Analyzing chiral EFT power counting by RG Analysis plot

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THE GEORGE WASHINGTON UNIVERSITY

WASHINGTON, DC

The Nuclear Force Problem: Is the Never-Ending Story Coming to an End?

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Table 1. Seven Decades of Struggle: The Theory of Nuclear Forces

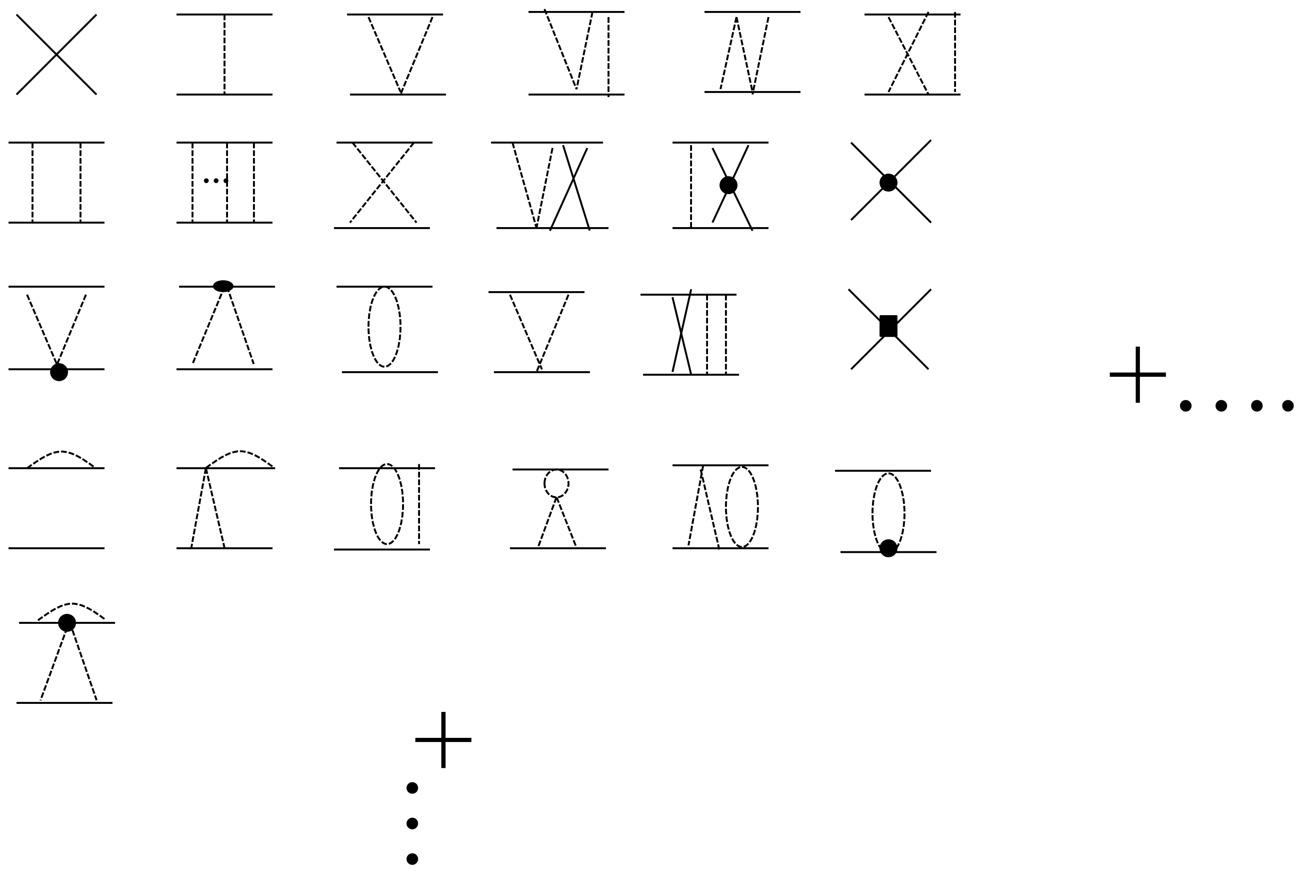
1935	**Yukawa: Meson Theory**
1950's	*The "Pion Theories"* One-Pion Exchange: o.k. Multi-Pion Exchange: disaster
1960's	Many pions $\equiv$ multi-pion resonances: $\sigma, \rho, \omega, \ldots$ The One-Boson-Exchange Model
1970's	Refine meson theory: Sophisticated 2π exchange models (Stony Brook, Paris, Bonn)
1980's	Nuclear physicists discover **QCD** Quark Cluster Models
1990's and beyond	Nuclear physicists discover **EFT** Weinberg, van Kolck **Back to Meson Theory!** *But, with Chiral Symmetry*

Answer: Yes, almost!

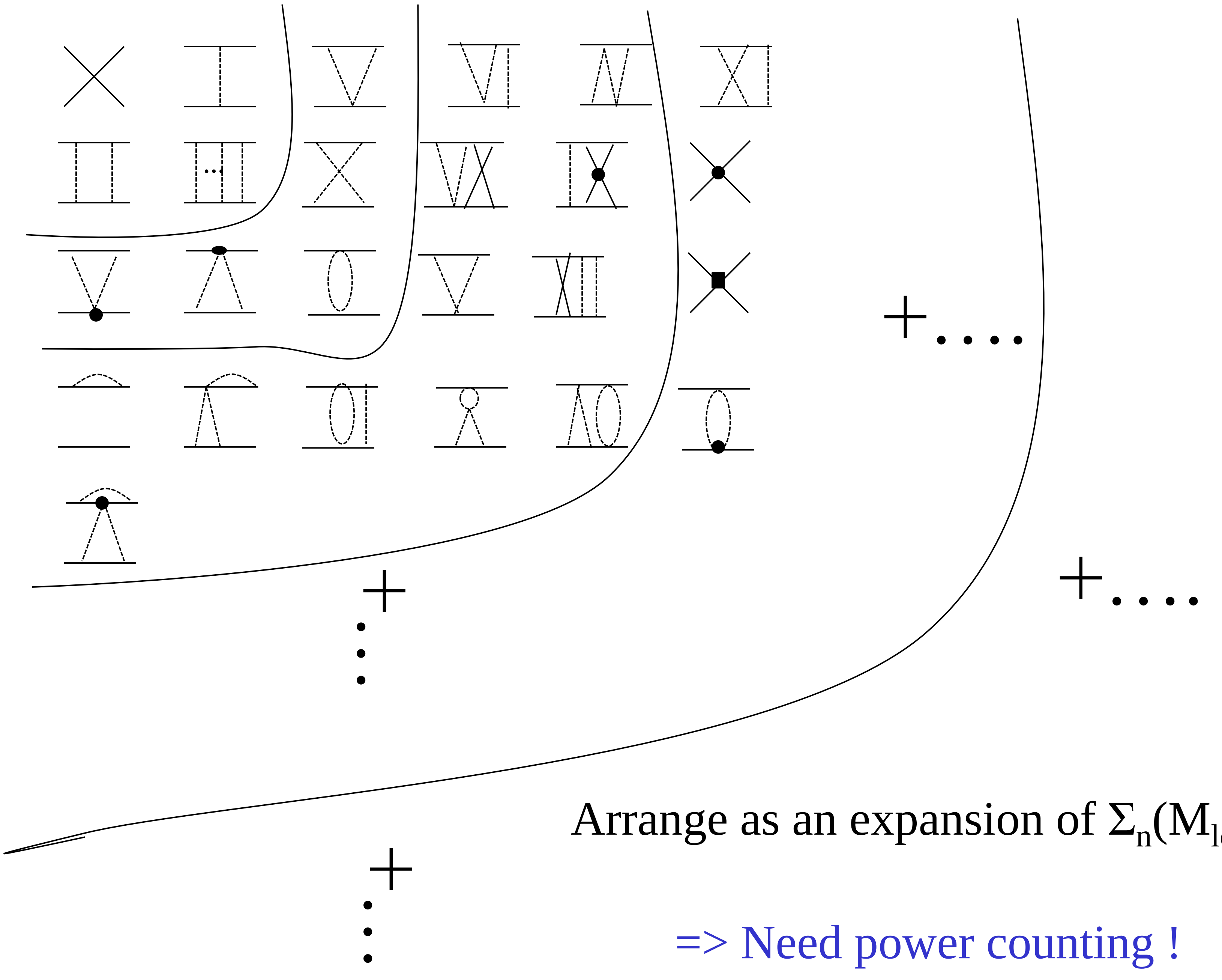
Nature of the problem:

Chiral EFT at NN sector

- Infinitely many diagrams contribute, most of them require renormalization.
- Need to arrange a way to include them based on their importance (there maybe more than one consistent way).
- Weinberg prescription can be used up to the potential level.
- Pure perturbation doesn’t work.



Infinity many diagrams.



Arrange as an expansion of $\Sigma_n(M_{lo}/M_{hi})^n$

=> Need power counting !

Conventional power counting

Epelbaum, Entem, Machleidt, Kaiser, Meissner, … etc., ~**90% of the people**

- **Arrange diagrams base on Weinberg's ~~power counting (WPC)~~:** each derivative on the Lagrangian terms is always suppressed by the underlying scale of chiral EFT, $M_{hi} \sim m_\sigma$.
- **Iterate potential to all order (in L.S. or Schrodinger eq.), with an ultraviolet Λ.**

Carried out to $N^5LO(Q^6/M^6_{hi})$

D. R. Entem, N. Kaiser, R. Machleidt and Y. Nosyk, PRC 92, 064001.

P. Reinert, H. Krebs and E. Epelbaum, arXiv:1711.08821.

$V(N^{n\geq 2}LO)$ performs as good as high accuracy $V_{\text{CDBonn, AV18, etc.,…}}$, if keep **500<Λ<875 MeV** (or, recently, **Λ=350~500** MeV).

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Carried out to $N^4LO(Q^5/M^5_{hi})$

D. R. Entem, N. Kaiser, R. Machleidt and Y. Nosyk, PRC 92, 064001.

P. Reinert, H. Krebs and E. Epelbaum, arXiv:1711.08821.

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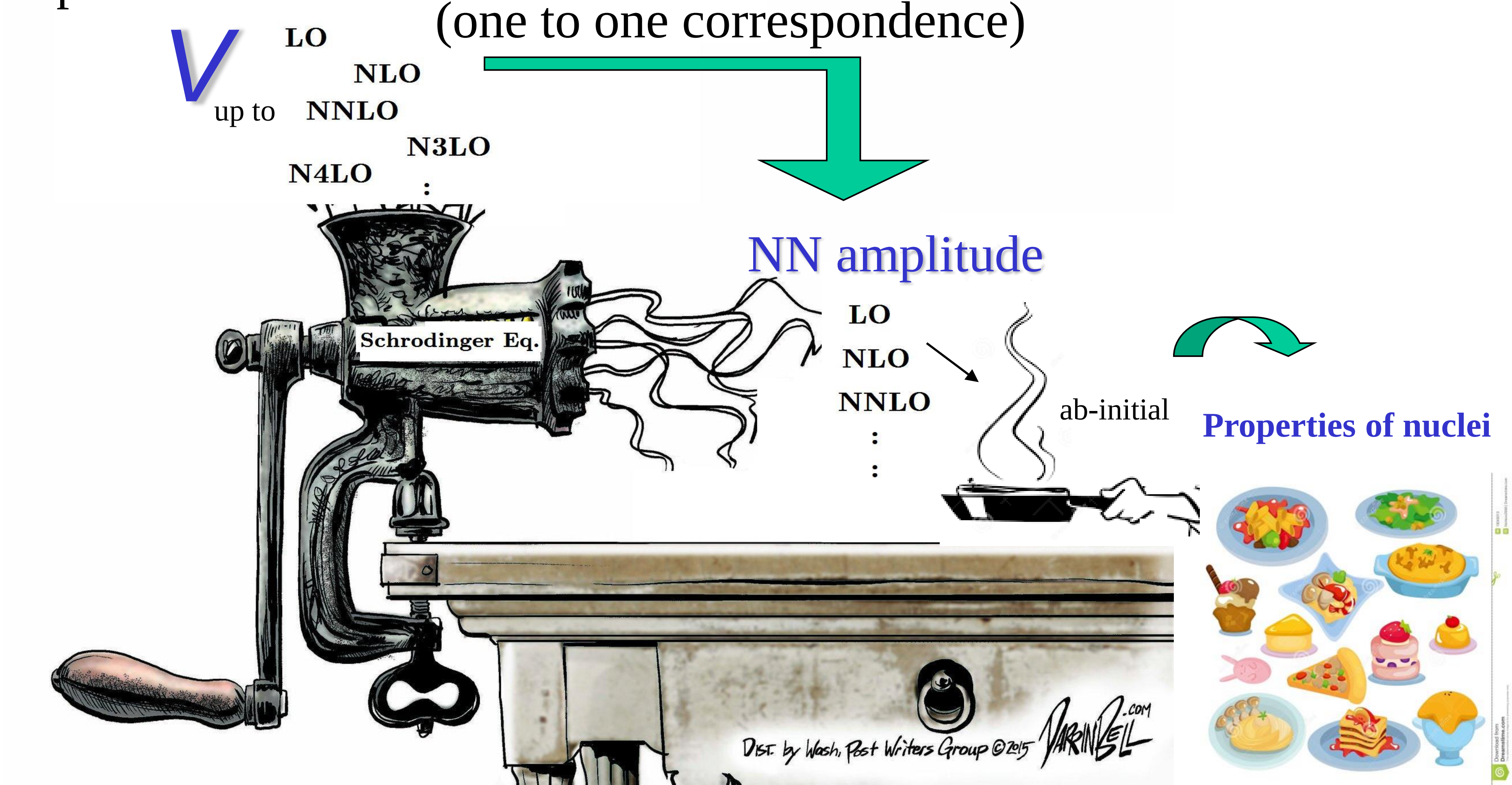
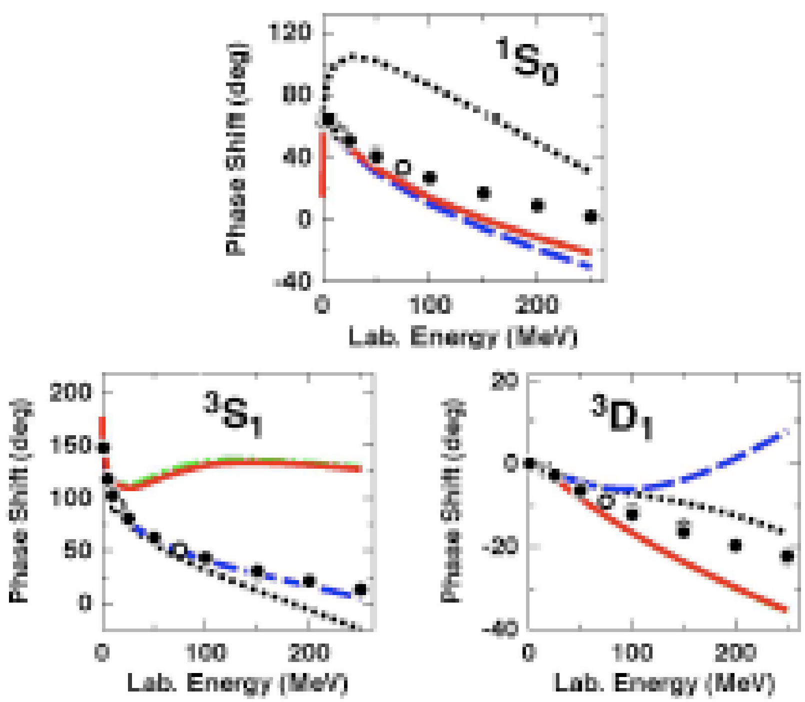
Hope:



Problems of WPC in RG

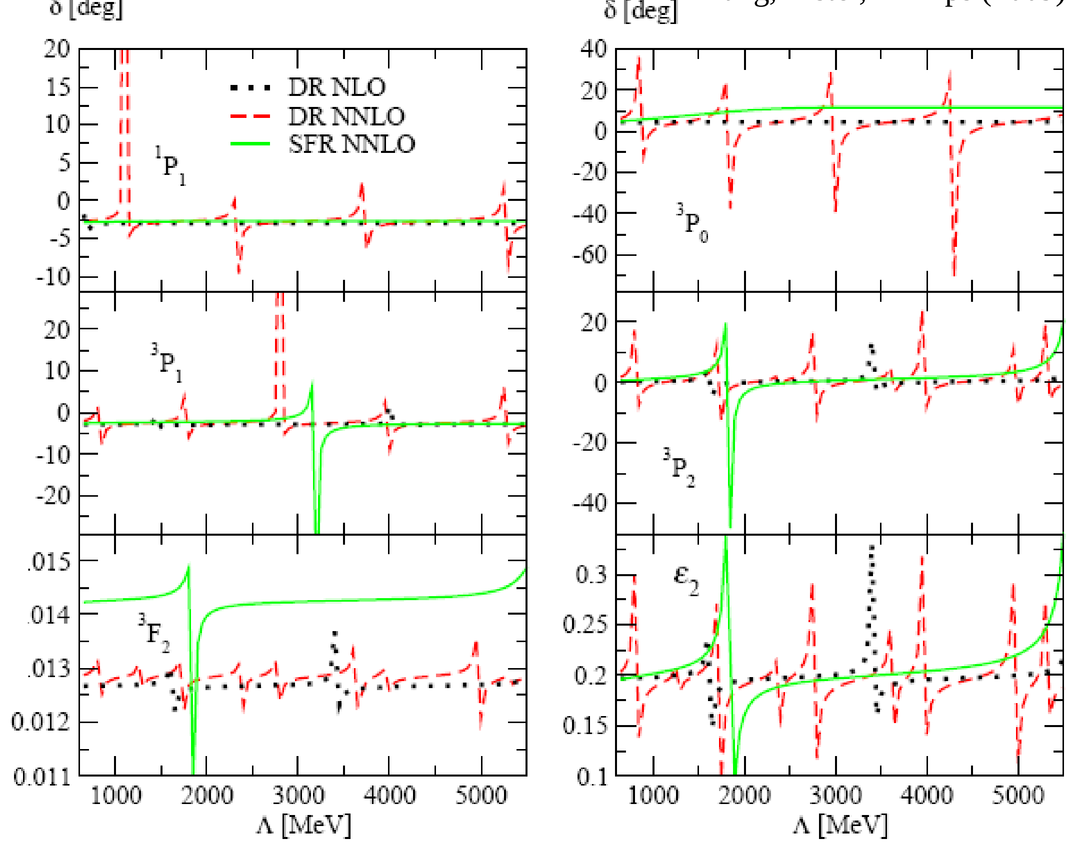
- Singular attractive potentials demand contact terms. (Nogga, Timmermans, van Kolck (2005))
- Beyond LO: Has RG problem at Λ>1 GeV (due to iterate to all order)



N^{3}LO(Q^4)

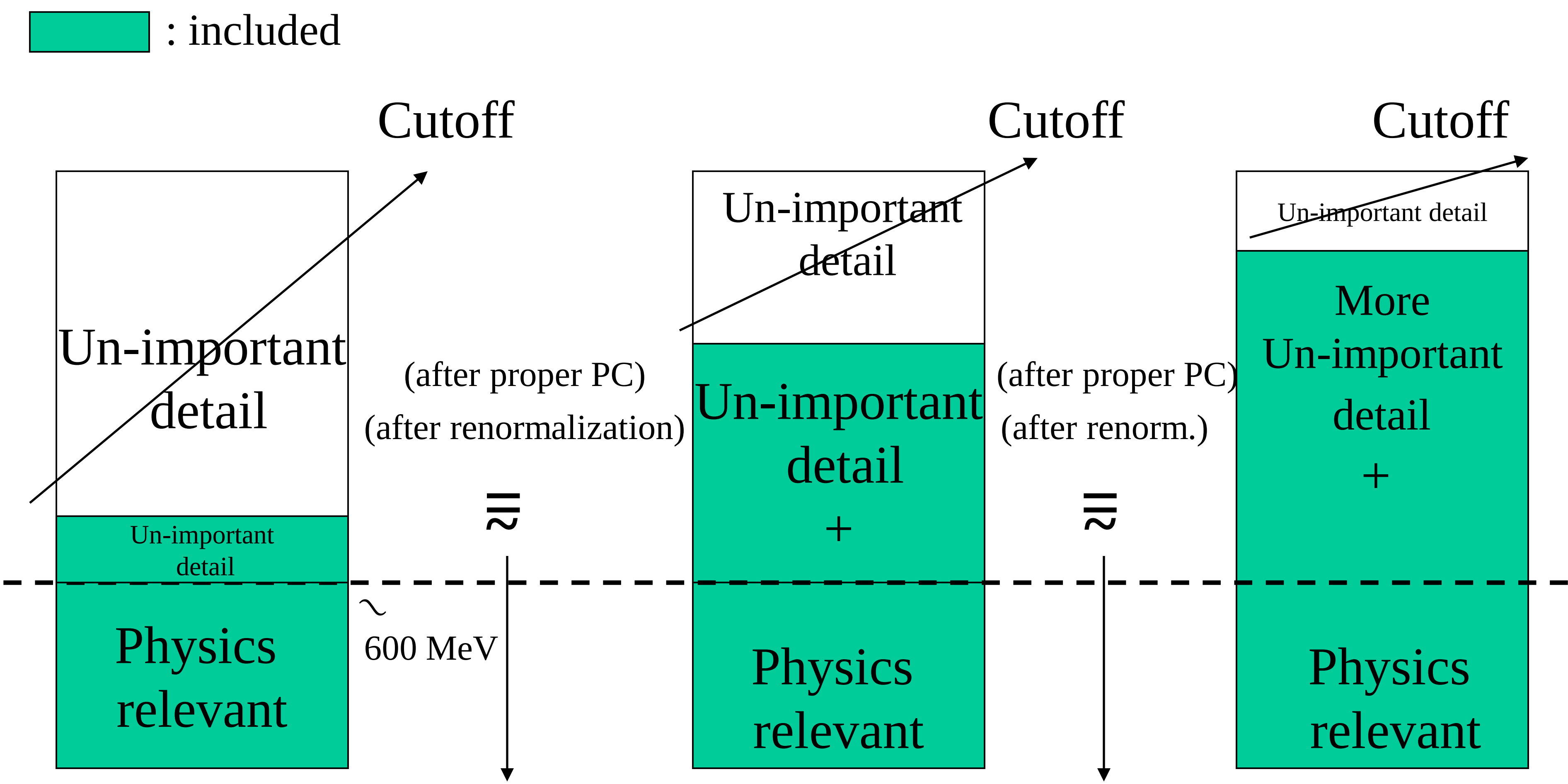
Ch. Zeoli R. Machleidt D. R. Entem (2012)

Yang, Elster, Phillips (2009)



Why is that a problem?

Renormalization group (RG)



***Only source of error**: given by the high order terms.

If not so, ➡ **the power counting isn't completely correct!**

(un-important are not really unimportant)

In the window of $500<\Lambda<875$ MeV

- Whether the conventional way happens to represent the reality, or, the problem just got hidden in the apparently o.k. fit of phase shifts ?
- It is safest/more reasonable, to develop a new power counting, which is more EFT.
- The ultimate way to check is through few-body and ab-initio nuclear structure calculations.

In the window of ~~$500<\Lambda<875$~~ MeV

350-500 MeV, recently

- Whether the conventional way happens to represent the reality, or, the problem just got hidden in the apparently o.k. fit of phase shifts ?
- It is safest/more reasonable, to develop a new power counting, which is more EFT.
- The ultimate way to check is through few-body and ab-initio nuclear structure calculations.

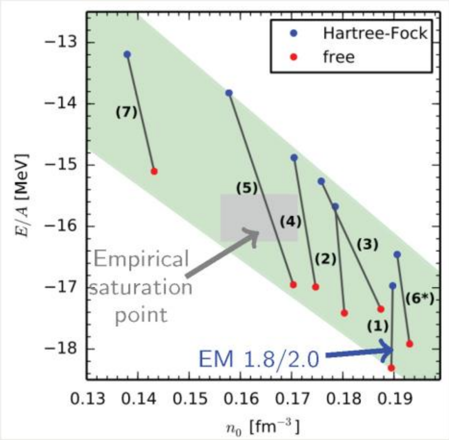
Motivation for developing new power counting

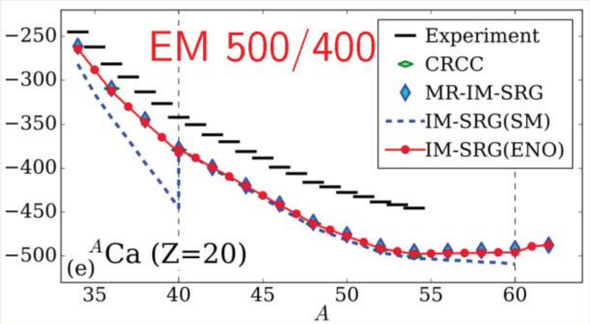
Some indications: nuclear structure

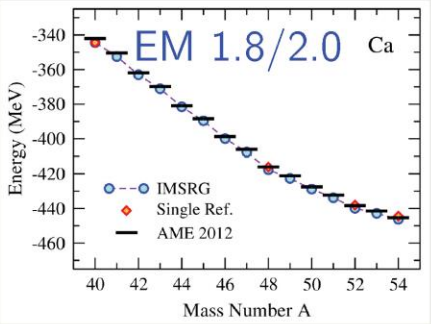
TRIUMF

One "magical" chiral (or chiral-inspired) interaction

	EM 500/400	EM 1.8/2.0
NN	N^3LO	same
	$\Lambda_{2N} = 500$ MeV	same
	non-local regulator	same
	fit to NN scattering, ^{2}H	same
	$\lambda_{SRG} = 1.88\ \mathrm{fm}^{-1}$	$\approx$ same
3N	N^2LO	same
	$\Lambda_{3N} = 400$ MeV	$\approx$ same
	local regulator	**non-local regulator**
	fit to ^{3}H BE, $t_{1/2}$	**fit to ^{3}H BE, ^{4}He r_{ch}**
	consistently SRG evolved	**no SRG for 3N**







- Neither interaction is fully consistent

  however...

- Saturation properties appear important for finite nuclei

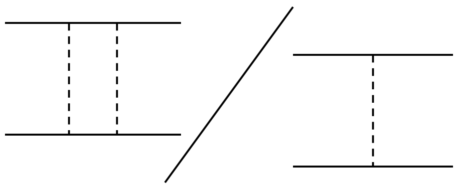
Hebeler et al. PRC(R) (2011), Drischler et al. PRC (2016), Simonis et al. (in prep.)

Talk by R. S. Stroberg, ESNT workshop 2017

New power counting
Long & Yang, (2010-2012)

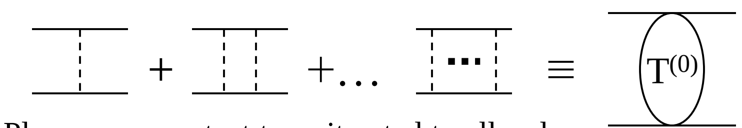
LO: Still iterate to all order (at least for most l<2).

Reason: van Kolck, Bedaque,… etc.



$\sim \frac{g_A^2 M}{8\pi f_\pi^2} Q$

Thus, at LO:



Plus proper contact term iterated to all order

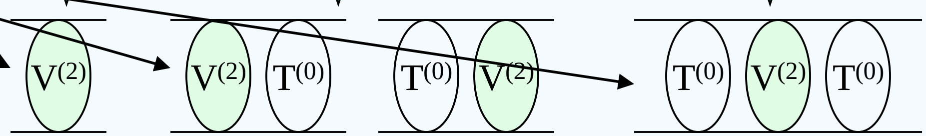
Start at NLO, do perturbation.

$(T = T^{(0)}+T^{(1)}+T^{(2)}+T^{(3)}+\ldots)$

If $V^{(1)}$ is absent:

$$T^{(2)} = V^{(2)} + 2V^{(2)}GT^{(0)} + T^{(0)}GV^{(2)}GT^{(0)}.$$

One insertion of $V^{(2)}$ in $T^{(0)}$



$$G \equiv \frac{2M_N}{\pi}\int_0^{\Lambda} \frac{p^2 dp}{p_0^2 - p^2 + i\varepsilon}$$

$$T^{(3)} = V^{(3)} + 2V^{(3)}GT^{(0)} + T^{(0)}GV^{(3)}GT^{(0)}.$$

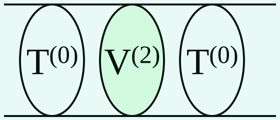
$$V^{(n)} = V^{(n)}_{Long} + V^{(n)}_{Short};$$

$$V^{(n)}_{Long}: \text{pion-exchange at } O\left((\frac{\mathrm{Q}}{\mathrm{M_{hi}}})^{\mathrm{n}} \right)$$

$$V^{(n)}_{Short}: \text{ counter terms, } \qquad \underbrace{C_0 + C_2 q^2 + C_4 q^4 +}_{\text{value of } C\text{'s decided from renormalization}} \qquad \ldots$$

3 types of counter terms (determined by RG)

1. Primordial: Those renormalize the pion-exchange diagrams.
   (always there if survived from partial-wave decomposition)
2. Distorted –wave counter terms

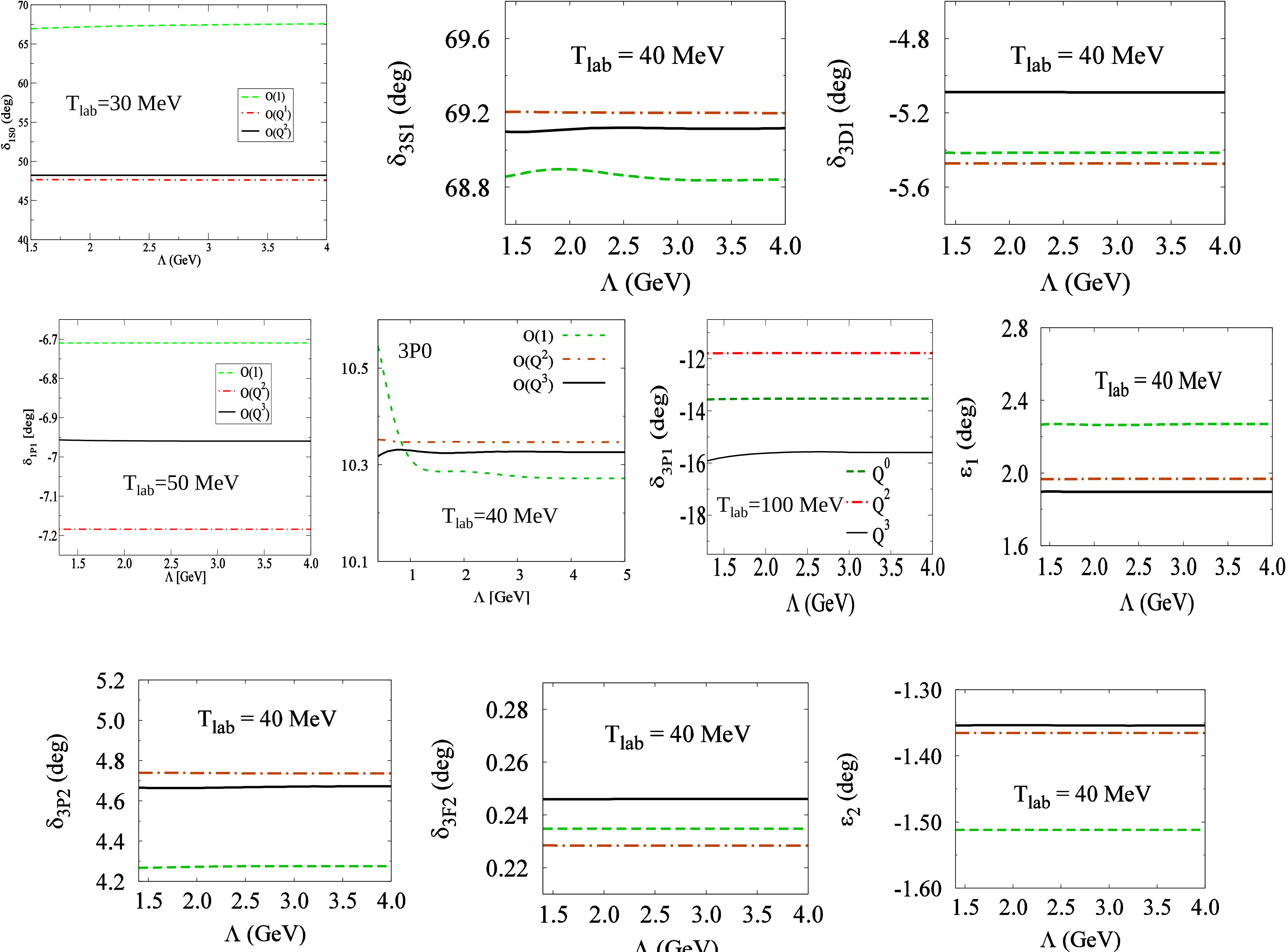


could diverge more than Q^2

3. Residual counter terms: Decided by the requirement from RG.

e.g., if $|T^{(n)}(k;\Lambda) - T^{(n)}(k;\infty)| \geq O(\frac{Q^{n+2}}{M_{hi}^{n+2}})$, then need V^{n+1}_{Short} at order n+1.

Results
(All RG-invariant)



δ1S0 (deg)
Λ (GeV)
Tlab=30 MeV
O(1)
O(Q1)
O(Q2)
δ3S1 (deg)
Tlab = 40 MeV
Λ (GeV)
δ3D1 (deg)
Tlab = 40 MeV
Λ (GeV)
δ1P1 [deg]
O(1)
O(Q2)
O(Q3)
Tlab=50 MeV
Λ [GeV]
3P0
O(1)
O(Q2)
O(Q3)
Tlab=40 MeV
Λ [GeV]
δ3P1 (deg)
Q0
Tlab=100 MeV
Q2
Q3
Λ (GeV)
ε1 (deg)
Tlab = 40 MeV
Λ (GeV)
δ3P2 (deg)
Tlab = 40 MeV
Λ (GeV)
δ3F2 (deg)
Tlab = 40 MeV
Λ (GeV)
ε2 (deg)
Tlab = 40 MeV
Λ (GeV)

Table of notations

L&Y	WPC	Potential enters
LO(Q^{-1})	WPC(1, or, Q^{-1*}) *E. Epelbaum, J. Gegelia, Ulf-G. Meißner Nucl. Phys. B925 (2017) 161-185	L&Y: $(\text{OPE+proper contact term})_{\text{iter-to-all}}$ WPC: $(\text{OPE+wpc contact term})_{\text{iter-to-all}}$
NLO(Q^0)		L&Y: Only 1S0: $D(p^2+p'^2)$
NNLO(Q^1)		L&Y: Leading TPE+ proper contact terms.
N3LO(Q^2)	WPC(Q^2)	L&Y: Subleading TPE+ proper contact terms. WPC: $(\text{Leading TPE+wpc contact term})_{\text{iter-to-all}}$
	WPC(Q^3)	L&Y: Haven't done. WPC: $(\text{Subleading TPE+wpc contact term})_{\text{iter-to-all}}$

Check power counting: Modified-Lepage plot

Any EFT the following must be true:

Residual, ~O(1) if: 1. EFT works
2. $\Lambda \geq \overline{\Lambda}_{EFT}$

observables

order

$$\mathcal{O}(k, p_{typ}; \Lambda; \overline{\Lambda}_{EFT}) = \sum_{i}^{n} \left(\frac{k, p_{typ}}{\overline{\Lambda}_{EFT}}\right)^{i} \mathcal{O}_i(k, p_{typ}; \overline{\Lambda}_{EFT}) + \mathscr{C}_n(\Lambda; k, p_{typ}, \overline{\Lambda}_{EFT}) \left(\frac{k, p_{typ}}{\overline{\Lambda}_{EFT}}\right)^{n+1}$$

cutoff

Breakdown scale
(given by 1st meson not included)

residual cutoff-dep.

No cutoff here! => *physics cannot dep. on cutoff !*

H. W. Griesshammer, arXiv:1511.00490v3 [nucl-th].

Lepage plot: subtract at two Λ's to extract "n+1"

- For a particular order n, choose (Λ_1, Λ_2), after renormalized the L.E.C.s

$\rightarrow$get (O_n (k,m_π;Λ_1), O_n(k, m_π;Λ_2)).

- After subtracting each other, the part independent of cutoff (i.e., the physical part) cancels out, and we are left with residual cutoff-dependence, which is of order "n+1" in the expansion.
- Divide the above by O_n (k,m_π;Λ_1):

=> Numerator ~$(k/\Lambda_{EFT})^{n+1}$. The denominator contains $(m_\pi/\Lambda_{EFT})^{n+1}$,which is $O(k^0)$, so that the slope we will get is "n+1".

$$\frac{\mathcal{O}_n(k,p_{typ};\Lambda_1)-\mathcal{O}_n(k,p_{typ};\Lambda_2)}{\mathcal{O}_n(k,p_{typ};\Lambda_1)}=\left(\frac{k,p_{typ}}{\overline{\Lambda}_{EFT}}\right)^{n+1}\frac{\mathscr{C}_n(\Lambda_1;k,p_{typ},\overline{\Lambda}_{EFT})-\mathscr{C}_n(\Lambda_2;k,p_{typ},\overline{\Lambda}_{EFT})}{\mathscr{C}_n(\Lambda_1;k,p_{typ},\overline{\Lambda}_{EFT})}$$

Question: how high should one fits the phase shift?

- If we renormalize L.E.C.s near $T_{lab}\rightarrow 0$, then:

=> The outcoming phase shifts becomes bad in the rest of the place. This is not what Pionful theory is designed for.

- If we renormalize L.E.C.s up to the breakdown scale (Λ_{EFT}), then:

$\Rightarrow$There is no region left for prediction.

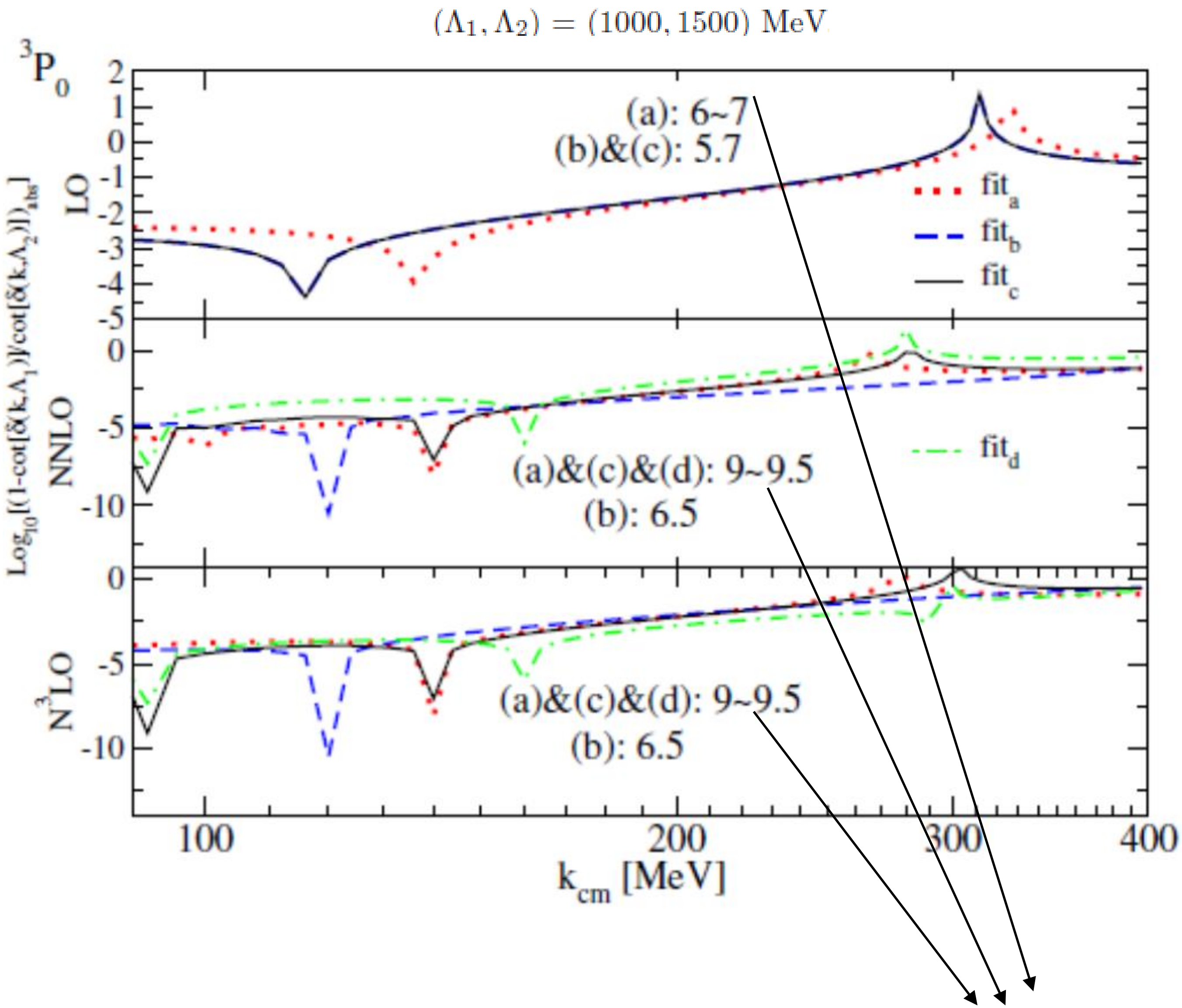
Thus, fit up to k_{cm}~140 MeV. And use $k_{cm}\subset[160,450]$ to extracted the slope.

Check effect of different fitting range

Take 3p0 as an example

In the modified PC (Long & Yang)

	LO (1 LEC)	NNLO and N^{3}LO (2 LECs)
fit$_a$	$k \sim 140$ MeV	$k \sim 68$ and 140 MeV
fit$_b$	$k \sim 116$ MeV	$k \sim 68$ and 120 MeV
fit$_c$	$k \sim 116$ MeV	$k \sim 92$ and 140 MeV
fit$_d$	$k \sim 116$ MeV	$k \sim 92$ and 160 MeV



Have small dependence if the highest fitting point ≥140 MeV.
Otherwise, the effect is not small.

(e.g., (b): highest fitting point ~120 MeV ⟶ behaves more like pionless theory).

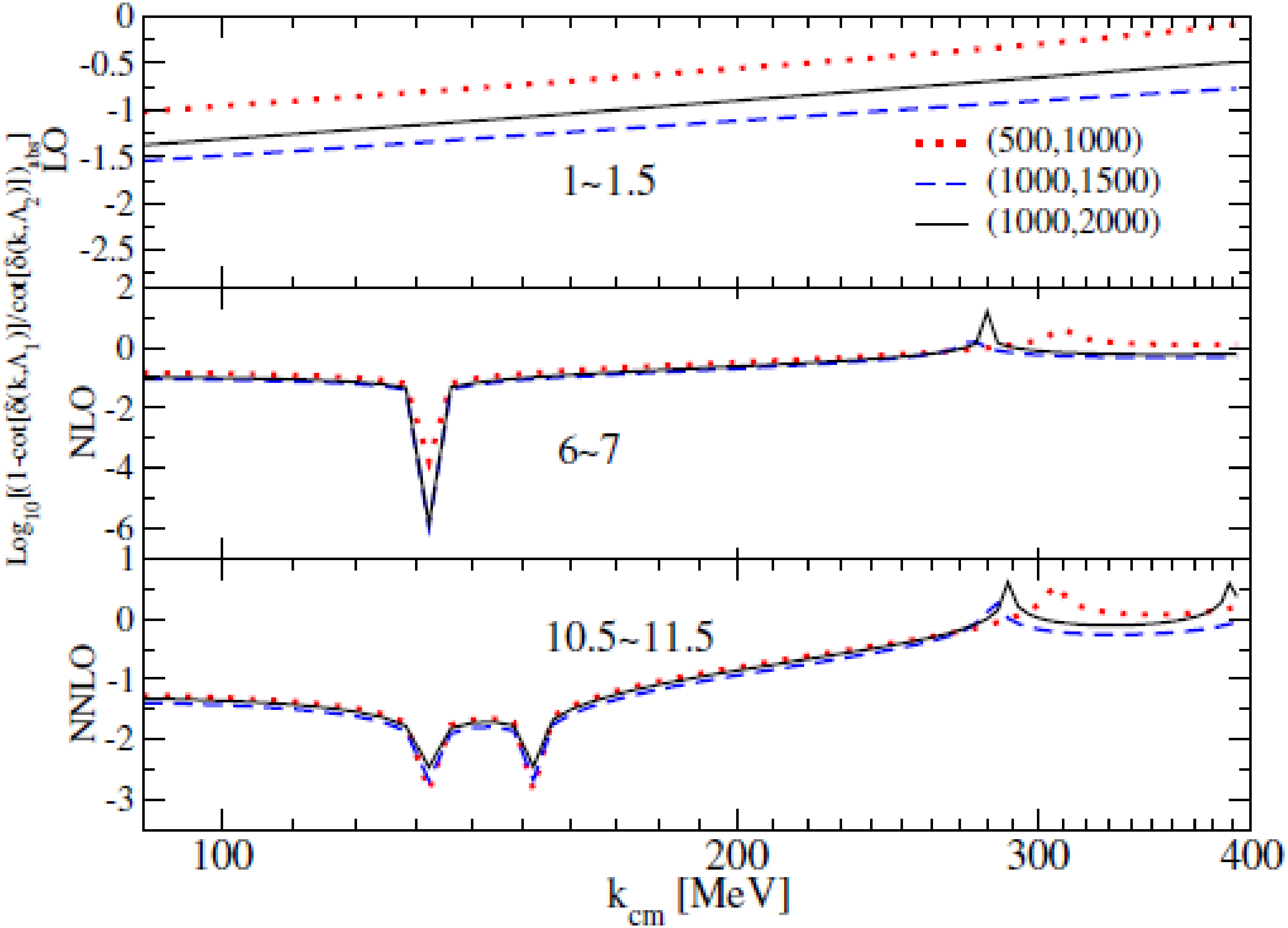
Fitting strategy

- In general, perform best fit up to k_{cm}=140 MeV.
- For $^1S_0$ at LO=>fit to a_0 (because for Λ>500 MeV, the phase shift deviate from Nijmegen a lot.)
- For subleading TPE, try both c_3=-4.7 and c_3 =-3.4 GeV^{-1}, turns out has almost no impact no the outcoming slope.

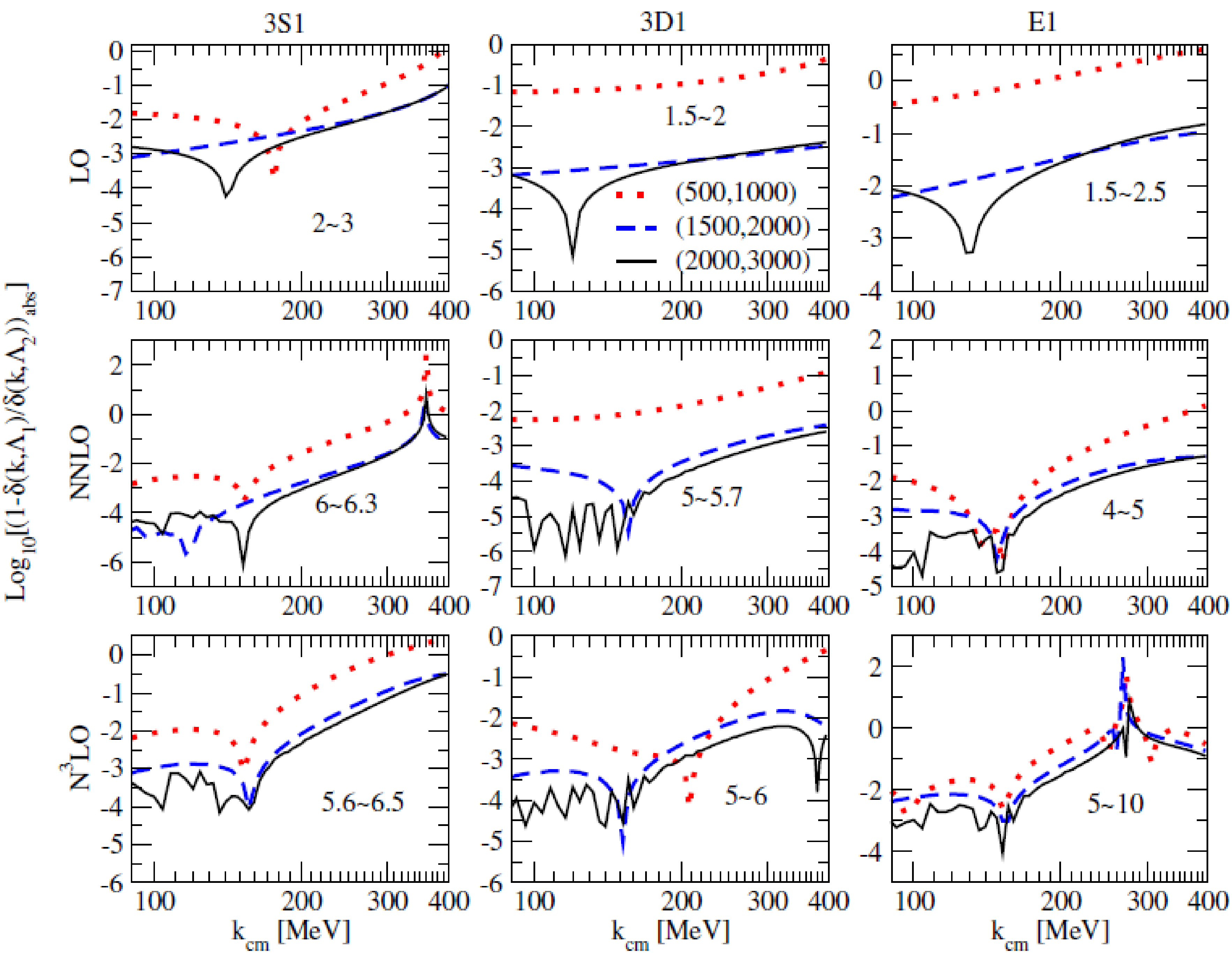
Result of the new power counting (Long & Yang)

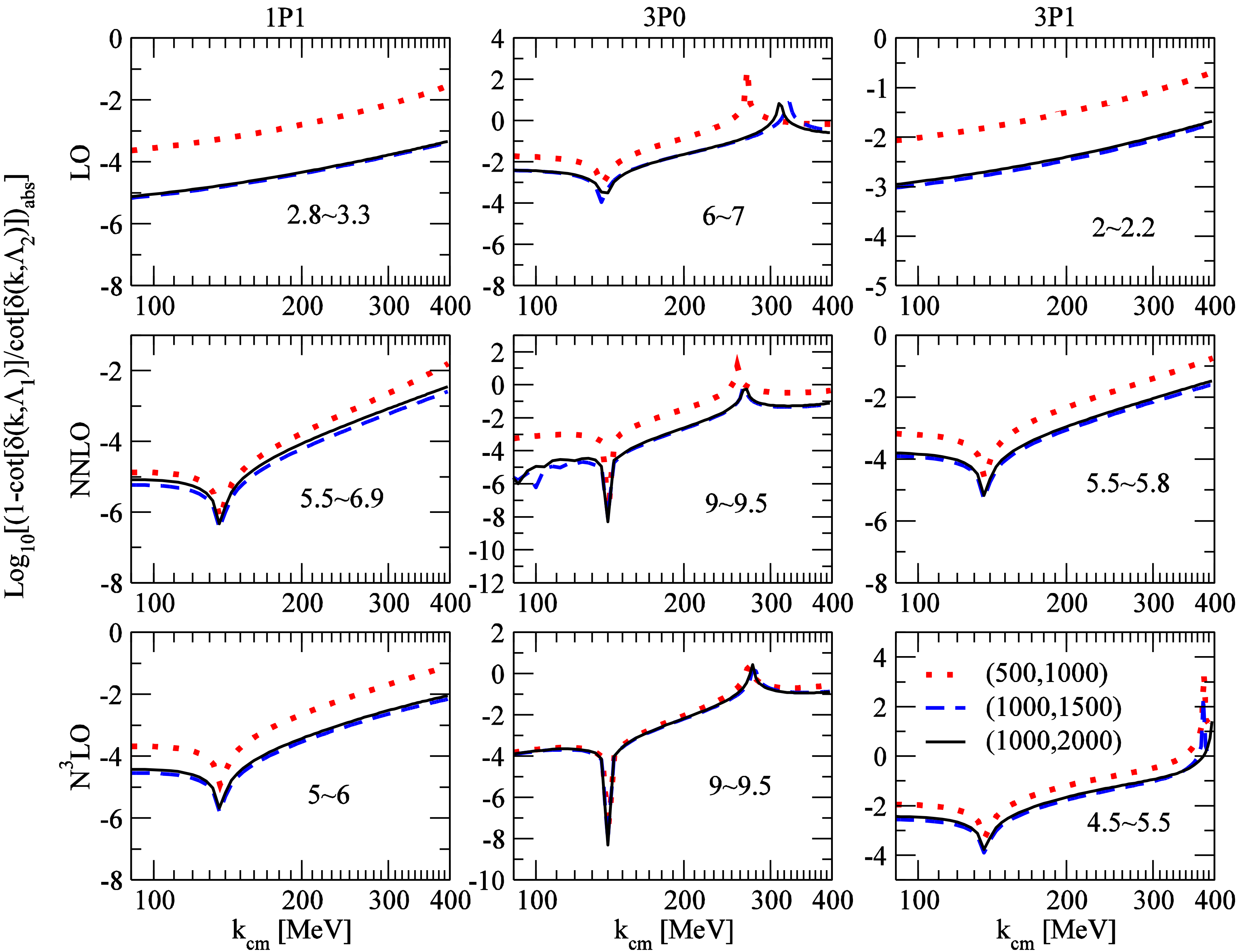
Phys. Rev. C84, 057001 (2011)
Phys. Rev. C85, 034002 (2012)
Phys. Rev. C86, 024001 (2012).

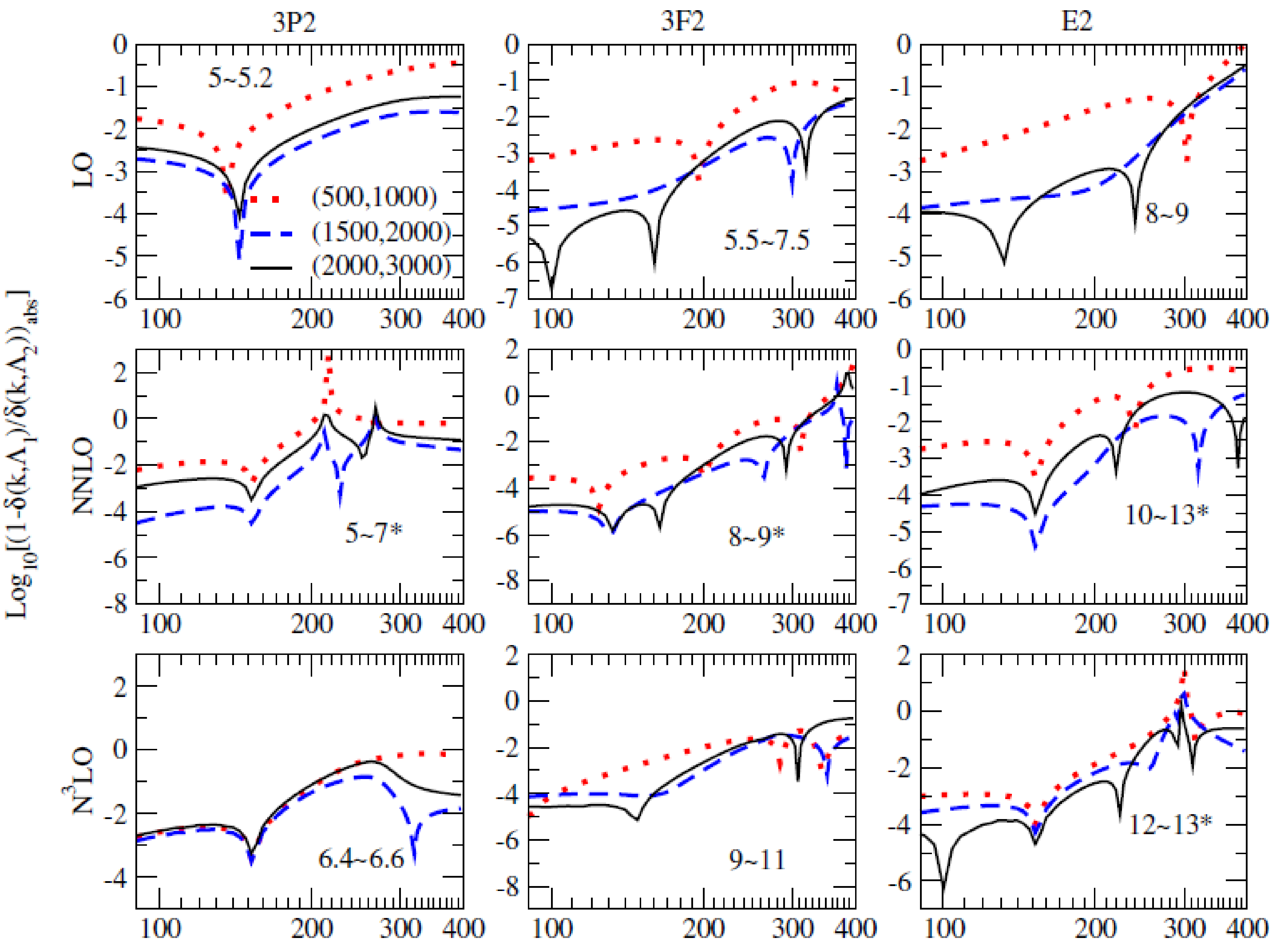
1S0



(500,1000)
(1000,1500)
(1000,2000)
1~1.5
6~7
10.5~11.5
LO
NLO
NNLO
$\mathrm{Log}_{10}[(1-\cot[\delta(k,\Lambda_1)]/\cot[\delta(k,\Lambda_2)])_{abs}]$
k_{cm} [MeV]


3S1
3D1
E1
LO
NNLO
N3LO
Log10[(1-δ(k,Λ1)/δ(k,Λ2))abs]
2~3
1.5~2
1.5~2.5
(500,1000)
(1500,2000)
(2000,3000)
6~6.3
5~5.7
4~5
5.6~6.5
5~6
5~10
kcm [MeV]


1P1
3P0
3P1
LO
NNLO
N3LO
Log10[(1-cot[δ(k,Λ1)]/cot[δ(k,Λ2)])abs]
2.8~3.3
6~7
2~2.2
5.5~6.9
9~9.5
5.5~5.8
5~6
9~9.5
4.5~5.5
(500,1000)
(1000,1500)
(1000,2000)
kcm [MeV]



3P2
3F2
E2
LO
NNLO
N^3LO
$Log_{10}[(1-\delta(k,\Lambda_1)/\delta(k,\Lambda_2))_{abs}]$
5~5.2
(500,1000)
(1500,2000)
(2000,3000)
5.5~7.5
8~9
5~7*
8~9*
10~13*
6.4~6.6
9~11
12~13*

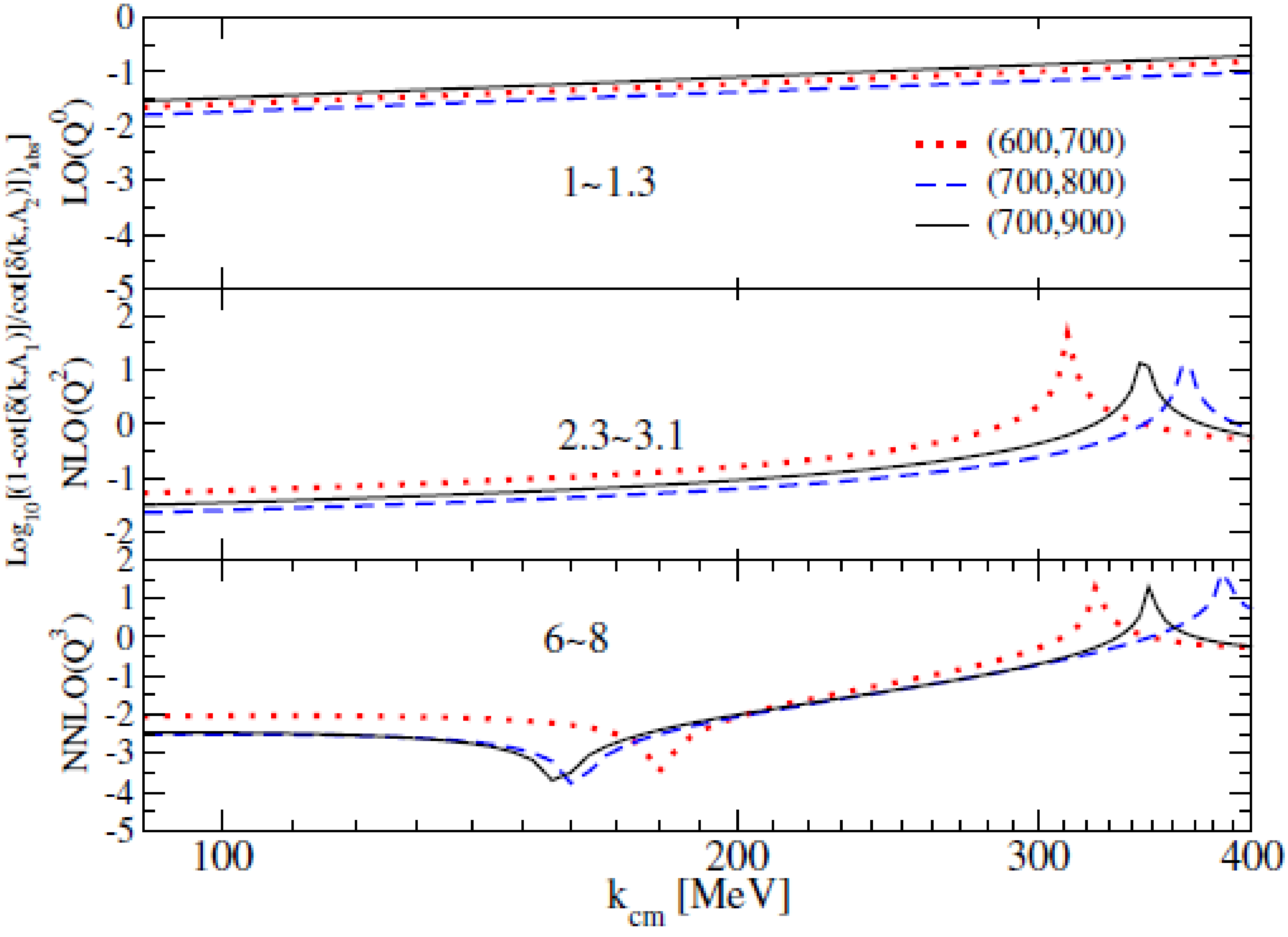
Try the same for WPC

- RG fails at Λ>1 GeV. Try Λ=500~900 MeV. Additionally, try Λ=350~500 MeV for uncoupled p-waves.

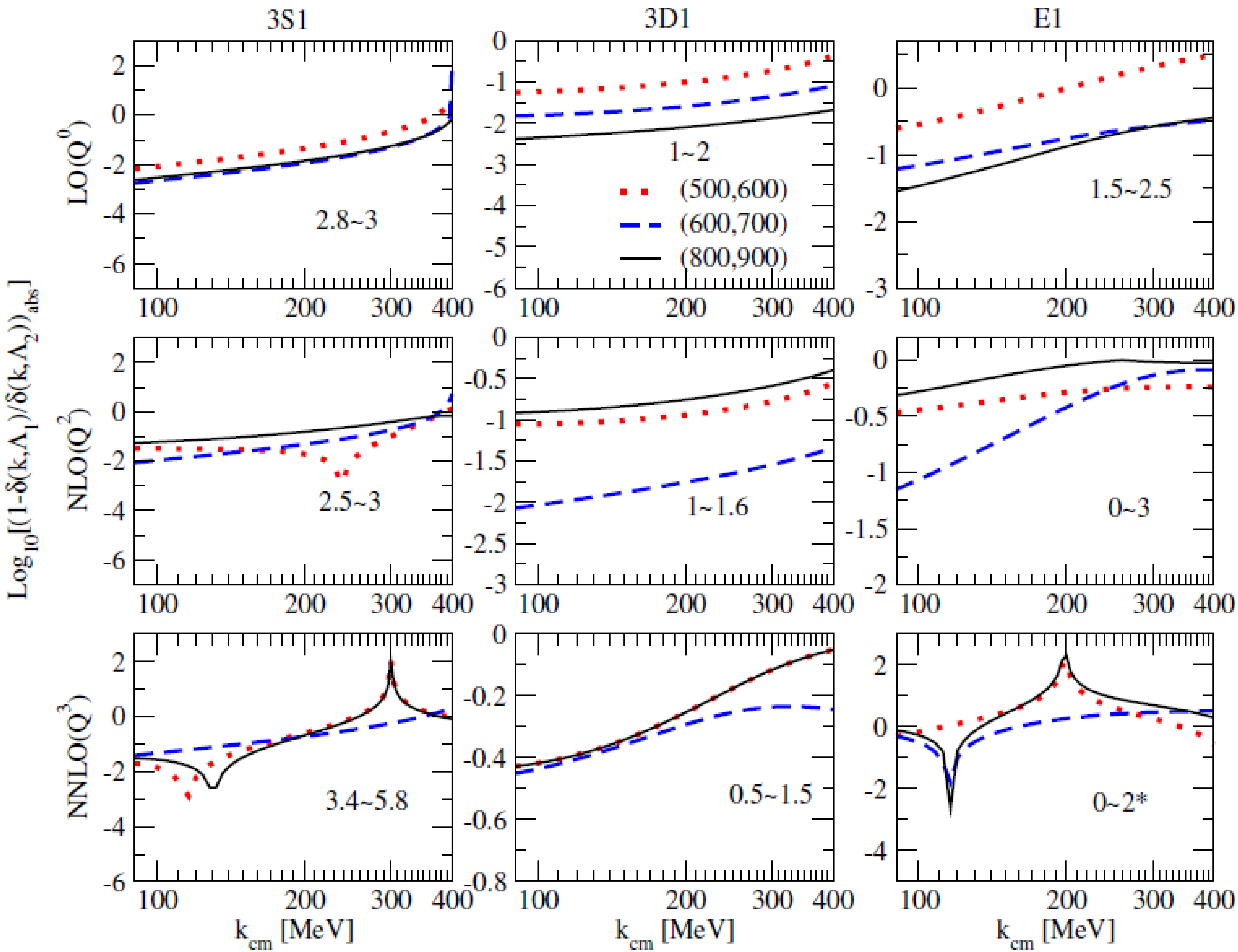
=>The pre-factor of residual cutoff dep. term may not be *O(1)*. Results might not be that meaningful.

- Fitting strategy: exactly the same as before.

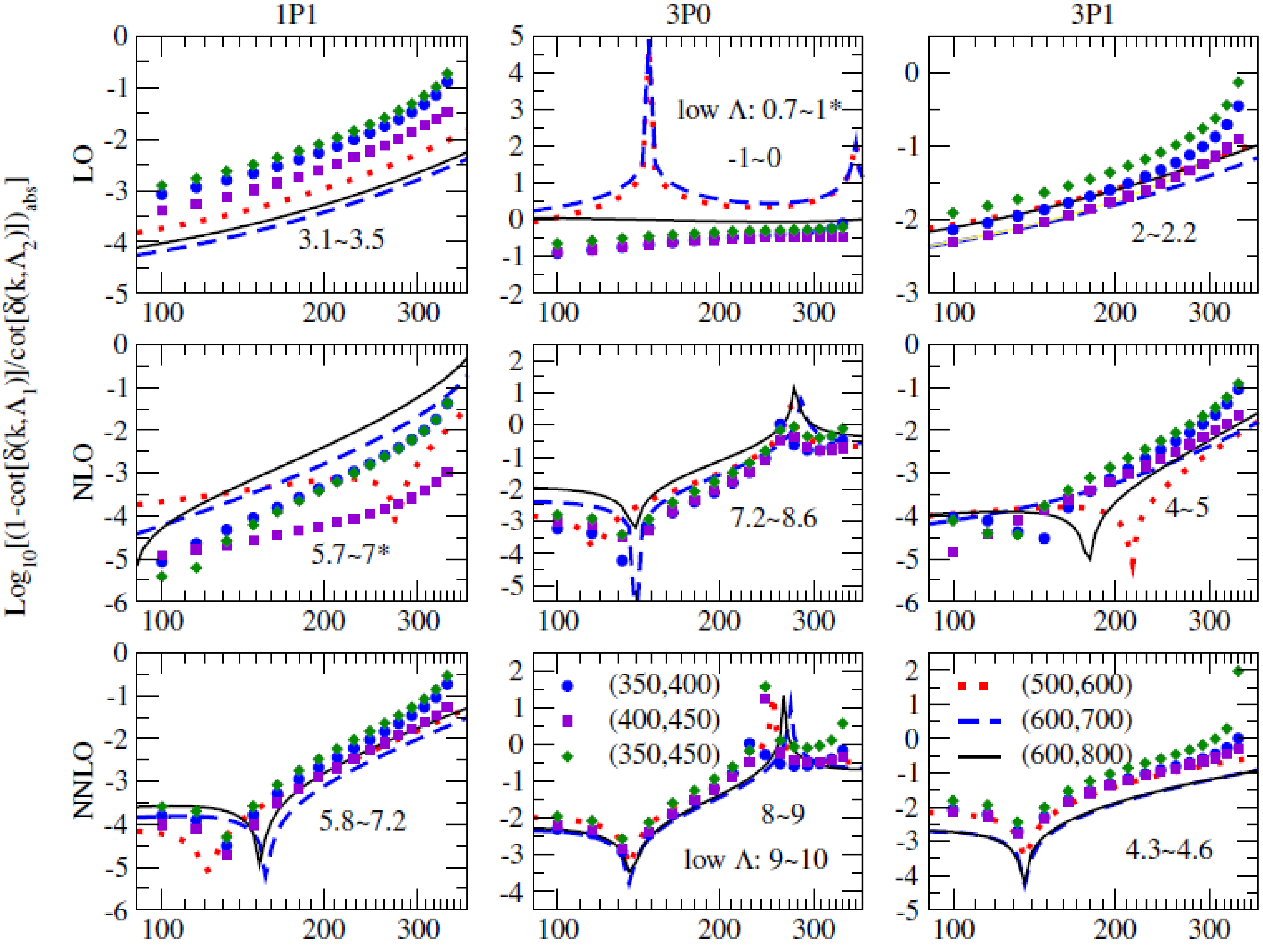
1S0



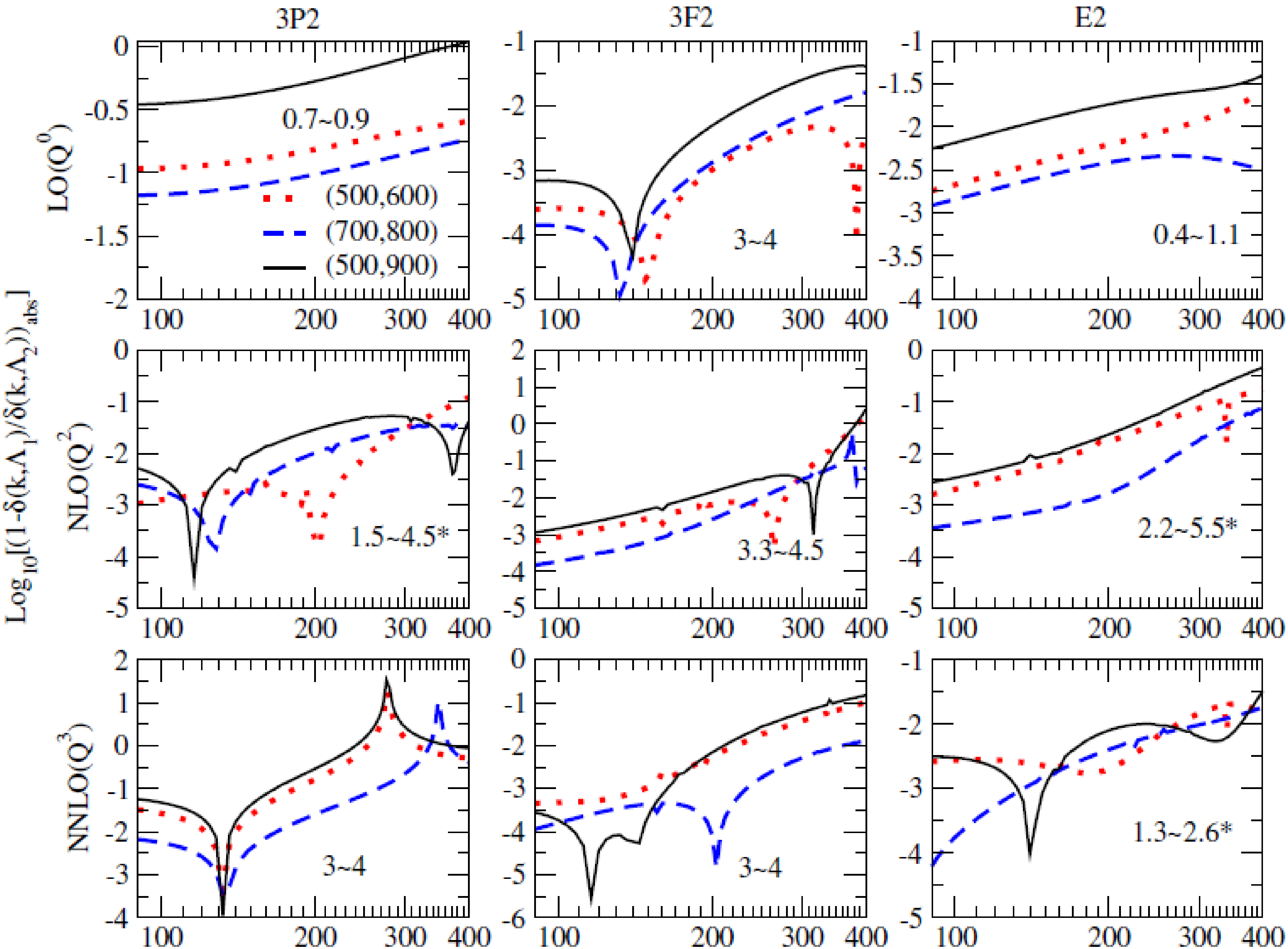
LO(Q^0)
NLO(Q^2)
NNLO(Q^3)
$\mathrm{Log}_{10}[(1-\cot[\delta(k,\Lambda_1)]/\cot[\delta(k,\Lambda_2)])_{abs}]$
1~1.3
2.3~3.1
6~8
(600,700)
(700,800)
(700,900)
k_{cm} [MeV]



3S1
3D1
E1
LO(Q^0)
NLO(Q^2)
NNLO(Q^3)
$Log_{10}[(1-\delta(k,\Lambda_1)/\delta(k,\Lambda_2))_{abs}]$
2.8~3
1~2
1.5~2.5
2.5~3
1~1.6
0~3
3.4~5.8
0.5~1.5
0~2*
(500,600)
(600,700)
(800,900)
k_{cm} [MeV]



1P1
3P0
3P1
LO
NLO
NNLO
Log10[(1-cot[δ(k,Λ1)])/cot[δ(k,Λ2)])abs]
3.1~3.5
low Λ: 0.7~1*
-1~0
2~2.2
5.7~7*
7.2~8.6
4~5
5.8~7.2
8~9
low Λ: 9~10
4.3~4.6
(350,400)
(400,450)
(350,450)
(500,600)
(600,700)
(600,800)



3P2
3F2
E2
LO(Q^0)
NLO(Q^2)
NNLO(Q^3)
$\mathrm{Log}_{10}[(1-\delta(k,\Lambda_1)/\delta(k,\Lambda_2))_{\mathrm{abs}}]$
(500,600)
(700,800)
(500,900)
0.7~0.9
3~4
0.4~1.1
1.5~4.5*
3.3~4.5
2.2~5.5*
3~4
3~4
1.3~2.6*

Summary of results

		LO(Q^{-1}),WPC(Q^{-1})	NLO(Q^0)	NNLO(Q^1)	N^3LO(Q^2),WPC(Q^2)	WPC(Q^3)
$^1S_0$ LY	n+1 extracted	1~1.5	6~7	10.5~11.5		
	n+1 predicted	0	1	2		
$^1S_0$ WPC	n+1 extracted	1~1.3			2.3~3.1 !	6~8
	n+1 predicted	0~2			3	4
$^3S_1$ LY	n+1 extracted	2~3		6~6.3	5.6~6.5	
	n+1 predicted	1		2	3	
$^3S_1$ WPC	n+1 extracted	2.8~3			2.5~3 !	3.4~5.8
	n+1 predicted	0~2			3	4

Black: Same or better than prediction.

Red: Worse than prediction.

Blue: +1 possible

$^3D_1$ LY	n+1 extracted	1.5~2		5~5.7	5~6	
	n+1 predicted	1		2	3	
$^3D_1$ WPC	n+1 extracted	1~2			1~1.6 !	0.5~1.5!
	n+1 predicted	0~2			3	4
E_1 LY	n+1 extracted	1.5~2.5		4~5	5~10	
	n+1 predicted	1		2	3	
E_1 WPC	n+1 extracted	1.5~2.5			0~3 !	0~2*!
	n+1 predicted	0~2			3	4

		LO(Q^{-1}),WPC(Q^{-1})	NLO(Q^{0})	NNLO(Q^{1})	N^3LO(Q^{2}),WPC(Q^{2})	WPC(Q^{3})
$^1P_1$ LY	n+1 extracted	2.8~3.3		5.5~6.9	5~6	
	n+1 predicted	1 plus 1~1.7		2 plus 1~1.7	3 plus 1~1.7	
$^1P_1$ WPC	n+1 extracted	3.1~3.5			5.7~7*	5.8~7.2
	n+1 predicted	0~2			3	4
$^3P_0$ LY	n+1 extracted	6~7		9~9.5	9~9.5	
	n+1 predicted	1 plus 1~1.7#		2 plus 1~1.7#	3 plus 1~1.7#	
$^3P_0$ WPC	n+1 extracted	-1~0 (0.7~1) !			7.2~8.6	8~9 (9~10)
	n+1 predicted	0~2			3	4
$^3P_1$ LY	n+1 extracted	2~2.2		5.5~5.8	4.5~5.5	
	n+1 predicted	1 plus 1~1.7#		2 plus 1~1.7#	3 plus 1~1.7#	
$^3P_1$ WPC	n+1 extracted	2~2.2			4~5	4.3~4.6
	n+1 predicted	0~2			3	4
$^3P_2$ LY	n+1 extracted	5~5.2		5~7*	6.4~6.6	
	n+1 predicted	1 plus 1~1.7#		2 plus 1~1.7#	3 plus 1~1.7#	
$^3P_2$ WPC	n+1 extracted	0.7~0.9			1.5~4.5 !	3~4 !
	n+1 predicted	0~2			3	4
$^3F_2$ LY	n+1 extracted	5.5~7.5		8~9*	10~11	
	n+1 predicted	1 plus 1~1.7#		2 plus 1~1.7#	3 plus 1~1.7#	
$^3F_2$ WPC	n+1 extracted	3~4			3.3~4.5	3~4 !
	n+1 predicted	0~2			3	4
E_2 LY	n+1 extracted	8~9		10~13*	12~13*	
	n+1 predicted	1 plus 1~1.7#		2 plus 1~1.7#	3 plus 1~1.7#	
E_2 WPC	n+1 extracted	0.4~1.1			2.2~5.5*	1.3~2.6*!
	n+1 predicted	0~2			3	4

Plus 1~1.7 => relative demotion (1p1 w.r.t. 1s0) due to the centrifugal barrier.

M. P. Valderrama, et al, PRC95 054001 (2017).

Blue: Assuming the same demotion for other p-waves.

Observations & Lessons

- In general, new power counting has larger slope than WPC => imply faster convergence, and less cutoff dependence.
- No improvement from the order where leading-TPE enters w.r.t. NLO-TPE, regardless new PC or WPC.=> need Δ(1232).
- WPC fails in some singular attractive channels (within the limited range Λ=500-900 MeV).

You can do better than prediction, but not worse.

Road Map for the future

A power counting scheme

Choose relevant data to fit: NN, 3H, 4He, etc…

Renormalization-group analysis

+

power counting check

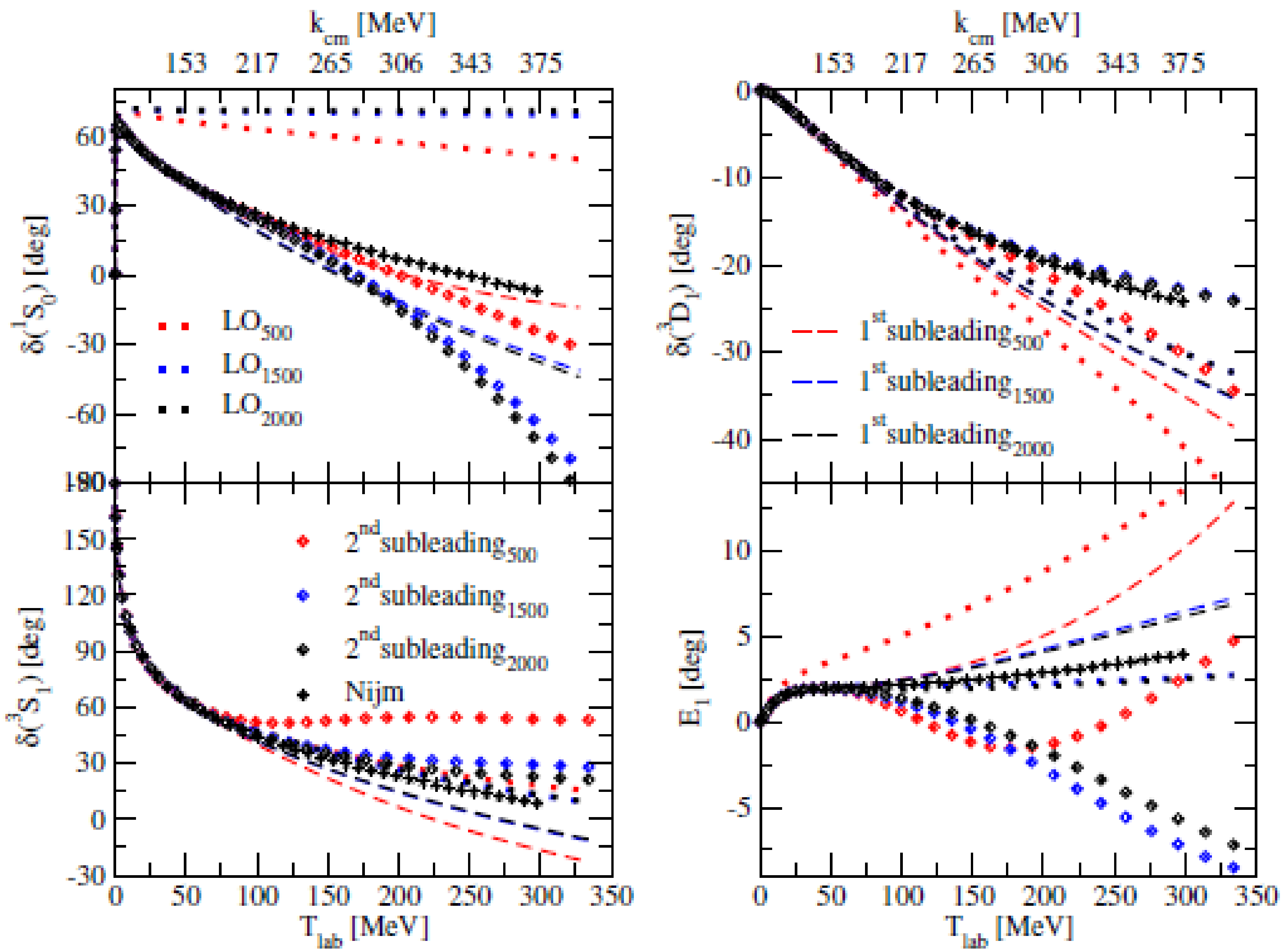
At each order: varying the cutoffs

Obtain the correct power counting.

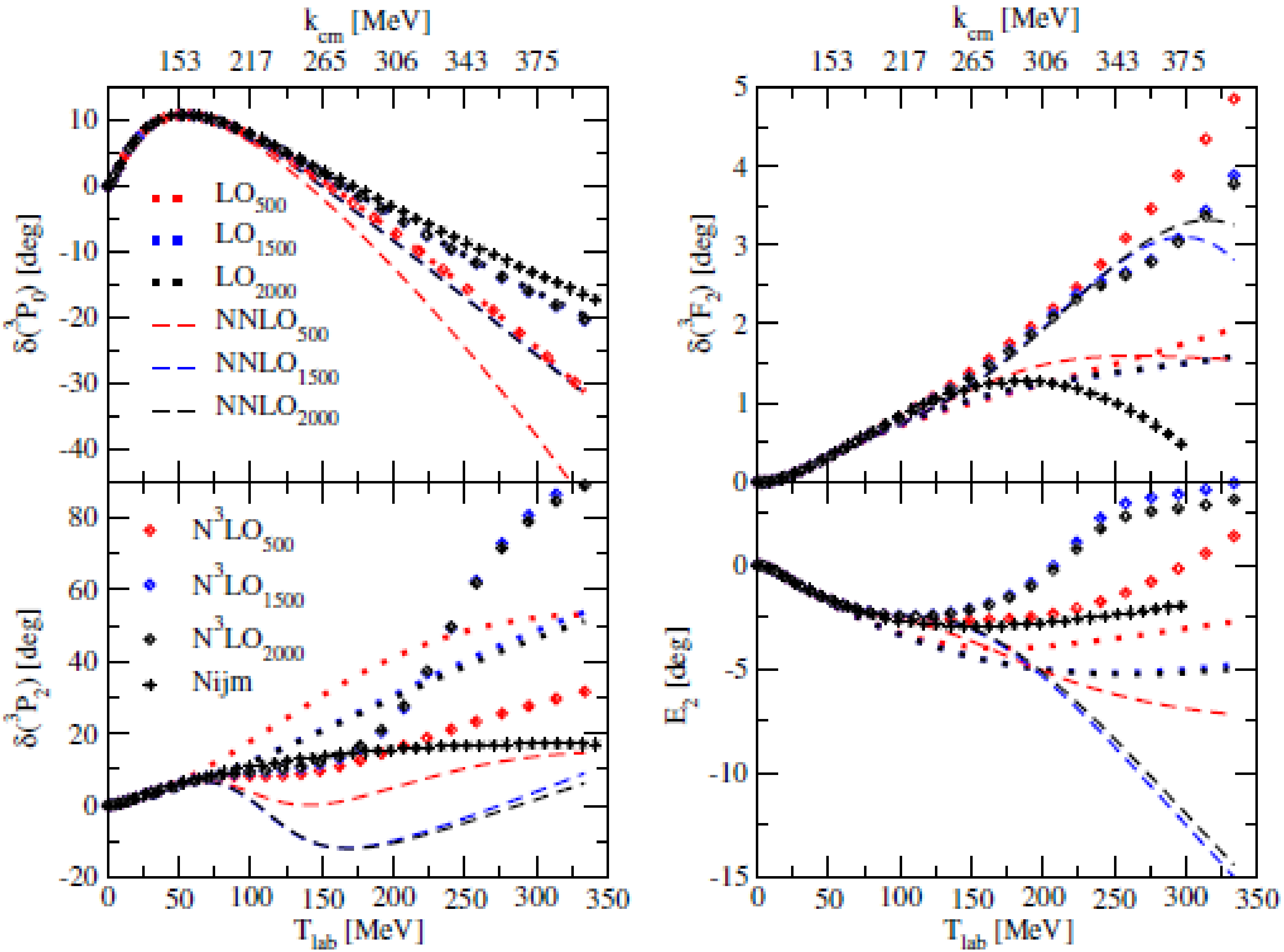
Optimize the fit.

Nuclear physics with true EFT.

Thank you!

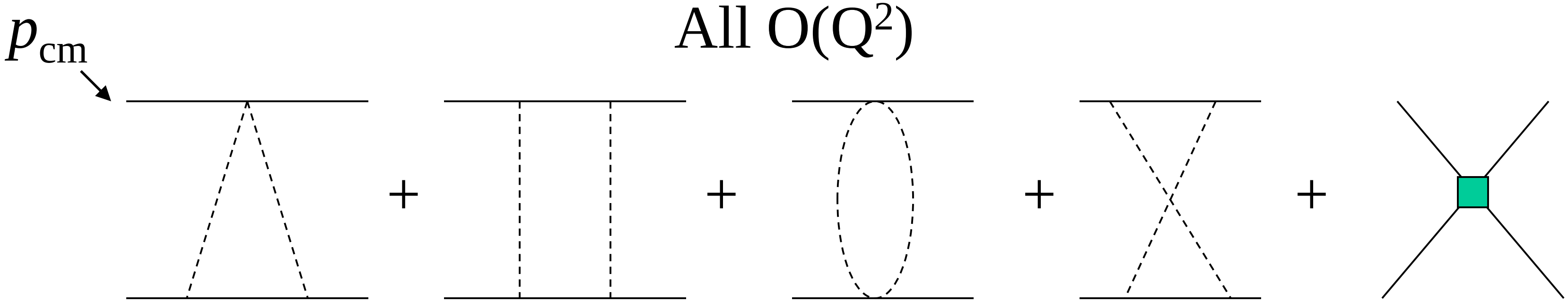


k_cm [MeV]
153 217 265 306 343 375
δ(¹S₀) [deg]
LO_500
LO_1500
LO_2000
δ(³D₁) [deg]
1st subleading_500
1st subleading_1500
1st subleading_2000
δ(³S₁) [deg]
2nd subleading_500
2nd subleading_1500
2nd subleading_2000
Nijm
E₁ [deg]
T_lab [MeV]



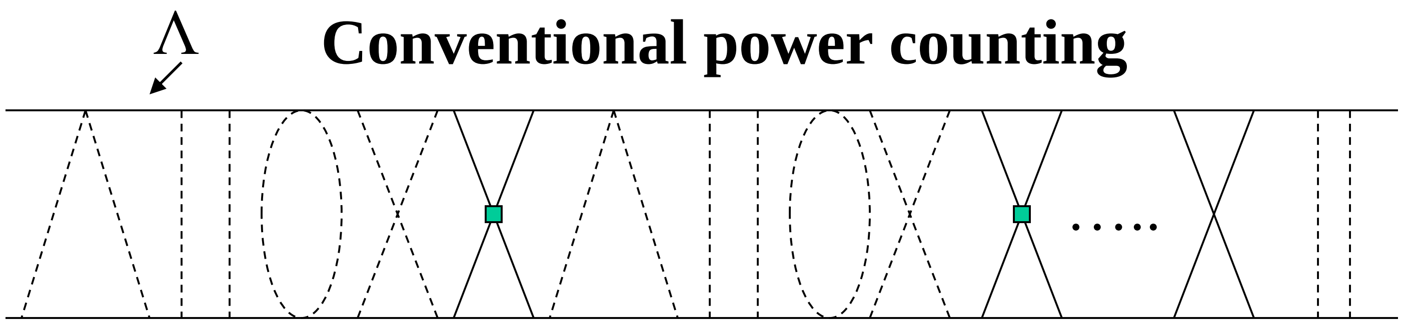
k_cm [MeV]
153 217 265 306 343 375
δ(³P₀) [deg]
δ(³P₂) [deg]
δ(³F₂) [deg]
E₂ [deg]
T_lab [MeV]
LO500
LO1500
LO2000
NNLO500
NNLO1500
NNLO2000
N3LO500
N3LO1500
N3LO2000
Nijm

All $O(Q^2)$



O.k., as long as p_{cm} is small enough, so that $\frac{p_{cm}}{M_{hi}} < 1$

Conventional power counting



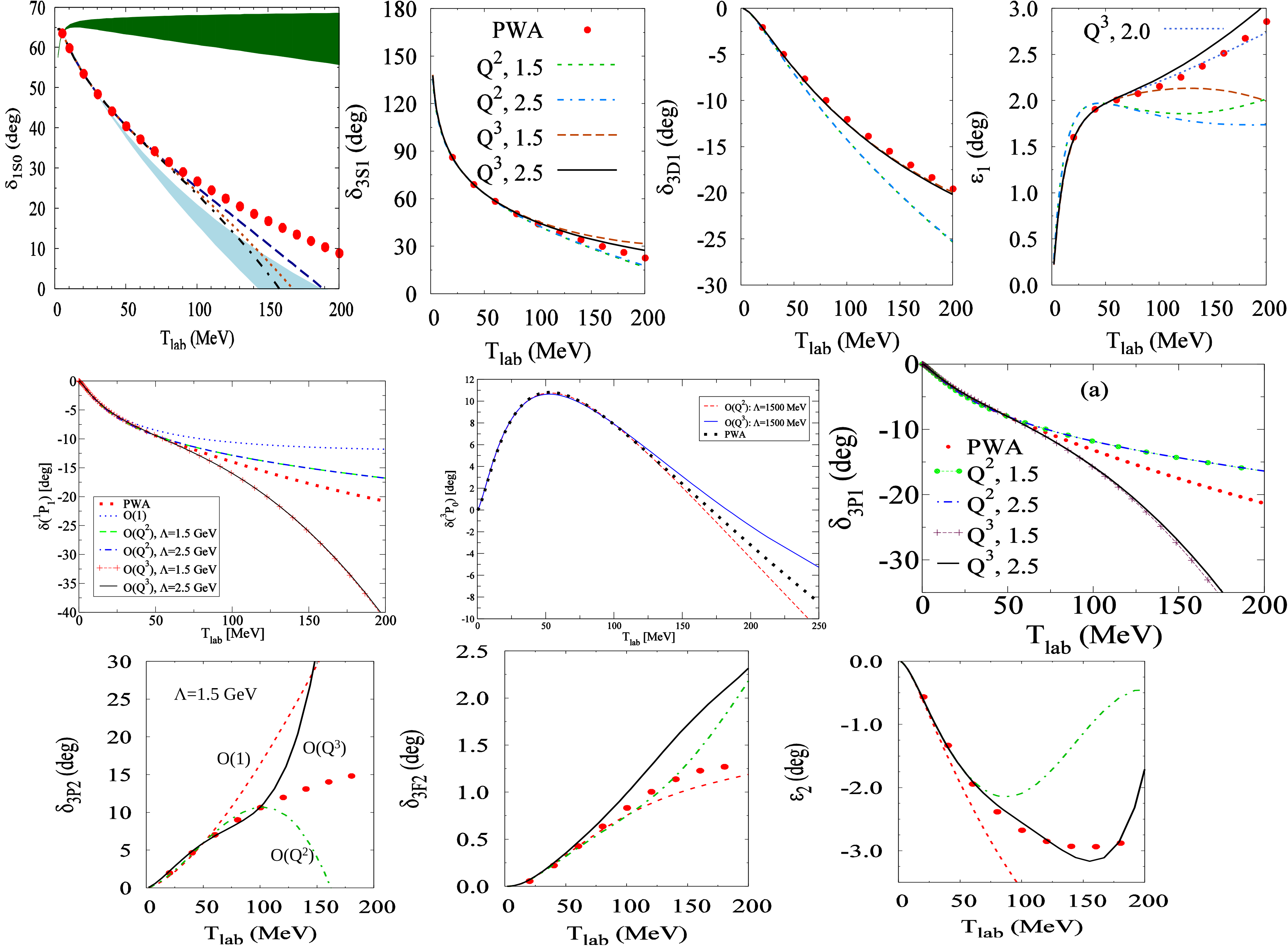
Has problem, as Λ-dependence enter here; contact term aren't enough.

The expansion parameter is no longer $\frac{p_{cm}}{M_{hi}}$!

Problems at Λ>1 GeV also imply that **WPC might not give you (to make use of) all the counter terms which are legitimate (according to a truly RG-invariant theory) to be used.**

Quality of the fits

(comparable to WPC at the same order)



δ_{1S0} (deg)
T_{lab} (MeV)
PWA
Q^2, 1.5
Q^2, 2.5
Q^3, 1.5
Q^3, 2.5
δ_{3S1} (deg)
T_{lab} (MeV)
δ_{3D1} (deg)
T_{lab} (MeV)
Q^3, 2.0
ε_1 (deg)
T_{lab} (MeV)
PWA
O(1)
$O(Q^2)$, Λ=1.5 GeV
$O(Q^2)$, Λ=2.5 GeV
$O(Q^3)$, Λ=1.5 GeV
$O(Q^3)$, Λ=2.5 GeV
$\delta(^1P_1)$ [deg]
T_{lab} [MeV]
$O(Q^2)$: Λ=1500 MeV
$O(Q^3)$: Λ=1500 MeV
PWA
$\delta(^3P_0)$ [deg]
T_{lab} [MeV]
(a)
PWA
Q^2, 1.5
Q^2, 2.5
Q^3, 1.5
Q^3, 2.5
δ_{3P1} (deg)
T_{lab} (MeV)
Λ=1.5 GeV
O(1)
$O(Q^3)$
$O(Q^2)$
δ_{3P2} (deg)
T_{lab} (MeV)
δ_{3F2} (deg)
T_{lab} (MeV)
ε_2 (deg)
T_{lab} (MeV)

Perturbation: Difficulties and solution

- Practical problem: Need to evaluate all E_n to do perturbation theory, ☹.

$$E_n^{nlo} = <\psi_n^{LO}|V_{nlo}|\psi_n^{LO}>, E_n^{nnlo} = \sum_{i\neq n} \frac{|<\psi_i^{LO}|V_{nnlo}|\psi_n^{LO}>|^2}{E_n^{LO} - E_i^{LO}}$$

- Solution (Back to starting point, thanks Dean Lee & Nir Barnea)

$$H = \underbrace{(H_0 + V_{LO})}_{non-per.} + \lambda V_{nlo} + \lambda^2 V_{nnlo} + \ldots$$

$$E_i = E_i^{LO} + \lambda E_i^{nlo} + \lambda^2 E_i^{nnlo} + \ldots$$

Still diagonal the matrix, but vary λ to extract contribution at each higher order. ☺