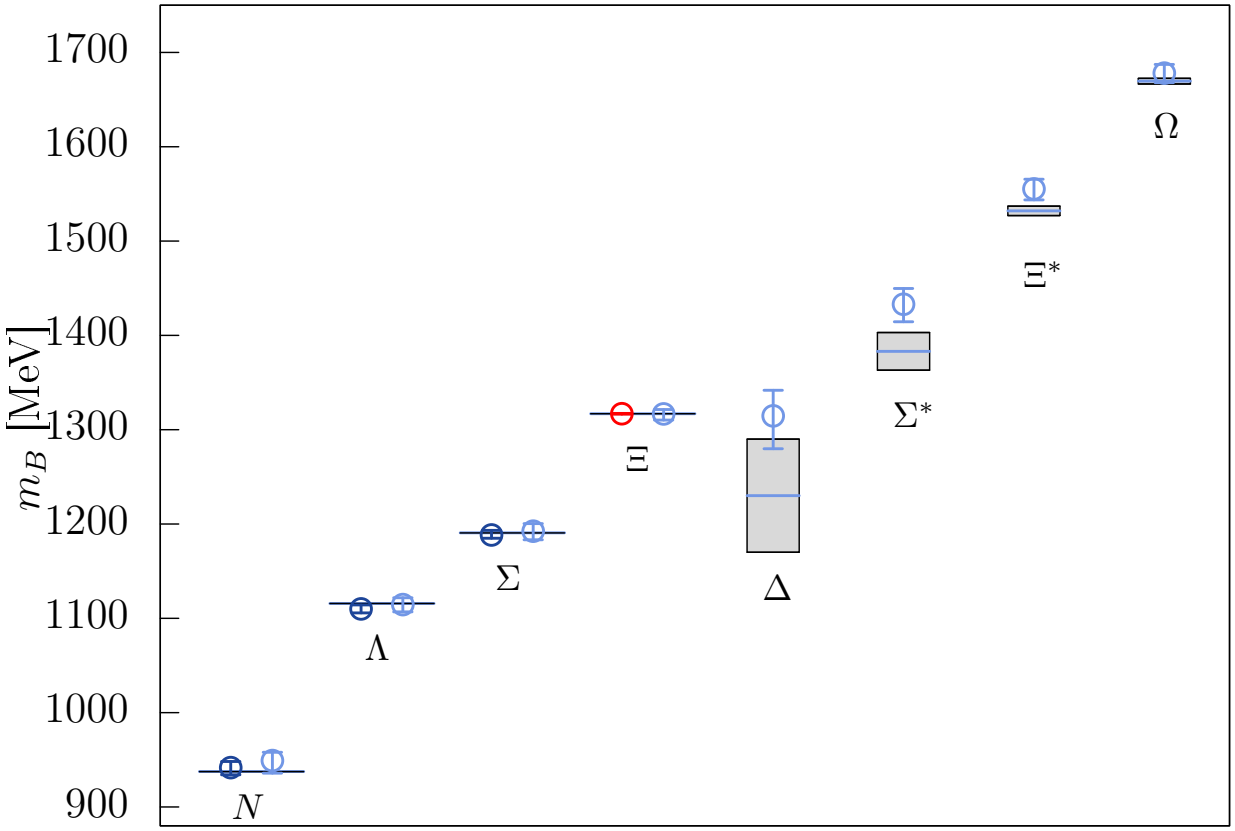


Unstable baryons: expt. masses not reproduced.

Low lying baryon spectrum

Octet baryon spectrum from BChPT fits + FV terms Agreement with corrected expt. masses within 1% overall uncertainty.

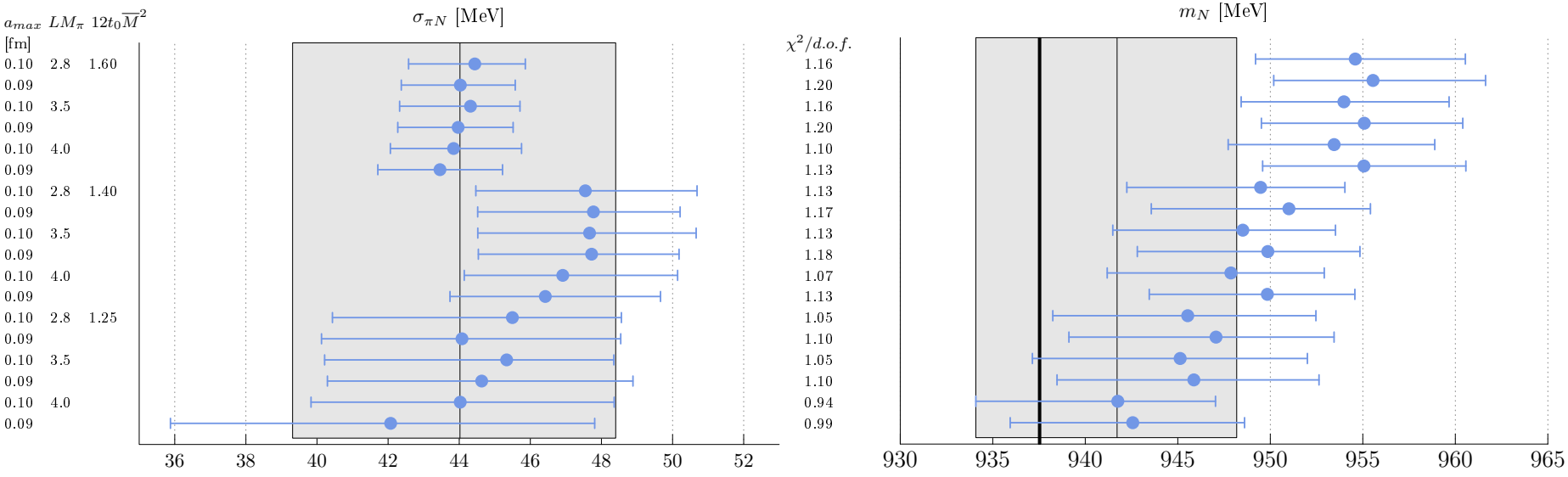
Octet and decuplet masses from BChPT + SSE + FV terms.



Unstable decuplet baryons: grey bands indicate the expt. Breit-Wigner width. Proper treatment via the Lüscher formalism required.

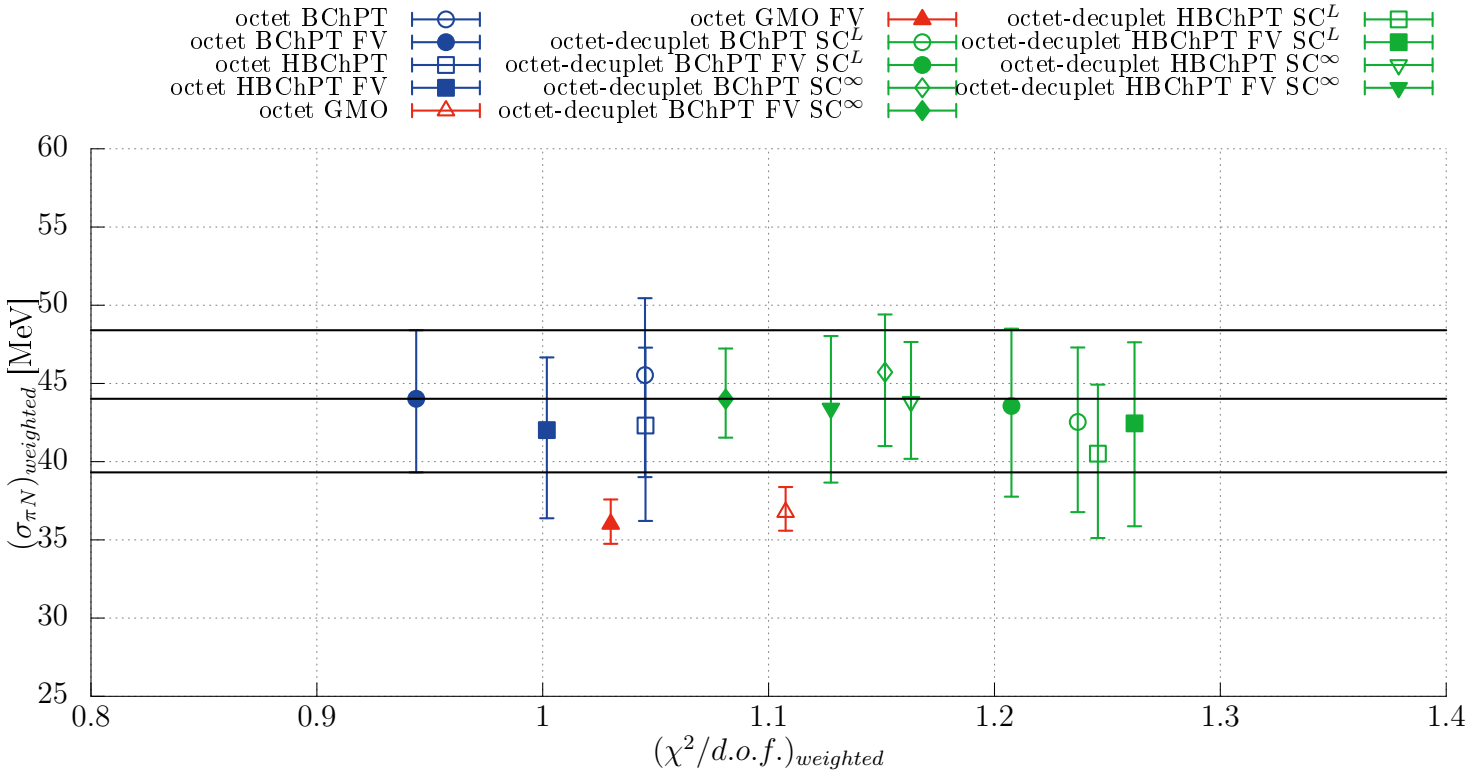
Variation of $\tilde{\sigma}_{\pi N}$ with cuts on the data

Using NNLO BChPT for $\mathrm{m}_O(\mathbb{M}_\pi, \mathbb{M}_K, \infty, 0)$.



Weighted average of the fits: $\tilde{\sigma}_{\pi N} = 44.0^{(4.4)}_{(4.7)}$ MeV.

Variation of $\tilde{\sigma}_{\pi N}$ with continuum fit form for m_B



Baryon octet sigma terms

	$\tilde{\sigma}^N_{u+d}$	$\tilde{\sigma}^\Lambda_{u+d}$	$\tilde{\sigma}^\sigma_{u+d}$	$\tilde{\sigma}^\Xi_{u+d}$
MeV	$44.0^{(4.4)}_{(4.7)}$	$27.6^{(4.3)}_{(4.9)}$	$24.9^{(4.6)}_{(5.0)}$	$10.1^{(4.4)}_{(5.4)}$
	$\tilde{\sigma}^N_s$	$\tilde{\sigma}^\Lambda_s$	$\tilde{\sigma}^\sigma_s$	$\tilde{\sigma}^\Xi_s$
MeV	$3.7^{(59.3)}_{(60.8)}$	$112.7^{(63.3)}_{(59.9)}$	$194.1^{(67.8)}_{(60.7)}$	$266.69^{(70.3)}_{(68.4)}$

cf. [BMWc,1109.4265] (more precise results for the nucleon in [BMWc,1510.08013] and [BMWc,2007.03319])

	σ^N_{u+d}	σ^Λ_{u+d}	σ^Σ_{u+d}	σ^Ξ_{u+d}
MeV	$39(4)^{(18)}_{(7)}$	$29(3)^{(11)}_{(5)}$	$23(3)^{(19)}_{(3)}$	$15(2)^{(8)}_{(3)}$
	σ^N_s	σ^Λ_s	σ^Σ_s	σ^Ξ_s
MeV	$67(27)^{(55)}_{(47)}$	$180(26)^{(48)}_{(77)}$	$245(29)^{(50)}_{(72)}$	$312(32)^{(72)}_{(77)}$

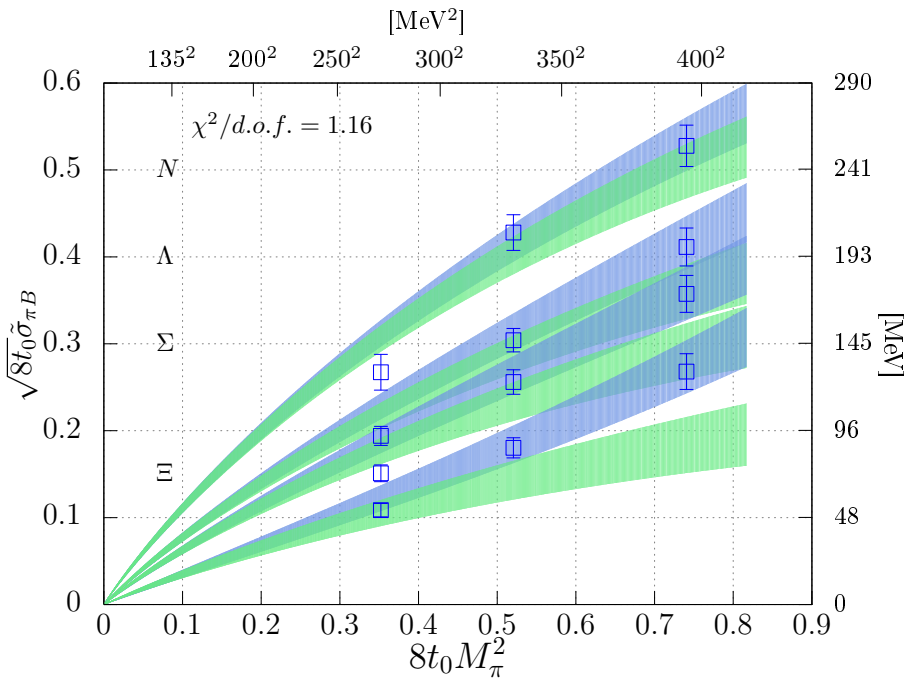
and [Shanahan et al.,1205.5365] (single lattice spacing, not all systematics under control).

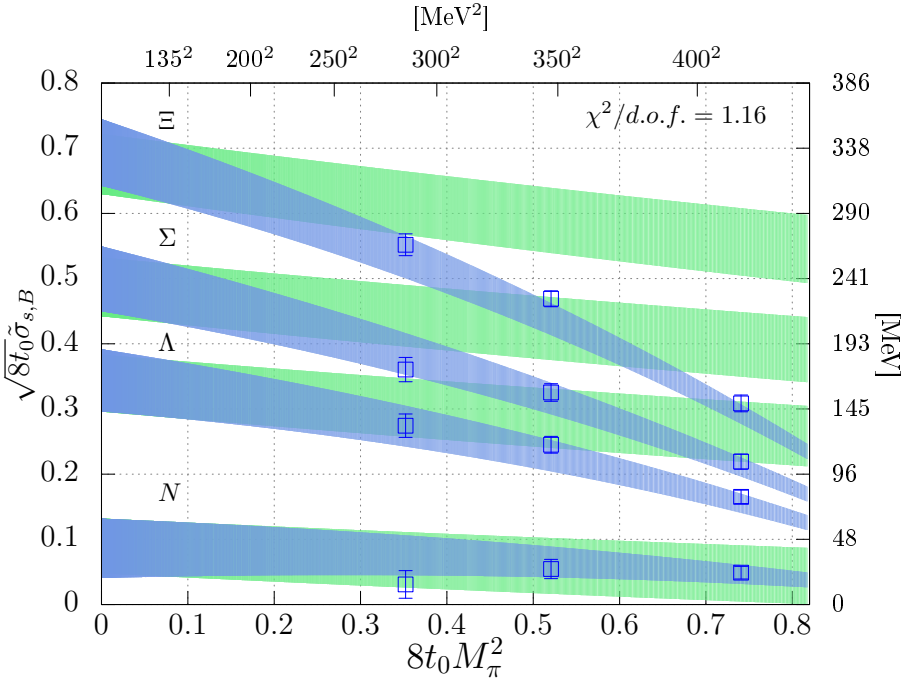
	σ^N_{u+d}	σ^Λ_{u+d}	σ^Σ_{u+d}	σ^Ξ_{u+d}
MeV	47(6)(5)	26(3)(2)	20(2)(2)	8.9(7)(4)
	σ^N_s	σ^Λ_s	σ^Σ_s	σ^Ξ_s
MeV	22(6)(0)	141(8)(1)	172(8)(1)	239(8)(1)

Consistency with direct determinations

Quark mass dependence of $\sigma_{\pi B}$ and σ_{sB} for the baryon octet using NNLO BChPT fit + FV terms.

Direct determination: Pia Petrak, Jochen Heitger + RQCD.

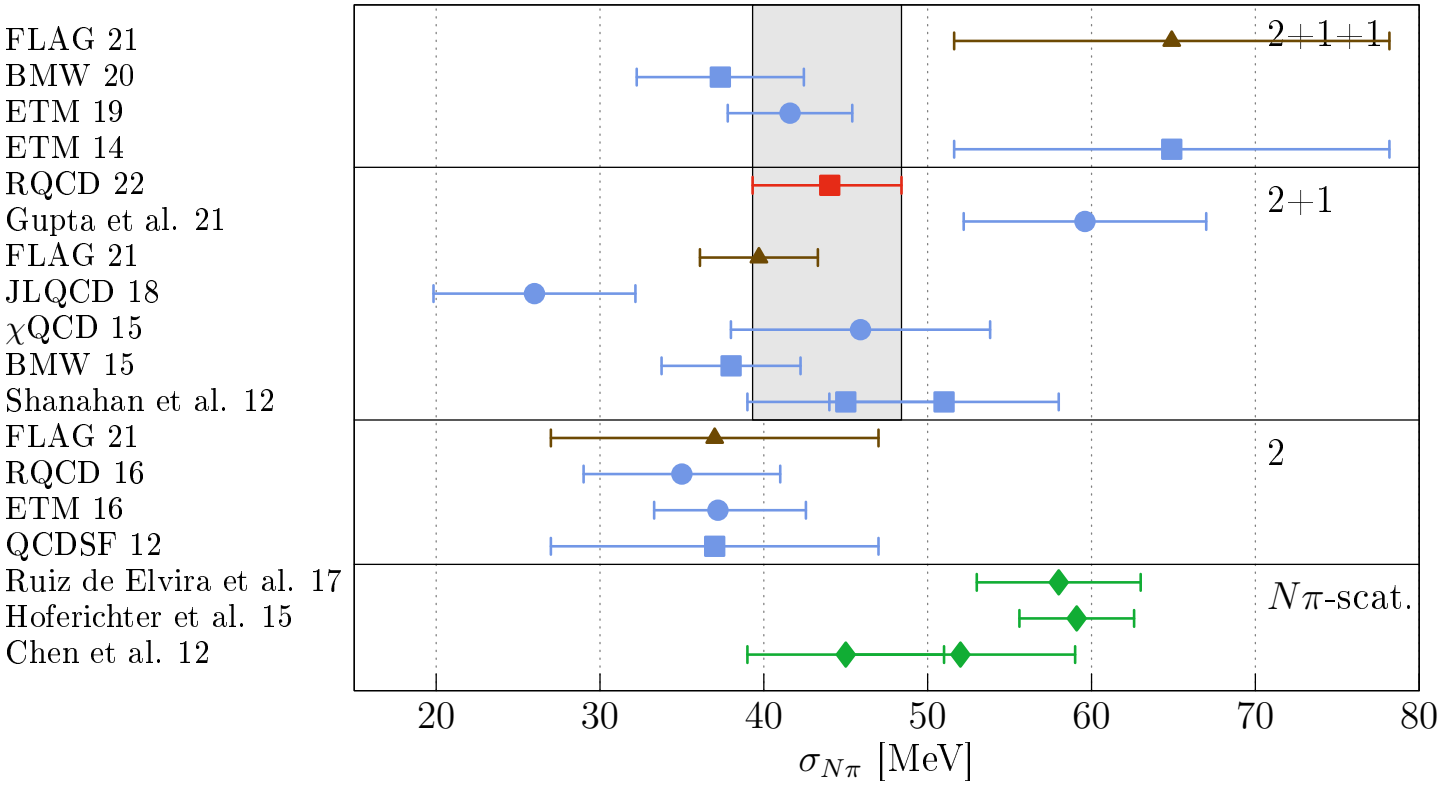




Note: curves show quark mass dependence in the continuum (not from the best fit!). Direct results are at finite $a = 0.064$ fm.

Future: fit m_B and direct determinations of $\sigma_{\pi B}$ and σ_{sB} together.

Comparison with other determinations of $\sigma_{\pi N}$



Some tension between the lattice results and the phenomenological results of [Hoferichter et al.,1506.04142] obtained using $N\pi$ scattering data.

Direct determination by [Gupta et al.,2105.12095] is also larger.

Baryon octet mass decompositions

For a scale $\mu \ll m_c$:

$$\langle H_m \rangle_H = \sum_{q \in \{u,d,s\}} m_q \langle \overline{q} q \rangle_H ,$$

$$\langle H_m \rangle_N \sim 48(60)\text{MeV} \qquad \langle H_m \rangle_\Lambda \sim 140(60)\text{MeV}$$

$$\langle H_m \rangle_\Sigma \sim 219(64)\text{MeV} \qquad \langle H_m \rangle_\Xi \sim 277(70)\text{MeV}$$

$N_f = 3$: very approximate

$$M_N \approx (0.08\, M_N)_m + (0.69\, M_N)_{\rm kin} + (0.23\, M_N)_{\rm a}$$

$$M_\Lambda \approx (0.13\, M_\Lambda)_m + (0.66\, M_\Lambda)_{\rm kin} + (0.21\, M_\Lambda)_{\rm a}$$

$$M_\Sigma \approx (0.19\, M_\Sigma)_m + (0.61\, M_\Sigma)_{\rm kin} + (0.20\, M_\Sigma)_{\rm a}$$

$$M_\Xi \approx (0.21\, M_\Xi)_m + (0.59\, M_\Xi)_{\rm kin} + (0.20\, M_\Xi)_{\rm a}$$

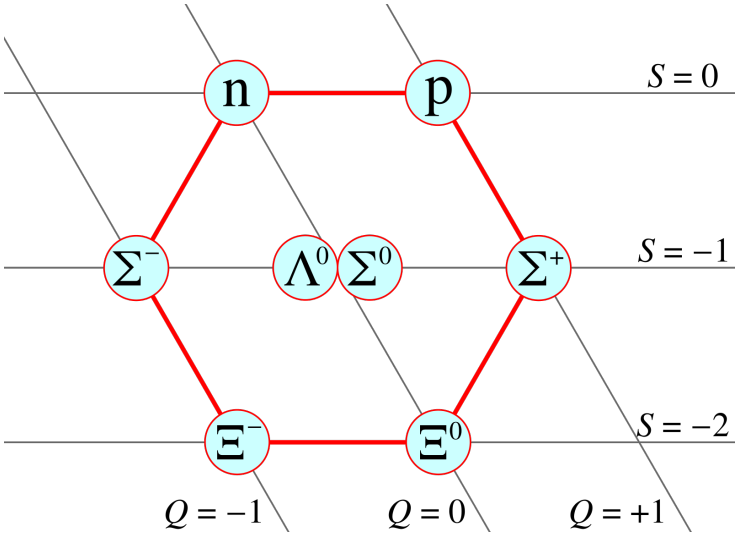
Axial charges of the baryon octet: $m_{u,d} = m_s$

For neutron β-decay, axial charge:

$$g_A = a_3 = \Delta u - \Delta d = \langle n|(\overline{u}\gamma_0\gamma_5 d)|p\rangle = \langle p|(\overline{u}\gamma_0\gamma_5 u - \overline{d}\gamma_0\gamma_5 d)|p\rangle = \langle n|(\overline{d}\gamma_0\gamma_5 d - \overline{u}\gamma_0\gamma_5 u)|n\rangle$$

Define axial charges for other members of the octet.

Characterize weak decays: $\Sigma \to \Sigma$, $\Xi \to \Xi$, $\Xi \to \Sigma$, $\Xi \to \Lambda$, $\Lambda \to N$ ....



Isospin $I = 1, \frac{1}{2}$ and hypercharge $Y = S + 1$.

$m_{u/d} = m_s$:

$$\begin{aligned} g_A^B &= (\Delta u - \Delta d)^B \\ &= \langle B|(\overline{u}\gamma_0\gamma_5 u - \overline{d}\gamma_0\gamma_5 d)|B\rangle \\ &= 2I_3(\overline{F} + Y\overline{D}) \end{aligned}$$

$I = 0$

$$\begin{aligned} g_A^\Lambda &= \langle \Lambda|(\overline{u}\gamma_0\gamma_5 d)|\Sigma\rangle \\ &= 2\overline{D} \end{aligned}$$

$$g_A^p = \overline{F} + \overline{D},\ g_A^{\Xi^0} = \overline{F} - \overline{D},\ g_A^{\Sigma^+} = 2\overline{F} \Rightarrow (g_A^p + g_A^{\Xi^0})/g_A^{\Sigma^+} = 1.$$

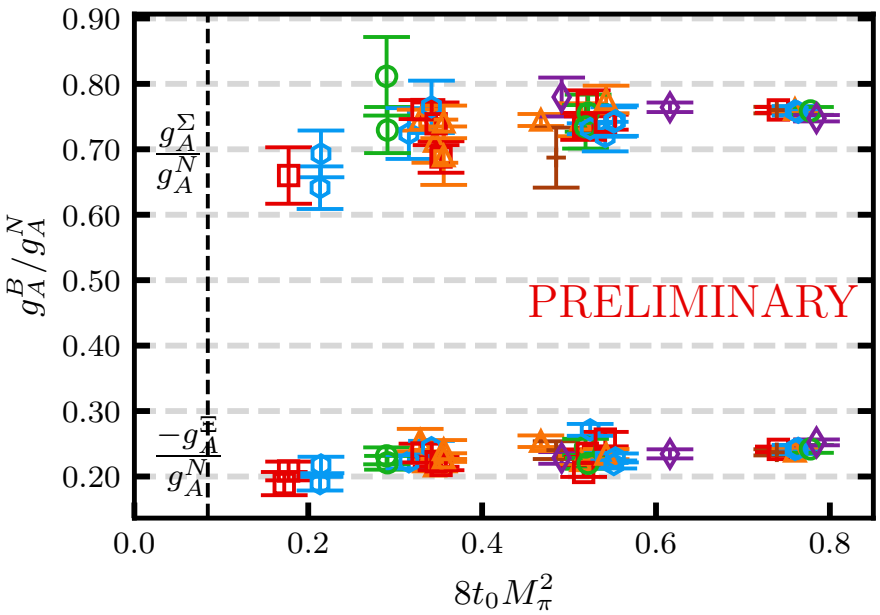
Hyperons: axial charge

Replacing $\overline{F}$ and $\overline{D}$ by their values in the SU(3) chiral limit $F = 0.447(7)$ and $D = 0.730(11)$ [RQCD,2201.05591] gives

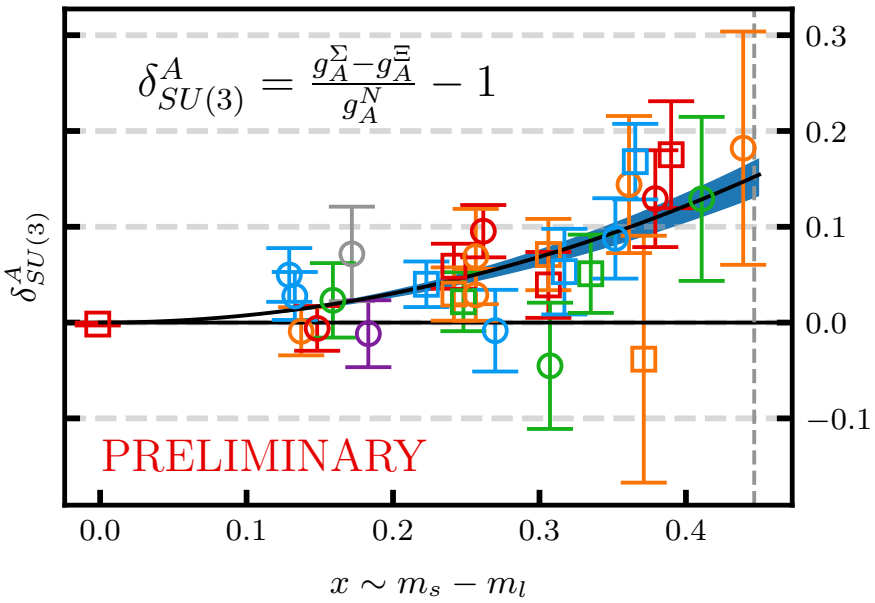
$$\frac{g_A^{\Sigma}}{g_A^N} \approx 0.76, \quad -\frac{g_A^{\Xi}}{g_A^N} \approx 0.24.$$

Away from this limit: corrections start at $O(\delta m_l)$, $\delta m_l = m_s - m_{u/d} \propto M_K^2 - M_\pi^2$.

g_A^{Σ}/g_N and $-g_A^{\Xi}/g_N$



$\delta^A_{SU(3)} = (g_A^p + g_A^{\Xi^0})/g_A^{\Sigma^+} - 1$



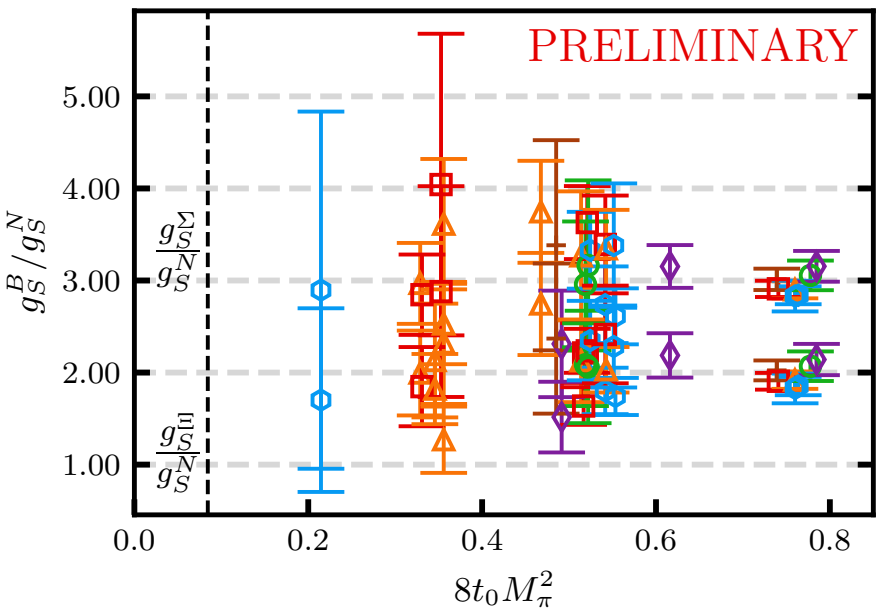
Hyperons: scalar charge

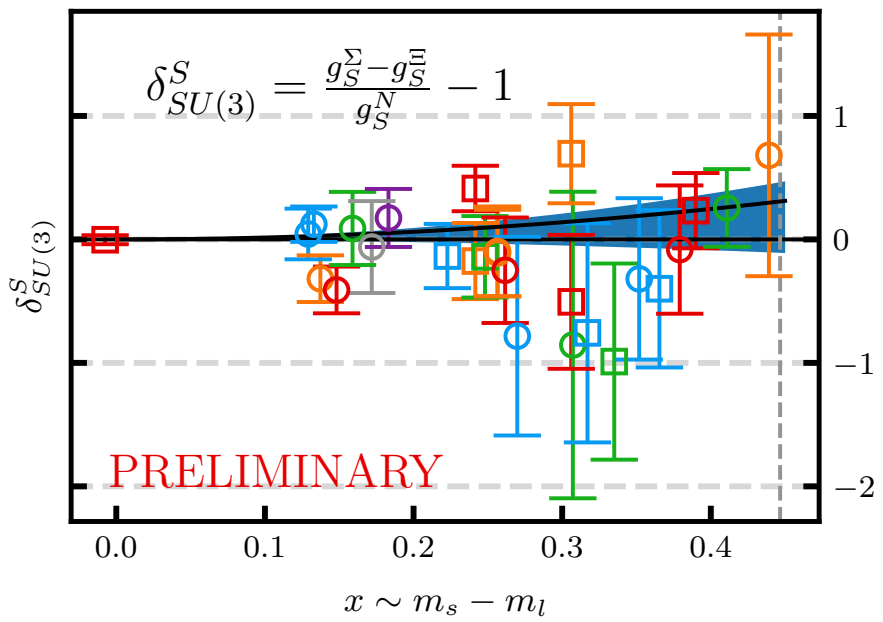
Conserved vector current relation: $\partial_\mu(\bar{u}\gamma_\mu d) = -i(m_u - m_d)\bar{u}\mathbb{1}d$ applied to $\langle p(p_\mathrm{f})|\bar{u}\gamma_\mu d|n(p_\mathrm{i})\rangle$ etc. [Gonzalez-Alonso,1309.4434].

$$g_S^N = \frac{M_p - M_n}{m_d - m_u} = \frac{\delta M_N^{QCD}}{\delta m_q} \quad g_S^\Sigma = \frac{M_{\Sigma^-} - M_{\Sigma^+}}{m_d - m_u} = \frac{\delta M_\Sigma^{QCD}}{\delta m_q} \quad g_S^\Xi = \frac{M_{\Xi^-} - M_{\Xi^0}}{m_d - m_u} = \frac{\delta M_\Xi^{QCD}}{\delta m_q}$$

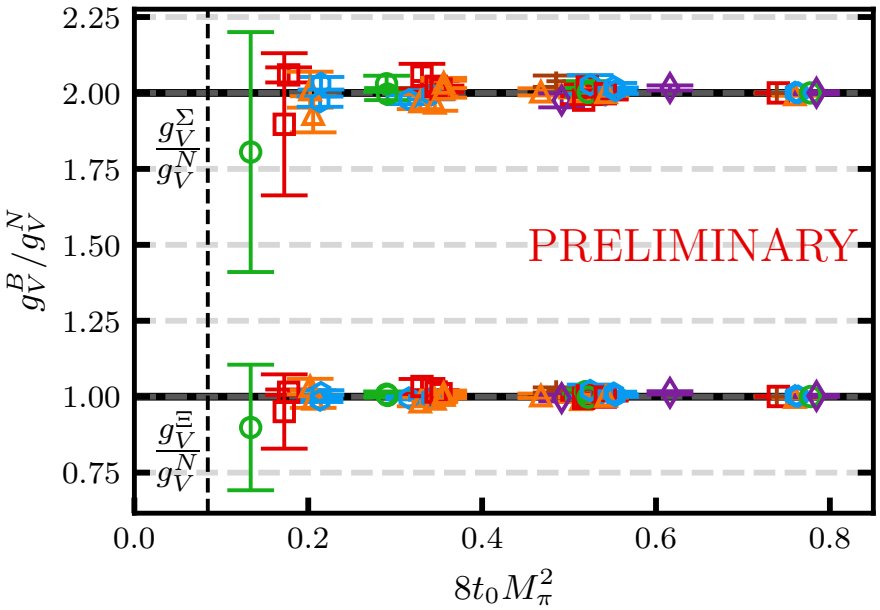
From [BMWc,1406.4088]: $\delta M_N^{QCD} = 2.52(29)$, $\delta M_\Sigma^{QCD} = 8.09(19)$, $\delta M_\Xi^{QCD} = 5.53(24)$.

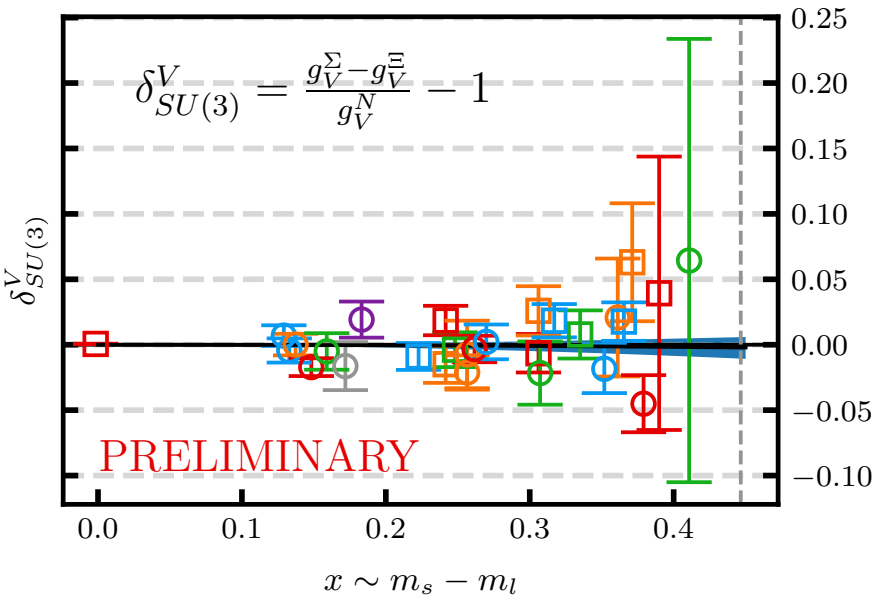
i.e. $g_S^\Sigma/g_S^N \sim 3.2(4)$ and $g_S^\Xi/g_S^N \sim 2.2(3)$ at the physical point.





Hyperons: vector charge





Summary and outlook

On-going project to determine the flavour structure of the baryon octet.

- ⋆ Performed continuum, finite volume, quark mass simultaneous fits to the baryon octet masses.
- ⋆ Covariant BChPT provided a reasonable description of the data for our range of M_π and M_K ($\overline{M}^2 < 440^2$ [MeV^2]).
- ⋆ Corrected expt. octet baryon spectrum reproduced to within 1% uncertainties.
- ⋆ Extracted the sigma terms $\sigma_{\pi B}$ and σ_{sB}. Results are consistent with a direct determination.
- ⋆ Flavour symmetry breaking in the hyperon charges investigated. Deviations for the axial charge are around 15% at the physical point. Scalar charge in agreement with CVC expectations.

In the future:

- ⋆ Test ChPT further through determination of LECs via simultaneous fits to related observables, e.g. $M_{\pi,K}$, $m_{\ell,s}$ and $F_{\pi,K}$ and m_B and g_A^B.