

Nuclear physics around the unitarity limit

Sebastian König

New Ideas in Constraining Nuclear Forces

ECT*, Trento, Italy

June 7, 2018

SK, H.W. Grießhammer, H.-W. Hammer, U. van Kolck, PRL 118 202501 (2017)

SK, J Phys. G 44 064007 (2017)



European Research Council
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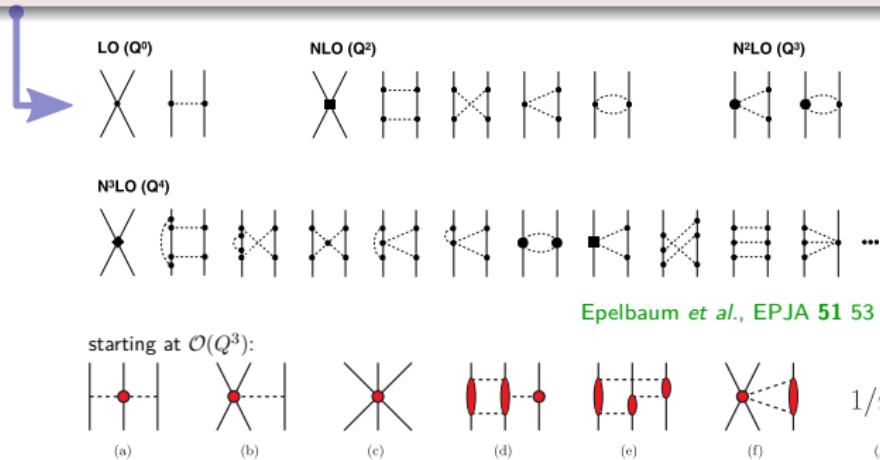
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DARMSTADT



Prelude

Typical nuclear *ab initio* calculation

chiral potential → SRG → many-body method → result

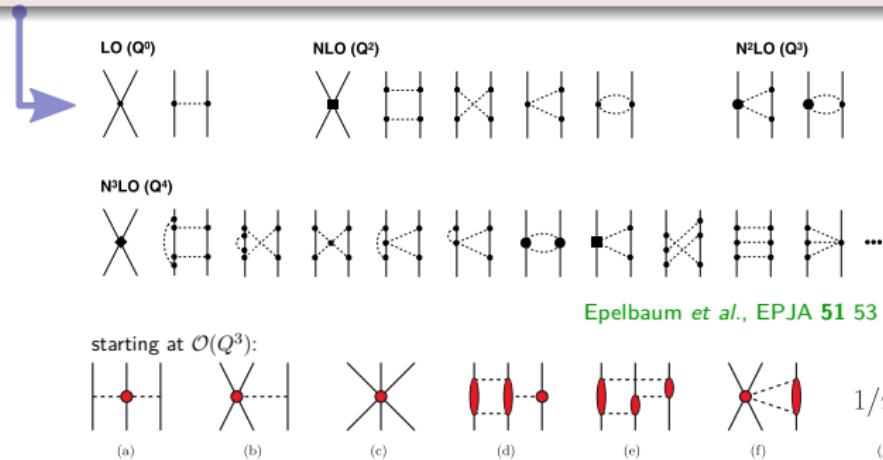


- power counting → hierarchy of forces: $Q \sim m_\pi \ll M_{\text{QCD}}$
- Weinberg approach:** diagrams → potential → iterate...

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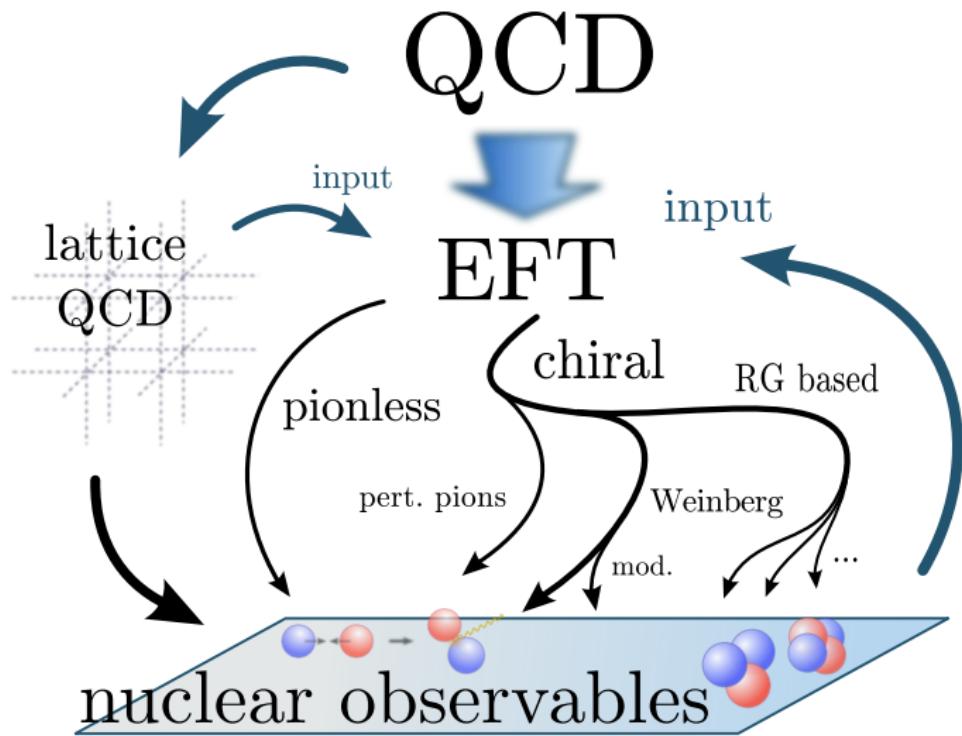


Epelbaum *et al.*, EPJA 51 53 (2015)

Hebeler *et al.*, PRC 91 044001 (2015)

- power counting → hierarchy of forces: $Q \sim m_\pi \ll M_{\text{QCD}}$
- **Weinberg approach:** diagrams → potential → iterate...
- **Note:** need at least $\mathcal{O}(Q^3)$ for reasonable triton!

Prelude



hierarchy of forces (natural in EFT)

many-body forces \leftrightarrow two-body off-shell tuning

Various approaches depart from focusing on two-body input...

- **JISP16** Shirokov *et al.*, PLB **644** 33 (2007)
 \hookrightarrow two-body only, but input from nuclei up to ^{16}O
- **N2LO_{opt}, N2LO_{sat}** Ekstöm *et al.*, PRL **110** 192502 (2013), PRC **91** 051301 (2015)
 simultaneous fit to NN + light nuclei, saturation properties
- **SRG-evolved 2N + N2LO 3N** Simonis *et al.*, PRC **93** (2016)
 \hookrightarrow predict realistic saturation properties
- **nuclear lattice calculations** Elhatisari *et al.*, PRL **117** 132501 (2016)
 \hookrightarrow use input from α - α scattering
- ...

Outline

The unitarity expansion

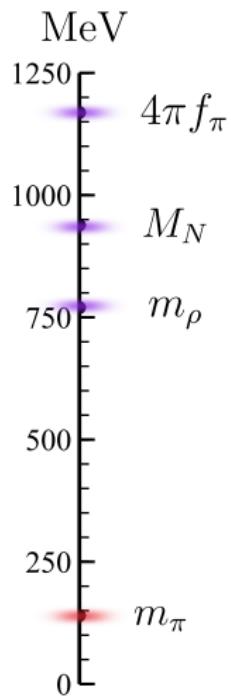
Bound states and resonances

Second order

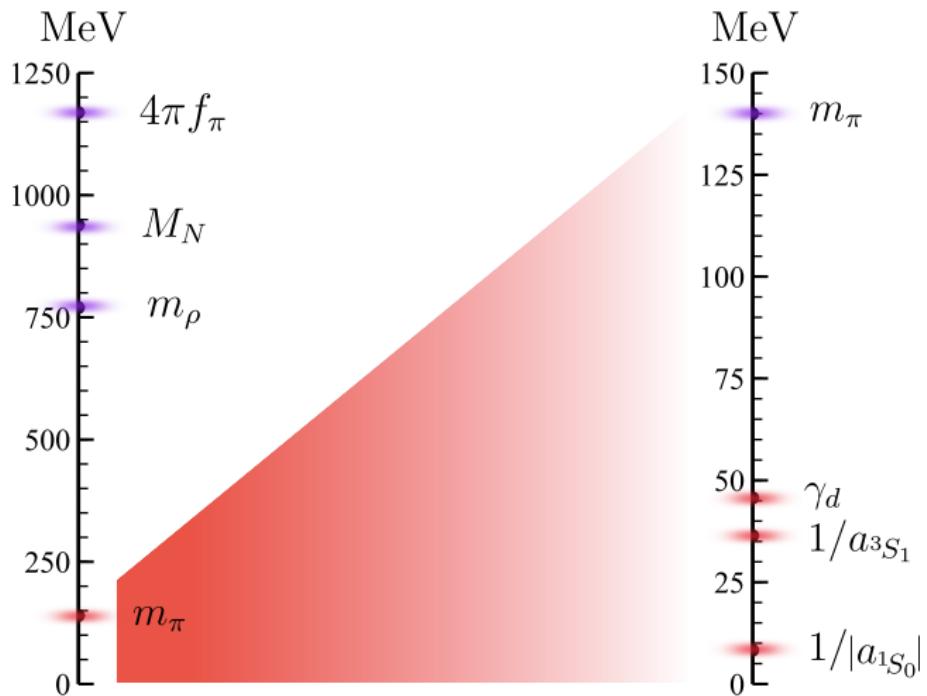
Current work

Summary

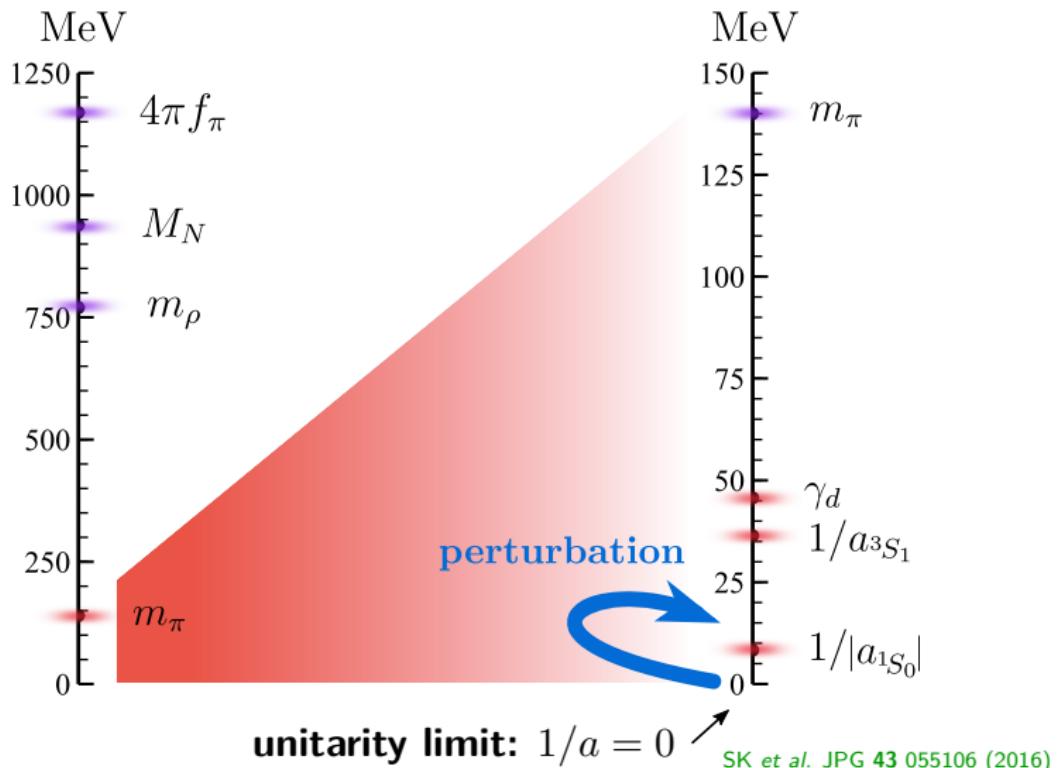
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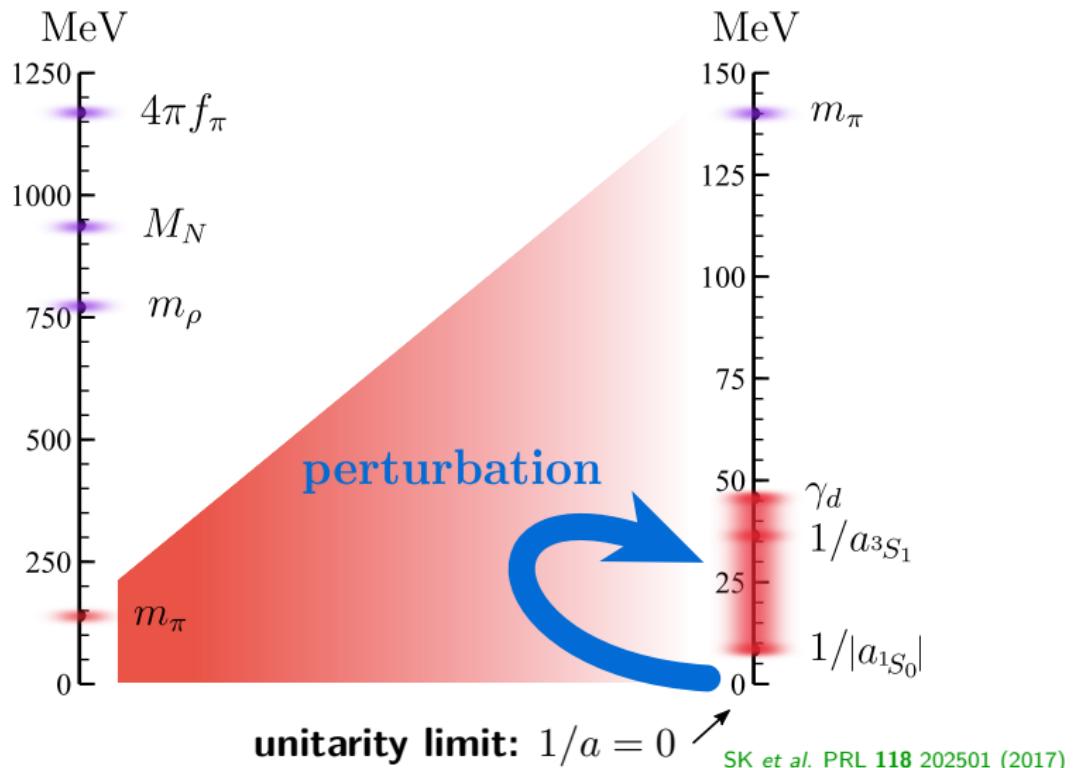
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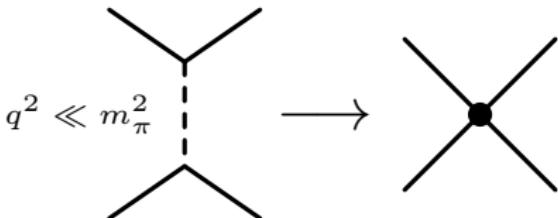
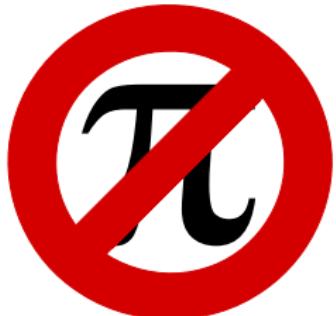
The unitarity expansion



The unitarity expansion



The unitarity expansion



Basic setup

- two-body physics (LECs) \leftrightarrow effective range expansion
- assume $a_{s=^1S_0,t=^3S_1} = \infty \iff 1/a_{s,t} = 0$ at leading order
- **need pionless LO three-body force!**
 \hookrightarrow reproduce triton energy exactly
- finite a , Coulomb, ranges \rightarrow perturbative corrections!

The unitarity expansion

Capture gross features at leading order, build up the rest as perturbative “fine structure!”

- shift focus away from two-body details
- note: zero-energy deuteron at LO and NLO
- exact $SU(4)_W$ symmetry at LO *cf. Vanasse+Phillips, FB Syst. 58 26 (2017)*
- universality regime: Efimov effect, bosonic clusters, . . .

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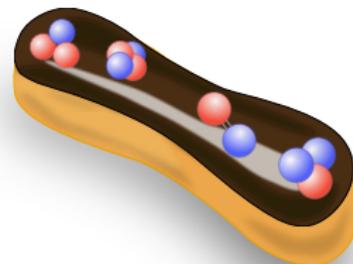
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Conjecture

Nuclear sweet spot

$$1/a_{s,t} < Q_A < 1/R$$

$$Q_A \sim \sqrt{2M_N B_A/A}$$

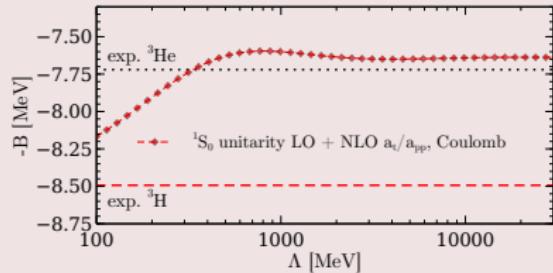


original eclair by Herve1729 (via Wikimedia Commons)

Helium results

^3He at 1S_0 and full unitarity

- good NLO established for 1S_0 unitarity



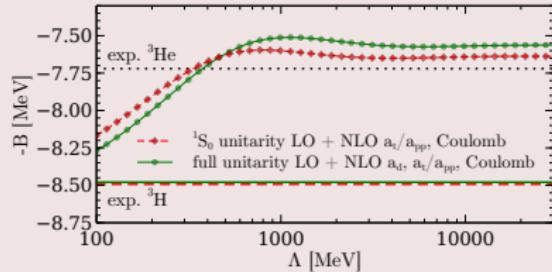
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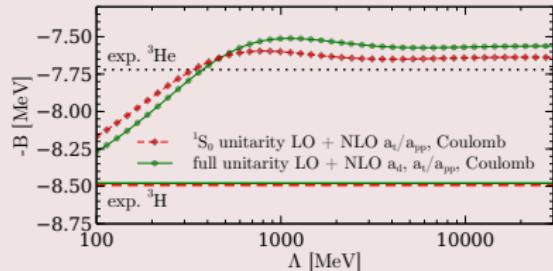
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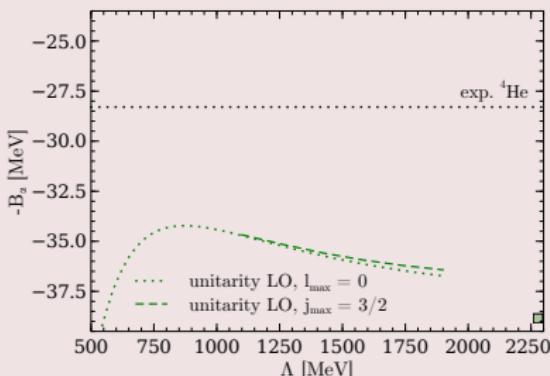
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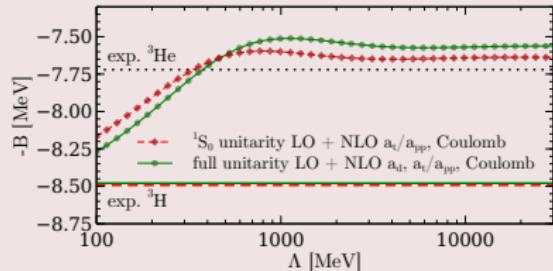


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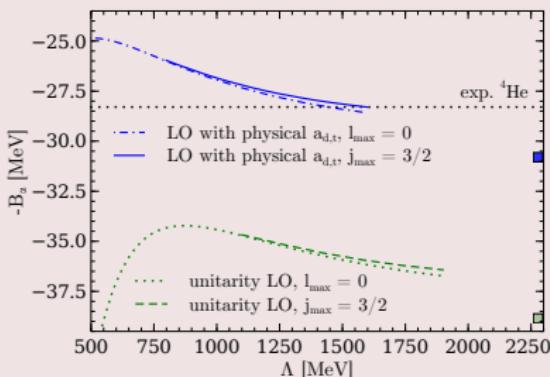
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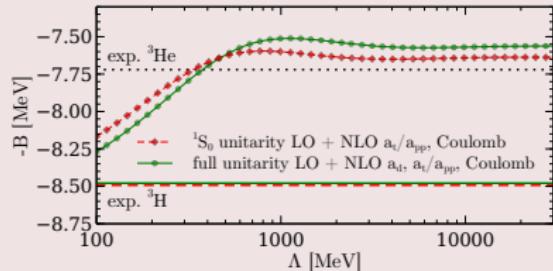


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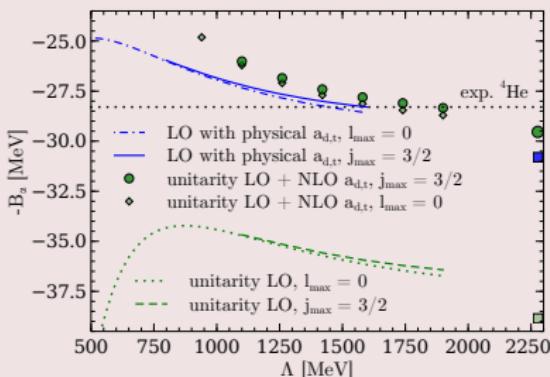
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Some details

- binding energies at LO: find zeros of $\det(\mathbf{1} - K(E))$,
 $K(E)$ = Faddeev(-Yakubowsky) kernel
- NLO energy shift: $\Delta E = \langle \Psi | V^{(1)} | \Psi \rangle$, $|\Psi\rangle$ = LO wavefunction

$$|\Psi\rangle = (\mathbf{1} - P_{34} - PP_{34})(1 + P)|\psi_A\rangle + (\mathbf{1} + P)(\mathbf{1} + \tilde{P})|\psi_B\rangle$$

wavefunction convergence slower than eigenvalue convergence!

→ need more mesh points and partial-wave components...

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Energy balance

- sample calculation with physical scattering lengths at LO:

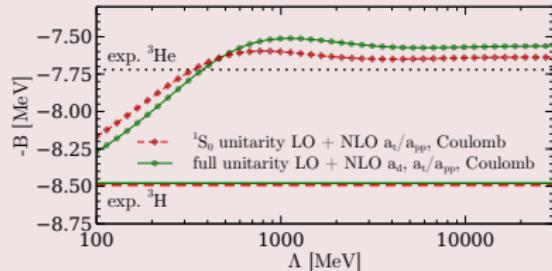
Λ / MeV	800	1000	1200	1400
$E_{\text{kin}} / \text{MeV}$	113.67	140.58	168.44	197.09
$E_{\text{pot}} / \text{MeV}$	-139.77	-167.41	-195.76	-224.62

- E_{kin} and E_{pot} not individually observable
- sum converges as cutoff is increased, individual values do not!

Helium results

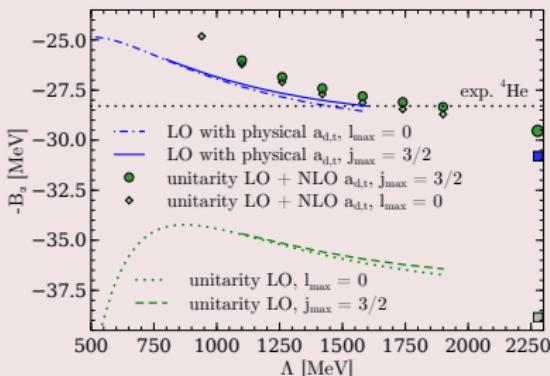
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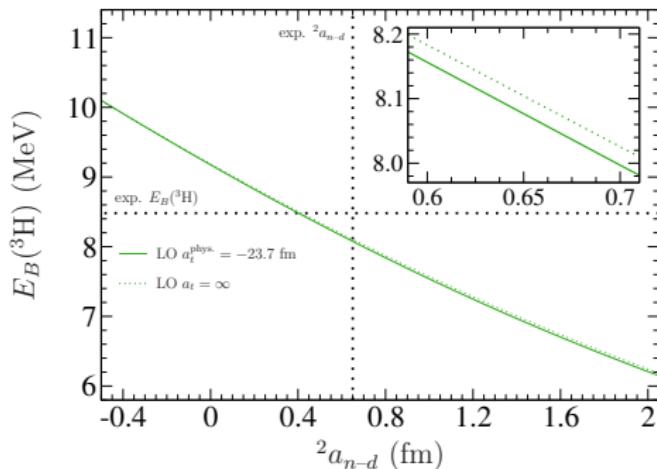
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Few-nucleon correlations

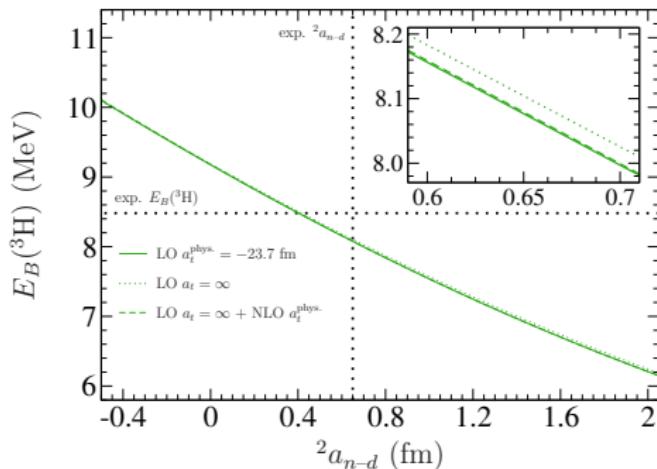


Phillips line

$$= f(\bullet + \bullet \bullet)$$

(1S_0 unitarity only)

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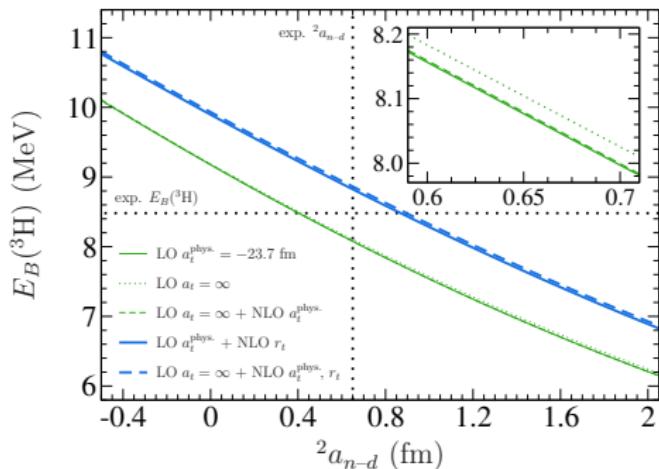


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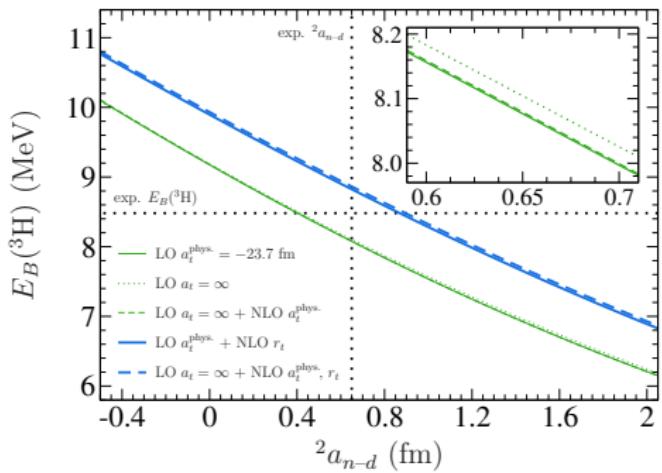
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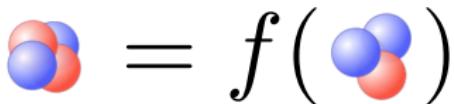
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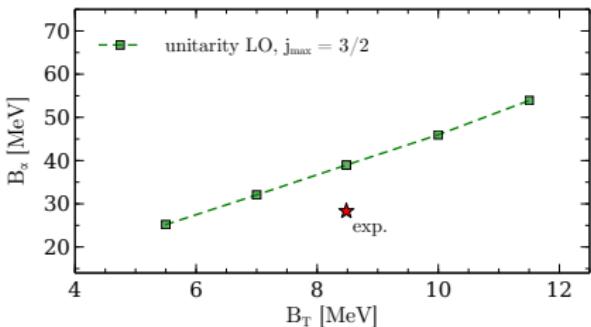
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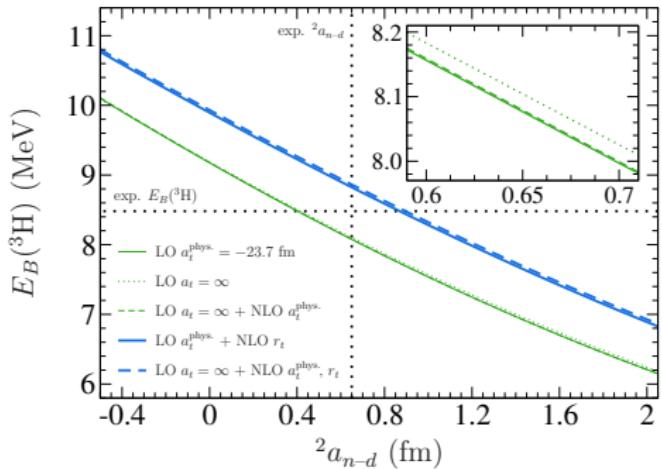
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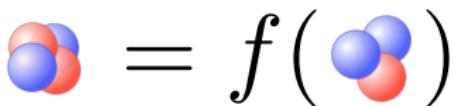
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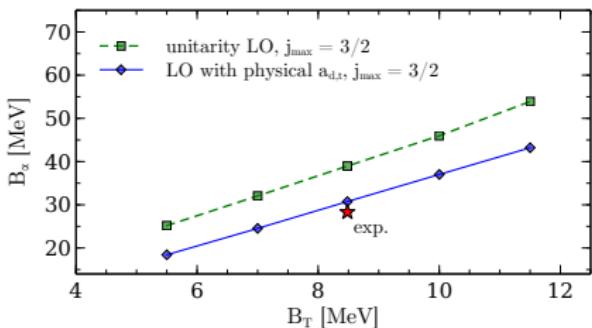
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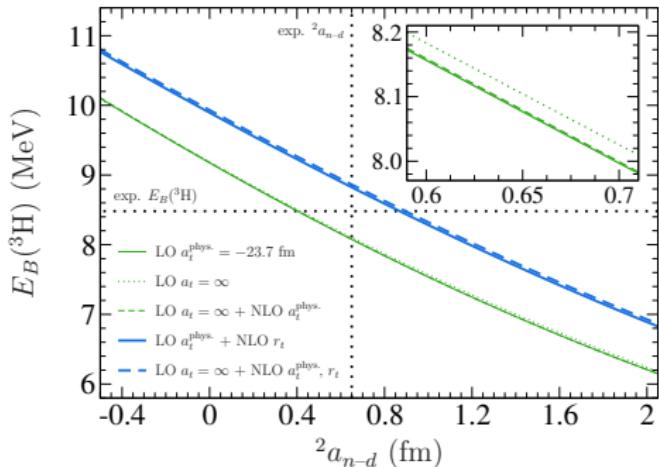
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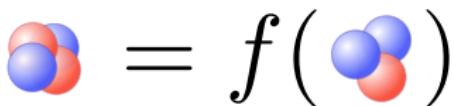
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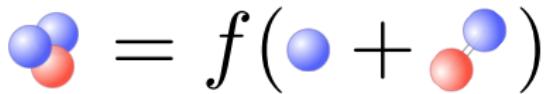
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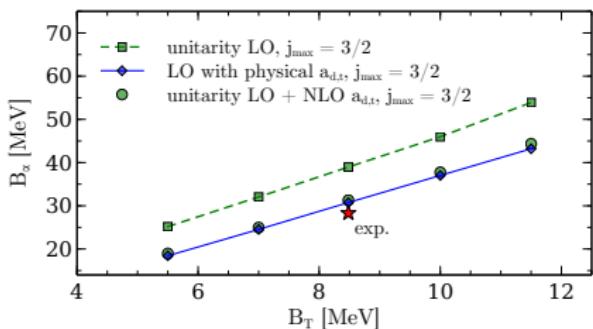
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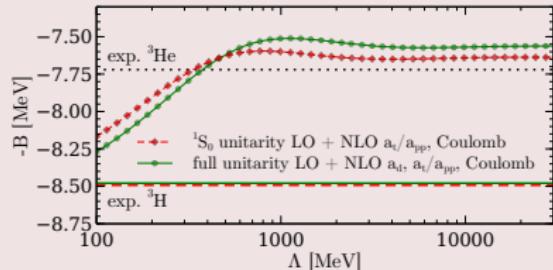
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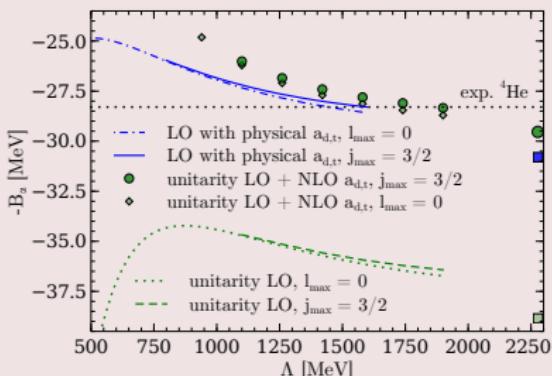
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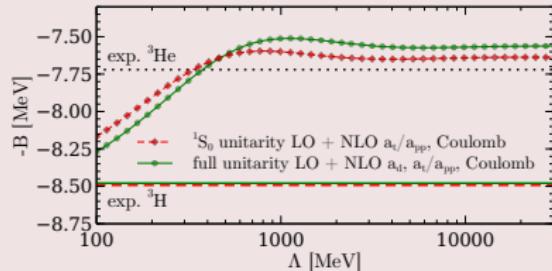


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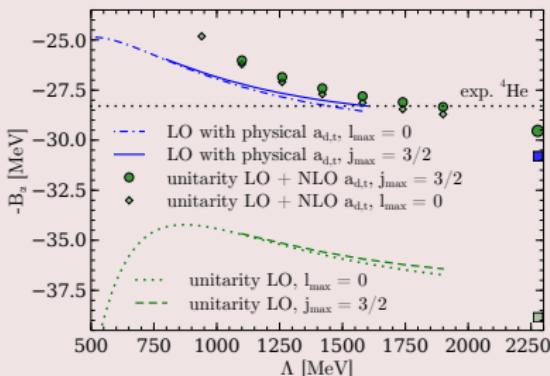
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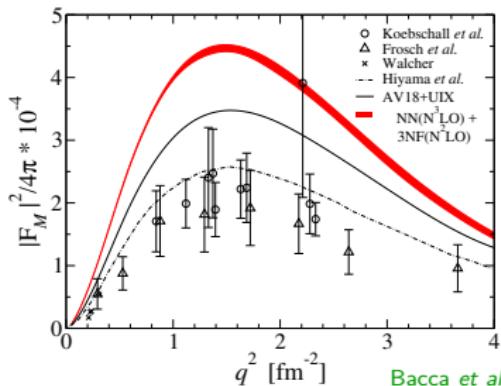
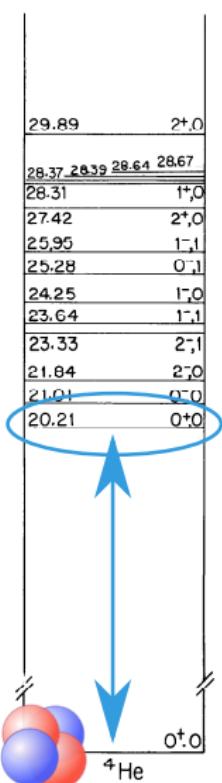
^4He (zero-range, no Coulomb)



- ${}^4\text{He}$ resonance state ~ 0.3 MeV above ${}^3\text{H} + p$ threshold
- just below threshold at unitarity LO
- boson calculations with nuclear scales
~~ shift by about $0.2 - 0.5$ MeV

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^4He monopole resonance



theoryA \neq theoryB
 \neq experiment!

"a prism to nuclear Hamiltonians"

Bacca *et al.*, PRL 110 042503 (2013)

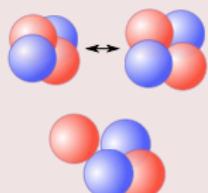
Structure of the 0⁺ resonance

- suggested to be a “breathing mode”

Bacca *et al.*, PRC 91 024303 (2015)

- indications for $p+^3\text{H}$ cluster structure

this work



TUNL nuclear data

Unitarity expansion(s) at second order

Various contributions at N²LO...

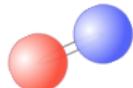
SK, J Phys. G 44 064007 (2017)

① quadratic scattering-length corrections

- at NLO, the deuteron remains at zero energy...
- ...but it moves to $\kappa^{(1)} = 1/a_t$ at N²LO

$$B_0 = \frac{(\kappa^{(0)})^2}{M_N} , \quad B_1 = \frac{2\kappa^{(0)}\kappa^{(1)}}{M_N} , \quad B_2 = \frac{(\kappa^{(1)})^2}{M_N} , \quad \kappa^{(0)} \rightarrow 0$$

- expansion in momentum, not energy



The perturbative deuteron

Efficient method to calculate T-matrix in pert. theory: Vanasse, PRC 88 044001 (2013)

T-matrix perturbation theory for $C_0 = C_0^{(0)} + C_0^{(1)} + C_0^{(2)} + \dots$

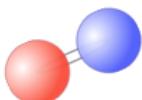
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↪ same integral kernel at each order!

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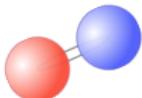
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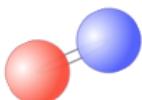
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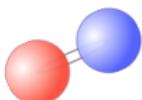
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$$B_0 = \frac{(\kappa^{(0)})^2}{M_N} , \quad B_1 = \frac{2\kappa^{(0)}\kappa^{(1)}}{M_N} , \quad B_2 = \frac{(\kappa^{(1)})^2}{M_N} , \quad \kappa^{(0)} \rightarrow 0$$

Unitarity expansion(s) at second order

Various contributions at N²LO...

SK, J Phys. G 44 064007 (2017)

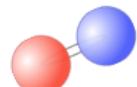
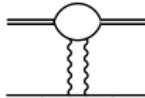
① quadratic scattering-length corrections

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- **expansion in momentum, not energy**

② two-photon exchange



③ quadratic range corrections

④ isospin-breaking effective ranges: $r_{pp} \neq r_{np}$

Unitarity expansion(s) at second order

Various contributions at $N^2\text{LO}\dots$

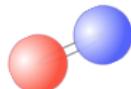
SK, J Phys. G 44 064007 (2017)

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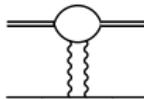
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- expansion in momentum, not energy



② two-photon exchange



③ quadratic range corrections

④ isospin-breaking effective ranges: $r_{pp} \neq r_{np}$

⑤ mixed Coulomb and range corrections!

e.g.

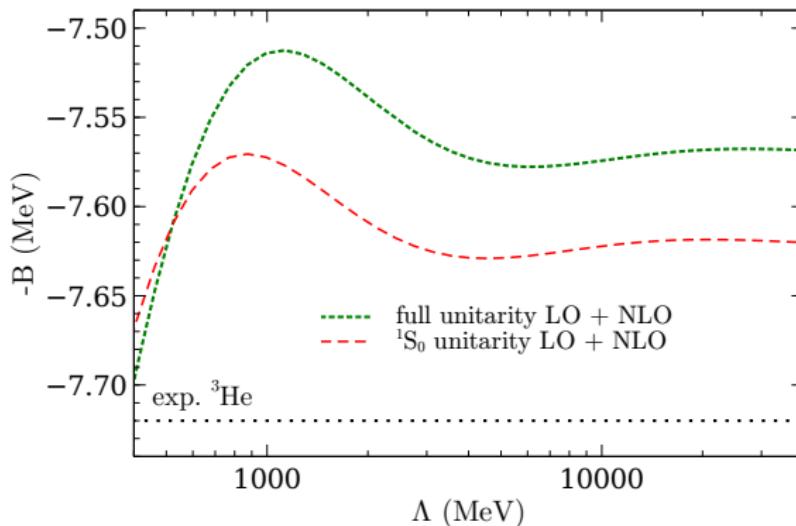


↔ log. divergence, new pd counterterm!

More ^3He results

SK, J Phys. G 44 064007 (2017)

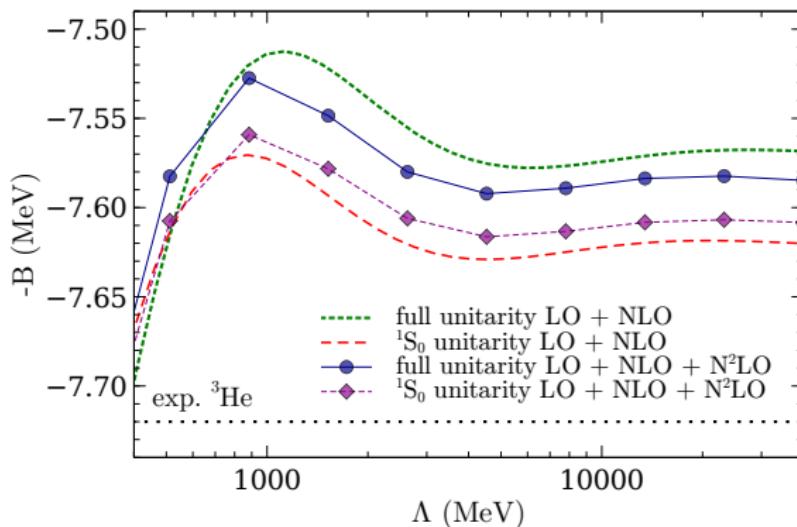
- with range corrections, there is a new pd three-body force at $\text{N}^2\text{LO}\dots$
- ... but the convergence of the unitarity expansions can be checked for the zero-range case



More ^3He results

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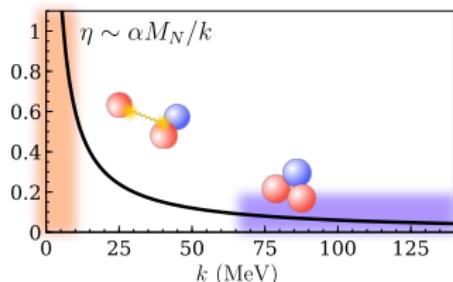


↪ good convergence of half- and full-unitarity expansions

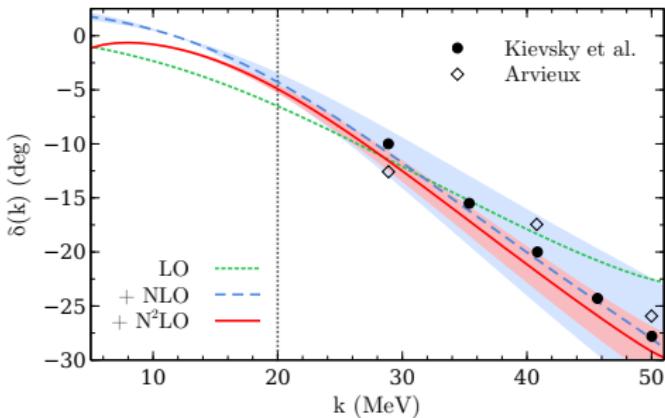
Perturbative p - d phase shifts

At intermediate energies, Coulomb is perturbative for pp/pd scattering!
(counting scheme here: 1S_0 unitarity)

SK *et al.* (2015); SK (2017)



$$\eta \leq 1/3 \text{ for } k \geq 20 \text{ MeV}$$



Perturbative subtracted phase shifts

$$\delta(k) \equiv \delta_{\text{full}}(k) - \delta_c(k)$$

$$= \delta_{\text{full}}^{(0)}(k) - \cancel{\delta_c^{(0)}(k)} + \delta_{\text{full}}^{(1)}(k) - \delta_c^{(1)}(k) + \delta_{\text{full}}^{(2)}(k) - \delta_c^{(2)}(k) + \dots$$

cf. also SK, Hammer (2014)

Current work

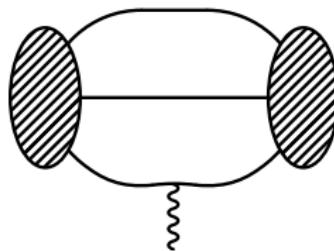
in progress: look at observables beyond binding energies

→ radii and charge form factors

Point charge radius

$$F_C(q^2) = \langle \Psi | J_0(q^2) | \Psi \rangle$$

$$\langle r^2 \rangle = -\frac{1}{6} \frac{d}{d(q^2)} F_C(q^2) \Big|_{q^2=0}$$



+ perm.

—preliminary—

LO charge radii

	unit.	phys. $a_{s,t}$	exp.
${}^3\text{H}$	1.05	1.17	1.60

fixed $\Lambda = 2000$ MeV, no extrapolation

Atomic Data and Nuclear Data Tables **99** 69 (2013)

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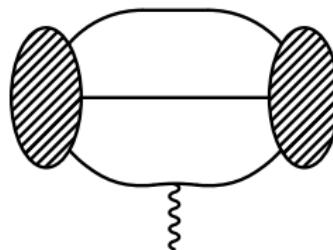
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Atomic Data and Nuclear Data Tables **99** 69 (2013)

${}^3\text{H}$ calculation up to N²LO (a_s , $B({}^2\text{H})$):

1.14(19) fm → 1.59(8) fm → 1.62(3) fm

Vanasse, PRC **95** 024002 (2017)

LO unitarity trinucleon @ 7.62 MeV:

1.10 fm

Vanasse+Phillips, Few-Body Syst. **58** 26 (2017)

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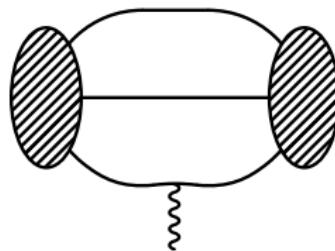
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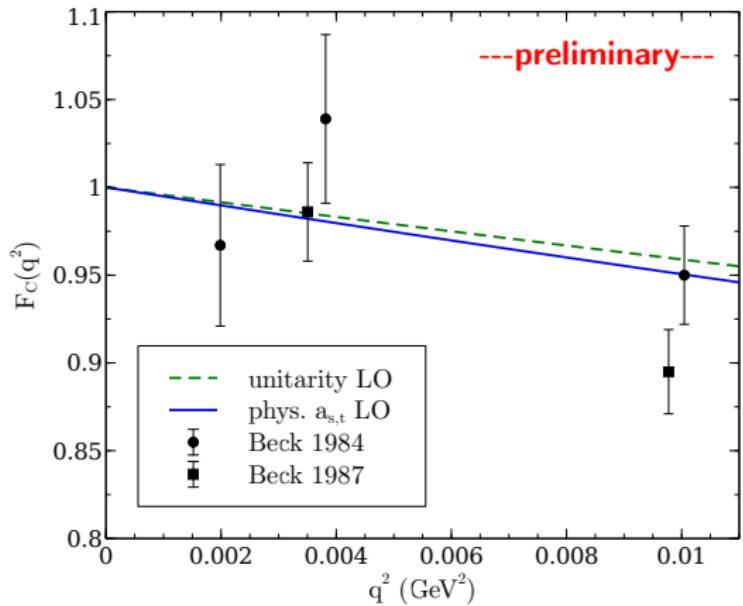


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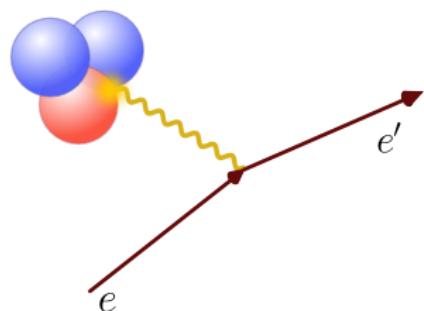
${}^3\text{H}$ radius cutoff convergence

Λ	1200	1600	2000
unit.	1.11	1.06	1.05
phys.	1.20	1.18	1.17

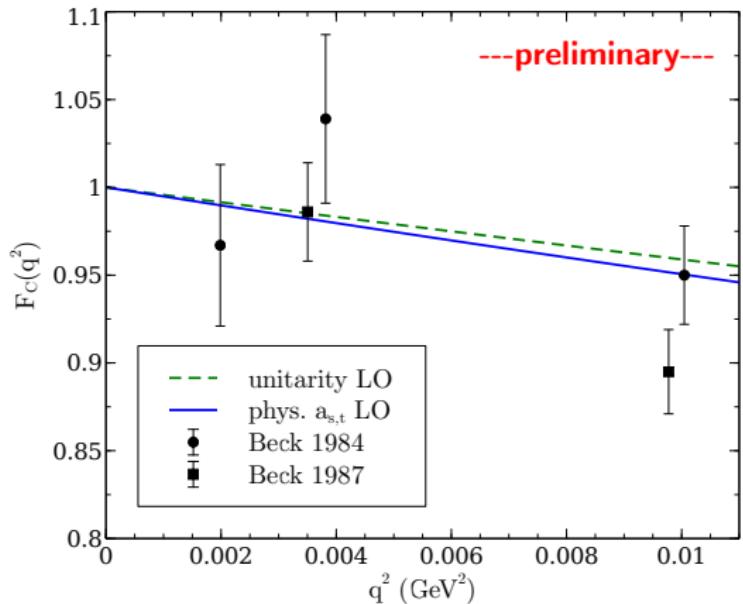
Triton charge form factor



data from Beck, PRC 30 1403 (1984)
Beck *et al.*, PRL 59 1537 (1987)

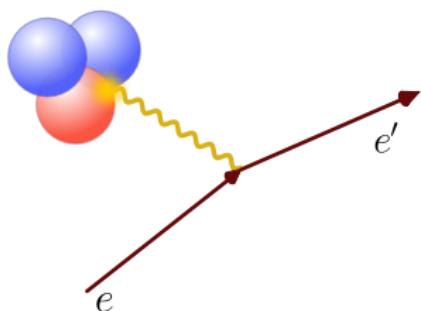


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Next steps: perturbation theory, range corrections, Coulomb corrections

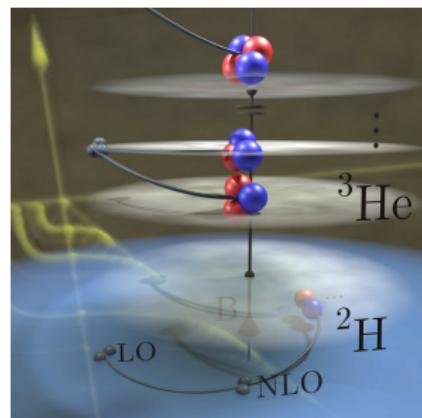
Summary

Novel approach to few-nucleon systems

SK *et al.*, PRL 118 202501 (2017)

	LO	NLO	N ² LO	exp.
² H	0	0	1.41	2.22
³ H	8.48	8.48	8.48	8.48
³ He	8.48	7.56		7.72
⁴ He	38.86	29.50		28.30

four-body: no Coulomb, zero-range
NLO uncertainties: 0.2 MeV (³He), 9 MeV (⁴He)



- emphasize three-body sector over two-body precision
- enhanced symmetry and only one parameter at leading order
- conjecture: unitarity expansion useful beyond four nucleons

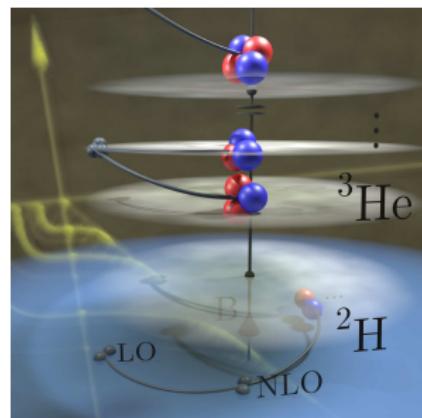
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*** Thank you! ***