Non-perturbativeness in Nuclear EFT: A Potential Compromise?

Manuel Pavon Valderrama

Beihang University

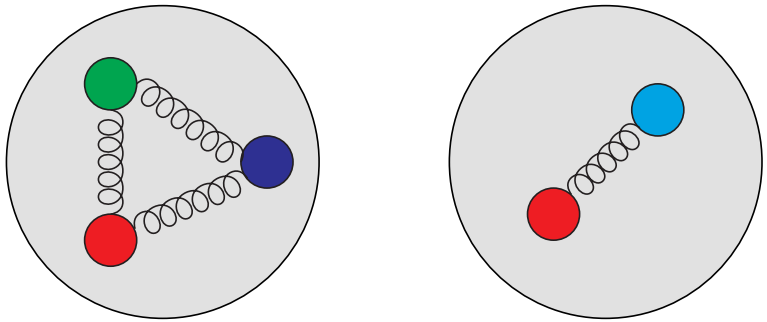
New Ideas in Constraining Nuclear Forces, June 2018, Trento

Contents

- What is an effective field theory?
- Power counting: potentials vs amplitudes
  - Nuclear physics is non-perturbative
  - EFT cannot be completely non-perturbative
  - But this is not a problem, because:
    Nuclear physics is not necessarily completely non-perturbative
- A potential compromise
  - To perturbe or not to perturbe?
  - The perturbing price of not perturbing:
    Power counting extravaganza
  - A pretty good deal for potential-based EFT:
    Trading (your non-existent) RG invariance for power counting

What is an effective field theory?

- Hadrons are particles composed of quarks and gluons.



- What is the problem with this?
  - We know pretty well the dynamics of quarks and gluons: **Quantum Chromodynamics**
  - But explaining hadrons in terms of quarks and gluons is **not exactly trivial**

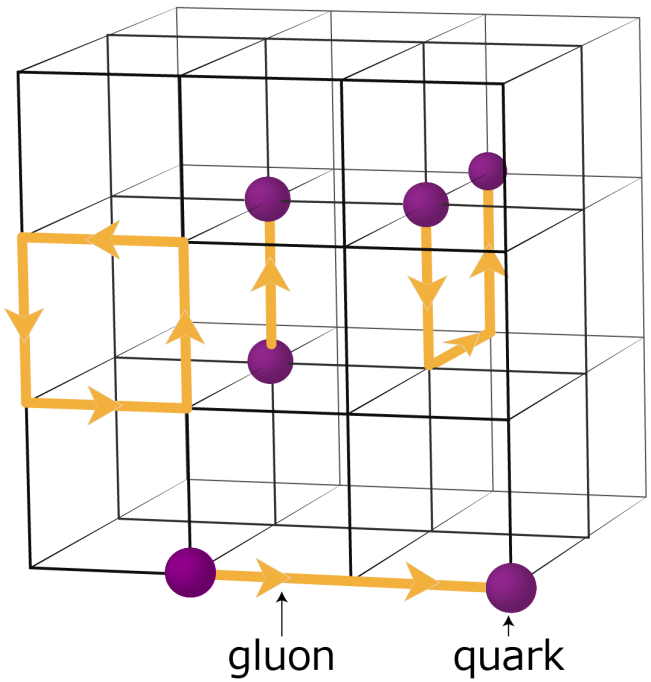
Why?: **Asymptotic Freedom**

What is an effective field theory

QCD description of hadrons: how? Two strategies come to mind:

Lattice QCD:

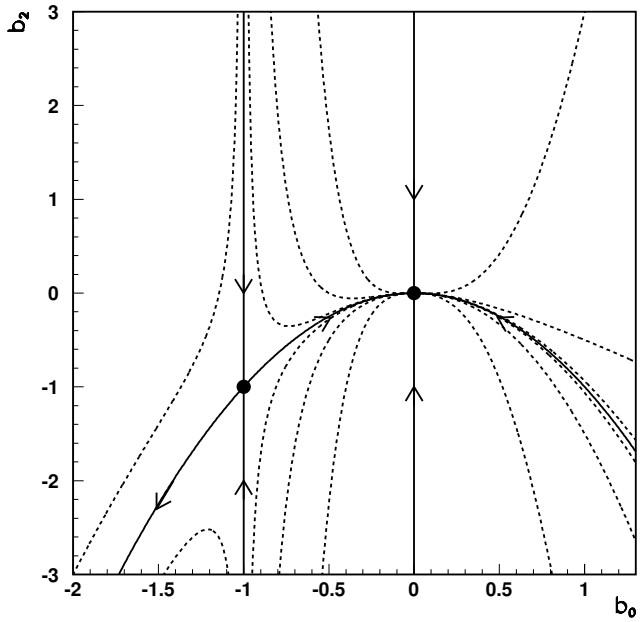
- Supercomputer
- Solve QCD, directly



Source: JICFuS webpage

Effective field theory:

- Renormalization group
- Solve QCD, indirectly



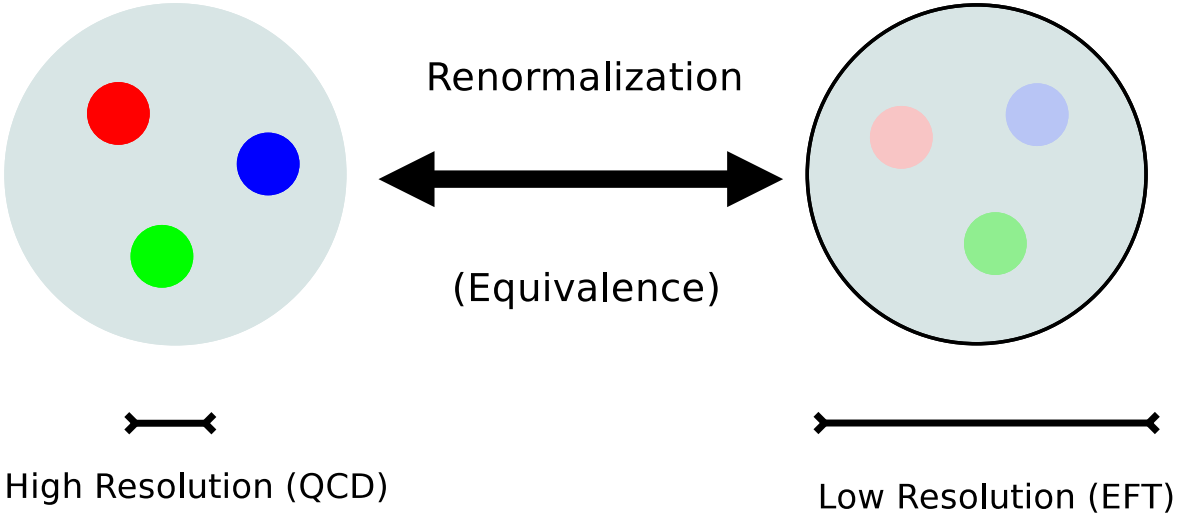
Source: Birse, McGovern, Richardson 98

What is an effective field theory

If no supercomputer, the right tool is Effective Field Theory:

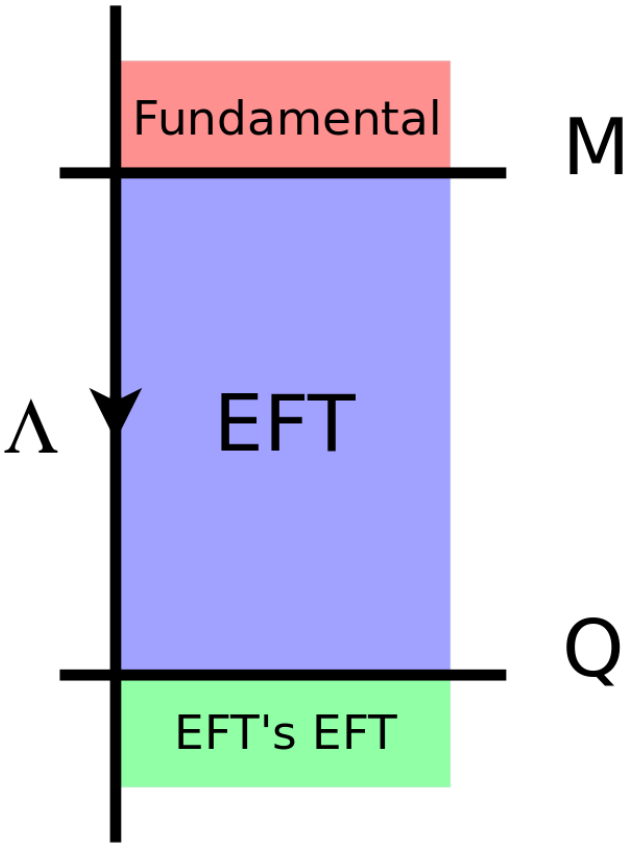
> Physics at long distances does not depend on the short distance details

Rigorous implementation of this principle: renormalization



The actual problem is how to implement this idea

Renormalization Group & EFT



Physics is unique, but choice of theory depends on resolution Λ:

- $\Lambda \geq M$: Fundamental
- $M \geq \Lambda \geq Q$: EFT

For equivalent descriptions:

$$\boxed{\frac{d}{d\Lambda}\langle\Psi|\mathcal{O}|\Psi\rangle = 0}$$

Renormalization group invariance

Renormalization Group & EFT

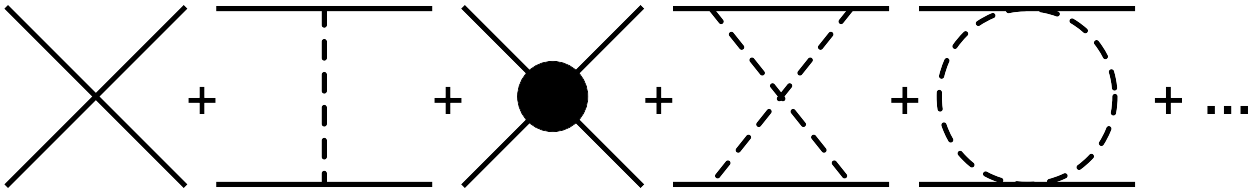
Begin at $\Lambda = M$, two equivalent descriptions

$$\underbrace{\text{quarks \& gluons}}_{\text{high energy}} \quad \Longleftrightarrow \quad \underbrace{\text{hadrons}}_{\text{low energy}}$$

The hadron description equivalent if and only if

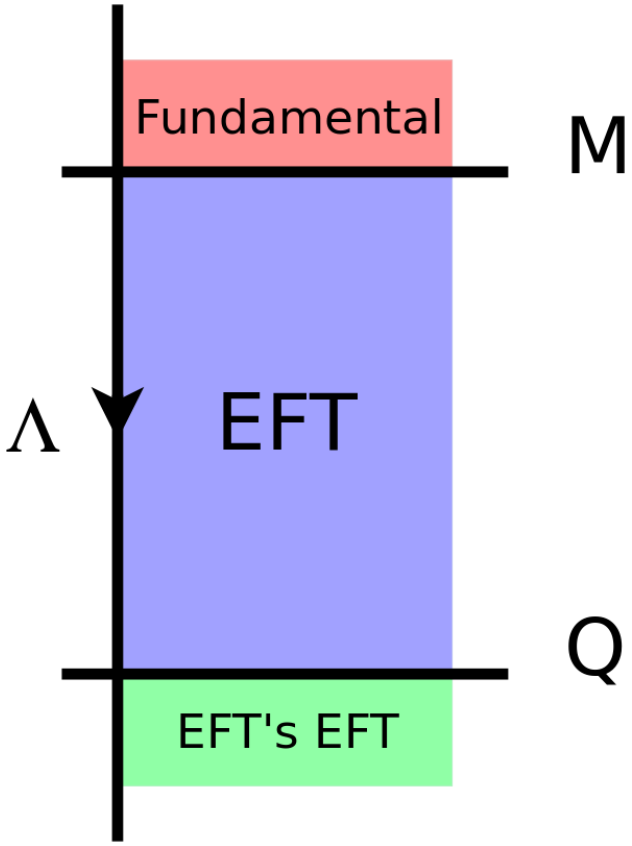
(1) Include correct low energy symmetries

(2) Consider infinite set of Feynman diagrams consistent with (1)

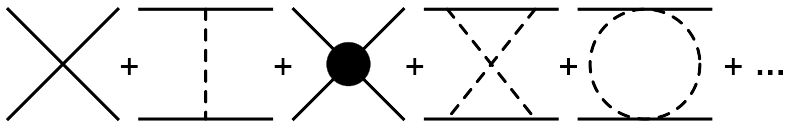


Problem: **infinite diagrams imply no predictive power**

Renormalization Group & EFT: Power Counting

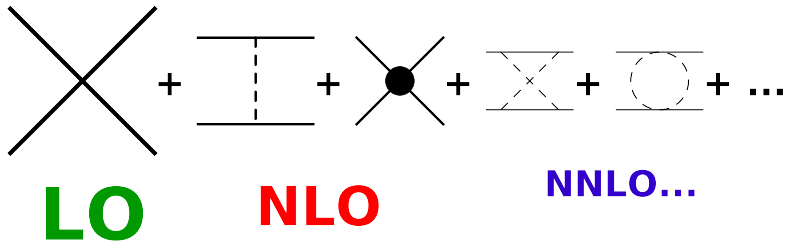


(1) At $\Lambda \sim M$ there is no order



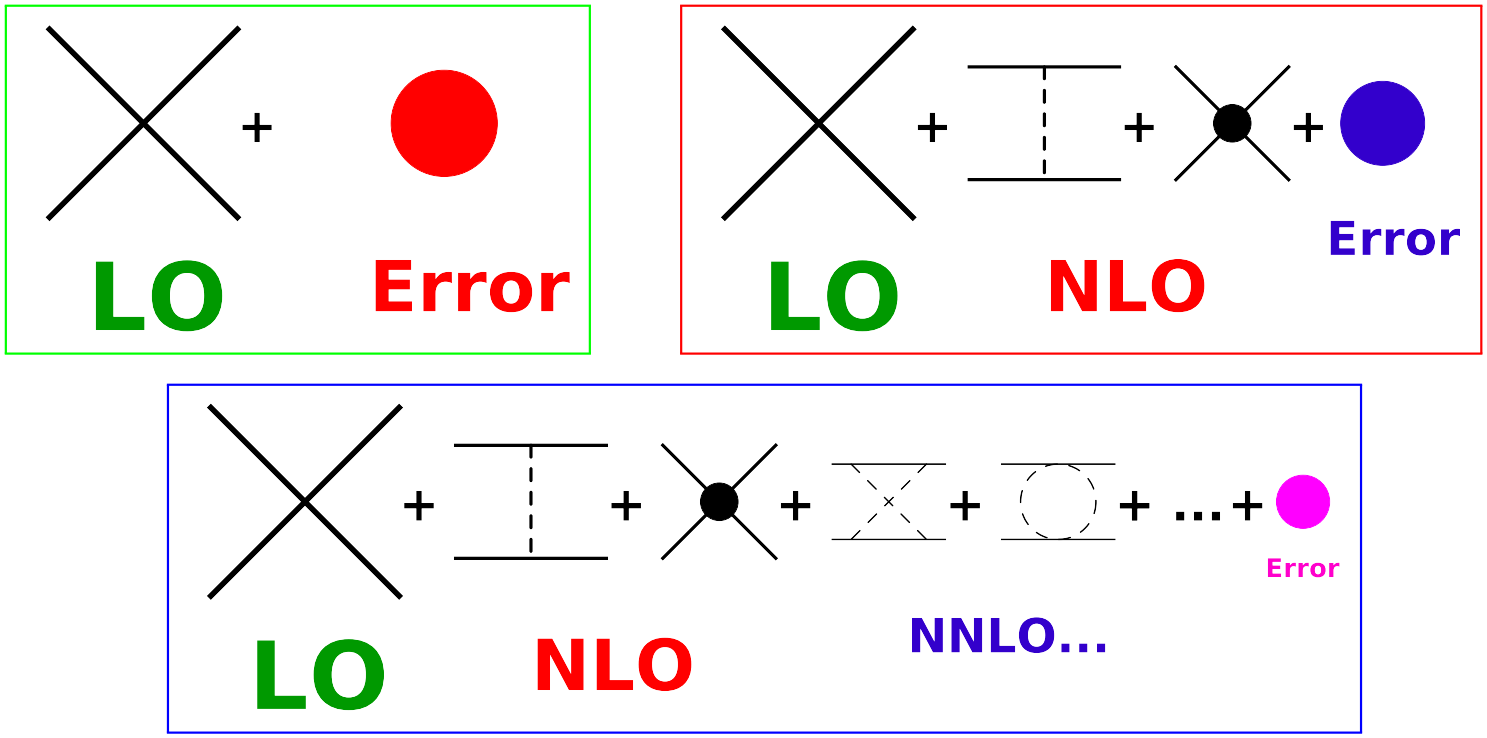
$$\frac{d}{d\Lambda}\langle\Psi|\mathcal{O}|\Psi\rangle = 0$$

(2) while at $\Lambda \sim Q$ there is order



Renormalization Group & EFT: Power Counting

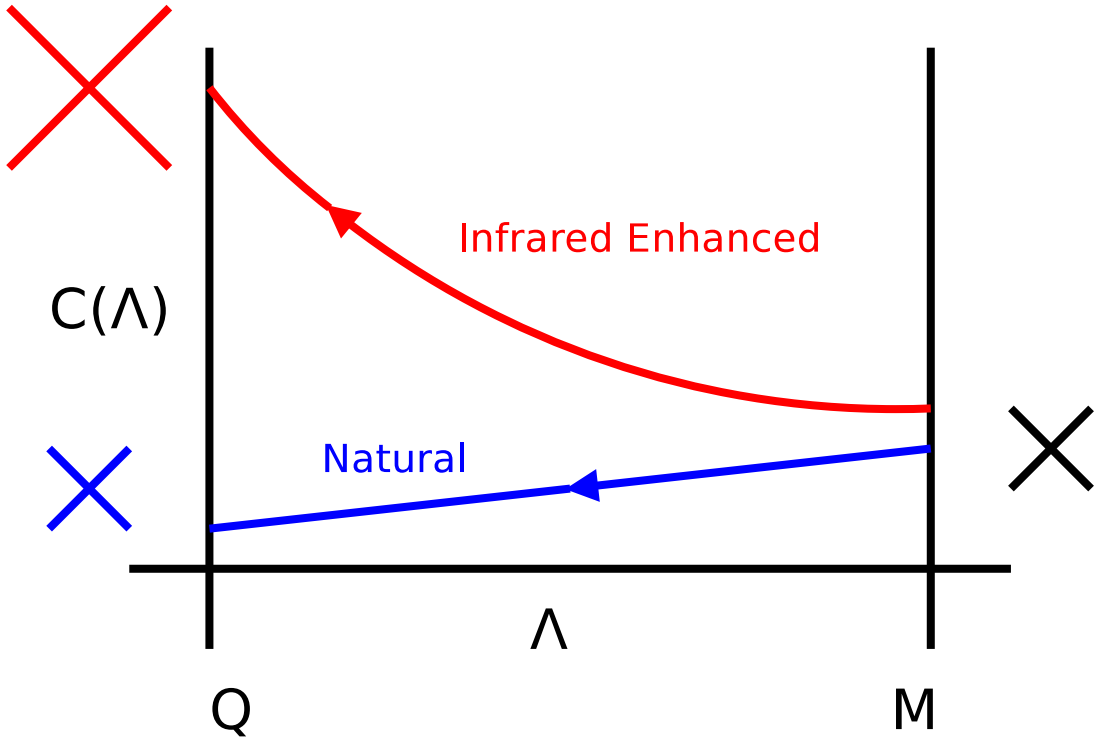
Predictive power: cut the expansion ⇒ systematic error estimations



Caveat: Power counting is not unique. Example above: KSW

Renormalization Group & EFT: Power Counting

RG Equations: no unique solution, rather **families of solutions**



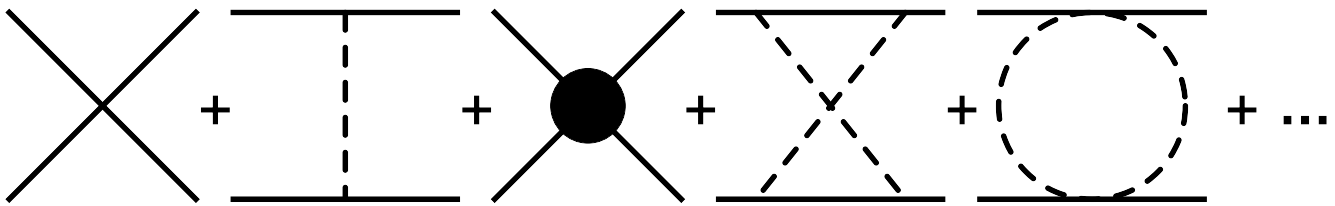
And this implies that **power counting is not unique**

Nuclear Physics & EFT: How to build it

Nuclear EFT: what's inside?

- Low energy fields: **pions** & **nucleons** (& optionally **deltas**)
- Low energy symmetries:
  - **chiral symmetry** (main low energy remnant of QCD)
  - standard symmetries: parity, time reversal, rotational...

First step: write the diagrams

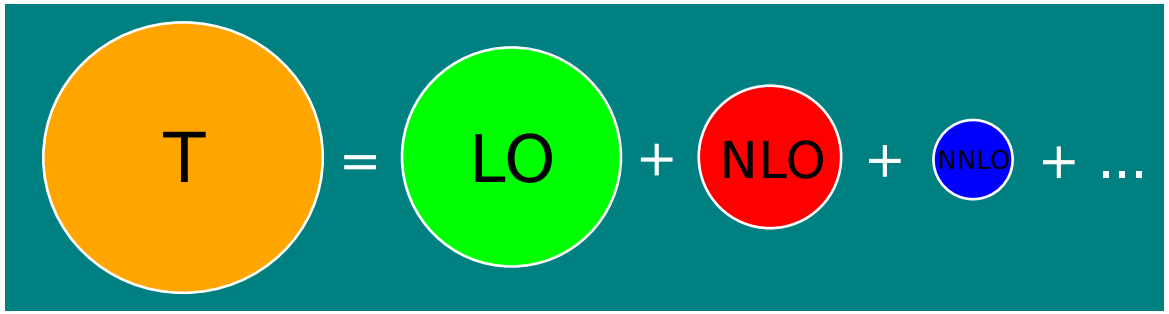


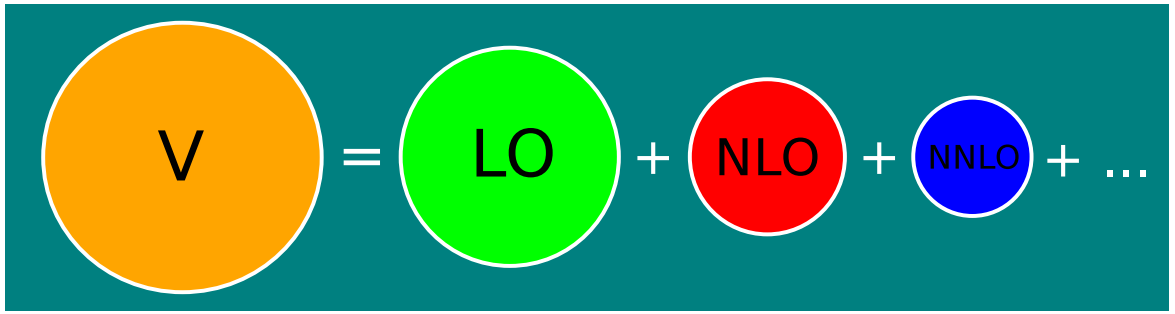
Second step: sort the diagrams (i.e. apply RG evolution)

(Power Counting)

Nuclear Physics & EFT: Why do we do it?

- Everything within EFT is an **expansion**:





  Calculations are amenable to **error estimations** (that is, the expected size of the next blob)

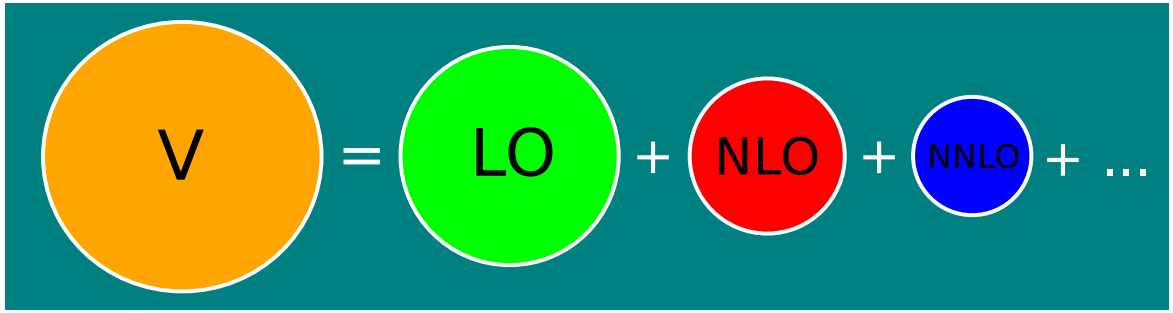
- Connection to QCD

Nuclear Physics & EFT: history and path-dependence

Traditional EFT / RG knowledge is perturbative, but nuclear physics is not.

We have to iterate something...

Weinberg's idea (90): we know how to count the potential

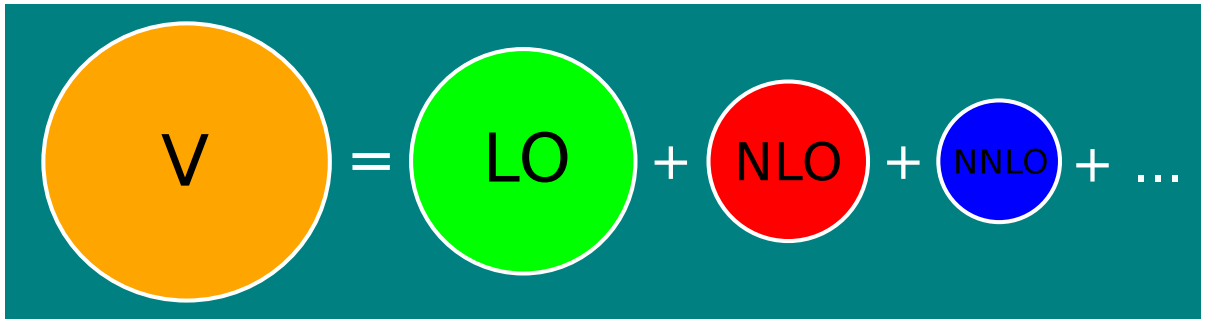


Simply iterate the EFT potential (or part of it)

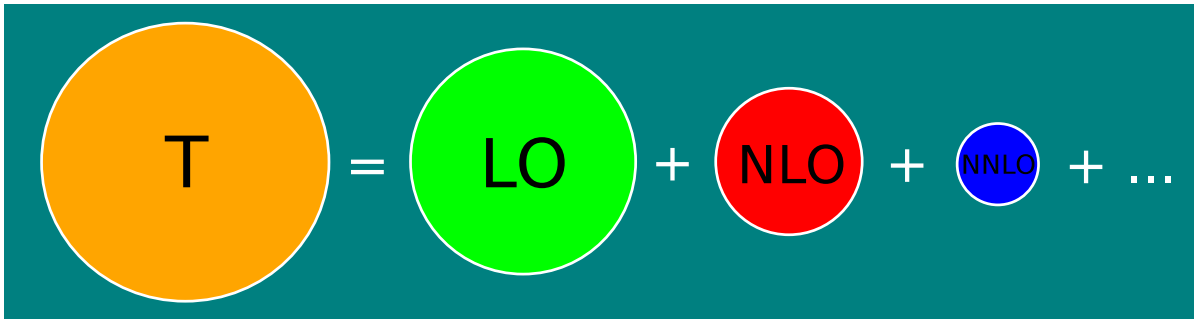
Nuclear Physics & EFT: General idea

Nucleons are heavy: the use of potentials is justified (and useful)

- We begin with



- which we put into $T = V + VG_0T$ (or a reexpansion of it)
- we want the following to happen

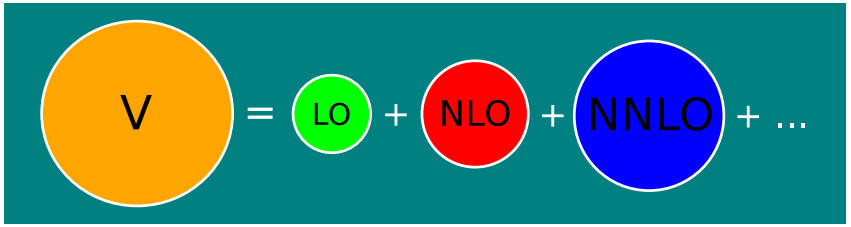


Well, it doesn't automatically happen. **Requires hard work**

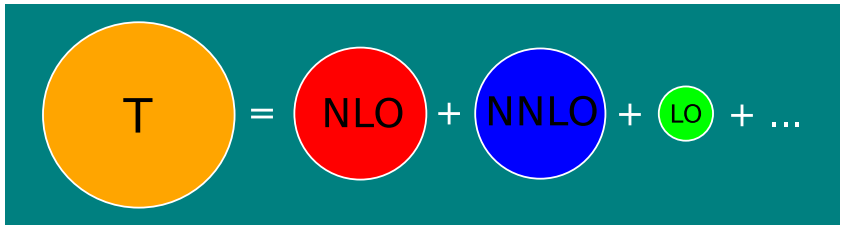
Nuclear Physics & EFT: What can fail?

The loops will fail, if not properly renormalized.

- At short distances, the potential is not counting-friendly



- Loops probe the short distance region above.
- For cut-offs probing short distances, the T-matrix might violate power counting (if not properly renormalized):



No counting, no error estimates, no QCD connection

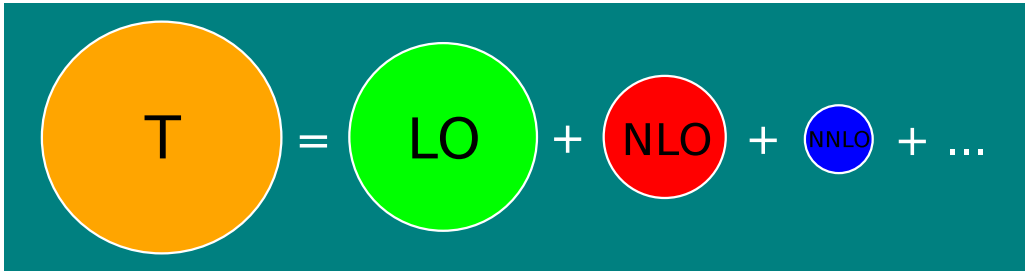
(Lepage (98); Epelbaum and Gegelia (09), though in a different context with a different message.)

Nuclear Physics & EFT: The Recipe

Beware iterations: non-perturbative EFT eventually fails

(MPV & Arriola (05), again in a different context and with a different message.)

Fool proof recipe for a T-matrix with correct counting:



(1) LO potential **non-perturbative**. We renormalize it.

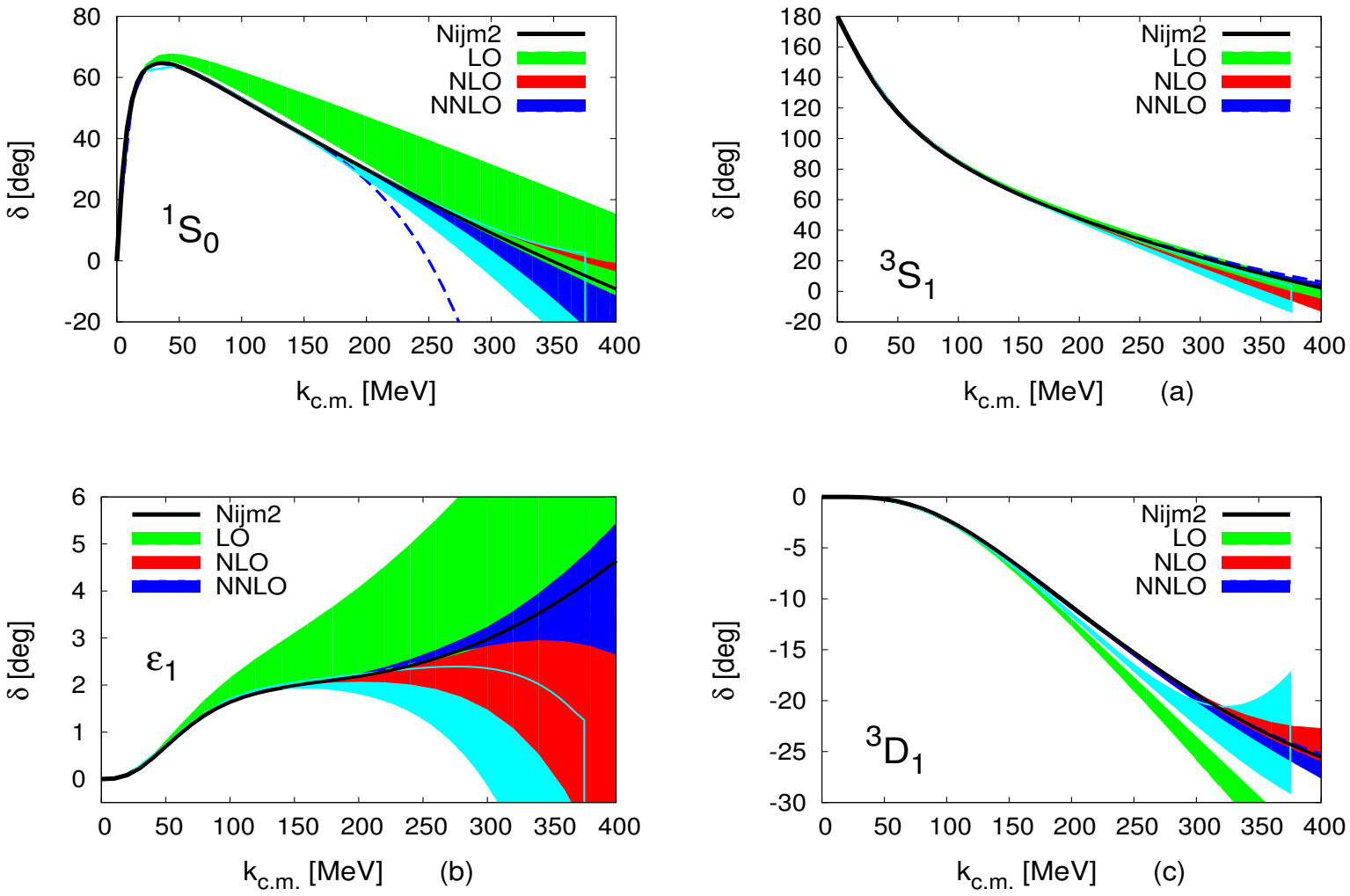
(2) NLO, NNLO ... potential: **perturbative**

(3) We renormalize NLO, NNLO... **perturbatively**

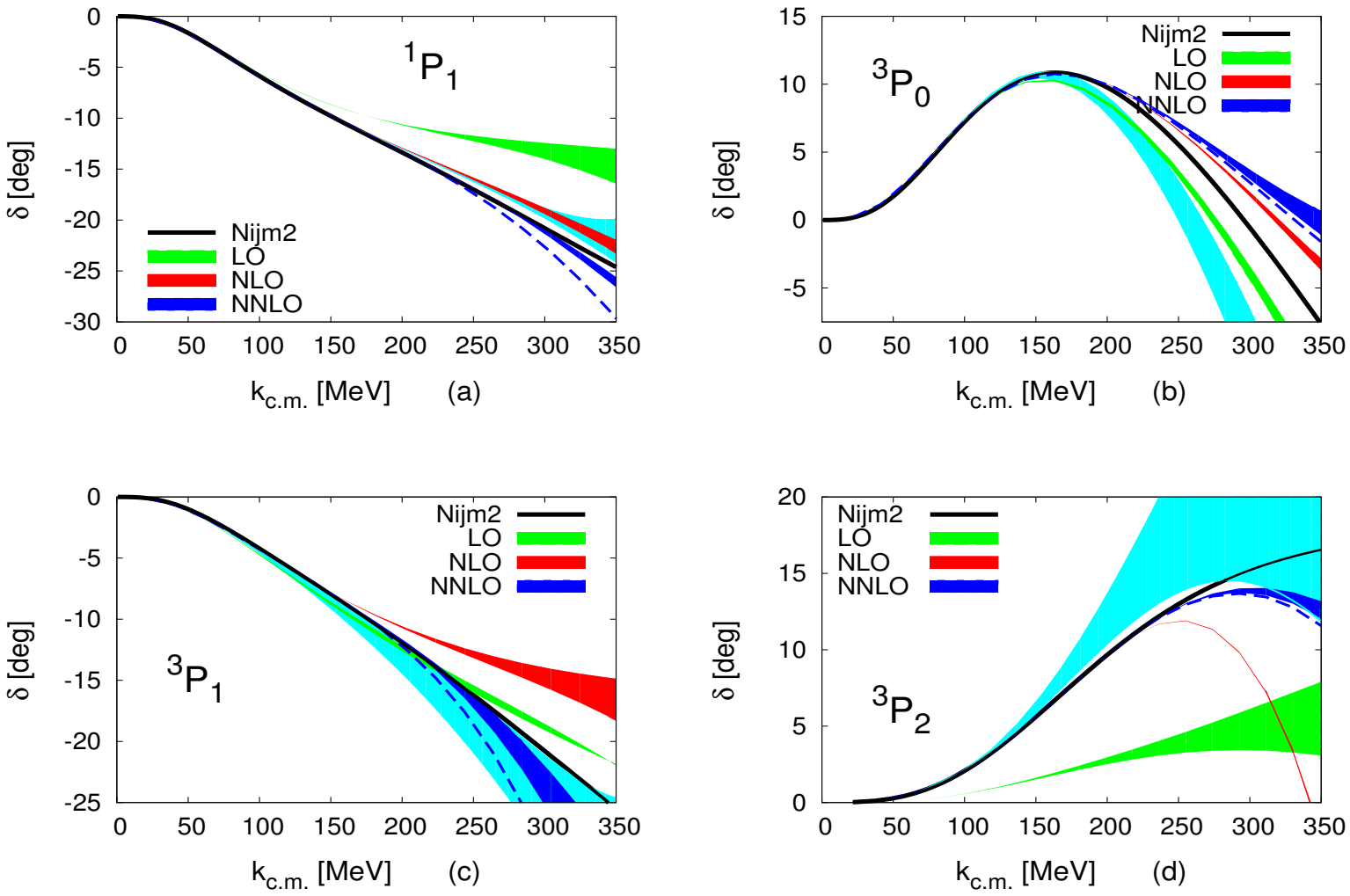
Mix of perturbative / non-perturbative compatible with EFT

(MPV 11, 12; Long, Yang 12)

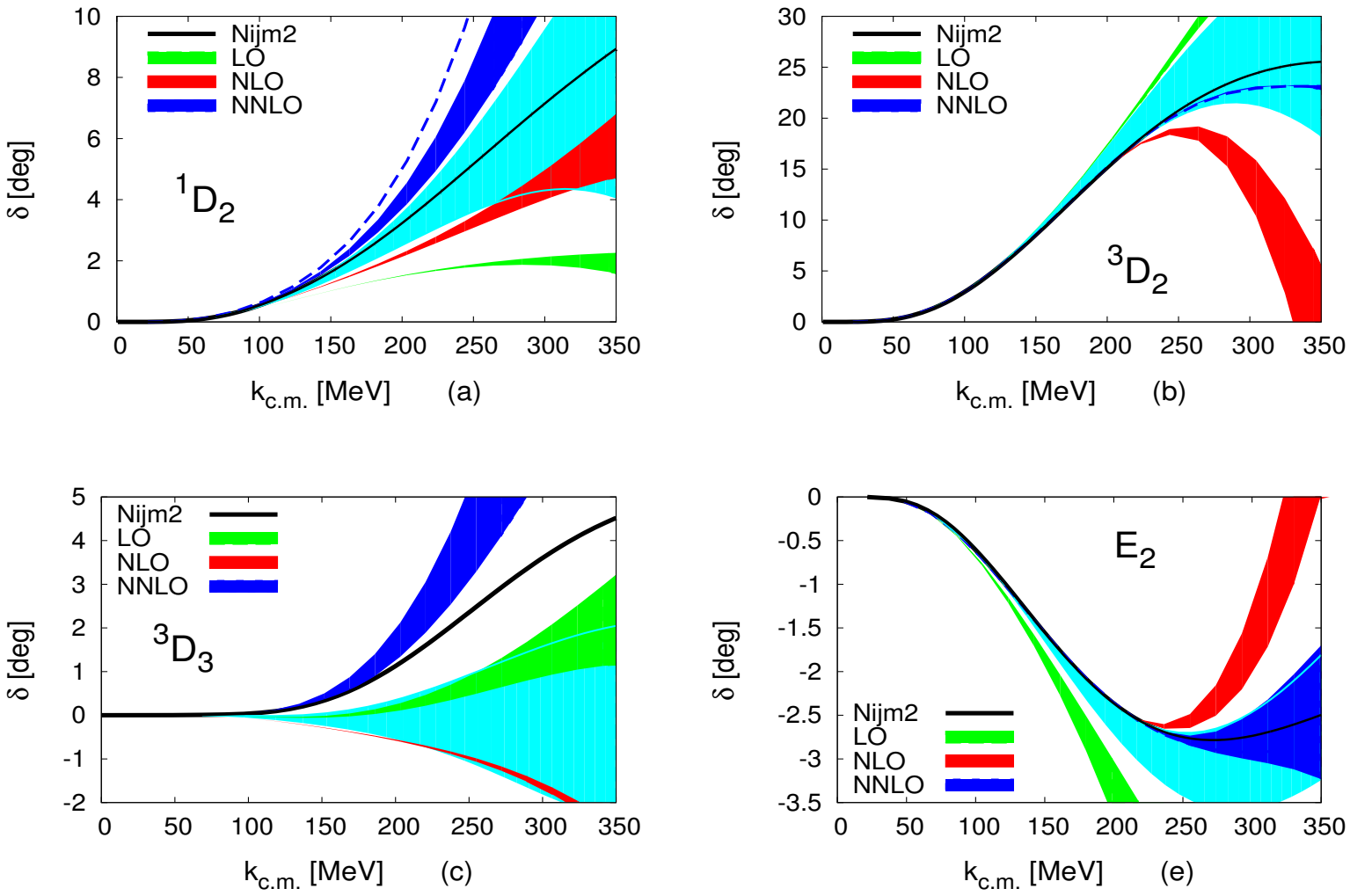
Nuclear Physics & EFT: two-body phase shifts



Nuclear Physics & EFT: two-body phase shifts



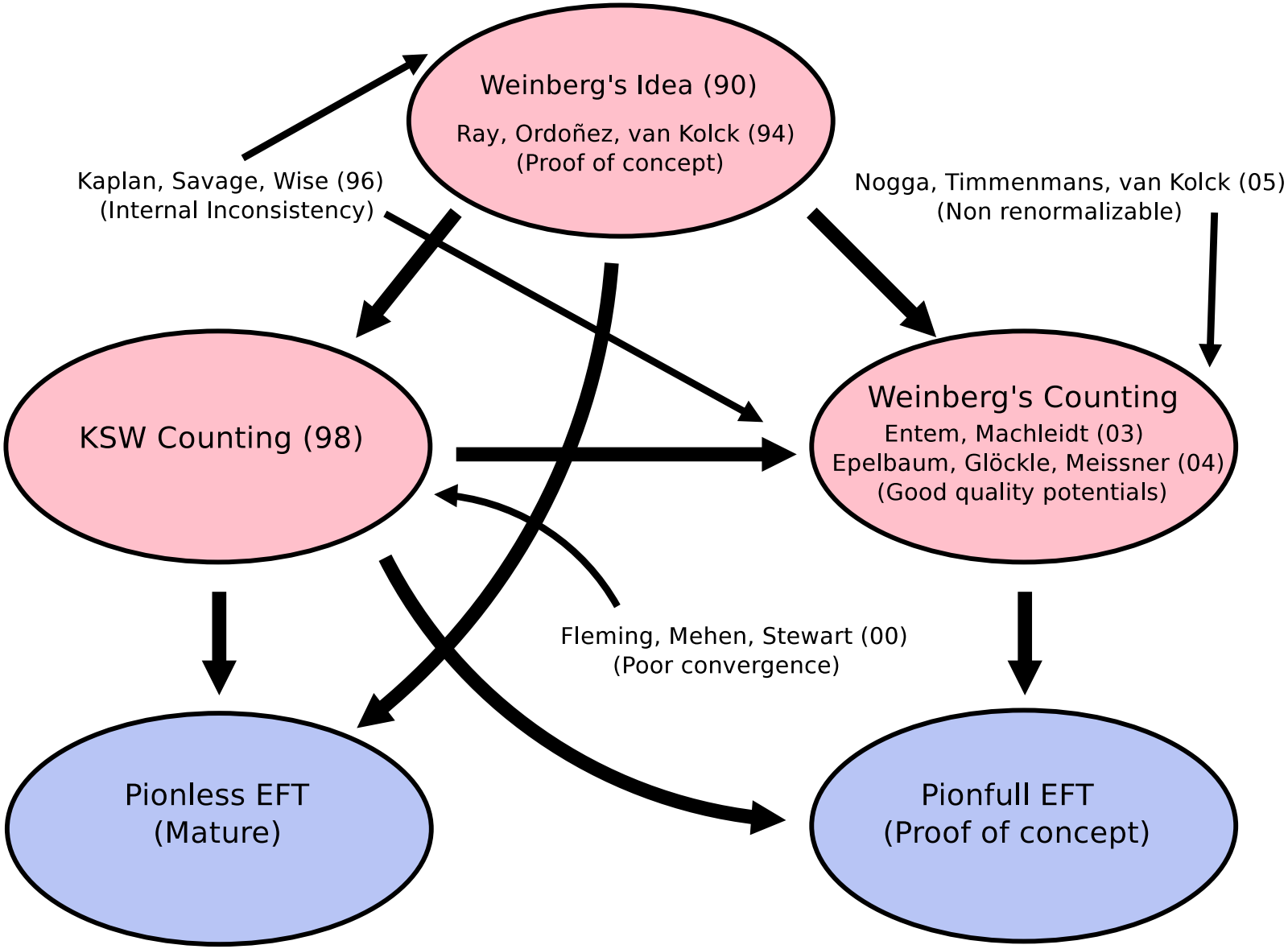
Nuclear Physics & EFT: two-body phase shifts



Nuclear Physics & EFT: Long story short

The problem of path dependence...

Nuclear Physics & EFT: Long story short



Nuclear Physics & EFT: the Outlook

A Perturbing Match

Nuclear Physics & EFT: the Outlook

Fundamental tension:

- Nuclear EFT: **mixture of perturbative & non-perturbative**
- Computational nuclear physics: **usually non-perturbative**
  - A few exceptions: **Lattice EFT** (Lee, Epelbaum, Meißner, Lähde...) Proof of compatibility. Current implementations use inconsistent power counting though. **GFMC**

Computational nuclear physics & EFT **usually incompatible**

We are confronted with a dilemma

(a) Use my old nuclear codes: **Give up (rigorous) EFT**

(b) Redo a lot of computational nuclear physics: **Panic!**

Nuclear Physics & EFT: a potential compromise

Findind a balance: $V = V_{LO} + \delta V$

- Two-body sector: direct comparison possible
  - $T = V + V\, G_0\, T$
  - $T_{\rm EFT} = T_{\rm LO} + \delta T$
    - $T_{\rm LO} = V_{\rm LO} + V_{\rm LO}\, G_0\, T_{\rm LO}$
    - $\delta T = (1 + T_{\rm LO}\, G_0)\, \delta V\, (G_0\, T_{\rm LO} + 1)$

  Then we **check** that $|T - T_{\rm EFT}|$ is **small**
- Many-body sector: **indirect comparison** possible

  Define $V(\lambda) = V_{\rm LO} + \lambda\, \delta V$ and compute $\mathcal{O}(\lambda)$:
  - $\mathcal{O} = \mathcal{O}(\lambda = 1)$
  - $\mathcal{O}_{\rm EFT} = \mathcal{O}_{\rm LO} + \delta\mathcal{O}$
    - $\mathcal{O}_{\rm LO} = \mathcal{O}(\lambda = 0)$
    - $\delta\, \mathcal{O}_{\rm EFT} = \frac{d}{d\lambda}\mathcal{O}(\lambda)\Big|_{\lambda=0}$

  Then we **check** that $|\mathcal{O} - \mathcal{O}_{\rm EFT}|$ is **small**

Nuclear Physics & EFT: checking the counting

Illustration of power counting breakdown in **Weinberg's counting**

- naive dimensional analysis

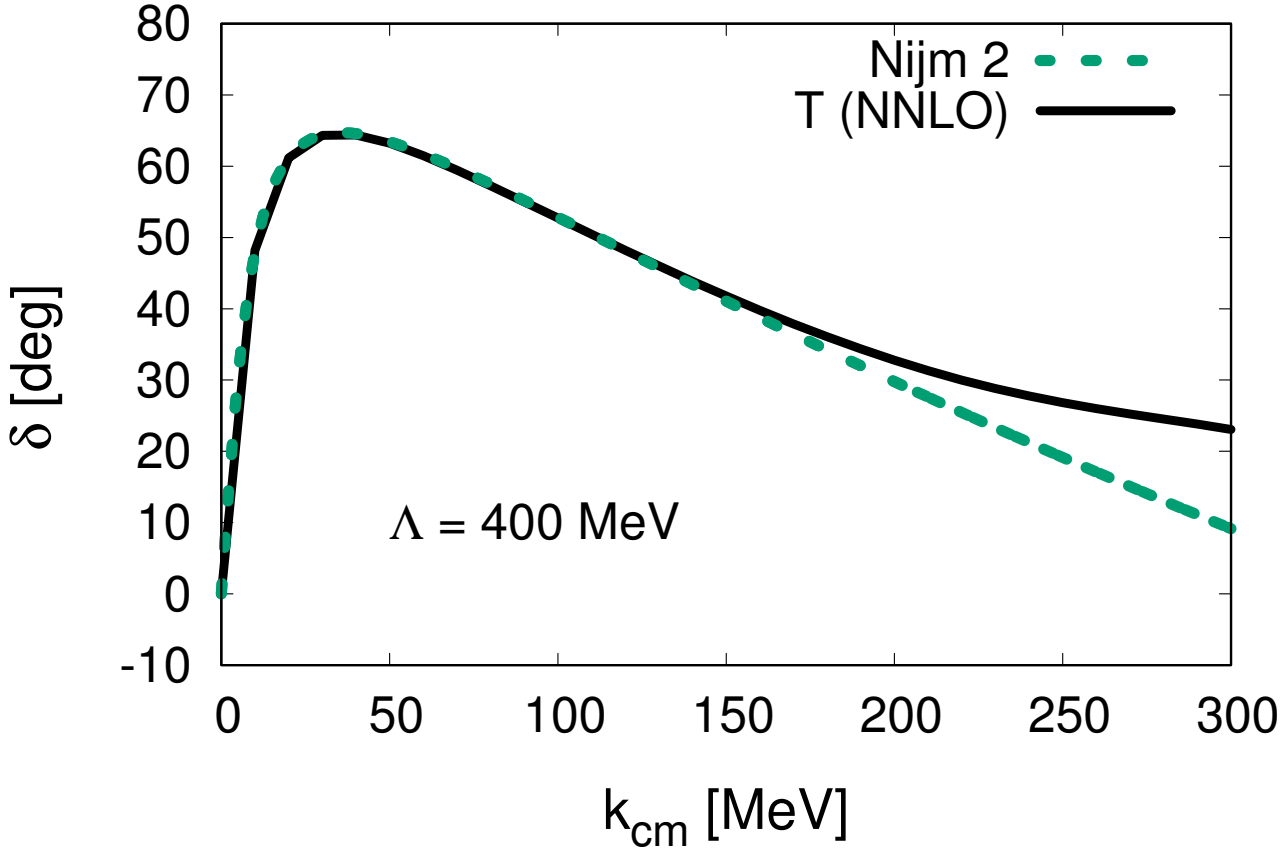
$$V_{\mathrm{EFT}}(\nu_{\mathrm{max}}) = V^{(0)} + V^{(2)} + V^{(3)} + \cdots + V^{(\nu_{\mathrm{max}})}$$

- fully non-perturbative

$$T(\nu_{\mathrm{max}}) = V_{\mathrm{EFT}}(\nu_{\mathrm{max}}) + V_{\mathrm{EFT}}(\nu_{\mathrm{max}}) G_0 T(\nu_{\mathrm{max}})$$

Now we do a calculation at $\nu_{\mathrm{max}} = 3$ a.k.a. $\mathrm{N^2LO}$

Nuclear Physics & EFT: checking the counting



$\mathrm{N^2LO}$ Weinberg counting calculation vs Nijmegen phase shifts

Nuclear Physics & EFT: checking the counting

Now we analyze **Weinberg's counting**. What we do is:

- The $\mathrm{N^2LO}$ potential is $V_{\mathrm{EFT}} = V^{(0)} + V^{(2)} + V^{(3)}$
- The $\mathrm{N^2LO}$ T-matrix is $T = V_{\mathrm{EFT}} + V_{\mathrm{EFT}} G_0 T$

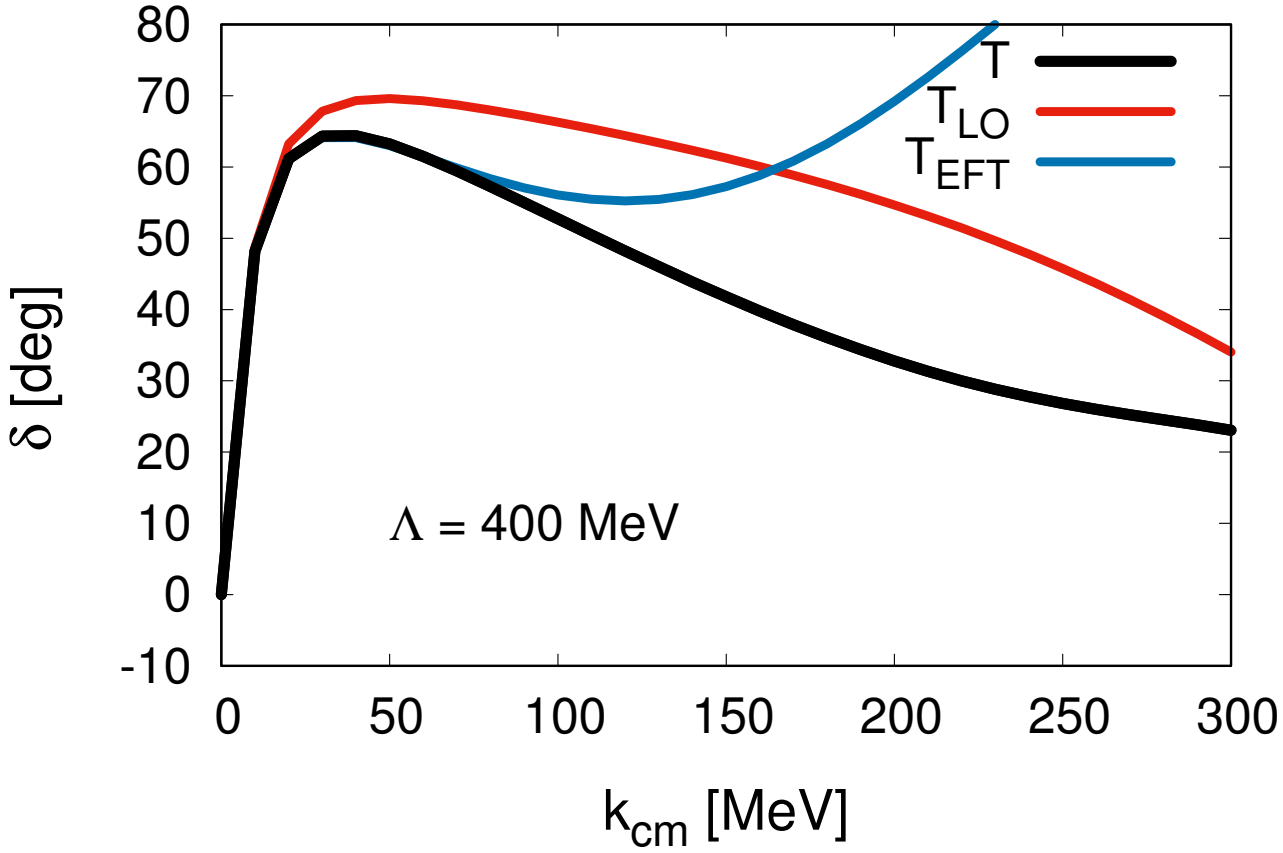
What we should have is:

- Power counting: $V^{(0)} \gg V^{(2)}, V^{(3)}$
- Subleading potential is small and thus a perturbation
- Compute $T_{\mathrm{EFT}} = T_{\mathrm{LO}} + \delta T$ and compare with T
  - $T_{\mathrm{LO}} = V_{\mathrm{LO}} + V_{\mathrm{LO}} G_0 T_{\mathrm{LO}}$
  - $\delta T = (1 + T_{\mathrm{LO}} G_0)\, \delta V\, (G_0 T_{\mathrm{LO}} + 1)$

  where $V_{\mathrm{LO}} = V^{(0)}$, $\delta V = V^{(2)} + V^{(3)}$

Now we check it...

Nuclear Physics & EFT: checking the counting



Matching $T_{\rm EFT}$ with $T_{\rm LO} + \delta\, T$: counting works up to $k \sim 100\,{\rm MeV}$

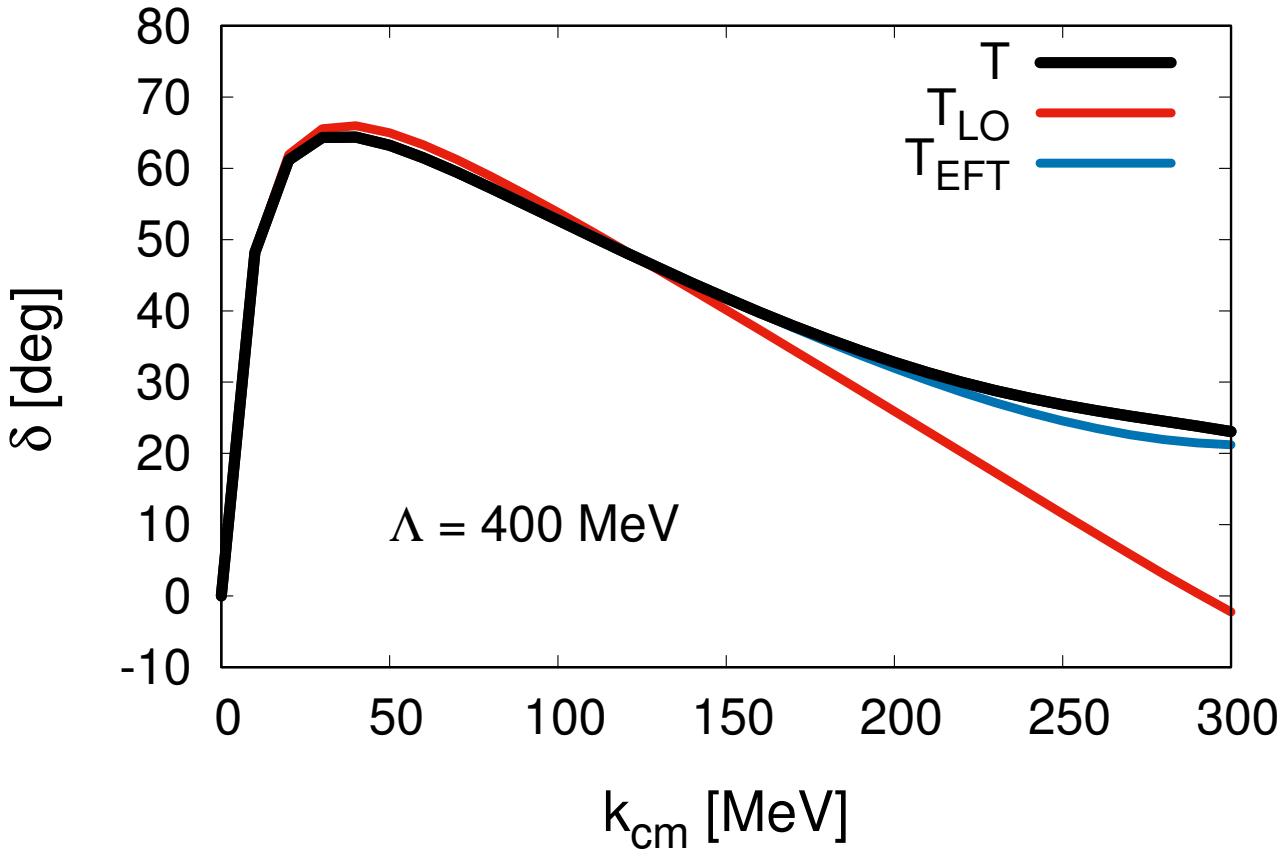
Nuclear Physics & EFT: power counting extravaganza

We can play another game to discover the implicit counting in T

- **Original assumptions:**
  - $V^{(0)} = V_{\mathrm{OPE}} + C_0$
  - $V^{(2)} + V^{(3)} = V_{\mathrm{TPE}} + C_2\,(p^2 + p'^2)$
- **New assumptions:**
  - $V^{(0)} = V_{\mathrm{TPE}} + C_0$
  - $V^{(2)} + V^{(3)} = V_{\mathrm{OPE}} + C_2\,(p^2 + p'^2)$

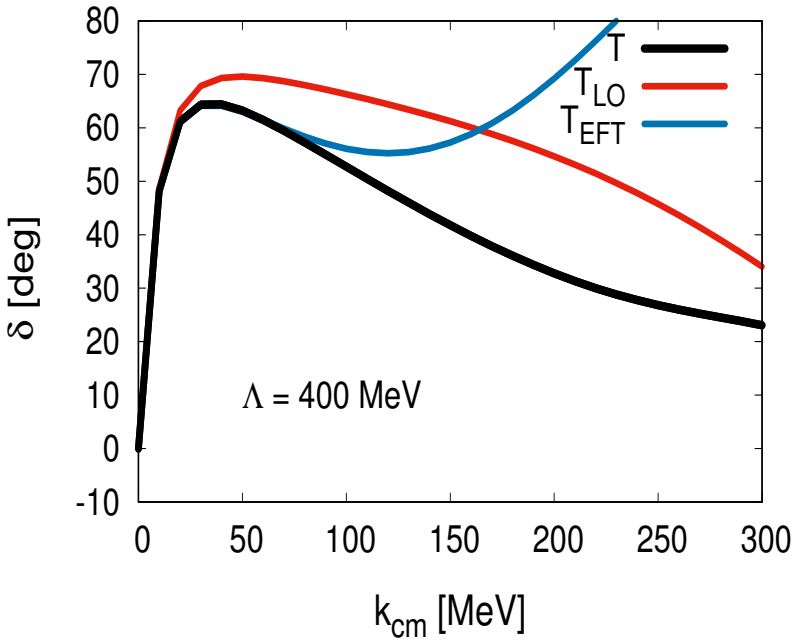
That is, we are **swapping OPE by TPE**

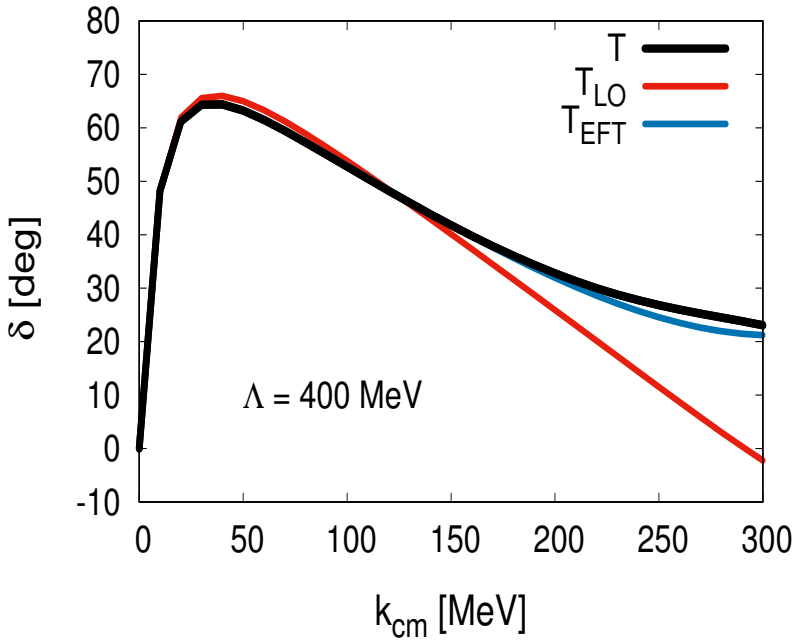
Nuclear Physics & EFT: power counting extravaganza



Comparison of $T_{\rm EFT} = T_{\rm LO} + \delta\, T$ with T after exchanging OPE and TPE. Power counting now works up to $k \sim 300\,{\rm MeV}$

Nuclear Physics & EFT: power counting extravaganza





Power Counting vs **Power Counting Extravaganza**

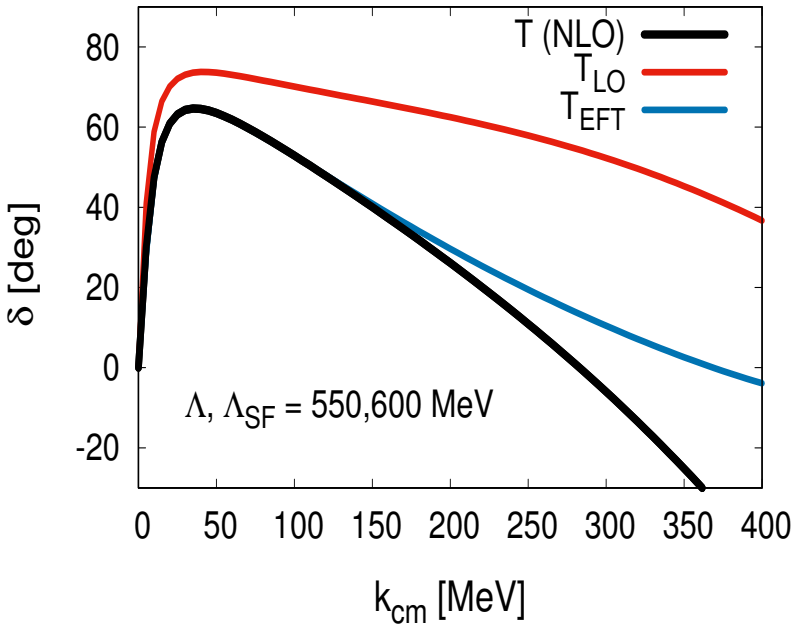
Relation with Epelbaum and Gegelia's **Peratization**

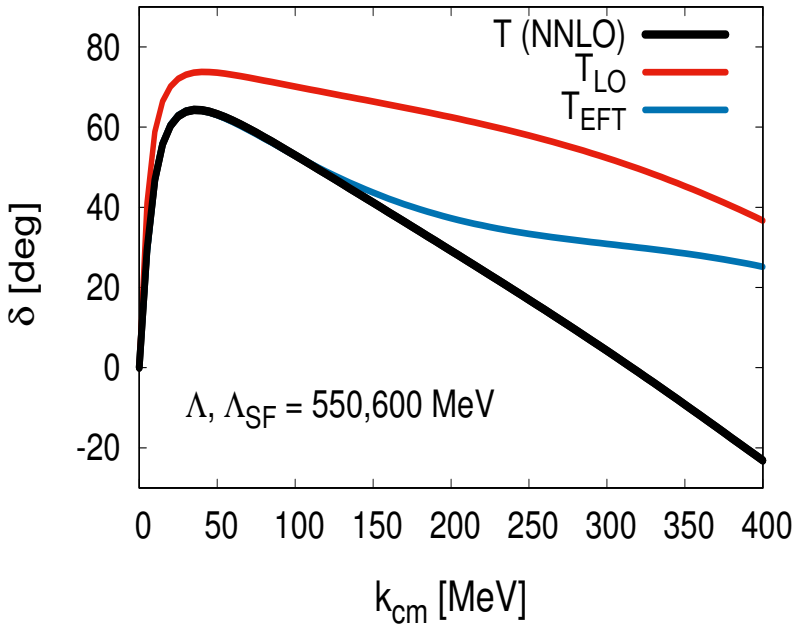
(again, in a different context and with a different intention)

Nuclear Physics & EFT: a potential compromise

Now we can test this with a few of the chiral potentials in the market

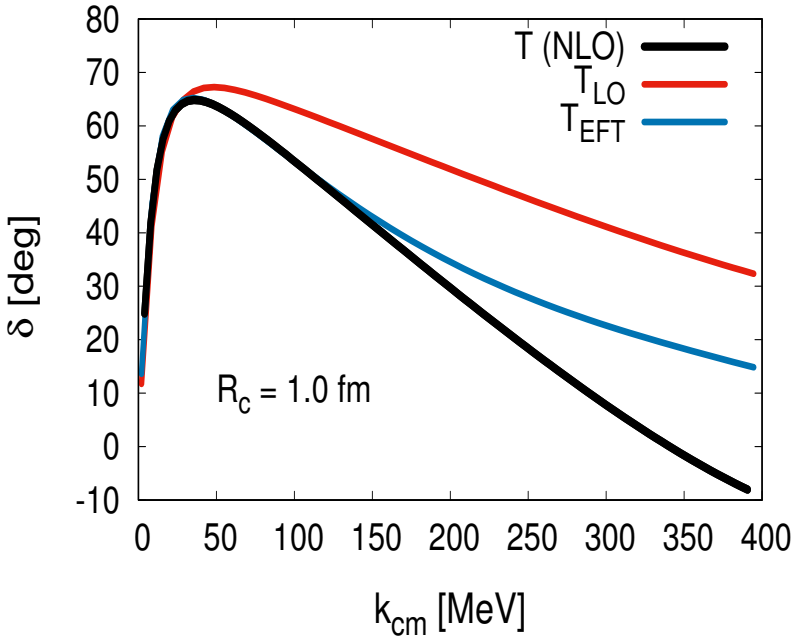
Nuclear Physics & EFT: a potential compromise

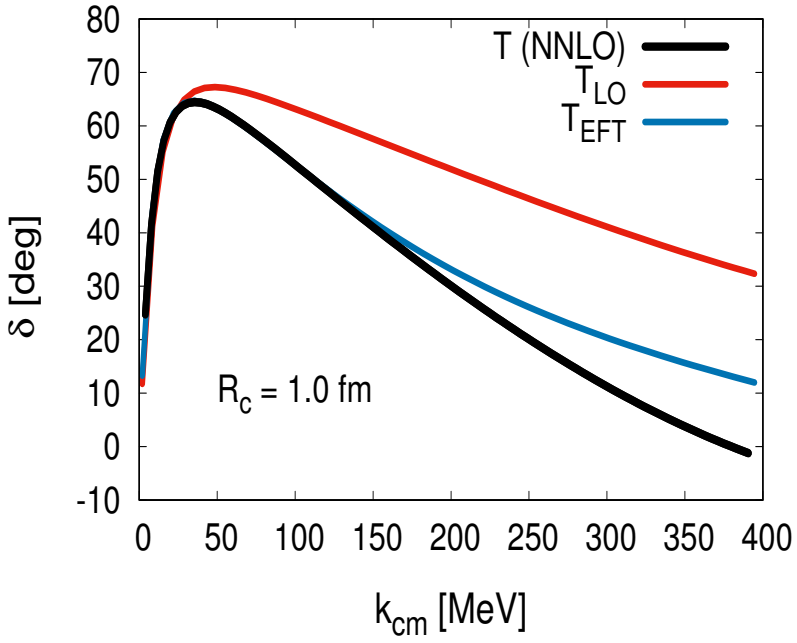




Epelbaum, Glöcke, Meißner (2003): **moderate breakdown**

Nuclear Physics & EFT: a potential compromise





Gerzelis, Tews, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk (2014): **this looks definitely better**

Nuclear Physics & EFT: a potential compromise

Now we do it with a real power counting

Nuclear Physics & EFT: now we use the correct counting

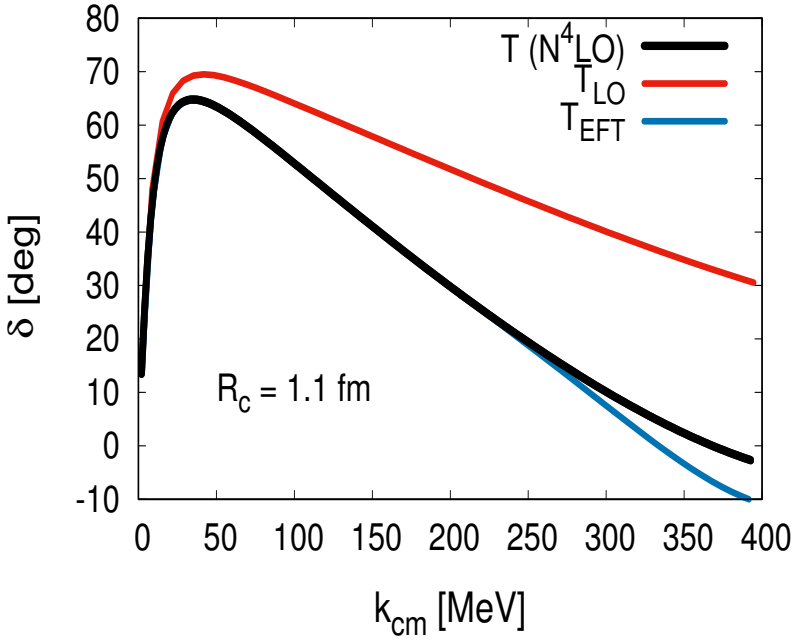
Previous calculations: **naive dimensional analysis**

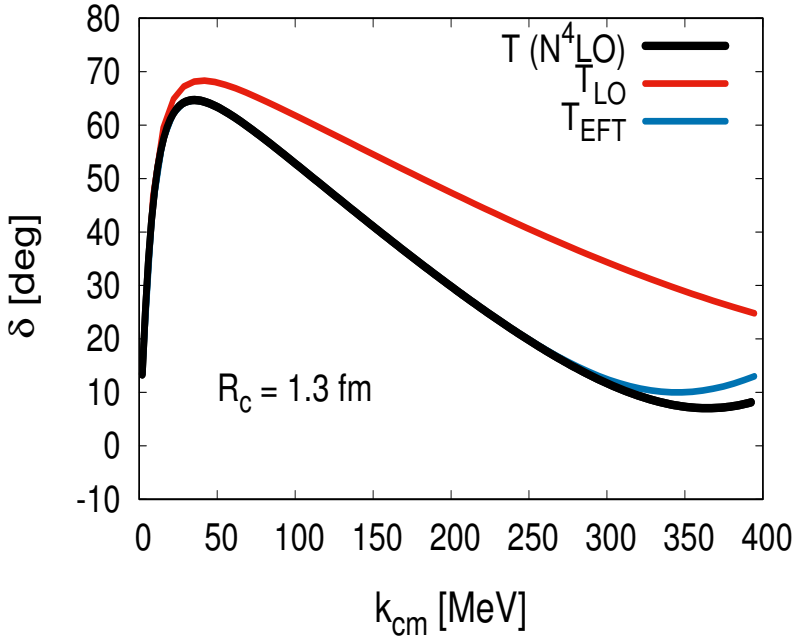
- not correct counting: large scattering length implies enhancement of derivatives by two orders (KSW 98, van Kolck 98)
- not renormalizable (NTvK 05, MPV, Arriola 05)

Power counting for the singlet indeed looks like this:

Order	NDA	MPV	MPV'	Long & Yang
Q^{-1}	-	$C_0,\ V_{\mathrm{OPE}}$	C_0	$C_0,\ V_{\mathrm{OPE}}$
Q^0	$C_0,\ V_{\mathrm{OPE}}$	C_2	$C_2,\ V_{\mathrm{OPE}}$	C_2
Q^1	-	-	-	$C_4,\ V_{\mathrm{TPE,L}}$
Q^2	$C_2,\ V_{\mathrm{TPE,L}}$	$C_4,\ V_{\mathrm{TPE,L}}$	$C_4,\ V_{\mathrm{TPE,L}}$	$C_6,\ V_{\mathrm{TPE,SL}}$
Q^3	$V_{\mathrm{TPE,SL}}$	$V_{\mathrm{TPE,SL}}$	$V_{\mathrm{TPE,SL}}$	$C_8,\ V_{3\pi}$

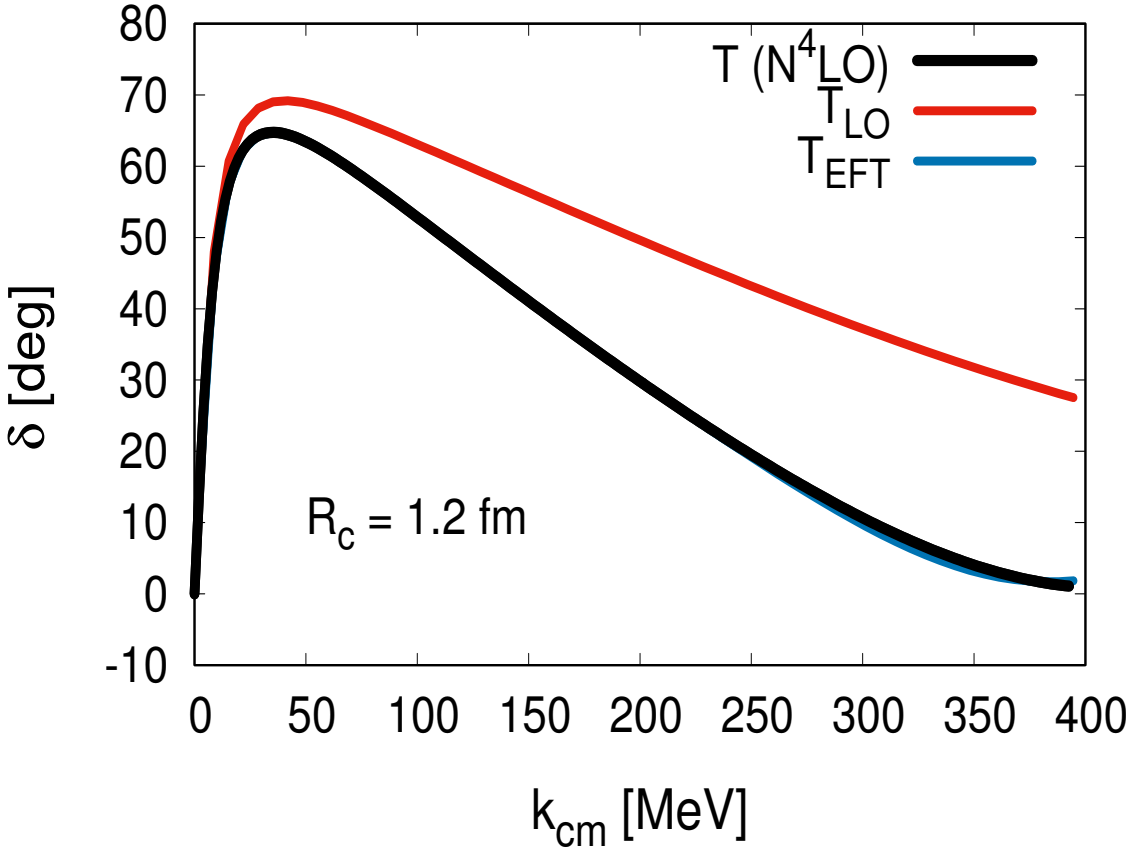
Nuclear Physics & EFT: now we use the correct counting





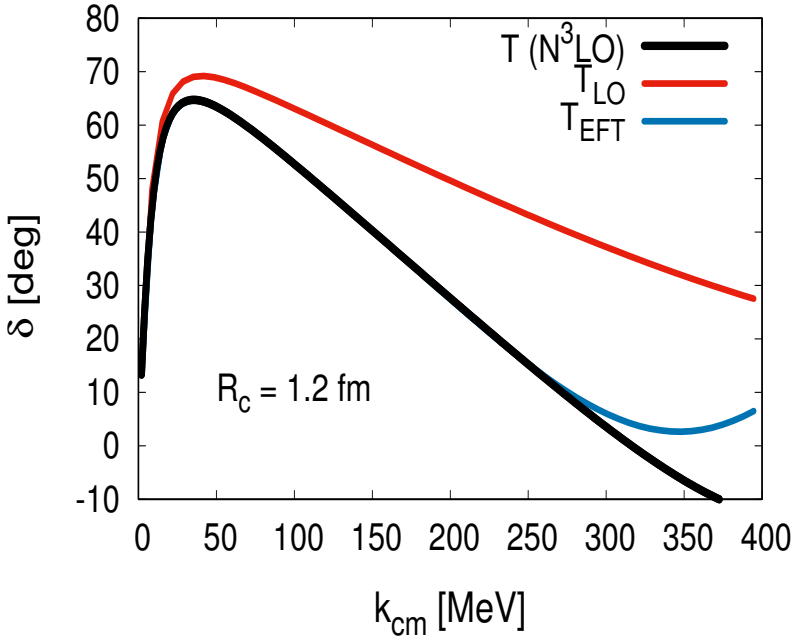
only supposed to work for a particular cut-off range

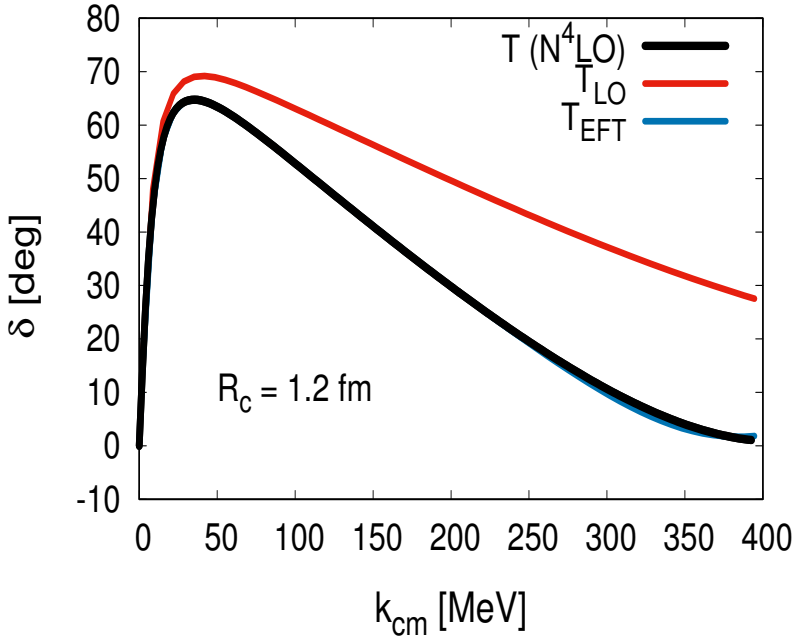
Nuclear Physics & EFT: now we use the correct counting



$R_c = 1.2\mathrm{fm}$ **best cut-off for gaussian regulator**

Nuclear Physics & EFT: now we use the correct counting





comparing the orders for $R_c = 1.2\,\mathrm{fm}$

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$$C^F \quad \text{versus} \quad C^{\mathrm{EFT}} = C^{\mathrm{LO}} + \delta C$$

Coupling	N^3LO (F)	LO	N^3LO (EFT)
C_0	$0.153\,\mathrm{fm}^2$	$-2.204\,\mathrm{fm}^2$	$-1.804\,\mathrm{fm}^2$
$10\,C_2$	$3.535\,\mathrm{fm}^4$	-	$4.933\,\mathrm{fm}^4$
$100\,C_4$	$2.938\,\mathrm{fm}^6$	-	$2.606\,\mathrm{fm}^6$

Coupling	N^4LO (F)	LO	N^4LO (EFT)
C_0	$3.051\,\mathrm{fm}^2$	$-2.204\,\mathrm{fm}^2$	$2.229\,\mathrm{fm}^2$
$10\,C_2$	$1.054\,\mathrm{fm}^4$	-	$1.874\,\mathrm{fm}^4$
$100\,C_4$	$1.374\,\mathrm{fm}^6$	-	$1.713\,\mathrm{fm}^6$

Questions

- Is **power counting extravaganza** a serious issue in potential-based approaches (e.g. Weinberg prescription)?
  - Is the division of **implicit vs explicit counting** relevant?
  - Should existing potentials be **diagnosed**?
- Are potential approximations to EFT amplitudes acceptable?
  - Implicitly **preserve power counting**
  - Explicitly **break RG invariance**:
    The compromise only works in a specific cut-off window

The End

Thanks For Your Attention!