Emergence of mass in the gauge sector of QCD

Joannis Papavassiliou

Department of Theoretical Physics and IFIC
University of Valencia-CSIC

J. P., Chinese Physics C, Vol. 46, No. 11 (2022) 112001

REVEALING EMERGENT MASS THROUGH STUDIES OF HADRON SPECTRA AND STRUCTURE

Emergence in QCD

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{1}{2\xi}(\partial^\mu A^a_\mu)^2 - \bar{c}^a\partial^\mu D^{ab}_\mu c^b + \mathcal{L}_{\rm quarks}$$

- All fundamental fields are massless at the level of the Lagrangian
- Properly regularized perturbation theory cannot generate mass at any finite order

And yet, there is a proliferation of masses in the wolrd

Emergent Hadron Mass (EHM)

C.D.Roberts, Symmetry 12, no.9, 1468 (2020)

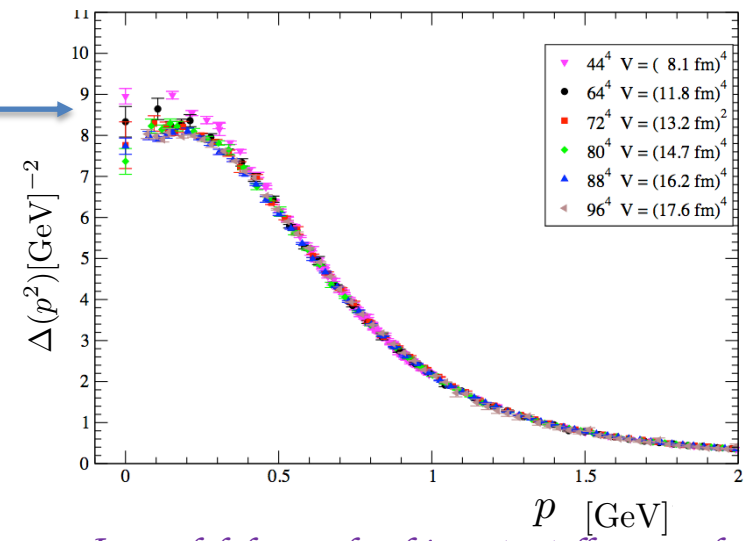
Dynamical generation of a mass scale in pure Yang-Mills

Emergence of mass in the gauge sector of QCD

- Gluon self-interactions generate a dynamical mass J. M. Cornwall, Phys. Rev. D26, 1453 (1982)

- Lattice QCD: The gluon propagator saturates in the deep infrared (circa 2007)```

- Unequivocal signal of gluon mass generation



I. Bogolubsky, E. Ilgenfritz, M. Muller-Preussker, and A. Sternbeck, Phys. Lett. B676, 69 (2009).

What is the underlying mechanism ?

The mass generation proceeds through the action of the Schwinger mechanism

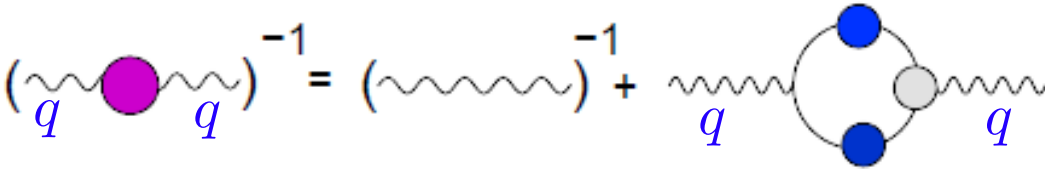
Schwinger mechanism

J. S. Schwinger, Phys. Rev.125, 397 (1962); Phys.Rev.128, 2425 (1962)

Gauge Invariance and Mass

A gauge boson may acquire a mass, even if the gauge symmetry forbids a mass term at the level of the fundamental Lagrangian, provided that its vacuum polarization function develops a pole at zero momentum transfer.

Schwinger-Dyson equation for gauge boson propagator



$$\Delta^{-1}(q^2) = q^2[1 + \Pi(q^2)]$$

If, for some reason

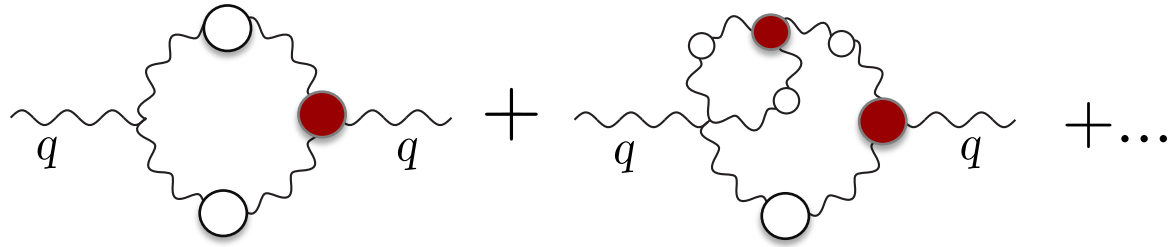
$$\lim_{q^2 \to 0} \Pi(q^2) = \frac{c}{q^2}\,, \qquad c > 0$$

$$\Delta^{-1}(0) = c > 0$$

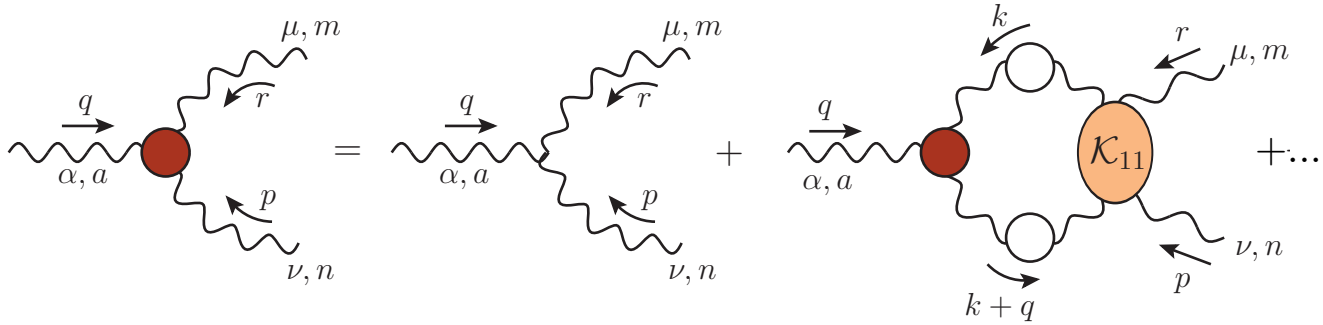
Schwinger mechanism in QCD

Consider the coupled system of Schwinger-Dyson equations:

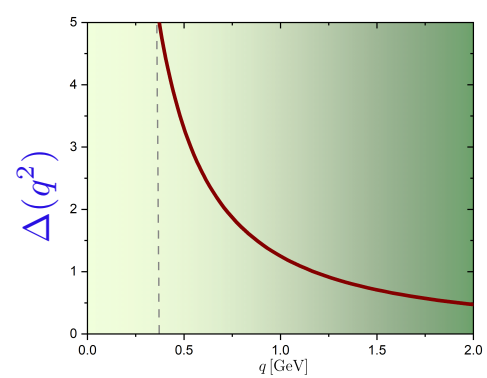
$$\Delta^{-1}(q^2) = q^2 +$$



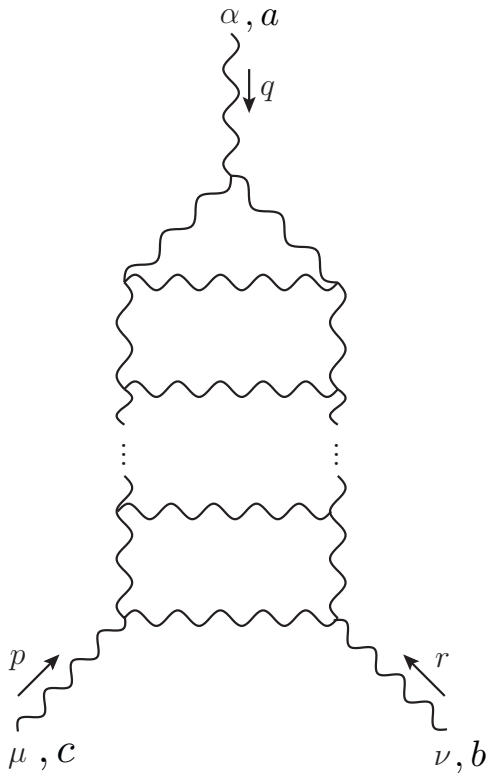
$$\mathbb{\Gamma}_{\alpha\mu\nu}(q,r,p) =$$



In the absence of masses, the system has no infrared stable solutions:

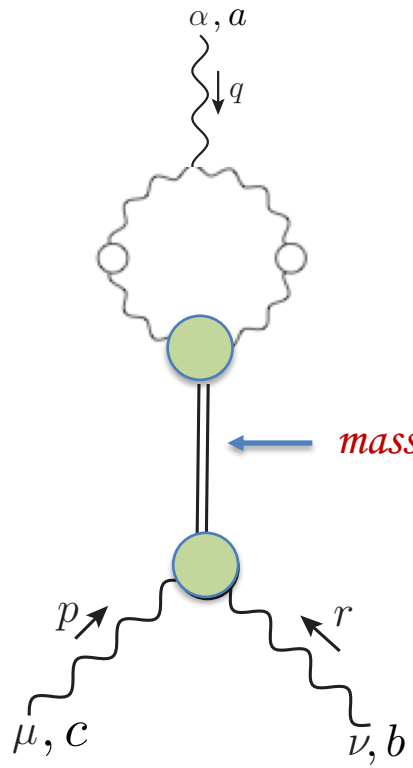


But, if the coupling gets sufficiently strong → *formation of bound states*



strong coupling

$q \to 0$



massless colored excitation $\dfrac{\delta^{ab}}{q^2}$

$$V^{abc}_{\alpha\mu\nu}(q,r,p) = f^{abc}\left(\frac{q_\alpha}{q^2}\right)\left[g_{\mu\nu}\underbrace{C_1(q,r,p)}_{\textit{residue}} + \cdots\right]$$

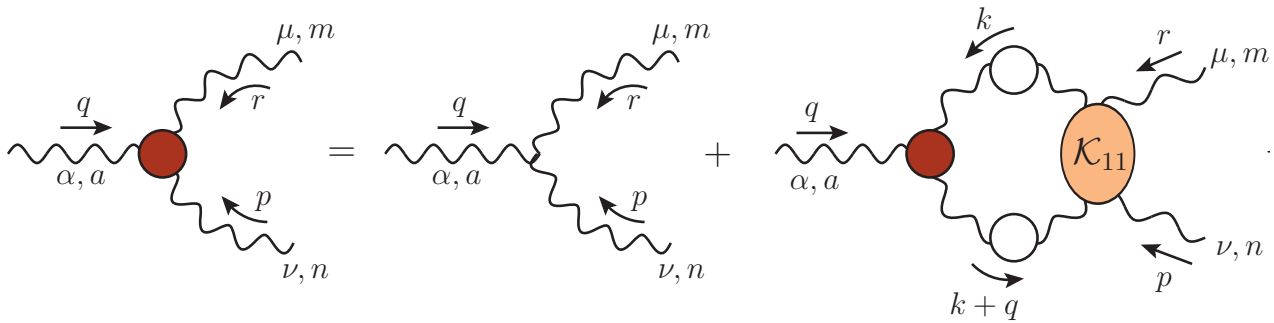
longitudinally coupled

E.Eichten and F.Feinberg, Phys. Rev. D 10, 3254-3279 (1974)
A.C.Aguilar, D. Ibanez, V. Mathieu, and J. P., Phys. Rev. D 85, 014018 (2012)
A.C.Aguilar, D.Binosi and J.P., Phys. Rev. D 95, no.3, 034017 (2017)

Bethe-Salpeter equation

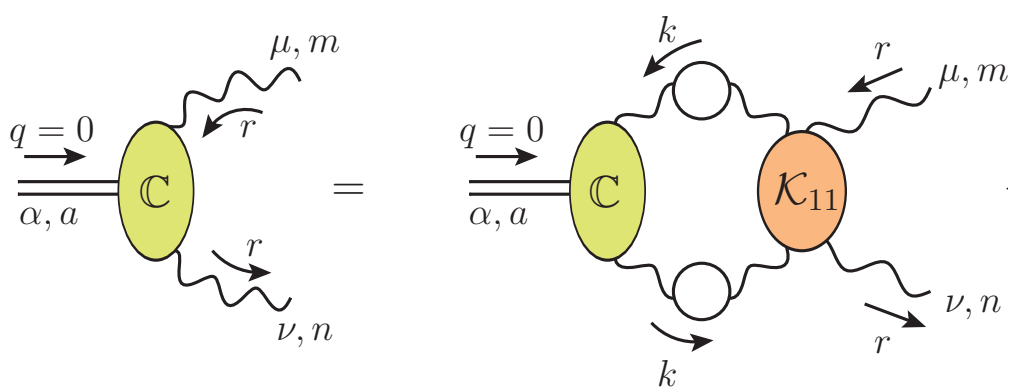
Bose symmetry $\Longrightarrow$ $C_1(0, r, -r) = 0$

$$\lim_{q\to 0} C_1(q, r, p) = 2(q\cdot r)\underbrace{\left[\frac{\partial C_1(q, r, p)}{\partial p^2}\right]_{q=0}}_{\mathbb{C}(r^2)} + \mathcal{O}(q^2)$$



$q \to 0$

BSE

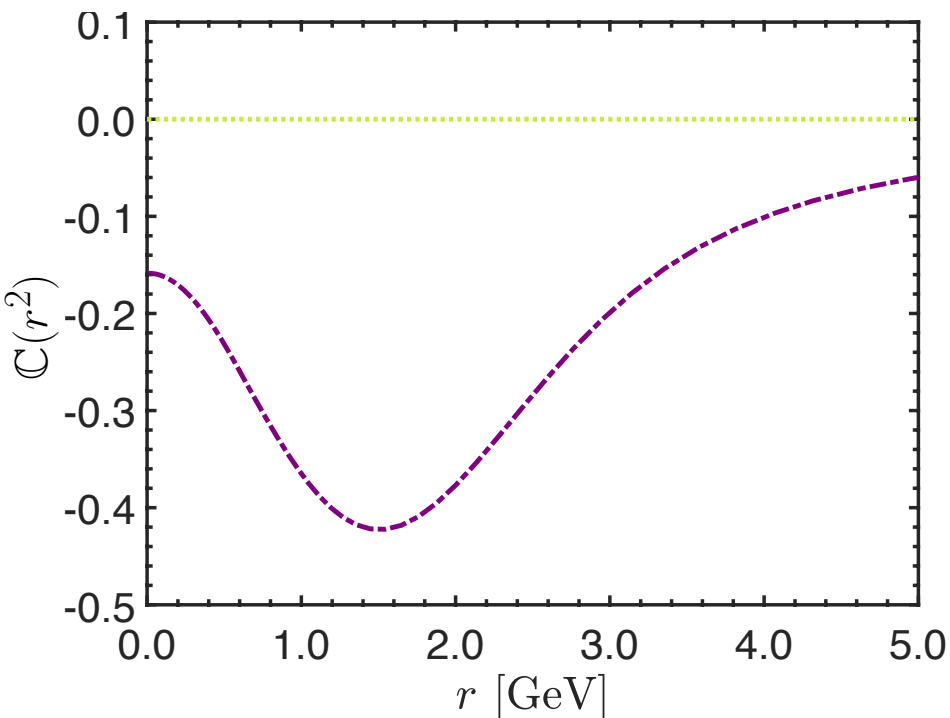


Solving the BSE

$$\mathbb{C}(r^2) = \alpha_s \int_k \mathbb{C}(k^2)\Delta^2(k^2)\mathcal{K}_{11}(k,r)$$

Eigenvalue problem

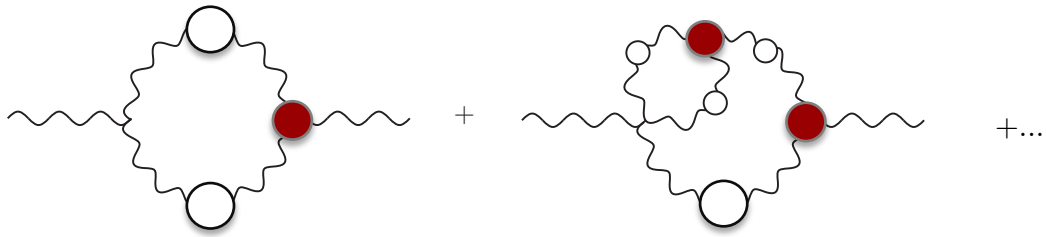
$\alpha_s \approx 0.3$ @ $\mu = 4.3\,\mathrm{GeV}$ *Non-trivial solution*



BS amplitude

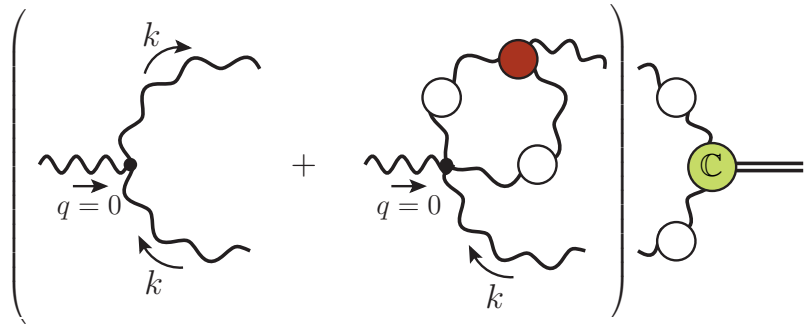
Schwinger mechanism in QCD : massless poles originate from the three-gluon vertex

$$\Delta^{-1}(q^2) = q^2 +$$



$$q \to 0$$

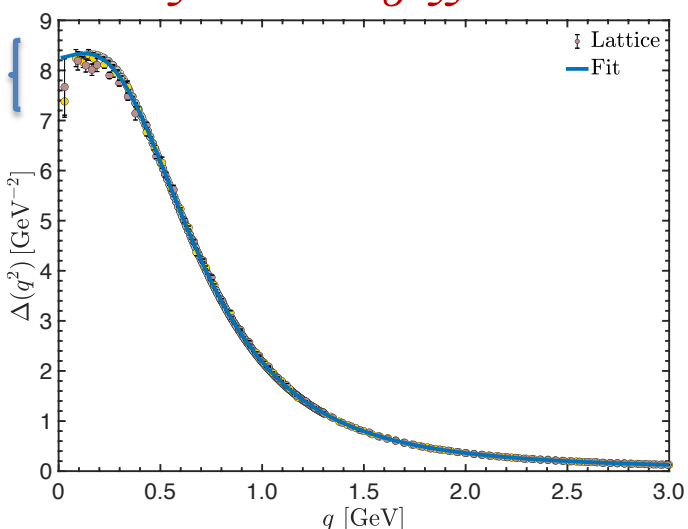
$$m^2 =$$



Determination of the saturation point of the gluon propagator

Gluon propagator acquires a mass
Self-stabilizing effect

$\frac{1}{m^2}$



- *Value of the mass depends on the subtraction point* μ

$$\mu = 4.3\,\text{GeV} \longrightarrow m = 350 \pm 25\,\text{MeV}$$

- *QCD generalization of the process-independent Gell-Mann-Low effective charge known from QED*

$$\Longrightarrow m_{\text{RGI}} \approx 430\,\text{MeV}$$

D.Binosi et al, Phys. Rev. D 96, no.5, 054026 (2017)

The Schwinger mechanism has a clear dynamical origin,
and is compatible with all field theoretic principles and requirements,
such as BRST symmetry and renormalizability

QUESTION

Is there some smoking-gun signal associated with its onset
(other than the infrared finiteness of the gluon propagator) ?

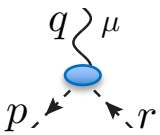
ANSWER: YES

The displacement of the Ward identities satisfied by the vertices,
in conjunction with lattice simulations, confirms the Schwinger mechanism

A.C.Aguilar, M.N.Ferreira and J.P, Phys. Rev. D 105, no.1, 014030 (2022)

Displacement of the Ward identity

Schwinger mechanism off



Takahashi identity

$$q^\mu \underbrace{\Gamma_\mu(q,r,p)}_{\text{pole-free}} = D^{-1}(p^2) - D^{-1}(r^2)$$

$$\xrightarrow[p \to -r]{q \to 0} \quad \text{Taylor expansion}$$

Ward identity

$$\Gamma_\mu(0,r,-r) = \frac{\partial D^{-1}(r^2)}{\partial r^\mu}$$

Tensorial decomposition

$$\Gamma_\mu(0,r,-r) = L(r^2) r_\mu$$

$$\Downarrow$$

$$L(r^2) = 2\frac{\partial D^{-1}(r^2)}{\partial r^2}$$

Schwinger mechanism on

$$\mathbb{\Gamma}_\mu(q,r,p) = \underbrace{\Gamma_\mu(q,r,p)}_{\text{pole-free}} + \frac{q_\mu}{q^2} C(q,r,p)$$

The Takahashi identity does not change

$$q^\mu \mathbb{\Gamma}_\mu(q,r,p) = q^\mu \Gamma_\mu(q,r,p) + C(q,r,p)$$
$$= D^{-1}(p^2) - D^{-1}(r^2)$$

$$\xrightarrow{q \to 0} \quad \text{Taylor expansion}$$

Ward identity

$$\Gamma_\mu(0,r,-r) = \frac{\partial D^{-1}(r^2)}{\partial r^\mu} - 2 r_\mu \underbrace{\left[\frac{\partial C(q,r,p)}{\partial p^2}\right]_{q=0}}_{\mathbb{C}(r^2)}$$

$$\Rightarrow \quad L(r^2) = 2\frac{\partial D^{-1}(r^2)}{\partial r^2} - \underbrace{2\,\mathbb{C}(r^2)}_{\text{displacement}}$$

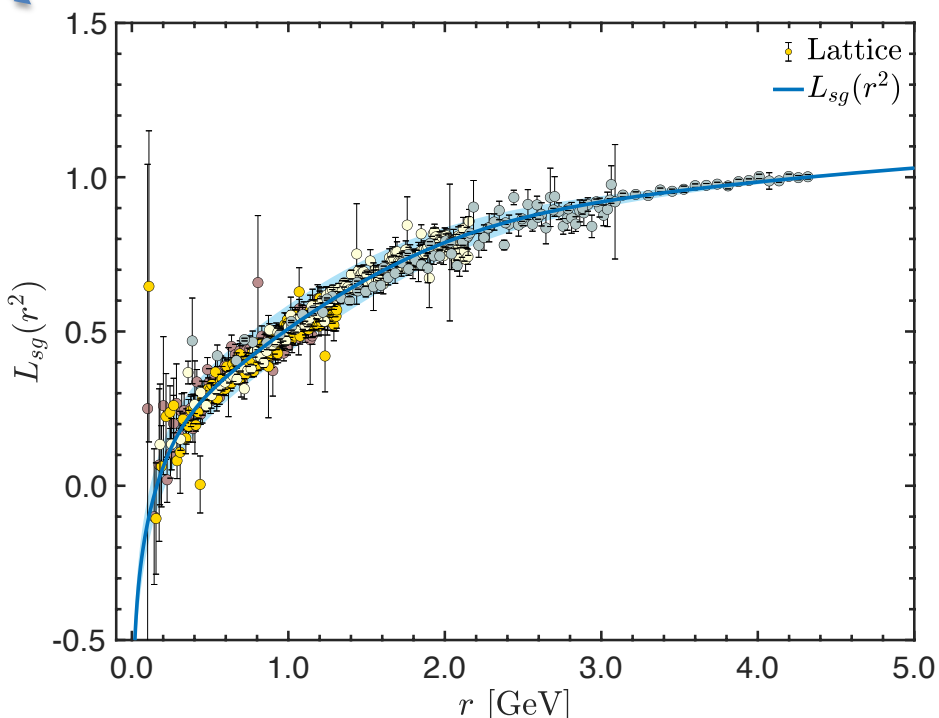
Displacement of the WI of the three-gluon vertex

$$\underbrace{\mathbb{C}(r^2)}_{\textit{displacement}} = L_{sg}(r^2) - F(0)\left\{\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right\}$$

Displacement of the WI of the three-gluon vertex

$$\underbrace{\mathbb{C}(r^2)}_{\text{displacement}} = L_{sg}(r^2) - F(0)\left\{\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right\}$$

Form factor of the three-gluon vertex

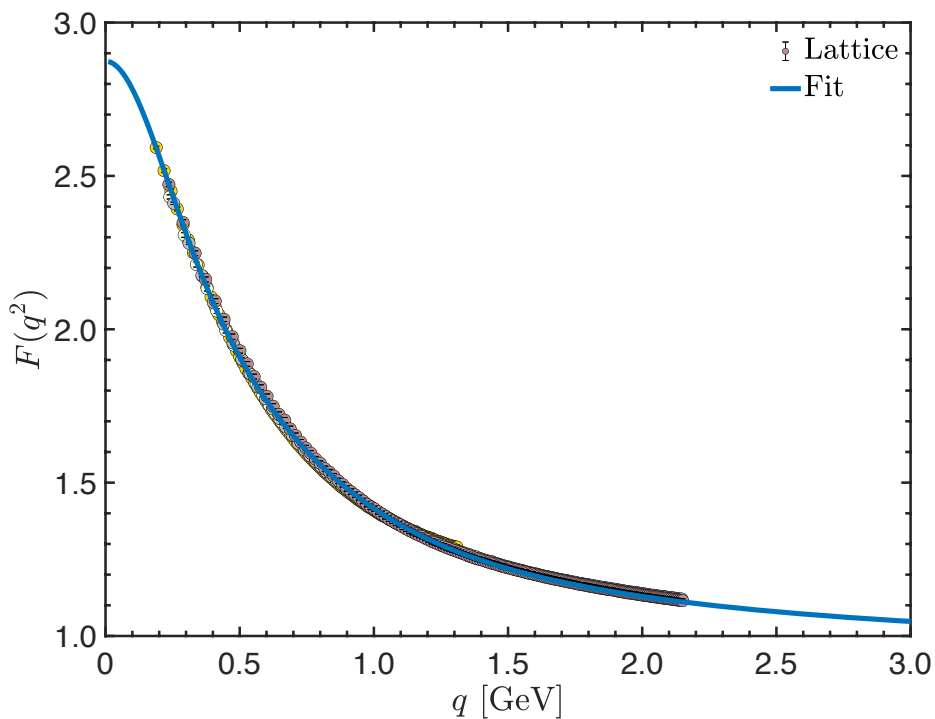


A.C. Aguilar, F. De Soto, M.N.Ferreira, J.P. , J.Rodriguez-Quintero, Phys. Lett. B818 (2021) 136352

Displacement of the WI of the three-gluon vertex

$$\underbrace{\mathbb{C}(r^2)}_{\text{displacement}} = L_{sg}(r^2) - F(0)\left\{\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right\}$$

Ghost dressing function



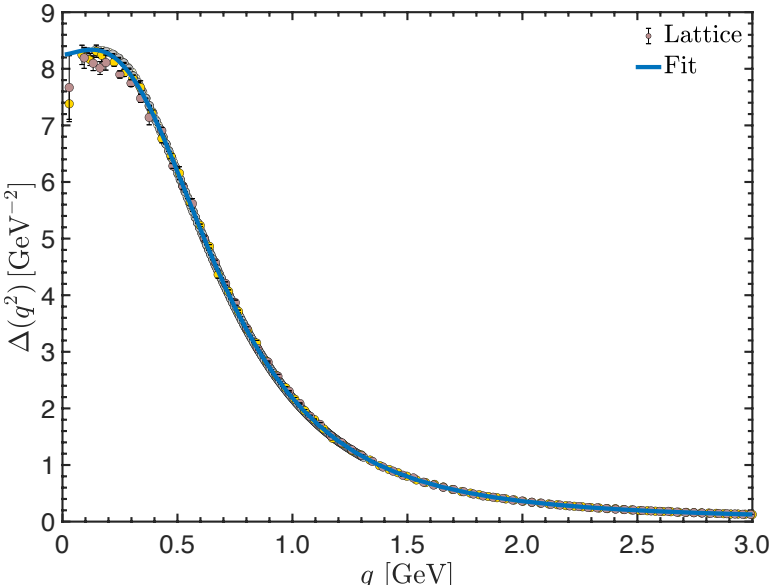
A.C. Aguilar, C.O. Ambrosio, F. De Soto, M.N. Ferreira, B.M. Oliveira, J.P and J. Rodriguez-Quintero

Phys. Rev. D 104 no.5, 054028, (2021)

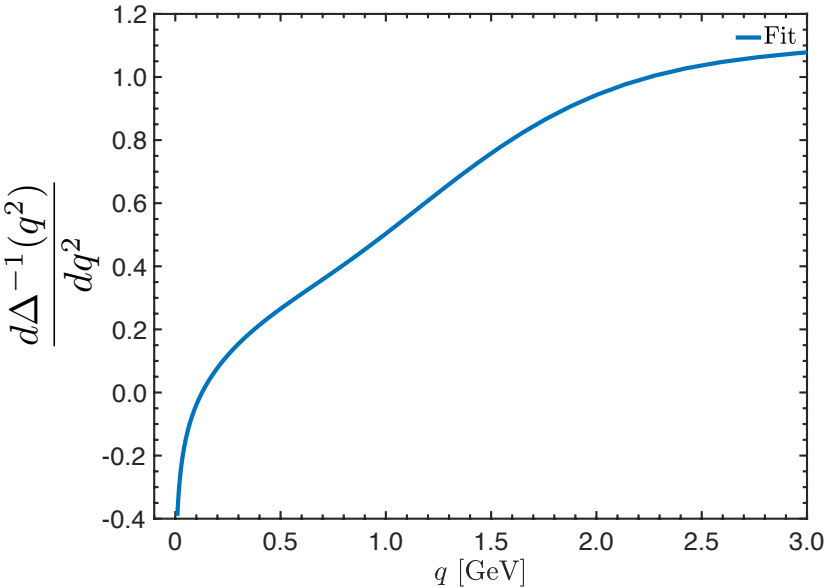
Displacement of the WI of the three-gluon vertex

$$\underbrace{\mathbb{C}(r^2)}_{\textit{displacement}} = L_{sg}(r^2) - F(0)\left\{\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right\}$$

gluon propagator



derivative of inverse



Displacement of the WI of the three-gluon vertex

$$\underbrace{\mathbb{C}(r^2)}_{\text{displacement}} = L_{sg}(r^2) - F(0)\left\{\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right\}$$

Nearly model independent determination of the displacement function

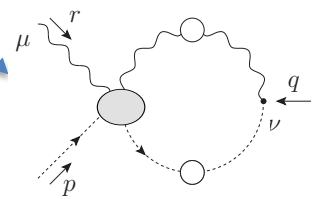
The lattice is "blind" to specific dynamical mechanisms

Displacement of the WI of the three-gluon vertex

$$\underbrace{\mathbb{C}(r^2)}_{\textit{displacement}} = L_{sg}(r^2) - F(0)\left\{\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right\}$$

Displacement of the WI of the three-gluon vertex

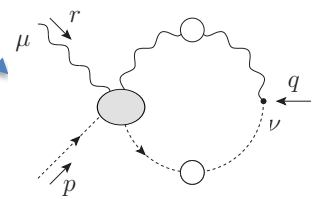
$$\underbrace{\mathbb{C}(r^2)}_{\text{displacement}} = L_{sg}(r^2) - F(0)\left\{\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right\}$$



partial derivative of the ghost-gluon kernel

Displacement of the WI of the three-gluon vertex

$$\underbrace{\mathbb{C}(r^2)}_{\text{displacement}} = L_{sg}(r^2) - F(0)\left\{\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right\}$$

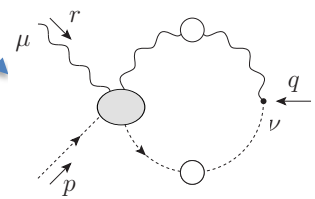


partial derivative of the ghost-gluon kernel

- *No lattice results for* $\mathcal{W}(r^2)$

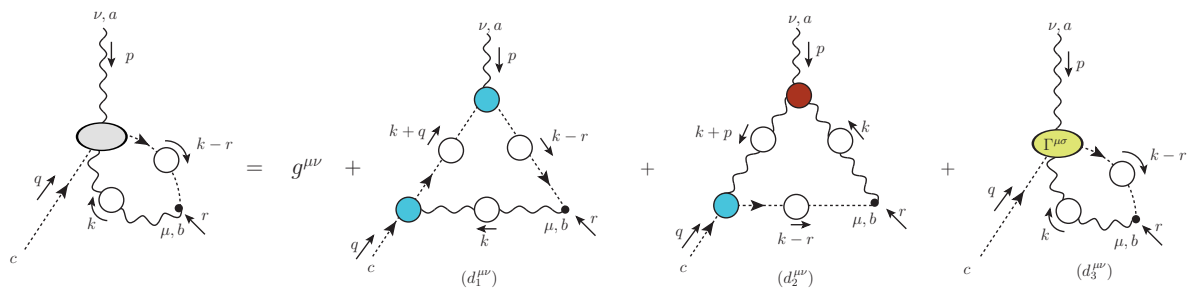
Displacement of the WI of the three-gluon vertex

$$\underbrace{\mathbb{C}(r^2)}_{\text{displacement}} = L_{sg}(r^2) - F(0)\left\{\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right\}$$



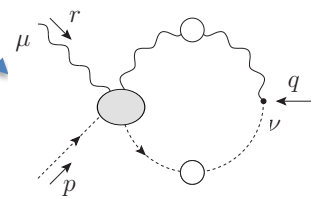
partial derivative of the ghost-gluon kernel

- *No lattice results for* $\mathcal{W}(r^2)$
- *Computed from its own Schwinger-Dyson eq.*



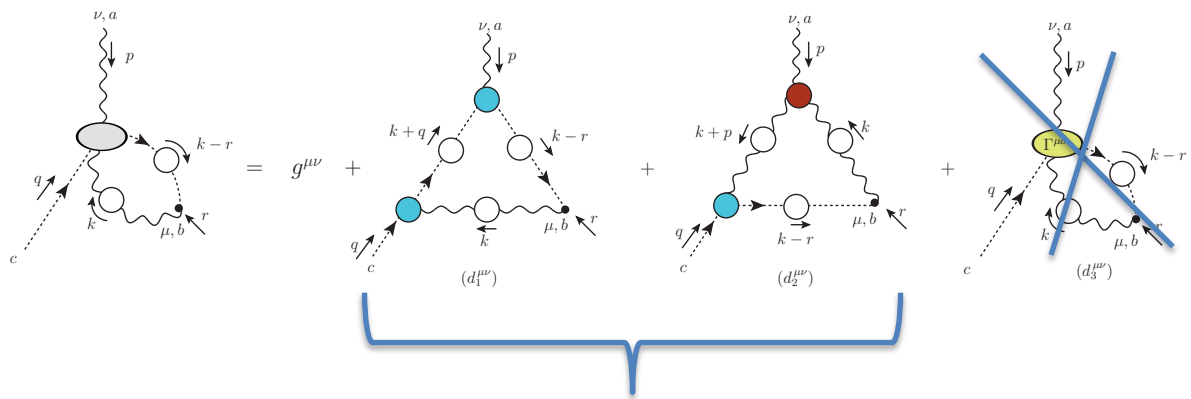
Displacement of the WI of the three-gluon vertex

$$\underbrace{\mathbb{C}(r^2)}_{\text{displacement}} = L_{sg}(r^2) - F(0)\left\{\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right\}$$



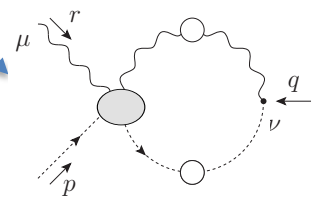
partial derivative of the ghost-gluon kernel

- *No lattice results for* $\mathcal{W}(r^2)$
- *Computed from its own Schwinger-Dyson eq.*



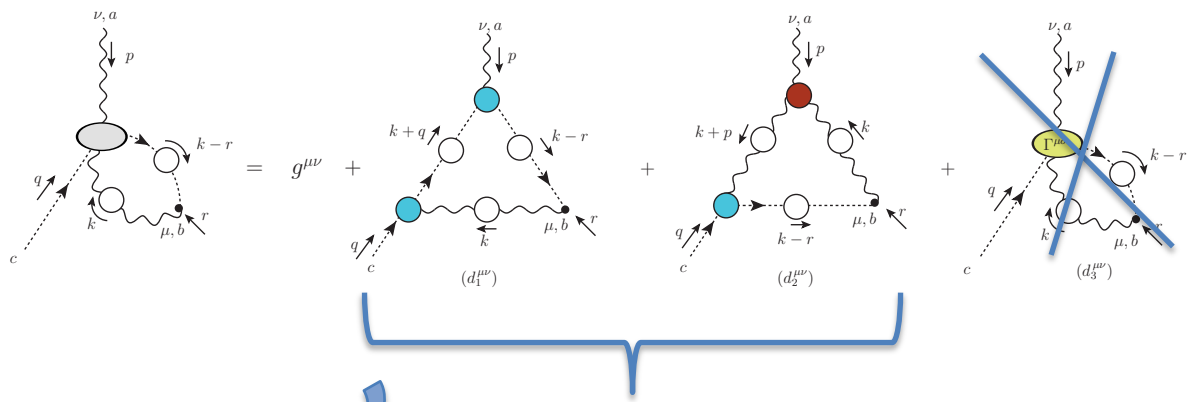
Displacement of the WI of the three-gluon vertex

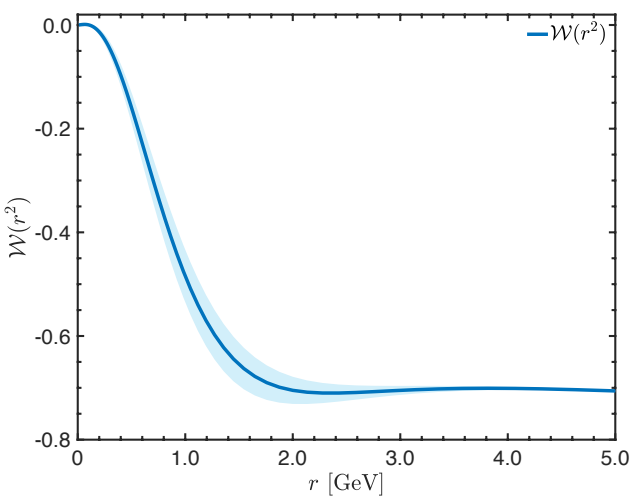
$$\underbrace{\mathbb{C}(r^2)}_{\text{displacement}} = L_{sg}(r^2) - F(0)\left\{\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right\}$$



partial derivative of the ghost-gluon kernel

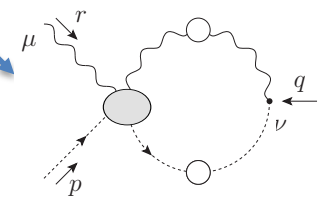
- *No lattice results for* $\mathcal{W}(r^2)$
- *Computed from its own Schwinger-Dyson eq.*





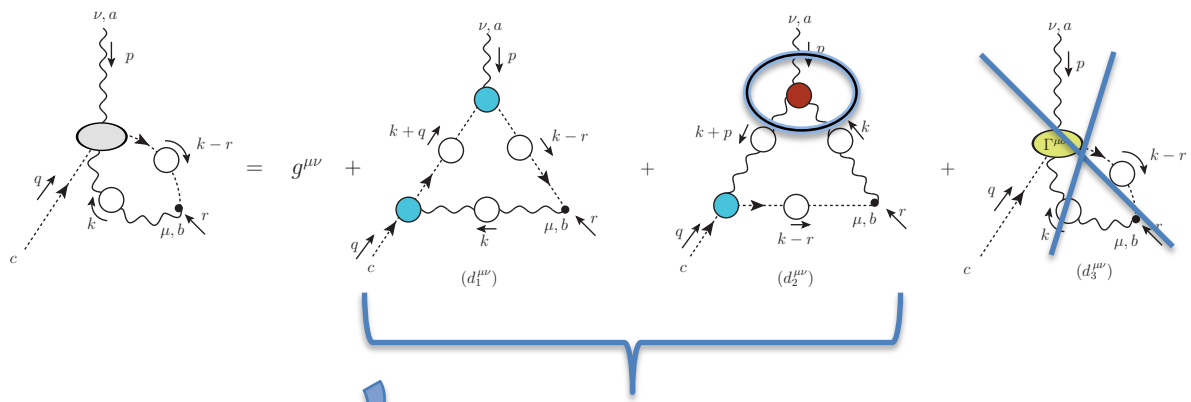
Displacement of the WI of the three-gluon vertex

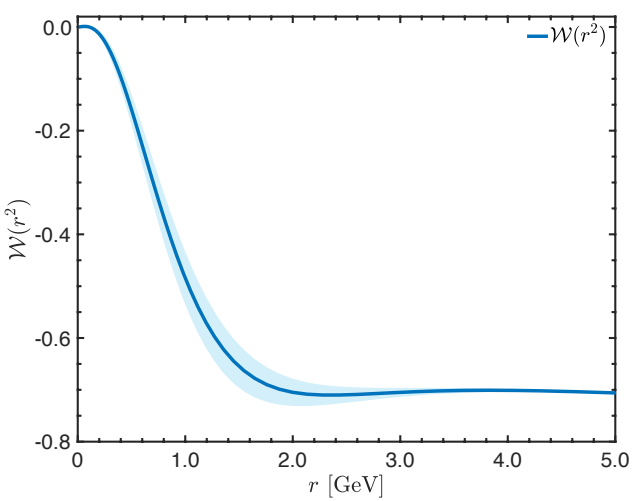
$$\underbrace{\mathbb{C}(r^2)}_{\text{displacement}} = L_{sg}(r^2) - F(0)\left\{\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right\}$$



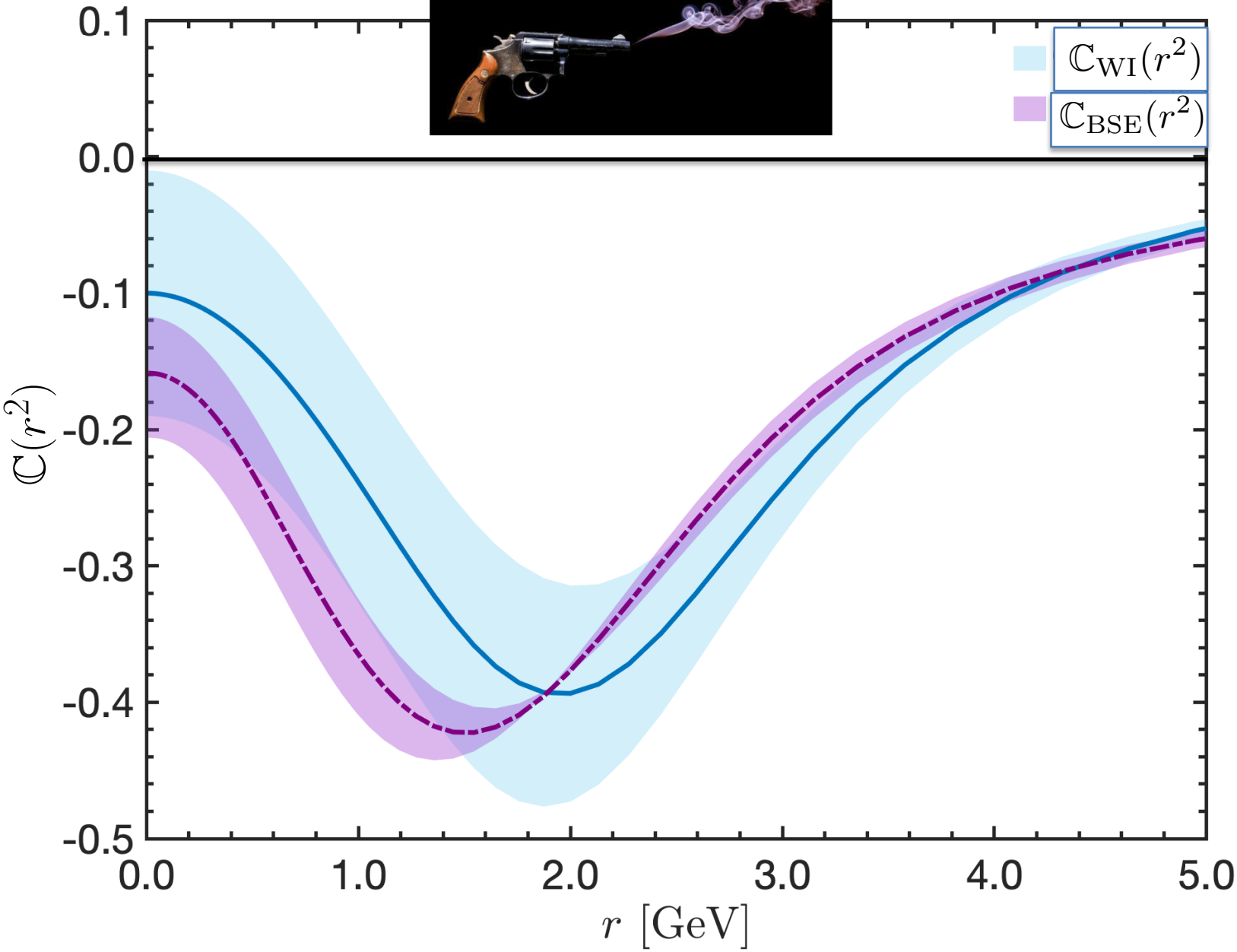
partial derivative of the ghost-gluon kernel

- *No lattice results for* $\mathcal{W}(r^2)$
- *Computed from its own Schwinger-Dyson eq.*





The result is dominated by a particular projection of the three-gluon vertex



A.C.Aguilar, M.N.Ferreira, and J.P., Phys. Rev. D 105, no.1, 014030 (2022)

Conclusions

- *The apparent simplicity of the QCD Lagrangian conceals an enormous wealth of dynamical patterns, giving rise to a vast array of complex emergent phenomena*

Conclusions

- *The apparent simplicity of the QCD Lagrangian conceals an enormous wealth of dynamical patterns, giving rise to a vast array of complex emergent phenomena*

- *The emergence of a hadronic mass (EHM) is the unifying concept underlying all these phenomena*

Conclusions

- The apparent simplicity of the QCD Lagrangian conceals an enormous wealth of dynamical patterns, giving rise to a vast array of complex emergent phenomena

- The emergence of a hadronic mass (EHM) is the unifying concept underlying all these phenomena

- Gluon self-interactions generate a dynamical mass scale in the gauge sector of QCD

Conclusions

- The apparent simplicity of the QCD Lagrangian conceals an enormous wealth of dynamical patterns, giving rise to a vast array of complex emergent phenomena

- The emergence of a hadronic mass (EHM) is the unifying concept underlying all these phenomena

- Gluon self-interactions generate a dynamical mass scale in the gauge sector of QCD

  - Dynamics and symmetry are tightly intertwined:

Displacement of the Ward identity = BSE amplitude for pole formation

Conclusions

- The apparent simplicity of the QCD Lagrangian conceals an enormous wealth of dynamical patterns, giving rise to a vast array of complex emergent phenomena

- The emergence of a hadronic mass (EHM) is the unifying concept underlying all these phenomena

- Gluon self-interactions generate a dynamical mass scale in the gauge sector of QCD

  - Dynamics and symmetry are tightly intertwined:

Displacement of the Ward identity = BSE amplitude for pole formation

- Smoking gun signal corroborates the action of the Schwinger mechanism in QCD

Conclusions

- The apparent simplicity of the QCD Lagrangian conceals an enormous wealth of dynamical patterns, giving rise to a vast array of complex emergent phenomena

- The emergence of a hadronic mass (EHM) is the unifying concept underlying all these phenomena

- Gluon self-interactions generate a dynamical mass scale in the gauge sector of QCD

  - Dynamics and symmetry are tightly intertwined:

Displacement of the Ward identity = BSE amplitude for pole formation

- Smoking gun signal corroborates the action of the Schwinger mechanism in QCD

  - A major success for the Schwinger-Dyson community

Absence of poles from lattice "observables"

$$\mathcal{A}(q,r,p) = \frac{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\,\mathbb{\Gamma}^{\alpha\mu\nu}(q,r,p)}{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\,\Gamma_0^{\alpha\mu\nu}(q,r,p)}$$

with $P_{\mu\nu}(q) := g_{\mu\nu} - q_\mu q_\nu / q^2$

$$\mathbb{\Gamma}^{\alpha\mu\nu}(q,r,p) = \underbrace{\Gamma^{\alpha\mu\nu}(q,r,p)}_{\textit{pole-free}} + \underbrace{V^{\alpha\mu\nu}(q,r,p)}_{\textit{poles}}$$

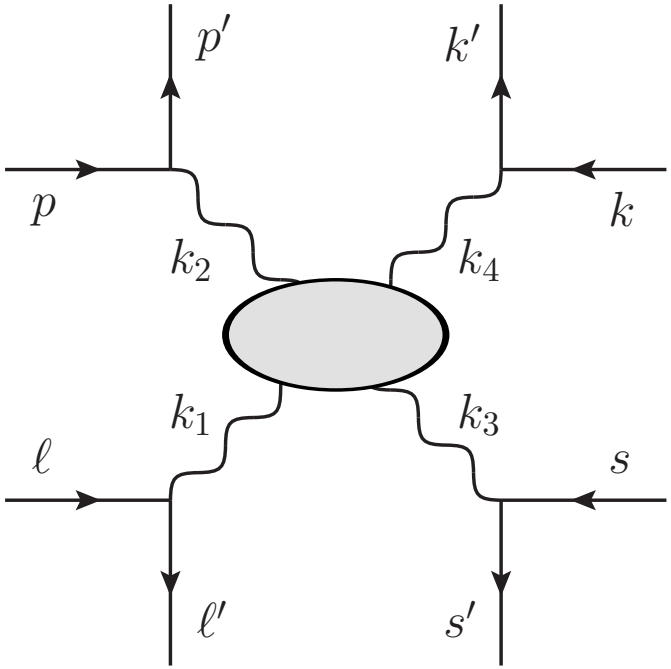
Given that the poles are longitudinally coupled:

$$P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\,V^{\alpha\mu\nu}(q,r,p) = 0$$

$$\mathcal{A}(q,r,p) = \frac{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\,\Gamma^{\alpha\mu\nu}(q,r,p)}{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\,\Gamma_0^{\alpha\mu\nu}(q,r,p)}$$

⇒ *the lattice extracts the pole-free part of the vertex*

Absence of pole divergences in the S-matrix

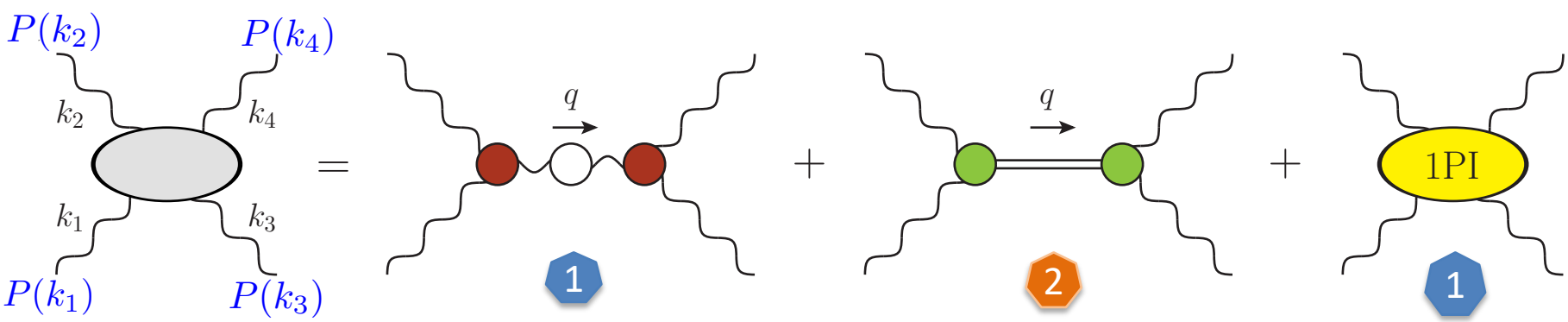


Two key properties:

1. $$P^{\alpha}_{\alpha'}(q)P^{\mu}_{\mu'}(r)P^{\nu}_{\nu'}(p)V_{\alpha\mu\nu}(q,r,p) = 0$$

2. $$\lim_{q\to 0} C_1(q,r,p) = 2(q\cdot r)\mathbb{C}(r^2)$$

The poles either get killed or are evitable



- Effective low-energy field theory describing the gluon mass:
  massive gauge-invariant Yang-Mills

$$\mathcal{L}_{MYM} = \frac{1}{2} G_{\mu\nu}^2 - m^2 \mathrm{Tr}\left[A_\mu - g^{-1} U(\theta)\partial_\mu U^{-1}(\theta)\right]^2$$

$U(\theta) = \exp\left[i\frac{1}{2}\lambda_a\theta^a\right]$, θ_a: scalar (Goldstone-like) fields

- Locally gauge-invariant under combined

$$A'_\mu = VA_\mu V^{-1} - g^{-1}\left[\partial_\mu V\right] V^{-1}, \qquad U' = U(\theta') = VU(\theta)$$

- Gauged non-linear sigma model:
  $\Rightarrow$ non-renormalizable (because $m = \text{const}$).
- But, from the SD analysis, $m = m(q^2)$, vanishes in the UV
  $\Rightarrow$ renormalizability restored .

$$\text{EOM}: \quad i\theta = -\frac{1}{\Box}\partial\cdot A - \frac{1}{2}\left[\frac{1}{\Box}\partial\cdot A, \partial\cdot A\right] + \frac{1}{\Box}\left[A_\mu, \partial^\mu \frac{1}{\Box}\partial\cdot A\right] + \mathcal{O}(A^3)$$