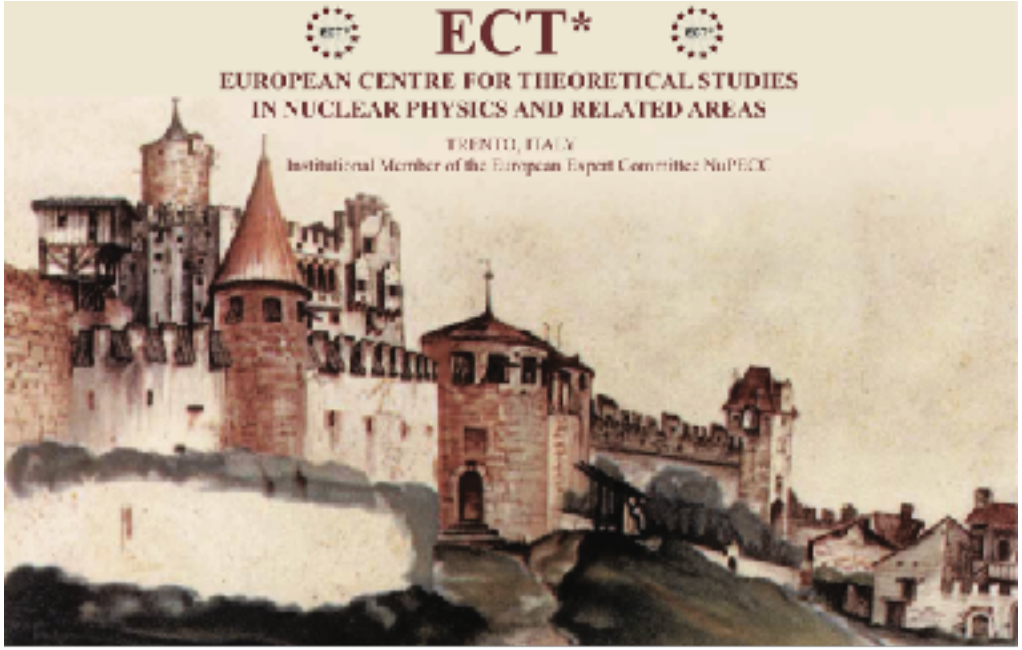
Evgeny Epelbaum, RUB

New ideas in constraining nuclear forces, ECT*, Trento
June 4-8, 2018

Chiral nuclear forces at the precision frontier

— Patrick Reinert, Hermann Krebs, EE, EPJA 54 (2018) 86 —



The framework

- For a general discussion of renormalization and power counting see materials of the KITP Program Frontiers in Nuclear Physics (2016):
  - **tutorial on nuclear EFT (EE)**, http://online.kitp.ucsb.edu/online/nuclear16/epelbaum/
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  - **nuclear EFT-the crux of the matter II (EE)**, http://online.kitp.ucsb.edu/online/nuclear16/epelbaum2/
- **EE, Gegelia, Meißner, NPB 925 (2017) 161:** identified renorm. conditions ($\mu_1 \sim \Lambda_b$, $\mu_i \sim Q$) yielding a consistent expansion for systems close to the unitary limit with NDA scaling of LECs (W. counting). No contradiction with KSW/RG-based counting (different renorm. cond.)!

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From chiral Lagrangians to nuclear systems

Step 1: Derive (and renormalize) nuclear potentials and currents in ChPT **[Method of UT, S-matrix matching, TOPT]**. We assume NDA for contacts…

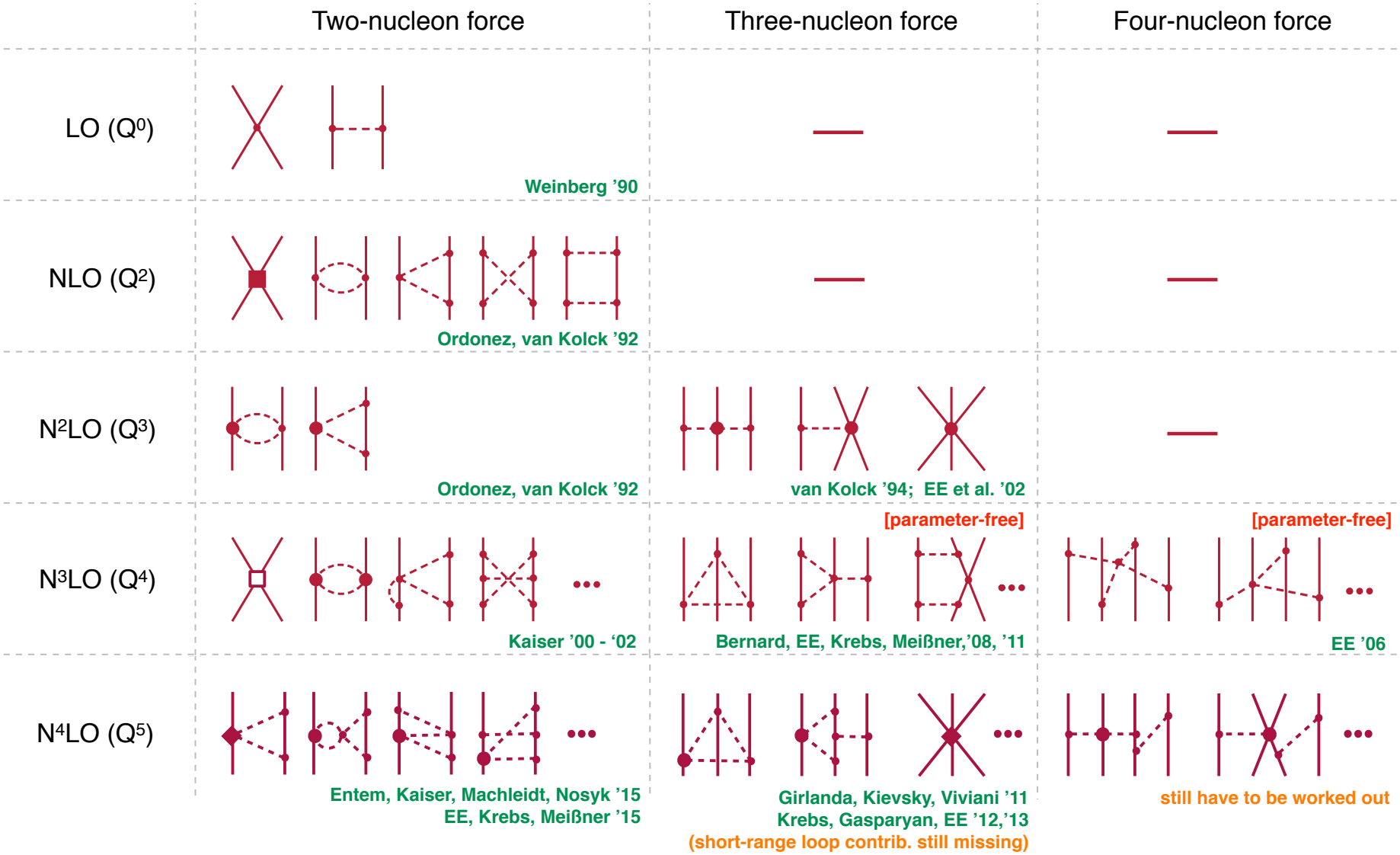
Step 2: Introduce a cutoff Λ which in a nonrelativistic approach must be kept finite, $\Lambda \sim \Lambda_b$ **[Lepage '97; EE, Meißner '06; EE, Gegelia '09]**. Nontrivial: maintaining the symmetries…

$\longrightarrow$ **talk by Hermann Krebs**

Step 3: Analyze NN scattering data to fix <u>bare</u> LECs $X_i(\Lambda)$ (i.e. implicit renormalization)

Step 4: Use ab-initio methods to calculate observables **[FY, Lattice, NCSM, GFMC, CC, IMSRG, …]** and estimate uncertainty

Chiral expansion of the nuclear forces [W-counting]



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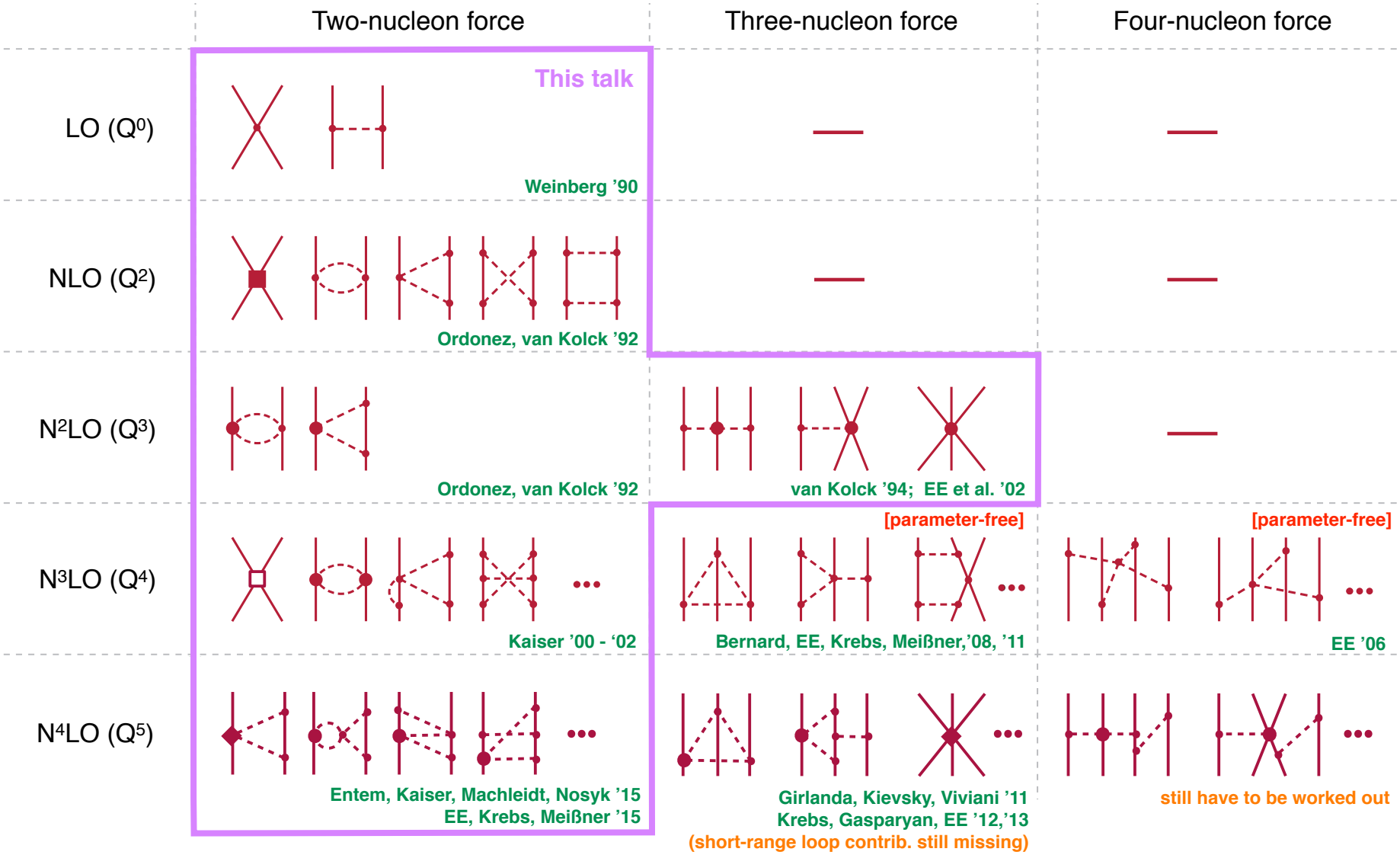

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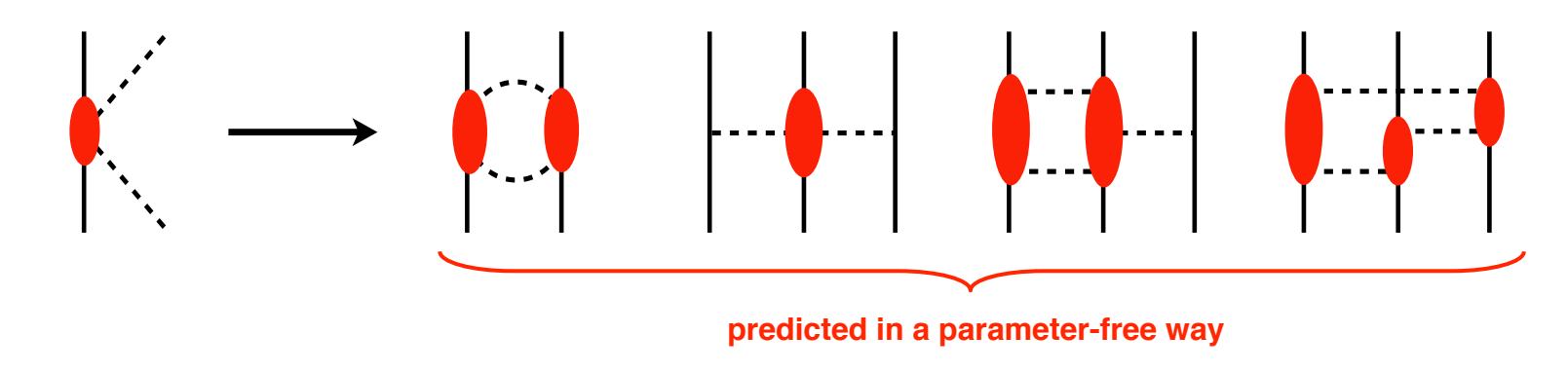
The long-range part of the nuclear force

The long-range part of nuclear forces and currents is completely determined by the chiral symmetry of QCD + experimental information on πN scattering



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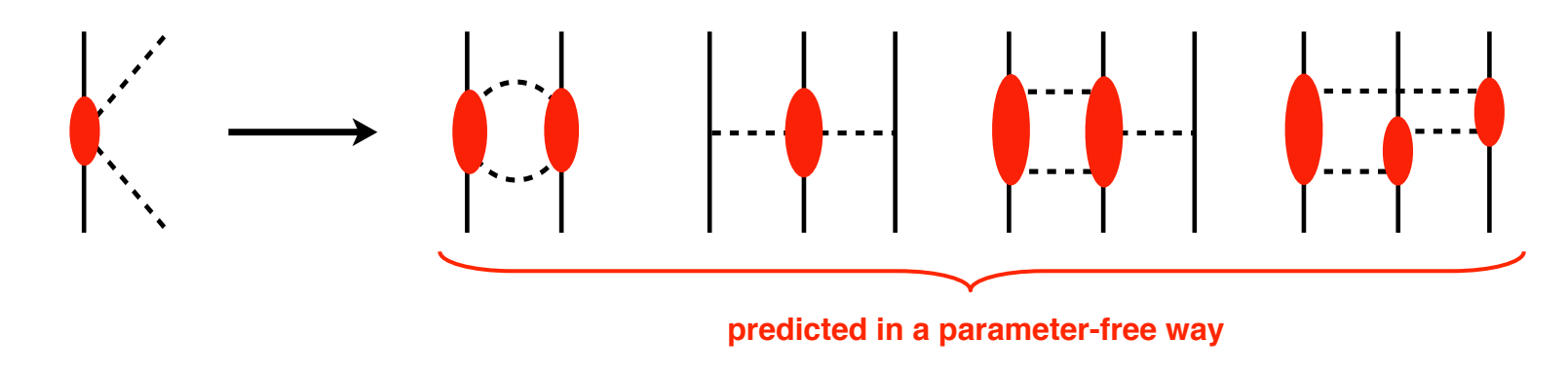
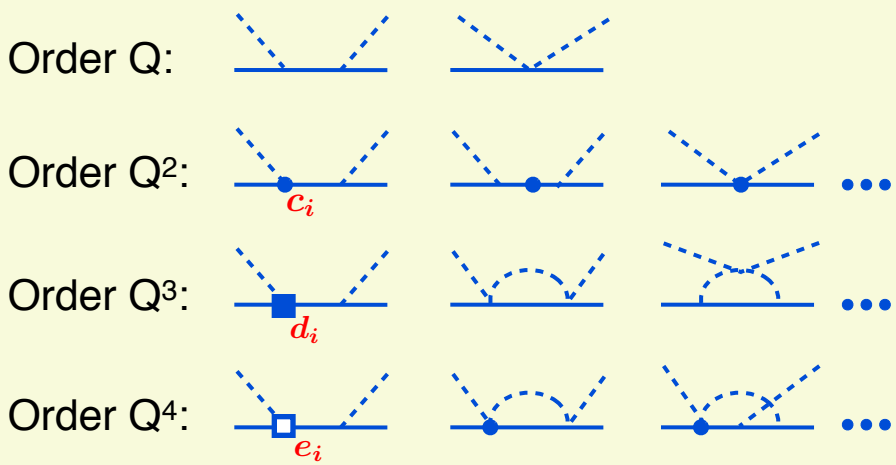
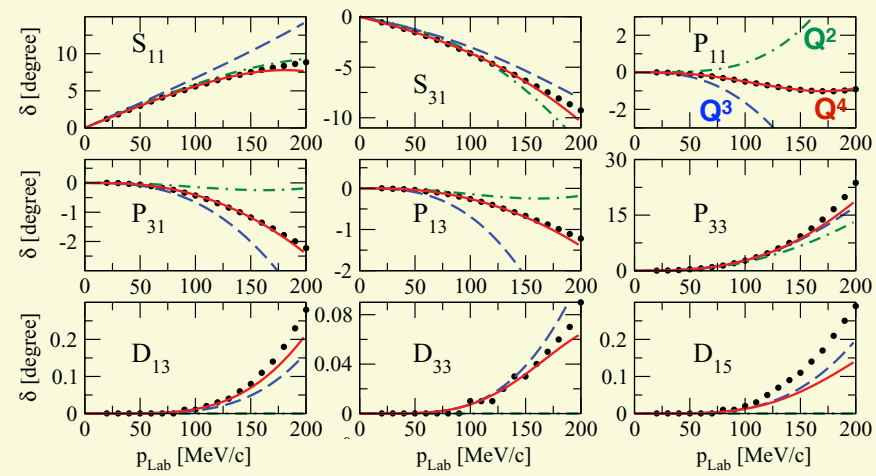


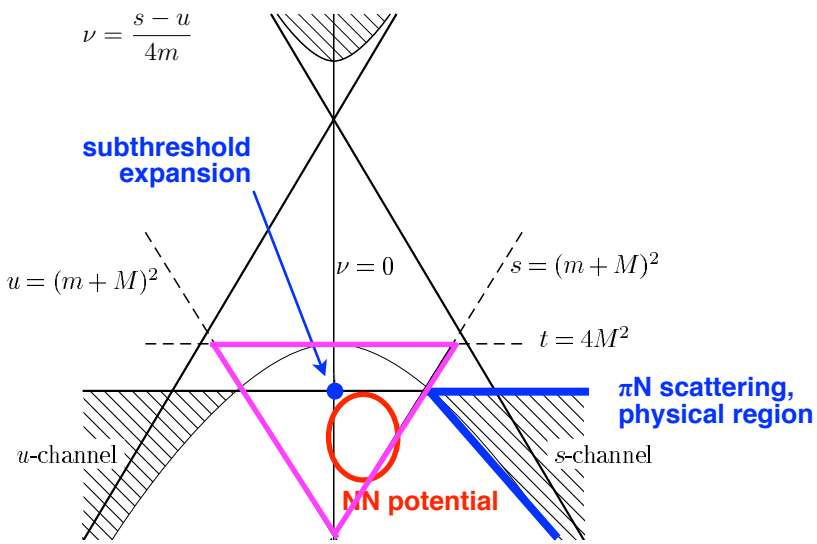
Pion-nucleon scattering up to Q^4 in heavy-baryon ChPT

Fettes, Meißner '00; Krebs, Gasparyan, EE '12





Determination of πN LECs



Matching ChPT to πN Roy-Steiner equations

Hoferichter, Ruiz de Elvira, Kubis, Meißner, PRL 115 (2015) 092301

- χ expansion of the πN amplitude expected to converge best within the Mandelstam triangle
- Subthreshold coefficients (from RS analysis) provide a natural matching point to ChPT

$$\bar{X} = \sum_{m,n} x_{mn}\, \nu^{2m+k} t^n, \qquad X = \{A^\pm,\, B^\pm\}$$

- Closer to the kinematics relevant for nuclear forces…

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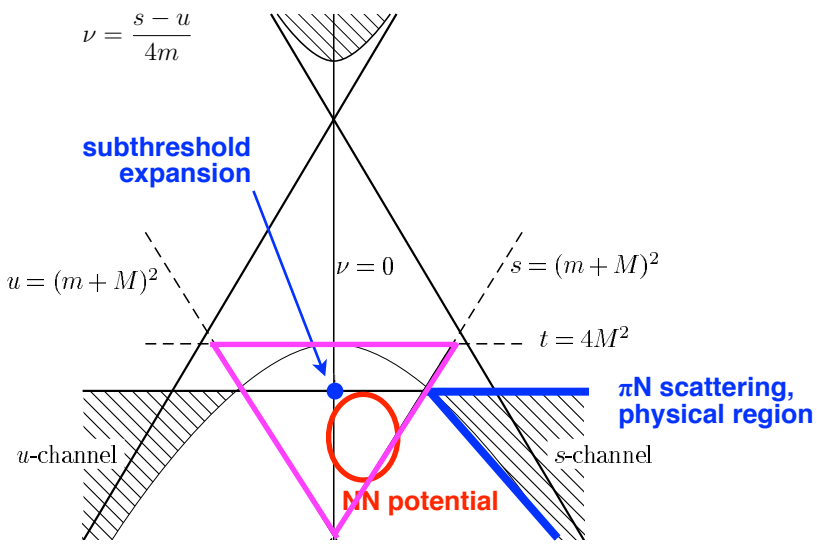

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Relevant LECs (in GeV^{-n}) extracted from πN scattering

	c_1	c_2	c_3	c_4	$\bar{d}_1+\bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14}-\bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{17}$	
$[Q^4]_{\mathrm{HB,NN}}$, GW PWA	−1.13	3.69	−5.51	3.71	5.57	−5.35	0.02	−10.26	1.75	−0.58	Krebs, Gasparyan, EE, PRC85 (12) 054006
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— Some LECs show sizable correlations (especially c_1 and c_3)…

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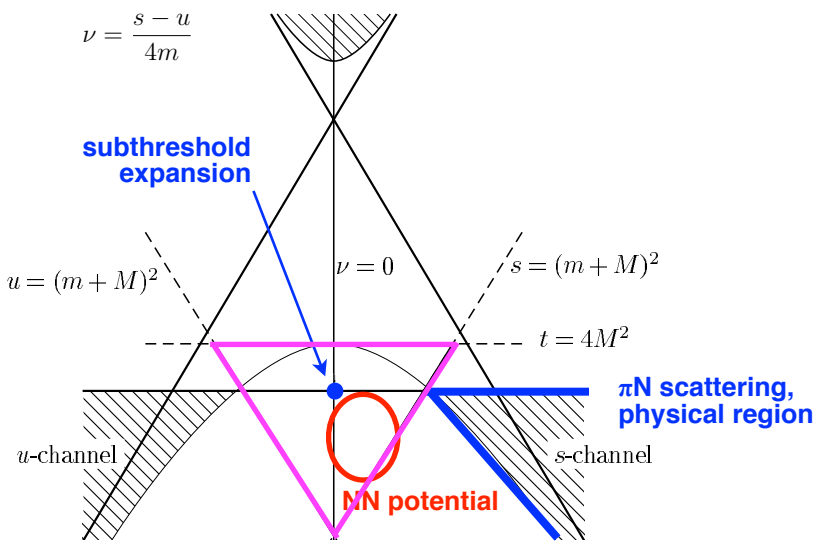

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— EKM N^{4}LO [EE, Krebs, Meißner, PRL 115 (2015) 122301]: Q^4 fit to KH PWA

— RKE N^{4}LO [Reinert, Krebs, EE, EPJA 54 (2018) 88]: Q^4 fit to RS and Q^4 fit to KH PWA

With the LECs taken from πN, the long-range NN force is completely fixed (parameter-free)

Regularization

The cutoff Λ has to be kept finite, $\Lambda \sim \Lambda_b$ (unless all counterterms are taken into account in the calculations) **[Lepage '97; EE, Gegelia '09]**. In practice, low values of Λ are preferred:

— many-body methods require soft interactions,

— spurious deeply-bound states for $\Lambda > \Lambda^{\rm crit}$ make calculations for $A > 3$ unfeasible…

$\longrightarrow$ it is crucial to employ a regulator that minimizes finite-Λ artifacts!

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Nonlocal: $$V_{1\pi}^{\rm reg} \propto \frac{e^{-\frac{p'^4+p^4}{\Lambda^4}}}{\vec{q}^{\,2}+M_\pi^2} \longrightarrow \frac{1}{\vec{q}^{\,2}+M_\pi^2}\underbrace{\left(1-\frac{p'^4+p^4}{\Lambda^4}+\mathcal{O}(\Lambda^{-8})\right)}_{\textit{affect long-range interactions…}}$$

EE, Glöckle, Meißner '04;
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[inspired by Thomas Rijken]

Reinert, Krebs, EE '18;

⟶ does not affect long-range physics **at any order** in $1/\Lambda^2$-expansion

— Application to 2π exchange does not require re-calculating the corresponding diagrams:

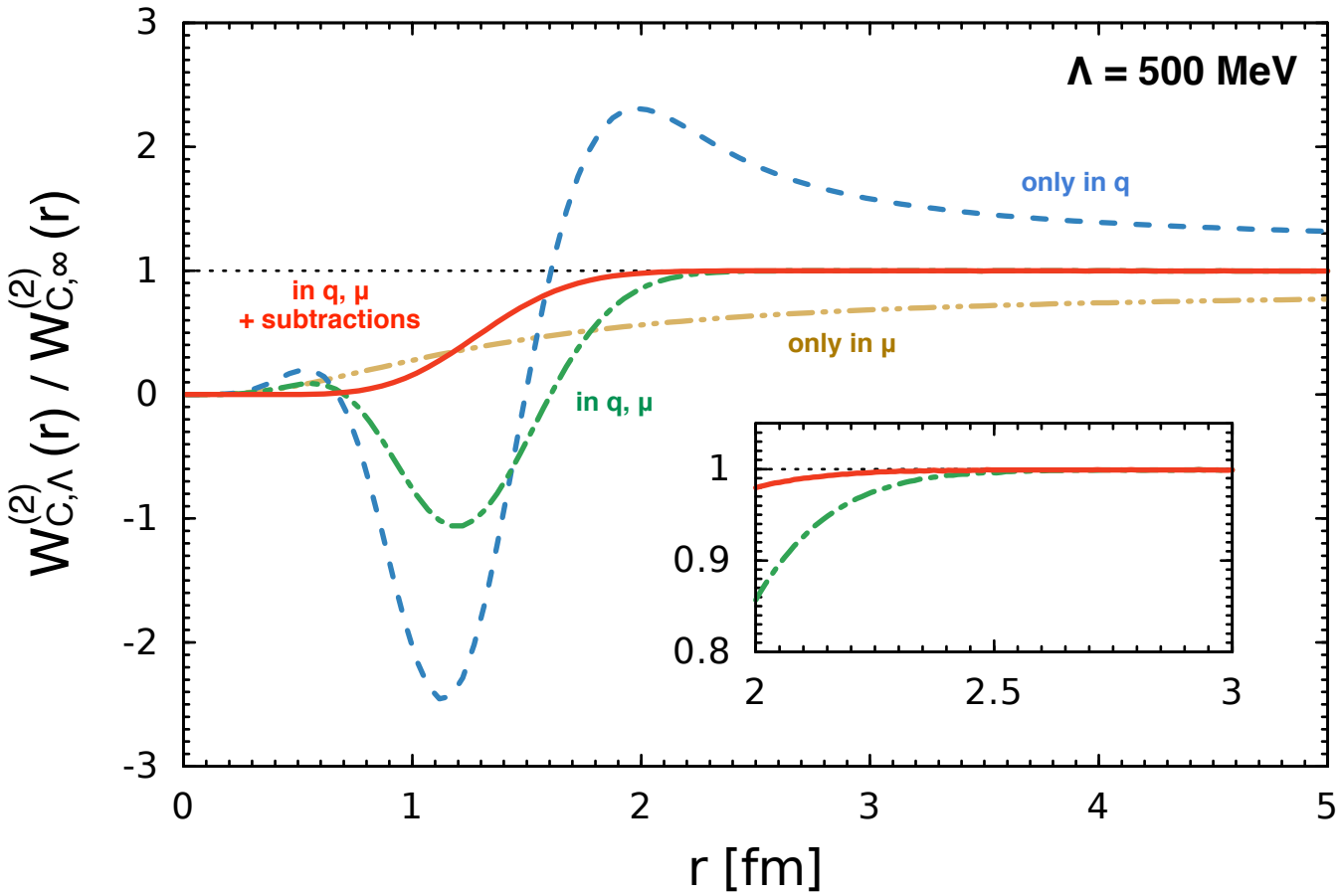
$$V(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} \mu\, d\mu \frac{\rho(\mu)}{q^2+\mu^2} + \ldots \xrightarrow{\text{reg.}} V_\Lambda(q) = e^{-\frac{q^2}{2\Lambda^2}}\,\frac{2}{\pi}\int_{2M_\pi}^{\infty} \mu\, d\mu \frac{\rho(\mu)}{q^2+\mu^2}\, e^{-\frac{\mu^2}{2\Lambda^2}} + \underbrace{\ldots}_{\textit{polynomial in } q^2,\, M_\pi}$$

— Convention: choose polynomial terms such that $\Delta^n V_{\Lambda,\,\rm long}(\vec{r})\big|_{r=0} = 0$

Regularization

Regularized 2π-exchange potential: $W_{\mathrm{C},\Lambda}(q) = e^{-\frac{q^2}{2\Lambda^2}} \frac{2}{\pi} \int_{2M_\pi^2}^{\infty} \mu \, d\mu \, \frac{\rho(\mu)}{q^2+\mu^2} \, e^{-\frac{\mu^2}{2\Lambda^2}}$

Various regularization approaches



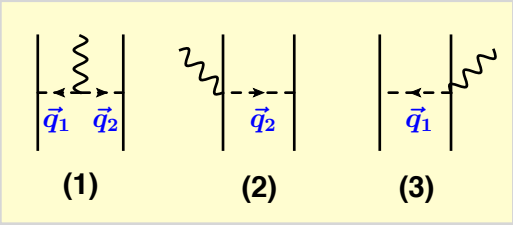
Does it matter in practice?

Regularization

- Can be straightforwardly applied to 3NF and currents up to N²LO, e.g.:

Leading electromagnetic 2N current

$$\vec{J}_{1\pi}^{\rm LO} \;=\; ie\frac{g_A^2}{4F_\pi^2}\,[\vec{\tau}_1\times\vec{\tau}_2]^3\,\frac{\vec{\sigma}_2\cdot\vec{q}_2}{\vec{q}_2{}^2+M_\pi^2}\left(\vec{q}_1\frac{\vec{\sigma}_1\cdot\vec{q}_1}{\vec{q}_1{}^2+M_\pi^2}-\vec{\sigma}_1\right)\;+\;1\leftrightarrow 2$$



Unregularized current fulfills the continuity equation:

$$\vec{k}_\gamma\cdot\vec{J}_{1\pi}^{\rm LO} \;=\; (\vec{q}_1+\vec{q}_2)\cdot\vec{J}_{1\pi}^{\rm LO} = ie\frac{g_A^2}{4F_\pi^2}\,[\vec{\tau}_1\times\vec{\tau}_2]^3\,\frac{\vec{\sigma}_1\cdot\vec{q}_1\;\vec{\sigma}_2\cdot\vec{q}_1}{\vec{q}_1{}^2+M_\pi^2}\;+\;1\leftrightarrow 2 = \;[V_{1\pi},\;\rho^{\rm LO}]$$

Introducing FFs in $V_{1\pi}$ requires (phenomenological) Riska prescription to maintain current conservation [Riska '84].

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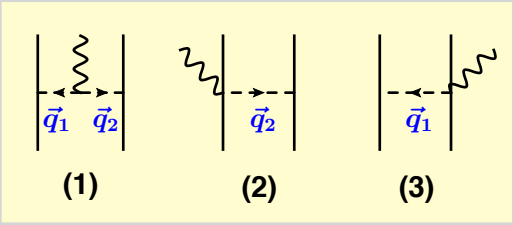

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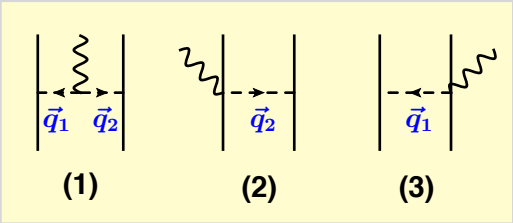

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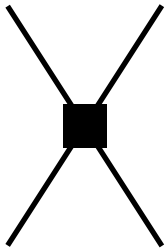
- Application to > 2NF and currents beyond N²LO is nontrivial
(contrary to NN, short-range 3NF, 4NF and currents are constrained by chiral symmetry…)

→ **talk by Hermann Krebs**

Contact interactions

- Weinberg's counting:



LO [Q^0]:	2 operators (S-waves)
NLO [Q^2]:	+ 7 operators (S-, P-waves and ε_1)
N^{2}LO [Q^3]:	no new isospin-conserving operators
N^{3}LO [Q^4]:	+ 15 operators (S-, P-, D-waves and ε_1, ε_2)
N^{4}LO [Q^5]:	no new isospin-conserving operators
N^{4}LO$^+$ [Q^6]:	+ 4 operators (F-waves)

- Use a simple nonlocal Gaussian regulator for contacts

- Fits to data at N^{3}LO & beyond tend to converge extremely slow indicating some redundancy
  [Hammer, Furnstahl '00, Beane, Savage '01, Wesolowski et al.'16]

$$\begin{aligned}
\langle {}^1S_0,\, p'|V_{\rm cont}|{}^1S_0,\, p\rangle &= \tilde{C}_{1S0} + C_{1S0}(p^2+p'^2) + D_{1S0}\, p^2p'^2 + D^{\rm off}_{1S0}\,(p^2-p'^2)^2 \\
\langle {}^3S_1,\, p'|V_{\rm cont}|{}^3S_1,\, p\rangle &= \tilde{C}_{3S1} + C_{3S1}(p^2+p'^2) + D_{3S1}\, p^2p'^2 + D^{\rm off}_{3S1}\,(p^2-p'^2)^2 \\
\langle {}^3S_1,\, p'|V_{\rm cont}|{}^3D_1,\, p\rangle &= C_{\epsilon 1}p^2 + D_{\epsilon 1}\, p^2p'^2 + D^{\rm off}_{\epsilon 1}\, p^2(p^2-p'^2)
\end{aligned}$$

(Short-range) UTs $U = e^{\gamma_1 T_1 + \gamma_2 T_2 + \gamma_3 T_3}$ with

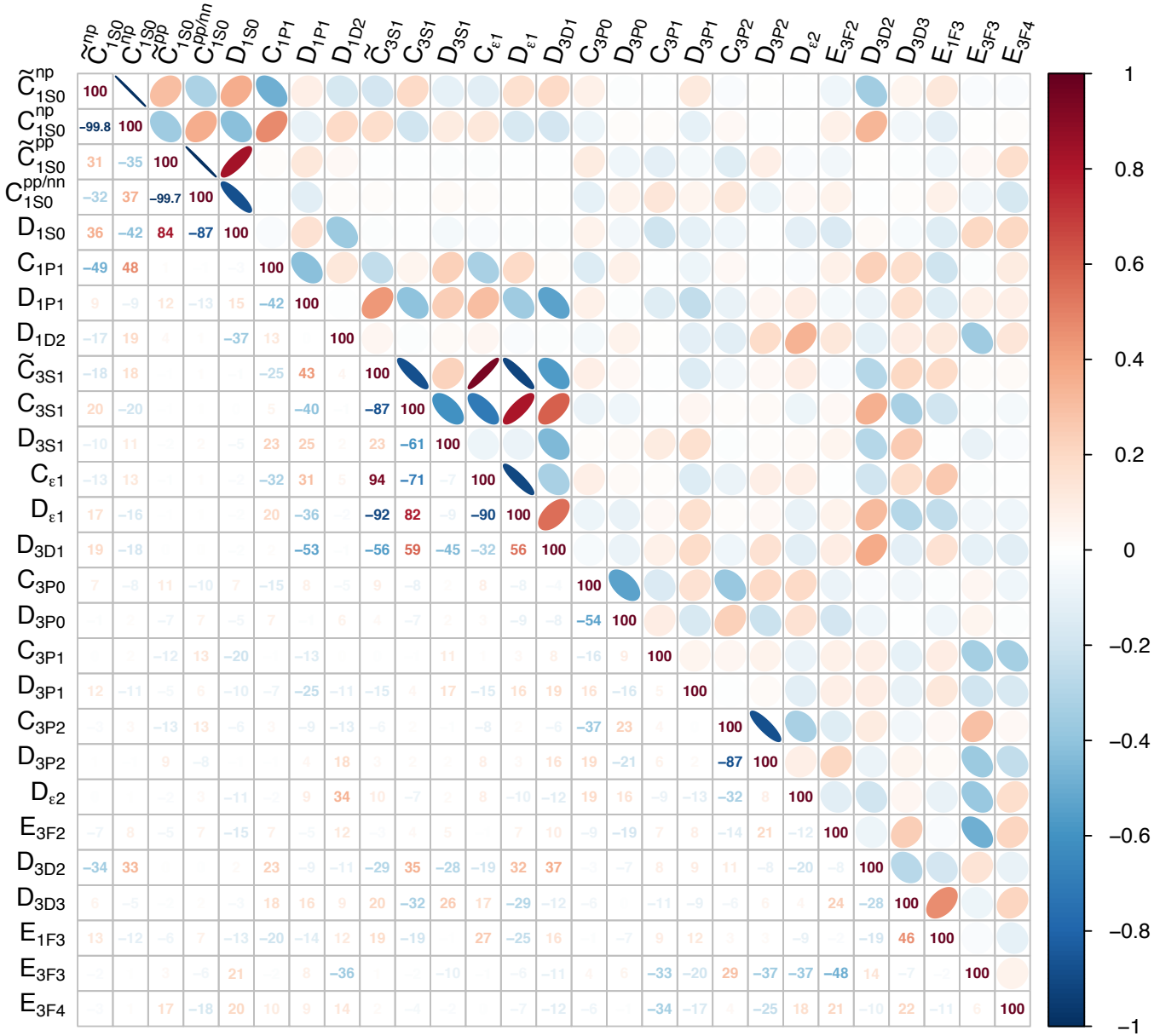
$$T_1 = \vec{k}\cdot\vec{q}, \qquad T_2 = \vec{k}\cdot\vec{q}\,\vec{\sigma}_1\cdot\vec{\sigma}_2, \qquad T_3 = \vec{\sigma}_1\cdot\vec{k}\,\vec{\sigma}_2\cdot\vec{q} + 1\leftrightarrow 2.$$

Induced terms in the Hamiltonian: $\delta H = U^\dagger H^{(0)} U = \underbrace{\sum_i \gamma_i \left[H^{(0)}_{\rm kin},\, T_i\right]}_{} + \ldots$

have the form of $V^{(4)}_{\rm cont}$ → *3 terms can be eliminated (modulo higher-order terms…)*

The UT also affects short-range 3NFs and current operators starting from N^{4}LO.

Correlations between various LECs



Contact interactions

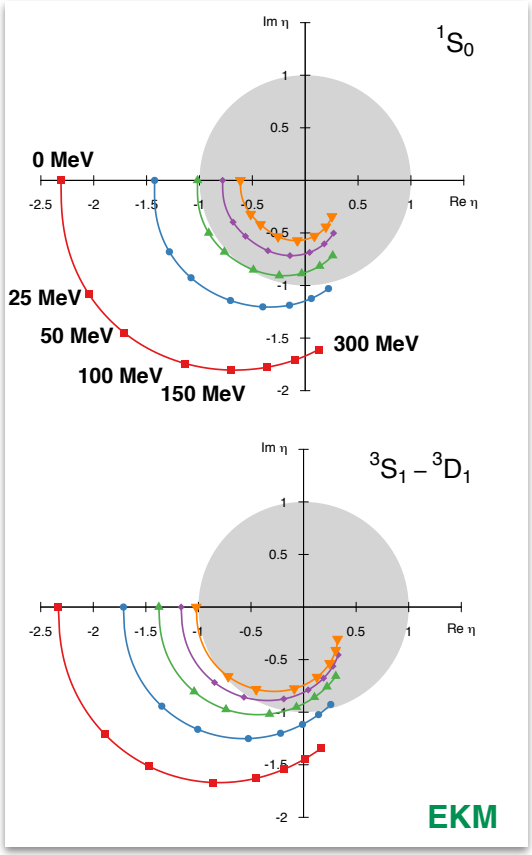
Removal of the redundant terms leads to softer potentials (good for many-body!)

Tool: Weinberg's eigenvalue analysis: $G_0(E^+)V|\Psi\rangle = \eta_i(E^+)\,|\Psi\rangle$

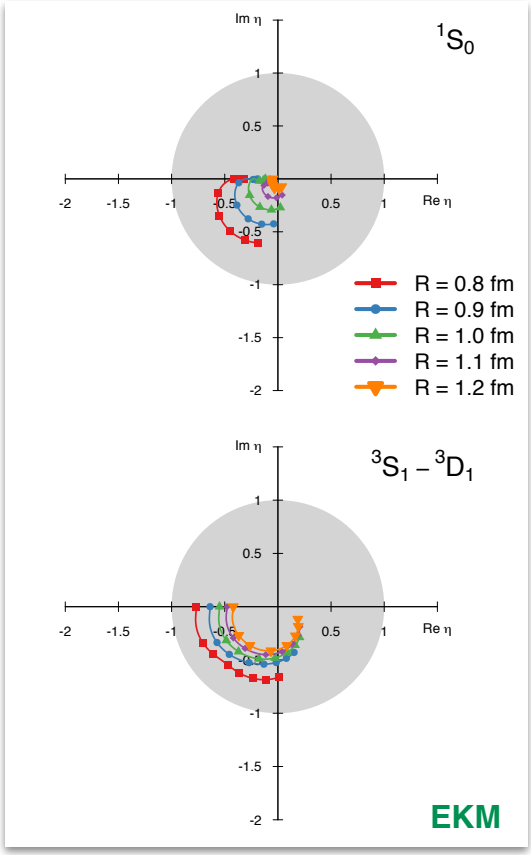
$$T(\underbrace{E^+}_{E+i\epsilon}) = V + V\,G_0(E^+)\,T(E^+) = \sum_{n=0}^{\infty} V\,(G_0(E^+)\,V)^n$$

← *converges at E iff* $\max\left(|\eta_i(E^+)|\right) < 1$

The largest repulsive Weinberg eigenvalues in S-waves



eliminate redundant terms



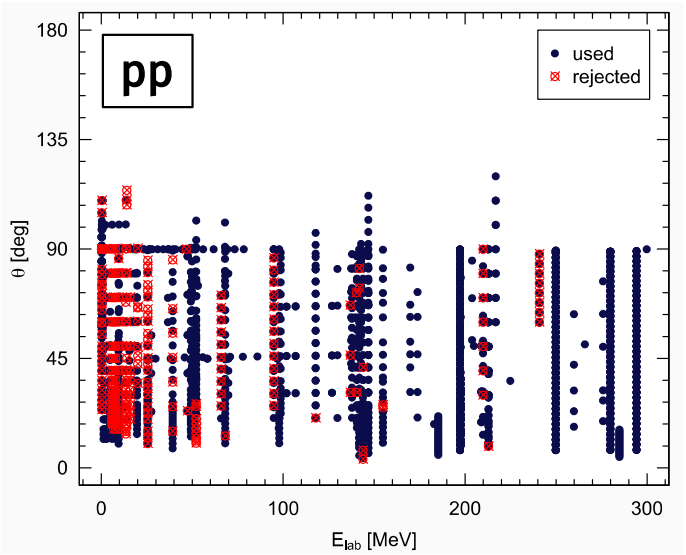
NN data analysis

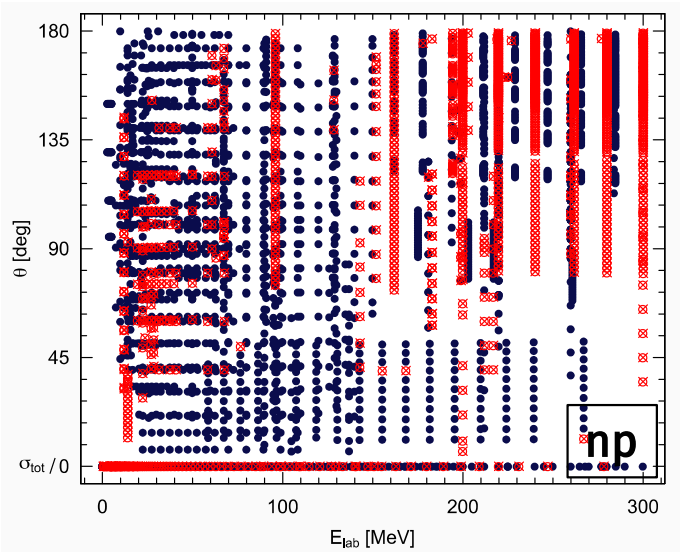
P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

- To fix NN contact interactions, use scattering data together with $B_d = 2.224575(9)$ MeV and $b_{np} = 3.7405(9)$ fm.
- Since 1950-es, ~ 3000 proton-proton + 5000 neutron-proton scattering data below 350 MeV have been measured.
- However, certain data are mutually incompatible within errors and have to be rejected.

  2013 Granada database [Navarro-Perez et al., PRC 88 (2013) 064002], rejection rate: 31% np, 11% pp:

  2158 proton-proton + 2697 neutron-proton data below $E_{lab} = 300$ MeV

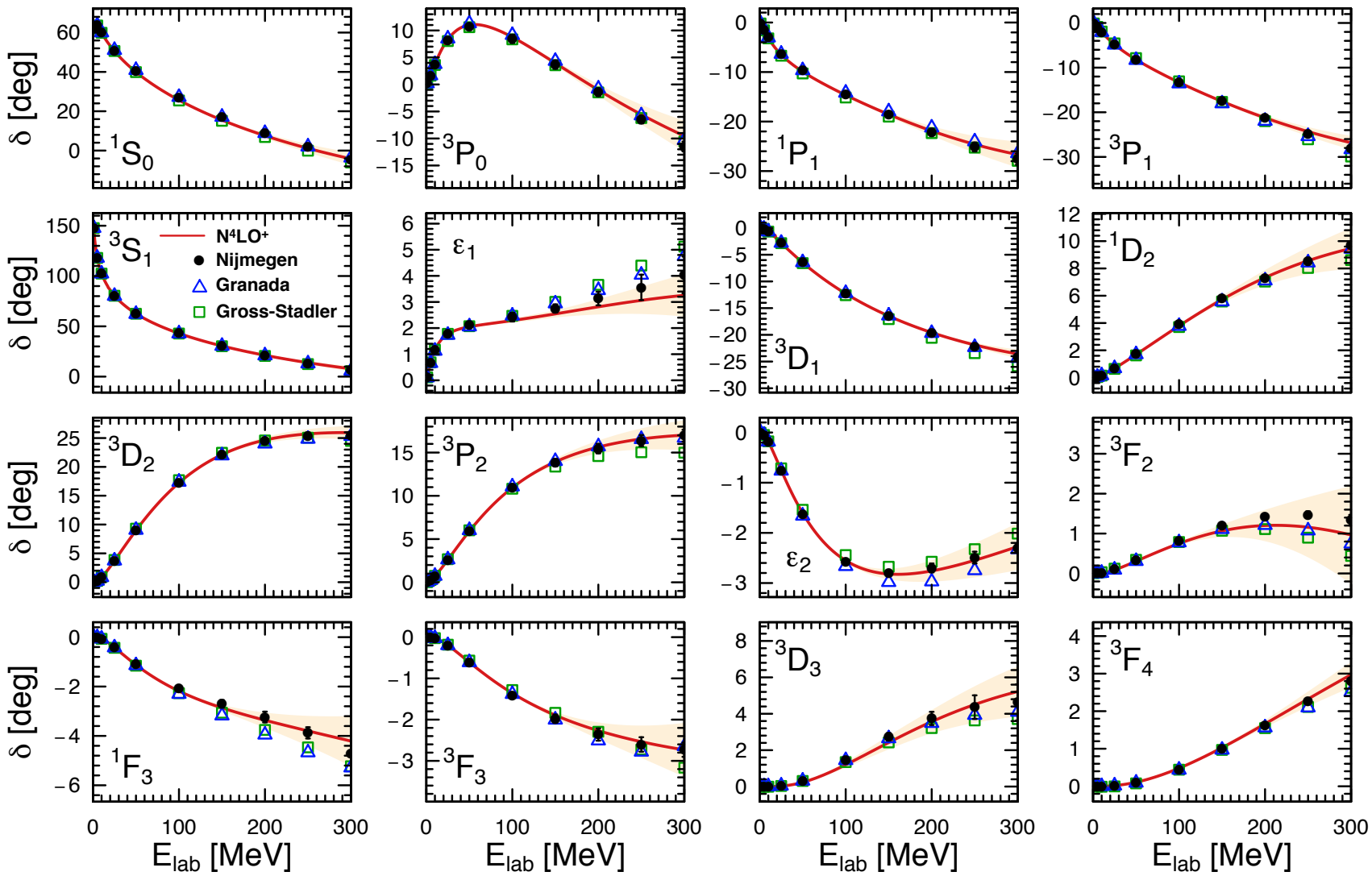




- After removal of the redundant contact terms find essentially „unique“ minima in the χ^2.
- Significant correlations in the $^1S_0$, $^3S_1$-$^3D_1$ channels. Still, all LECs are accurately determined…

Partial wave analysis

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88



Our results at N⁴LO⁺ can be regarded as partial wave analysis

Partial wave analysis

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

Description of the np & pp data at various chiral orders

$E_{\rm lab}$ bin	LO (Q^0)	NLO (Q^2)	N^2LO (Q^3)	N^3LO (Q^4)	N^4LO (Q^5)	N^4LO^+
neutron-proton scattering data						
$0-100$	73	2.2	1.2	1.08	1.08	1.07
$0-200$	62	5.4	1.8	1.09	1.08	1.07
$0-300$	75	14	4.4	1.99	1.18	1.06
proton-proton scattering data						
$0-100$	2300	10	2.1	0.91	0.88	0.86
$0-200$	1780	91	33	2.00	1.42	0.95
$0-300$	1380	89	38	3.42	1.67	1.00
	2 LECs	+ 7 + 1 IB LECs		+ 12 LECs	+ 1 LEC (np)	+ 4 LEC

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Clear evidence of the (parameter-free) chiral 2π-exchange!

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no new LECs → (NLO → N^2LO)

no new LECs → (N^3LO → N^4LO)

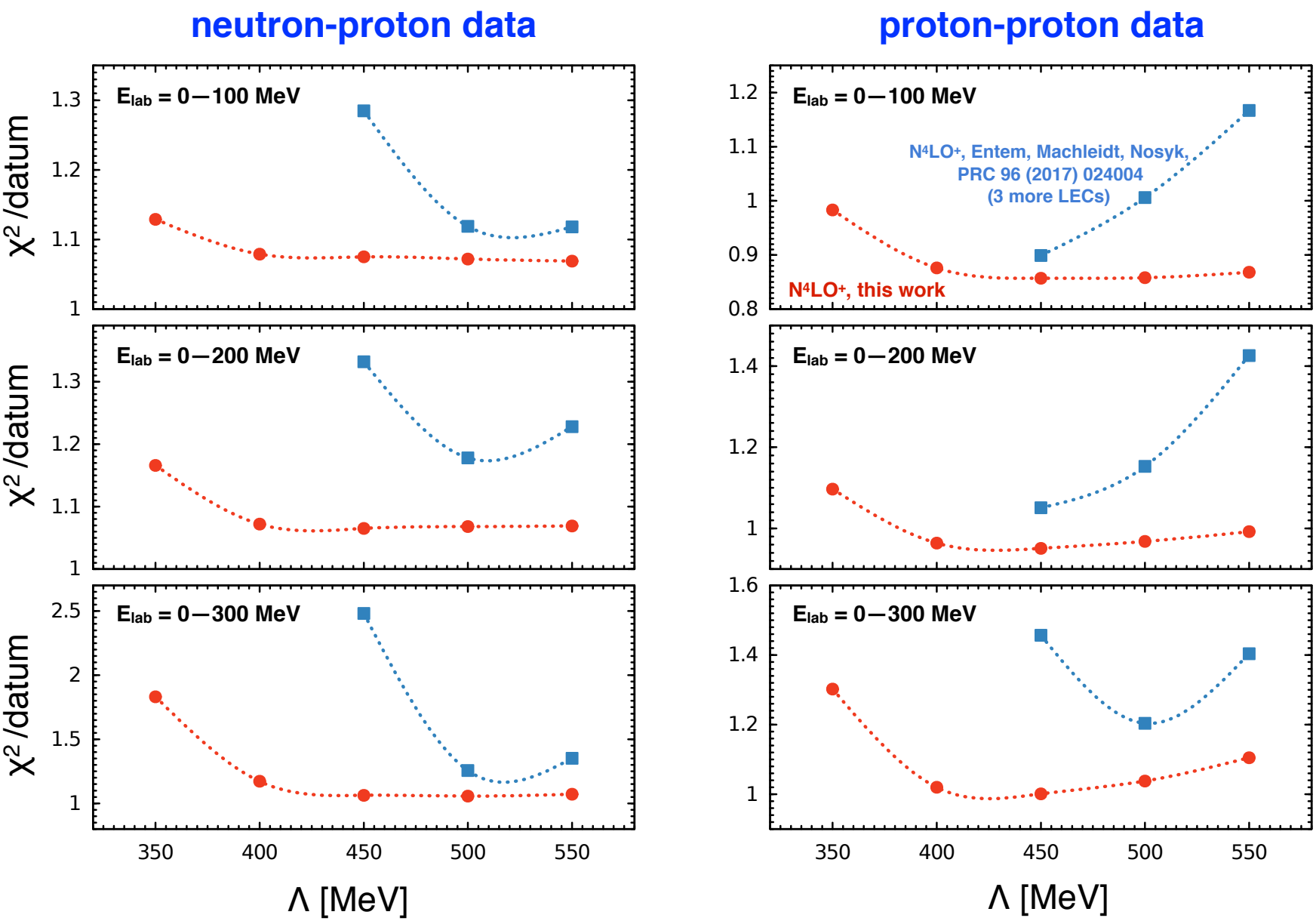
Clear evidence of the (parameter-free) chiral 2π-exchange!

Chiral nuclear forces versus high-precision phenomenological potentials

$E_{\rm lab}$ bin	CD Bonn$_{(43)}$	Nijm I$_{(41)}$	Nijm II$_{(47)}$	Reid93$_{(50)}$	$N^4LO^+{}_{(27+1)}$, this work
neutron-proton scattering data					
0 – 100	1.08	1.06	1.07	1.08	1.07
0 – 200	1.08	1.07	1.07	1.09	1.07
0 – 300	1.09	1.09	1.10	1.11	1.06
proton-proton scattering data					
0 – 100	0.88	0.87	0.87	0.85	0.86
0 – 200	0.98	0.99	1.00	0.99	0.95
0 – 300	1.01	1.05	1.06	1.04	1.00

State-of-the-art NN potentials

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88



Error analysis

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

1. Truncation error [use the algorithm of EE, Krebs, Meißner, EPJA 51 (2015) 53]

2. Statistical uncertainties

Assume $\chi^2(c) \approx \chi^2_{\min} + \frac{1}{2}(c - c_{\min})^T H (c - c_{\min})$ where $H_{ij} = \left.\frac{\partial^2 \chi^2}{\partial c_i \partial c_j}\right|_{c=c_{\min}}$

Quadratic approximation is employed to propagate statistical errors in observables

$$O(c) = O(c_{\min}) + J_O(c - c_{\min}) + \frac{1}{2}(c - c_{\min})^T H_O (c - c_{\min})$$

see also: Carlsson et al., PRX 6 (16) 011019

3. Uncertainties due to πN LECs $c_{1,2,3,4}$, $d_{1,2,3,5,14,15}$ and $e_{14,17}$

Estimated based on the results using a different set of LECs (KH PWA of πN scattering)

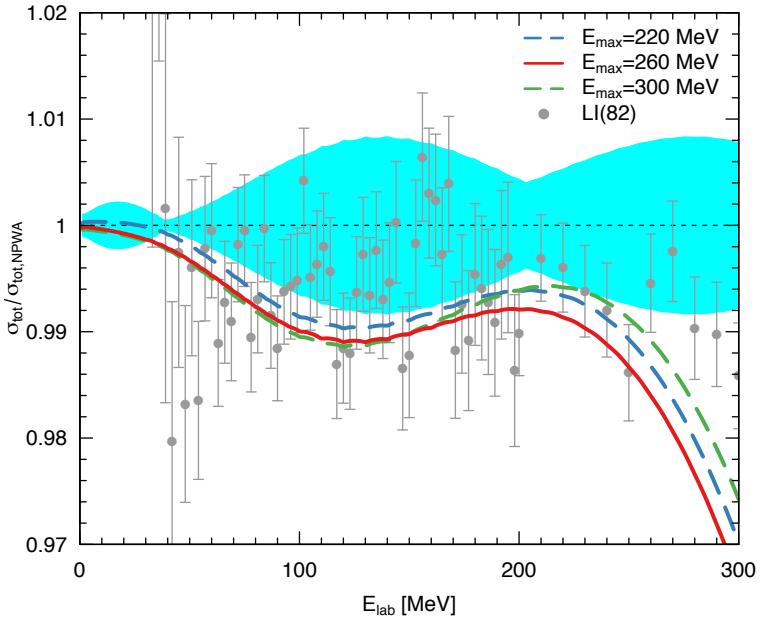
see EE, Krebs, Meißner, PRL 115 (15) 122301

4. Choice of E_{max} in the fits

Uncertainty estimated at N⁴LO/N⁴LO⁺ by performing fits with E_{max} = 220…300 MeV

$E_{\rm lab}$ bin	220 MeV	260 MeV	300 MeV
neutron-proton scattering data			
0 – 100	1.07	1.07	1.08
0 – 200	1.06	1.07	1.07
0 – 300	1.10	1.06	1.06
proton-proton scattering data			
0 – 100	0.86	0.86	0.87
0 – 200	0.95	0.95	0.96
0 – 300	1.00	1.00	0.98

N⁴LO⁺, Λ = 450 MeV



Error analysis

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

In most cases, the uncertainty is dominated by truncation errors. At N^{4}LO and at very low energies, other sources of errors become comparable (especially πN LECs…).

Example: deuteron asymptotic normalizations (relevant for nuclear astrophysics)

Our determination:

truncation error — πN LECs — statistical error — variation of E_{max}

$$A_S = 0.8847^{(+3)}_{(-3)}(3)(5)(1)\ \mathrm{fm}^{-1/2}$$

$$\eta \equiv \frac{A_D}{A_S} = 0.0255^{(+1)}_{(-1)}(1)(4)(1)$$

Exp: $A_S = 0.8781(44)\ \mathrm{fm}^{-1/2}$, $\eta = 0.0256(4)$

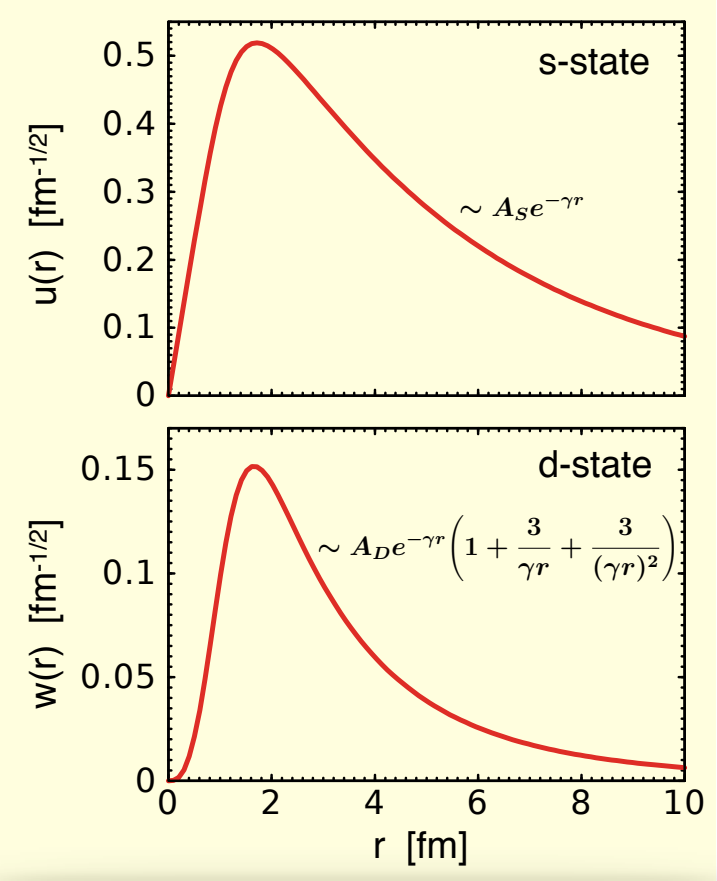
Borbely et al. '85 — Rodning, Knutson '90

Nijmegen PWA [errors are „educated guesses“] Stoks et al. '95

$$A_S = 0.8845(8)\ \mathrm{fm}^{-1/2}, \quad \eta = 0.0256(4)$$

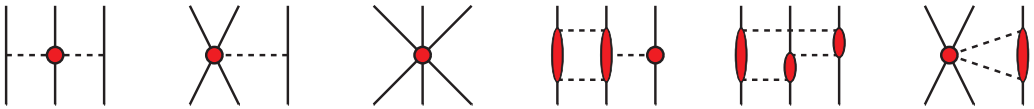
Granada PWA [errors purely statistical] Navarro Perez et al. '13

$$A_S = 0.8829(4)\ \mathrm{fm}^{-1/2}, \quad \eta = 0.0249(1)$$



Three-nucleon forces

N²LO: tree-level graphs, 2 new LECs
van Kolck '94; EE et al '02



N³LO: leading 1 loop, parameter-free
Ishikawa, Robilotta '08; Bernard, EE, Krebs, Meißner '08, '11

N⁴LO: full 1 loop, almost completely worked out, several new LECs
Girlanda, Kievski, Viviani '11; Krebs, Gasparyan, EE '12,'13; EE, Gasparyan, Krebs, Schat '14

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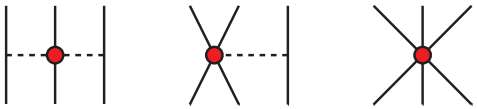

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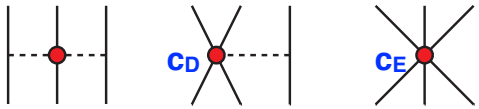
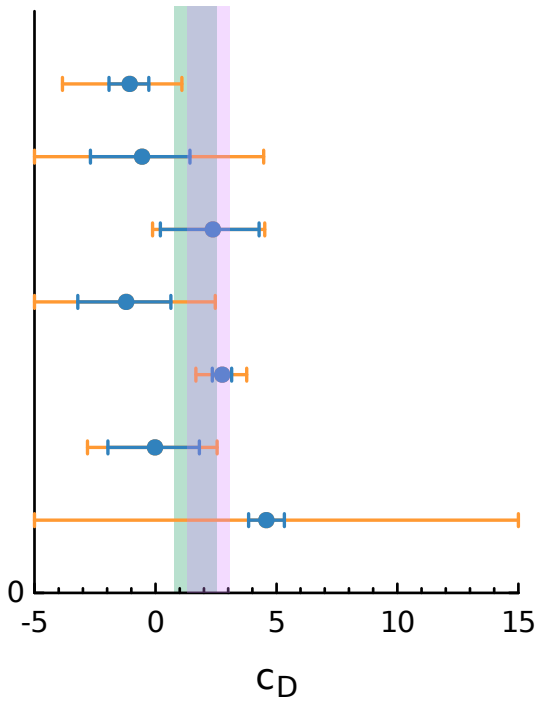


Determination of the LECs c_D, c_E

— Triton BE (c_D-c_E correlation)

— Explore various possibilities and let theory and/or data decide…



pd minimum of dσ/dθ at 135 MeV [Sekiguchi et al.'02]

nd σ_{tot} at 135 MeV [Abfalterer et al.'01]

pd minimum of dσ/dθ at 108 MeV [Ermisch et al.'03]

nd σ_{tot} at 108 MeV [Abfalterer et al.'01]

pd minimum of dσ/dθ at 70 MeV [Sekiguchi et al.'02]

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nd scattering length ^2a [Schoen et al.'03]

LENPIC, to appear
(based on r-space-regularized potentials, R = 0.9 fm)

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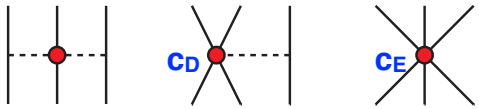

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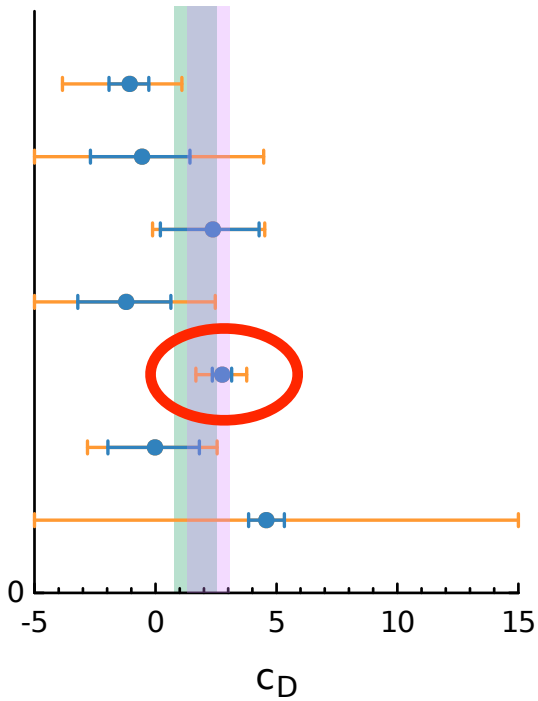

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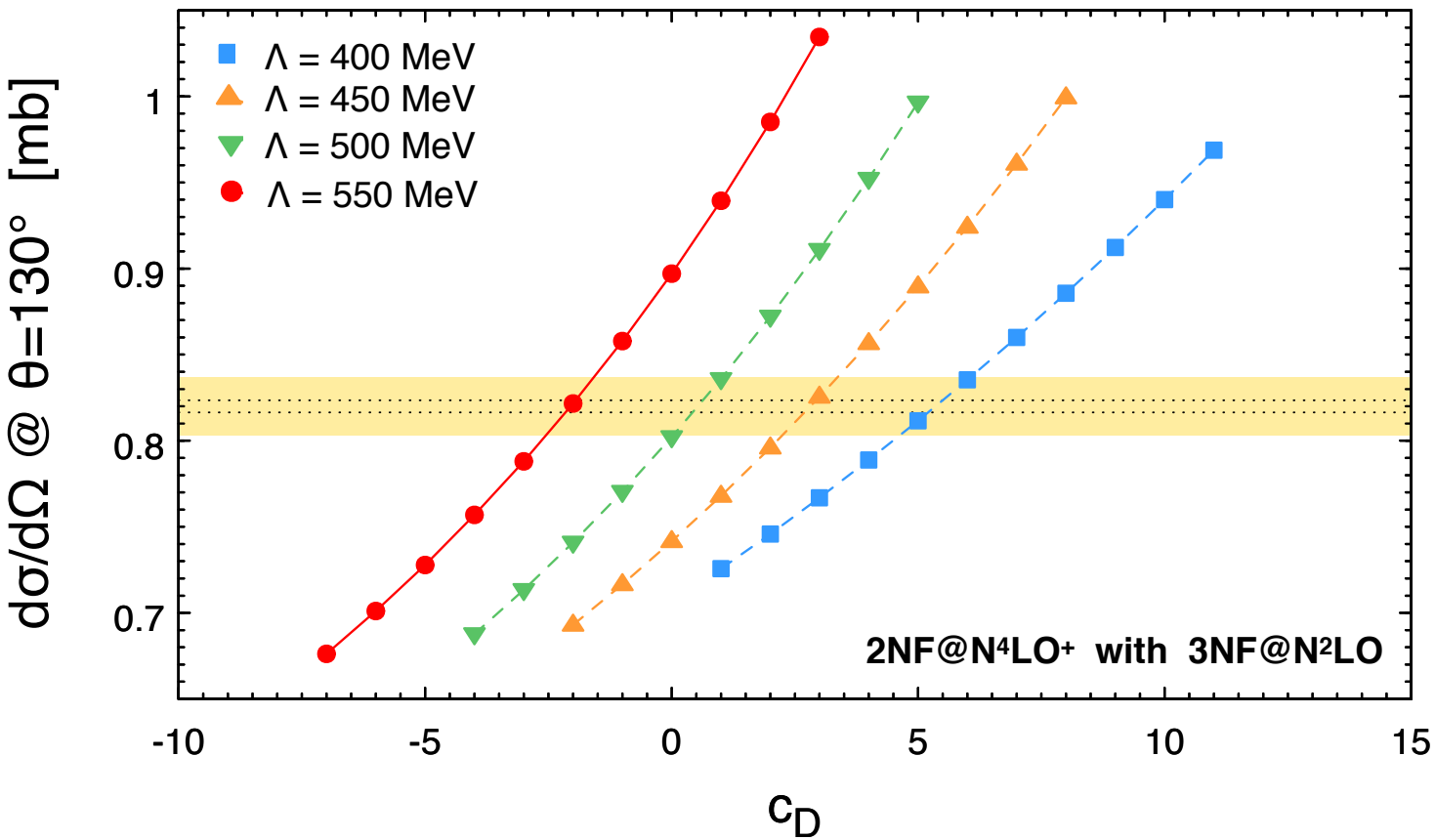
yields the strongest constraint…

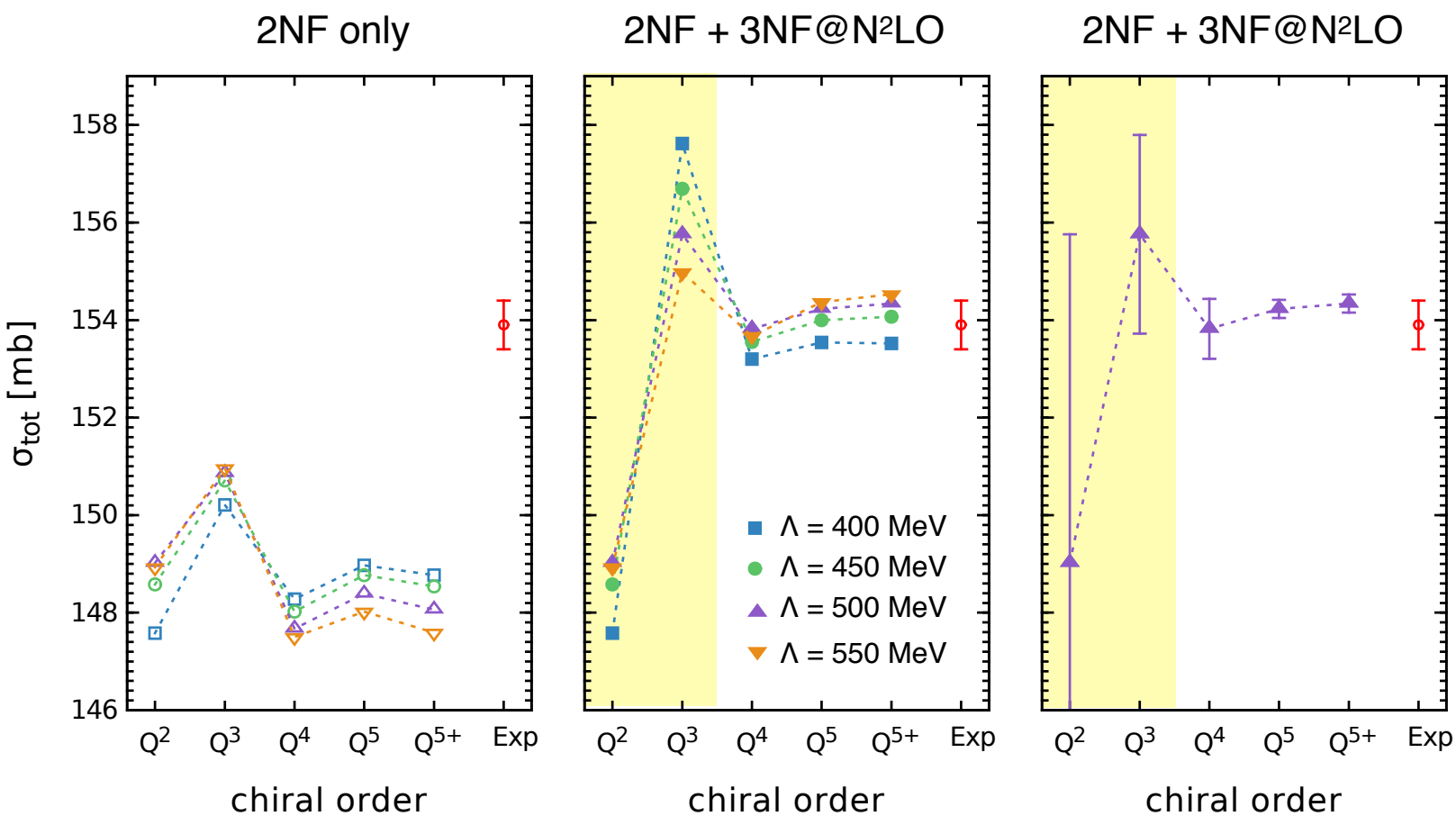
LENPIC: Low Energy Nuclear Physics International Collaboration

Determination of c_D, c_E (preliminary)



Sensitivity to the ^{3}H BE: changing E_{3H} = 8.482 MeV by +-70 keV significantly affects c_E (e.g. for Λ = 450 MeV: δc_E ~15%) but has almost no effect on c_D (δc_D ~ 0.05%), so that δE_{3H} is almost completely generated by δc_E → no sizable correlations between c_D, c_E

Nd total cross section at 70 MeV (preliminary)



— Similar improvement is found for many other few-N observables

— Radii of medium-mass nuclei seem to be underestimated (by ~ 15% for ^{16}O)

LENPIC: Low Energy Nuclear Physics International Collaboration

Radii of medium-mass nuclei

- Calculations are incomplete: 3NFs and MECs are missing…

Radii of medium-mass nuclei

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	r_D, ^{2}H (fm)	r_p, ^{3}H (fm)	r_p, ^{4}He (fm)
AV18/AV18+UIX	1.967 (−0.4%)	1.584 (−1%)	1.44 (−2%)

from: Lazauskas, Carbonell, PRC 70 (2004) 044002

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NNLO$_{sat}$: r_D = 1.978 fm (+0.13%) ΔNNLO(450): r_D = 1.982 fm (+0.3%)

However, NN data seem to prefer smaller r_D:

	RKE N^{4}LO$^+$	Granada PWA (δ-shell)	Nijm I	Nijm II	Reid93	CD-Bonn	Exp.
r_D, ^{2}H (fm)	1.965 . . . 1.968	1.965	1.967	1.968	1.969	1.966	1.975

Using r_D as a constraint in the fits increases χ^2/datum considerably (standard fit protocol)…

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- Work in progress with the Darmstadt group: using alternative choices for redundant contact terms can reshuffle some N^{4}LO contributions from 3NF and MECs into the 2NF. Hope to better understand the impact of missing contributions…

Summary

— derivation of the nuclear Hamiltonian at N^3LO completed already in 2011; derivation of N^4LO corrections done for V_{2N} & almost done for V_{3N} (new LECs…) and V_{4N}

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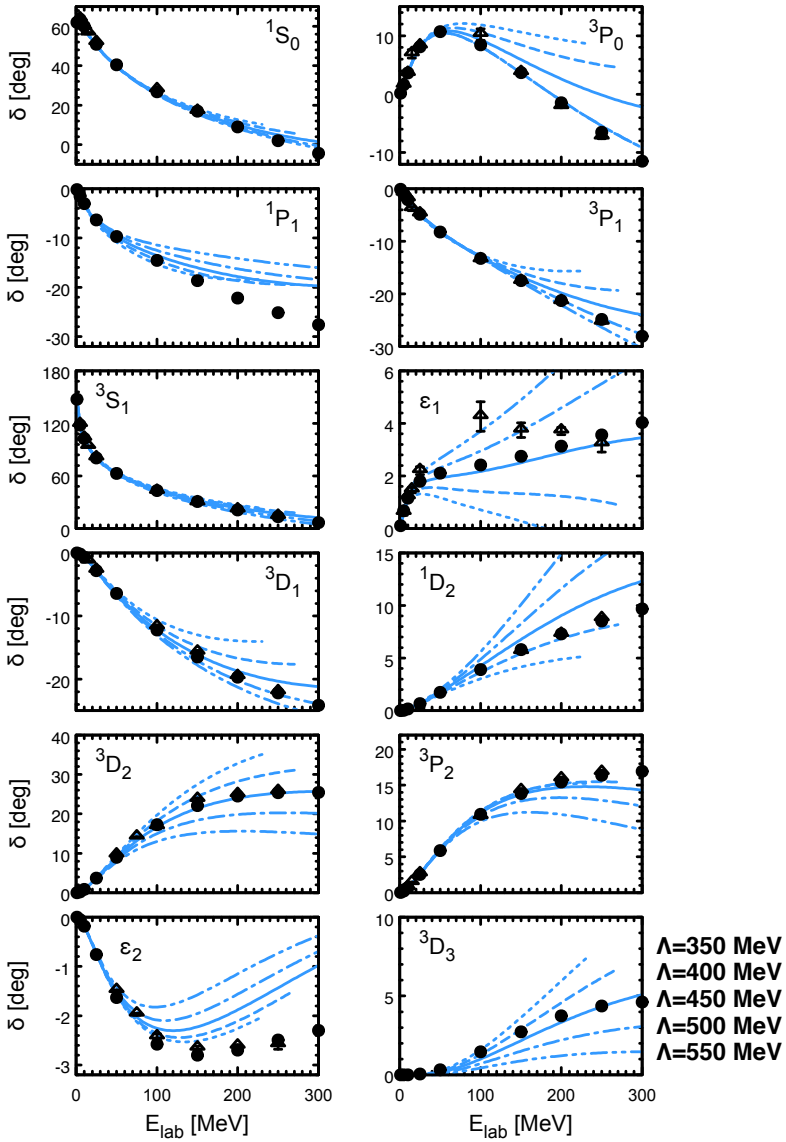
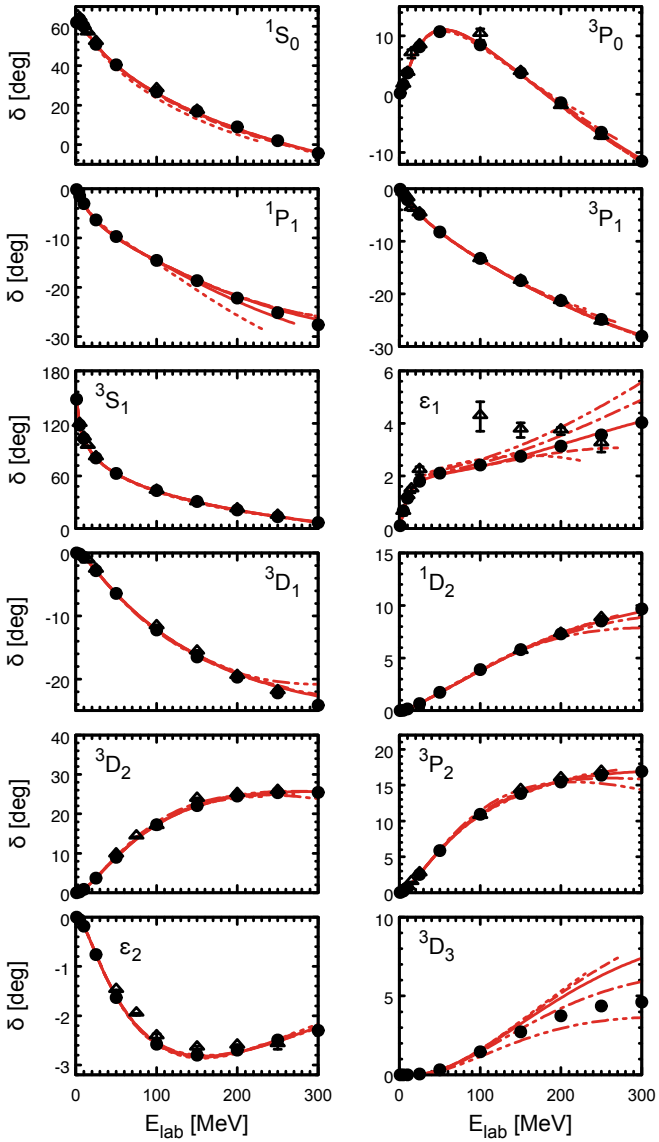


N⁴LO [$C_0 + C_2 p^2 + C_4 p^4$]



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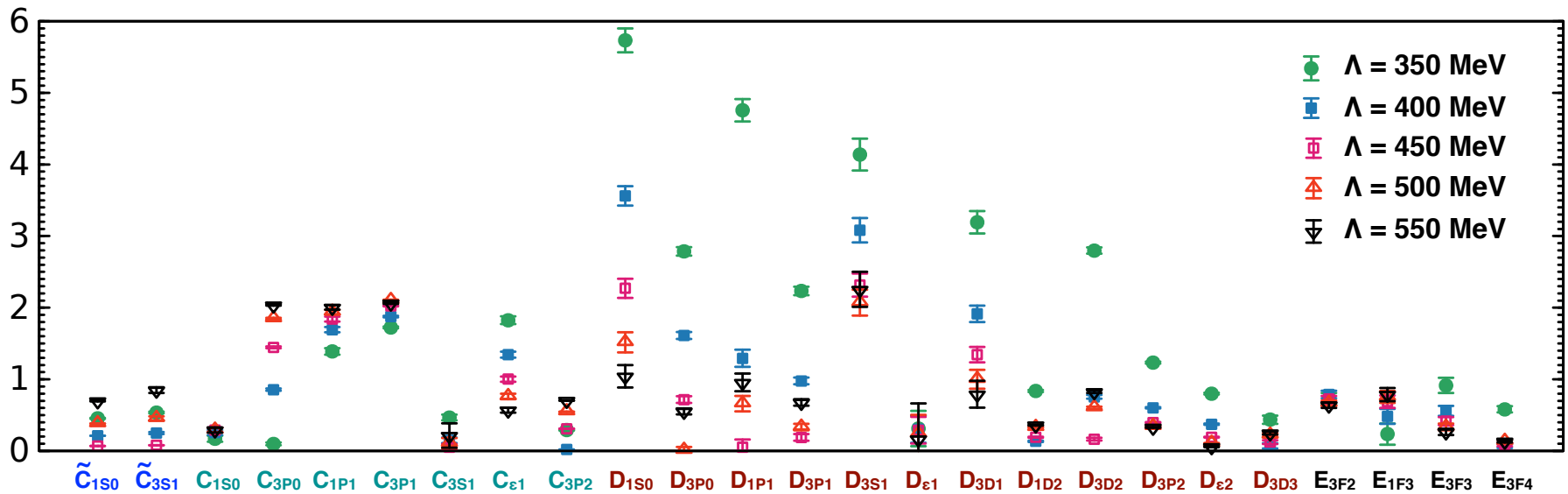
Summary

Natural units for the LECs according to NDA:

$$|\tilde{C}_i| \sim \frac{4\pi}{F_\pi^2}, \qquad |C_i| \sim \frac{4\pi}{F_\pi^2 \Lambda_b^2}, \qquad |D_i| \sim \frac{4\pi}{F_\pi^2 \Lambda_b^4}, \qquad |E_i| \sim \frac{4\pi}{F_\pi^2 \Lambda_b^6}$$

Assuming Λ_b = 600 MeV [EE, Krebs, Meißner EPJA 51 (15) 53; Furnstahl, Klco, Phillips, Wesolowski, PRC 92 (15) 024005], all LECs come out of a natural size.

Absolute values of the LECs in natural units



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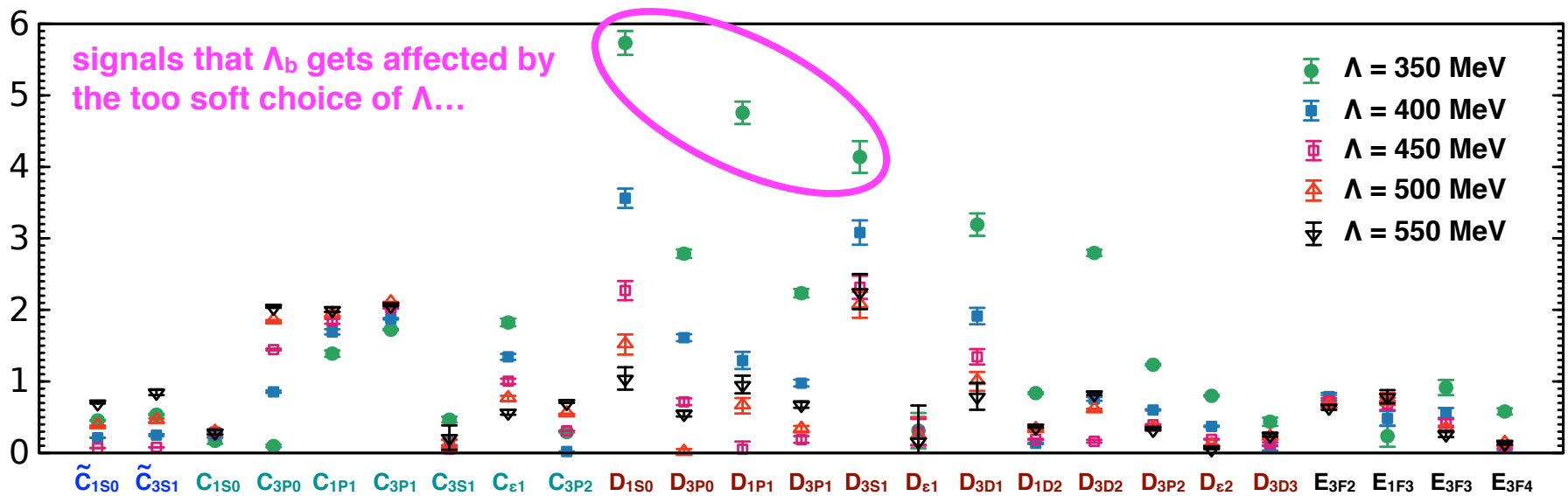

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— so far, all LECs come out of a natural size

— the covariant matrix has no large eigenvalues $\longrightarrow$ no redundancy in the contact terms

Summary

Eigenvalues of the covariance matrix

$$\Sigma = 2\frac{\chi^2}{N_{\mathrm{dof}}}H^{-1}$$

for LECs taken in natural units (N^4LO^+, Λ = 450 MeV)

```
4.274396e-02
2.474783e-02
1.902965e-02
1.035190e-02
6.300807e-03
3.912243e-03
2.902483e-03
2.251440e-03
1.902579e-03
1.089075e-03
9.322493e-04
5.588222e-04
3.562153e-04
1.610448e-04
1.409259e-04
1.229603e-04
8.654795e-05
4.958497e-05
4.316301e-05
3.576713e-05
1.911708e-05
1.448694e-05
8.518138e-06
8.268942e-07
4.213655e-10
2.063609e-11
1.614358e-11
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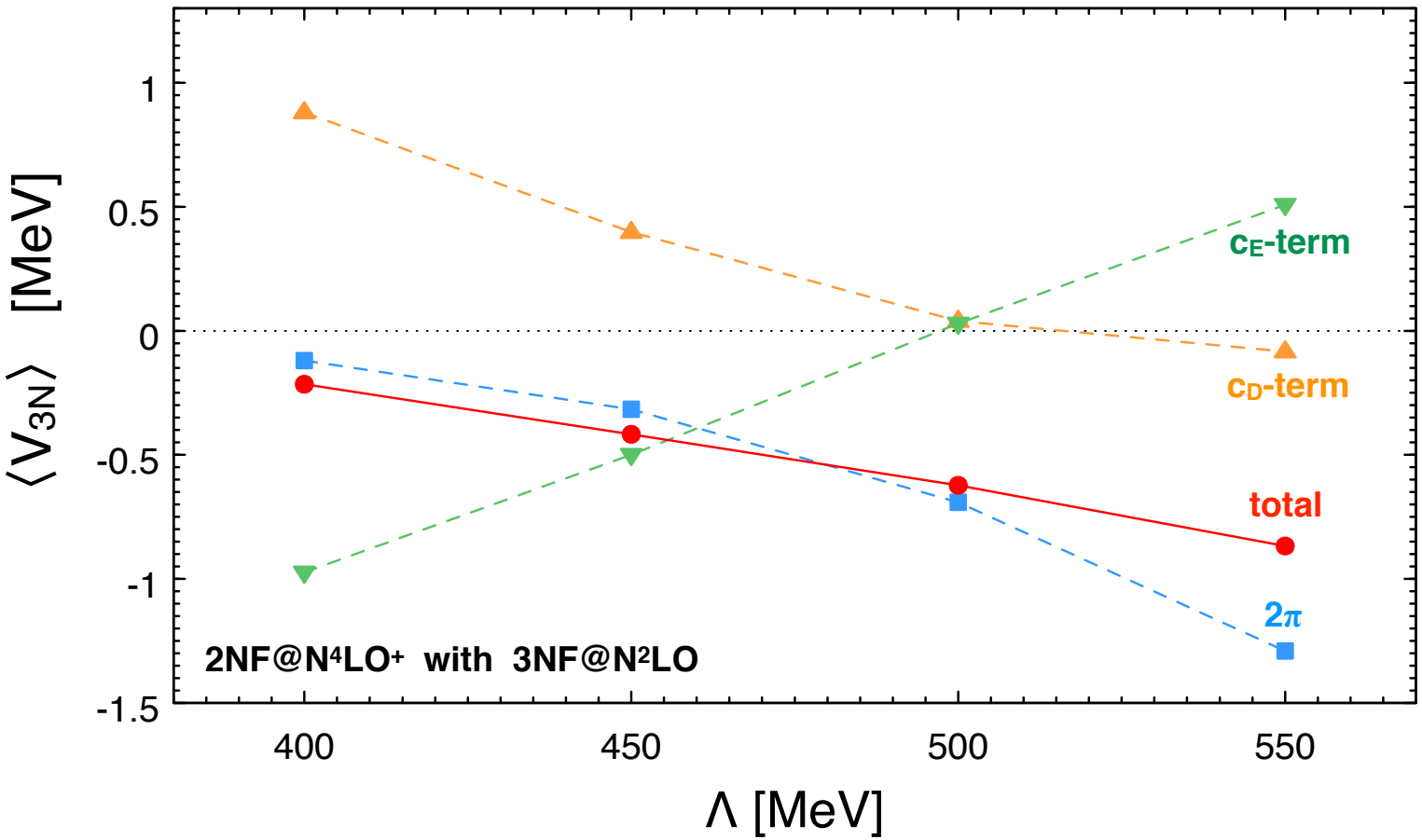

— all contributions are of natural size: $\langle V_{3N}\rangle \sim (M_\pi/\Lambda_b)^3 \langle V_{2N}\rangle \sim 650$ keV

— no support of the RG analysis by Birse: $V_{2\pi} \sim Q^3$, $V_D \sim Q^{5/4}$, $V_E \sim Q^{(>3)}$

[M. Birse, Phil. Trans. Roy. Soc. Lond. A369 (2011) 2662-2678]

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