

POWER COUNTING WHAT? WHERE?

U. van Kolck

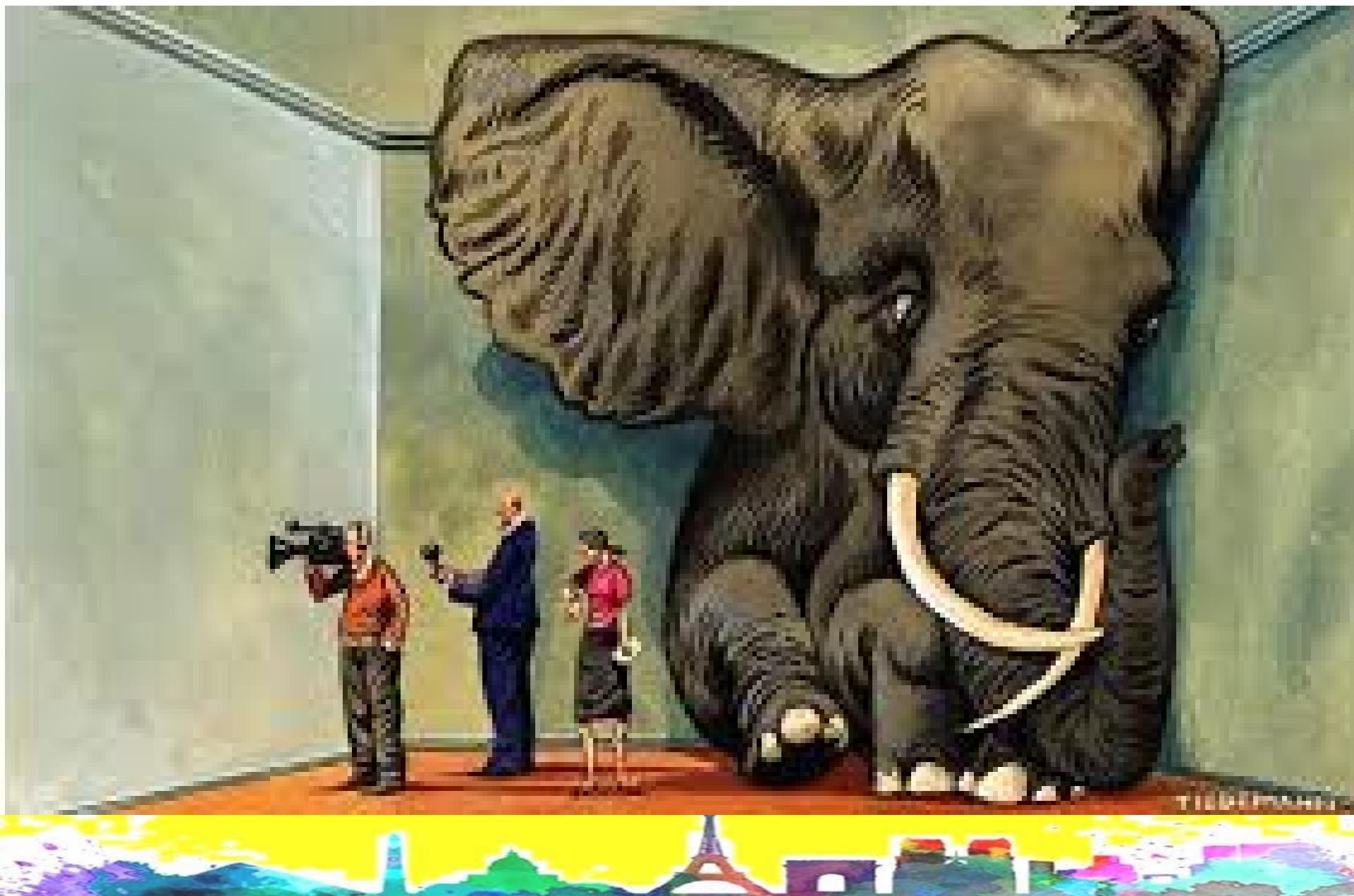
Institut de Physique Nucléaire d'Orsay
and
University of Arizona



Supported by CNRS and US DOE







Outline

- Meeting the elephant
 - What? Where?
 - Walking out of McDonald's
 - Conclusion

with

Y.-H. Song
& R. Lazauskas

The elephant

Thursday, June 7: Power counting: Beyond Weinberg (?)

9:00	H. W. Griesshammer: TBA
9:40	U. van Kolck: TBA
10:20	Coffee break
10:50	B. Long: TBA
11:30	S. König: Nuclear physics around the unitarity limit
12:10	Lunch
14:10	C.-J. Yang: Examining chiral EFT potentials with Lepage plot
14:50	M. P. Valderrama: TBA
15:30	Coffee break
16:00	Discussion: Power counting, regularization, renormalization, and related topics
17:00	End of day 4

Friday, June 8: Constraining nuclear forces from lattice QCD calculations

9:00	7. DeGrand: Nuclear and hypernuclear forces from lattice QCD
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- 1996: Chiral-symmetry breaking interaction, absent according to Weinberg's power counting, needed in 1S_0 channel at LO

$$\mathcal{L}_{\chi EFT} = \dots + D_2 \textcolor{red}{m_\pi^2} N^\dagger P_{^1S_0} N N^\dagger P_{^1S_0} N \left(1 - \frac{\pi^2}{F_\pi^2} + \mathcal{O}\left(\frac{\pi^4}{F_\pi^4}\right) \right) + \dots$$

NDA $D_2 = \mathcal{O}\left(\frac{1}{\textcolor{red}{f_\pi^2} \textcolor{blue}{M_{QCD}^2}}\right)$ **vs** nonperturbative renormalization $D_2 = \mathcal{O}\left(\frac{1}{\textcolor{red}{f_\pi^4}}\right)$

→ D.B. Kaplan, M.J. Savage, M.B. Wise, *Nucl. Phys. B* **478** (1996) 629

S.R. Beane, P.F. Bedaque, M.J. Savage, U. van Kolck, *Nucl. Phys. A* **700** (2002) 377

B. Long, C.-J. Yang, *Phys. Rev. C* **86** (2012) 024001

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→ D.B. Kaplan, M.J. Savage, M.B. Wise, *Nucl. Phys. B* **478** (1996) 629

- 1997: Nonperturbative treatment of higher-order interactions beset by RG problems

e.g.

$$\mathcal{L}_{\pi EFT} = \dots + C_2 N^\dagger P_{^1S_0} N N^\dagger P_{^1S_0} \vec{\nabla}^2 N + \dots$$

→ Wigner bound

T.D. Cohen, D.R. Phillips, *Phys. Lett. B* **390** (1997) 7

K.A. Sculdeferri, D.R. Phillips, C.W. Kao, T.D. Cohen,
Phys. Rev. C **56** (1997) 679

...

- 2005: Derivative contact interactions, down **two or more** orders according to Weinberg's power counting, needed at LO in (triplet) channels with attractive tensor force where pions are iterated to all orders

e.g.

$$\mathcal{L}_{\chi EFT} = \dots + C_2' N^\dagger P_{^3P_0} N (\vec{\nabla} N)^\dagger P_{^3P_0} \vec{\nabla} N + \dots$$

NDA

$$C_2' = \mathcal{O}\left(\frac{1}{f_\pi^2 M_{QCD}^2}\right)$$

vs

nonperturbative
renormalization

$$C_2' = \mathcal{O}\left(\frac{1}{f_\pi^4}\right)$$

A. Nogga, R.G.E. Timmermans, U. van Kolck, *Phys. Rev. C* **72** (2005) 054006

M. Pavón Valderrama, E. Ruiz Arriola, *Phys. Rev. C* **74** (2006) 064004

C.-J. Yang, C. Elster, D.R. Phillips, *Phys. Rev. C* **77** (2008) 014002

...

- 2012: Two-derivative contact interaction, down **two** orders according to Weinberg's power counting, needed in 1S_0 channel at NLO (i.e. ONE, *ONE*, **ONE**, **ONE** power down)

$$\mathcal{L}_{\chi EFT} = \dots + C_2 N^\dagger P_{^1S_0} N N^\dagger P_{^1S_0} \vec{\nabla}^2 N + \dots$$

NDA $C_2 = \mathcal{O}\left(\frac{1}{f_\pi^2 M_{QCD}^2}\right)$ **vs** nonperturbative renormalization $C_2 = \mathcal{O}\left(\frac{1}{f_\pi^3 M_{QCD}}\right)$

B. Long, C.-J. Yang, *Phys. Rev. C* **86** (2012) 024001

Y.-H. Song, R. Lazauskas, U. van Kolck, *Phys. Rev. C* **96** (2017) 024002

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BREAKING
NEWS

Reactions

➤ Pionless EFT

Power counting has VERY LITTLE to do with Weinberg's

e.g.

$$C_{2n} = \begin{cases} \mathcal{O}\left(\frac{4\pi}{m_N M_{lo}^{1+n} M_{hi}^n}\right), & S \rightarrow S \\ \mathcal{O}\left(\frac{4\pi}{m_N M_{lo} M_{hi}^{2n}}\right), & S \leftrightarrow \times \\ \mathcal{O}\left(\frac{4\pi}{m_N M_{hi}^{1+2n}}\right), & \times \leftrightarrow \times \end{cases}$$

$$M_{lo} \sim \sqrt{2m_N B_A/A}$$

$$M_{hi} \sim m_\pi$$

P.F. Bedaque, U. van Kolck, *Phys. Lett. B* **428** (1998) 221

U. van Kolck, *Lect. Notes Phys.* **513** (1998) 62

D.B. Kaplan, M.J. Savage, M.B. Wise, *Phys. Lett. B* **424** (1998) 390

D.B. Kaplan, M.J. Savage, M.B. Wise, *Nucl. Phys. B* **534** (1998) 329

For more, see talk by
König

➤ Chiral EFT with perturbative pions

Same with

$$M_{lo} \sim m_\pi$$

$$M_{hi} \sim M_{NN} \equiv \frac{4\pi f_\pi}{m_N} f_\pi \sim f_\pi$$

D.B. Kaplan, M.J. Savage, M.B. Wise, *Phys. Lett. B* **424** (1998) 390

D.B. Kaplan, M.J. Savage, M.B. Wise, *Nucl. Phys. B* **534** (1998) 329

...

➤ Chiral EFT with *partly* perturbative pions

$$M_{lo} \sim M_{NN}, m_\pi$$

$$M_{hi} \sim M_{QCD} \equiv 4\pi f_\pi, m_N, \dots$$

NDA except when running dominated by pions,
i.e. low waves where OPE tensor force is attractive

A. Nogga, R.G.E. Timmermans, U. van Kolck, *Phys. Rev. C* **72** (2005) 054006

...

For more, see talks by
Long, Pavón, Yang (?)

Walking out of McDonald's...



Mid-term quizz

1) We should power counting where?

- a) The potential
- b) The scattering amplitude
- c) Both
- d) None of the above

2) We should count powers of what?

- a) Q/f_π
- b) $Q/(4\pi f_\pi)$
- c) Both
- d) It doesn't matter which





$$+ \dots = \frac{1}{f_\pi^4} \int \frac{d^4 l}{(2\pi)^4} \frac{(l, k, m_\pi)^2}{l^2 - m_\pi^2 - i\varepsilon} \frac{(l, k, m_\pi)^2}{(l+k)^2 - m_\pi^2 - i\varepsilon}$$

forbidden by
chiral sym

$$\downarrow \quad \uparrow \quad \text{absorbed in}$$

$$\simeq \frac{1}{f_\pi^2} (\# k^2 + \# m_\pi^2) \sim \frac{Q^2}{f_\pi^2}$$

$$\simeq \frac{1}{f_\pi^2} (\# c_{1,2} k^4 + \# c_3 m_\pi^2 k^2 + \# c_4 m_\pi^4) \sim c_i(\Lambda) \frac{Q^4}{f_\pi^2}$$

$$c_i(\Lambda) = -\frac{\#}{(4\pi f_\pi)^2} \ln\left(\frac{\Lambda}{m_\pi}\right) + c_i^{(R)}$$

four parameters; if omitted:

- cutoff becomes physical
- only one parameter = model

$$c_i(\alpha\Lambda) = \frac{\#}{(4\pi f_\pi)^2} \ln\left(\frac{\Lambda}{m_\pi}\right) + \frac{\#\ln\alpha}{(4\pi f_\pi)^2} + c_i^{(R)}$$

➡ $c_i^{(R)} = \mathcal{O}((4\pi f_\pi)^{-2}) = \mathcal{O}(M_{QCD}^{-2})$

NDA: naïve
dimensional
analysis

$$\sim \frac{Q^4}{f_\pi^2 (4\pi f_\pi)^2}$$

~~$\frac{1}{f_\pi^2 (4\pi f_\pi)^2} \left\{ \# \cancel{\Lambda^4} + \Lambda^2 (\# k^2 + \# m_\pi^2) + (\# k^4 + \# m_\pi^2 k^2 + \# m_\pi^4) \left[\ln\left(\frac{\Lambda}{m_\pi}\right) + \# \ln\left(\frac{k}{m_\pi}\right) \right] + \mathcal{O}\left(\frac{Q^6}{\Lambda^2}\right) \right\}$~~

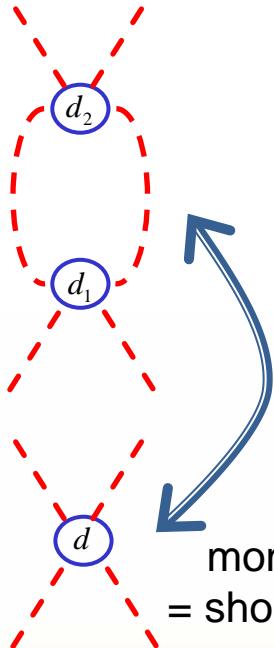
non-analytic

error
not dominant
as long as
 $\Lambda \gtrsim M_{QCD}$

cf.

$$\simeq \frac{Q^6}{f_\pi^2 M_{QCD}^4}$$

NDA, more generally



$$\begin{aligned} &\sim \frac{Q^4}{(4\pi)^2} \frac{Q^{d_1+d_2}}{Q^4} C_{d_1}(\Lambda) C_{d_2}(\Lambda) \\ &\sim \underbrace{\frac{\Lambda^{d_1+d_2-d}}{(4\pi)^2} C_{d_1}(\Lambda) C_{d_2}(\Lambda)}_{\sim C_d(\Lambda)} Q^d + \dots \end{aligned}$$

$d_1 = d_2 = d \rightarrow C_d(\Lambda) \sim \frac{(4\pi)^2}{\Lambda^d}$
“strong” naturalness
 $\rightarrow C_d(M) \sim \frac{(4\pi)^2}{M^d}$

arbitrary diagram

fermions
more loops, vertices
other interactions

Moral:
NDA comes from
the renormalization of
perturbative amplitudes

number of fields
in operator

$$c_i = \mathcal{O}\left(\frac{(4\pi)^{N-2}}{M^{D-4}} c_i^{\text{red}}\right)$$

dimension
of operator

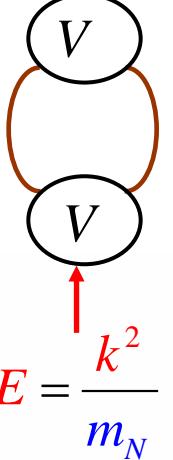
$$c_i^{\text{red}} = \mathcal{O}\left((g^{\text{red}})^\#\right)$$

reduced
underlying theory parameter

Georgi +
Manohar '86

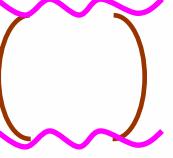
reduced
coupling
insertions

Are nuclear amplitudes perturbative?



$$= \int \frac{d^3 l}{(2\pi)^3} V \frac{m_N}{l^2 - k^2} V + \dots \sim \mathcal{O}\left(\frac{m_N Q}{4\pi} V V\right)$$

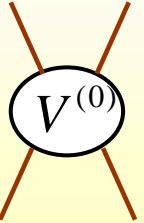
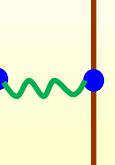
$E = \frac{k^2}{m_N}$


 $\sim \frac{m_N Q}{4\pi}$

Weinberg's IR enhancement instead of $\sim \frac{Q^2}{(4\pi)^2}$

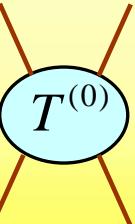
4π enhancement compared to ChPT

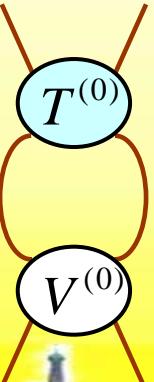
Does it make sense?


 $=$

 $\sim \frac{e^2}{Q^2} \equiv \frac{4\pi}{mQ} \frac{m\alpha}{Q}$


expansion in $\frac{m\alpha}{Q}$

$Q \lesssim m\alpha$


 $=$

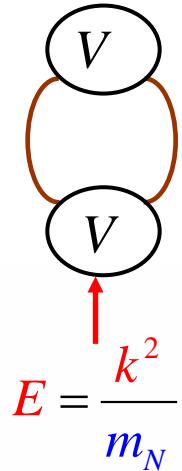
 $+ \quad$


b.s. at

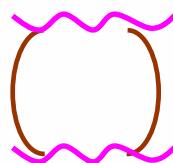
$$B \sim \frac{(\alpha m)^2}{m} \sim \alpha^2 m$$



Are nuclear amplitudes perturbative?



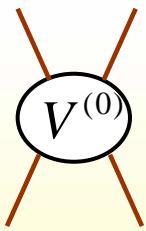
$$= \int \frac{d^3 l}{(2\pi)^3} V \frac{m_N}{l^2 - k^2} V + \dots \sim \mathcal{O}\left(\frac{m_N Q}{4\pi} V V\right)$$



$$\sim \frac{m_N Q}{4\pi}$$

Weinberg's IR enhancement instead of $\sim \frac{Q^2}{(4\pi)^2}$

4 π enhancement compared to ChPT



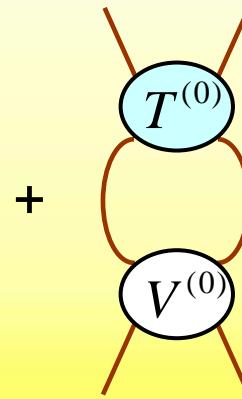
$$= \text{---} + \text{---} \sim \frac{1}{f_\pi^2} \equiv \frac{4\pi}{m_N M_{NN}}$$

expansion in $\frac{Q}{M_{NN}}$

$$M_{NN} \equiv \frac{4\pi f_\pi}{m_N} f_\pi \sim f_\pi$$

Resum in S wave when $Q \gtrsim M_{NN}$

$$T^{(0)} = V^{(0)}$$

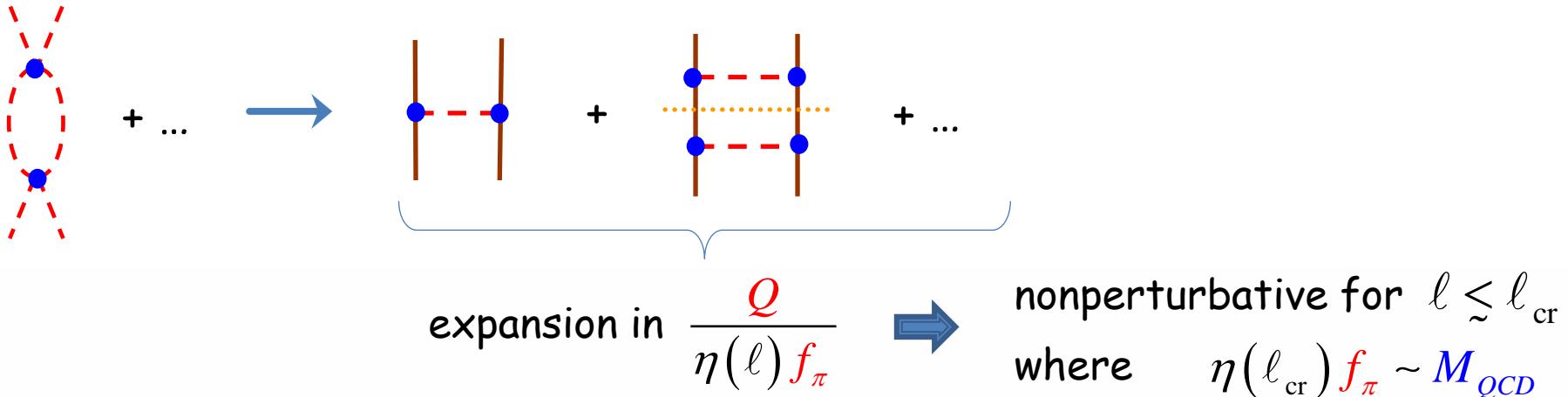


b.s. at

$$B \sim \frac{M_{NN}^2}{m_N} \sim \frac{f_\pi}{4\pi} \simeq 10 \text{ MeV}$$

but still keep perturbative expansion in Q/M_{QCD}

CANNOT JUST COUNT POWERS OF Q



singlets

$$\eta_s(\ell) \sim (M_{QCD}/f_\pi)^\ell$$

M. Pavón Valderrama *et al.*,
Phys. Rev. C **95** (2017) 054001

but regular pot, so can be resummed without RG penalty

triplets

Channel	p_{cr}/MeV	Channel	p_{cr}/MeV	Channel	p_{cr}/MeV
$^3S_1 - ^3D_1$	66	3P_0	182	3P_1	365
$^3P_2 - ^3F_2$	470	3D_2	403	$^3D_3 - ^3G_3$	382
3F_3	2860	$^3F_4 - ^3H_4$	2330	3G_4	1870

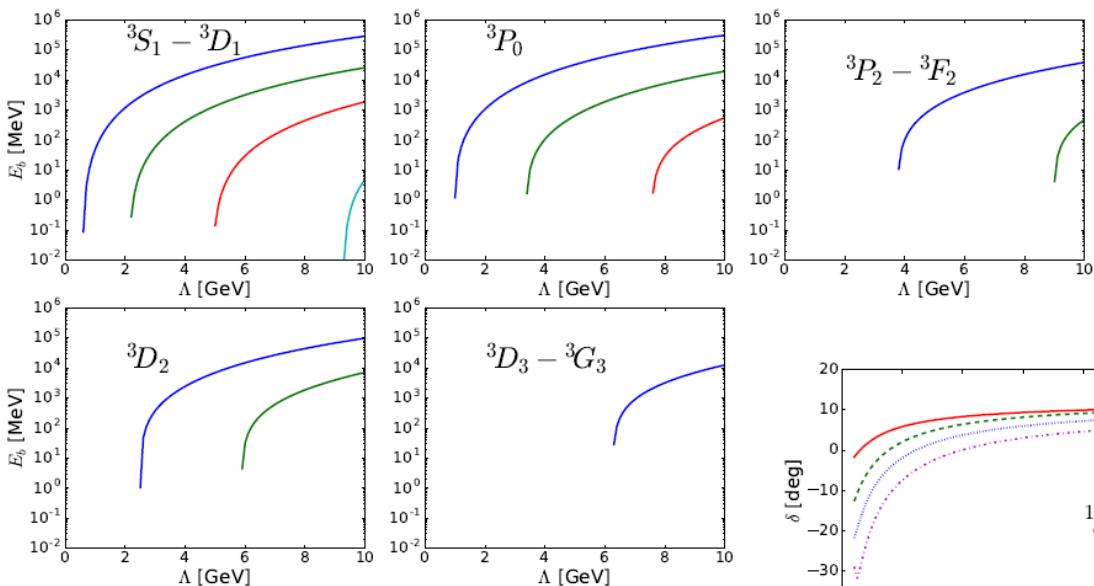
M.C. Birse, *Phys. Rev. C* **74** (2006) 014003

For more, see Long's talk

When OPE's tensor force is attractive and resummed,

$$V(r) \propto -r^{-3} \quad \rightarrow \quad \psi(r) \propto r^{-\frac{1}{4}} \cos\left(\frac{1}{\sqrt{M_{NN}} r} + \delta\right) + \dots$$

if no counterterm, will depend on cutoff



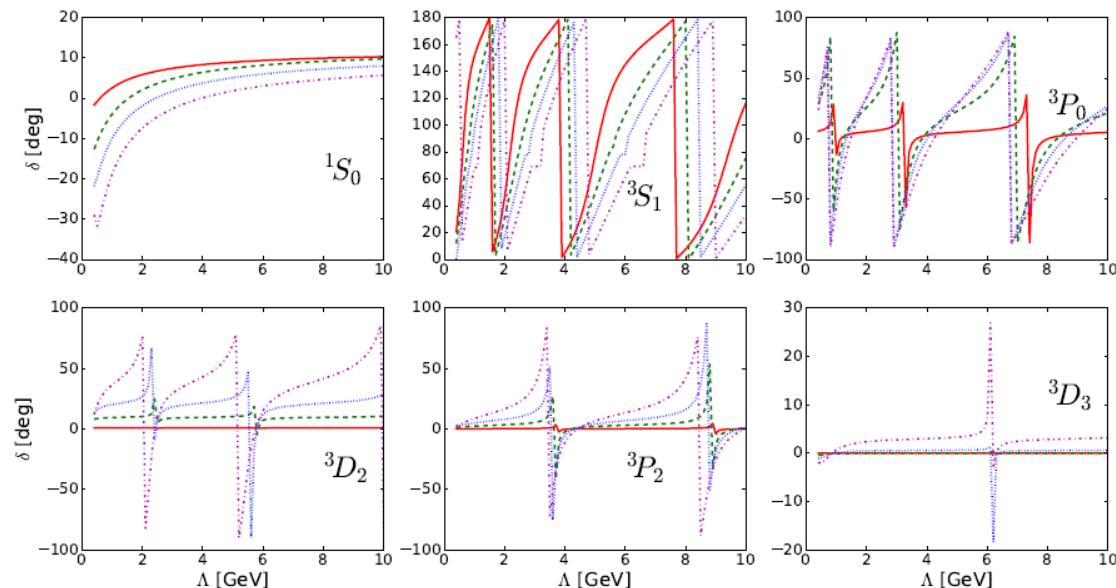
$$V(\vec{p}', \vec{p}) \rightarrow f_n(\vec{p}'/\Lambda) V(\vec{p}', \vec{p}) f_n(\vec{p}/\Lambda)$$

$$f_n(x) = \exp(-x^n), n = 2, 4, 6$$

Y.-H. Song, R. Lazauskas, U. van Kolck,
Phys. Rev. C 96 (2017) 024002

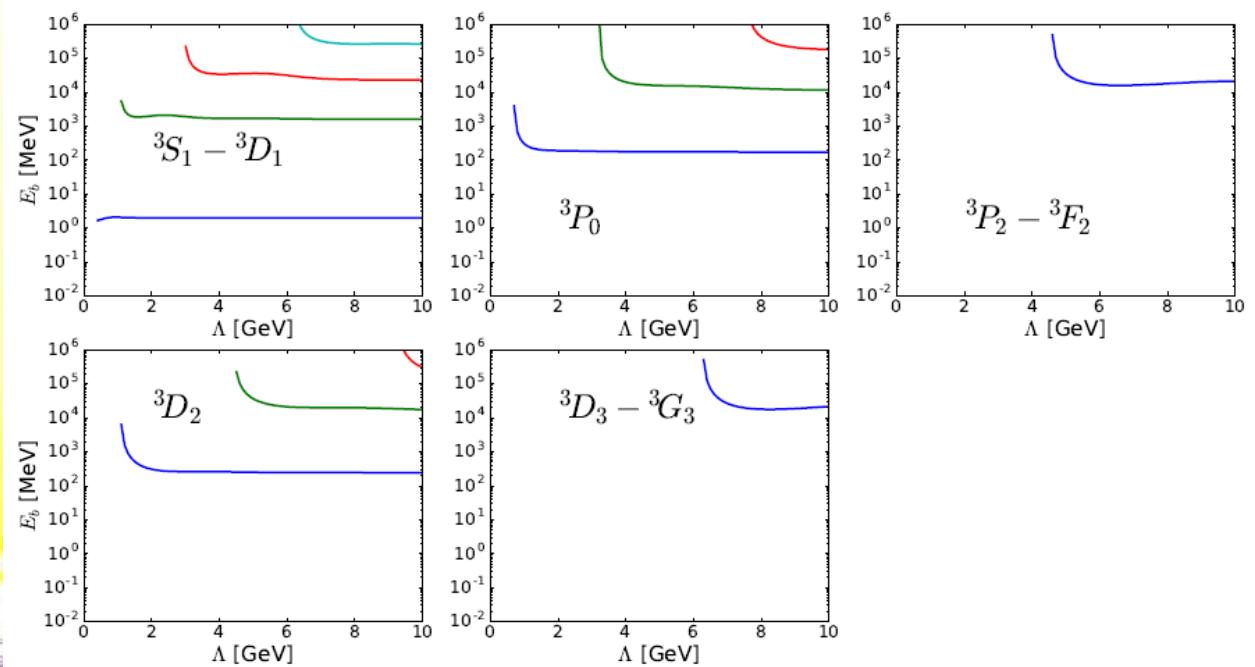
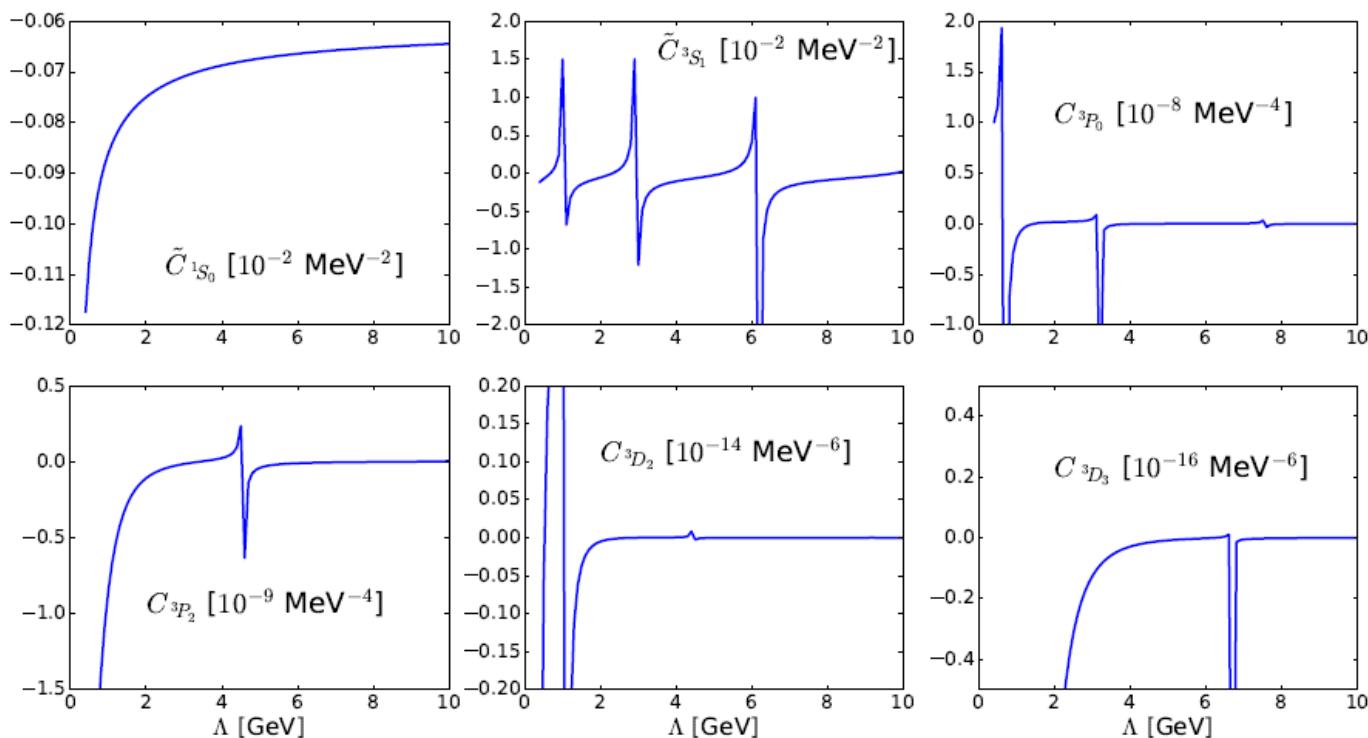
$n = 4$ (similar for others)

$E_{\text{lab}} = 10, 50, 100, 200$ MeV



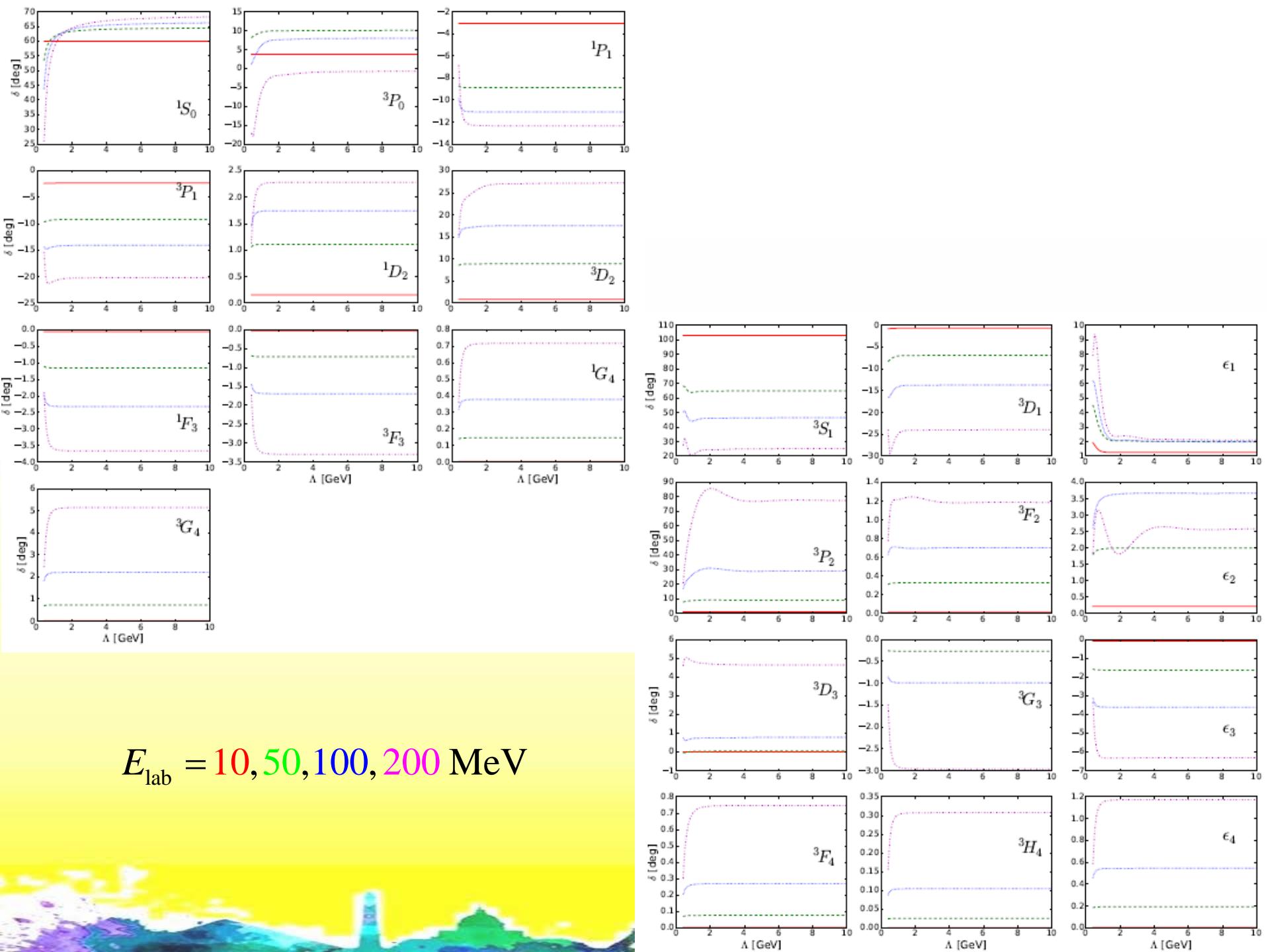
Following Birse

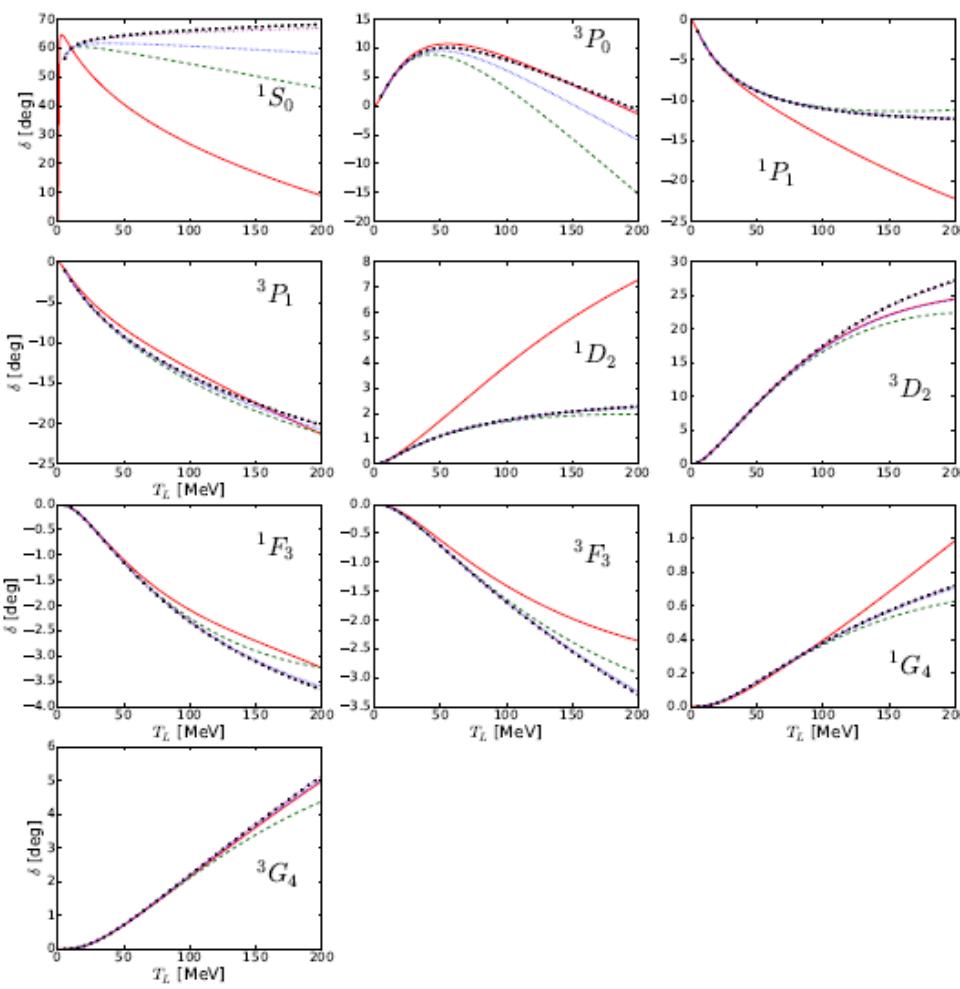
LO



index	1S_0 LO	3S_1 LO	$l > 0$ LO	1S_0 NLO
(I)	δ_{10}	δ_{10}	δ_{10}	δ_{10}, δ_{20}
(II)	δ_5	δ_5	δ_5	δ_5, δ_{10}
(III)	a_s	a_t	δ_{10}	a_s, r_s
(IV)	a_s	E_d	δ_5	a_s, r_s

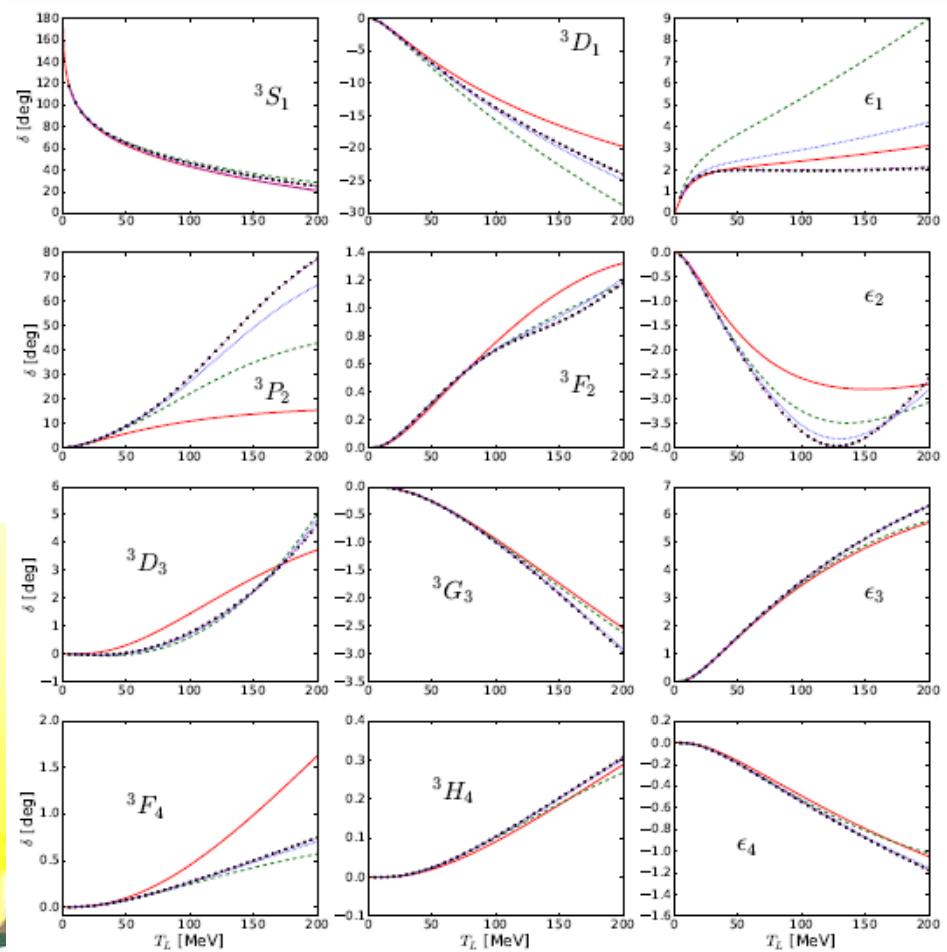
(similar for all)





Nijmegen PWA

$$\Lambda = 0.6, 1, 4, 10 \text{ GeV}$$



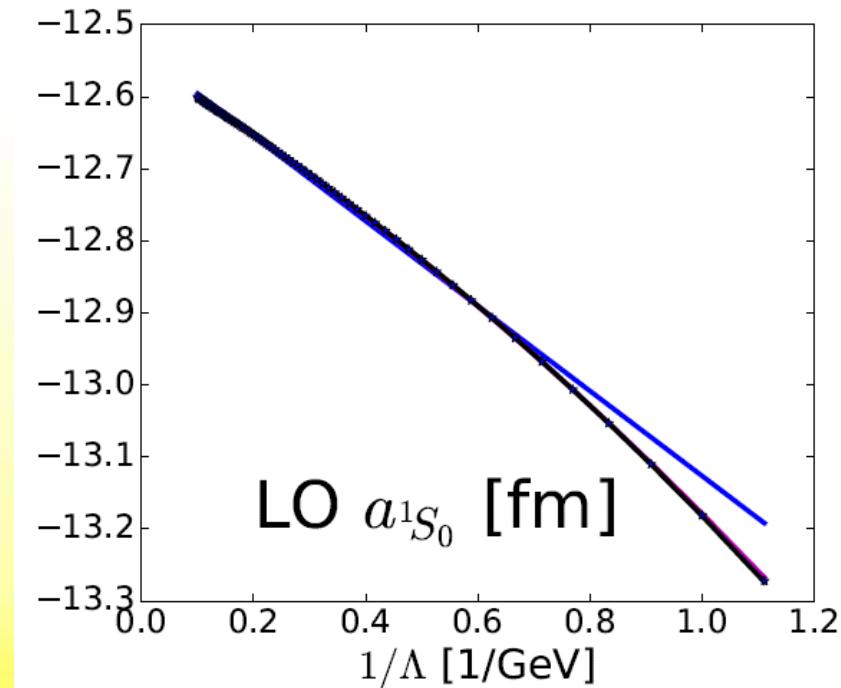
Extension of similar results from

- A. Nogga, R.G.E. Timmermans, U. van Kolck,
Phys. Rev. C **72** (2005) 054006
 E. Epelbaum, U.-G. Meißner,
Few-Body Syst. **54** (2013) 2175

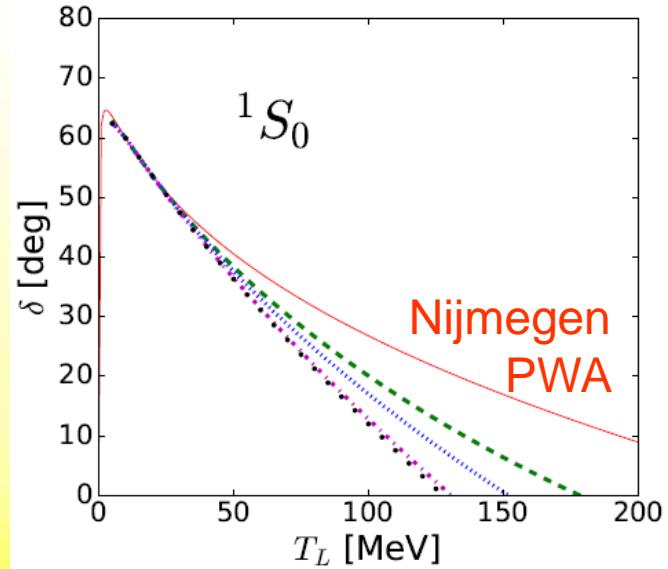
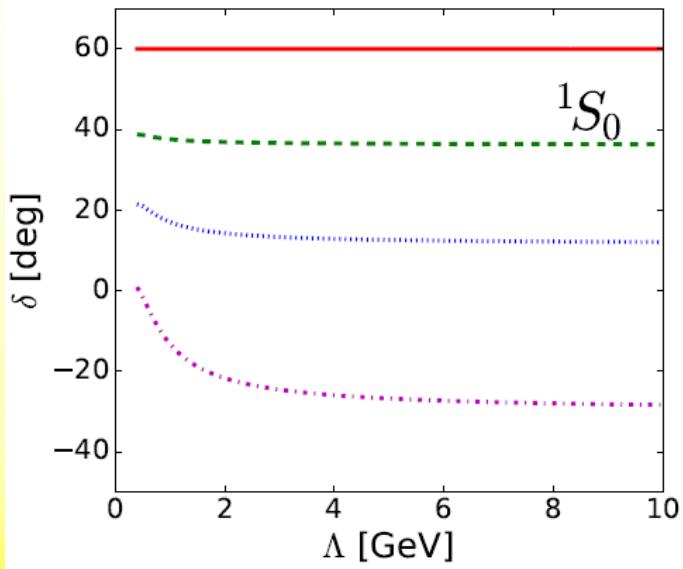
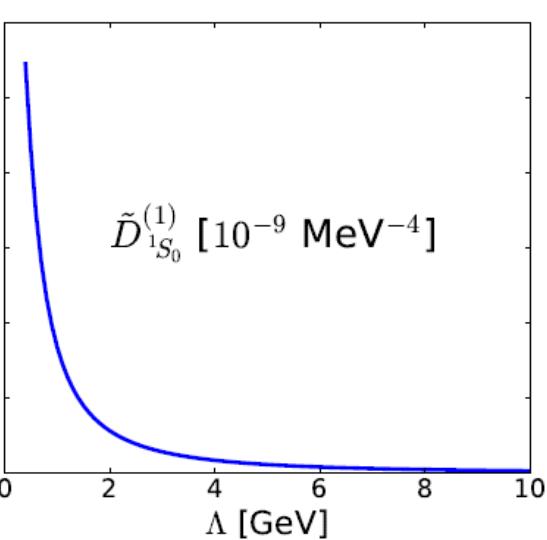
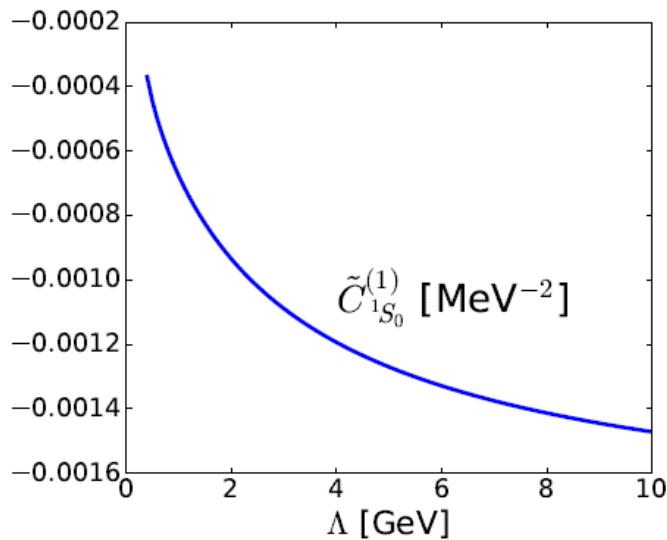
1S_0 is special

$$a_s^{(0)}(\Lambda) = a_s^{(0)}(\infty) \left[1 + \frac{p_{s1}^{(0)}}{\Lambda} + \left(\frac{p_{s2}^{(0)}}{\Lambda} \right)^2 + \left(\frac{p_{s3}^{(0)}}{\Lambda} \right)^3 + \dots \right],$$

LO				
$a_s^{(0)}(\infty)$	$p_{s1}^{(0)}$	$p_{s2}^{(0)}$	$p_{s3}^{(0)}$	
-12.5	47.0(0.3)	-	-	blue
-12.6	37.5(0.1)	111(11)	-	magenta
-12.6	38.3(0.3)	100(28)	124(86)	black



NLO



$E_{\text{lab}} = 10, 50, 100, 200$ MeV

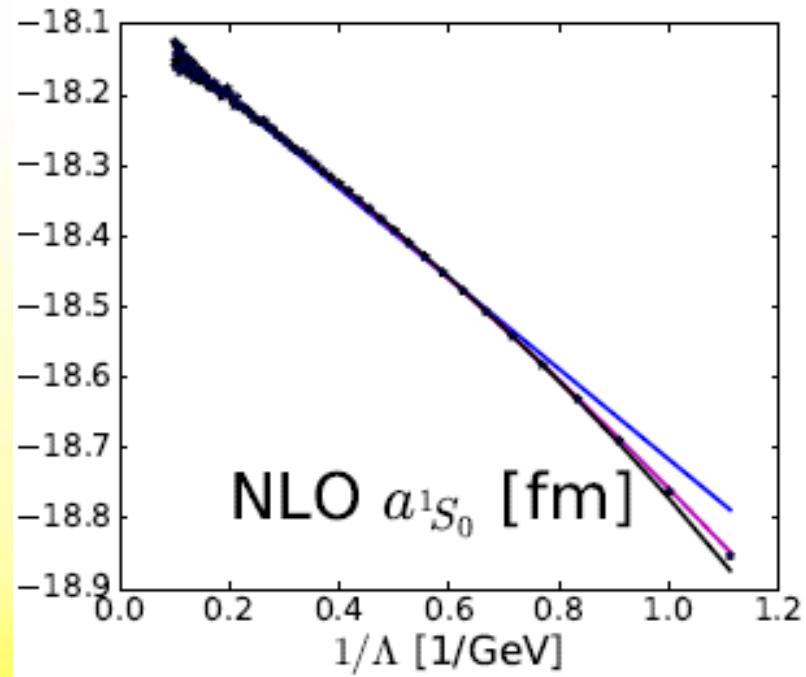
$\Lambda = 0.6, 1, 4, 10$ GeV

$$a_s^{(1)}(\Lambda) = a_s^{(1)}(\infty) \left[1 + \frac{p_{s1}^{(1)}}{\Lambda} + \left(\frac{p_{s2}^{(1)}}{\Lambda} \right)^2 + \left(\frac{p_{s3}^{(1)}}{\Lambda} \right)^3 + \dots \right]$$

NLO

$a_s^{(1)}(\infty)$	$p_{s1}^{(1)}$	$p_{s2}^{(1)}$	$p_{s3}^{(1)}$
-18.1	35.4(0.3)	-	-
-18.1	30.2(0.9)	82.8(34)	-
-18.1	32.5(2.8)	8.1(88)	176(183)

blue
magenta
black



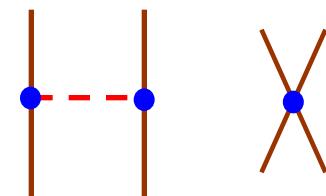
2-body

3-body

...

LO

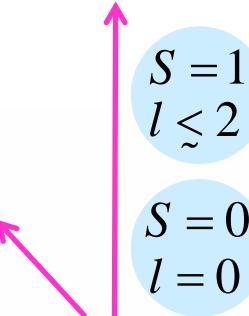
$$\mathcal{O}\left(\frac{4\pi}{m_N Q}\right)$$



NLO

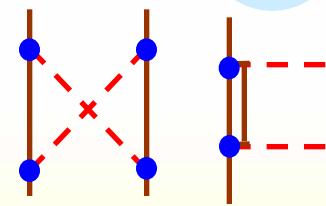
$$\mathcal{O}\left(\frac{4\pi}{m_N M_{QCD}}\right)$$

$$S = 0 \\ l = 0$$



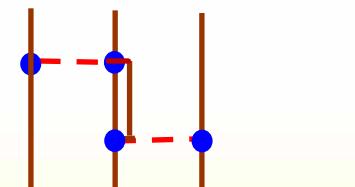
N^2LO

$$\mathcal{O}\left(\frac{4\pi Q}{m_N M_{QCD}^2}\right)$$



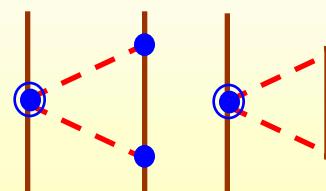
$$S = 0 \\ l = 0$$

$$S = 1 \\ l \lesssim 2$$



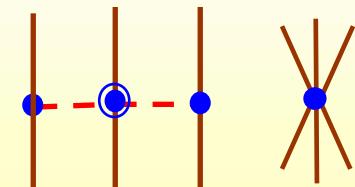
N^3LO

$$\mathcal{O}\left(\frac{4\pi Q^2}{m_N M_{QCD}^3}\right)$$



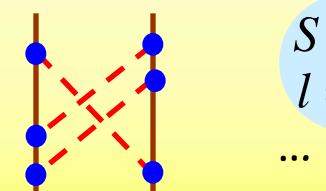
$$S = 0 \\ l = 0$$

$$S = 1 \\ l \lesssim 2$$



N^4LO

$$\mathcal{O}\left(\frac{4\pi Q^3}{m_N M_{QCD}^4}\right)$$



$$S = 0 \\ l = 0$$

$$S = 1 \\ l \lesssim 2$$

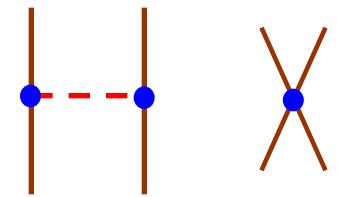
$$... \\ ...$$

etc.

(Details still being worked out!)

LO

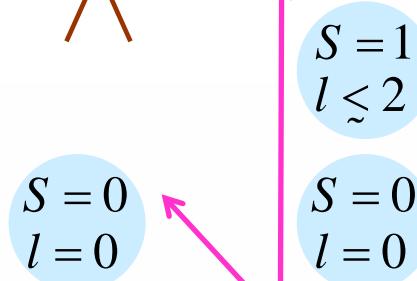
$$\mathcal{O}\left(\frac{4\pi}{m_N Q}\right)$$



3-body

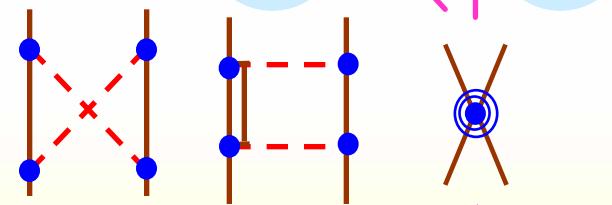
NLO

$$\mathcal{O}\left(\frac{4\pi}{m_N M_{QCD}}\right)$$



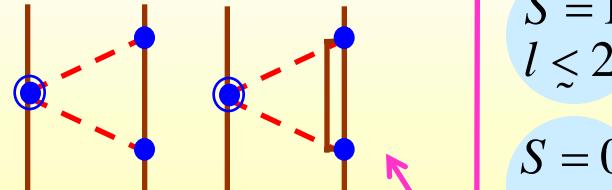
N^2LO

$$\mathcal{O}\left(\frac{4\pi Q}{m_N M_{QCD}^2}\right)$$



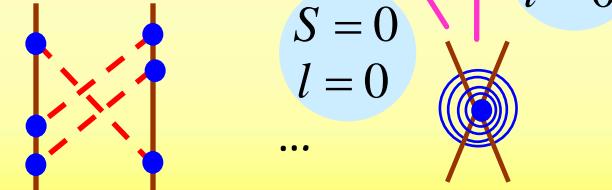
N^3LO

$$\mathcal{O}\left(\frac{4\pi Q^2}{m_N M_{QCD}^3}\right)$$



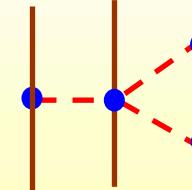
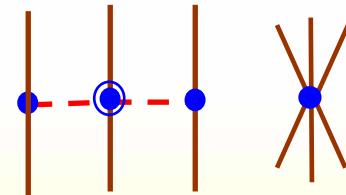
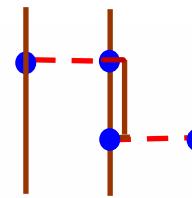
N^4LO

$$\mathcal{O}\left(\frac{4\pi Q^3}{m_N M_{QCD}^4}\right)$$



etc.

2-body



Friar counting
(includes factors
of 4π)

Deltaless

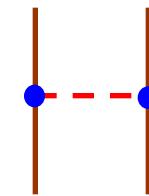
2-body

3-body

...

LO

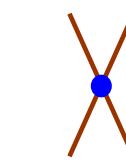
$$\mathcal{O}\left(\frac{4\pi}{m_N Q}\right)$$



NLO

$$\mathcal{O}\left(\frac{4\pi}{m_N M_{QCD}}\right)$$

$$S = 0 \\ l = 0$$



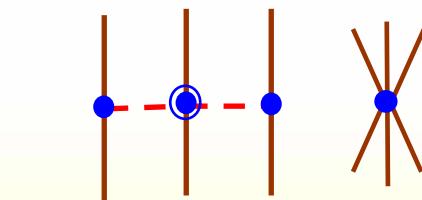
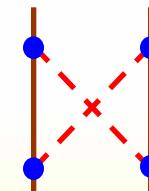
$$S = 1 \\ l \lesssim 2$$

$$S = 0 \\ l = 0$$

$$S = 1 \\ l \lesssim 2$$

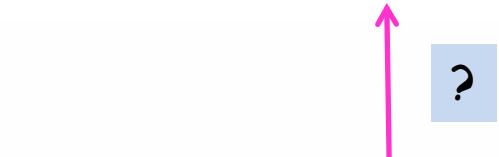
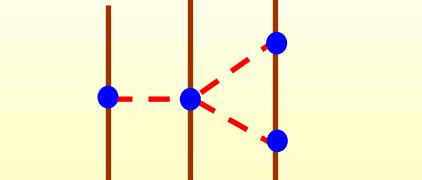
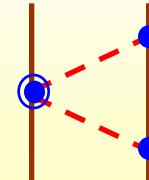
N^2LO

$$\mathcal{O}\left(\frac{4\pi Q}{m_N M_{QCD}^2}\right)$$



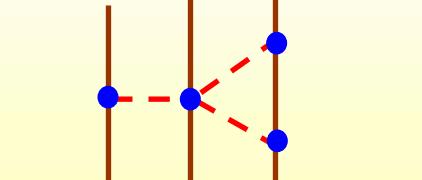
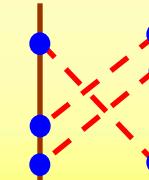
N^3LO

$$\mathcal{O}\left(\frac{4\pi Q^2}{m_N M_{QCD}^3}\right)$$



N^4LO

$$\mathcal{O}\left(\frac{4\pi Q^3}{m_N M_{QCD}^4}\right)$$



etc.

e.g. A. Kievsky *et al.*,
Phys. Rev. C 95 (2017) 024001

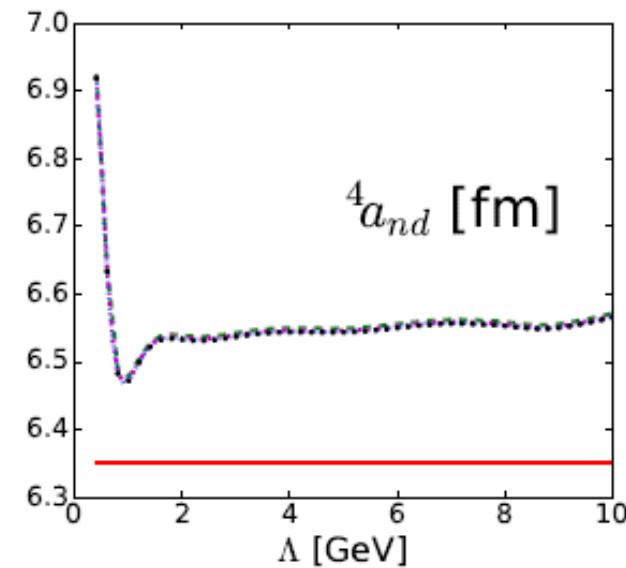
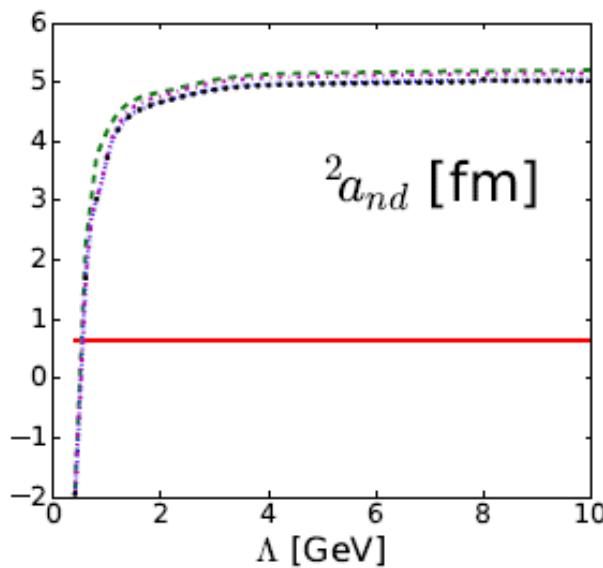
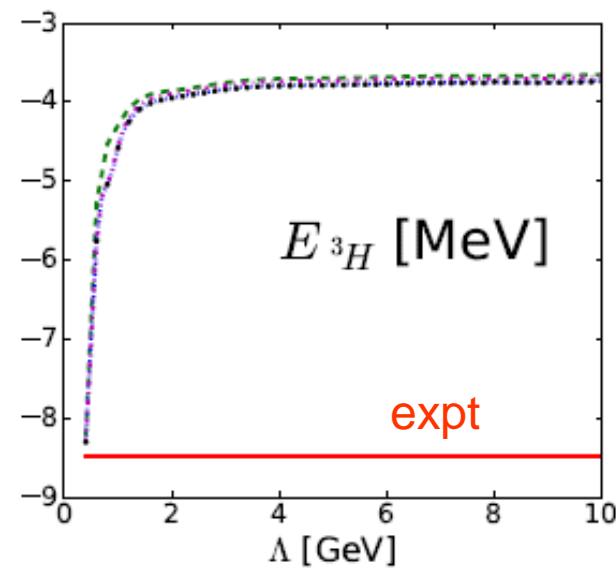
LO

$$V_{\Lambda}^{(0)} \rightarrow \tilde{V}_{\Lambda}^{(0)} = V_{\Lambda}^{(0)} + \sum_n |\psi_n\rangle \lambda_n \langle \psi_n|,$$

wavefunctions of
deep 2B bound states

large and
positive

V.I. Kukulin, V.N. Pomerantsev, Ann. Phys. 111 (1978) 330

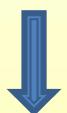


$j_{\max} = 1, 2, 3, 4$

$$E_{^3\text{H}}^{(0)}(\Lambda) = E_{^3\text{H}}^{(0)}(\infty) \left[1 + \frac{p_{t1}^{(0)}}{\Lambda} + \left(\frac{p_{t2}^{(0)}}{\Lambda} \right)^2 + \left(\frac{p_{t3}^{(0)}}{\Lambda} \right)^3 + \dots \right]$$

LO

$E_{^3H}^{(0)}(\infty)$	$p_{t1}^{(0)}$	$p_{t2}^{(0)}$	$p_{t3}^{(0)}$	
-3.82	146(3)	-	-	blue
-3.88	35.8(4.6)	377(75)	-	magenta
-3.88	29.6(14)	399(194)	243(310)	black

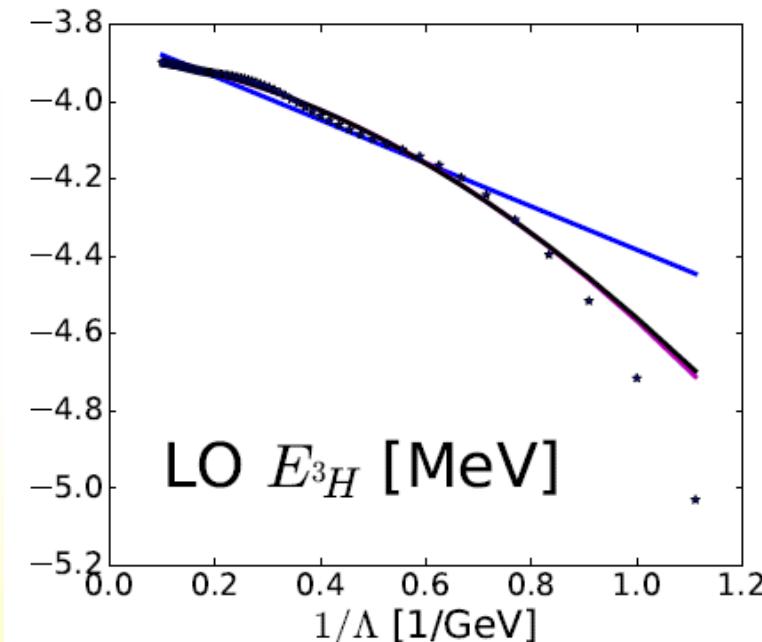


finite

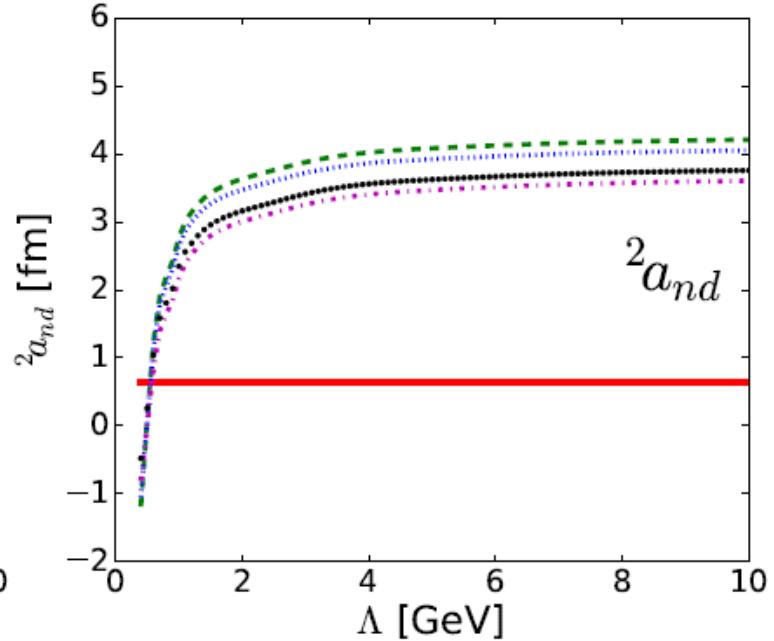
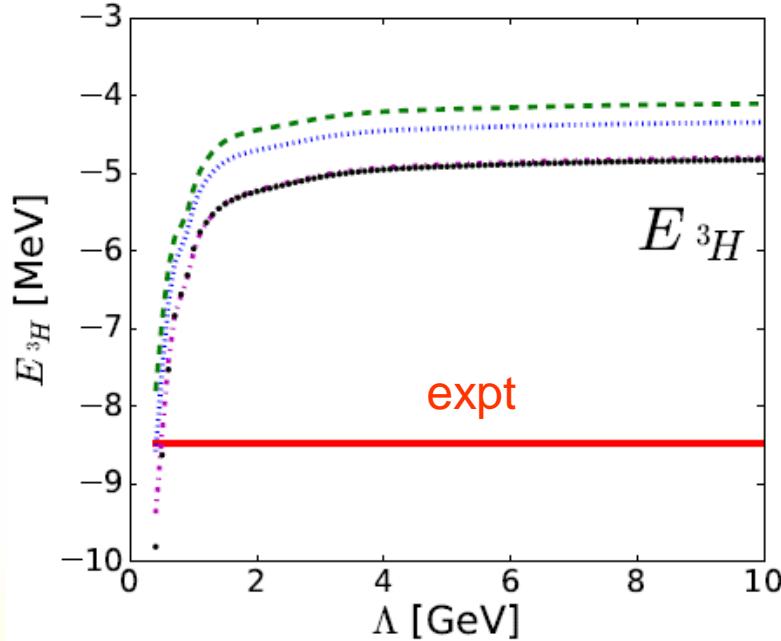
but small

NO RG NEED FOR LO 3BF

because triton energy too low, ie, amenable to Pionless EFT?
 or motivation to promote 3B contact? cf. Kievsky *et al.* (2017)



NLO



	index	1S_0 LO	3S_1 LO	$l > 0$ LO	1S_0 NLO
green	(I)	δ_{10}	δ_{10}	δ_{10}	δ_{10}, δ_{20}
blue	(II)	δ_5	δ_5	δ_5	δ_5, δ_{10}
magenta	(III)	a_s	a_t	δ_{10}	a_s, r_s
black	(IV)	a_s	E_d	δ_5	a_s, r_s

Corrections in the
right direction
but small

$$E_{^3\text{H}}^{(1)}(\Lambda) = E_{^3\text{H}}^{(1)}(\infty) \left[1 + \frac{p_{t1}^{(1)}}{\Lambda} + \left(\frac{p_{t2}^{(1)}}{\Lambda} \right)^2 + \left(\frac{p_{t3}^{(1)}}{\Lambda} \right)^3 + \dots \right]$$

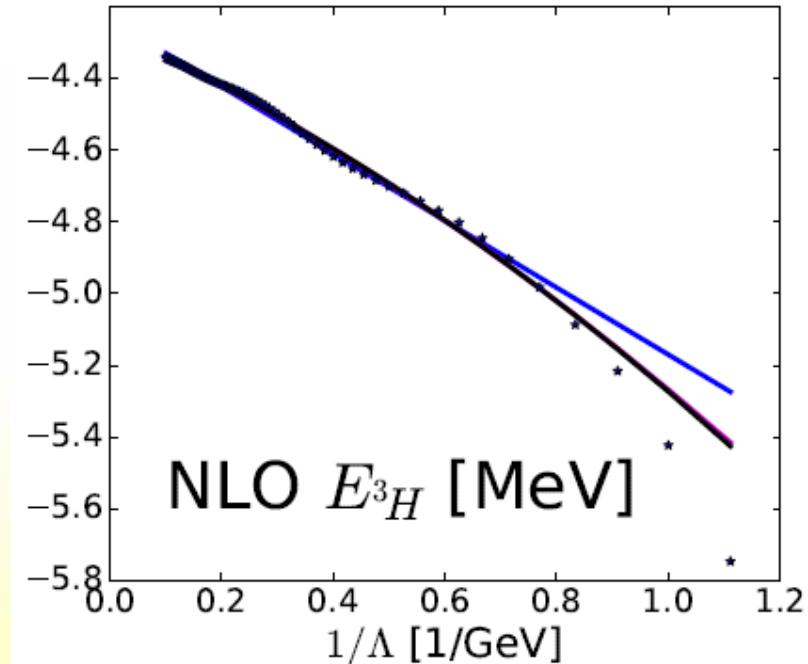
NLO

$E_{^3\text{H}}^{(1)}(\infty)$	$p_{t1}^{(1)}$	$p_{t2}^{(1)}$	$p_{t3}^{(1)}$
-4.24	221(2)	-	-
-4.27	167(5)	262(80)	-
-4.26	171(16)	239(208)	208(325)

blue

magenta

black



finite
↓

but small

somewhat larger than before

NO RG NEED FOR NLO 3BF

indication of relatively
larger N²LO corrections?

Conclusion

- ◆ Power counting: beyond Weinberg 
 - Pionless EFT
 - Chiral EFT with perturbative pions
 - Chiral EFT with *partly* perturbative pions
- ◆ No RG need for LO and NLO 3BFs with partly perturbative pions ---
but more work needed to understand three- (and more-) nucleon systems