From light-nuclei to neutron stars within chiral dynamics

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The "*basic model*" of nuclear theory

One of the major goals of nuclear theory is to achieve a comprehensive description of the wealth of data and peculiarities exhibited by nuclear systems. We would like to have a "good" description of:

Nucleon-nucleon (NN) scattering data: "thousands" of experimental data available such as differential and total cross sections, polarizations, asymmetries, etc…

The spectra, properties, and transition of nuclei: binding energies, radii, magnetic moments, beta decays rates, weak/radiative captures, electroweak form factors, etc.

The nucleonic matter equation of state: neutrons stars with masses of order twice the solar mass

Inputs for the *basic model*:

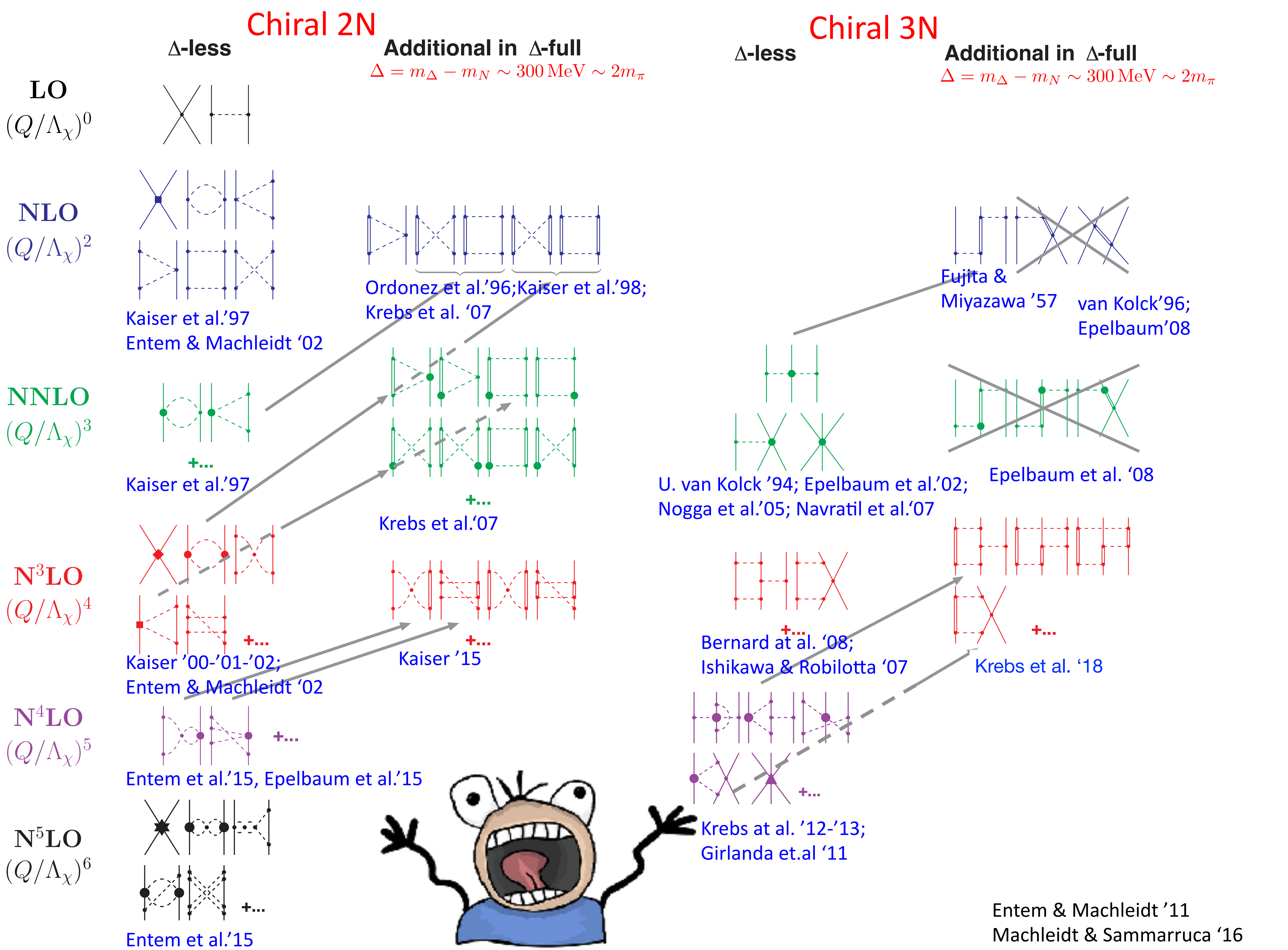
$\boxed{\text{Many-body interactions between the constituents}}$

$$H = \underbrace{\sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2m_i}}_{\text{One-body}} + \underbrace{\sum_{i<j=1}^{A} v_{ij}}_{\text{Two-body (NN)}} + \underbrace{\sum_{i<j<k=1}^{A} V_{ijk}}_{\text{Three-body (3N)}} + \ldots..$$

$\boxed{\text{Electroweak current operators:}}$

$$j^{\rm EW} = \underbrace{\sum_{i=1}^{A} j_i}_{\text{One-body}} + \underbrace{\sum_{i<j=1}^{A} j_{ij}}_{\text{Two-body}} + \underbrace{\sum_{i<j<k=1}^{A} j_{ijk} + \ldots.}_{\text{Many-body}}$$

Nuclear interactions in chiral EFT



Chiral 2N potentials: recent developments

- First generation of chiral NN potential up to N3LO:
  Entem-Machleidt PRC **68**, 041001 2003; Epelbaum-Gloeckle-Meissner JNP **A747**, 362 2005

- Improved nuclear matter calculations from chiral low-momentum interactions
  Hebeler et al. PRC **83**, 031301(R) (2011).

- Optimized N2LO NN potential (πN LECs are tuned to NN peripheral scattering):
  Ekström et al. PRL **110**, 192502 2013; JPG **42**, 034003 2015

- N2LO potential: a simultaneous fit of NN and 3N forces to low NN data (E_{lab}=35 MeV), deuteron BE, BE and CR of hydrogen, helium, carbon and oxygen isotopes:
  Carlsson et al. PRC **91**, 051301(R) 2015

- New generation of chiral NN potentials up to N4LO: improved choice of the regulator, no SFR:
  Epelbaum et al. PRL **112**, 102501 2014; EPJ **A51**, 53 2015; PRL **115**, 122301 2015

- Chiral 2π and 3π exchange up to N4LO and up to N5LO in NN peripheral scattering:
  Entem et al. PRC **91**, 014002 2015; PRC **92**, 064001 2015, PRC **96**, 024004 2017

- Semilocal momentum-space regularized chiral NN potentials up to N4LO and N4LO+ (LENPIC collaboration):
  arXiv: 1705.01530v1 2017, arXiv:1711.08821 2017, arXiv:1802.08584 2018

- N2LO potential with Δ-isobar; Ekström et al. PRC **97**, 024332 2018

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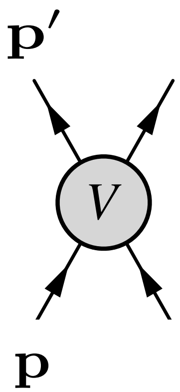
Chiral potentials and QMC

Note: Many of the available versions of chiral potentials are formulated in momentum-space and are strongly nonlocal: ⇒ $\mathbf{p} \rightarrow -i\boldsymbol{\nabla}$ hard to use in QMC methods

Nonlocalities due to contact interactions and to regulator functions



Nonlocal regulator

$$\boxed{V_{\rm NN}(\mathbf{p},\mathbf{p}') \rightarrow \exp\left[-[(\mathbf{p}^2+\mathbf{p}'^2)/\Lambda^2]^n\right] V_{\rm NN}(\mathbf{p},\mathbf{p}')}$$

Local regulator

$$\boxed{V_{\rm NN}(\mathbf{p},\mathbf{p}') \rightarrow \exp\left[-[(\mathbf{p}'-\mathbf{p})^2/\Lambda^2]^n\right] V_{\rm NN}(\mathbf{p},\mathbf{p}')}$$

- Local NN potential up to N2LO:
  Gezerlis et al. PRL **111**, 032501 2013; PRC **90**, 054323 2014; Lynn et al. PRL **113**, 192501 2014
- Minimally nonlocal/local NN potentials including N2LO Δ-contributions and N3LO contacts:
  Piarulli et al. PRC **91**, 024003 2015; PRC **94**, 054007 2016

We use QMC (VMC, GFMC, AFDMC) and HH methods to solve the many-body Schrödinger equation

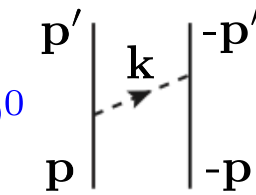
$$H\,\Psi(\mathbf{R};s_1,..,s_A;t_1,..,t_A) = E\,\Psi(\mathbf{R};s_1,..,s_A;t_1,..,t_A)$$

A “Semi-phenomenological” local chiral NN potential with Δ’s

$$\boxed{v_{12} = v_{12}^{\mathrm{EM}} + v_{12}^{\mathrm{L}} + v_{12}^{\mathrm{S}}}$$

v_{12}^{EM} : EM component including corrections up to α^2

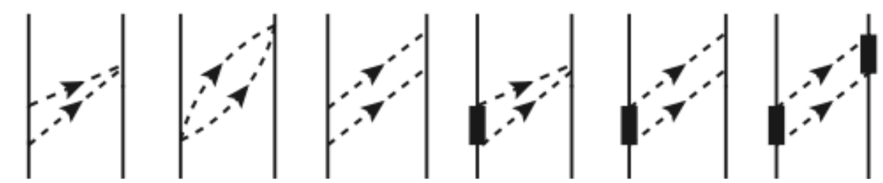
$\mathrm{LO}: \; Q^0$



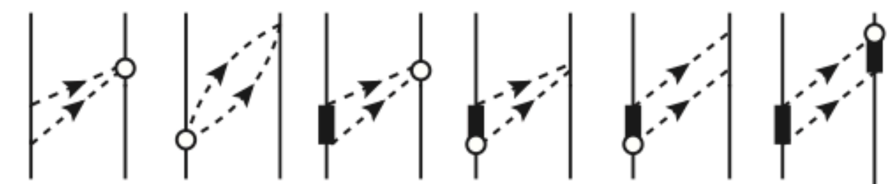
v_{12}^{L} : chiral OPE and TPE component with Δ’s

- dependence only on the momentum transfer **k=p**′**-p**

$\mathrm{NLO}: \; Q^2$



$\mathrm{N2LO}: \; Q^3$



v_{12}^{S} : short-range contact component up to order N3LO (Q^4) parametrized by (2+7+11) CI and (2+4) IB LECs

- the functional form taken as $C_{R_S}(r) \propto e^{-(r/R_S)^2}$ with $R_S = 0.8 \; (0.7) \; \mathrm{fm}$ a (b) models

In coordinate-space it reads as:

$$\boxed{v_{12} = \sum_{l=1}^{16} v^l(r) O_{12}^l}$$

$$O_{12}^{l=1,\dots,6} = [\mathbf{1}\,,\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\,,\, S_{12}] \otimes [\mathbf{1}\,,\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2]$$

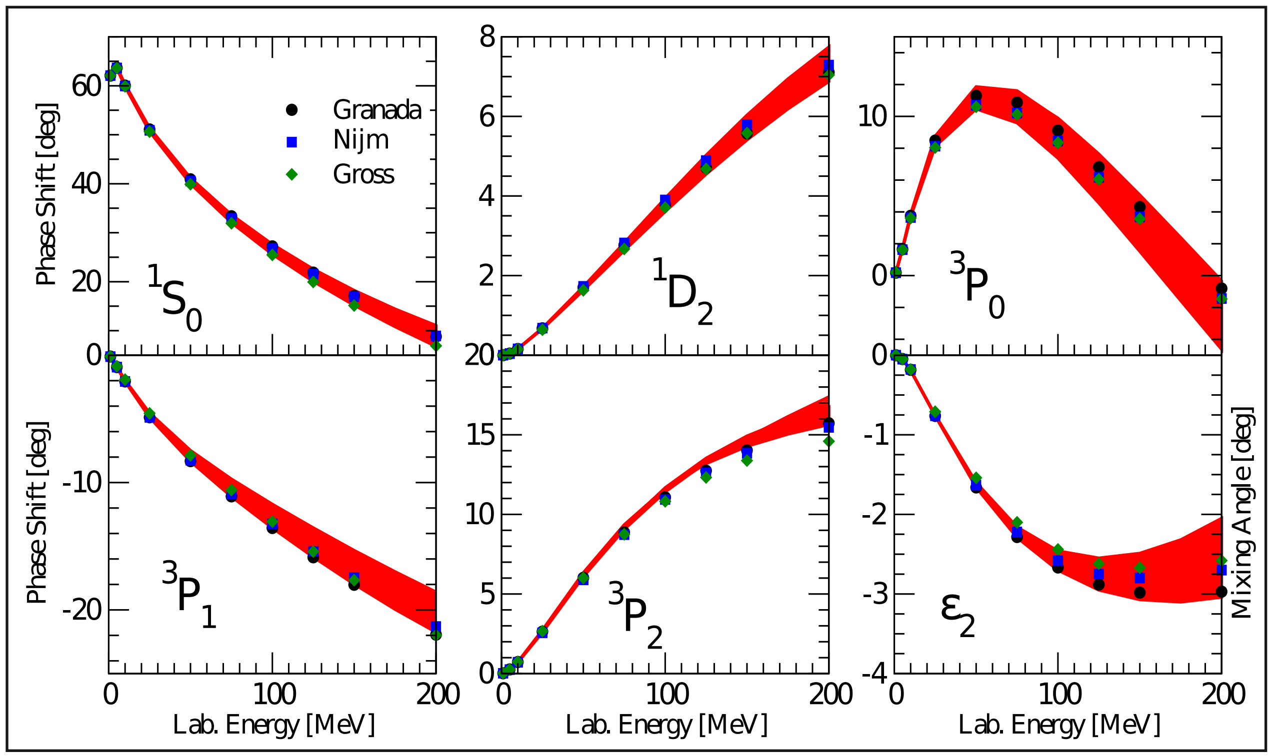
$$O_{12}^{l=7,\dots,11} = \mathbf{L}\cdot\mathbf{S}\,,\, \mathbf{L}\cdot\mathbf{S}\,\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\,,\, (\mathbf{L}\cdot\mathbf{S})^2\,,\, \mathbf{L}^2\,,\, \mathbf{L}^2\,\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

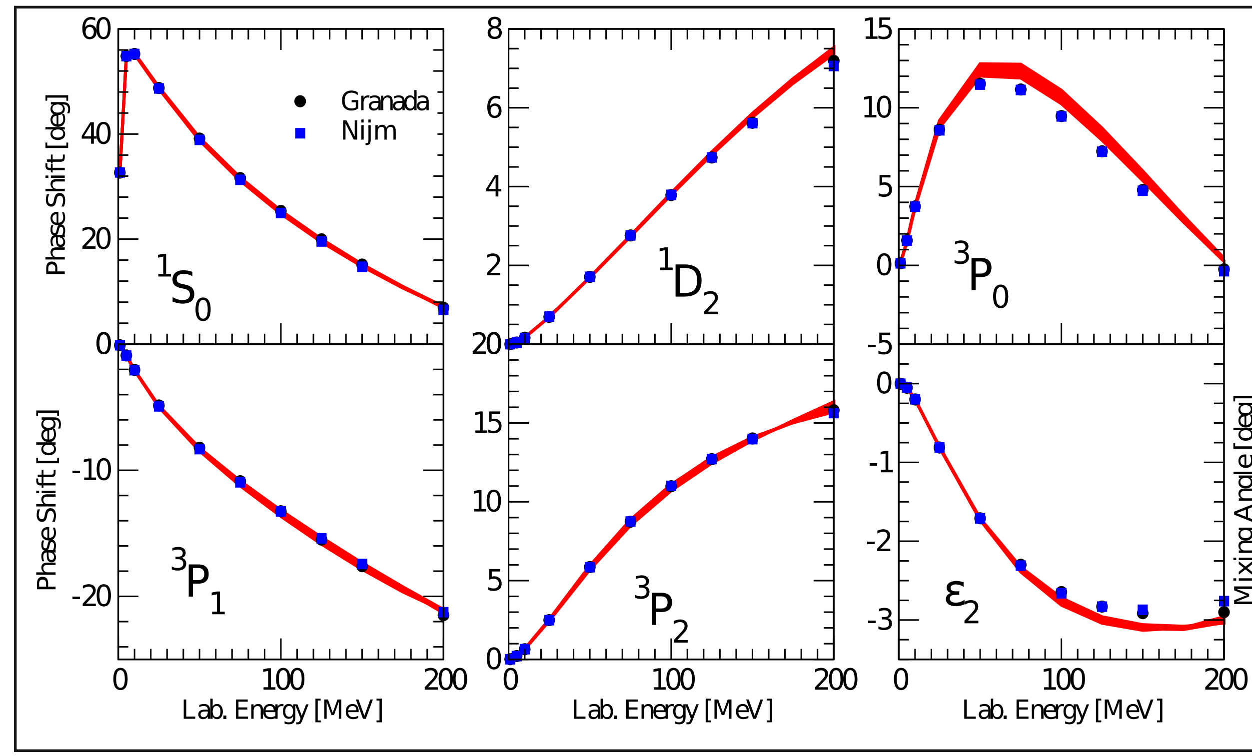
$$O_{12}^{l=12,\dots,16} = T_{12}\,,\, (\tau_1^z + \tau_2^z)\,,\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\, T_{12}\,,\, S_{12}\, T_{12}\,,\, \mathbf{L}\cdot\mathbf{S}\, T_{12}$$

Fitting procedure: NN PWA and database

The 26 LECs are fixed by fitting the pp and np Granada database up to two ranges of E_{lab} = 125 MeV and 200 MeV, the deuteron BE and the nn scattering length

To minimizing the χ^2 we have used the Practical Optimization Using No Derivatives (for Squares), POUNDers (M. Kortelainen, PRC **82**, 024313 2010)



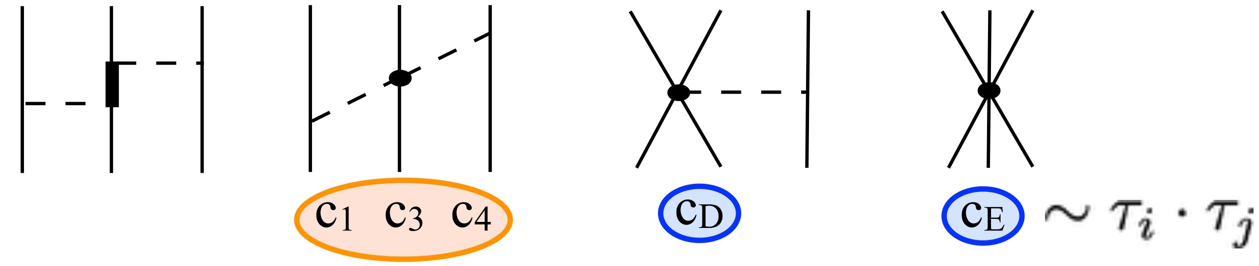


model	order	$E_{\rm Lab}$ (MeV)	N_{pp+np}	χ^2/datum
Ia	N3LO	0–125	2668	1.05
Ib	N3LO	0–125	2665	1.07
IIa	N3LO	0–200	3698	1.37
IIb	N3LO	0–200	3695	1.37

Models a (b) cutoff~500 MeV (600 MeV) in momentum-space

Local chiral 3N potential with Δ's

Inclusion of 3N forces at N2LO:



1) Fit to:

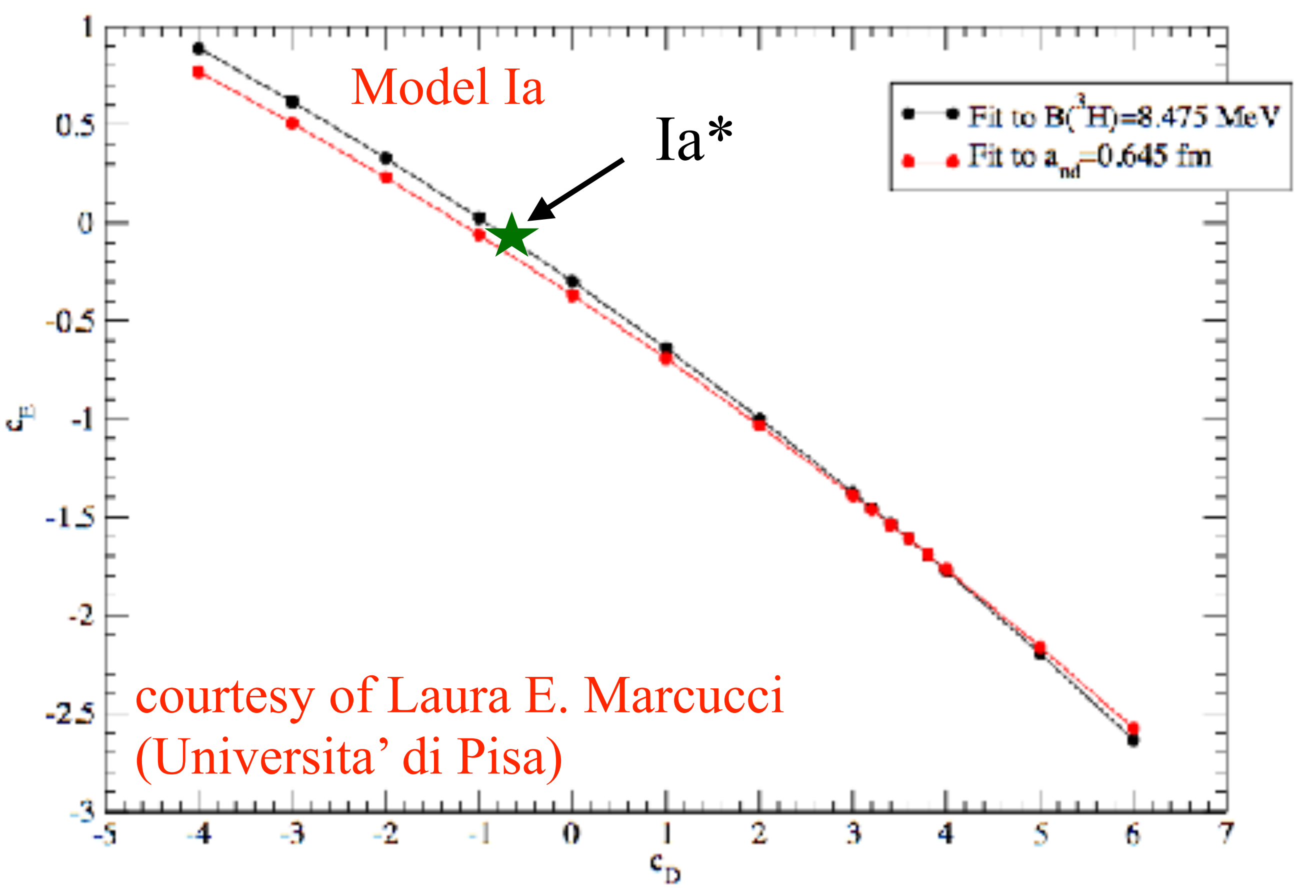
- $E_0(^3\mathrm{H}) = -8.482$ MeV
- $^2a_{nd} = (0.645 \pm 0.010)$ fm

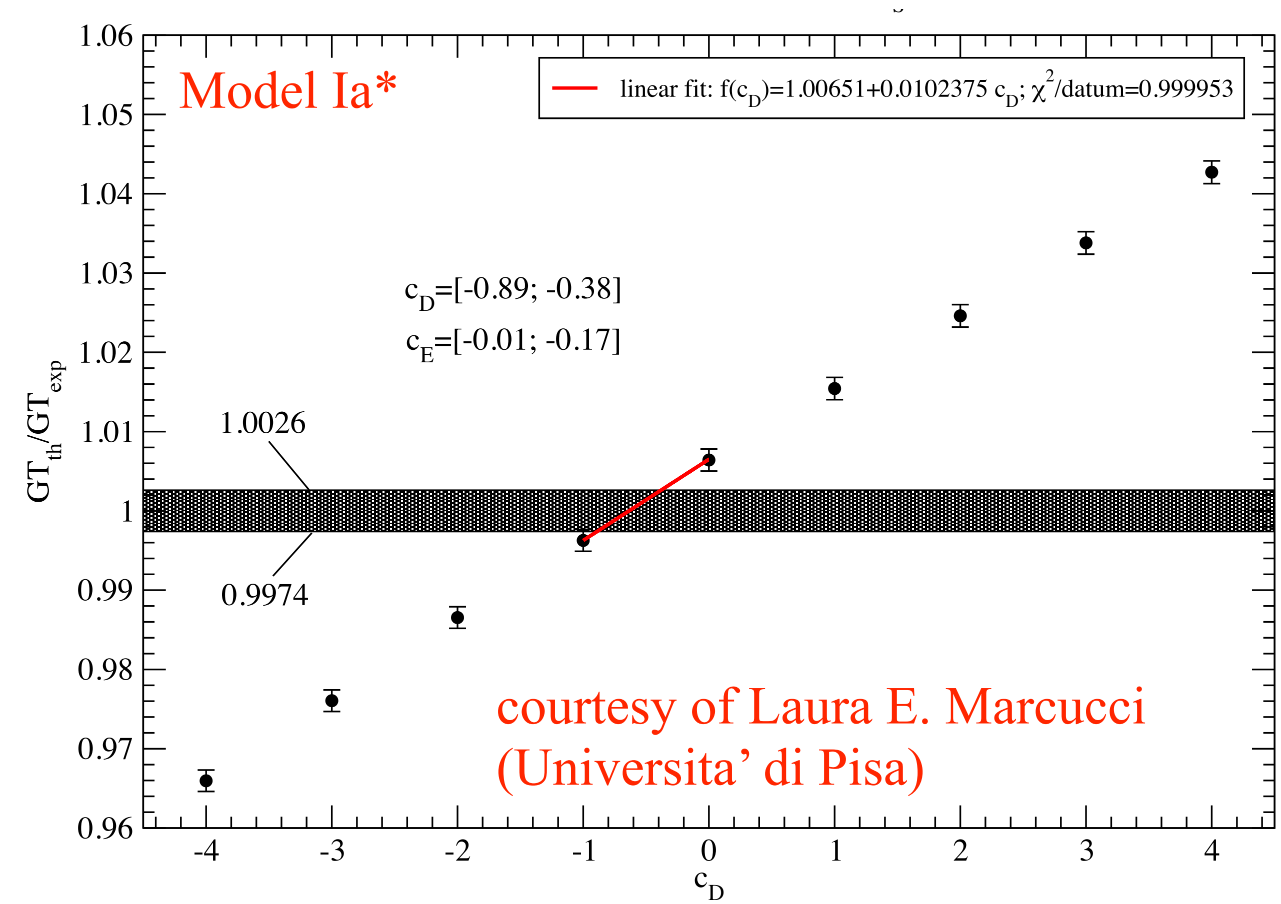
Model	c_D	c_E
Ia	3.666	–1.638
Ib	–2.061	–0.982
IIa	1.278	–1.029
IIb	–4.480	–0.412

2) Fit to:

- $E_0(^3\mathrm{H}) = -8.482$ MeV
- GT m.e. in ^{3}H β-decay

Model	c_D	c_E
Ia*	–0.635(255)	–0.09(8)
Ib*	–4.705(285)	0.550(150)
IIa*	–0.610(280)	–0.350(100)
IIb*	–5.250(310)	0.05(180)





Ab initio methods: HH and QMC

Hyperspherical Harmonics (HH) expansion for A=3 and 4 bound and continuum states

$$|\Psi\rangle = \sum_\mu c_\mu \underbrace{|\Phi_\mu\rangle}_{\text{HH basis}} \qquad c_\mu \qquad \text{from} \qquad E = \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}$$

Kievsky *et al.*, JPG: NPP **35,** 063101 (2008)

Quantum Monte Carlo (QMC) methods encompass a large family of computational methods whose common aim is the study of complex quantum systems

VMC, GFMC: sampling in coordinate space

→ limited number of nucleons A=12 (new developments for A=13)

R.B. Wiringa, PRC **43**, 1585 (1991)
Carlson, et al., Rev. Mod. Phys. 87, 1067 (2015)

AFDMC: sampling in coordinate space + spin-isospin coordinate

→ larger nuclei & neutron matter

Schmidt and Fantoni, Phys. Lett. B **446**, 99 (1999)
Carlson, *et al.*, Rev. Mod. Phys. **87**, 1067 (2015)

CVMC: sampling in coordinate space + cluster expansion

→ closed shell nuclei (+/- 1): A=40

Pieper, *et al.*, Phys. Rev. C **46**, 1741 (1992)
Phys. Rev. Lett. **120**, 122502 (2018)

QMC: Variational Monte Carlo (VMC)

R.B. Wiringa, PRC **43**, 1585 (1991)

Minimize the expectation value of H:

$$\boxed{E_T = \frac{\langle \Psi_T | H | \Psi_T \rangle}{\langle \Psi_T | \Psi_T \rangle} \geq E_0}$$

Trial wave function (involves variational parameters):

$$\boxed{|\Psi_T\rangle = \left[1 + \sum_{i<j<k} U_{ijk}\right] \left[S \prod_{i<j} (1 + U_{ij})\right] |\Psi_J\rangle}$$

$|\Psi_J\rangle = \left[\prod_{i<j} f_c(r_{ij})\right] |\Phi(JMTT_z)\rangle$ (s-shell nuclei): Jastrow wave function, fully antisymmetric

$S \prod_{i<j}$: represents a symmetrized product

$U_{ij} = \sum_{p=2,6} u_p(r_{ij})\, O^p_{ij}$: pair correlation operators

$U_{ijk} = \sum_x \epsilon_x\, V^x_{ijk}$: three-body correlation operators

$|\Psi_T\rangle$ are spin-isospin vectors in 3A dimension with $2^A \begin{pmatrix} A \\ Z \end{pmatrix}$

The search in the parameter space is made using COBYLA (Constrained Optimization BY Linear Approximations) algorithm available in NLopt library

QMC: Diffusion Monte Carlo (DMC)

J. Carlson et al., Rev. Mod. Phys. **87**, 1067 (2015)

The diffusion Monte Carlo (DMC) method (ex. GFMC or AFDMC) overcomes the limitations of VMC by using a projection technique to determine the true ground-state

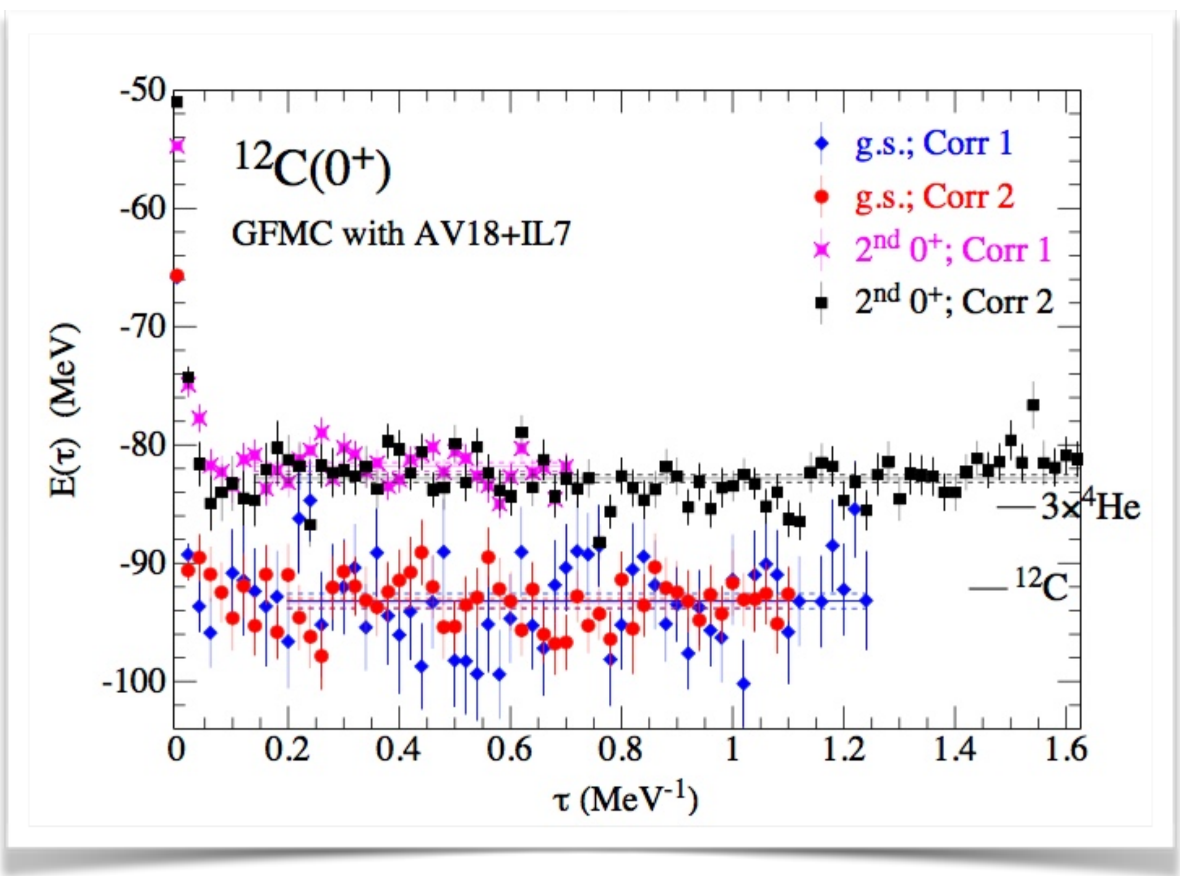
The method relies on the observation that Ψ_T can be expanded in the complete set of eigenstates of the Hamiltonian according to

$$|\Psi_T\rangle = \sum_n c_n|\Psi_n\rangle \qquad H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

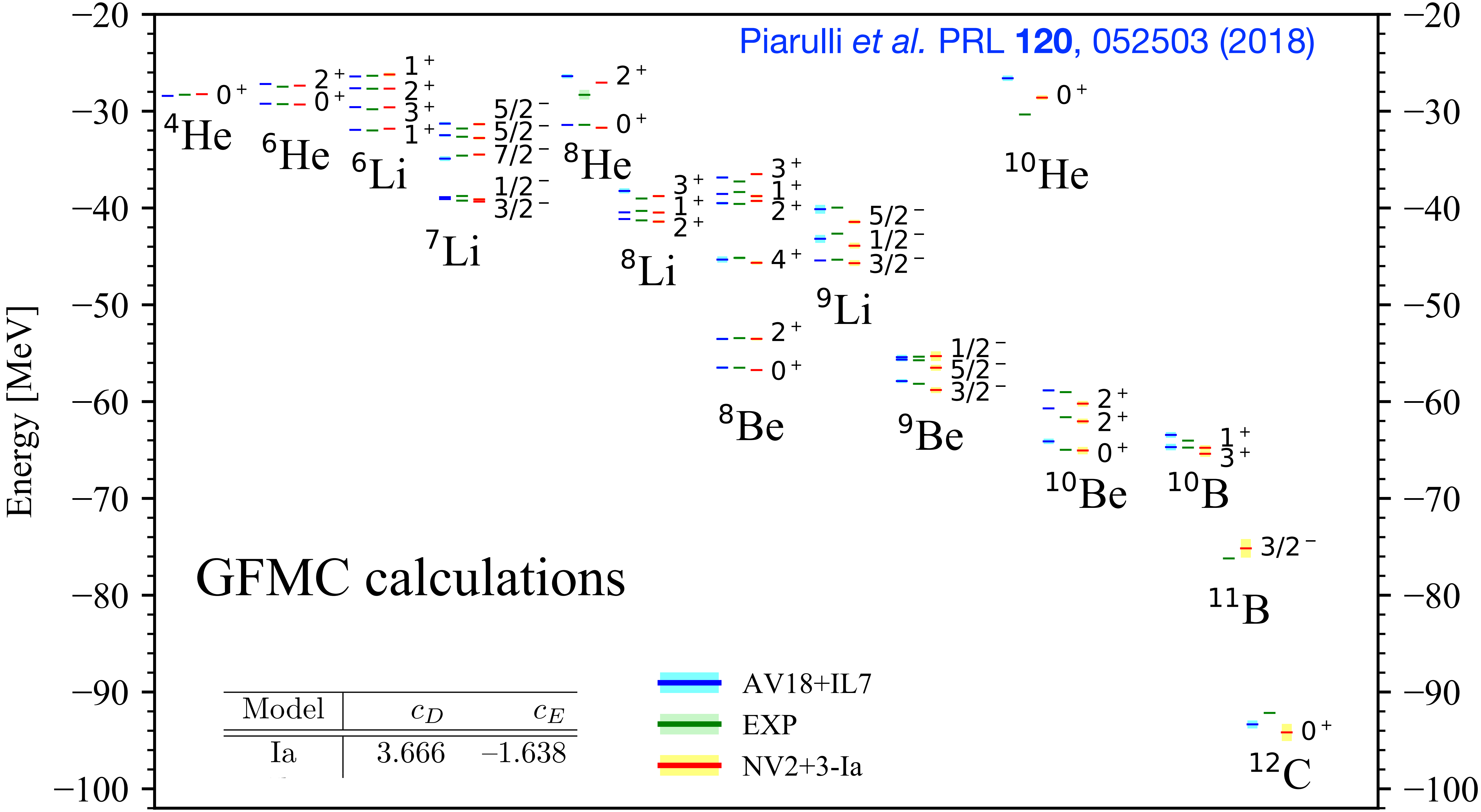
$$\boxed{\lim_{\tau\to\infty} |\Psi(\tau)\rangle = \lim_{\tau\to\infty} e^{-(H-E_0)\,\tau}\,|\Psi_T\rangle = c_0|\Psi_0\rangle} \qquad |\Psi(\tau=0)\rangle = |\Psi_T\rangle$$

where τ is the imaginary time

The evaluation of $\Psi(\tau)$ is done stochastically in small time steps $\Delta\tau$ ($\tau = n\,\Delta\tau$) using a Green's function formulation



Spectra of Light Nuclei: Phenomenology vs χEFT

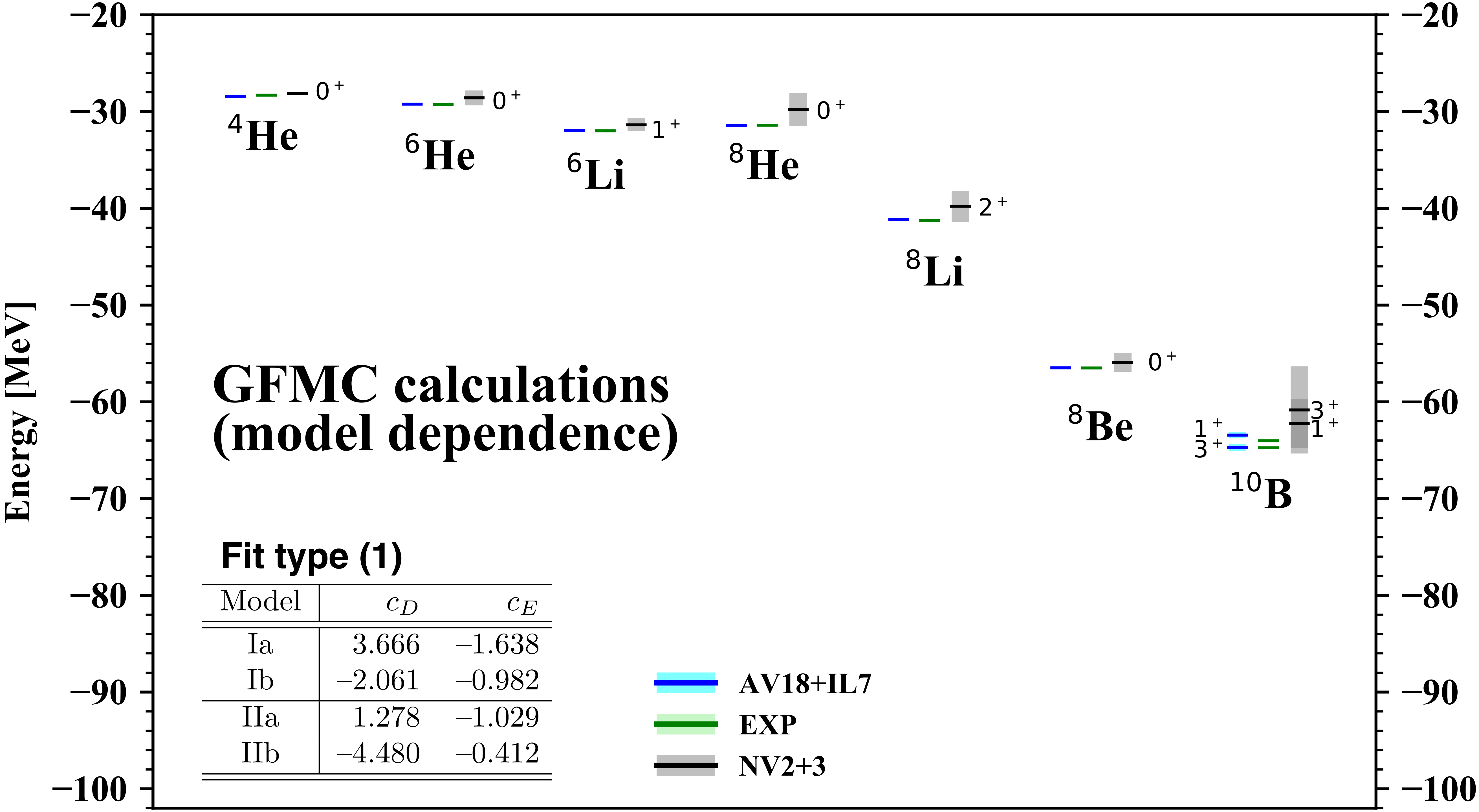


The rms from experiment is 0.72 MeV for NV2+3-Ia compared to 0.80 MeV for AV18+IL7

$c_E < (>) 0$: repulsion (attraction) in light-nuclei (the opposite effect in PNM)

$c_D < (>) 0$: repulsion (attraction) in light-nuclei (same effect in PNM but very small)

Energies of Light Nuclei: Model-dependence



Fit type (1)

Model	c_D	c_E
Ia	3.666	–1.638
Ib	–2.061	–0.982
IIa	1.278	–1.029
IIb	–4.480	–0.412

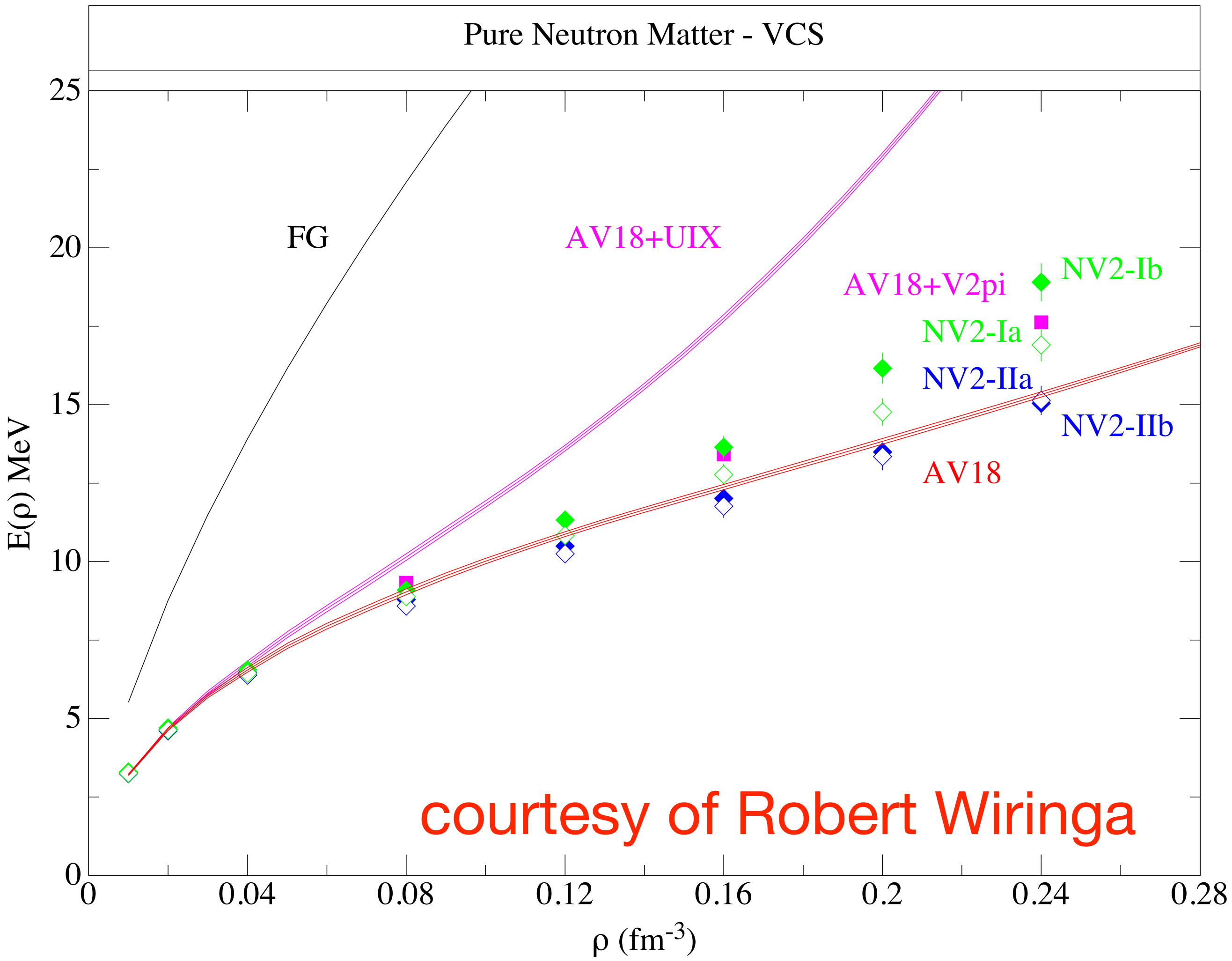
Model-dependence for NV2+3 up to 7-8% of the total binding energy

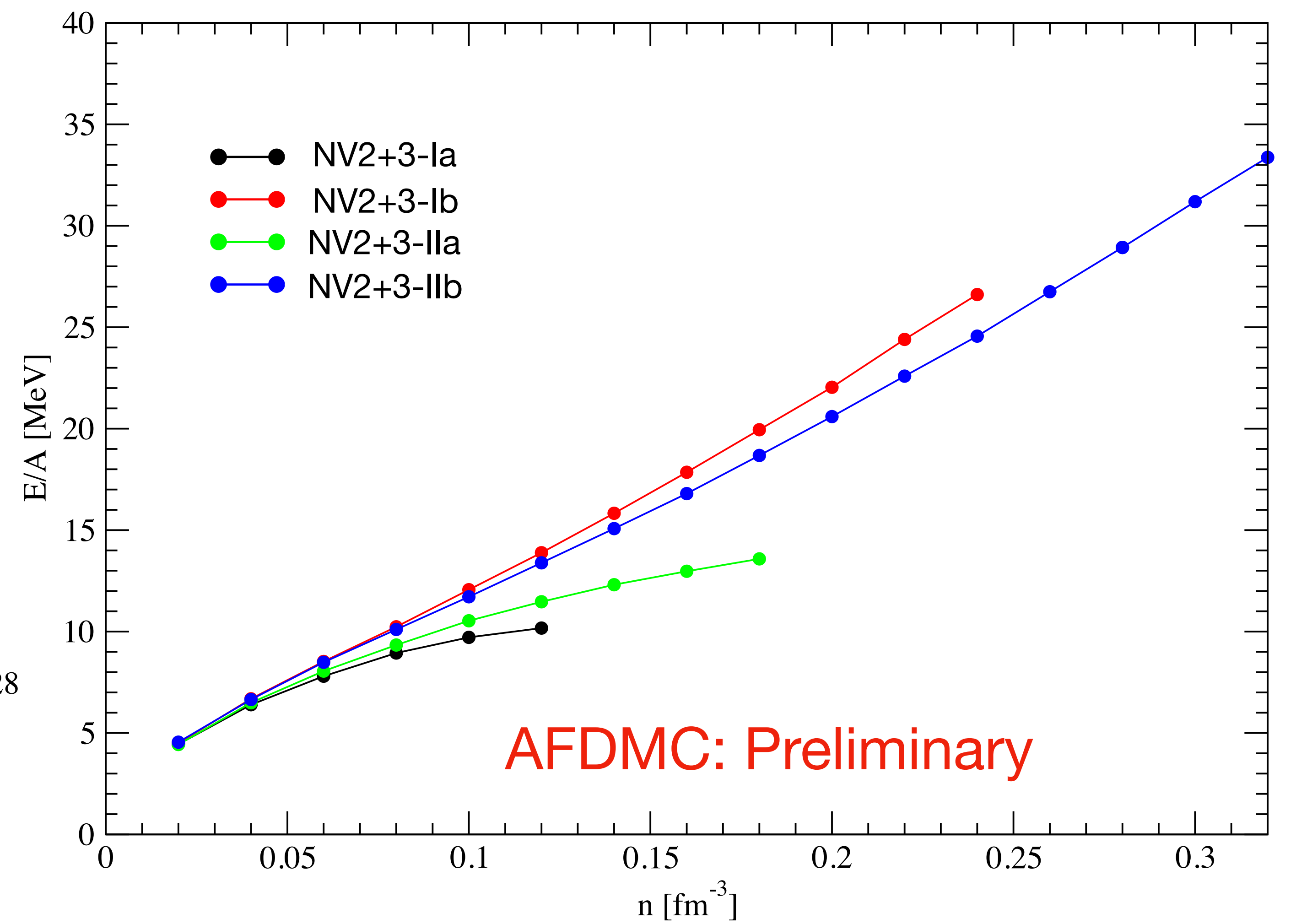
$c_E < (>) 0$: repulsion (attraction) in light-nuclei (the opposite effect in PNM)

$c_D < (>) 0$: repulsion (attraction) in light-nuclei (same effect in PNM but very small)

Equation of State of Pure Neutron Matter in χEFT

EoS of PNM is very sensitive to the choice of the 3N force; particularly the short-range part of the 3N which is the less understood





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Ia	3.666	−1.638
Ib	−2.061	−0.982
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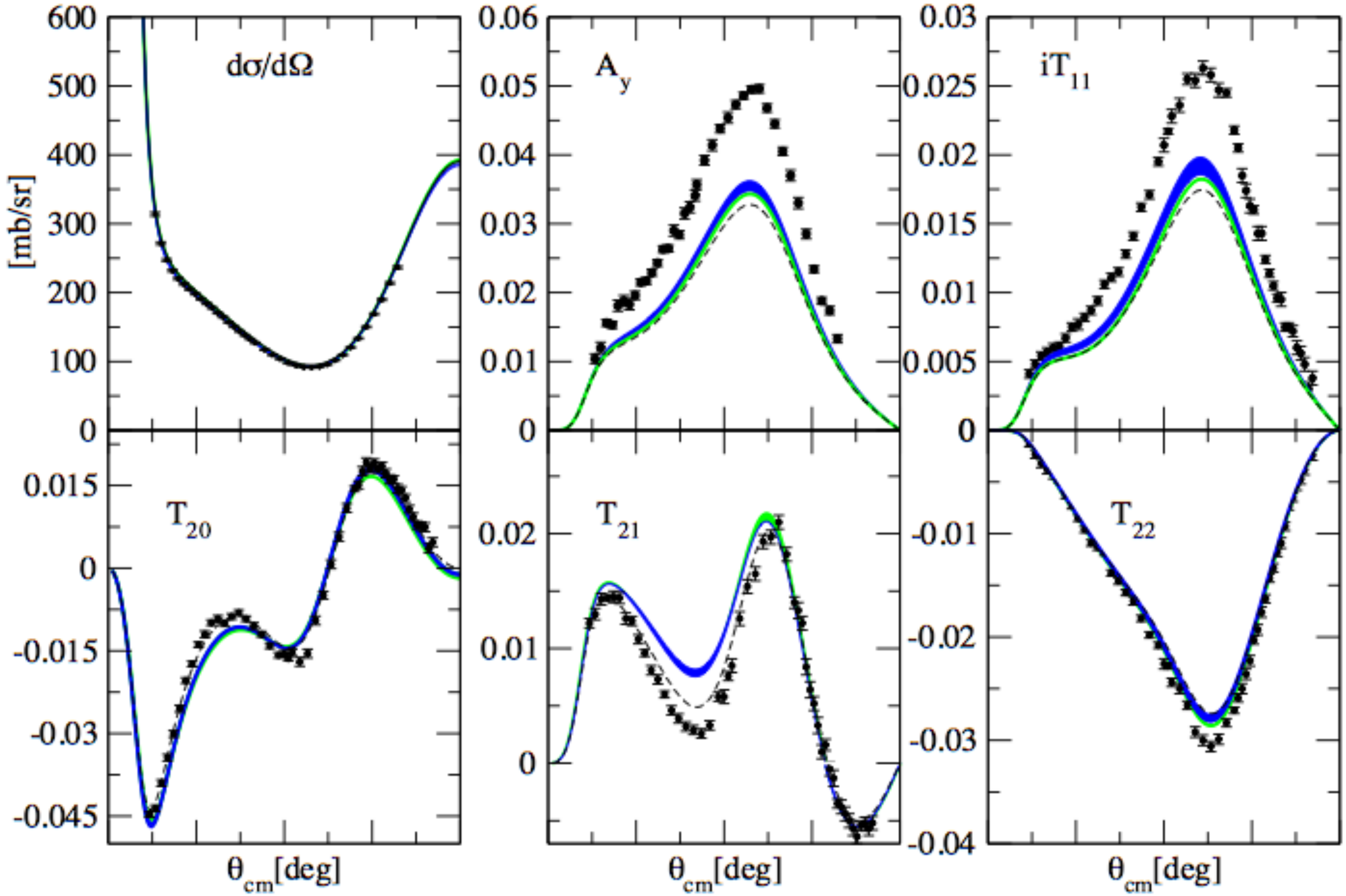
IIb	−4.480	−0.412

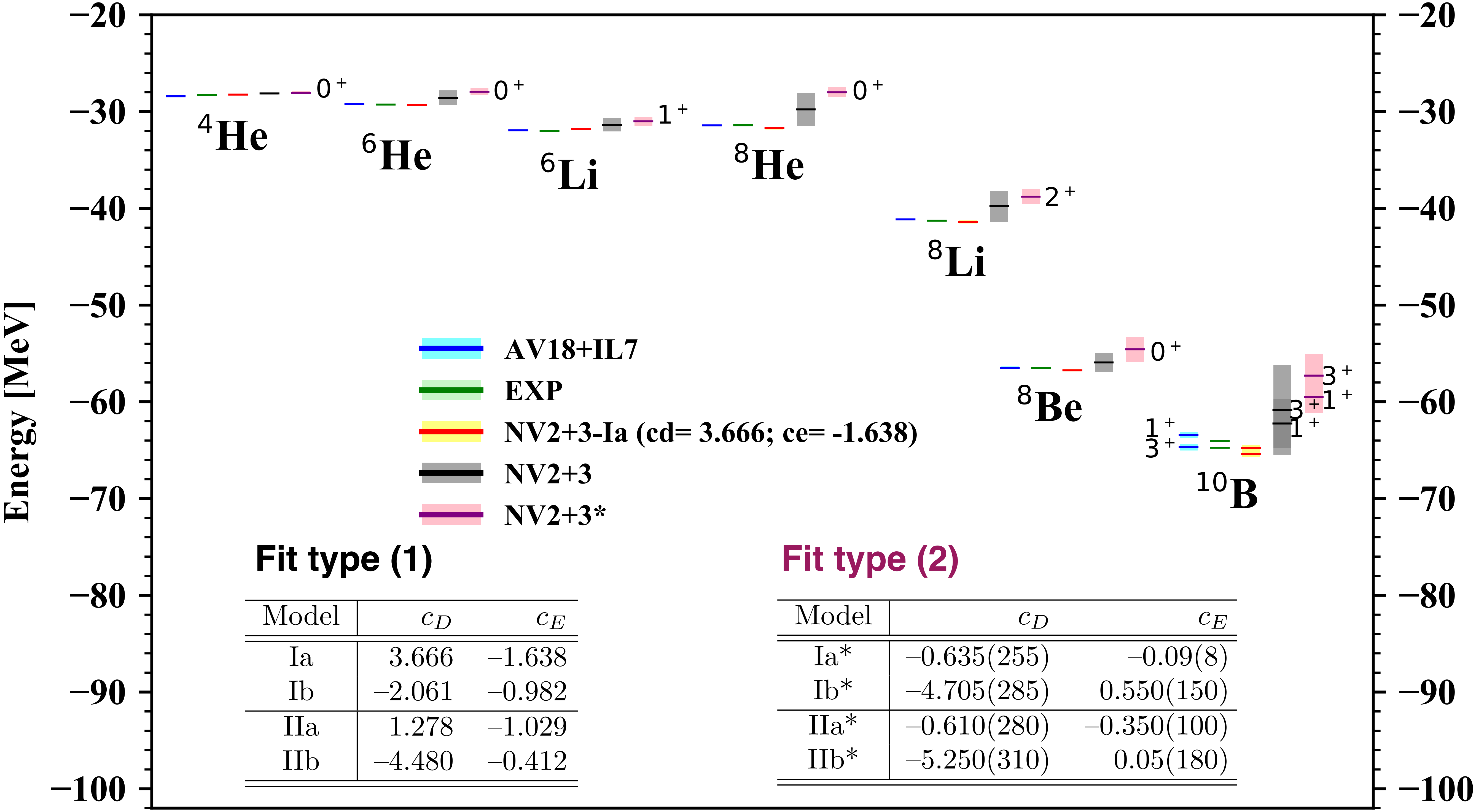
Cutoff sensitivity: modest in NV2 models; large in NV2+3 models

Polarization observables in pd elastic scattering at 3 MeV: HH calculations with the NV2+3 models Ia-Ib (IIa-IIb), are shown by the green (blue) band. The black dashed line are results obtained with only the two-body interaction NV2-Ia



subleading contact terms in 3N interaction???

Energies of Light Nuclei: Model-dependence



Fit type (1)

Model	c_D	c_E
Ia	3.666	–1.638
Ib	–2.061	–0.982
IIa	1.278	–1.029
IIb	–4.480	–0.412

Fit type (2)

Model	c_D	c_E
Ia*	–0.635(255)	–0.09(8)
Ib*	–4.705(285)	0.550(150)
IIa*	–0.610(280)	–0.350(100)
IIb*	–5.250(310)	0.05(180)

Model-dependence for NV2+3 up to 7-8% of the total binding energy

Model-dependence for NV2+3* up to 2-3% of the total binding energy

3N subleading contact terms

There are 146 3N contact operators with two derivatives; but Fierz identities lead to 10 independent operator structure; a possible choice

$$\sum_{n=1}^{4} V_{ijk}^{(n)} = \left(E_1 + E_2\, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + E_3\, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + E_4\, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\right)$$

$$\times \left[C''_{R_S}(r_{ij}) + 2\, \frac{C'_{R_S}(r_{ij})}{r_{ij}}\right] C_{R_S}(r_{jk}) + (j \rightleftharpoons k)$$

$$\sum_{n=5}^{6} V_{ijk}^{(n)} = \left(E_5 + E_6\, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\right) S_{ij} \left[C''_{R_S}(r_{ij}) - \frac{C'_{R_S}(r_{ij})}{r_{ij}}\right] C_{R_S}(r_{jk}) + (j \rightleftharpoons k)$$

$$\sum_{n=7}^{8} V_{ijk}^{(n)} = -2\left(E_7 + E_8\, \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k\right) \frac{C'_{R_S}(r_{ij})}{r_{ij}} \left\{(\mathbf{L}\cdot\mathbf{S})_{ij}\,,\, C_{R_S}(r_{jk})\right\} + (j \rightleftharpoons k)$$

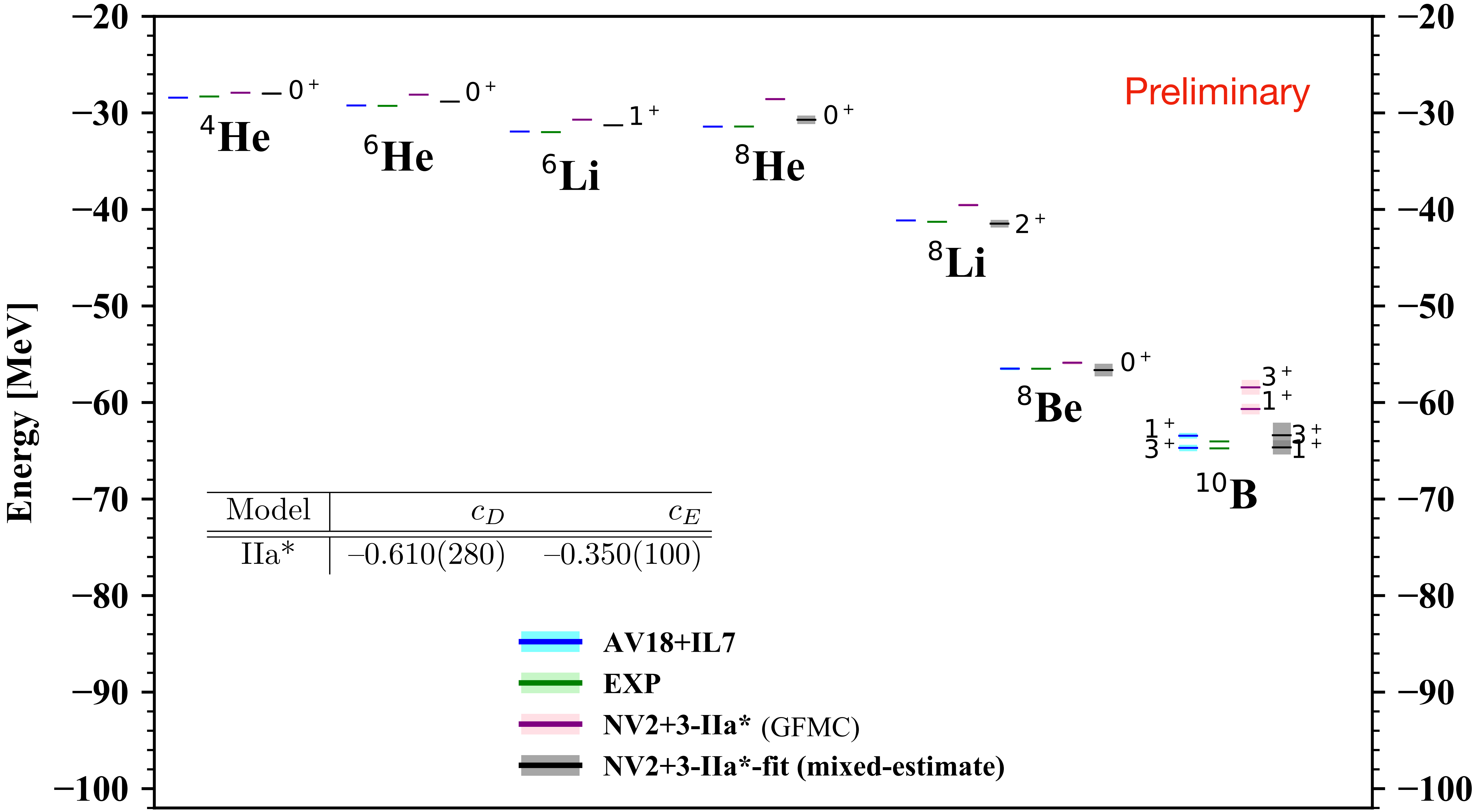
$$\sum_{n=9}^{10} V_{ijk}^{(n)} = \left(E_9 + E_{10}\, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\right) \boldsymbol{\sigma}_i \cdot \hat{\mathbf{r}}_{ik}\, \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{jk}\, C'_{R_S}(r_{ik})\, C'_{R_S}(r_{jk}) + (j \rightleftharpoons k)$$

For consistency these operators should go along with NN[1] and (multi-pion exchange) 3N potentials at N4LO[2]

However it is worth the effort to test them in calculations of few-body reactions (p-d and p-^{3}He A_y) and spectra of light-nuclei

[1]Entem et al. (2015) and Epelbaum et al. (2015); [2]Bernard et al. (2008) and (2011)

Energies of Light Nuclei: inclusion-subleadings



- E5 and E6 are helping mostly with 10B even if the splitting is not quite solved
- E7 and E8 are helping mostly to get A=8 nuclei

Conclusions

We are testing our models of NN+3N interactions with Δ-isobar based on chiral EFT framework in both light-nuclei and infinite nuclear matter

We mainly focused our attention on studying the spectra of nuclei up to A=12 and EoS of infinite neutron matter

For the time being, we are interested in studying the model-dependence of the nuclear observables by exploring different cutoffs and range of energies used to fit the NN interactions as well as analyzing different strategies fo fit the TNI

It looks like that the formulation of the TNI with only c_D and c_E terms is too simplistic if we want to have a good descriptions of spectra, properties of light-nuclei, infinite nuclear matter, three-body observables with a certain degree of accuracy

We are investigating the effect of subleading 3N contact interactions in light-nuclei (we will do so also for infinite nuclear matter)

Outlook

QMC calculations of beta-decays using local interactions with Δ-isobar and corresponding axial currents (on going project in collaboration with S. Pastore et al.)

- Axial currents without Δ-isobar up to one loop have been fully derived in r-space (A. Baroni et al. PRC93(2016)015501)
- tree-level contributions with Δ-isobar are fully developed in r-space
- one loop contributions with Δ-isobar need to be workout in r-space & in the codes (on going project)

QMC calculations of the electromagnetic structure (and reactions) using local interactions with Δ-isobar and corresponding currents (in collaboration with S. Pastore et al.)

- EM currents with Δ-isobar up to one loop have been derived in momentum space (Pastore et al. PRC78(2008)064002)
- tree-level contributions with Δ-isobar are fully developed in r-space
- one loop contributions with Δ-isobar need to be workout in r-space & in the codes (on going project)