What's the Issue – and Why Do We Care?

H. W. Grießhammer

Institute for Nuclear Studies
The George Washington University, DC, USA

1. The Promise of Reliable Error Bars
2. Live Up To Your Promises!
3. The Promise of Being Systematic
4. Concluding Questions

Providing reliable theoretical uncertainties,
testing non-perturbative EFTs.

hg: *Nucl. Phys.* **A744** (2004) 192;
hg: NNPSS 2008, Saclay workshop 04.03.2013, Benasque workshop 24.07.2014;
hg: Chiral Dynamics proceedings [arXiv:1511.00490 [nucl-th]]

Highly readable & even-handed: Phillips 1302.5959

1. The Promise of Reliable Error Bars

(a) (Dis)Agreement Significant Only When All Error Sources Explored

Editorial PRA 83 (2011) 040001

PHYSICAL REVIEW A **83**, 040001 (2011)

Editorial: Uncertainty Estimates

The purpose of this Editorial is to discuss the importance of including uncertainty estimates in papers involving theoretical calculations of physical quantities.

It is not unusual for manuscripts on theoretical work to be submitted without uncertainty estimates for numerical results. In contrast, papers presenting the results of laboratory measurements would usually not be considered acceptable for publication

The question is to what extent can the same high standards be applied to papers reporting the results of theoretical calculations. It is all too often the case that the numerical results are presented without uncertainty estimates. Authors sometimes say that it is difficult to arrive at error estimates. Should this be considered an adequate reason for omitting them? In order to answer this question, we need to consider the goals and objectives of the theoretical (or computational) work being done. Theoretical papers

physical effects not included in the calculation from the beginning, such as electron correlation and relativistic corrections. It is of course never possible to state precisely what the error is without in fact doing a larger calculation and obtaining the higher accuracy. However, the same is true for the uncertainties in experimental data. The aim is to estimate the uncertainty, not to state the exact amount of the error or provide a rigorous bound.

Theoretical uncertainty: Truncation of Physics

Does Nuclear Structure emerge from QCD?
Beyond-Standard-Model Physics from Supernovae?

EFT claim: systematic in $Q = \dfrac{\textit{typ. low scale } p_{typ}}{\textit{typ. high scale } \overline{\Lambda}_{EFT}}$

Religion — Thou Shalt Believe!

Science: Degree of Belief — ***Conjecture*** ⇄ ***Evidence***

Scientific Method: Quantitative results with corridor of theoretical uncertainties for *falsifiable predictions.*

Need procedure which is established, economical, reproducible: room to argue about "error on the error".

"Double-Blind" Theory Errors: Assess with pretense of no/very limited data.

2. Live Up To Your Promises!

Wilson, Weinberg, ...

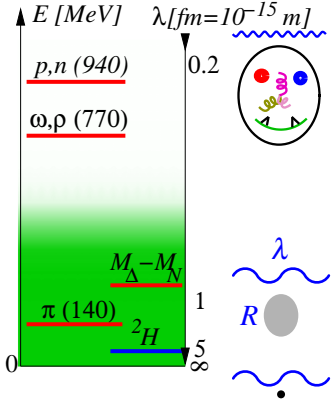
(a) Chiral Effective Field Theory of Nuclear Physics

Correct long-range + symmetries: Chiral SSB, gauge, iso-spin,...

$\Longrightarrow$ ***Write most general Interaction Lagrangean permitted.***

Short-range: ignorance into *minimal* parameter-set at given order.

Coefficients from experiment or QCD or...



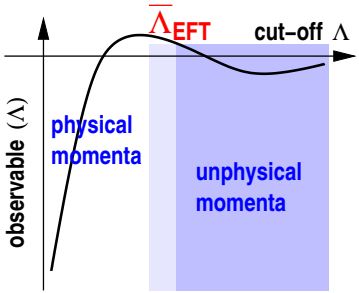
"The Power Counting":

Systematic ordering in $Q = \dfrac{\text{typ. momentum } p_{\text{typ}}}{\text{breakdown scale } \overline{\Lambda}_{\text{EFT}}} \ll 1$

Controlled approximation: model-independent, error-estimate.

Space for improvement.

$\Longrightarrow$ **Chiral Effective Field Theory χEFT $\equiv$ low-energy QCD**



$\Longrightarrow$ saturated at $\overline{\Lambda}_{\text{EFT}} \lesssim \Lambda$.

NN: $\chi^2/\text{d.o.f} \approx 1$ for χ "N^{3}LO " and AV18 – 24 vs 40 parameters.

(b) NN χEFT Power Counting Comparison

prepared for Orsay Workshop by Grießhammer 7.3.2013
based on and approved by the authors in private communications

Derived with explicit & implicit assumptions; contentious issue.

All but WPP: RGE as construction principle, but different approximations at short-range lead to variant interpretations.

Proposed order Q^n at which counter-term enters *differs*. $\Longrightarrow$ Predict *different* accuracy, # of parameters.

order	**Weinberg** (modified) PL**B251** (1990) 288 etc.	**Birse** PR**C74** (2006) 014003 etc.	**Pavon Valderrama et al.** PR**C74** (2006) 054001 etc.	**Long/Yang** PR**C86**(2012) 024001 etc.
Q^{-1}	LO of $^1S_0$, $^3S_1$, OPE			
		plus $^3D_1$, $^3SD_1$	plus $^3P_{0,2}$, $^3D_2$	plus $^3P_{0,2}$
$Q^{-\frac{1}{2}}$	none	LO of $^3P_{0,1,2}$, $^3PF_2$, $^3F_2$, $^3D_2$	LO of $^3SD_1$, $^3D_1$, $^3PF_2$, $^3F_2$	none
Q^0	none	NLO of $^1S_0$		
$Q^{\frac{1}{2}}$	none	NLO of $^3S_1$, $^3D_1$, $^3SD_1$	none	none
Q^1	LO of $^3SD_1$, $^1P_1$, $^3P_{0,1,2}$; NLO of $^1S_0$, $^3S_1$	none	none	LO of $^3SD_1$, $^1P_1$, $^3P_1$, $^3PF_2$; NLO of $^3S_1$, $^3P_0$, $^3P_2$; N^{2}LO of $^1S_0$
# at Q^{-1}	2	4	5	4
# at Q^0	$+0$	$+7$	$+5$	$+1$
# at Q^1	$+7$	$+3$	$+0$	$+8$
total at Q^1	9	14	10	13

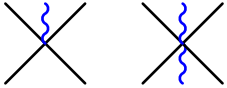
With same $\chi^2/$d.o.f., the *self-consistent* proposal with least parameters *wins*: minimum information bias.

(c) EFT and Information Theory: Lossless Compression vs. Data Reproduction

Number of parameters at Q^1 for some attractive partial waves:

wave	Weinberg (modified) PL**B251** (1990) 288 etc.	Birse PR**C74** (2006) 014003 etc.	Pavon Valderrama et al. PR**C74** (2006) 054001 etc.	Long/Yang PR**C86**(2012) 024001 etc.
$^3P_2$-$^3F_2$	1, very small	3 of similar size	3 of different orders	2 of different orders
$^3P_0$	1, very small	1 just below LO	1 non-perturbative (LO)	2 of different orders

Predict different importance also for gauge currents: $\vec{p}\cdot\vec{p}\,' C_P \rightarrow$

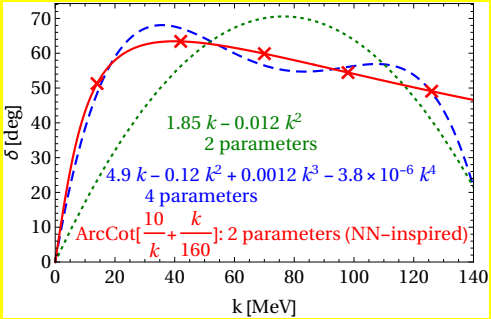


The EFT Promise:

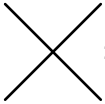
Encode information about unresolved short-range at given resolution and *at given order* into smallest number of independent CTs:

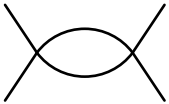
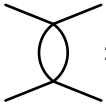
minimal set of parameters for *lossless compression*.

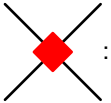
$\Longrightarrow$ **Falsifiability; robust predictions to uncover new Physics, Alternative Worlds, hidden symmetries (unitarity $\rightarrow$König, large-N_c Schindler/Springer 2018,...).**



(d) Ordering Perturbative EFTs: Example Fermi's Theory

LO:  : $G_F \sim Q^0$

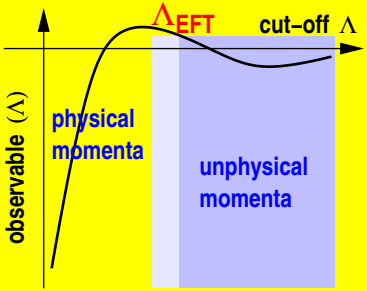
  : $G_F^2 \int^{\text{cutoff } \Lambda} \frac{d^4q}{\not{q}\not{q}} \sim G_F^2 \left(\Lambda^2 + p^2 \ln\Lambda + \frac{1}{\Lambda} + \ldots\right) \to \infty$

$\Longrightarrow$ Counter term  : $p^2\, g_2(\Lambda)$ at $Q^2 =$N^{2}LO absorbs UV divergence. Initial condition: datum.

Perturbative EFT: Simply count powers of p: N^{n}LO $\sim p^n$ counter terms.

"UV divergence" is code for "Physics Nonsense".

("Sensitive to momenta $q \gtrsim \overline{\Lambda}_{\text{EFT}}$ at which EFT is not supposed to work/unphysical")

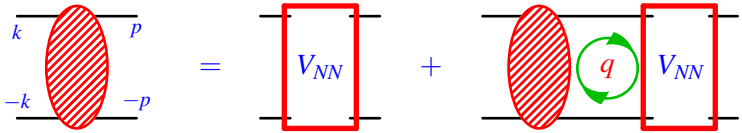


(e) Unusual Scales Obscure the Chiral Power-Counting

Weinberg 1991, van Kolck 1992-;
cf. hg 1511.00490 [nucl-th]

Phenomenology: Non-relativistic system with shallow (real/virtual) bound-state $\Longrightarrow$ LO *non-perturbative.*

$$T_{NN}(E \sim \tfrac{p^2,k^2}{M}) \sim Q^{-1}$$

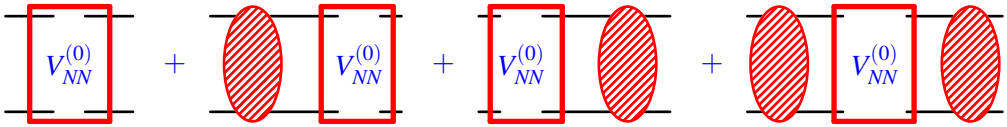


$$T_{\rm LO} = V_{\rm LO} + V_{\rm LO}\, G_{NN}^{\rm nonrel.}\, T_{\rm LO}$$

Power-Counting: $Q^m \qquad Q^m \qquad Q^{2m+3-2} \overset{!}{=} Q^m \Longrightarrow m = -1$

All corrections in "strict perturbation" about LO (Distorted-Wave Born; efficient way: Vanasse 1305.0283):
$\Longrightarrow$ Power-counting of amplitudes (observables); simple, no resummation artefacts.

Example NLO:



$$T_{\rm NLO} = (\mathbb{1} + T_{\rm LO}^\dagger)\, V_{\rm NLO}\, (\mathbb{1} + T_{\rm LO})$$

Popular Alternative: Power-count & iterate $V_{NN} \Longrightarrow T = \dfrac{V_{\rm LO} + V_{\rm NLO} + \ldots}{1 - (V_{\rm LO} + V_{\rm NLO} + \ldots)\, G_{NN}}$. Weinberg 1990

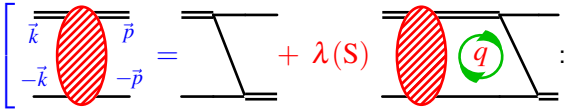
$\Longrightarrow$ Obscures PC in observables, unphysical poles around $\overline{\Lambda}_{\rm EFT}$: artefacts, wrong causal structure.

$\Longrightarrow$ Limited to small cutoff variation range $\Lambda \approx \overline{\Lambda}_{\rm EFT} \pm 20\%$, implementation & numerics more difficult.

(f) EFT($\not{\pi}$): Order of 3N Interactions from UV Behaviour

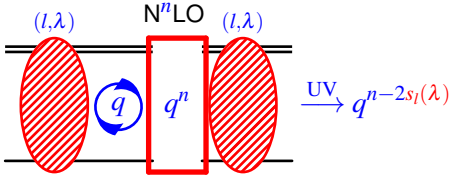
hg NPA**760** (2005) 110

LO: lth partial wave



: UV limit analytic $\mathcal{A}_{(l,S)}(0, q \to \infty) \propto \dfrac{1}{q^{s_l(\lambda)+1}}$

Include 3BF only if needed as counter-term to cancel cut-off dependence of low-energy observables.

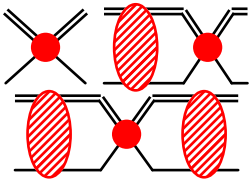


If UV divergent

$\Longrightarrow$

then Counter Term

3-body interaction



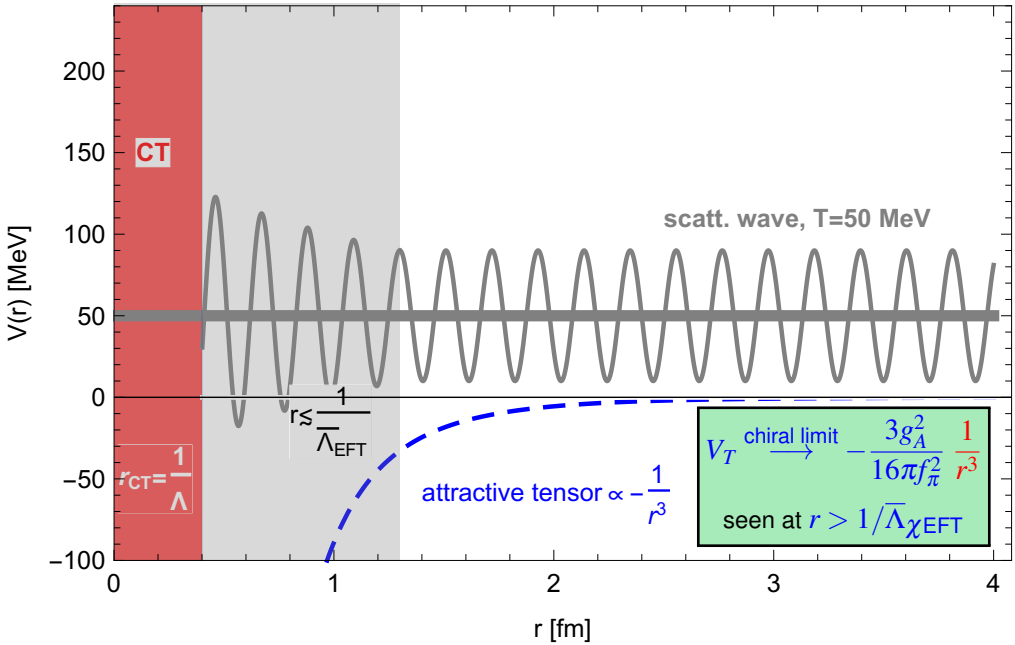
partial wave	asymptotic exponent $s_l(\lambda)$	first 3-body interaction: m derivatives	first 3-body interaction: $\mathrm{Re}[2s_l(\lambda)]$	simplistic $s_l = l+1$
Doublet-S	$\pm 1.0062\,\mathrm{i}$	0: H_0	LO	N^{2}LO: Promoted
		2: $p^2 H_2$	N^{2}LO	N^{4}LO: Promoted
Doublet-P	2.86	2	N$^{5.7}$LO	N^{4}LO: Demoted
Quartet-S	2.16	2	N$^{6.3}$LO	N^{4}LO: Demoted
Quartet-P	1.77	2	N$^{3.5}$LO	N^{4}LO
higher	$\sim l+1$	$2l+2$	N^{2l+2}LO	

EFT non-perturbative at LO $\Longrightarrow$ Ordering of CTs *not* straightforward.

Non-analytic exponents, higher orders in strict perturbation: Expect same in χEFT.

(g) χEFT NN: Intuitive Argument for Attractive Triplet Partial Waves

cf. Birse 2006

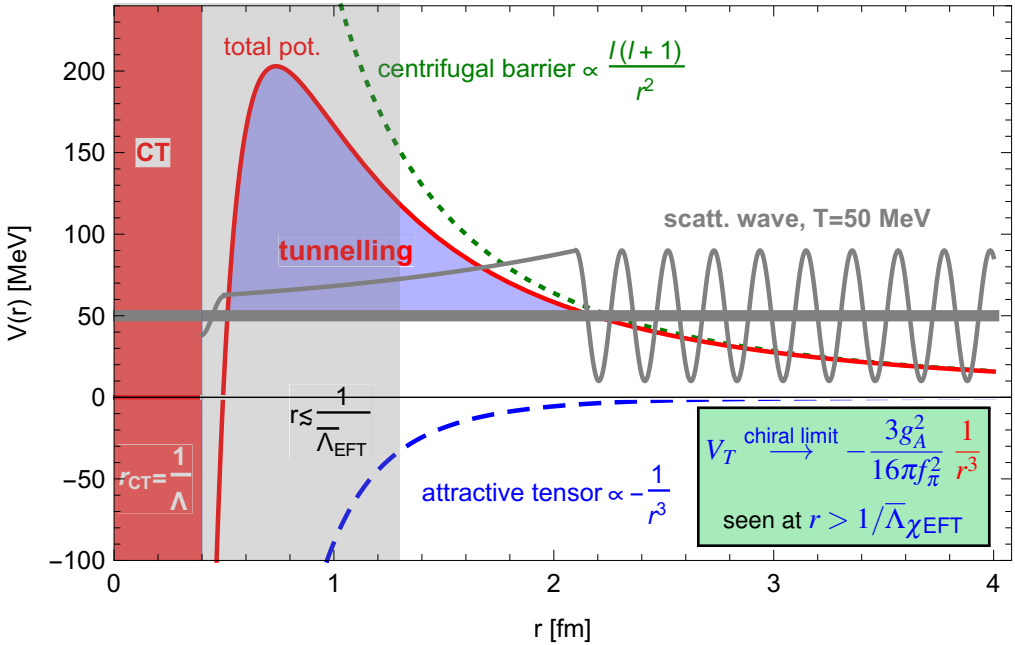


RGE: Adjust strength of CT $c(R = \frac{1}{\Lambda})$ **with** $R = \frac{1}{\Lambda \lesssim \overline{\Lambda}_\chi}$ **such that observables cutoff-independent.**

Initial condition set by one datum, e.g. scatt. length. ***One k-independent CT suffices – predict k-dependence.***

(g) χEFT NN: Intuitive Argument for Attractive Triplet Partial Waves

cf. Birse 2006



Higher PWs: Tunnelling through centrifugal barrier reduces sensitivity to details of short-distance Physics.

Growing centrifugal barrier $l \nearrow$ shields CT. $\Longrightarrow$ Higher partial waves perturbative. Kaiser/Brockmann/Weise 1997 Birse 2006: quantify estimate

Disputes: Cutoff $\Lambda \sim \overline{\Lambda}_{\rm EFT}$ breakdown scale, or all $\Lambda \gtrsim \overline{\Lambda}_{\rm EFT}$ equally acceptable, including $\Lambda \to \infty$?

Effect of higher orders (Distorted-Wave Born or resumming into Schrödinger eq.)?

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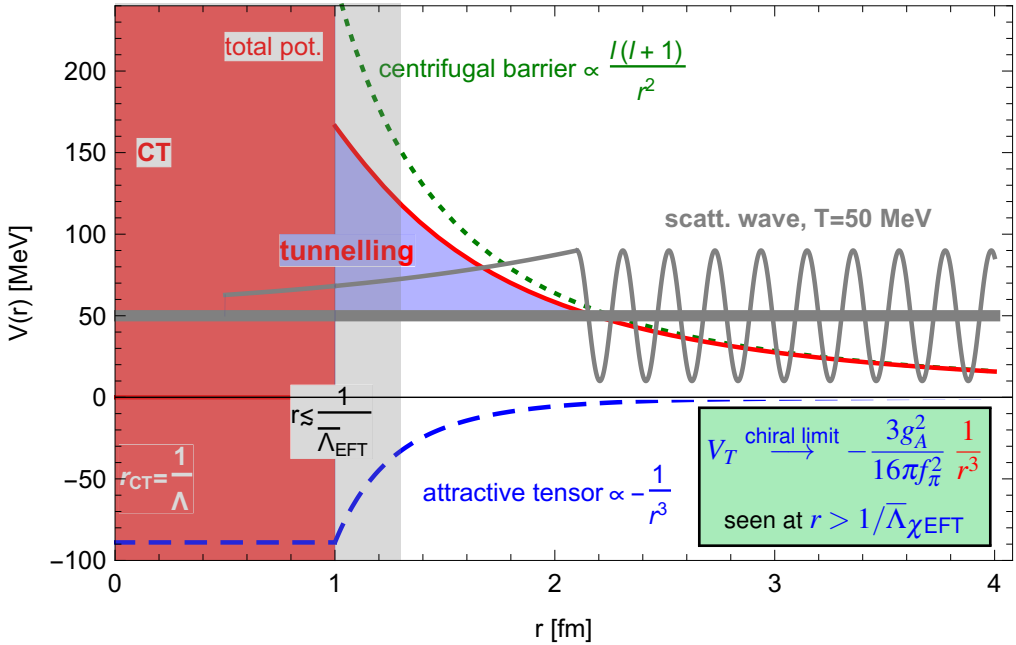

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(h) Even Weinberg Can Be Wrong

Beane/. . . 2002, Nogga/Timmermans/van Kolck 2005, Birse 2005-07;
NLO: Song/Lazauskas/van Kolck 1612.09090

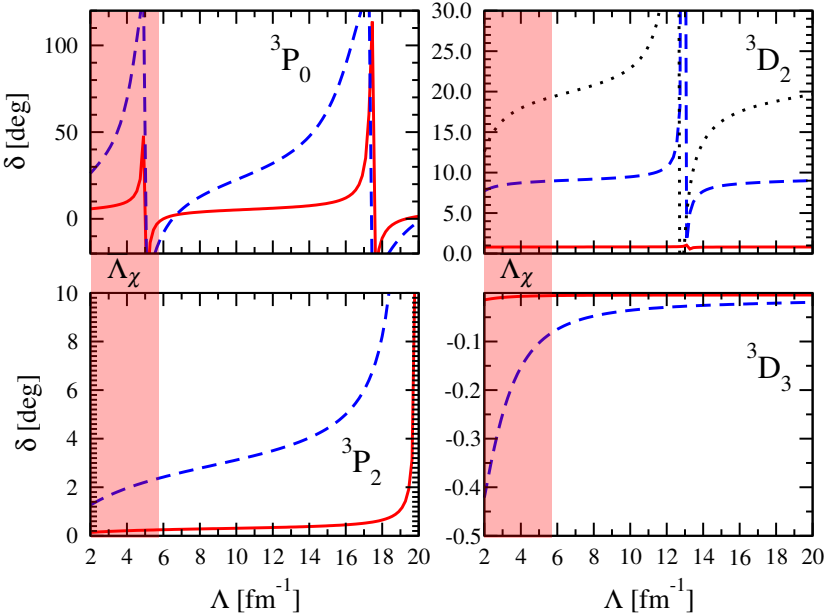
Check consistency of Weinberg's proposal: Observables cut-off dependent at LO?

Low attractive P/D-wave triplets: Weinberg predicts zero LECs at LO (momentum-independence).

phase-shift δ(cut-off Λ):

$E_{\rm lab} = 10$ MeV
50 MeV
100 MeV
190 MeV

$$V(r) \propto -\frac{\#}{r^3} + \frac{l(l+1)}{r^2}$$



Cutoff-dependent, even for $\Lambda \approx \overline{\Lambda}_\chi$ $\Longrightarrow$ Short-distance missing!
$\Longrightarrow$ ***Not renormalised!***

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Check consistency of Weinberg's proposal: Observables cut-off dependent at LO?
Need 4 *new*, momentum-dependent LECs for low attractive triplets: $^3P_{0,2}$, $^3D_{2,3}$.

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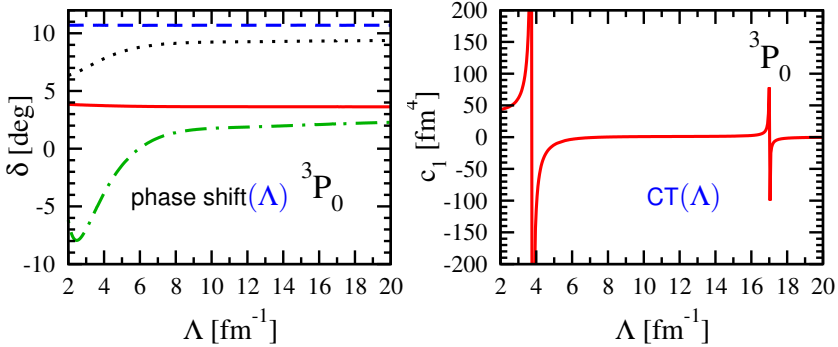


(i) “Mene, Tekel, Upharsin”: Weinberg’s Pragmatic Proposal

Pragmatic, widely used (“Everybody Does It”).

But *conceptually inconsistent*:

– Not renormalised in low partial waves with attractive tensor.

Nogga/Timmermans/van Kolck 2005

– $^1\mathrm{S}_0$: m_π^2-dependence of CT and divergence do not match.

$V_C \to \dfrac{m_\pi^2}{\pi^2\, r}$ but $C_0 \sim m_\pi^0$ bites you for $m_\pi \neq 140\ \mathrm{MeV}$!

Kaplan/Savage/Wise 1996, Beane/Bedaque/Savage/van Kolck 2002

Not just LO problem: LO Reg/Ren ricochets through all orders.

$\Longrightarrow$ WPP underestimates number of CTs per order.

$\Longrightarrow$ Gauged & gauge-invariant currents earlier.

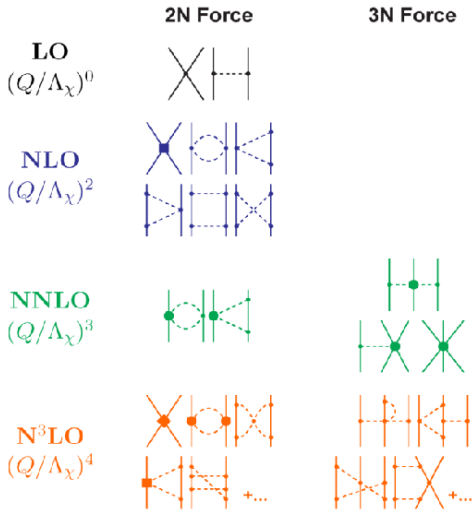
Phillips/Valderrama 2015

$\Longrightarrow$ WPP at alleged order Q^n not as accurate as thought:

Accurate only to lower order $Q^{n-1,2,3,\ldots}$.

Fitting may obscure the problem. . . ;

but final PC will contain many of WPP’s features (we’re already close).



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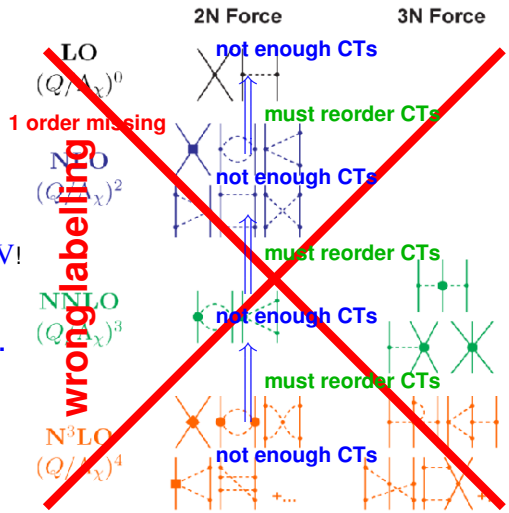


underlying plot by Epelbaum, butchered by hg

We may be unable to say whose PC is right, but we have evidence whose is wrong. WPP is; it's *In-Effective*.

Evgeny conjectures there is no problem.

Still, use it pragmatically to develop numerics & first glimpses at final theory – with *caveat on systematics*!

(j) There's a War Going On

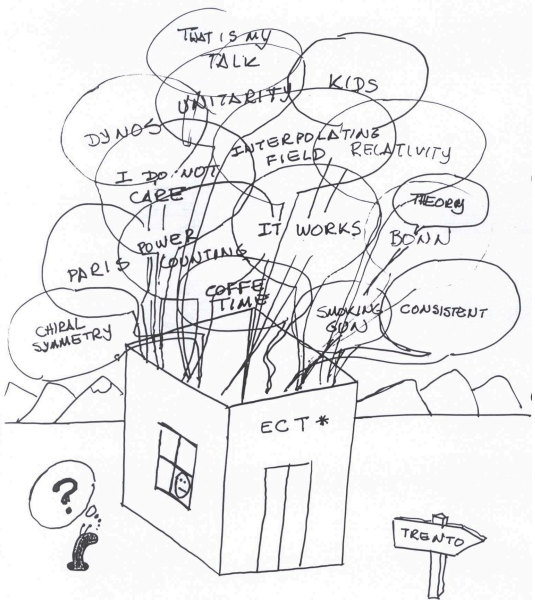
Reminder: Power-counting for non-perturbative EFTs is *not* straightforward.

Contentious is the *short-range* part, (mostly) *not* the long-range one.

Issue would not arrive if we could *derive* PC from underlying theory.

For the sake of this talk, I will be *agnostic* about who is right – if anyone.

But I want to test self-consistency of the proposals.



M. Robilotta: Impression of the Workshop on Nuclear Forces at the ECT*, Trento 1999

The Three Big Lies of Nuclear Physics

Nuclear Power is Safe.

They have Weapons of Mass Destruction.

The Three Big Lies of Nuclear Physics

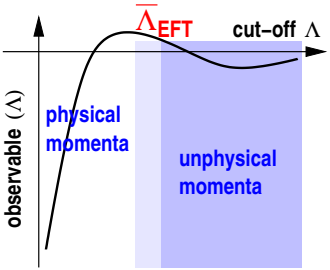
Nuclear Power is Safe.

They have Weapons of Mass Destruction.

My Power-Counting is Systematic.

(a) Quantitative Predictions of Your PC: Advantage of Cut-Offs

hg 2004-; 1511.00490



Observable $\mathcal{O}(k)$ at momentum k, order Q^n in EFT, breakdown $\overline{\Lambda}_{\text{EFT}} \lesssim$ cut-off Λ:

$$\mathcal{O}_n(k;\Lambda) = \underbrace{\sum_i^n \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^i \mathcal{O}_i(k, p_{\text{typ.}})}_{\text{renormalised, } \Lambda\text{-indep.}} + \underbrace{\mathcal{C}(\Lambda; k, p_{\text{typ}}, \overline{\Lambda}_{\text{EFT}}) \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1}}_{\substack{\text{residual } \Lambda\text{-dependence} \\ \text{parametrically small} \\ \mathcal{C} \text{ “of natural size”}}}$$

$\Longrightarrow$ **Difference between *any two* cut-offs:** $\dfrac{\mathcal{O}_n(k;\Lambda_1) - \mathcal{O}_n(k;\Lambda_2)}{\mathcal{O}_n(k;\Lambda_1)} = \left(\dfrac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1} \times \dfrac{\mathcal{C}(\Lambda_1) - \mathcal{C}(\Lambda_2)}{\mathcal{C}(\Lambda_1)}$

Isolate breakdown scale $\overline{\Lambda}_{\text{EFT}}$, order n by double-ln plot of “derivative of observable w. r. t. cut-off”.

Ideally, *no resort to Data!* – Test consistency: Does numerics match predicted convergence pattern?

***After that*, quantitative test of EFT assumptions *against data*.**

Renormalisation Group Evolution: $\Lambda_1 \to \Lambda_2 \Longrightarrow \dfrac{\Lambda}{\mathcal{O}} \dfrac{\mathrm{d}\mathcal{O}}{\mathrm{d}\Lambda} = \left(\dfrac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1} \dfrac{\mathrm{d}\ln\mathcal{C}(\Lambda)}{\mathrm{d}\ln\Lambda} \to 0$ if exact RGE.

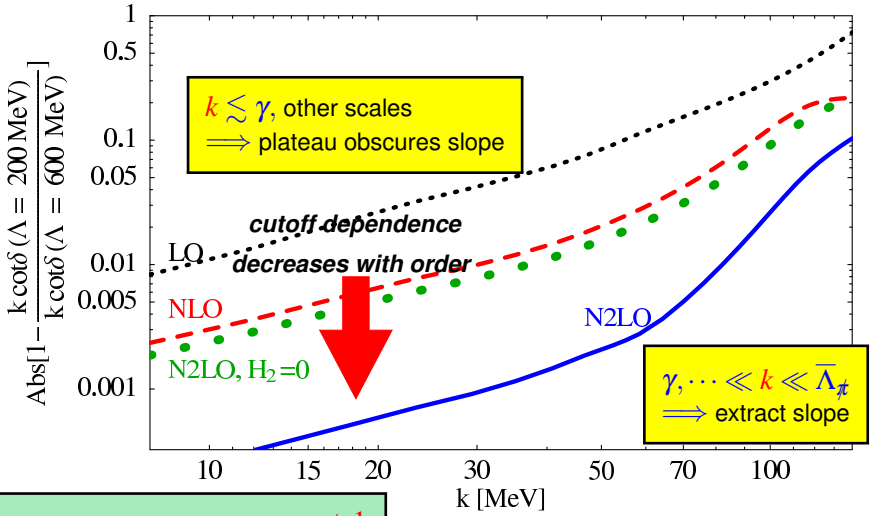
Residual Λ-dependence should “usually” decrease parametrically order-by-order.

Complication: Several intrinsic low-energy scales in few-N EFT:

scattering momentum k, m_π, inverse NN scatt. lengths $\gamma(^3\mathrm{S}_1) \approx 45$ MeV, $\gamma(^1\mathrm{S}_0) \approx 8$ MeV,...

(b) "Toy Model": RG Plot of nd Doublet-S Wave in EFT($\not\pi$)

Bedaque/hg/Hammer/Rupak 2002
hg 2004



$$\left|1 - \frac{k\cot\delta(\Lambda = 200\ \mathrm{MeV})}{k\cot\delta(\Lambda = \infty)}\right| \sim \underbrace{\left(\frac{k, p_{\mathrm{typ.}}}{\overline{\Lambda}_{\not\pi}}\right)^{n+1}}_{Q^{n+1}}$$

	LO	NLO	N^{2}LO	N^{2}LO without H_2
$n+1$ fitted	~ 1.9	2.9	4.8	3.1
$n+1$ predicted	2	3	4	**not renormalised**

$\Longrightarrow$ Fit to $k \in [70; 100 \ldots 130]$ MeV $\gg \gamma, \ldots$: H_2 **is indeed N^{2}LO.**

Slope confirms Power Counting; estimates $\overline{\Lambda}_{\not\pi} \approx 140$ MeV.

(c) Comments: It's Not The Golden Bullet, but Worth A Try

$$\frac{\mathcal{O}_n(k;\Lambda_1)-\mathcal{O}_n(k;\Lambda_2)}{\mathcal{O}_n(k;\Lambda_1)} = \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1} \times \frac{\mathcal{C}(\Lambda_1)-\mathcal{C}(\Lambda_2)}{\mathcal{C}(\Lambda_1)}$$

- Estimate k-dependence of expansion parameter $Q(k) = \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)$

  $\Longrightarrow$ Lower limit of residual theoretical uncertainties.
- ***"Window of Opportunity"***: Fit is most transparent for $p_{\text{typ}} \ll k \ll \overline{\Lambda}_{\text{EFT}}$.
- Any two cutoffs Λ_1, Λ_2 – Numerical leverage?! Cutoff $\Lambda \to \infty$ not necessary.
- Order n, $\overline{\Lambda}_{\text{EFT}}$ regulator independent. – But not $\mathcal{C}$: flexible regulator...

$\Longrightarrow$ Test robustness: cutoff range & schemes, fit window,...

- Non-integer powers, non-analyticities: $n+1 \to n+\text{Re}[\alpha]$ with $n \notin \mathbb{Z}$, $\text{Re}[\alpha] > 0$.

Some Limitations:

– Cannot see LECs which do *not absorb cutoff-dependence*.

– Can be numerically indecisive (e.g. small coefficients).

Test is necessary but not sufficient for consistency.

(c) Comments: It's Not The Golden Bullet, but Worth A Try

$$\frac{\mathcal{O}_n(k;\Lambda_1)-\mathcal{O}_n(k;\Lambda_2)}{\mathcal{O}_n(k;\Lambda_1)} = \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1} \times \frac{\mathcal{C}(\Lambda_1)-\mathcal{C}(\Lambda_2)}{\mathcal{C}(\Lambda_1)}$$

What observable to choose?: Avoid Accidental Zeroes $\mathcal{O}(\Lambda_1)-\mathcal{O}(\Lambda_2)=0$ & Infinities $\mathcal{O}(\Lambda)=0$.

Best if unconstrained: Isolate dynamics!

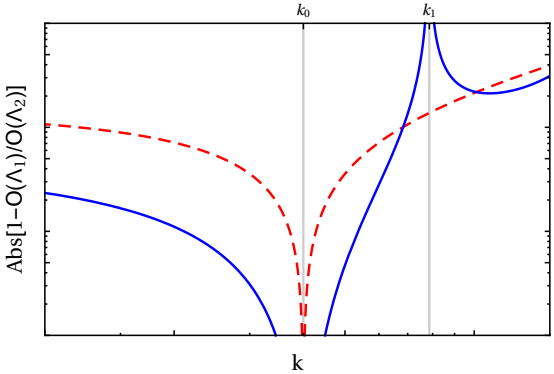
e.g. $k^{2l+1}\cot\delta_l(k)$ for lth scattering wave.

Not $\delta_l(k)$: $\delta_l(k\to 0)\propto k^{2l+1}$: constrained.

Best if same sign for all $k \lesssim \overline{\Lambda}_{\text{EFT}} \implies$ Peruse Λ_1, Λ_2.

If LECs need fitting, the fit for $k \lesssim p_{\text{typ}}$.

Slope may still emerge for $k \nearrow \overline{\Lambda}_{\text{EFT}}$; larger LEC fit error.



Goal: Test Self-Consistency, not Convergence to Data. $\implies$ *"RG Plots"* with minimal resort to experiment.

These are *not* "Lepage plots" which compare to data nucl-th/9706029. – EFT may converge but not to data.

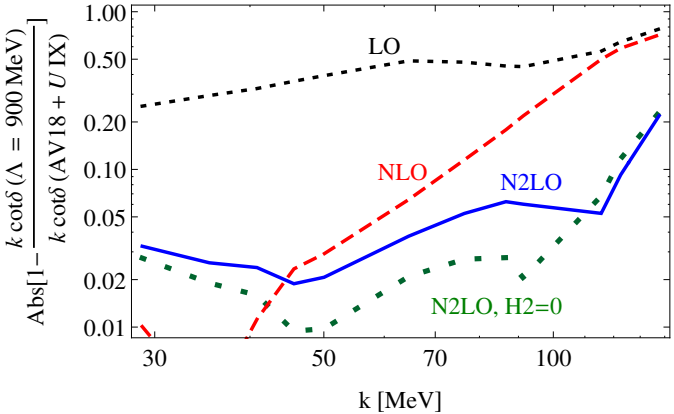
(c) Comments: It's Not The Golden Bullet, but Worth A Try

$$\frac{\mathcal{O}_n(k;\Lambda_1)-\mathcal{O}_n(k;\Lambda_2)}{\mathcal{O}_n(k;\Lambda_1)} = \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1} \times \frac{\mathcal{C}(\Lambda_1)-\mathcal{C}(\Lambda_2)}{\mathcal{C}(\Lambda_1)}$$

These Are *Not* "Lepage-Plots" $\dfrac{\mathcal{O}_n(k;\Lambda)-\mathcal{O}(\textbf{data})}{\mathcal{O}(\textbf{data})}$.

Lepage: nucl-th/9706029; Steele/Furnstahl: nucl-th/9802069; ...

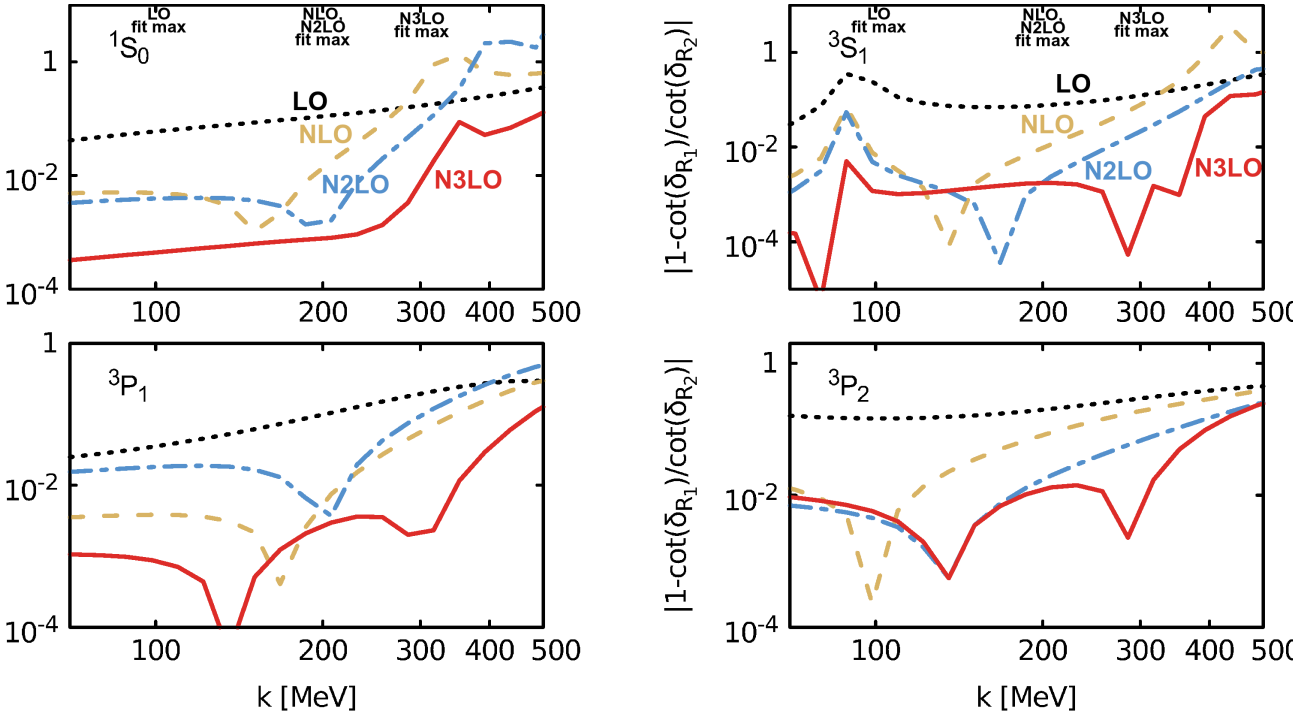
"Lepage" needs data/pseudo-data. $\Longrightarrow$ No consistency test; not double-blind; compromise predictive power.



EFT may converge by itself, but not to data. – Example χEFT without dynamical $\Delta(1232)$ at $k \sim 300$ MeV.

(d) Case of Interest: NN in χEFT: Fitting Parameters Obscures Slopes

Weinberg's Hunch is wrong, but nobody else published: Plot stolen from Epelbaum/Krebs/Meißner EPJA51 (2015) 5, 53.



Inconclusive: Breakdown $400-500$ MeV, fit- & slope-regions not clearly separated.

$k \gtrsim 200$ MeV, but no $\Delta(1232)$ degree of freedom.

Coupled channels; NLO & N^{2}LO parallel? Slopes?

(e) (Some) More Ways to Estimate Theoretical Uncertainties at fixed k

Expansion parameter $Q = \dfrac{\textbf{typ. low scale } p_{\text{typ}}}{\textbf{typ. high scale } \overline{\Lambda}_{\text{EFT}}} \Longrightarrow \mathcal{O} = Q^m \sum_{i=0}^{k-1} c_i(\Lambda)\, Q^i$ **complete at** $\mathcal{O}(Q^{k-1})$ **(N^{k}LO).**

- A priori: Q^k of LO.
- Convergence pattern of series: smaller corrections LO $\rightarrow$ NLO $\rightarrow$ N^{2}LO $\rightarrow \ldots$

  $\Longrightarrow$ **Bayesian estimate:** error $Q^k \times \max_i |c_i|$ captures corridor with $\dfrac{k}{k+1} \times 100\%$ degree of belief.

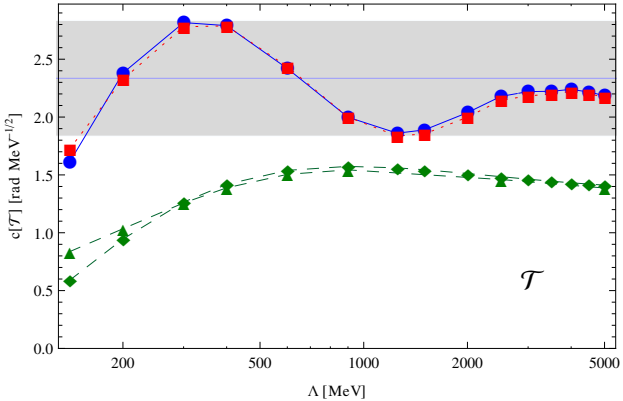
  "Since Time Immemorial" (before 6 July 1189); statistical interpretation by Furnstahl/Klco/Phillips/Wesolowski (BUQEYE) 2015
- Less dependence on particular low-E data taken for LECs. (e.g. Z-param. vs. ERE; fit H_0 to a_3 vs. $B_3, \ldots$)
- Include selected higher-order RG- & gauge-invariant effects: *This does not increase accuracy.*
- ***Any*** Λ **between** $\overline{\Lambda}_{\text{EFT}}$ **and** ∞ **is equally acceptable.** contended by some

  $\Longrightarrow$ Corridor mapped by Λ *in wide range*.

  Should decrease order-by-order.

  Appears often to under-estimate error.

  Example: PV coefficient in nd at $k = 0$.



hg/Schindler/Springer 2012

Choose most conservative/worst-case error for final estimate! Clearly state your choice!

4. Concluding Questions

We have not quite followed through on EFT's promises.

– **Quantitative, falsifiable predictions test EFT's assumptions**: symmetries, constituents, naturalness,...

An EFT may be consistent and converge, but not with & to Nature.

– If **non-perturbative EFT** not derived from underlying theory, finding a consistent Power-counting is *non-trivial*.

$\Longrightarrow$ **Much debate, but agreement that Weinberg is wrong:** no RG-invariance,...

Consistency Test "Momentum-dependent Renormalisation Group flow of observable with cut-off":

$$\frac{\mathcal{O}_n(k;\Lambda_1)-\mathcal{O}_n(k;\Lambda_2)}{\mathcal{O}_n(k;\Lambda_1)} \propto \left(\frac{k, p_{\text{typ.}}}{\overline{\Lambda}_{\text{EFT}}}\right)^{n+1} \quad \textbf{for any two cut-offs } \Lambda_1, \Lambda_2 \gtrsim \overline{\Lambda}_{\textbf{EFT}}.$$

- For ***order*** $\mathcal{O}(Q^n)$ ***to which result is complete***: slope at $k \gg$low scales;
- For ***breakdown scale*** $\overline{\Lambda}_{\textbf{EFT}}$: k at which different orders show same-size variations;
- For lower bound on ***expansion parameter*** Q: vary Λ_1, Λ_2 over wide range.

Minimal resort to data, but may be inconclusive. – One of hopefully many arrows in the quiver.

– ***EFT results must have reproducible, defensible assessment of theoretical uncertainties!!*** *Bayes helps.*

Goal: World Domination by Uncertainty Quantification. – *Error Bars for Nuclear Theory!* –

The efficient person gets the job done right. The effective person gets the right job done.

There is always a well-known solution
to every human problem —
neat, plausible, and wrong.

H. L. Mencken

5. The EFT Promise: Serious Theorists Have Error Bars

(a) Physical Models vs. Physical Theories – Sliding Scale

Model: Capture *some* aspects with lots of data – no "fail" but "tuned". Cargo Cult mode.

The Trouble With Nuclear Physics

In fact the trouble in the recent past has been a surfeit of different *models* [of the nucleus], each of them successful in explaining the behavior of nuclei *in some situations*, and each in *apparent contradiction with other successful models* or with our ideas about nuclear forces.

Rudolph E. Peierls: "The Atomic Nucleus", *Scientific American* **200 (1959)**, no. 1, p. 75; emphasis added

Theory: Comprehensive, prescriptive, predictive, may fail. Explain-All-To-Some-Degree mode.

Gelman's Totalitarian Principle/Swiss Basic Law/ Weinberg's "Folk Theorem": Throw In the Kitchen Sink

As long as you let it be the most general possible Lagrangian consistent with the symmetries of the theory, you're simply writing down the most general theory you could possibly write down.

Original: Weinberg: Physica 96A (1979) 327 – here 1997 version

"EFT = Symmetries + Parametrisation of Ignorance"?? WHAT CAN POSSIBLY GO WRONG???

(b) (Some) More Ways to Estimate Theoretical Uncertainties at fixed k

Expansion parameter $Q = \dfrac{\textbf{typ. low scale } p_{\text{typ}}}{\textbf{typ. high scale } \overline{\Lambda}_{\text{EFT}}} \Longrightarrow \mathcal{O} = Q^m \sum_{i=0}^{k-1} c_i(\Lambda)\, Q^i$ **complete at** $\mathcal{O}(Q^{k-1})$ **(N^{k}LO).**

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- Less dependence on particular low-E data taken for LECs. (e.g. Z-param. vs. ERE; fit H_0 to a_3 vs. $B_3, \ldots$)
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  Should decrease order-by-order.

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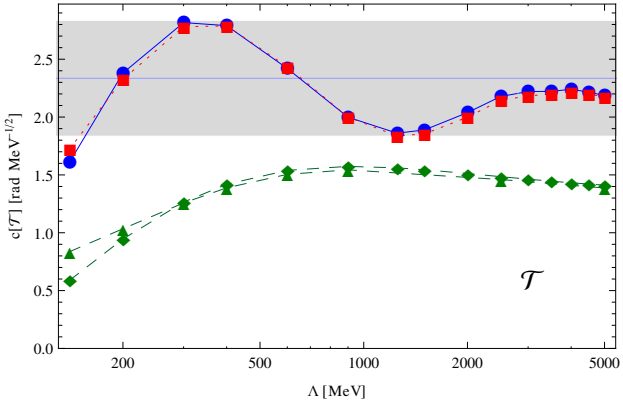
  Example: PV coefficient in nd at $k = 0$.



hg/Schindler/Springer 2012

Choose most conservative/worst-case error for final estimate! Clearly state your choice!

(c) EFTs Can Go Wrong: Check & Follow Assumptions

Expand observables as $\mathcal{O} = c_0 + c_1 Q^1 + c_2 Q^2 + \ldots$

with $Q = \dfrac{\text{typ. momentum } p_{\text{typ.}}}{\text{breakdown scale } \overline{\Lambda}_{\text{EFT}}} < 1.$

– No separation/jungle of scales? e.g. N^* at $2\ \text{GeV}$

– Incorrect usage: $p_{\text{typ.}} \nearrow \overline{\Lambda}_{\text{EFT}} \implies Q \not\ll 1$?

"EFTs carry seed of own destruction." D. R. Phillips

Check EFT's Fundamental Building Blocks

– **Which constituents?** – **The Elephant in the Room:**

Results at $k \gtrsim 200\ \text{MeV}$ **without** $\Delta(1232)$ **inconsistent.**

Breakdown of χEFT without it: $M_\Delta - M_N \approx 300\ \text{MeV}$.

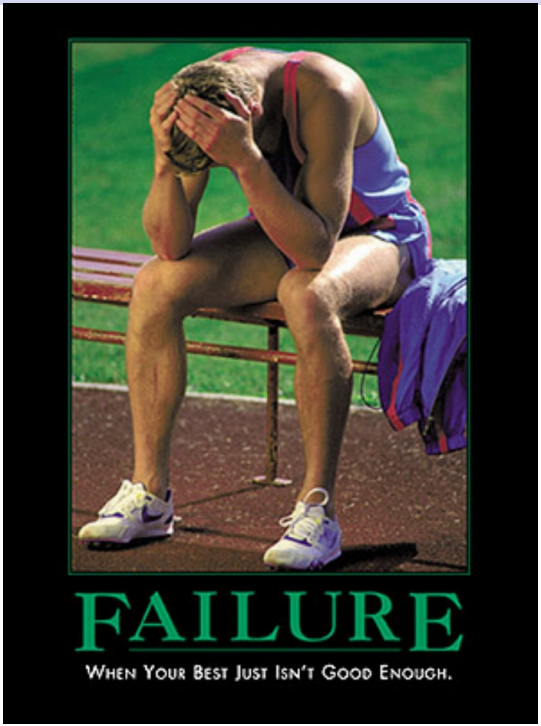
Often not considered (phase shift fits), although available.

UvK 1993, Krebs/...2007/8, Piarulli/Navarro Pérez/Amaro/Ruiz Arriola/...2016,...

– **Which symmetries?** e.g. impose Parity in weak processes

– **Check Quantitatively Predicted Convergence Pattern:**

Order by order smaller corrections & cut-off dependence.



– **EFT may converge, but not to Nature:** Wrong ordering scheme (e.g. perturbative in NN) – or any of the above.

Convergence to Nature **tests assumptions. –** ***After*** **theoretical consistency & uncertainties determined.**

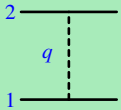
Humans abhor failure, but if an EFT fails, ***"you have learned a lot"*** **UvK Saclay 2017.**

(d) Long-Range Interaction: One Pion Exchange

One Pion Exchange Potential (OPE)

like mag. dipole-dipole int., **parameters fixed by πN.**

$$V_{OPE} = -\frac{g_A^2}{4f_\pi^2}\frac{(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})}{\vec{q}^{\,2}+m_\pi^2}\,\tau_1^a\tau_{2a}$$



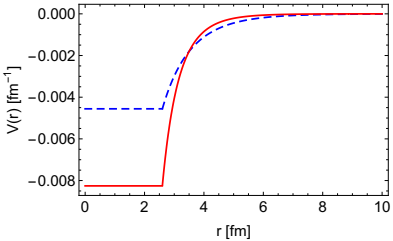
Central part Yukawa 1935:
$$V_C(r) = -\frac{g_A^2 m_\pi^2}{16\pi f_\pi^2}\frac{e^{-m_\pi r}}{r} < 0 \quad \xrightarrow{\text{chiral limit}} \quad 0 + \text{CT}$$

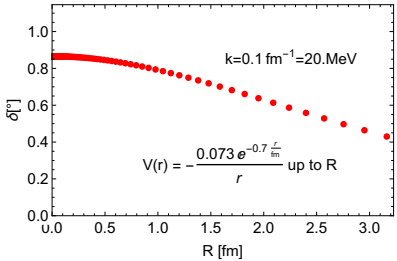
Tensor part cf. mag. dipoles:
$$V_T(r) = -\frac{g_A^2 m_\pi^2}{16\pi f_\pi^2}\left(1+\frac{3}{m_\pi r}+\frac{3}{(m_\pi r)^2}\right)\frac{e^{-m_\pi r}}{r} < 0 \quad \xrightarrow{\text{chiral limit}} \quad -\frac{3g_A^2}{16\pi f_\pi^2}\frac{1}{r^3}$$

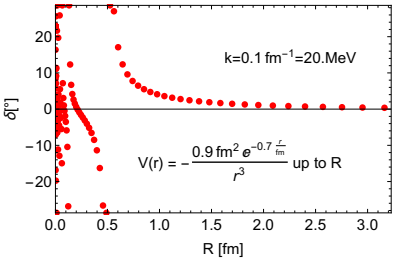
$$V_{OPE}[S=0] = V_C(r)\times\begin{cases}-3 & \text{: repulsive for } I=0\text{, i.e. } L \text{ odd}\\ +1 & \text{: attractive for } I=1\text{, i.e. } L \text{ even}\end{cases}$$

$$V_{OPE}[S=1] = \frac{1}{3}\left[V_C(r)+\left[6(\vec{S}\cdot\vec{e}_r)^2-4\right]V_T(r)\right]\times\begin{cases}+3 & \text{: attractive for } I=0\text{, i.e. } L \text{ even}\\ -1 & \text{: repulsive for } I=1\text{, i.e. } L \text{ odd}\end{cases}$$

Regularise attractive $V_{C/T}$, study $R=\frac{1}{\Lambda}\to 0$:







$$V_{C/T} = \begin{cases} V_{C/T}(r) & \text{unchanged}\\ V_{C/T}(R) & \textbf{cut off at } R\end{cases}$$

Coulombic "stable" as expected.

Thomas Collapse to $r=0$.
Unstable, RG mandates CT.

(e) Statistical Interpretation of the Max-Criterion: A Simple Example

I take this table of πN scattering parameters in χEFT with effective $\Delta(1232)$ degrees of freedom from the talk by Jacobo Ruiz de Elvira. Here, I am not interested in the Physics, but use it as series $c_i = c_{i0} + c_{i1}\epsilon^1 + c_{i2}\epsilon^2$ in a small expansion parameter.

parameter $[\mathrm{GeV}^{-1}]$	LO total	NLO total	N^{2}LO total	expansion $= c_{i0} + c_{i1}\epsilon^1 + c_{i2}\epsilon^2$	perturbative expansion $\epsilon \approx 0.4$ (guess)
c_1	-0.69	-1.24	-1.11	$= -0.69 + 0.55 - 0.13$	$= -0.69 + 1.38\epsilon^1 - 0.81\epsilon^2$
c_2	$+0.81$	$+1.13$	$+1.28$	$= +0.81 - 0.32 - 0.15$	$= +0.81 - 0.80\epsilon^1 - 0.94\epsilon^2$
c_3	-0.45	-2.75	-2.04	$= -0.45 + 2.30 - 0.71$	$= -0.45 + 5.75\epsilon^1 - 4.44\epsilon^2$
c_4	$+0.64$	$+1.58$	$+2.07$	$= +0.64 - 0.94 - 0.49$	$= +0.64 - 2.35\epsilon^1 - 3.06\epsilon^2$

Now pick the largest absolute coefficient to estimate typical size of next-order correction $c_{i(n+1)} = c_{i3}$ in our case:

Max-Criterion: $c_{i(n+1)} \lesssim \max_{n\in\{0;1;2\}} \{|c_{in}|\} =: R$ **is labelled as red in the table.**

This criterion has been applied since "Time Immemorial"
See example on the next slide which predates EKM by 4 years.

Multiply that number with ϵ^3 to finally get a corridor of uncertainty/typical size of the ϵ^3 contribution.

For c_1: $\max_{n\in\{0;1;2\}} \{|-0.69|;|1.38|;|-0.81|\} = 1.38 \implies$ error $\pm 1.38 \times (\epsilon = 0.4)^3 \approx 0.09 \implies c_1 = -0.69 \pm 0.09$.

Similar: $c_2 = 1.28 \pm 0.06$, $c_3 = -2.04 \pm 0.37$, $c_4 = 2.07 \pm 0.20$ (round significant figures conservatively).

But what's the statistical interpretation? $\implies$ Next slide!

Notes: (1) Provide a theoretical error *estimate* that is *reproducible*. You can then discuss with others who have different opinions. No estimate, no discussion possible. – **(2)** Sometimes, one discards the LO$\rightarrow$NLO correction if it's anomalously large. That is a "prior information" you need to disclose as "bias" of your estimate. – **(3)** Coefficients c_{in} appear "more natural" for c_1 and c_2 than for c_4 – c_4 not that well-converging? – **(4)** The uncertainty estimate is agnostic about the Physics details. Somebody just handed me a table. – **(5)** If you are not happy with the input "$\epsilon \approx 0.4$", pick another number. BUQEYE 1511.03618 developed the Bayesian technology to extract degrees of belief on what value of the expansion parameter the series suggests. – **(6)** The c_i are not observables, but they are renormalised couplings which – according to Renormalisation – should follow a perturbative expansion.

(e) Statistical Interpretation of the Max-Criterion: A Simple Example

The Bayesian interpretation of the max-criterion on the next slide will provide probability distribution (pdf)/degree-of-belief functions using a "reasonable" set of assumptions ("priors") which give nice, analytic expressions. That's one choice of assumptions, but other reasonable assumptions provide very similar pdf's see BUQEYE: 1506.01343, 1511.03618,. . . .

But before that, let's do something intuitive which gives the same statistical likeliness interpretation of the max-criterion as the Bayesian one. The Bayesian analysis formalises the example and provides actual pdf's.

Estimating a Largest Number: Given a finite set of (finite, positive) numbers in an urn. You get to draw one number at a time.

Your mission, should you choose to accept it: Guess the largest number in the urn from a limited number of drawings.

For c_1, we first draw $c_{10} = 0.69$. I would say it's "natural" to guess that there is a 1-in-$2 = 50\%$ chance that the next number is lower. But there is also a pretty good chance that if it is higher, then its distribution up there is not Gauß'ian but with a stronger tail.

Next, we draw $c_{11} = 1.38$ which is larger. So I revise my largest-number projection to $R = 1.38$, but I also get more confident that this may be pretty high (if not he highest already). After all, I already found one number which is lower, namely $c_{10} = 0.69$. With 2 pieces of information (0.69 and 1.38), it's "natural" that the 3rd drawing has a 2-in-3 or $2/3$ chance to be lower.

Next, we draw $c_{12} = 0.81 < R$. Looking at my set of 3 numbers, I am even more confident that $R = c_{11} = 1.38$ is the largest number, with 3-in-4 or 75% confidence. For c_1, evil forces interfere and we have no more drawings to draw information from.

But if we could reach into the urn k times and look at the collected k results, every time revising our max-estimate, it's "natural" to assign a $100\% \times k/(k+1)$ confidence that I have actually gotten the largest number R.

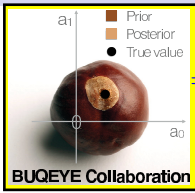
The Bayesian procedure on the next slide provides the same result. Read the BUQEYE papers for details and formulae!

In our example, we had $k = 3$ terms (drawings) for c_1. So the confidence that $R = 1.38$ is indeed the highest number is $3/4 = 75\%$, which is larger than $p(1\sigma) \approx 68\%$. For a 1σ corridor, I reasonably assume that the numbers are equi-distributed between 0 and the maximum R. Then, the 68%-error corridor is set by $\pm 68\% \times (k+1)/k \times R$ amongst the known numbers.

Now, I multiply that number with 3 powers of the expansion parameter $\epsilon \approx 0.4$ (estimate N^{3}LO terms!) (but see **Note (5)** on the previous slide): $\pm 1.38 \times (68\%/75\%) \times 0.4^3 = \pm 0.08$ is a good uncertainty estimate for a traditional 68% confidence region. I also get a feeling that the probabilities outside the interval $[0; R]$ may not be Gauß'ian-distributed. Bayes will confirm that.

(f) What Does "Conservative" Theory Uncertainty Mean?

BUQEYE 1506.01343
hg/JMcG/DRP 1511.01952



Observable as series: $\mathcal{O} = c_0 + c_1\delta^1 + c_2\delta^2 +$ unknown $\times\,\delta^3$: $|$**unknown** $c_3| \lesssim \max\{|c_i|\}$.

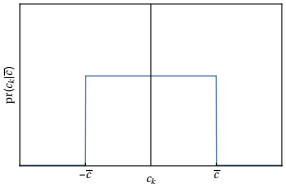
$\Longrightarrow$ information on coefficients $\rightarrow$ reasonable bound of theory uncertainty: $\pm R := \pm\delta^3 \max\{|c_i|\}$

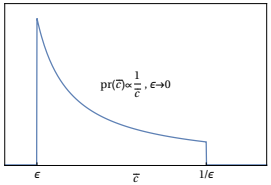
Bayes: Clearly state premises/assumptions. New information increases level of confidence.

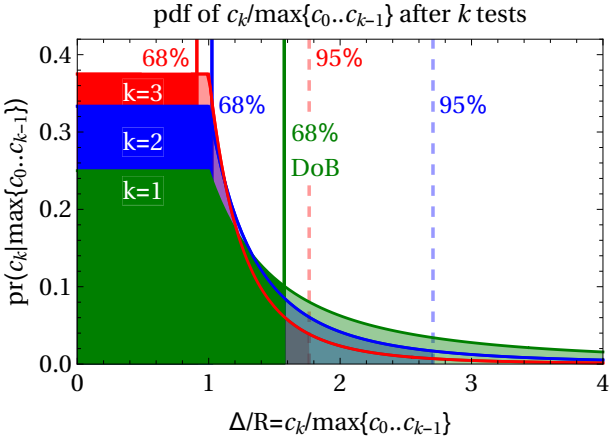
$\Longrightarrow$ **Smaller corrections, more reliable uncertainties.**

To arrive at the quoted theory error on our results, we note that we perform an $\mathcal{O}(e^2\delta^3)$ fit in a framework in which the polarisabilities first enter at $\mathcal{O}(e^2\delta^2)$. We would expect corrections to be of order $\delta^2 \sim 16\%$ of the lowest-order result, or $\delta \sim 40\%$ of the shift between the LO and NLO results; taking $(\alpha^{(p)}_{E1} + \beta^{(p)}_{M1})/2 \approx 7$ to set the scale for the first approach gives 1.1, while taking the shifts in the values of $\alpha^{(p)}_{E1}$ and $\beta^{(p)}_{M1}$ from third order to fourth order to be ≈ 2 gives 0.8 in the second approach. In view of the similarity between our third- and fourth-order results (see later), the stability under

PPNP67 **(2012)** 871 sect. 4.4 – used as widely accepted 4 years before EKM







Priors: all c_n "equally likely", "any upper bound" $\bar{c}$.

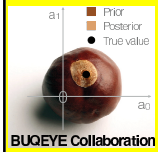
order	DOB in $\pm R$	σ: 68%	$\Delta(95\%)$
LO	$\frac{1}{2} = 50\%$	$1.6\,R$	$11R = 7\sigma$
NLO	$\frac{2}{3} = 66.7\%$	$1.0\,R$	$2.7R = 2.6\sigma$
N^{2}LO	$\frac{3}{4} = 75\%$	$0.9\,R$	$1.8R = 1.9\sigma$
k	$\frac{k}{k+1}$	$0.68\frac{k+1}{k}R(k \geq 2)$	
Gauß	68.27%	$1.0\,R$	2.0σ

Posterior pdf *not Gauß'ian:* Plateau & power-law tail.

(g) What Does "Conservative" Theory Uncertainty Mean?

hg/JMcG/DRP
1511.01952

Observable as series $\mathcal{O} = \delta^n \left(c_0 + c_1\delta^1 + c_2\delta^2 + \text{unknown} \times \delta^3\right) \implies$
Estimate next term *"most conservatively"* as $|\text{unknown } c_3| \lesssim \max\{|c_0|;|c_1|;|c_2|\}$.



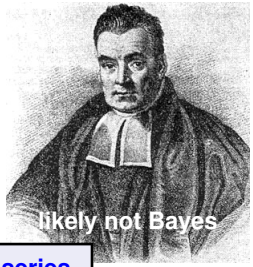
No infinite sampling pool; data fixed; more data changes confidence.

$\implies$ **Call upon the Reverend Bayes!**

New information (new order) increases level of confidence.

$\implies$ **Smaller corrections, more reliable uncertainties.**

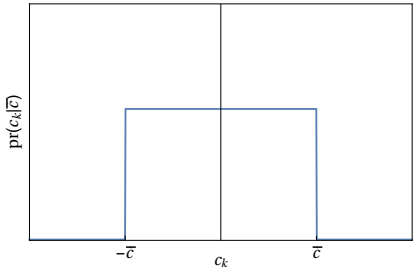
see e.g. **BUQEYE collaboration** Furnstahl/Phillips/...1506.01343



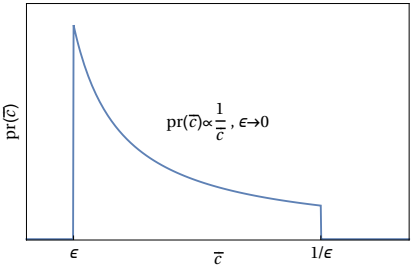
Bayes makes you specify your premises/assumptions about series.

Priors: leading-omitted term dominates ($\delta \ll 1$); putative distributions of *all* c_k's and of largest value $\bar{c}$ in series.

"Least informed/informative": All values c_k equally likely, given upper bound $\bar{c}$ of series.



"Any upper bound": $\ln$-uniform prior sets no bias on scale of $\bar{c}$.

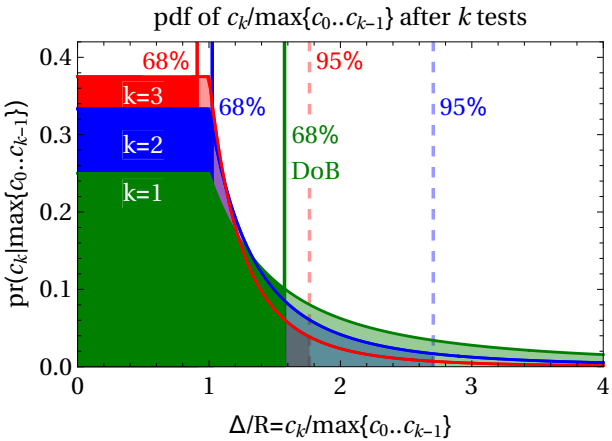


Quantifying One's Beliefs in $\mathcal{O} = Q^n(c_0 + c_1 Q^1 + c_2 Q^2 + \dots)$

Information: Convergence LO→NLO→N^{2}LO gives **probable "largest number"** $R = \delta^k \max\{|c_0| \dots |c_{k-1}|\}$.

Result: Posterior $\equiv$ ***Degree of Belief (DoB)*** that next term $c_k \delta^k$ differs from order-k central value by δ.

$$\mathrm{pr}(\Delta|\max.\ R, \text{order } k) \propto \int_0^\infty d\bar{c}\ \mathrm{pr}(\bar{c})\ \mathrm{pr}\left(c_k = \frac{\Delta}{\delta^k}|\bar{c}\right) \prod_n^{k-1} \mathrm{pr}(c_n|\bar{c}) \to \frac{k}{k+1}\ \frac{1}{2R} \begin{cases} 1 & |\Delta| \le R \\ \left(\frac{R}{|\Delta|}\right)^{k+1} & |\Delta| > R \end{cases}$$



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For "high enough" order, largest number R limits $\gtrsim 68\%$ **degree-of-belief interval.**

Varying priors: When $k \ge 2$ orders known, DoBs with different assumptions about $\bar{c}$, c_n vary by $\lesssim \pm 20\%$.

Posterior pdf *not Gauß'ian:* Plateau & power-law tail.– Do not add in quadrature in convolution!

$\Longrightarrow$ ***Interpretation of all theory uncertainties, with these priors; "$A \pm \sigma$": 68% DoB interval*** $[A - \sigma; A + \sigma]$.