From microscopic to effective interactions using Quantum Monte Carlo

Alex Gezerlis



"New Ideas in Constraining Nuclear Forces" ECT*, Trento, Italy June 5, 2018

Outline



Credit: Dany Page



Motivation

Nuclear methods



Recent results

Outline



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Motivation

Nuclear methods



Recent results

Physical systems studied

Nuclear forces



Nuclear structure



Nuclear astrophysics



Physical systems studied

Nuclear forces



Nuclear structure



Nuclear astrophysics







Physical systems studied

Few nucleons



Many nucleons





Outline



Credit: Dany Page

Motivation





Recent results

Historically

"Effective Interactions" were employed in the context of mean-field theory.

Phenomenological

NN interaction fit to N-body experiment

Non-microscopic

NN interaction does not claim to (and will not) describe np scattering

Nuclear physics is difficult

Scattering phase shifts: different "channels" have different behavior.

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Scattering phase shifts: different "channels" have different behavior.



Any potential that reproduces them must be spin (and isospin) dependent

Different approach: phenomenology treats NN scattering without connecting with the underlying level



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$$V_2 = \sum_{j < k} v_{jk} = \sum_{j < k} \sum_{p=1}^8 v_p(r_{jk}) O^{(p)}(j,k)$$
$$O^{p=1,8}(j,k) = (1, \sigma_j \cdot \sigma_k, S_{jk}, \mathbf{L}_{jk} \cdot \mathbf{S}_{jk}) \otimes (1, \tau_j \cdot \mathbf{S}_{jk})$$



Different approach: phenomenology treats NN scattering without connecting with the underlying level

60

30

0

-30

-60

-90

-120^L

0.5

V(r) [MeV]

$$V_2 = \sum_{j < k} v_{jk} = \sum_{j < k} \sum_{p=1}^8 v_p(r_{jk}) O^{(p)}(j,k)$$

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Such potentials are hard, making them non-perturbative at the many-body level (which is a problem for most methods on the market).

Softer, momentum-space formulations like CD-Bonn very popular

1.5

r [fm]

1

2

2.5

How to go beyond?

- Historically, fit NN interaction to N-body experiment
- Parallel approach, fit NN interaction to 2-body experiment, ignoring underlying level of quarks and gluons

How to go beyond?

Historically, fit NN interaction to N-body experiment

Parallel approach, fit NN interaction to 2-body experiment, ignoring underlying level of quarks and gluons

Natural goal: fit NN interaction to 2-body experiment, without ignoring underlying level

Effective field theory



- Attempts to connect with underlying theory (QCD)
- Lowmomentum expansion
- Naturally emerging many-body forces
- Low-energy constants from experiment or lattice QCD
- Now available in non-local, local, or semi-local varieties
- Power counting's relation to renormalization still an open question

Local chiral EFT



A. Gezerlis, I. Tews, E. Epelbaum, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, Phys. Rev. Lett. 111, 032501 (2013).

A. Gezerlis, I. Tews, E. Epelbaum, M. Freunek, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, Phys. Rev. C 90, 054323 (2014).

J. E. Lynn, J. Carlson, E. Epelbaum, S. Gandolfi, A. Gezerlis, K. E. Schmidt, A. Schwenk, I. Tews, Phys. Rev. Lett. 113, 192501 (2014)

I. Tews, S. Gandolfi, A. Gezerlis, A. Schwenk, Phys. Rev. C 93, 024305 (2016)

J. E. Lynn, I. Tews, J. Carlson, S. Gandolfi, A. Gezerlis, K. E. Schmidt, A. Schwenk, I. Tews, Phys. Rev. Lett. 116, 062501 (2016)

P. Klos, J. E. Lynn, I. Tews, S. Gandolfi, A. Gezerlis, H.-W. Hammer, and A. Schwenk, Phys. Rev. C, 94, 054005 (2017)

But even with the interaction in place, how do you solve the many-body problem?

Nuclear many-body problem

$H\Psi = E\Psi$

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where $H = \sum_i K_i + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \cdots$

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where
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SO

$$H\Psi(\mathbf{r}_1,\cdots,\mathbf{r}_A;s_1,\cdots,s_A;t_1,\cdots,t_A)=E\Psi(\mathbf{r}_1,\cdots,\mathbf{r}_A;s_1,\cdots,s_A;t_1,\cdots,t_A)$$

i.e. $2^A \begin{pmatrix} A \\ Z \end{pmatrix}$ complex coupled second-order differential equations

Main many-body methods employed (by me)

Quantum Monte Carlo

- Microscopic
- Computationally demanding (3N particle coordinates + spins)
- Limited to smallish N

$$\Psi(\tau \to \infty) = \lim_{\tau \to \infty} e^{-(\mathcal{H} - E_T)\tau} \Psi_V$$
$$\to \alpha_0 e^{-(E_0 - E_T)\tau} \Psi_0$$



Credit: Steve Pieper



Credit: W. Nazarewicz

Density Functional Theory

- More phenomenological (to date, but see major developments)
- Easier in crude form (orbitals → density → energy density)

• Can do any large N

$$E = \int d^3r \left\{ \mathcal{E}[\rho(\mathbf{r})] + \rho(\mathbf{r}) V_{\text{ext}}(\mathbf{r}) \right\}$$

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Research Strategies

i) Use QMC as a benchmark with which to compare DFT results ii) Constrain DFT with QMC, then use DFT to make predictions

Outline



Credit: Dany Page



Motivation

Nuclear background





QMC with chiral EFT

From low to high density



- Ab initio results for low-density matter under control
- Doubly-magic input better constrained at higher density



B. A. Brown and A. Schwenk, Phys. Rev. C 89, 011307 (2014)

Chiral EFT in QMC



- Use Auxiliary-Field Diffusion Monte Carlo to handle the full interaction
- First ever non-perturbative systematic error bands
- Band sizes to be expected
- Many-body forces will emerge systematically



A. Gezerlis, I. Tews, E. Epelbaum, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, Phys. Rev. Lett. 111, 032501 (2013).
A. Gezerlis, I. Tews, E. Epelbaum, M. Freunek, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, Phys. Rev. C 90, 054323 (2014).

Nuclear Hamiltonian: chiral EFT



Leading three-nucleon force

- Two-pion exchange (parameter-free)
- One-pion exchange-contact (c_D)
- Three-nucleon contact (c_E)

3NF TPE in PNM

Overall error bands



• NN error band already published

NEUTRONS

I. Tews, S. Gandolfi, A. Gezerlis, A. Schwenk, Phys. Rev. C 93, 024305 (2016)

Overall error bands



- NN error band already published
- Now vary 3NF cutoff within plateau



I. Tews, S. Gandolfi, A. Gezerlis, A. Schwenk, Phys. Rev. C 93, 024305 (2016)

Overall error bands



- NN error band already published
- Now vary 3NF cutoff within plateau
- 3NF cutoff dependence tiny in comparison with NN cutoff one
- 3NF contribution 1-1.5 MeV, cf. with MBPT 4 MeV with EGM



I. Tews, S. Gandolfi, A. Gezerlis, A. Schwenk, Phys. Rev. C 93, 024305 (2016)
Compare with other calculations at N2LO



- Overall agreement across methods
- QMC band result of using more than one cutoff
- Band width essentially understood



I. Tews, S. Gandolfi, A. Gezerlis, A. Schwenk, Phys. Rev. C 93, 024305 (2016)

Overall error bands



- Updated way of producing error estimate (using LO, NLO, NNLO)
- Fierz ambiguity also in 3NF



J. E. Lynn, I. Tews, J. Carlson, S. Gandolfi, A. Gezerlis, K. E. Schmidt, A. Schwenk, I. Tews, Phys. Rev. Lett. 116, 062501 (2016)

Now turn to light nuclei

Nuclear GFMC with chEFT: 4He



- Binding energy of ⁴He
- Non-perturbative systematic error bands
- All results are strong force + Coulomb, no NNN



J. E. Lynn, J. Carlson, E. Epelbaum, S. Gandolfi, A. Gezerlis, A. Schwenk, Phys. Rev. Lett. 113, 192501 (2014)

Nuclear GFMC with chEFT: 4He



- Binding energy of ⁴He
- All results are strong force + Coulomb, no NNN
- Example of doing one interaction perturbatively and another one nonperturbatively



J. E. Lynn, J. Carlson, E. Epelbaum, S. Gandolfi, A. Gezerlis, A. Schwenk, Phys. Rev. Lett. 113, 192501 (2014)

Nuclear GFMC with chEFT: NN+3NF



- Use c_D and c_E we fit
- Shown are both binding energies and point proton radii
- Things look reasonably good



J. E. Lynn, I. Tews, J. Carlson, S. Gandolfi, A. Gezerlis, K. E. Schmidt, A. Schwenk, I. Tews, Phys. Rev. Lett. 116, 062501 (2016)
J. E. Lynn, I. Tews, J. Carlson, S. Gandolfi, A. Gezerlis, K. E. Schmidt, A. Schwenk, I. Tews, Phys. Rev. C 96, 054007 (2017)

AFDMC with chEFT: NN+3NF



- AFDMC with better wave function
- Same local chiral EFT interactions as above
- Can be pushed to heavier masses



D. Lonardoni et al, Phys. Rev. C 97, 044318 (2018)

Nuclear GFMC with chEFT



- GFMC with very good wave functions
- Different local chiral EFT interactions than above
- 37 states shown, with 60 more probed



M. Piarulli et al, Phys. Rev. C 120, 052503 (2018)

AFDMC with pionless **EFT**

4He

Λ	$m_{\pi} = 140 \text{ MeV}$
2 fm^{-1}	-23.17 ± 0.02
4 fm^{-1}	-23.63 ± 0.03
6 fm^{-1}	-25.06 ± 0.02
8 fm^{-1}	-26.04 ± 0.05
$\rightarrow \infty$	$-30^{\pm 0.3 (sys)}_{\pm 2 (stat)}$
Exp.	-28.30

Λ	$m_{\pi} = 140 \text{ MeV}$
2 fm ⁻¹	-97.19 ± 0.06
4 fm^{-1}	-92.23 ± 0.14
6 fm^{-1}	-97.51 ± 0.14
8 fm^{-1}	-100.97 ± 0.20
$\rightarrow \infty$	$-115^{\pm 1}_{\pm 8}(\text{sys})_{\pm 8}$
Exp.	-127.62

160

- AFDMC with simple wave function
- LO pionless EFT interaction (with 3NF
- 160 tends to break up into 4He clusters



L. Contessi, A. Lovato, F. Pederiva, A. Roggero, J. Kirscher, U. van Kolck, Phys. Lett. B 07, 048 (2017)

Inhomogeneous matter









M. Buraczynski and A. Gezerlis, Phys. Rev. Lett. **116**, 152501 (2016)







M. Buraczynski and A. Gezerlis, Phys. Rev. Lett. **116**, 152501 (2016)



Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2m} \sum_{i} \nabla_i^2 + \sum_{i < j} V_{ij} + \sum_{i < j < k} V_{ijk} + \sum_{i} 2v_q \cos(\mathbf{q} \cdot \mathbf{r}_i)$$

Hamiltonian



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Trial wave function

$$|\Psi_T\rangle = \prod_{i < j} f(r_{ij}) \mathcal{A}\left[\prod_i |\phi_i, s_i\rangle\right]$$

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single-particle orbitals:

- plane waves
- Mathieu functions

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 sing

single-particle orbitals:

- plane waves
- Mathieu functions

Approach: Carry out microscopic QMC calculations for ~100 particles

One periodicity, one strength



M. Buraczynski and A. Gezerlis, Phys. Rev. Lett. **116**, 152501 (2016)

- Periodic potential in addition to nuclear forces
- Energy trivially decreased



One periodicity, one strength



M. Buraczynski and A. Gezerlis, Phys. Rev. Lett. **116**, 152501 (2016)

- Periodic potential in addition to nuclear forces
- Energy trivially decreased
- Considerable dependence on wave function (physics input)
- Microscopic input for energy-density functionals



Background on DFT

Standard functional in PNM

$$\mathcal{E} = \frac{\hbar^2}{2m}\tau + s_1 n^2 + s_2 n^{\sigma+2} + s_3 n\tau + s_4 (\nabla n)^2$$

Background on DFT

Standard functional in PNM

$$\mathcal{E} = \frac{\hbar^2}{2m}\tau + s_1 n^2 + s_2 n^{\sigma+2} + s_3 n\tau + s_4 (\nabla n)^2$$

Skyrme functional in isospin representation

$$\mathcal{E}_{\text{Skyrme}} = \sum_{T=0,1} \left[(C_T^{n,a} + C_T^{n,b} n_0^{\sigma}) n_T^2 + C_T^{\Delta n} (\nabla n_T)^2 + C_T^{\tau} n_T \tau_T \right]$$

Background on DFT

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Approach: Use QMC results to constrain DFT gradient term(s) (which then apply to terrestrial nuclei and neutron-stars more broadly)



M. Buraczynski and A. Gezerlis, Phys. Rev. Lett. **116**, 152501 (2016) $n = 0.10 \text{ fm}^{-3}$

• Try to disentangle bulk from isovector gradient contribution





M. Buraczynski and A. Gezerlis, Phys. Rev. Lett. **116**, 152501 (2016) $n = 0.10 \text{ fm}^{-3}$

• Try to disentangle bulk from isovector gradient contribution (homogeneous EOSs also differ)



Relationship between homogeneous EOSs depends on the density





M. Buraczynski and A. Gezerlis, Phys. Rev. C 95, 034012 (2017)



Many densities

- Repeat exercise at lower density
- Homogeneous relation is reversed
- Find density-dependent isovector coefficient, analogously to what is seen with DME (Holt, Kaiser)



M. Buraczynski and A. Gezerlis, Phys. Rev. C 95, 034012 (2017)



$n = 0.10 \text{ fm}^{-3}$

• Using chiral EFT interactions as input to AFDMC (and from there to the Skyrme fitting)



preliminary

Finite-size effects

Free non-interacting gas



M. Buraczynski and A. Gezerlis, Phys. Rev. C 95, 034012 (2017)

Finite-size effects

Free non-interacting gas

Modulated non-interacting gas





M. Buraczynski and A. Gezerlis, Phys. Rev. C 95, 034012 (2017)

Neutron matter density response

Non-interacting gas: Lindhard function

$$\chi_L = -\frac{mq_F}{2\pi^2\hbar^2} \left[1 + \frac{q_F}{q} \left(1 - \left(\frac{q}{2q_F}\right)^2 \right) \ln \left| \frac{q + 2q_F}{q - 2q_F} \right| \right]$$

Neutron matter density response

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$$\frac{E_{\text{tot}}}{N} = \frac{E_0}{N} + \frac{\chi(q)}{n_0}v_q^2 + C_4v_q^4 + \cdots$$

Neutron matter density response

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Three-dimensional electron gas





S. Moroni, D. M. Ceperley, G. Senatore, Phys. Rev. Lett. 75, 689 (1995)

Many periodicities, many strengths



$n = 0.10 \text{ fm}^{-3}$

- First ever ab initio density-density response for neutron matter
- Neither Lindhard nor Coulomb
- Results on this plot derived from several strengths and periodicities



M. Buraczynski and A. Gezerlis, Phys. Rev. Lett. 116, 152501 (2016)

M. Buraczynski and A. Gezerlis, Phys. Rev. C 95, 034012 (2017)

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M. Buraczynski and A. Gezerlis, Phys. Rev. Lett. 116, 152501 (2016)M. Buraczynski and A. Gezerlis, Phys. Rev. C 95, 034012 (2017)


Impact on neutron stars



Core-crust boundary

- Thermodynamic instability determines transition from inhomogeneous to homogeneous matter
- Modified isovector coefficients compared with large class of other results

Y. Lim and J. W. Holt, Phys. Rev. C 95, 065805 (2017)

Quasiparticle energy dispersion

Finite-size effects

Effective mass for non-interacting gas for finite N



preliminary

Interacting gas results

Novel prescription to reach thermodynamic limit when subtracting



preliminary

Interacting gas results

Study imbalanced gas with extra particle placed at different excitation energies



preliminary

Conclusions

- Rich connections between physics of nuclei and that of compact stars
- Exciting time in terms of interplay between nuclear interactions, QCD, and many-body approaches
- Ab initio and phenomenology are mutually beneficial

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Collaborators

Guelph

- Mateusz Buraczynski
- Will Dawkins
- Nawar Ismail
- Nicholas van Heijst

Darmstadt

- Joel Lynn
- Achim Schwenk

INT

• Ingo Tews

LANL

- Joe Carlson
- Stefano Gandolfi

IPN Orsay

• Denis Lacroix

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Extra slide 1

Big-picture questions

- Functionals tailored to neutron stars or universal density functional theory?
- Functional fit only to *ab initio* (as per Fayans and Orsay) or fit to any available data point?
- How will LIGO data constrain functionals? How will this propagate to *ab initio* and nuclear forces?



Little-picture questions

Extra slide 2a

Something wrong with Skyrme response?



A. Boulet and D. Lacroix, Phys. Rev. C 97, 014301 (2017)

Extra slide 2b

Isovector coefficient density-dependent or not?



S. Gandolfi, J. Carlson, S. Pieper, Phys. Rev. Lett. 106, 012501 (2011)

M. Buraczynski and A. Gezerlis, Phys. Rev. C **95**, 034012 (2017)

